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ABSTRACT

We study a competitive credit market in which lenders with partial knowledge of loan repayment use one of three decision criteria – maximization of expected utility, maximin, or minimax regret – to make lending decisions. Lenders allocate endowments between loans and a safe asset, while borrowers demand loans to undertake investments. Borrowers may incompletely repay their loans when investment productivity turns out to be low ex post. We characterize market equilibrium, the contracted repayment rate being the price variable that equilibrates loan supply and demand. Supposing that a public Authority wants to maximize the net social return to borrowing, we study two interventions in the credit market to achieve this objective. One intervention manipulates the return on the safe asset and the other guarantees a minimum loan return to lenders. In a simple scenario, we find that manipulation of the return on the safe asset can be an effective way to achieve the socially desired outcome if lender beliefs about the return to lending are not too pessimistic relative to the beliefs of the Authority. Contrariwise, guaranteeing a minimum loan return can be effective if lender beliefs are not too optimistic relative to the beliefs of the Authority.

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1. Introduction

A common informational problem in credit markets is partial knowledge of loan repayment. A borrower may repay in full, in part, or not at all. At the time that a loan is negotiated, a lender may not know the amount that will be repaid.

It has been standard to assume that lenders place subjective probability distributions on loan repayment and maximize subjective expected utility (SEU). Indeed, the norm has been to go further and assume that expectations are rational, in the sense that subjective distributions are objectively correct conditional on available information. However, a subjective probability distribution is a form of knowledge. There are realistic circumstances in which a lender may have no credible basis for asserting one at all, never mind one that is objectively correct. These circumstances pose problems of lending under ambiguity (aka Knightian uncertainty).

Ambiguity may be particularly prevalent when a previously stable credit market experiences a significant unanticipated shock. Lenders may be unsure how to interpret the shock—it may have been temporary or it may signal a regime change. Lenders in a stable credit market may have rational expectations for loan repayment, but they may feel unable to update them afterwards.

This paper studies credit markets in which lenders lack rational expectations about loan repayment, or may not have probabilistic expectations at all. Our modeling of behavior under ambiguity builds in part on our earlier work. Brock (2006) considered the behavior of an isolated lender facing repayment ambiguity. Manski (2005, 2006, 2007, 2009) analyzed various problems of social planning under ambiguity. Whereas our earlier work studied the behavior of a single agent, be it a lender or planner, here we analyze a market in which agents may have to cope with ambiguity.

We consider a setting that is highly simplified in many respects, to enable a relatively straightforward analysis. Our credit market is small relative to the economy as a whole and is competitive. Lenders interact only through this market; thus, there are no derivatives markets. Borrowers are observational identical to lenders and, hence, face the same price of loans. Inflation is anticipated, so we denote all monetary quantities in real terms.

Loans are one-period contracts specifying that a borrower will receive one dollar today and repay r dollars tomorrow. Loans are securitized rather than held by individual lenders, repayment to each lender being proportional to his share of the aggregate supply of loans. Whereas r denotes the contracted loan repayment rate, let $\lambda(r)$ denote the loan return per dollar that would actually occur under contracted rate r. Lenders know r when they provide loans and they know that $0 \leq \lambda(r) \leq r$, but they may not know the magnitude of $\lambda(r)$.

Loan supply is determined by lenders who allocate monetary endowments between loans and a safe asset. Borrowers demand loans to enable them to undertake potentially productive investments. Incomplete repayment occurs when persons borrow with partial knowledge of the productivity of their investments. When productivity turns out to be low ex post, they sometimes lack the resources to fully repay their loans. Bankruptcy law limits their liability for repayment. The market is in equilibrium when the contracted repayment rate r equates loan supply and demand.

We study market equilibrium under several alternative assumptions about loan supply. We suppose that lenders use one of a trinity of decision criteria to allocate their endowments: maximization of SEU, the maximin (MM) criterion, or the minimax-regret (MMR) criterion. The SEU criterion chooses an allocation that performs well on average across the loan returns λ that the lender thinks feasible. The MM and MMR criteria make allocations that, in different senses, perform uniformly well across the feasible loan returns. Here, as in our earlier work, we find it illuminating to compare the behavior of agents who use the SEU, MM, and MMR criteria to make decisions with partial knowledge.

Our general analysis of market equilibrium requires only weak qualitative assumptions about loan demand and repayment. However, the general results are too abstract to yield much intuition. We obtain explicit, easily interpretable findings when we assume that lender beliefs about the loan return $\lambda(\cdot)$ vary directly with r. We use a simple borrowing scenario to motivate this assumption. In this scenario each borrower faces only two states of nature, a good state with positive investment productivity and a bad state in which the investment does not payoff.

Section 2 studies the operation of a credit market without government intervention except for the limited liability provided borrowers by bankruptcy law. In Section 3, we consider a more activist government that aims to maximize the net social returns to borrowing. Acting through a public Authority, the government may intervene to affect the competitive equilibrium. The Authority, like lenders and borrowers, may have incomplete information about the productivity of loan-financed investments.

We formalize the Authority's decision problem and study the use of two policy instruments. One intervention manipulates the return on the safe asset and the other guarantees a minimum loan return to lenders. Focusing on the simple borrowing scenario, we find that manipulation of the return on the safe asset can be an effective way to achieve the socially desired outcome if lender beliefs about the return to lending are not too pessimistic relative to the beliefs of the Authority. Contrariwise, guaranteeing a minimum loan return can be effective if lender beliefs are not too optimistic relative to the beliefs of the Authority. Thus, the two interventions have somewhat distinct domains of application. Successful implementation of either policy requires that the Authority know lender beliefs and decision criteria.

The concluding Section 4 comments on recent suggestions that ambiguity about investment returns is a negative influence on the operation of financial markets, inducing investors to allocate more of their portfolios to safe assets than is socially optimal. Our analysis suggests that investor ambiguity need not be the driving force behind a "flight to safety" in credit markets. In the market we study, the force driving a flight to safety is increased lender pessimism about the return to lending. Pessimism is distinct from ambiguity and is easily formalized within the SEU framework.

