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### TRADE AND GROWTH WITH HETEROGENOUS FIRMS

Richard E. Baldwin  
Frédéric Robert-Nicoud

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Cambridge, MA 02138  
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This paper explores the impact of trade on growth when firms are heterogeneous. We find that greater openness produces anti- and pro-growth effects. The Melitz-model selection effects raises the expected cost of introducing a new variety and this tends to slow the rate of new-variety introduction and hence growth. The pro-growth effect stems from the impact that freer trade has on the marginal cost of innovating. The balance of the two effects is ambiguous with the sign depending upon the exact nature of the innovation technology and its connection to international trade in goods and ideas. We consider five special cases (these include the Grossman-Helpman, the Coe-Helpman and Rivera-Batiz-Romer models) two of which suggest that trade harms growth; the others predicting the opposite.

Richard E. Baldwin  
Graduate Institute of International Studies  
Av. De la Paix 11a  
CH-1202 Geneva  
SWITZERLAND  
and NBER  
[baldwin@hei.unige.ch](mailto:baldwin@hei.unige.ch)

Frédéric Robert-Nicoud  
Department of Geography and Environment  
London School of Economics  
Houghton Street  
London WC2A 2AE  
UNITED KINGDOM  
[f.l.robert-nicoud@lse.ac.uk](mailto:f.l.robert-nicoud@lse.ac.uk)

# Trade and growth with heterogeneous firms

*Richard E. Baldwin and Frédéric Robert-Nicoud\**

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## ABSTRACT

This paper explores the impact of trade on growth when firms are heterogeneous. We find that greater openness produces anti-and pro-growth effects. The Hopenhayn-Melitz model selection-effect raises the expected cost of introducing a new variety and this tends to slow the rate of new-variety introduction and hence growth. The pro-growth effect stems from the impact that freer trade has on the marginal cost of innovating. The balance of the two effects is ambiguous with the sign depending upon the exact nature of the innovation technology and its connection to international trade in goods and ideas. We consider five special cases (these include the Grossman-Helpman, the Coe-Helpman and Rivera-Batiz-Romer models) two of which suggest that trade harms growth; the others predicting the opposite.

**JEL** H32, P16.

**Keywords:** trade and endogenous growth, heterogeneous firms, dynamic versus static efficiency.

## 1. INTRODUCTION

Until the 1980s, trade theory assumed away intra-industry trade for convenience, but empirical evidence revealed that much of world trade was exactly of the assumed-away kind (Grubel and Lloyd 1975). In response, the so-called new trade theory (Helpman and Krugman 1985) incorporated imperfect competition and increasing returns to account for intra-industry trade. The modelling choices made by new trade theorists assumed away, again for convenience, differences among firms. Recent empirical evidence, however, shows that differences among firms are crucial to understanding world trade. For example, firm differences within sectors may be more pronounced than differences between sector averages, and most firms – even in traded-goods sectors – do not export at all (Bernard and Jensen 1995, 1999a,b, 2001; Clerides, Lach and Tybout 1998, Aw, Chung, and Roberts 2000, Bernard, Eaton, Jensen and Kortum 2003, Eaton, Kortum, and

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Kramarz 2004; see Tybout 2003 for a survey). In response, what might be called the ‘new new’ trade theory incorporated firm-level heterogeneity to account for the many of the new firm-level facts. The main theoretical papers in this rapidly expanding literature are Bernard, Eaton, Jensen and Kortum (2003), Melitz (2003), Helpman, Melitz and Yeaple (2004), Bernard, Redding and Schott (2004), Bernard, Jensen, and Schott (2003), Melitz and Ottaviano (2003), Yeaple (2005), Luttmer (2005a, b) and Chaney (2005).

Our paper studies the growth effects of greater openness by embedding a heterogeneous-firms trade model in a series of product-innovation endogenous growth models (Grossman and Helpman 1989, 1991, Romer 1986, 1990, Rivera-Batiz and Romer 1991a, b, Coe and Helpman 1995, Dinopoulos and Segerstrom 1999a, b, Keller 2004). We show that openness may either slow or speed growth depending upon the impact of openness on the marginal cost of innovating. Since growth is too slow from a welfare perspective in our model (as usual in product-innovation models), there can be a tension between the dynamic and static welfare effects when greater openness slows growth. We show, however, that the overall welfare impact may be positive even when openness slows growth.

To make these points as simply as possible, we work with a new-new trade model related to Helpman, Melitz and Yeaple (2004) and a ‘portmanteau’ product-innovation growth model that includes as special cases Grossman and Helpman’s product innovation model (Grossman and Helpman 1991 chapter 4), Rivera-Batiz and Romer’s ‘lab-equipment’ model, as well as three other models. This task is made easier by the fact that these models are based on the well-known Helpman-Krugman trade model. Indeed, it is insightful to think of our model as the Krugman (1980) model with three important additions. First, we allow for knowledge spillovers in variety-creation so that the cost of developing a new variety falls at the same pace as the value of introducing a new variety. This eliminates the mass of varieties from the equilibrium conditions with the growth rate of the mass of varieties taking its place as a key equilibrating variable; it also allows us to endogenise the growth rate. Second, we add heterogeneity in firms’ marginal production costs, using the Hopenhayn-Melitz variety generation/selection set-up where firms are randomly assigned a marginal cost after having paid a start-up cost. Third, we add sunk market-entry costs as in the ‘hysteresis and trade’ literature (Baldwin 1988, Baldwin and Krugman 1989, Dixit 1989). The sunk costs of establishing a marketing ‘beachhead’ interact with the heterogeneous marginal costs to produce two types of active firms – firms that sell only locally and firms that export as well as sell locally.

The paper is organised in five sections after the introduction. The next two present the model (section 2) and work out the long-run growth path (section 3). The subsequent two sections work

out the growth effects of greater openness (Section 4) and the welfare implications (Section 5). Section 6 presents our concluding remarks.

## 2. A HETEROGENEOUS FIRMS TRADE MODEL WITH GROWTH

The foundation of our model is the well-known Helpman-Krugman monopolistic competition model. We work with two identical nations, a single primary factor  $L$ , and a single Dixit-Stiglitz goods sector ('manufactures') with differentiated varieties that consumers view as symmetric. Competition takes the form of Dixit-Stiglitz monopolistic competition among firms facing iceberg trade costs. Each manufacturing firm's production cost function is linear, involving a sunk, variety-development cost (we denote this 'start-up cost' as  $F_I$  where  $I$  is a mnemonic for 'innovation') and a constant marginal production cost (we denote the unit labour input coefficient as 'a').

Manufacturing firms are assumed to be heterogeneous with respect to their marginal production cost (Hopenhayn 1992a,b). A typical firm's 'a' is drawn from a density function  $G[a]$  whose support is  $0 \leq a \leq a_0$ . The firm learns its 'a' after it has paid the start-up cost,  $F_I$ .

