Competition for Access and Full Revelation of Evidence

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Abstract

A decision maker must divide a prize between multiple agents. The prize may be divisible (e.g., a budget, pork-barrel spending) in which case he prefers to award larger shares of the prize to relatively more-qualified agents, or it may be non-divisible (e.g., jobs, college admissions) in which case he prefers to award the limited number of prizes to the most-qualified agents. He is, however, ex ante uncertain about agent qualifications. Agents may directly reveal verifiable evidence about their qualifications to the decision maker only if the decision maker grants them "access" (e.g., he takes time to review their applications, hold interviews, or conduct an investigation). The time-constrained decision maker must decide which agents receive access, as he cannot grant access to everyone.

One way to award access is through a competition, where agents submit payments (e.g., time, money) and higher payments correspond to a greater likelihood of receiving access. The analysis shows that there always exists competition for access mechanisms in which the decision maker becomes fully informed about the qualifications of *all* agents (even though only *some* agents reveal their qualifications through access). That is, the decision maker can award access in such a way that he always becomes fully informed and chooses his preferred prize allocation. The paper derives such full-revelation mechanisms, and determines when awarding access through a traditional all-pay auction is sufficient for full revelation. (*JEL* D44, D78, D82)

Keywords: Verifiable evidence disclosure, hard information, all-pay auction, handicap auction, access, lobbying

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1 Introduction

An administrator must decide how to allocate a budget across different departments or projects. A politician must decide which initiatives to devote time and energy, or which projects in his district to direct pork spending. An employer must decide which applicants to hire. A university must decide whom to grant admission or award scholarships. Each of these examples can be modeled as a game in which a decision maker is responsible for allocating a limited resource across multiple potential recipients. In the first two examples (and in the first part of the paper), the decision maker must split a divisible resource across multiple agents. In the second two examples (and in the later part of the paper), the decision maker must decide which agents receive a limited number of non-divisible prizes.

In each of the examples, the decision maker's ideal allocation likely depends on the merits of the potential recipients, although he may be uncertain about these merits. To learn about his ideal allocation, the decision maker may review applications, hold interviews, call references, or otherwise conduct an investigation. Due to the cost of these activities, however, he may not be able to directly learn the qualifications of all agents. To get noticed and increase the odds of the decision maker taking a closer look at their qualifications, agents may make monetary payments or undertake costly actions observed by the decision maker.[1]

This paper considers an allocation game in which individual agents either vie for a larger share of the total, or for one of a limited number of prizes. Agents have verifiable evidence regarding their own qualifications (e.g., their ability to use financing effectively, their skill level, their past achievements, their project's support amongst a politician's constituents), which they can disclose to the decision maker only if he grants them "access." The decision maker is time constrained and cannot grant access to all agents before he chooses an allocation.

Unlike other models of verifiable information disclosure, the decision maker determines which agents receive access. The analysis considers whether the way in which the decision maker awards access impacts his ability to learn and choose his preferred allocation. In doing so, we focus on a class of mechanisms in which the decision maker auctions off access to the agents. In these *competition for access* mechanisms, agents bid by providing payments (e.g., campaign contributions, effort) that are observed by the decision maker (e.g., politician, employer), where submitting a higher bid increases the expected likelihood of gaining access.

[1]Monetary payments may include political contributions or bribes. Costly actions may include putting extra effort into the development of application materials (e.g., writing a personalized cover letter, then printing it on high-quality paper), buying an expensive interview suit, spending extra hours in the office leading up to the award of bonuses, asking well-known colleagues to call on one's behalf, hounding a decision maker with phone calls and emails, or otherwise kissing up.

Remarkably, the analysis shows that there always exists mechanisms for awarding access under which the decision maker becomes fully informed. That is, the decision maker can award access in such a way that he *always* learns the evidence of *all* agents, even though he only gives access to *some* of the agents.

When agents only differ in terms of their qualifications, simply awarding access to the highest bidders results in full revelation. However, when agents differ in terms of their wealth, valuations, or other factors, it is no longer sufficient to award access to the highest bidders. Instead, full revelation requires that the auction mechanism adjust the bids to account for agent heterogeneity before awarding access. An auction mechanism that fully adjusts agent bids to account for heterogeneity (such that in equilibrium agents with the highest qualifications always win access) is a full revelation mechanism.[2]

In equilibrium, under a full revelation access mechanism, agents provide payments or bids according to strictly increasing bid functions. This means that for each agent there exists a one-to-one mapping between the agent's qualifications and its bid. The rational decision maker, upon observing an agent's payment, can correctly infer the agent's qualifications, even if the agent does not receive access. This means that in equilibrium the decision maker becomes fully informed about the qualifications of all agents, even though not all agents receive access. Access still plays an important role in determining equilibrium strategies, however, as it provides the needed disincentive to prevent agents from overbidding in an attempt to exaggerate their true qualifications. Without some amount of access in the model, there is a pooling equilibrium in which agent payments do not correspond to their qualifications. In the access mechanisms considered in this paper, an agent that increases its payment both signals higher qualifications, and increases the probability it wins access. When an agent wins access, it reveals its true qualifications and any money spent to exaggerate its qualifications is waisted. In equilibrium, the expected benefit of increasing one's bid to exaggerate qualifications is fully offset by the increased payment.

The paper begins with a model in which the decision maker must split a perfectly divisible prize between multiple agents. In this setting, it is natural to think of the decision maker as a politician, the agents as special interest groups, and their payments as political contributions or in-kind payments such as providing canvasing or advertisements during a campaign. The analysis suggests that providing access to special interests based on contributions (rather than randomly or based on some other factor) may result in a better-informed politician who more effectively allocates her time and effort across projects.[3] This result provides an

[2]Such a mechanism is not the unique full-revelation mechanism. This is considered in Section 4.3.