While the credit market we study is highly idealized, we nevertheless think that this paper makes contributions that advance the understanding of credit markets and that warrant the attention of policy makers. We advance understanding of credit markets by studying competitive equilibrium when lenders use various decision criteria to make lending decisions with partial knowledge of loan repayment. We characterize equilibrium in abstraction and we report simple analytical findings that hold in illustrative settings. Considering policy, we study interventions that the government may use to achieve a normatively satisfactory market equilibrium.

Credit markets are complex, and the theoretical literature studying them is diverse and vast, with different authors emphasizing different aspects of market operation. We are aware of two other recent studies of financial markets that assume agents face some sort of ambiguity and then ask how government intervention might mitigate unpalatable market outcomes. Easley and O'Hara (2009) study the sub-optimal asset pricing that may occur when a subset of "ambiguity averse" investors choose not to participate in the market. They suggest a possible corrective role for regulation that limits the occurrence of extreme events. Caballero and Krishnamurthy (2008a) consider an environment in which agents face ambiguity about the timing of liquidity shocks. They find a salutary role for a Central Bank as a lender of last resort.

These precedent studies share our broad concern with the positive and normative analysis of competitive financial markets under ambiguity, but they differ greatly from our work in their specifics. Whereas they use the maximin expected utility model to express agent behavior under ambiguity, we study maximin and minimax-regret behavior. Whereas they pose relatively abstract general equilibrium models of financial markets, we develop a partial-equilibrium model of a credit market with relatively explicit institutional features. In particular, we differentiate lenders who choose how to allocate asset endowments from borrowers who demand loans to make productive investments and who have limited liability for repayment. We locate the source of ambiguity as lender inability to interpret a productivity or other shock that reduces loan returns relative to an initial steady state. There also are differences across studies in the type of normative analysis performed. We pose an explicit social welfare function in which the objective is to maximize the net social return to borrowing.

2. Credit Market Equilibrium

To begin, Section 2.1 formalizes the asset allocation problem that yields the supply of loans. Maintaining minimal assumptions on the demand for loans, Section 2.2 characterizes the market equilibrium in generality and when lenders have specific beliefs about the loan return. Section 2.3 presents a simple borrowing scenario that motivates the specific beliefs studied in Section 2.2.

2.1. The Supply of Loans

The supply of loans is determined by the portfolio choices of lenders. A dollar invested in the safe asset yields a known return $\rho \geq 0$. A dollar invested in loans yields a possibly unknown return $\lambda(r)$. A lender must choose a fraction $\delta \in [0, 1]$, implying that he allocates the fraction δ of his endowment to loans and $1 - \delta$ to the safe asset. An allocation is said to be *singleton* if $\delta = 0$ or 1 and is *fractional* if $0 < \delta < 1$. [1]

If a lender chooses allocation δ, his portfolio return is $m[\delta\lambda(r) + (1 - \delta)\rho]$, where m is the size of his endowment. We suppose that the lender wants to maximize this quantity. Thus, he wants to solve the problem

$$(1) \qquad \max_{\delta \in [0, 1]} \; m[\delta\lambda(r) + (1 - \delta)\rho],$$

The unique optimal allocation is $\delta = 1$ if $\lambda(r) > \rho$ and $\delta = 0$ if $\lambda(r) < \rho$. All allocations are optimal if $\lambda(r) = \rho$.

Our concern is asset allocation when the lender knows ρ and r but not $\lambda(r)$. Let $\Gamma(r)$ index the feasible states of nature; that is, the loan returns the lender thinks feasible. Let $\lambda_0(r) \equiv \min_{\gamma \in \Gamma(r)} \lambda_\gamma(r)$ and $\lambda_1 \equiv \max_{\gamma \in \Gamma(r)} \lambda_\gamma(r)$. The lender cannot solve problem (1) if $\lambda_0(r) < \rho < \lambda_1(r)$.

Although a lender may not know the optimal allocation, he must somehow choose one. We

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[1] The requirement that $\delta \in [0, 1]$ prohibits the lender from exercising leverage. If we were to permit $\delta > 1$, the lender would be able to borrow the safe asset in order to lend an amount larger than his endowment. If we were to permit $\delta < 0$, the lender would be able to borrow from the public in order to invest in the safe asset.

consider loan supply when the lender maximizes SEU or uses the maximin or minimax-regret criterion. We do not argue that lenders "should" use a particular decision criterion. If the optimal allocation is determinate, all of the criteria considered here yield it. If it is indeterminate, there is no unique "right" way to choose an allocation.

Maximization of SEU: A lender may place a subjective probability distribution on the states of nature, compute the subjective expected return under each allocation, and choose an allocation that maximizes this quantity. Let $\pi(r)$ denote the subjective distribution on $\Gamma(r)$. Then the lender solves the optimization problem

$$(2) \qquad \max_{\delta \in [0, 1]} \; m[\delta\lambda_{\pi}(r) + (1 - \delta)\rho],$$

where $\lambda_{\pi}(r) \equiv \int \lambda_{\gamma}(r)d\pi(r)$ denotes the subjective mean of $\lambda(r)$. The solution to (2) is generically singleton, being $\delta = 0$ when $\lambda_{\pi}(r) < \rho$ and $\delta = 1$ when $\lambda_{\pi}(r) > \rho$. All $\delta \in [0, 1]$ are solutions when $\lambda_{\pi}(r) = \rho$. Thus, a lender maximizing subjective expected return chooses the same allocation as does a lender who knows the loan return to be $\lambda_{\pi}(r)$.

Maximin Loan Supply: To determine the maximin allocation, one first computes the minimum return attained by each allocation across all states of nature. One then chooses an allocation that maximizes this minimum return. Thus, the criterion is

$$(3) \qquad \max_{\delta \in [0, 1]} \; \min_{\gamma \in \Gamma(r)} \; m[\delta\lambda_{\gamma}(r) + (1 - \delta)\rho].$$

The solution is $\delta = 0$ if $\lambda_0(r) < \rho$ and $\delta = 1$ if $\lambda_0(r) > \rho$. All $\delta \in [0, 1]$ are solutions if $\lambda_0(r) = \rho$. Thus, a lender using the MM criterion chooses the same allocation as does a lender who knows the loan return to be $\lambda_0(r)$.