Selling a new variety in a particular market also requires the firm to pay a sunk cost that reflects the cost of adapting the variety to market-specific standards, regulations and norms. These costs – which we refer to as 'beachhead' costs – may differ for locally produced varieties and imported varieties. The cost of establishing a 'beachhead' for locally made products is  $F_D$  ( $D$  is a mnemonic for domestic) and  $F_X$  for imported varieties ( $X$  is a mnemonic for exports).

The three fixed costs – the start-up cost  $F_I$  and the two beachhead costs  $F_D$  and  $F_X$  – involve units of 'knowledge'. The start-up cost requires  $\kappa_I$  units of knowledge, while adapting it to local and export market conditions requires  $\kappa_D$  units and  $\kappa_X$  units of knowledge, respectively. Thus:

$$(1) \quad F_I = \kappa_I P_K, \quad F_D = \kappa_D P_K, \quad F_X = \kappa_X P_K$$

where  $P_K$  is the price of a unit of knowledge ( $K$  for knowledge), and the  $\kappa$ 's are expressed in units of knowledge. Knowledge is created by a perfectly competitive 'innovation sector' ( $I$ -sector for short) and sold to potential entrants in the manufacturing sector.<sup>1</sup> The  $I$ -sector produces knowledge using labour; its technology can be written implicitly as:

$$(2) \quad Q_K = \frac{S}{P_K}; \quad P_K = c[w, \bar{a}, n]$$

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<sup>1</sup> The Hopenhayn-Melitz approach folds variety development into manufacturing, but innovation is the engine of growth in our model, so we separate innovation and manufacturing by introducing an explicit innovation sector.

where  $Q_K$  is the flow of new knowledge (the I-sector's output), and  $S$  is total spending on new knowledge ( $S$  is a mnemonic for savings since in equilibrium all savings goes to buy new knowledge). Competition ensures  $P_K$  equals  $c[w, \vec{a}, n]$  which is the I-sector's marginal cost function; its arguments are the price of labour  $w$ , the vector of all unit-input coefficients of manufactured varieties  $\vec{a}$ , and 'n' is the number (mass) of varieties produced in equilibrium. This allows the cost of producing knowledge to be influenced by the cost of direct inputs (labour) as well as the general level of efficiency in the economy as captured by  $\vec{a}$  and  $n$ . The only restriction we impose at this point is that  $c[\cdot]$  is homogenous of degree minus one in  $n$  due to technological spillovers. This means that the cost of knowledge falls as the I-sector learns from the experience of making successful varieties as usual in endogenous growth models. We leave  $c[\cdot]$  implicit for the moment.

### 3. THE EQUILIBRIUM

This section works out the steady state growth rate and characterises the equilibrium distribution of firm-level productivity. We first work out the instantaneous equilibrium, i.e. the economy's equilibrium at a point on the long-run growth path taking the growth rate as given.

#### 3.1. Instantaneous equilibrium

Entry decisions are the fulcrum of this model and there are two types: 1) the start-up decision, and 2) the two market-entry decisions. Although these are linked, we treat them in sequence.

A firm that has already sunk its start-up cost will enter a particular market if the benefit of doing so exceeds the cost. The benefit of entering a market depends upon the firm's anticipated sales and these, in turn, depend upon the firm's marginal selling cost relative to that of its competitors.

Relying on the standard logic of fixed costs, we can already anticipate that only firms with sufficiently low marginal costs will enjoy market shares that are high enough to justify sinking the beachhead costs. Since there are two beachhead costs ( $F_X$  and  $F_D$ ) there will be two thresholds, one for domestic sales, which we denote as  $a_D$ , and one for export sales, which we denote as  $a_X$ . These two thresholds define three types of firms. The most efficient firms, i.e. those with the lowest unit labour requirements ( $a < a_X$ ) sell locally and also export (we call these X-types, short for export firms). The least efficient firms ( $a > a_D$ ) do not produce (we call these N-types, short for non-producers). Firms with intermediate efficiency ( $a_X < a < a_D$ ) will produce but only sell locally (we call these D-types, short for domestic firms).

To flesh out this intuition, we characterise the operating profit that firms would earn if they decided to enter a given market. Under Dixit-Stiglitz competition, firms find it optimal to charge a constant

operating profit margin,  $1/\sigma$ , where  $\sigma > 1$  is the constant elasticity of substitution among varieties.<sup>2</sup> Thus the operating profit that a firm earns in a particular market equals the value of the firm's sales times  $1/\sigma$ , where the value of its sales are its market-specific market-share times total spending in the market,  $E$ .

By symmetry, wages are equalised internationally, so taking labour as numeraire ( $w=1$ ) and using well-known Dixit-Stiglitz results, firm- $j$ 's price in its local market is  $a_j/(1-1/\sigma)$ . If it exports, its price in the export market is  $\tau a_j/(1-1/\sigma)$ , where  $\tau \geq 1$  represents the usual iceberg trade costs ( $\tau$  units must be shipped to sell one unit in the export market). Thus, a firm's market share as a function of its marginal selling cost is:

$$(3) \quad s[m] = \frac{1}{n} \left(\frac{m}{\bar{m}}\right)^{1-\sigma}; \quad \bar{m}^{1-\sigma} = \int_0^{a_D} a^{1-\sigma} dG[a|a_D] + \phi \int_0^{a_X} a^{1-\sigma} dG[a|a_D]; \quad 0 \leq \phi \equiv \tau^{1-\sigma} \leq 1$$

where 'm' is the firm's marginal selling cost ('a' for local sales and 'ta' for export sales),  $\bar{m}$  can be thought of as a weighted average of firms' marginal selling costs in a particular market; technically  $\bar{m}^{1-\sigma}$  is the  $(1-\sigma)^{\text{th}}$  moment of the distribution function. Also,  $G[a|a_D]$  is the conditional distribution function of the a's for producing firms, i.e. firms with a's less than  $a_D$ .<sup>3</sup> A critical parameter in our paper is the level of trade freeness (phi-ness);  $\phi$  ranges from zero when trade is perfectly closed ( $\tau=\infty$ ) to unity when trade is perfectly free ( $\tau=1$ ), thus trade gets freer as  $\phi$  rises. Equation (3) is pivotal; six observations facilitate intuition and subsequent analysis: i) The ratio,  $(m/\bar{m})^{1-\sigma}$ , is a measure of the firm's market-specific competitiveness, i.e. its marginal selling cost relative to an average of its competitors' marginal selling costs; ii) A firm's market share exceeds  $1/n$  to the extent its m is below the average  $\bar{m}$ ; iii) A firm's market share (and thus its operating profit) increases as its marginal cost falls, ceteris paribus; iv) The average marginal selling costs,  $\bar{m}$ , depends on the two cut-off marginal costs,  $a_D$  and  $a_X$ , and the distribution G; v) As the mass of active firms 'n' grows (as it will along the steady state growth path), the market shares of all existing firms decline in tandem and at the rate at which n grows; vi) Freer trade (i.e.  $\phi$  rises) reduces the firms sales in the protected market.