[3]There is substantial evidence that political contributions do influence a group's ability to gain access. See for example the empirical analyses by Langbein (1986), Ansolabehere et al. (2002), and Hall and Wayman (1990), as well as the surveys by Herndon (1982) and Makinson (2003).

example of how political contributions may play a positive role in the political process, and is largely in contrast to models of influence in which politicians sell policies rather than access (e.g., Grossman and Helpman 1994, Coate 2004a).[4]

The model assumes that agents have verifiable evidence regarding their qualifications.[5] This means, for example, that job applicants can provide information about their background and experience. Similarly, interest groups can provide information about the importance of their issues or proposed projects for a politician's constituents. To our knowledge, only two other papers, Austen-Smith (1998) and Cotton (2009), require one to gain access before presenting verifiable information.[6] Austen-Smith (1998) assumes that a decision maker sets a price for access, then awards an agent access if the agent pays the announced price. Although such a method for awarding access may increase the decision maker's information (i.e., he knows that any agent that doesn't pay for access must have low-enough quality evidence), it is not a full revelation mechanism as the decision maker remains less than fully informed about the qualifications of agents that do not buy access. Cotton (2009) presents a stylized model in which politicians can sell either policy favors or access through an all-pay auction with two agents and a one-dimensional policy space. Given the symmetry of the interest groups in that game, the all-pay auction for access is a full revelation mechanism. The present paper considers full revelation mechanisms in a general framework.

Although other papers consider the incentives agents have to disclose private information to a decision maker, these papers almost uniformly ignore the need to gain *access* to the decision maker, and the possibility that access is limited. It matters whether access is limited. Milgrom and Roberts (1986), Seidmann and Winter (1997), and Ben-Porath and Lipman (2009) establish conditions under which agents will disclose private information.[7] But, if the

[4]See also the policy-for-sale models by Grossman and Helpman (1996), and Prat (2002a,b). In such policy-for-sale models, politicians typically know which course of auction is best for their constituents, but choose to do something else in order to attract political contributions. Others including Bennedsen and Feldmann (2006) and Dahm and Porteiro (2008a,b) allow interest groups to influence policy through both the disclosure of evidence and the explicit exchange of contributions for favors. These papers do not consider limited access, and contributions are only provided in exchange for favors. We are not the first to suggest that political contributions may play a beneficial role. In Coate (2004b), for example, contributions fund informative advertising, which improves voter ability to choose the most qualified candidate.

[5]This is distinct from games of non-verifiable information, e.g., Crawford and Sobel (1982), Austen-Smith (1994).

[6]Levy and Razin (2009) consider a game in which interest groups compete to put items on a politician's agenda. Although their model does not include verifiable information disclosure, it does incorporate a similar notion of access as our paper.

[7]Ben-Porath and Lipman (2009) show that a decision maker can use a threat of taxes and transfers to make agents fully reveal their evidence in equilibrium. In their setting, in contrast to the current paper, the decision maker can listen to everyone, and he has the power to tax the agents. In our paper, the decision maker cannot listen to everyone, and any payments made by the agents are made voluntarily. See also Green and Laffont (1986), Lipman and Seppi (1995), and Bull and Watson (2004, 2007).

decision maker doesn't have time to meet with all of the agents (i.e., limited access), then he will not become fully informed, even if all agents are willing to share their information. An employer, for example, only has time to interview some applicants, even though all applicants may be willing to come in for an interview. Where other papers consider when agents *with* access disclose evidence, we ask what the decision maker can learn about the evidence of those *without* access. In particular, we show that the decision maker can always award a limited amount of access in such a way that he becomes fully informed about the evidence of everyone, even those without access.

Competition for access in this paper is a variation of a "handicap" all-pay auction.[8] In a handicap auction, the auction mechanism favors certain bidders who may not have to bid as much as others to win a prize (e.g., Feess et al. 2008). For example, a handicap auction may inflate the bids of poor agents to allow them to better compete against rich agents.[9] Eso and Szentes (2007) show that an auctioneer can use a handicap auction to maximize revenue in a game in which he can reveal information to the bidders about the value of the item.[10] The present paper assumes that bidders reveal information to the auctioneer (not the other way around), and the auctioneer's goal is to collect as much information as possible (rather than revenue maximization). We show how a handicap auction can also be used to maximize the revelation of information, not only revenue.

Section 2 describes the model for the case of a divisible prize, and walks through preliminary results that allow the remainder of the analysis to focus on the access game. Section 3 considers the access game in detail, and presents the main result of the paper. We show that a decision maker can always award access in such a way that he becomes fully informed about the qualifications of all agents, and we describe such a mechanism. Section 4 discusses the main result, including how access drives the result. Section 4.2 shows that a standard all pay auction is a full revelation mechanism when agents differ only in terms of their qualifications (a special case of the general game). Section 4.3 shows that the full revelation mechanism found earlier is not unique, and provides a more-general result describing a larger class of full revelation mechanisms. Section 5 shows that the results continue to hold when prizes are non-divisible. Section 6 concludes the paper.

[8]For models of the all pay auction, see Baye et al. (1993) and Che and Gale (1998).

[9]Think of a "head start" in a foot race, or a golf handicap, which allows golfers of different ability to compete on more equal terms.

[10]Others also consider games in which an auctioneer can affect bidder information (e.g., Bergemann and Pesendorfer 2007).

2 Preliminaries

2.1 Setting

An individual decision maker is responsible for splitting a prize between N independent agents. Denote an arbitrary agent by i, and a vector of all other agents besides i by the subscript $-i$. The total size of the prize equals 1, and the share assigned to agent $i = \{1, ..., N\}$ equals p_i, where $p = (p_1, ..., p_N)$ and $\sum_{i=1}^{N} p_i \leq 1$. (Section 5 allows for non-divisible prizes.) Agents differ in terms of their qualifications, where q_i denotes the qualifications of agent i, and $q = (q_1, ..., q_N)$. When the decision maker knows q, he can choose an allocation to maximize his payoff (these payoffs will be described below). However, he is ex ante uncertain about agent qualifications, and is therefore less than fully informed about q.

Before choosing an allocation p, the decision maker can grant access, giving agents the ability to verifiably communicate their own q_i. Due to time constraints, however, the decision maker is unable to award access to all agents. Formally, the politician may provide access to no more than K agents, where $K \in \{1, ..., N-1\}$. This paper is concerned with whether the decision maker's choice of *how* to award access affects his information and his ability to choose his preferred policy.[11] In particular, the analysis is concerned with a specific type of mechanism in which agents make payments (e.g., money, effort, time commitments) in *competition for access*. Let $t_i \geq 0$ denote any payment made by agent i, where $t = (t_1, ..., t_N)$ is observed by the decision maker.