Minimax-Regret Loan Supply: By definition, the regret of allocation δ in state of nature γ is the difference between the maximum achievable portfolio return and the return achieved with this allocation. The MMR rule computes the maximum regret of each allocation over all states of nature and chooses an allocation to minimize maximum regret. Thus, the criterion is

$$(4) \quad \min_{\delta \in [0, 1]} \ \max_{\gamma \in \Gamma(r)} \ \max\,[m\lambda_\gamma(r), m\rho] - m[\delta\lambda_\gamma(r) + (1-\delta)\rho].$$

Manski (2007, 2009) showed that the MMR allocation is

$$(5) \quad \delta_{MMR} = \min\left[\max\left(\frac{\lambda_1(r) - \rho}{\lambda_1(r) - \lambda_0(r)}, 0\right), 1\right].$$

Thus, the MMR allocation is always fractional when the lender faces ambiguity; that is, when $\lambda_0(r) < \rho < \lambda_1(r)$. In contrast, we found above that the SEU and MM allocations are singleton except when ρ takes particular values.

Maximization of Monotone Functions of the Portfolio Return

Rather than seek to solve optimization problem (1), a lender may want to solve

$$(1') \qquad \max_{\delta \in [0,1]} \; f\{m[\delta\lambda(r) + (1 - \delta)\rho]\}$$

for some strictly increasing function $f(\cdot)$. He would consequently apply the SEU, MM, or MMR criterion to this transformation of the portfolio return. The shape of $f(\cdot)$ does not affect the MM allocation, but it does affect the SEU and MMR allocations. It is well known that the SEU allocation may be fractional if $f(\cdot)$ is sufficiently concave and the subjective distribution π has sufficient dispersion. The MMR allocation is fractional under ambiguity for all continuous $f(\cdot)$, but the value of δ_{MMR} varies with $f(\cdot)$; see Manski (2009).

Our analysis of market equilibrium takes $f(\cdot)$ to be the identity function. This case is simple in various respects. In particular, the allocation chosen using each of the three decision criteria is invariant with respect to the size m of the lender's endowment.

2.2. Equilibrium Contracted Repayment Rates

The credit market is in equilibrium when the contracted repayment rate r equates the aggregate supply of and demand for loans. The analysis in this section places only weak qualitative assumptions on the demand for loans. We suppose that the demand function $D(\cdot)$ is continuous and strictly decreasing in r, with $\lim_{r \to \infty} D(r) = 0$.

We are much more specific in our characterization of supply. We suppose that lenders have homogeneous beliefs and use the same decision criterion to allocate their endowments. These assumptions, coupled with the fact that the size of the endowment does not affect the chosen allocation, enable us to aggregate lenders into a representative lender who allocates the combined endowment of all lenders.

Let $\delta(r)$ be the asset allocation that the representative lender would choose if the contracted repayment rate were r. Section 2.1 showed that $\delta(r)$ depends on beliefs about $\lambda(r)$ and on the decision criterion used. We found that $\delta(r)$ is always determinate under the MMR criterion and typically so under the SEU and MM criteria, but it is indeterminate for specific values of ρ under the latter criteria. Hence, the aggregate supply of loans at rate r is the possibly set-valued quantity

(6) $S(r) = M\delta(r)$,

where M is the combined endowment of all lenders. The market is in equilibrium if

(7) $D(r) \in M\delta(r)$.

Equilibrium with Known Loan Return

Given specification of the representative lender's beliefs and decision criterion, we can determine $\delta(\cdot)$ and characterize equilibrium condition (7). Suppose first that $\lambda(\cdot)$ is known. We showed in Section 2.1 that the unique optimal allocation is $\delta(r) = 1$ if $\lambda(r) > \rho$, $\delta(r) = 0$ if $\lambda(r) < \rho$, and indeterminate if $\lambda(r) = \rho$. Hence, an equilibrium value of r satisfies the inequality

$$(8) \qquad M \cdot 1[\lambda(r) > \rho] \leq D(r) \leq M \cdot 1[\lambda(r) \geq \rho].$$

Here and elsewhere, $1[\cdot]$ is the indicator function taking the value one if the logical condition in the brackets holds, and zero otherwise.

Two types of equilibria may occur, which we label *full-supply* and *indifferent-supply* equilibria. Consider the set $[1, \infty)$ of all feasible values of r. Lenders supply their full asset endowment M to the credit market when r lies in the set $R_f \equiv [r: \lambda(r) > \rho]$. They are indifferent among all loan supplies $[0, M]$ when r lies in $R_i \equiv [r: \lambda(r) = \rho]$. They supply nothing to the credit market when r is in $R_z \equiv [r: \lambda(r) < \rho]$. Hence, a full-supply equilibrium occurs if $r \in R_f$ and $D(r) = M$. An indifferent-supply equilibrium occurs if $r \in R_i$ and $D(r) \in [0, M]$. Contract rates in R_z cannot be equilibria: they imply that $S(r) = 0$, but we have assumed that $D(r) > 0$ for all r.

The credit market has at most one full-supply equilibrium, the reason being that there exists at most one value of r such that $D(r) = M$. Co-existence of a full-supply equilibrium with one or more indifferent-supply equilibria can occur in principle. However, the credit market has a unique equilibrium given a weak shape restriction on $\lambda(\cdot)$.

Suppose that $\lambda(\cdot)$ satisfies the single-crossing property at ρ. That is, suppose there exists a unique r^* such $\lambda(r^*) = \rho$, with $\lambda(r) < \rho$ for $r < r^*$ and $\lambda(r) > \rho$ for $r > r^*$. If $D(r^*) \leq M$, then r^* is the unique equilibrium. If $D(r^*) > M$, then equilibrium occurs at the unique $r > r^*$ such that $D(r) = M$.