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<sup>2</sup> See Guide to Calculations for details available on [www.hei.unige.ch/~baldwin/](http://www.hei.unige.ch/~baldwin/)

<sup>3</sup> Since only varieties with  $a < a_D$  are produced,  $G[a|a_D]$  equals  $G[a]/G[a_D]$ . See Melitz (2003) for a proof.

**Cut-off conditions.** Given the sunk-ness of the beachhead costs,  $F_D$  and  $F_X$ , the relevant benefit of market entry is the present value of the market-specific operating profit. Defining the discount rate as  $\gamma$  and using (3), firm- $j$ 's benefit from entering its local and export market are, respectively,  $s[a_j]E/\sigma\gamma$  and  $\phi s[a_j]E/\sigma\gamma$ , where  $E$  is total spending on manufactured goods in each (symmetric) market. The market-entry cut-off conditions are thus:

$$(4) \quad s[a_D] \frac{E}{\sigma\gamma} = P_K \kappa_D; \quad \phi s[a_X] \frac{E}{\sigma\gamma} = P_K \kappa_X$$

where  $a_D$  and  $a_X$  are the cut-off marginal costs for entering the local market and the export market respectively (D stands for domestic and X for export).

**Free entry condition.** A potential entrant to the manufacturing sector must pay  $F_I$  to develop a new variety, learning the variety's associated 'a' only after sinking  $F_I$ . The potential entrant then decides whether to invest in the further knowledge that is necessary to enter the local market and the export market. Potential entrants that draw N-type varieties ( $a > a_D$ ) abandon their project. Those lucky enough to get  $a < a_D$  sink  $F_D$  and  $F_X$ , depending upon the revealed 'a'. Plainly, the value of the resulting variety depends on its 'a'; we derive these values in section 4.2.

A firm's start-up decision can be cast in terms of the expected present value of a 'winning' variety (winning in the sense that it is actually produced, i.e. is a D- or X-type). Because we focus on steady states, the ex ante likelihood of getting a winner with any particular 'a' is exactly the same as the actual distribution of a's for winners already in the market. In other words, the expected operating profit of a winner must exactly match the average operating profit earned in the market. This average is  $E/\sigma n$  because total operating profit worldwide equals  $2E/\sigma$  (due to the constancy of the Dixit-Stiglitz operating profit margin) and the worldwide mass of produced varieties is  $2n$ . Thus:<sup>4</sup>

$$\text{Expected value of a 'winner'} = \frac{E}{\sigma n \gamma}.$$

Of course, innovators do not come up with a winner every try; only new varieties with  $a < a_D$  will be 'winners'. It is straightforward to calculate the expected fixed cost of getting a winner (i.e. developing a D- or X-type patent). The answer is:

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<sup>4</sup> This 'expected=observed' result marks the steady state in any model with a memory-less firm-birth and death process. Eaton and Kortum (2002) use a similar approach.



$$(5) \quad \bar{F} \equiv P_K \bar{\kappa}; \quad \bar{\kappa} \equiv \kappa_D + \kappa_X \frac{G[a_X]}{G[a_D]} + \kappa_I \frac{1}{G[a_D]}$$

where  $\bar{F}$  is the expected fixed cost, which depends upon the price of knowledge  $P_K$  and the expected units of knowledge required to get a ‘winner.’ The first term in  $\bar{\kappa}$  is the fixed cost for local sales – an expense that every winner incurs. The second term reflects the fact that a fraction of winners are X-types and so incur  $\kappa_X$  as well;  $G[a_X]/G[a_D]$  is the probability of being an X-type conditional on being a winner. The third right-hand term reflects the expected variety development cost, i.e.  $\kappa_I$  times the inverse probability of getting a winner on a random draw since, i.e.  $1/G[a_D]$  is the number of ‘tries’ needed to get a winner.

The expected pure profit from developing a new variety is  $(E/\sigma n \gamma - \bar{F})$ . Free entry drives this to zero, so the free entry condition for variety introduction is:<sup>5</sup>

$$(6) \quad \frac{E/\sigma}{n\gamma} = P_K \bar{\kappa}$$

If we took the ratio of  $(E/\sigma n \gamma)$  and  $P_K \bar{\kappa}$ , (6) would be Tobin’s  $q=1$  equation since  $E/\sigma n \gamma$  is akin to the expected stock market value and  $P_K \bar{\kappa}$  is the expected replacement cost. As we shall see, all growth effects of trade work through the replacement cost.<sup>6</sup>

Since  $c[\cdot]$  from (2) is homogeneous of degree minus one in  $n$ , we can use an ‘intensive form’ of  $P_K$ ,  $p_K \equiv n P_K$ . With this notation, the cut-off and free entry conditions, (4) and (6), are:

$$(7) \quad \frac{E}{\sigma \gamma} \left(\frac{a_D}{m}\right)^{1-\sigma} = p_K \kappa_D, \quad \phi \frac{E}{\sigma \gamma} \left(\frac{a_X}{m}\right)^{1-\sigma} = p_K \kappa_X, \quad \frac{E}{\sigma \gamma} = p_K \bar{\kappa}; \quad p_K \equiv n P_K = c[w, \bar{a}, 1]$$

Observe that ‘ $n$ ’ drops out of the three conditions (as it must if  $n$  is to continue rising forever on the growth path). In an endogenous growth model, the discount factor  $\gamma$  – which varies with the growth rate – adjusts as ‘ $n$ ’ would in a no-growth model.

### 3.2. Saving, investment and growth

To finish our characterisation of the growth path, we need to address two further issues: the equilibrium discount rate, and the utility maximising division of income between consumption and saving/investment. To this end, we work with simple intertemporal preferences, namely:

$$(8) \quad U = \int_0^{\infty} e^{-\rho t} \ln D_t dt; \quad D = \left( \int_{i \in \Theta} d_i^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}; \quad Y = E + S; \quad Y = L + \frac{E}{\sigma}$$

<sup>5</sup> See Guide to Calculations for details of the correspondence between ours and the Melitz (2003) approach.

<sup>6</sup> See the Guide to Calculations for details.

where  $\rho$  is the rate of pure time preference,  $D$  is the CES consumption composite,  $\Theta$  is the (time-varying) set of consumed varieties, and  $d_i$  is consumption of variety- $i$ . Consumers divide income  $Y$  between current expenditure  $E$  and saving/investment  $S$ , where  $Y$  equals labour income,  $L$ , plus all manufacturing operating profit  $E/\sigma$  (I-sector profit is zero). Utility optimisation implies a transversality condition and the Euler equation<sup>7</sup>:

$$(9) \quad \dot{E}/E = r - \rho$$

where ‘ $r$ ’ is the rate of return on saving/investment. Expenditure on goods  $E$  equals income less spending on new units of knowledge,  $S$ . Since there are zero pure profits in the I-sector,  $S$  equals the value of inputs, i.e. labour employed in the I-sector,  $L_I$  (recall  $w=1$ ). Thus  $E=L+E/\sigma-L_I$ , so:

$$(10) \quad E = \frac{L - L_I}{1 - 1/\sigma} \quad \Leftrightarrow \quad L_I = L - E(1 - 1/\sigma)$$

Thus determining the utility-maximising path of expenditure  $E$  also pins down the savings/investment path  $S=L_I$ . The implied growth rate of ‘ $n$ ’ given  $L_I$  is:

$$(11) \quad \dot{n} = \frac{Q_K}{\bar{K}} \quad \Leftrightarrow \quad g = \frac{L_I}{p_K \bar{K}}; \quad g \equiv \frac{\dot{n}}{n}$$

where the first and second expressions are related by (2),  $S=L_I$  and  $p_K \equiv P_K n$ ;  $g$  is the growth rate of  $n$ . Note that a constant  $L_I$  implies a constant growth in the mass of varieties  $n$ .