Preferred Allocation and Payoffs–The decision maker earns utility $W(p; q)$ from his choice of p.[12] Let $p^*(q)$ define the decision maker's preferred allocation given q, where $p^* = (p_1^*, ..., p_N^*)$. Thus,

$$p^*(q) \equiv \arg\max_{p} W(p; q) \text{ s.t. } \sum_{i=1}^{N} p_i \leq 1.$$

All else equal, the decision maker prefers to award larger shares of the prize to more-qualified

[11] This is equivalent to a setting in which the decision maker has access to some verification service which reveals q. This verification service may be scarce, so he cannot verify all N qualifications. How does he decide which agent's qualifications to verify?

[12] The analysis assumes that the decision maker cares only about the allocation, and not about payments received from the agents. When this is the case, the main result in the paper, that the decision maker can award access in a way that he always becomes fully informed, makes the most sense because awarding access in such a way maximizes his utility. If the decision maker also cares about payments, there continues to exist a full revelation access mechanism. The decision maker can award access in a way that he becomes fully informed; however, he may not care to use such a mechanism, as he is not longer only concerned with allocation utility. He prefers the mechanism that optimally trades off expected allocation utility and payments.

agents. That is, W is such that $\frac{\partial p_i^*}{\partial q_i} > 0$ and $\frac{\partial p_i^*}{\partial q_j} < 0$ for all $j \neq i$ and any possible q. Furthermore, the analysis assumes that $p_i^*(q) > 0$ for all i and any possible q; this condition simplifies the analysis, but is not required for the main results of the paper to hold.[13] When the decision maker knows q, he sets $p = p^*$. However, he is ex ante uncertain about q, and is therefore uncertain about the identity of p^*.

Agents strictly prefer receiving higher allocations, and they find payments in competition for access costly. Therefore, agent i earns utility

$$U_i(p_i, t_i) = V_i(p_i) - c_i t_i,$$

where V_i is twice continuously differentiable, $V_i'(p_i) > 0$, and $c_i > 0$. An agent's utility does not directly depend on the payments made or allocation received by others. Furthermore, V_i is independent of q_i. By keeping agent valuations independent of their qualifications, the model is able to highlight how qualifications alone impact one's willingness to contribute. An agent's utility is linear in its payment t_i; assuming linearity allows for a closed-form solution for the equilibrium payment function.[14] The functions U_i are common knowledge.[15]

Qualifications and Evidence–The state of the world is given by vector q. Before the game, each agent observes its own q_i, but not the qualifications of the other agents. The decision maker does not observe q. Each agent's q_i is the independent realization of a random variable continuously distributed on R_+ according to distribution function F_i and density function f_i. The distributions are common knowledge.

Although the decision maker does not observe q ex ante, he can provide agents access before he chooses p. An agent with access can provide verifiable evidence to the decision maker regarding their own qualifications. Formally, the evidentiary structure meets the requirements of Lipman and Seppi (1995)'s full reports condition and Bull and Watson (2007)'s evidentiary normality condition, which require that higher-type agents can always provide evidence not available to lower-type agents. An agent can underrepresent but not

[13]This assumption holds when, for example, $\left.\frac{\partial W}{\partial p_i}\right|_{p_i=0} = \infty$. Various forms of W meet the requirements described in the paper, including $W(p;q) = \sum_{i=1}^{N} q_i \ln p_i$ and $W(p;q) = \prod_{i=1}^{N} p_i^{q_i}$.

[14]Alternatively, $-c_i t_i$ may be replaced by a function $C : \mathrm{R}_+ \to \mathrm{R}$. So long as C is strictly decreasing in t_i, agent equilibrium payment functions are strictly increasing in q_i. Therefore, the main result of the analysis holds and the decision maker learns the qualifications of all agents even when he only grants access to some of the agents.

[15]Most auction models assume that valuations or costs are unknown. This paper makes the alternative assumption that V_i and c_i are known, but that agents have private information about their own qualifications. This is consistent with an example in which a politician knows whether an interest group (e.g., the NRA or the Village of Fremont Community Center) is rich or poor and he knows how desirable the group would find a change in policy or increased funding, but the politician doesn't know whether the interest group can make a strong argument that the change in policy or increase in funding will benefit the politician's constituents as a whole.

exaggerate its evidence. If agent i has access to the decision maker, he can present any evidence amount $e_i \in [0, q_i]$, or he can refuse to present any evidence in which case $e_i = \emptyset$. Let $\omega = \{e_i\}_{\forall i \ni a_i = 1}$ denote the vector of evidence revealed by those with access.

Game Order–Given the access mechanism Γ, the agents and the decision maker participate in a one-shot game that takes place in the following order:

1. The decision maker awards access to up to K agents. (Section 3 considers this stage of the game in detail.)

2. Each of the agents that receive access simultaneously chooses evidence e_i to reveal.

3. The decision maker chooses allocation p.

States and Beliefs–The realized state of the world is defined by the vector of realized qualifications q. Let $S = \mathrm{R}_+^N$ denote the state space, and $\hat{q} \in S$ denote an arbitrary state in S. The function $\mu(\cdot \mid t, \omega)$ defines the decision maker's beliefs about the state of the world given payment vector t and the vector of evidence revealed through access ω. These beliefs may be fully represented by the vector of updated density functions $\left\{\hat{f}_i(\cdot \mid t, \omega)\right\}_{\forall i}$, where $\mu(\hat{q} \mid t, \omega) = \prod_i \hat{f}_i(\hat{q}_i \mid t, \omega)$.

Solution Concept–The analysis considers the pure-strategy Perfect Bayesian Equilibrium of the game. A formal description of the equilibrium must include strategy profiles for the decision maker and agents, as well as the decision maker's beliefs about the state of the world at the time he chooses p. The beliefs μ must be consistent with using Bayes' Rule on the ex ante distribution of evidence quality given the interest group strategies. Each player's strategy must be a best response to the strategies of the other players, given the player's beliefs.[16]

The analysis, however, is primarily concerned with the stage of the game in which the decision maker awards access. Therefore, when we formalize the equilibrium concept in Section 3, we treat the strategies and belief functions of the players at the later stages of the game (i.e., at the point of evidence disclosure and allocation decision) as fixed. This in no way weakens the analysis since, as Section 2.2 shows, the equilibrium strategies in these later stages are independent of the way in which the decision maker awards access.