Equilibrium with Partially Known Loan Return

Suppose now that the lender does not know $\lambda(\cdot)$. If he maximizes SEU or uses the MM criterion, the equilibrium condition has the same form as (8) with $\lambda_\pi(\cdot)$ or $\lambda_0(\cdot)$ replacing $\lambda(\cdot)$. Thus, an equilibrium value of r respectively satisfies the inequality

$$\text{(9)} \qquad M\cdot 1[\lambda_\pi(r) > \rho] \;\le\; D(r) \;\le\; M\cdot 1[\lambda_\pi(r) \ge \rho].$$

or

$$\text{(10)} \qquad M\cdot 1[\lambda_0(r) > \rho] \;\le\; D(r) \;\le\; M\cdot 1[\lambda_0(r) \ge \rho].$$

If he uses the minimax-regret criterion, an equilibrium solves the equation

$$\text{(11)} \qquad D(r) \;=\; M\cdot\min\left[\max\left(\frac{\lambda_1(r) - \rho}{\lambda_1(r) - \lambda_0(r)}, 0\right), 1\right].$$

Equilibrium When Loan-Return Beliefs Vary Directly with r

Equilibrium conditions (9) – (11) are too abstract to offer much intuition. However, they have simple and readily interpretable forms when lender beliefs about the loan return vary directly with r. We give the findings here. Section 2.3 will present a simple borrowing scenario that motivates these beliefs.

Suppose lenders believe that the lowest feasible loan return is $\alpha_0 r$ and the highest feasible return is $\alpha_1 r$, where $0 \le \alpha_0 \le \alpha_1 \le 1$. Thus, lenders set $\lambda_0(r) = \alpha_0 r$ and $\lambda_1(r) = \alpha_1 r$ as the lowest and highest feasible loan returns at contracted repayment rate r. The boundary case $\alpha_0 = \alpha_1$ occurs if lenders know the loan return. The boundary case $(\alpha_0 = 0, \alpha_1 = 1)$ expresses the belief that all logically possible loan returns are feasible. Suppose as well that if lenders maximizes SEU, they place a uniform subjective distribution on $\lambda(r)$. Then we have these findings for equilibrium under the trinity of decision criteria.

Maximization of SEU: The subjective mean return is $\lambda_\pi(r) = (\alpha_0 + \alpha_1)r/2$, so (9) takes the form

$$M \cdot 1[r > 2\rho/(\alpha_0 + \alpha_1)] \le D(r) \le M \cdot 1[r \ge 2\rho/(\alpha_0 + \alpha_1)]. \quad (12)$$

Thus, the equilibrium value of r equals $2\rho/(\alpha_0 + \alpha_1)$ if $D[2\rho/(\alpha_0 + \alpha_1)] \le M$ and solves the equation $D(r) = M$ if $D[2\rho/(\alpha_0 + \alpha_1)] > M$.

Consider the two boundary cases mentioned above. Suppose lenders know the loan return, with $\alpha_0 = \alpha_1 = \alpha$ for some $\alpha \in (0, 1]$. Then the equilibrium value of r equals ρ/α if $D(\rho/\alpha) \le M$ and solves the equation $D(r) = M$ if $D(\rho/\alpha) > M$. Suppose lenders think that all loan returns are possible, so $(\alpha_0 = 0, \alpha_1 = 1)$. Then the equilibrium value of r equals 2ρ if $D(2\rho) \le M$ and solves the equation $D(r) = M$ if $D(2\rho) > M$.

Maximin Loan Supply: If $\lambda_0(r) = \alpha_0 r$, (10) becomes

(13) $$M\cdot 1[r > \rho/\alpha_0]m \;\le\; D(r) \;\le M\cdot 1[r \ge \rho/\alpha_0].$$

Thus, the equilibrium repayment rate is ρ/α_0 if $D(\rho/\alpha_0) \le M$ and solves the equation $D(r) = M$ if $D(\rho/\alpha_0) > M$.

Consider the two boundary cases. If lenders know the loan return, the equilibrium r again equals ρ/α if $D(\rho/\alpha) \le M$ and solves the equation $D(r) = M$ if $D(\rho/\alpha) > M$. If lenders think that all loan returns are possible, then $\rho/\alpha_0 = \infty$. Hence, the credit market collapses in this case.

Minimax-Regret Loan Supply: If $\lambda_0(r) = \alpha_0 r$ and $\lambda_1(r) = \alpha_1 r$, (11) becomes

(14) $$D(r) \;=\; M\cdot\min\left[\max\left(\frac{\alpha_1 r - \rho}{(\alpha_1 - \alpha_0)r}, 0\right), 1\right].$$

If $\alpha_0 < \alpha_1$, the factor multiplying M on the right-hand side of (14) is an increasing continuous function of r, whose value increases strictly from zero at $r = \rho/\alpha_1$ to one at $r = \rho/\alpha_0$. If $D(\rho/\alpha_0) > M$, the equilibrium repayment rate solves the equation $D(r) = M$. If $D(\rho/\alpha_0) \le M$, the equilibrium rate is the $r \in (\rho/\alpha_1, \rho/\alpha_0)$ that solves the equation

(15) $$D(r) \;=\; M\cdot\frac{\alpha_1 r - \rho}{(\alpha_1 - \alpha_0)r}.$$

In the boundary case where lenders think that all loan returns are possible, the equilibrium rate solves this special case of (15): $D(r) = M(r - \rho)/r$.

In the boundary case where lenders know the loan return, the right-hand-side of equation (14) is a step mapping whose value equals 0 for $r < \rho/\alpha$, M for $r > \rho/\alpha$, and the set [0, M] for $r = \rho/\alpha$. Hence, the equilibrium value of r again equals ρ/α if $D(\rho/\alpha) \leq M$ and solves the equation $D(r) = M$ if $D(\rho/\alpha) > M$.