**Solving for the time-path of  $E$ .** The economy can be described as a dynamic system consisting of two differential equations, the Euler equation (9) and the growth of varieties equation (11). To solve these for the steady state, we must specify the state variable. Although the steady state is unaffected by the choice of state variables, the ease of calculation depends greatly on this choice. Following Baldwin and Forslid (2000), we choose  $L_I$  as the state variable.<sup>8</sup> The fruit of this choice is immediately apparent. First,  $r=\rho$  given expressions (9) and (10) and the fact that  $\dot{L}_I=0$  by definition of steady state. Moreover, if  $L_I$  stops evolving, (11) tells us that  $g$  is constant along the steady-state growth path. The discount rate  $\gamma$  equals  $g+\rho$ . Intuitively,  $\gamma$  reflects pure time preference  $\rho$  and the rate at which the typical variety’s market share falls, namely  $g$ .<sup>9</sup>

Summarising, expenditure  $E$ , savings/investment  $S=L_I$ , the growth rate of varieties  $g$ , and the discount rate  $\gamma$  are time-invariant along the steady state growth path. The dynamic system therefore

<sup>7</sup> See the Guide to Calculations for details.

<sup>8</sup> Our use of ‘state variable’ is somewhat unconventional in the standard terminology of economic dynamics; see the Guide to Calculations for a detailed discussion on this.

reduces to three equations – the two cut-off conditions and the free entry condition – with three unknowns,  $g$ ,  $a_D$  and  $a_X$ . The only thing left to find is the utility maximising level of  $E$ .

Using  $\gamma=\rho+g$ , (10), (11), and the third expression in (7) yields the intuitive result that the utility-maximising  $E$  equals ‘permanent income’, i.e. the income from labour  $L$  plus the rental rate on the steady-state value of the nation’s capital stock (which equals  $p_K \bar{K}$ ):<sup>10</sup>

$$E = L + \rho p_K \bar{K}$$

Using this and  $\gamma=\rho+g$  in (7), the cutoff and free entry conditions are:

$$(12) \quad \frac{L + \rho p_K \bar{K}}{\sigma(\rho + g)} \left(\frac{a_D}{\bar{m}}\right)^{1-\sigma} = p_K \kappa_D, \quad \phi \frac{L + \rho p_K \bar{K}}{\sigma(\rho + g)} \left(\frac{a_X}{\bar{m}}\right)^{1-\sigma} = p_K \kappa_X, \quad \frac{L + \rho p_K \bar{K}}{\sigma(\rho + g)} = p_K \bar{K}$$

These three expressions implicitly define the three equilibrating variables,  $a_D$ ,  $a_X$  and  $g$ .

### 3.3. Discussion of the growth path

The growth path in our model shares many similarities with the growth path of the canonical Grossman-Helpman product-innovation growth model that assumes homogenous varieties. The long-run growth path is characterised by a time-invariant division of labour between the production of capital goods (knowledge in our model) and consumption goods; the division is  $L_1$  versus  $L-L_1$ . The constant application of  $L_1$  to knowledge creation yields a steadily rising mass of varieties due to the learning curve assumed in the I-sector,  $c[\cdot]$ . Observe that ‘ $g$ ’ here is the equilibrating variable much as ‘ $n$ ’ is in no-growth heterogenous firm trade (HFT hereafter) models. Because knowledge spillovers remove  $n$  from the calculation, it is the speed of innovation that ensures zero expected pure profit from variety introduction. The economic logic is that a faster growth of  $n$  drives down the expected benefit of introducing a new variety since it boosts the rate at which innovators expect the operating profit of a new variety to decline as per (3).

The constant flow of new varieties together with the time invariant employment of labour in the production of consumption goods means that consumption/production of each variety continuously declines at a rate of  $g$ . The representative consumer sees her real consumption,  $E/P$ , rise at:

$$(13) \quad g_c = \frac{g}{\sigma - 1}$$

where  $g_c$  is the growth rate of real consumption.<sup>11</sup> In the working paper version of our paper (Baldwin and Robert-Nicoud, 2005), we proved that the laissez-faire mass of varieties and cut-off

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<sup>9</sup> See Guide to Calculations for discussion.

points along the steady-state growth path are socially optimal, but the laissez-faire growth rate is too low from the planner's perspective.

#### 4. GROWTH EFFECTS OF MARKET OPENNING

Solving the third expression in (12):

$$(14) \quad g = \frac{L/\sigma}{p_K \bar{\kappa}} - \frac{\rho(\sigma-1)}{\sigma}$$

Since everything here is a parameter except  $p_K \bar{\kappa}$ , all growth effects of trade must come through the expected sunk cost of getting a new variety, i.e. the denominator of Tobin's  $q$ . Thus:

**Result 1 : Closer international economic integration is pro-growth if and only if it lowers the expected sunk cost of developing a new, produced variety (namely,  $p_K \bar{\kappa}$ ). This allows us to define two growth-effect channels: the  $p_K$ -channel and the  $\bar{\kappa}$ -channel.**

##### 4.1. Pro- and anti-growth effects

**Anti-growth.** As it turns out, the  $\bar{\kappa}$ -channel is unambiguously anti-growth since the expected sunk cost of getting a 'winner' rises with freer trade. Intuitively, freer trade makes it easier to export, so the threshold marginal cost for exporting,  $a_X$ , rises. The increased competition from imports makes both markets more competitive and this lowers the common  $a_D$  (the extra competition induces the least efficient D-type firms to cease production). From the definition of  $\bar{\kappa}$  in (5), this means that the typical winner is more likely to be an X-type and a successful entry will require more tries before getting a winner; both factors raise  $\bar{\kappa}$ .<sup>12</sup> In short trade may be anti-growth since it raises the fixed knowledge-requirement of new varieties conditional on entry.

**Pro-growth.** Freer trade in goods and ideas may alter the price of new knowledge,  $p_K$ . We cannot be more precise without an explicit functional form for  $c[w, \bar{a}, n]$ , but two well-known economic mechanisms suggest that the  $p_K$ -channel will usually be pro-growth. One concerns the price-lowering impact of freer trade on goods prices and the indirect impact of this on  $p_K$  when the knowledge-producing sector uses intermediate inputs – 'lab equipment.' An entirely separate line of reasoning concerns international knowledge spillovers. If tighter integration makes the flow of ideas freer then we would expect learning in each nation to have a greater cost-lowering impact in both nations and this would show up in a lower  $p_K$ .