2.2 Preliminary Analysis

The main focus of this paper is on the competition between agents for access to the decision maker. To better focus on the access mechanism, it is helpful to first consider the agents'

[16] For a detailed description of Perfect Bayesian Equilibrium belief requirements, see Fudenberg and Tirole (1991).

evidence revelation decision, and the decision maker's allocation decision and beliefs, before the paper analyzes the competition for access.

Allocation Choice–The decision maker chooses an allocation in the final stage of the game before payoffs are realized. The decision maker chooses p to maximize $E_\mu W(p,q) = \int_{\hat{q}\in S} \mu(\hat{q} \mid t,\omega) W(p,\hat{q}) d\hat{q}$, where the operator E_μ represents the decision maker's expectations given his beliefs μ at the time he chooses allocation p.

Given that the decision maker's payoffs do not directly depend on t or ω, the contributions and revealed evidence can only influence p by influencing the decision maker's beliefs μ. If the decision maker's beliefs place probability 1 on the true state of the world q, then he will choose the optimal allocation, p^*.

Agent Evidence Revelation and Principal Beliefs–Any agent with access reveals all of its evidence, $e_i = q_i$. This is a standard result in the hard evidence literature (e.g., Milgrom 1981, Bull and Watson 2004). If agents with access revealed $e_i < q_i$, then an agent could costlessly represent higher qualifications by revealing $e_i = q_i$ instead. Only when each i reveals $e_i = q_i$ do no agents have an incentive to deviate. Similarly, no agent with $q_i > 0$ will ever refuse to reveal evidence. If $e_i = \emptyset$, the decision maker updates his beliefs and puts probability 1 on $q_i = 0$; thus $\hat{f}_i(0 \mid \emptyset = e_i \in \omega) = 1$.[17]

In equilibrium, any agent i with access announces $e_i = q_i$. Therefore, when agent i reveals evidence e_i to the decision maker, the decision maker's beliefs μ must put probability 1 on $q_i = e_i$ and probability 0 on any state in which $q_i \neq e_i$. Thus, $\hat{f}_i(e_i \mid e_i \in \omega) = 1$.

3 The Access Game

The decision maker decides which agents receive access. The primary contribution of this paper is to establish that the decision maker can always award a limited amount of access in such a way that he becomes fully informed about the qualifications of all agents, even those who do not receive access.

The analysis focuses on a specific class of mechanisms for awarding access in which agents provide payments in *competition for access*. The decision maker assigns access according to a predefined competition for access mechanism $\Gamma = \{k, \theta\}$, which defines the number of agents to be awarded access $k \in \{0, 1, ..., K\}$, and a vector of functions $\theta = \{\theta_1, ..., \theta_N\}$. Function

[17] If instead the decision maker's beliefs were such that $E_\mu q_i > 0$ when $e_i = \emptyset$, then all agents with $q_i \leq E_\mu q_i$ (and no agents with $q_i > E_\mu q_i$) have an incentive to announce $e_i = \emptyset$. The decision maker recognizes this and his beliefs therefore must account for the types of agents that do announce $e_i = \emptyset$, which requires him to lower $E_\mu q_i$. Again however, only agents with actual qualifications lower than the updated expected qualifications refuse to reveal evidence. The reasoning repeats, and $E_\mu(q_i \mid e_i = \emptyset) \rightarrow 0$; only then do no agents have an incentive to deviate. See Milgrom (1981) for a formal proof.

$\theta_i : \mathrm{R}_+ \rightarrow \mathrm{R}_+$ maps agent i's payment t_i onto the real line. θ_i is a strictly-increasing, continuous function in t_i, where $\theta_i(0) = 0$ for each i. Agent i receives access if fewer than k other agents have $\theta_j(t_j)$ greater than $\theta_i(t_i)$. Tied agents receive access with equal probability.

When $\theta_i(t_i) = t_i$ for each i, access is awarded through a standard all-pay auction with the k highest bidders winning access. A more general function θ_i allows the access decision to take into account individual agent characteristics, including differences in V_i, F_i, c_i, and p_i^*.[18] The analysis considers a framework in which all agents pay their t_i, regardless of whether they win access.

In competition for access, agents simultaneously choose payments (e.g., money, effort), and the k agents with the highest $\theta_i(t_i)$ receive access. Let $T_i : \mathrm{R}_+ \rightarrow \mathrm{R}_+$ denote a payment strategy for agent i, where $T_i(q_i)$ is its payment when it has qualifications q_i. The vector of payment strategies for all agents $T = \{T_1, ..., T_N\}$ constitutes an "access equilibrium" of the game if no agent has an incentive to deviate from T_i given access mechanism Γ and the strategies of the other agents. This is formalized in the following definition, where $P_i(t)$ denotes the decision maker's equilibrium choice of p_i given the payments.

Definition 1 *Payment strategies $T = \{T_1, ..., T_N\}$ constitute an **access equilibrium** under mechanism Γ if for all $q \in S$, for each $i = 1, ..., N$,*

$$T_i(q_i) \in \arg\max_{t_i} \int_{\hat{q}_{-i}} \pi_i(\hat{q}_{-i}) U_i \left(P_i(t_i, \{T_j(\hat{q}_j)\}_{\forall j \neq i}), t_i \right) d\hat{q}_{-i},$$

where $\pi_i(\hat{q}_{-i}) \equiv \prod_{j \neq i} f_j(\hat{q}_j)$.

This defines the equilibrium of the competition for access taking as given the strategies and beliefs during the later stages of the game, as described in section 2.2.

3.1 Full Revelation Mechanisms

This section formalizes the concept of a full revelation mechanism, under which in equilibrium the decision maker becomes fully informed about the evidence of all agents, even though he can only give access to some of the agents. It then provides a sufficient condition for the equilibrium payment strategies in a competition for access game to result in full revelation of information.