2.3. A Simple Borrowing Scenario

Although equilibrium conditions (9) – (11) are abstract in general, we showed above that they take the simple and intuitive forms (12) – (14) when lender beliefs about the loan return vary directly with r. We now present a simple borrowing scenario that motivates these beliefs. This scenario is highly idealized, but we nonetheless think it instructive.

Borrowing to Finance Productive Investments

To begin, we formalize the idea that borrowers take loans to finance productive investments, but may be unable to repay in full when investment returns turn out to be low. Consider a population J who want to undertake productive investments. For example, a student might invest in a college education or a firm may open a new retail store. For each $j \in J$, let $g_j(x)$ give the gross return to an investment of size x, where $x \geq 0$. We assume that $g_j(0) = 0$ and that $g_j(\cdot)$ is increasing, differentiable, and concave. The gross-return function may vary with j, but we continue to assume that borrowers are observationally identical to lenders.

Suppose that members of J do not have their own endowments and, hence, must borrow to finance their investments. The tentative net return to a loan-financed investment of size x, subtracting full repayment of the loan, is $g_j(x) - rx$. However, bankruptcy law limits loan liability to the magnitude of the gross return $g_j(x)$. We assume that the borrower repays as much of the loan as he is able to, subject to the limit on liability given by bankruptcy law. Hence, the borrower actually repays $\min[rx, g_j(x)]$ and he realizes the net return $\max[0, g_j(x) - rx]$.

Borrower j wants to solve this optimization problem:

$$(16) \qquad \max_{x \geq 0} \ \max\,[0, g_j(x) - rx].$$

He can solve this problem if he knows $g_j(\cdot)$. Given that $g_{jt}(0) = 0$, the maximum value of $g_{jt}(x) - rx$ over $x \geq 0$ must be non-negative. Hence, the optimization problem reduces to

$$(17) \qquad \max_{x \geq 0} \ g_j(x) - rx.$$

Given that $g_j(\cdot)$ is increasing, differentiable, and concave, the optimal loan magnitude is positive if $dg_j(0)/dx > r$ and is zero otherwise.

A borrower with partial knowledge of $g_j(\cdot)$ may not be able to solve problem (17). He may place a subjective probability distribution on $g_j(\cdot)$ and choose x to maximize expected net return. Or, not having probabilistic expectations, he may use the MM or MMR criterion to choose x. In any case, borrowing $x > 0$ may be a reasonable decision ex ante but a poor one ex post. If the gross return turns out to be low, a borrower may not be able to fully repay his loan.[2]

[2] Another source of incomplete repayment, present in real credit markets but not in our model, is imperfect lender ability to enforce legal repayment claims.

Given our assumption that loans are securitized, lenders are concerned with the aggregate repayment of loans rather than with the repayment of a particular individual's loans. Let $x_j(r)$ be loan demand by borrower j at contracted repayment rate r. The loan return at this rate is

$$(18) \qquad \lambda(r) \equiv \frac{\sum_{j \in J} \min\{rx_j(r), g_j[x_j(r)]\}}{\sum_{j \in J} x_j(r)}.$$

The denominator is aggregate loan demand and the numerator is aggregate repayment.

Loan Returns with All-or-Nothing Investments

Loan return (18) is particularly simple if each borrower faces two possible states on nature, a good state yielding positive gross returns to investment and a bad state yielding no return. Formally, let borrower j know that

(a) $g_j(x) = \nu_j h(x)$, where $\nu_j \geq 0$, $h(0) = 0$, and $h(\cdot)$ is increasing, differentiable, and concave;

(b) there are two possible values of ν_j, being $\nu_{0j} = 0$ and $\nu_{1j} = 1$.

Then the net return to investing any amount x is zero in state 0 and $\max[0, h(x) - rx]$ in state 1. The borrower cannot affect his outcome in state 0, so his dominant strategy is to ignore state 0 and choose x optimally for state 1. Thus, $x_j(r) = \operatorname{argmax}_{x \geq 0} h(x) - rx$. [3]

[3] Bankruptcy protection is key to this result. The borrower knows that if he receives a bad draw on ν, he will realize a zero net return regardless of what magnitude loan he demands. Hence,

Although the borrower behaves as if he knew that state 1 will occur, either state may actually occur. If state 1 occurs, he repays his loan in full. If state 0 occurs, he repays nothing. Hence, the loan return (18) has the form

$$(19)\quad \lambda(r) = \frac{r\cdot\sum_{j\in J} x_j(r)\cdot 1[\nu_j = 1]}{\sum_{j\in J} x_j(r)}.$$

This expression simplifies further under the assumption that all borrowers faces the same gross-return function h(·) in state 1. This implies that loan demand is homogeneous across borrowers. Hence, (19) reduces to

$$(20)\quad \lambda(r) = pr,$$

where $p \equiv (1/|J|)\cdot\sum_{j\in J} 1[\nu_j = 1]$ is the fraction of borrowers who realize the good state of nature.

Equation (20) motivates the lender belief setting $\lambda_0(r) = \alpha_0 r$ and $\lambda_1(r) = \alpha_1 r$. In the borrowing scenario under discussion, the actual loan return varies directly with r. However, lenders may not know the fraction p of borrowers who realize the good state of nature and, hence, repay their loans. Thus, α_0 and α_1 are the lowest and highest values of p that lenders thinks feasible.

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he can ignore the possibility of a bad draw and optimizes for the case of a good draw.

Consider borrowing without bankruptcy protection. Suppose that a borrower knows p and maximizes expected net return, believing that he faces state 1 with probability p and state 0 with probability 1 − p. The expected net return is $p\cdot h(x) - rx$. Hence, loan demand at rate r in the absence of bankruptcy protection equals loan demand at rate r/p with bankruptcy protection.

3. Intervention to Maximize the Social Return to Borrowing

Section 2 considered the operation of a credit market without government intervention except for the limited liability provided borrowers by bankruptcy law. In this section, we consider a more activist government that aims to maximize the net social returns to borrowing. Acting through a public Authority, the government may intervene to affect the competitive equilibrium.