##### 4.2. Analytic solutions and microfoundations for I-sector technology

<sup>10</sup> From (6), the present value of all varieties, namely  $E/\sigma\gamma$ , equals  $p_K \bar{\kappa}$ , since  $p_K = nP_K$ .

<sup>11</sup> This follows from the fact that the ideal CES price index is  $P = \bar{m} \{n^{1/(1-\sigma)}\} / (1-1/\sigma)$ .

<sup>12</sup> See Melitz (2003) for proof with a general  $G$ ; below we demonstrate this for a specific functional form.

Analysis up to this point has been conducted without resort to a functional form for  $G$ . Much of the subsequent analysis can also be conducted in this manner, but the reasoning is clearer with explicit solutions for  $a_D$ ,  $a_X$  and  $g$ . We assume a Pareto distribution:

$$(15) \quad G[a] = (a/a_0)^k \quad 0 \leq a \leq a_0 \equiv 1$$

where  $k$  and  $a_0$  are the ‘shape’ and ‘scale’ parameters, respectively. By choice of units,  $a_0$  is normalised to unity without loss of generality. With (15), we solve  $\bar{m}$  and  $\bar{\kappa}$ , so (12) yields:<sup>13</sup>

$$(16) \quad g = \frac{L/\sigma}{p_K \bar{\kappa}} - \frac{\rho(\sigma-1)}{\sigma}, \quad a_D = \left( \frac{(\beta-1)\kappa_I}{(1+\Omega)\kappa_D} \right)^{1/k}, \quad a_X = \left( \frac{\Omega(\beta-1)\kappa_I}{(1+\Omega)\kappa_X} \right)^{1/k}$$

where  $p_K = c[1, \bar{a}, 1]$ ,  $\bar{\kappa} = \beta\kappa_D(1+\Omega)/(\beta-1)$  and<sup>14</sup>

$$\Omega \equiv \phi^\beta T^{1-\beta}, \quad T \equiv \kappa_X / \kappa_D, \quad \beta \equiv k/(\sigma-1) > 1$$

**Omega as a measure of openness.**  $\Omega$  (a mnemonic for ‘openness’) is a Cobb-Douglas average of the two types of trade barriers in the HFT model, variable trade cost (as measured by freeness  $\phi$ ) and fixed trade costs (as measured by the excessive beachhead cost facing imported varieties, namely  $T \equiv F_X/F_D$ ).  $\Omega$  displays several convenient features. It combines the protective effects of fixed and variable trade costs, and it is bound between zero and unity with zero indicating autarky and unity indicating free trade.

#### 4.2.1. Microfoundations for the I-sector marginal cost function

The strength of the  $p_K$ -channel depends upon  $c[w, \bar{a}, 1]$ . Five special cases are considered.<sup>15</sup>

**Grossman-Helpman model.** The canonical Grossman-Helpman product-innovation model assumes a learning curve where the marginal cost of creating knowledge falls as the I-sector’s experience in producing successful varieties rises (i.e. as  $n$  rises). Specifically:

$$(17) \quad c[w, \bar{a}, n] = \frac{w}{n + \lambda n^*} \quad \Rightarrow \quad p_K \bar{\kappa} = \frac{\beta\kappa_D(1+\Omega)}{(\beta-1)(1+\lambda)}$$

where  $0 \leq \lambda \leq 1$  ( $\lambda$  is a mnemonic for ‘learning’) measures the international dimension of spillovers; this is for the Home nation, but the Foreign nation’s is identical since  $n = n^*$  by symmetry. (The original Grossman-Helpman version considered the extremes of no and perfect

<sup>13</sup> Specifically,  $\bar{m}^{1-\sigma} = \beta \{ (1+\Omega)/(\beta-1) \}^{1+\beta} (\kappa_D / \kappa_I)^\beta$ .

<sup>14</sup> The restriction  $\beta > 1$  ensures the integrals converge.

<sup>15</sup> We thank one referee for suggesting the second and third.

spillovers, i.e.  $\lambda=0$  and  $\lambda=1$ .) Many justifications of this intertemporal externality are possible. Romer (1990) rationalizes it by referring to the non-rival nature of knowledge. Grossman and Helpman (1991) assert that it reflects the impact of ‘public knowledge’ that is created automatically along with the private, patentable knowledge that leads to new varieties.

**Coe-Helpman model.** A variant of (17) makes  $\lambda$  a function of trade flows by equating  $\lambda$  to the fraction of foreign varieties that are imported, namely  $(a_X/a_D)^k$ , i.e.:

$$(18) \quad \tilde{\lambda} \equiv (a_X / a_D)^k \quad \Rightarrow \quad p_K \bar{K} = \frac{\beta \kappa_D (1 + \Omega)}{(\beta - 1)(1 + \Omega/T)}$$

where  $\tilde{\lambda}$  replaces  $\lambda$  in (17). Coe and Helpman (1995) provide empirical evidence that knowledge spillovers are related to trade flows; see Keller (2002) for a critique.

**Efficiency-linked knowledge spillovers.** Expression (17) assumes that I-sector learning is equal for all varieties (the impact on  $p_K$  is the same for X- and D-types). An alternative is to assume that the extent of the spillovers is proportional to the average efficiency of the produced varieties (efficiency being inversely related to marginal cost), so the  $n$ 's are weighted by the average efficiency  $\bar{m}^{1-\sigma}$  and  $(\bar{m}^*)^{1-\sigma}$ . Noting  $n=n^*$  and  $\bar{m} = \bar{m}^*$ :

$$(19) \quad c[w, \bar{a}, n] = \frac{w}{[n\bar{m}^{1-\sigma} + n^*(\bar{m}^*)^{1-\sigma}]/2} \quad \Rightarrow \quad p_K \bar{K} = \frac{\{(\beta - 1)\kappa_I\}^\beta}{(1 + \Omega)^\beta \kappa_D^{\beta-1}}$$

**Reverse engineering.** The three previous special cases treat I-sector learning as proportional to the number of successful varieties currently produced. Varieties, however, contain different units of knowledge depending upon their export status. The average number of knowledge units in a locally produced variety is  $\bar{K}$ , as per (5), but it is higher for an imported variety, namely  $\kappa_D + \kappa_X + \kappa_I$ . Presuming that the local I-sector only learns from varieties that it can actually purchase (the idea being only these can be ‘reverse engineered’),  $c[w, \bar{a}, n]$  would be:

$$(20) \quad c[w, \bar{a}, n] = \frac{w}{n\bar{K} + \tilde{\lambda} n^* \bar{K}_X} \quad \Rightarrow \quad p_K \bar{K} = \left( 1 + \frac{(\beta - 1)\bar{K}_X \Omega}{\beta T (1 + \Omega)} \right)^{-1}$$

where  $\bar{K}_X$  equals  $\kappa_D + \kappa_X + \kappa_I$  and  $\tilde{\lambda}$  is from (18); again  $n=n^*$ .