Definition 2 *The decision maker is said to be **fully informed** if in equilibrium, his beliefs put probability 1 on the true state of the world at the time he chooses an allocation p.*

[18] That is, the paper allows for a "handicap auction" mechanism in which all payers' bids may not be treated equally. See for example Feess et al. (2008) and Eso and Szentes (2007).

Definition 3 *Mechanism Γ is said to be a* ***full revelation mechanism*** *if under Γ there exists an access equilibrium in which the decision maker is always fully informed.*

As section 2.2 shows, a fully informed decision maker always implements his first best allocation p^*. Therefore, a full revelation mechanism results in the decision maker's preferred allocation in equilibrium.

3.2 Main Result

The main result establishes that there always exists a competition for access mechanism that results in a fully informed decision maker. The analysis holds for any $k \in \{1, ..., K\}$; therefore, such a full revelation mechanism exists even when the decision maker can give access to a very small number of agents, including $K = 1$.

Proposition 1 *For each $k \in \{1, ..., K\}$, there always exists a competition for access mechanism $\hat{\Gamma} = \{k, \hat{\theta}\}$ with access equilibrium $\hat{T} = \{\hat{T}_1, ..., \hat{T}_N\}$, where for each agent, $\hat{T}_i(q_i) = \hat{\theta}_i^{-1}(t_i)$. Such a mechanism is a full revelation mechanism.*

In addition to establishing that a full revelation mechanism always exists, the proposition also establishes that in equilibrium, $\hat{T}_i(q_i) = \hat{\theta}_i^{-1}(t_i)$. Letting $\hat{Q}_i(t_i) \equiv \hat{T}_i^{-1}(q_i)$, this condition means $\hat{\theta}_i = \hat{Q}_i$.[19] That is, the k agents who's payments signal the highest qualifications receive access, and in equilibrium, access goes to the most qualified agents. This does not mean that the most qualified agents necessarily submit the highest payments to the decision maker in the competition for access game. Rather, it means that allocation of access takes into account differences in agent characteristics (e.g., valuation, wealth, evidence distribution) such that when bidding for access no agent is at an advantage or disadvantage compared to other agents with similar qualifications.

The proof to Proposition 1 in the appendix derives $\hat{\theta}$ and $\hat{T}$. In the following equation for $\hat{T}$, the function $\Omega_{ik}(q_i, q_{-i}) = 1$ if fewer than k other agents have qualifications greater than q_i; otherwise, $\Omega_{ik}(q_i, q_{-i}) = 0$.

$$\hat{T}_i(q_i) = \frac{1}{c_i} \int_0^{q_i} \int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \left[(1 - \Omega_{ik}(y; \hat{q}_{-i})) \times V_i'\left(p_i^*\left(y, \hat{q}_{-i}\right)\right) \times \frac{\partial p_i^*\left(y, \hat{q}_{-i}\right)}{\partial q_i} \right] d\hat{q}_{-i} dy. \quad (1)$$

For each i, $\hat{\theta}_i(t_i) = \hat{T}_i^{-1}(q_i)$. Note that $\hat{T}_i$ is invertible since it is strictly increasing in q_i. The derivative of $\hat{T}_i$ with respect to q_i is

$$\hat{T}_i'(q_i) = \frac{1}{c_i} \int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \left[(1 - \Omega_{ik}(q_i; \hat{q}_{-i})) \times V_i'\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) \times \frac{\partial p_i^*\left(q_i, \hat{q}_{-i}\right)}{\partial q_i} \right] d\hat{q}_{-i}, \quad (2)$$

[19] In other words, $\hat{Q}_i(t_i)$ is the qualifications of agent i if he gives t_i in equilibrium.

which is strictly positive.[20] If agent i receives access under $\hat{\Gamma}$, the decision maker observes q_i directly. If i does not receive access, the decision maker believes that $q_i = \hat{Q}_i(t_i)$, as is consistent with the Perfect Bayesian Equilibrium of the game.

4 Discussion of Main Result

In equilibrium, independent of other heterogeneity, the k agents with the highest qualifications receive access and the decision maker chooses his fully informed allocation. Since a relatively rich agent (e.g., one with a low c_i) finds any payment less costly, its equilibrium payment function is steeper than an otherwise similar poor agent. In equilibrium, a rich agent contributes more than an otherwise similar poor agent with the same qualifications, but the rich agent is not more likely to receive access since the access mechanism accounts for its higher ability to pay. Similarly, in equilibrium, differences in agent valuations (i.e., V_i), qualification distributions (i.e., F_i), or politician biases (i.e., p_i^*) do not give agents an advantage or disadvantage when trying to secure access, as $\hat{\Gamma}$ accounts for this heterogeneity.

4.1 Importance of Access

An agent can "exaggerate" its qualifications by paying $t_i > \hat{T}_i(q_i)$. However, doing so also increases the probability that the agent receives access and its true qualifications are discovered directly. In equilibrium, the slope of an agent's payment function must be steep enough such that any expected benefit from exaggerating its evidence is offset by the monetary costs of doing so. At the same time, the slope must be moderate enough that the agent doesn't want to pay less than its equilibrium payment. This implies Eq. 2, which is formally derived in proof to Proposition 1 in the appendix.

The main result requires that the decision maker award access to at least one agent; that is, it only holds for $k \in \{1, ..., K\}$. It is the possibility of winning access that keeps an agent from paying more than $\hat{T}_i(q_i)$. Without this disincentive to overpay the strict monotonicity of the equilibrium payment functions breaks down.

4.2 Homogeneous Agents

Section 3 solves for a full revelation mechanism for a model with potentially heterogeneous agents. This section considers the special case in which agents are completely homogeneous,

[20] It must be the case that $\Omega_{ik} = 0$ for some agents and $\Omega_{ik} = 1$ for others. This is guaranteed by $k \in \{1, ..., K\}$.

only differing in terms of their qualifications. In this game, c_i, V_i, p_i^*, and F_i are identical for all agents.

When agents are homogeneous, the full revelation mechanism described in Proposition 1 simplifies to a standard all-pay auction in which the agents that submit the highest payments receive access. Therefore, the decision maker becomes fully informed about agent qualifications if he awards access through a standard all-pay auction in which the k highest bidders receive access (i.e., an all-pay auction without handicaps).