Section 3.1 formalizes the Authority's decision problem. Sections 3.2 and 3.3 study the use of two policy instruments to promote the social objective. One intervention manipulates the return on the safe asset and the other guarantees a minimum loan return to lenders.

3.1. The Authority's Decision Problem

Recall that $g_j(x)$ is the gross return if agent j borrows x_j dollars to undertake an investment. Let the net social return to this investment be $g_j(x_j) - \rho^* x_j$, where $\rho^* > 0$ is the social return to an alternative use of a dollar. The value of ρ^* may but need not equal the private return ρ that lenders receive for investment in the safe asset. We assume that the government aims to maximize the aggregate net social return to the investments financed by borrowing. Formally, the public Authority wants to solve the optimization problem

$$(21) \qquad \max_{x_j \geq 0,\, j \in J} \sum_{j \in J} g_j(x_j) - \rho^* x_j .$$

We think maximization of the net social returns to borrowing to be a sensible public objective.

Observe that welfare function $\sum_{j \in J} g_j(x_j) - \rho^* x_j$ is not utilitarian. That is, the Authority does not aim to maximize a weighted sum of the private utilities achieved by lenders and borrowers. A utilitarian welfare function would depend on the loan return that lenders obtain. When a borrower cannot repay his loan in full, a transfer occurs from the lender, who receives less than the contracted repayment, to the borrower, who does not fulfill the contract. We take this zero-sum transfer to be entirely a private matter, not one of social import.

The Authority, like lenders and borrowers, may have only partial knowledge and so be unable to solve problem (21). To drastically simplify analysis of the Authority's problem, we will continue to study the borrowing scenario of Section 2.3. Then (21) becomes

$$(22) \qquad \max_{x_j \geq 0,\, j \in J} \sum_{j \in J} \nu_j h(x_j) - \rho^* x_j.$$

If the Authority were to know $(\nu_j, j \in J)$, it would want to have $x_j = 0$ when $\nu_j = 0$ and would want x_j to maximize $h(x) - \rho^* x$ when $\nu_j = 1$. Our concern is with settings where the Authority does not know $(\nu_j, j \in J)$.

We assume that, lacking knowledge of $(\nu_j, j \in J)$, the Authority seeks to maximize the expected net social return to borrowing; that is, to solve

$$(23) \qquad \max_{x_j \geq 0,\, j \in J} \sum_{j \in J} p \cdot h(x_j) - \rho^* x_j.$$

Here, as in Section 2.3, p is the fraction of borrowers who realize the good state of nature. The solution to (23) is to have every borrower invest the amount $\operatorname{argmax}_{x \geq 0} h(x) - (\rho^*/p)x$. This is the amount that borrowers in a competitive credit market would invest if the contracted repayment rate were ρ^*/p. Thus, given knowledge of p, the Authority can solve (23) if it can intervene in the credit market so that the equilibrium contracted repayment rate becomes ρ^*/p.

Suppose that the Authority does not know p, but believes that $p \in [p_0, p_1]$, where $0 \leq p_0 \leq p_1 \leq 1$. Then it might use one of the trinity of decision criteria that we previously considered for lenders. Application of the MMR criterion is complex in this scenario, but the SEU and MM criteria yield immediate modifications of problem (23). An Authority maximizing SEU would place a subjective distribution, say ψ, on $[p_0, p_1]$ and replace p in (23) with its subjective mean, denoted p_ψ. An Authority using the MM criterion would replace p with p_0. Hence, the Authority would want to intervene in the credit market so that the equilibrium contracted repayment rate becomes ρ^*/p_ψ or ρ^*/p_0 respectively.

3.2. Manipulation of the Return on the Safe Asset

Suppose that the Authority can augment lenders' endowment M and can manipulate the return ρ on the safe asset. If the Authority knows how lenders behave, it can set (M, ρ) to achieve the desired borrowing outcome. In what follows, we assume that the Authority maximizes SEU and, hence, wants the equilibrium value of r to be ρ^*/p_ψ. This encompasses cases where the Authority knows p or use the MM criterion, as p_ψ then equals p or p_0. As in Section 2.2, we assume that lenders' beliefs about the loan return vary directly with r.

The Authority first must determine if lenders' endowment M is large enough to support the desired magnitude of borrowing. Let $x^* \equiv \text{argmax}_{x \geq 0}\, h(x) - (\rho^*/p_\psi)x$ be the desired investment per borrower. Then the desired aggregate loan volume is $x^*|J|$. If $x^*|J| \le M$, the endowment suffices to supply this loan volume. Otherwise, the Authority must augment the endowment by the amount $x^*|J| - M$. There are many ways to accomplish this. For example, the Authority might establish a governmental or quasi-public lending agency with endowment $x^*|J| - M$. Henceforth, we take lenders' endowment to be $M^* \equiv \max(M, x^*|J|)$.

Given endowment M^*, the desired equilibrium can be achieved by manipulating the return on the safe asset. The required intervention depends on lenders' beliefs and decision criterion. We consider here the three cases examined in Section 2.2.

Maximization of SEU: In the absence of intervention, the equilibrium value of r equals $2\rho/(\alpha_0 + \alpha_1)$ if $D[2\rho/(\alpha_0 + \alpha_1)] \le M^*$. To make $r = \rho^*/p_\psi$, the Authority should set $\rho = \rho^*(\alpha_0 + \alpha_1)/(2p_\psi)$.

Maximin Loan Supply: In the absence of intervention, the equilibrium value of r equals ρ/α_0 if $D(\rho/\alpha_0) \le M^*$. To make $r = \rho^*/p_\psi$, the Authority should set $\rho = \rho^*(\alpha_0/p_\psi)$.