**Lab-equipment model.** Another well-known product-innovation model, Rivera-Batiz and Romer (1991), supposes that knowledge is produced using the final good CES composite, so:<sup>16</sup>

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<sup>16</sup> The same ‘output composite’ can either be consumed or invested as in traditional growth models.

$$(21) \quad c[w, \bar{a}, n] = P^\alpha; \quad P = \frac{n^{1/(1-\sigma)} \bar{m}}{1-1/\sigma}, \quad \alpha = \sigma - 1 \quad \Rightarrow \quad p_K \bar{K} = \frac{\kappa_D}{(1-1/\sigma)^{\sigma-1}} \left( \frac{(\beta-1)\kappa_I}{(1+\Omega)\kappa_D} \right)^\beta$$

where  $P$  is the CES price index, and the restriction on  $\alpha$  is implied by our homogeneity assumption on  $c[w, \bar{a}, n]$ . This provides microfoundations for a connection between the cost of goods and the productivity of the knowledge creating sector.<sup>17</sup> Plainly, these five special cases could be mixed and matched to create a wide range of possibilities.

### 4.3. Autarky to Free Trade liberalisation

The product  $p_K \bar{K}$  is the pivot for growth effects, as per Result 1. Having worked out the analytic solutions for  $p_K \bar{K}$  in the five cases, evaluation of the growth effects of freer trade is straightforward. We start with a base-case comparison of two conceptual extremes, complete autarky and complete integration. When nations are autarkic in terms of the flow of goods and ideas,  $\Omega=\lambda=1/T=0$ ; when they are fully integrated  $\Omega=\lambda=T=1$ .

The Grossman-Helpman and Coe-Helpman models display the same growth rate in the two conceptual extremes. Intuitively, this is because the shift to full goods-market openness ( $\Omega=0$  to  $\Omega=1$ ) increases the amount of knowledge per variety,  $\bar{K}$ , by exactly the same amount that the increase in ideas-openness ( $\lambda=0$  to  $\lambda=1$ ) lowers the per-unit price of knowledge. This result does not hold for the other models of innovation – the efficiency-linked, reverse-engineering and lab-equipment models. In these models zero-to-full integration is pro-growth. The stark contrast in results is due to the connection between manufacturing-sector average productivity and innovation-sector productivity. In the Grossman-Helpman and Coe-Helpman models there is no direct connection while there is in the other three models, so the well-known productivity enhancing effects of liberalisation in the HFT models (so-called shifting and sorting effects) stimulate innovation in the latter but not the former. Next, consider marginal changes in integration.

### 4.4. The growth effect of freer trade in goods and ideas

The term  $p_K \bar{K}$  is the pivot for growth effects, but observe that all ambiguity of growth effects stems from the  $p_K$ -channel. In all five cases,  $d\bar{K}/d\phi$  equals  $\{\beta\kappa_D/(\beta-1)\}(d\Omega/d\phi)>0$ , so the  $\bar{K}$ -channel is unambiguously anti-growth. If the overall effect is positive, it must be that the  $p_K$ -channel is pro-growth and sufficiently strong to overwhelm the negative  $\bar{K}$ -channel effect. We

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<sup>17</sup> In this case, the expression for  $E$  in operating profits is just  $Y$  since both  $E$  and  $S$  are spent on goods.

consider freer trade with respect to variable trade costs (i.e.  $d\phi > 0$ ), with respect to fixed trade costs (i.e.  $dT < 0$ ), and with respect to trade in ideas, (i.e.  $d\lambda > 0$ ).

**Lower variable trade-costs.** Reducing iceberg trade costs tends to slow growth in two cases, speeding it in three cases. Given Result 1,  $\Omega = \phi^\beta T^{1-\beta}$  and the simple analytic solutions in (17)-(21), the proofs are by inspection of the relevant expression for  $p_K \bar{\kappa}$ .

- Under the expression (17) knowledge-creating technology (Grossman-Helpman),  $d\phi > 0$  has no impact on  $p_K$ , so the  $p_K$ -channel does nothing to offset the negative  $\bar{\kappa}$ -channel effect. The overall impact is therefore anti-growth (our working paper version, Baldwin and Robert-Nicoud 2005, discusses this case at length).
- Under the expression (18) knowledge-creating technology (Coe-Helpman), the  $p_K$ -channel is positive, but it is not sufficiently large to offset the negative  $\bar{\kappa}$ -channel effect unless  $T=1$  in which case the net growth effect is zero (NB:  $T=1$  implies the beachhead cost is the same for a local and imported variety).
- Under the expression (19) knowledge-creating technology (efficiency-linked spillovers), the  $p_K$ -channel is positive. It is also strong enough to overcome the anti-growth  $\bar{\kappa}$ -channel. Formally, the result can be seen by inspection of (19). Intuitively, the pro-growth  $p_K$ -channel is driven by the productivity enhancing impact of freer trade, which is in turn driven by a selection effect (weaker firms are eliminated and more firms export) and a share-shifting effect (the market shares of the most efficient firms rise while those of inefficient firms falls). The anti-growth effect, however, is driven by the selection effect alone (i.e.  $d(a_X) > 0$  and  $d(a_D) < 0$ ) and so it is weaker.
- Under the expression (20) knowledge-creating technology (reverse engineering), the  $p_K$ -channel is positive. It dominates the anti-growth  $\bar{\kappa}$ -channel, so the overall effect is pro-growth as inspection of (20) reveals. Intuitively, we can think of this as the Coe-Helpman model with an extra fillip for international knowledge spillovers. This extra fillip is due to the fact that imported varieties systematically ‘embody’ more knowledge since they have been adapted to meet the beachhead costs in both markets, while some of the locally-produced varieties embody only  $\kappa_I$  and  $\kappa_D$  units of knowledge. Notice that this extra effect is strong enough to overcome the anti-growth  $\bar{\kappa}$ -channel even when  $T$  is very large but finite.
- Under the expression (21) knowledge-creating technology (lab-equipment), the  $p_K$ -channel is positive and strong enough to overcome the fact that freer trade raises the expected sunk cost of



getting a ‘winner.’ As in the efficiency-linked-spillovers model, the result is due to the powerful, pro-efficiency impact of freer trade in the HFT model.

**Lower fixed trade-costs.** Reducing  $F_X$  to the level of  $F_D$  (i.e.  $dT < 0$ ) has qualitatively identical effects on growth as  $d\phi > 0$  in three of the five cases since  $T$  and  $\phi$  appear bundled in  $\Omega$ . These are the cases where the degree of international spillovers,  $\lambda$ , is exogenous, viz. (17), (19) and (21).

When the international spillovers parameter  $\lambda$  is linked to the fraction of foreign varieties that are traded, then  $T$  has an independent effect on growth since this fraction is a function of  $T$ , i.e. from (16) and the definition of  $\tilde{\lambda}$ , we have  $\tilde{\lambda} = \Omega/T$ . It is easy to show that taking account of the  $T$  inside  $\Omega$  and the explicit  $T$  in the models where  $\lambda$  is endogenous, i.e. (18) and (20), lower fixed barriers to trade is always pro-growth. The intuition is obvious. Since lower  $T$  encourages spillovers by increasing the fraction of traded varieties, it tends to be pro-growth and this only adds to the pro-growth effect from  $d\Omega > 0$ .