Proposition 2 *In the game with homogeneous agents, f or each $k \in \{1, ..., K\}$, awarding access to the k agents that provide the highest payments in a standard all-pay auction constitutes a full revelation mechanism.*

4.3 Non-Uniqueness and a More General Result

Section 3.2 describes a specific competition for access mechanism that always results in full revelation of agent qualifications. Although the described full revelation mechanism is intuitively appealing (e.g., it always awards access to the most qualified agents), it is not unique.

Lemma 3 provides a sufficient condition for a mechanism to be a full revelation mechanism.

Lemma 3 *Suppose the decision maker awards access through Γ. If there exists an access equilibrium such that for each i the equilibrium payment function T_i is strictly increasing in q_i for all $q_i \in \mathrm{R}_+$, then Γ is a full-revelation mechanism.*

When all agents make payments according to strictly increasing functions, there exists a one-to-one mapping between each agent's qualifications and its payment. Therefore, the rational decision maker can correctly infer an agent's qualifications whenever he observes the agent's payment.

The equilibrium payment functions in Section 3.2 meet this requirement, but they are not the only set of functions that does so. Proposition 4 provides a more general result.

Proposition 4 *Let Z denote any vector of functions $\{Z_1, ..., Z_N\}$ where for each i, $Z_i : \mathrm{R}_+ \to \mathrm{R}_+$, $Z_i(0) = 0$, $Z_i'(x) > 0$ for all $x \in \mathrm{R}_+$, and there exists at least k other $j \neq i$ such that $\lim_{x\to\infty} Z_i(x) \geq \lim_{x\to\infty} Z_j(x)$. Then, there exists a competition for access mechanism $\tilde{\Gamma} = \{k, \tilde{\theta}\}$ with access equilibrium $\tilde{T} = \{\tilde{T}_1, ..., \tilde{T}_N\}$ such that for each i and all $q_i \in \mathrm{R}_+$, $\tilde{\theta}_i(\tilde{T}_i(q_i)) = Z_i(q_i)$, and mechanism $\tilde{\Gamma}$ is a full revelation mechanism.*

The function Z_i describes the relationship between agent i's qualifications and the probability that it receives access in equilibrium. The conditions imposed on Z by Proposition 4 imply that an agent's probability of receiving access is strictly increasing in its qualifications, an agent with the lowest possible qualifications never receives access,[21] and for any $q_i > 0$ the agent wins access with probability $\tilde{\theta}_i(q_i) \in (0, 1)$. The requirement that for each agent there must exist at least k other $j \neq i$ such that $\lim_{x\to\infty} Z_i(x) \geq \lim_{x\to\infty} Z_j(x)$ results in separating equilibrium over all possible q.[22]

5 Non-Divisible Prizes

This section shows that the main result of the paper continues to hold when prizes are non-divisible. Here, the decision maker must award $m \in \{1, ..., K\}$ identical prizes to agents. An agent may either receive one prize, or not win a prize; agents may not receive multiple prizes or fractions of prizes. The decision maker's utility is such that he strictly prefers to award a prize to each of the m most-qualified agents. The concept of qualifications, their distribution, and the evidentiary structure is unchanged from the earlier analysis. Let $v_i > 0$ denote the value of receiving a prize to agent i, where $v = \{v_1, ..., v_N\}$ is common knowledge.

In this alternative framework, the decision maker may still award access through a full revelation competition for access mechanism, similar to the one described by Proposition 1.

Proposition 5 *When the decision maker must award $m \in \{1, ..., K\}$ non-divisible prizes, there always exists a competition for access mechanism $\breve{\Gamma} = \{k, \breve{\theta}\}$ with access equilibrium $\breve{T} = \{\breve{T}_1, ..., \breve{T}_N\}$, where $k = m$ and for each agent, $\breve{T}_i(q_i) = \breve{\theta}_i^{-1}(t_i)$. Such a mechanism is a full revelation mechanism.*

As in the earlier analysis, when the decision maker awards access according to a full revelation mechanism, he becomes fully informed about the qualifications of all agents. In equilibrium, he gives prizes to the m most-qualified agents.

The proof to Proposition 5 in the appendix derives the $\breve{\theta}$ and $\breve{T}$ that satisfy Proposition 5. In the following equation for $\breve{T}_i$, the function $\Phi_{i,k}(q_i)$ denotes the ex ante probability that fewer than k other agents have qualifications greater than q_i. Given the characteristics of the qualification distributions $F = \{F_1, ..., F_N\}$, the function $\Phi_{i,k}$ is strictly increasing in q_i,

[21] Technically, an agent with the lowest qualifications may receive access if enough other agents also have the lowest qualifications. This happens with probability 0.

[22] If this condition is not met for agent i, then there exists a cut point for q_i such that any q_i greater than the cut point results in the agent winning access for sure, and there is pooling amongst any agent-type with q_i greater than the cut point.

and differentiable.

$$\breve{T}_i(q_i) = \frac{1}{c_i}\Phi_{i,k}(q_i)v_i. \tag{3}$$

For each i, $\breve{\theta}(t_i) = \breve{T}_i^{-1}(q_i)$. Note that $\breve{T}_i$ is invertible since it is strictly increasing in q_i. The derivative of $\breve{T}_i$ with respect to q_i is

$$\breve{T}_i'(q_i) = \frac{1}{c_i}\Phi_{i,k}'(q_i)v_i, \tag{4}$$

which is strictly positive. Although the analysis behind Eq. 3 and 4 is similar to the case when prizes are divisible, the intuition differs. When prizes were divisible, the slope of the payment function (Eq. 2) reflects the disincentive necessary to prevent agents from over representing their evidence. When prizes are non-divisible, and the decision maker awards access through the mechanism described in Proposition 5, agents would never overrepresent their evidence. This is because in equilibrium an agent always receives access and discloses its true qualifications before receiving a prize. Exaggerating one's evidence increases the agent's payment without increasing the likelihood it receives a prize. Here the slope of $\breve{T}_i$ is just steep enough that agents never want to *underrepresent* their qualifications.