Minimax-Regret Loan Supply: Suppose that $\alpha_0 < \alpha_1$, as MMR loan supply coincides with the above cases in the boundary case $\alpha_0 = \alpha_1$. In the absence of intervention, the equilibrium value of r solves equation (15) if $D(\rho/\alpha_0) \le M^*$. To make $r = \rho^*/p_\psi$, the Authority should set ρ to solve the equation

$$(24) \qquad x^*|J| = \frac{M^*(\alpha_1\rho^*/p_\psi - \rho)}{(\alpha_1 - \alpha_0)(\rho^*/p_\psi)} = \frac{M^*(\alpha_1 - p_\psi\rho/\rho^*)}{(\alpha_1 - \alpha_0)}.$$

Thus, the Authority should set $\rho = (\rho^*/p_\psi)[\alpha_1 - (\alpha_1 - \alpha_0)(x^*|J|/M^*)]$. The desired value of ρ equals the value with SEU lending if $x^*|J| = M^*/2$ and equals the value with maximin lending if $x^*|J| = M^*$.

Discussion

The above shows that augmentation of lenders' aggregate endowment and manipulation of the return to the safe asset can yield the socially desired equilibrium. However, two caveats temper the appeal of this intervention.

First, a successful intervention requires the Authority to know lenders' beliefs and decision criterion. If lenders use the SEU or MM criterion, the Authority needs to know (α_0, α_1) to effectively manipulate ρ. If lenders use the MMR criterion, it also needs to know the magnitude x^* of the optimal investment. In any case, the Authority needs to know enough about x^* to establish an endowment that is large enough to supply the desired level of borrowing. The Authority may or may not have the required information in practice.

Second, the intervention presumes that the Authority has the power to manipulate ρ as required. In practice, the safe asset often is a government-issued security. Hence, manipulation of ρ would appear to be feasible. However, the Authority may find it infeasible to set the desired value of ρ if lenders are more pessimistic about loan repayment than is the Authority.

Suppose in particular that fiat money is available as a store of value. Then ρ can be set no lower than one minus the inflation rate. If lenders are sufficiently pessimistic relative to the Authority, the desired return on the safe asset may be smaller than this lower bound.

Suppose, for example, that $\rho^* = 1.02$, $p_\psi = 0.90$, and $(\alpha_0 = 0.75, \alpha_1 = 0.95)$. In this plausible scenario, the desired value of ρ is 0.96 if lenders maximize SEU and 0.85 if they use the MM criterion. Setting ρ to these values is infeasible if the inflation rate is less than 4 and 18 percent respectively. If lenders use the MMR criterion, the desired value of ρ depends on x^*, equaling the SEU value if $x^*|J| = M^*/2$ and the maximin value if $x^*|J| = M^*$. Hence, setting ρ as desired may be infeasible here as well.

3.3. Guaranteeing a Minimum Loan Return

When lenders are pessimistic about the return to lending, the Authority may be able to achieve the desired outcome by guaranteeing a minimum loan return. Suppose as above that the Authority augments lenders' endowment if necessary, so the endowment is $M^* \equiv \max(M, x^*|J|)$. Suppose the Authority guarantees to lenders that they will receive a minimum return of $g \geq \alpha_0$ per dollar loaned. Then lenders should revise upward their beliefs about p, replacing (α_0, α_1) with $[g, \max(g, \alpha_1)]$.

The guarantee required to achieved the desired outcome depends on lenders' beliefs and decision criterion. We show here that a loan guarantee can succeed if lender beliefs are sufficiently pessimistic but not otherwise.

Maximization of SEU: In the absence of intervention, the equilibrium value of r equals $2\rho/(\alpha_0 + \alpha_1)$ if $D[2\rho/(\alpha_0 + \alpha_1)] \leq M^*$. To use a loan guarantee to make $r = \rho^*/p_\psi$, the Authority needs to select g to solve the equation

(25) $$\rho^*/p_\psi = 2\rho/[g + \max(g, \alpha_1)].$$

This equation has a unique solution if $(\alpha_0 + \alpha_1)/2 \leq p_\psi\rho/\rho^*$ and no solution otherwise. To show this, we consider in turn the ranges $\alpha_0 \leq g \leq \alpha_1$ and $g > \alpha_1$.

When $\alpha_0 < g \leq \alpha_1$, equation (25) becomes $\rho^*/p_\psi = 2\rho/(g + \alpha_1)$. Ignoring the constraint, this yields $g = 2p_\psi\rho/\rho^* - \alpha_1$. Thus, the guarantee achieves the desired outcome if $\alpha_0 \leq 2p_\psi\rho/\rho^* - \alpha_1 \leq \alpha_1$. Equivalently, the guarantee succeeds if $(\alpha_0 + \alpha_1)/2 \leq p_\psi\rho/\rho^*$ and $p_\psi\rho/\rho^* \leq \alpha_1$.

When $g > \alpha_1$, equation (25) becomes $\rho^*/p_\psi = \rho/g$. Ignoring the constraint, this yields $g = p_\psi\rho/\rho^*$. Thus, the guarantee succeeds if $p_\psi\rho/\rho^* > \alpha_1$.

Maximin Loan Supply: In the absence of intervention, the equilibrium value of r equals ρ/α_0 if $D(\rho/\alpha_0) \leq M^*$. To use a loan guarantee to make $r = \rho^*/p_\psi$, the Authority needs to select $g \geq \alpha_0$ to solve the equation

(26) $$\rho^*/p_\psi = \rho/g.$$

Ignoring the constraint $g \geq \alpha_0$, this yields $g = p_\psi\rho/\rho^*$. Thus, the guarantee succeeds if $\alpha_0 < p_\psi\rho/\rho^*$.

Minimax-Regret Loan Supply: Suppose that $\alpha_0 < \alpha_1$, as MMR loan supply coincides with the above cases in the boundary case $\alpha_0 = \alpha_1$. In the absence of intervention, solves equation (15) if $D(\rho/\alpha_0) \leq M^*$. To use a loan guarantee to make $r = \rho^*/p_\psi$, we consider in turn the ranges $\alpha_0 < g \leq \alpha_1$ and $g > \alpha_1$.