**Higher knowledge spillovers.** It is plain that raising the exogenous  $\lambda$  in isolation is pro-growth since it lowers the expected cost of getting a ‘winner’ without any direct impact on the reward to getting a ‘winner.’ In the Grossman-Helpman model, openness to ideas,  $\lambda$ , opposes openness to goods,  $\Omega$ , such that they cancel if they are equal, viz.  $\lambda = \Omega$ ; in this case, greater openness has no impact on growth.

#### 4.5. The Role of Heterogeneity and beachhead costs

The role of heterogeneity and beachhead effects in trade and growth links are quite obvious.<sup>18</sup> If the beachhead costs are eliminated, integration has no impact on the range of varieties and so the  $\bar{\kappa}$ -channel is eliminated; it turns  $\tilde{\lambda}$ ,  $\bar{m}$ ,  $\bar{\kappa}$  and  $\bar{\kappa}_X$  into parameters. It also eliminates most of the effects on the  $p_K$ -channel. Changes in the exogenous learning spillovers (as measured by  $\lambda$ ) still effect growth, and the price lowering impact of integration feeds directly into  $p_K$  in the lab-equipment model. Note that when  $T = F_X/F_D = 1$  the growth effects of freer-trade (in the traditional iceberg sense) are unchanged, except for the Coe-Helpman model. In this case,  $T = 1$  implies the anti-growth  $\bar{\kappa}$ -channel exactly offsets the pro-growth  $p_K$ -channel.

#### 4.6. Static versus dynamic productivity effects

A major finding of heterogeneous-firms trade theory is that trade openness can raise productivity via selection and share-shifting effects (Melitz 2003). Defining ‘measured productivity’ as the ratio

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<sup>18</sup> See our working paper version for the mathematical details.

of real manufacture output to manufacturing labour input, we note that openness also boosts productivity in our model since greater openness lowers the ideal CES price index. However, in cases such as (17), openness slows the rate at which the price index falls and so slows the rate of productivity improvement. This implies that there may be tension between the static and dynamic productivity effects of trade in an HFT model.

## 5. WELFARE

Most of the welfare issues in our model are simple. The present value of utility on the equilibrium growth path at time  $t=0$  is  $U_0=E/P_0(\rho+g)$ . Using the permanent income expression for  $E$ ,  $P$  from (21), and the analytic solution for  $\bar{m}$ , we have:

$$(22) \quad U_0 = \left( \frac{\sigma}{p_K \bar{K}} \right) \left( \beta \left( \frac{1+\Omega}{\beta-1} \right)^{1+\beta} \left( \frac{\kappa_D}{\kappa_I} \right)^\beta \right)^{1/(\sigma-1)}$$

The first term in parentheses reflects the dynamic welfare aspects, i.e.  $U_0=E/(\rho+g)$ , and the second captures the static aspects, i.e.  $1/P_0$ . Plainly greater openness (i.e.  $d\Omega>0$ ) always produces positive static effects. The sign of the dynamic normative effects of greater openness balances on the same fulcrum as did the positive effects, namely  $p_K \bar{K}$ .

Indeed, the decentralised growth rate  $g$  is suboptimal from a welfare perspective because innovators do not internalise the positive dynamic (learning) externalities they exert on future innovations (see Grossman and Helpman 1991 and Baldwin and Robert-Nicoud 2005). As a result, any policy that raises growth reduces the gap between the market growth rate and the desirable one is welfare improving in a dynamic sense. To summarise:

**Result 2: The static welfare effect of greater openness is always positive; since laissez-faire growth is sub-optimal from the social-welfare perspective, the dynamic welfare effect is positive if and only if greater openness raises the growth rate. Given Result 1, this means that a sufficient condition for greater openness to be welfare enhancing is that greater openness lowers the expected cost of a new produced variety, namely  $p_K \bar{K}$ .**

Thus for 3 of the 5 special cases for I-sector technology, namely (19), (20) and (21), we can say the welfare effects are unambiguously positive, given the analysis in Section 4. Welfare ambiguity arises when openness is anti-growth since it creates a tension between welfare-enhancing static effects and welfare-worsening dynamic effects. In the Grossman-Helpman case in which openness is the most strongly anti-growth, plugging (17) into (22) yields:

$$U_0 = \left( \frac{\sigma(1+\lambda)}{\beta\kappa_D} \right) \left( \beta \left( \frac{1+\Omega}{\beta-1} \right)^{2-\sigma+\frac{k}{\sigma-1}} \left( \frac{\kappa_D}{\kappa_I} \right)^{\frac{k}{\sigma-1}} \right)^{1/(\sigma-1)}$$

By inspection, this is unambiguously increasing in openness as measured by  $\lambda$  and as measured by  $\Omega$  as well only if  $\sigma$  is small enough; in such cases, openness raise welfare despite the negative impact on growth.

## 6. CONCLUDING REMARKS

This paper explores the growth effects of trade when firms are heterogeneous and face sunk market-entry costs. Our findings can be viewed as speaking to the trade and growth literature on one hand, and the new-new trade theory on the other. As far as trade and growth is concerned, our main finding - that freer trade has an ambiguous impact on growth - contrasts with most findings in the homogeneous-firms endogenous growth literature where positive effects are the standard result; this ambiguous finding squares better with the empirical evidence on the effects of trade openness on growth (see Berg and Krueger, 2003, for a survey). Specifically, our model stresses the need for empirical researchers to better investigate the link between trade and the dissemination of knowledge, in particular its impact on the efficiency of the innovation sector. As far as the new-new trade theory is concerned, our main finding is that there is tension between static and dynamic productivity effects. Although freer trade improves industry productivity in a level sense, it may harm it in a growth sense, at least when ideal price indices are used to measure real output.

One line of research that would be interesting would be to explore the impact of unilateral trade liberalisation. However this analysis becomes very involved, so we leave this for future research.