6 Conclusion

The analysis shows that the way in which a decision maker allocates access can affect his information. If he awards access through a full revelation competition for access mechanism, he becomes fully informed about the qualifications of all agents, even though he only gives access to some of them. Such mechanisms always exist, including one in which the decision maker awards access to the agents whose contributions signal the highest qualifications. The analysis holds for various prize structures, including when the decision maker splits a divisible resource (e.g., budget) between agents, and when the decision maker chooses which agents receive one of a limited number of prizes (e.g., jobs).

This paper contributes to the literature in at least three important ways. First and foremost, it extends the literature on verifiable evidence disclosure to allow for limited access. We show that even when the decision maker cannot give access to all privately informed agents, he can still become fully informed about agent evidence if agents compete for access. Second, it contributes to the literature on auctions and competitions, as the competition for access mechanisms studied in this paper are a variation of a handicap all-pay auction. Third, the paper makes a contribution to the applied fields addressed by the model. For example, the results provide an insight into political lobbying. They suggest that campaign contributions may play a beneficial role in a decision making process. By observing a special interest group's willingness to pay for access, a politician can become better informed about the merits of the group's position. Similarly, the results provide insights into the labor market where job applicants put in costly effort in an effort to get noticed and get invited for an interview.

Although the framework is already quite general, there is room to make it even more so. For example, the analysis focuses on evidentiary structures that meet the requirements of Lipman and Seppi (1995)'s full reports condition and Bull and Watson (2007)'s evidentiary normality condition, which require that higher-type agents can always provide evidence not available to lower-type agents. This ensures that an agent can underrepresent but not exaggerate its evidence, which is reasonable in many situations. It would be interesting, however, to consider how the full disclosure results in this paper hold up under more general evidentiary structures. Similarly, the model assumes that only agent qualifications are unobserved; all other characteristics, including valuations and cost parameters, are common knowledge. This assumption makes the analysis tractable, and may be reasonable for the case when agents are well established special interest groups. However, it is less realistic when agents are job applicants, for example. It would be interesting to consider how well the results hold up when there are multiple dimensions of uncertainty. We expect that both more general evidentiary structures and multidimensional uncertainty would mean the decision maker cannot become fully informed about agents who do not receive access, although competition for access should still help the decision maker become *better informed* compared to if he assigned access randomly or based on some ex ante observed agent characteristic. Its less clear if the competition for access mechanisms found here would continue to be optimal from the perspective of the decision maker. These are questions for future research.

7 Appendix

7.1 Proofs

We begin with a proof to Lemma 3, despite its location near the end of the paper. The result is used in the proof to Proposition 1.

Proof of Lemma 3. Given that T_i is strictly monotonic, there exists a one-to-one mapping between agent i's qualifications q_i and its contribution $t_i = T_i(q_i)$. Furthermore, it implies that T_i is invertible; let $Q_i(t_i) \equiv T_i^{-1}(q_i)$. The rational decision maker, upon seeing the agent's contribution will correctly infer that $q_i = Q_i(t_i)$. This is true for all i, and in equilibrium the decision maker is fully informed. ■

Proof of Proposition 1. In this proof, we walk through the derivation of the vector of strictly increasing functions $\hat{T}$ such that if the decision maker awards access according to mechanism $\hat{\Gamma} = \{k, \hat{\theta}\}$ (where $\hat{\theta}_i^{-1}(t_i) = \hat{T}_i(q_i)$) then for each i, making payments according to $\hat{T}_i$ is a best response when all other agents play according to $\hat{T}_{-i}$. Therefore, $\hat{T}$ constitutes an access equilibrium of mechanism $\hat{\Gamma}$. The derivation of $\hat{T}$ relies only on the initial conditions of the model, and such a $\hat{T}$ and $\hat{\Gamma}$ will always exist. Furthermore, the solution for $\hat{T}$ meets the requirements of Lemma 3, and therefore $\hat{\Gamma}$ is a full revelation mechanism.

Now, for the derivation of $\hat{T}$ and $\hat{\Gamma}$. Let $\hat{T} = \{\hat{T}_1, ..., \hat{T}_N\}$ denote a set of payment functions, where for each i, the function $\hat{T}_i$ is differentiable and strictly increasing in q_i. (Later, we show that such conditions are met in equilibrium.) Since $\hat{T}_i$ is strictly increasing, it is invertible. Define $\hat{Q}_i \equiv \hat{T}_i^{-1}$, and let $\hat{Q} = \{\hat{Q}_1, ..., \hat{Q}_N\}$. If an agent contributes according to $\hat{T}_i$, then the agent's qualifications are given by $\hat{Q}_i(t_i)$. Suppose that $\hat{T}$ is the access equilibrium of a competition for access mechanism $\hat{\Gamma} = \{k, \hat{\theta}\}$ that always awards access to the most qualified agents. Since $\hat{\Gamma}$ always awards access to the agents with the highest q_i, then it must be that for all i and j, $\hat{\theta}_i(\hat{T}_i(q_i)) > \hat{\theta}_j(\hat{T}_j(q_j))$ if and only if $q_i > q_j$. This will be the case if, for all agents, $\hat{\theta}_i(t_i) = \hat{Q}_i(t_i)$. When $\hat{\theta}_i(t_i) = \hat{Q}_i(t_i)$ for all i, the k agents that signal the highest qualifications receive access, and in equilibrium these k agents with the highest signals are the most qualified agents. Below, the analysis solves for $\hat{T}$ such that these conditions are satisfied.