Consider $\alpha_0 < g \leq \alpha_1$. The Authority needs g to solve the equation

(27) $$x^*|J| = M^*(\alpha_1 - p_\psi\rho/\rho^*)/(\alpha_1 - g).$$

Ignoring the constraint $\alpha_0 < g \leq \alpha_1$, this yields $g = \alpha_1 - M^*(\alpha_1 - p_\psi\rho/\rho^*)/(x^*|J|)$. Thus, the guarantee succeeds if $\alpha_0 < \alpha_1 - M^*(\alpha_1 - p_\psi\rho/\rho^*)/(x^*|J|) \leq \alpha_1$. The MMR guarantee coincides with the SEU guarantee if $x^*|J| = M^*/2$ and coincides with the MM guarantee if $x^*|J| = M^*$.

Consider $g > \alpha_1$. Then the guarantee yields a boundary case where lenders know the loan return is $g \cdot r$. Hence, the equilibrium value of r equals ρ/g if $D(\rho/g) \leq M^*$. Ignoring the constraint, setting $g = p_\psi\rho/\rho^*$ yields the desired outcome. Thus, this guarantee succeeds if $p_\psi\rho/\rho^* > \alpha_1$.

Discussion

As with manipulation of the return on the safe asset, successful implementation of a loan guarantee requires the Authority to know lenders' beliefs and decision criterion. To obtain a sense of the magnitude of the required guarantee, consider again the example with $\rho^* = 1.02$, $p_\psi = 0.90$, and ($\alpha_0 = 0.75$, $\alpha_1 = 0.95$). Suppose as well that the return ρ on the safe asset equals ρ^*. Then the required guarantee is $g = 0.85$ if lenders maximize SEU and $g = 0.90$ if they use the MM criterion. If lenders use the MMR criterion, the desired value of ρ depends on x^*, equaling the SEU value if $x^*|J| = M^*/2$ and the maximin value if $x^*|J| = M^*$. In all of these cases, the required guarantee is less than or equal to the subjective mean 0.90 that the Authority holds for the rate of loan repayment. Hence, it is plausible that the Authority would be willing to make the guarantee.

The potential advantage of a loan guarantee is that it can be implemented when lender beliefs are too pessimistic for manipulation of the return on the safe asset to be feasible. Contrariwise, a loan guarantee cannot achieve the desired outcome when lender beliefs are relatively optimistic. Hence, the two interventions have somewhat distinct domains of application.

In some settings it may be advantageous for the Authority to combine the two interventions, jointly setting (ρ, g) to achieve the desired borrowing outcome. We do not explicitly study joint interventions here, but combinations that yield the desired outcome can be determined in the same way that we have examined the interventions in isolation.

Finally, we note that discussions of loan guarantees often caution about moral hazard, the concern being that the guarantee provides an incentive for lenders to pool groups of borrowers whom lenders believe have different repayment rates. We have throughout this paper defined the credit market under consideration as providing loans to borrowers who are observationally identical to lenders. Hence, such pooling is not possible by assumption within the model.

Nevertheless, moral hazard may be a real concern with implementation of loan guarantees in the real world. In actuality, lenders may have information about borrowers that is not observable by the Authority and, hence, may be able to inappropriately pool loans without detection by the Authority. The potential for problem is exacerbated if lenders can leverage, borrowing the safe asset to finance a loan volume larger than their endowments. Loan guarantees must be designed with caution to mitigate such problems.

4. Conclusion

Ambiguity (or Knightian uncertainty) about investment returns has recently been asserted to be a negative influence on the operation of financial markets, inducing investors to allocate more of their portfolios to safe assets than is socially optimal. Government intervention to reduce ambiguity has been recommended as a suitable treatment. For example, Greenspan (2004, p. 38) has written:

> When confronted with uncertainty, especially Knightian uncertainty, human beings invariably attempt to disengage from medium- to long-term commitments in favor of safety and liquidity. . . The immediate response on the part of the central bank to such financial implosions must be to inject large quantities of liquidity.

Considering the recent credit crisis, Caballero and Krishnamurthy (2008b, p. 2) have written:

> The heart of the recent crisis is a rise in *uncertainty* – that is, a rise in unknown and immeasurable risk rather than the measurable risk that the financial sector specializes in managing. . . . What should central banks do in this case? They must find a way to re-engage the *private sector's liquidity*. Re-engagement will only occur as agents' uncertainty over outcomes is reduced.

Our analysis suggests that investor ambiguity need not be the driving force behind a "flight to safety" in credit markets. In the market we study, the force driving a flight to safety is increased lender pessimism about the return to lending. A lender who maximizes SEU becomes more pessimistic if his subjective mean for the loan return decreases. A decrease in the subjective mean return to lending induces a decrease in loan supply, the consequence being an increase in the equilibrium contracted repayment rate and a decrease in the equilibrium loan volume. Thus, a flight to safety may occur if lenders maximize SEU.[4]

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[4] Other phenomena associated with credit crises are more difficult to explain within the SEU framework. Easley and O'Hara (2008) study the "freezing" of financial markets, where investors are unwilling to trade at all. Freezing differs from a flight to safety, because the latter requires investors who hold risky assets to trade towards safe ones. To explain freezing, they conjecture that investors who lack subjective probability distributions for asset returns have incomplete preferences and, when unable to order alternative portfolios, choose to maintain the status quo as suggested by Bewley (2002).

While we do not see ambiguity as a prerequisite for a flight to safety, we do think it highly plausible that lenders lack rational expectations about loan repayment after unanticipated shocks. Indeed, they may not have probabilistic expectations at all. Lenders who lack probabilistic expectations may use the MM or MMR criterion, or they may cope with ambiguity in some other way.

In any case, the operation of credit markets certainly does depend on lender beliefs about loan returns and on the decision criteria they use to allocate their endowments. Hence, an Authority who wants to intervene in the credit market needs to understand lender beliefs and decision criteria. Our study of policies that manipulate the return on the safe asset or provide a loan guarantee shows that design of a successful intervention requires the Authority to know how pessimistic lender beliefs are relative to the beliefs of the Authority. Theoretical analysis of the type performed in this paper can suggest how lenders may behave, but it cannot reveal how they actually behave. Empirical analysis of lender beliefs and decision criteria is necessary.

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