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## Guide to Calculations by Footnote: “Trade and growth with heterogeneous firms”

*(not for publication, will be posted on the web)*  
Richard E. Baldwin and Frédéric Robert-Nicoud

2. The constant operating profit mark-up follows directly from the standard Dixit-Stiglitz result of constant cost-price mark-up; for example, rearranging the first order condition for local sales,  $p(1-1/\sigma) = a$  we get that operating profit earned on local sales is  $(p-a)c = pc/\sigma$ , where  $c$  is local sales.
5. The direct approach to formulating the condition for zero-expected-profit-from-innovation is to calculate the expected benefit net of market entry costs. Starting from the well-known CES demand function, the expected operating profit of a firm that draws its ‘ $a$ ’ from the distribution  $G[a]$  will be:

$$\int_0^{a_D} \left\{ \frac{a^{1-\sigma} (E / \sigma \gamma)}{\int_0^{a_D} a^{1-\sigma} ndG[a|a_D] + \phi \int_0^{a_X} a^{1-\sigma} ndG[a|a_D]} - F_D \right\} dG[a]$$

$$+ \int_0^{a_X} \left\{ \phi \frac{a^{1-\sigma} (E / \sigma \gamma)}{\int_0^{a_D} a^{1-\sigma} dG[a|a_D] + \phi \int_0^{a_X} a^{1-\sigma} dG[a|a_D]} - F_X \right\} dG[a]$$

since  $ndG[a|a_D]$  gives the frequency of  $a$ 's among varieties that are actually produced. Using our

notation  $\bar{m}^{1-\sigma}$ , this becomes:  $\int_0^{a_D} \left\{ a^{1-\sigma} \frac{E}{\sigma \bar{m}^{1-\sigma} \gamma n} - F_D \right\} dG[a] + \int_0^{a_X} \left\{ \phi a^{1-\sigma} \frac{E}{\sigma \bar{m}^{1-\sigma} \gamma n} - F_X \right\} dG[a]$

which simplifies to  $(E / \sigma \bar{m}^{1-\sigma} \gamma n) (A[a_D] + \phi A[a_X])$  minus  $(G[a_D]F_D + G[a_X]F_X)$ , where

$A[x] \equiv \int_0^x a^{1-\sigma} dG[a]$ . Notice, however, that (3) implies  $G[a_D] = (A[a_D] + \phi A[a_X]) / \bar{m}^{1-\sigma}$ , since

$G[a|a_D] = G[a] / G[a_D]$ . Thus the benefit less market-entry costs is  $G[a_D](E/n\sigma\gamma) - G[a_D]F_D -$

$G[a_X]F_X$ . This is set equal to  $F_I$  in the direct approach. Dividing this through by  $G[a_D]$  validates our indirect approach in (6).

We adopt the indirect approach since it allows us to deal more clearly with growth and corresponds more closely to Tobin’s insightful approach to characterising investment in a general equilibrium setting. It also allows a direct comparison with standard endogenous growth models (which do not have market-entry costs) and it facilitates analysis of growth effects by concentrating the impact of openness and parameter changes in the expected cost of getting a winner.

6. See Baldwin and Forslid (2000), Proposition 1 for a general proof that the direct impact on Tobin's  $q$  is a sufficient statistic for growth effects in this sort of model. Intuitively, this is obvious since anything that raise Tobin's  $q$  encourages more investment and this, in an endogenous growth model, results in faster growth. The faster growth, in turn, returns Tobin's  $q$  back to its steady state value of unity. All the variables in the numerator of Tobin's  $q$  are either endogenous (e.g.  $E$ ) or are parameters unrelated to trade liberalisation.

7. Here we work out the whole dynamic system. In the course of the discussion, the naturalness of using  $L_1$  as the state variable becomes clear, but intuitively, the key to constant equilibrium growth is that a time-invariant amount of the primary factor, labour, is devoted to knowledge creation and this knowledge creation process is not subject to diminishing returns with respect to the amount of knowledge created. Since the amount of labour involved in knowledge creation is the key, it is natural to take it as the variable that stops evolving in steady state, i.e. as the state variable.

Formally, income is  $L+rW$ , where  $W$  is wealth,  $r$  is the rate of return, and  $\dot{W}=L+rW-E$  describes wealth accumulation. All variables are measured in terms of the numeraire, i.e. labour. The Hamiltonian is  $e^{-\rho t} \ln(E/P) + \omega(L+rW-E)$  and  $r$  and the path of  $P$  is exogenous to the consumer/saver. The necessary conditions are  $e^{-\rho t}/E = \omega$  and  $-\omega r = \dot{\omega}$  plus a transversality condition. Manipulation involving the time derivative of the first condition and substitution of the second condition yields the Euler equation.

The two differential equations are:

$$\dot{E}/E = r - \rho, \quad \frac{\dot{n}}{n} = \frac{L_1}{p_K \bar{K}}$$

Doing a standard change-of-variables transformation, we take  $L_1$  and  $g \equiv \dot{n}/n$  as the state variables and, using (10), the system becomes:

$$\frac{-\dot{L}_1}{L - L_1} = r - \rho, \quad \frac{\dot{n}}{n} = \frac{L_1}{p_K \bar{K}}$$

As usual, changing state variables has no impact on the system dynamics. As is well known from decades of growth models, this system is saddle path stable. Also, as in most endogenous growth models, there is no transitional dynamics since the saddle path is a point, namely the steady state equilibrium. The system jumps immediately to the steady state since otherwise the system would violate the transversality conditions. All these assertions are proved at great length and generality in Grossman and Helpman (1991) and other textbooks on growth. The exact proof in this case can be

found in Baldwin and Forslid (2000) which uses Tobin's q approach to evaluate the growth effects of trade in a homogenous goods model.

8. Our use of 'state variable' is somewhat unconventional in the standard terminology of economic dynamics (where state variable means a non-jumper and co-state and control variables are jumpers). Mathematically speaking, the state vector is the set of variables that defines the state of the system fully. This includes variables that can jump or only move smoothly. Thus what economists usually call control variables, co-state variable and state variables are, mathematically speaking, all state variables. After all, one needs to know them all in order to fully characterise the state of the system. For example, in the simple differential equation  $\dot{x} = \rho x$   $x$  is the state variable whether it is a jumper or not. The trick of taking  $L$  as numeraire and  $L_1$  as a state variable opens the door to the intuitive simplification of working with the static economy representation of the dynamic path, i.e. focusing on the share of primary resources devoted to creation of new 'capital' and the creation of consumption goods. Anything that raises the share of  $L$  devoted to knowledge creation is pro-growth.

9. The only asset in this model is the ownership of a variety. The income stream, measured in terms of the numeraire, corresponds to the flow of operating profits. For example, the present value of a D-type's operating profit is discounted back to  $t=0$  at the rate of pure time preference and the stream of operating profit is falling at the rate that  $n$  rises, since higher  $n$  means lower sales and thus lower operating profit for an existing variety. In symbols,  $\int_0^{\infty} e^{-\rho t} \pi[a, n(t)] dt$  where  $\pi[a, n(t)]$  is the function that relates the firm's 'a' and the time-varying  $n$  to the level of operating profit. Using (3), this becomes:  $\int_0^{\infty} \left\{ e^{-(\rho+g)t} \left( \frac{a}{\bar{m}} \right)^{1-\sigma} \frac{E}{\sigma n_0} \right\} dt$ . Noting that  $\bar{m}$  is time-invariant, and  $n(t) = n_0 e^{gt}$  on the steady state growth path ( $n_0$  is the initial  $n$ ), we solve the integral to get:  $(a/\bar{m})^{1-\sigma} E/\sigma n_0 \gamma$ , where  $\gamma = \rho + g$ . In other words, discounting at the rate of  $\rho$  an income stream that falls at a rate of  $g$ , is like discounting a constant income stream at the rate of  $\rho + g$ .