Since $\hat{T}$ satisfies the requirements of Lemma 3, $\hat{\Gamma}$ is a full revelation mechanism. The decision maker rationally puts probability 1 on $q_i = \hat{Q}_i(t_i)$ when agent i does not receive access. If agent i does receive access, the decision maker learns q_i directly. Let $\Omega_{ik}\left(\theta_i(t_i); \{\theta_j(t_j)\}_{\forall j \neq i}\right) \in \{0, 1\}$ indicate whether fewer than k other agents have $\theta_j(t_j) > \theta_i(t_i)$. Therefore, $\Omega_{ik}\left(\theta_i(t_i); \{\theta_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i}\right) = 1$ if agent i receives access in state $\hat{q}$ given i's own payment t_i.[23]

To derive the equilibrium payment function $\hat{T}_i$, the analysis considers the payment of agent i taking as given that all other agents make payments according to $\hat{T}$. Agent i chooses t_i to maximizes his expected utility:

$$\begin{aligned} \int_{\hat{q}_{-i}} \pi_i(\hat{q}_{-i}) \Big[&\Omega_{ik}(\hat{Q}_i(t_i); \{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i}) \times V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) \\ &+ \left(1 - \Omega_{ik}(\hat{Q}_i(t_i); \{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j \neq i})\right) \times V_i(p_i^*(\hat{Q}_i(t_i), \hat{q}_{-i}))\Big] d\hat{q}_{-i} - c_i\ t_i. \end{aligned} \tag{5}$$

Since the decision maker expects that he is fully informed, he chooses policy according to $p^*(q)$; for each i plugging in $q_i = \hat{Q}_i(t_i)$ when he only observes agent i's payment (i.e., when i does not receive access), and plugging in $q_i = q_i$ when i has access. When $\Omega_{ik} = 1$ the agent receives access, and the decision maker awards i allocation $p_i^*\left(q_i, q_{-i}\right)$ based on his true qualifications, as revealed through the presented evidence. When $\Omega_{ik} = 0$ the agent does not receive access, and the decision maker awards i allocation $p_i^*(\hat{Q}_i(t_i), q_{-i})$ based on the equilibrium qualifications that correspond to payment t_i.

[23]It is straightforward, but unnecessary for the analysis to derive Ω_{ik} from the qualification distributions $\{F_j\}_{j=1}^N$. We leave that exercise to the reader.

First order conditions for the agent's problem are given by

$$\begin{aligned}\int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \Bigg[\left(1-\Omega_{ik}(\hat{Q}_i(t_i);\{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j\neq i})\right) \times V_i'\left(p_i^*\left(\hat{Q}_i(t_i),\hat{q}_{-i}\right)\right) \times \frac{\partial p_i^*(q_i,\hat{q}_{-i})}{\partial q_i}\Big|_{q_i=\hat{Q}_i(t_i)} \times \frac{\partial \hat{Q}_i(t_i)}{\partial t_i} \\ + \frac{\partial \Omega_{ik}(q_i;\{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j\neq i})}{\partial q_i}\Big|_{q_i=\hat{Q}_i(t_i)} \times \frac{\partial \hat{Q}_i(t_i)}{\partial t_i} \times \left[V_i\left(p_i^*\left(q_i,\hat{q}_{-i}\right)\right) - V_i(p_i^*(\hat{Q}_i(t_i),\hat{q}_{-i}))\right]\Bigg] d\hat{q}_{-i} - c_i = 0.\end{aligned} \tag{6}$$

The first row of notation in the above condition represents the marginal impact that a change in t_i has on the decision maker's beliefs about q_i and the resulting change in agent policy utility when i does *not* receive access. The second row represents the marginal impact of a change in t_i on on the probability the agent wins access.

The strict monotonicity of $\hat{T}_i$ means that in equilibrium, $1/\hat{Q}_i'(t_i) = \hat{T}_i'(q_i)$. Also in equilibrium all agents including i contribute according to their equilibrium payment functions, therefore $\hat{Q}_i(t_i) = q_i$ and $\hat{Q}_i(\hat{T}_i(q_i)) = q_i$. The first order conditions simplify to Eq. 2 in Section 3.2. The initial conditions regarding V_i and W imply that $\hat{T}_i'(q_i) > 0$ for all $q_i \in \mathrm{R}_+$. Integrating with respect to q_i gives the closed-form solution for the equilibrium payment function, which is given by Eq. 1.

It is possible to verify the concavity of the agents' maximization problem, given the competition for access mechanism $\hat{\Gamma}$. To do so, plug in $1/\hat{T}_i'(q_i)$ for the first occurrence of $\hat{Q}_i'(t_i)$ in Eq. 6. Then substitute in Eq. 2 for $\hat{T}_i'(q_i)$. Simplifying the expression gives revised first derivative

$$c_i \int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right) \times \frac{\partial \Omega_{ik}(q_i;\{\hat{Q}_j(\hat{T}_j(\hat{q}_j))\}_{\forall j\neq i})}{\partial q_i}\Bigg|_{q_i=\hat{Q}_i(t_i)} \times \frac{\partial \hat{Q}_i(t_i)}{\partial t_i} \times \left[V_i\left(p_i^*\left(q_i,\hat{q}_{-i}\right)\right) - V_i(p_i^*(\hat{Q}_i(t_i),\hat{q}_{-i}))\right] d\hat{q}_{-i}. \tag{7}$$

It should be clear that Eq. 7 is positive iff $V_i\left(p_i^*\left(q_i,\hat{q}_{-i}\right)\right) - V_i(p_i^*(\hat{Q}_i(t_i),\hat{q}_{-i})) > 0$, which is true iff $\hat{Q}_i(t_i) < q_i$ or equivalently $t_i < \hat{T}_i(q_i)$ (which includes the possible case where $t_i = 0$). It is negative iff $t_i > \hat{T}_i(q_i)$. Thus, under mechanism $\hat{\Gamma}$, the agent maximization problem is strictly concave, and achieves its maximum at $t_i = \hat{T}_i(q_i)$.

When an agent increases his payment, doing so signals higher qualifications, but it also increases the probability that the agent wins access (in which case, the decision maker ignores the signal and depends instead on the evidence revealed through access). When the equilibrium condition given by expression 2 holds, the monetary costs of "exagerating"' one's evidence (i.e., signaling higher qualifications than one actually has) is greater than the expected benefit of doing so. Therefore, an agent does not have an incentive to contribute more than $\hat{T}_i(q_i)$. Similarly, he also prefers to contribute $\hat{T}_i(q_i)$ to any lower t_i. This follows because agent expected utility is strictly increasing in all $t_i < \hat{T}_i(q_i)$. This also rules out the possibility that an agent prefers not to participate. Not participating is equivalent to setting $t_i = 0$, which results in $p_i = 0$ for sure. When each agent i's payment strategy $\hat{T}_i$ is given by equation 1, no agent has an incentive to deviate from playing their respective strategy. The set of payment strategies $\hat{T}$ for all agents constitutes an access equilibrium under competition for access mechanism $\hat{\Gamma} = \{k, \hat{Q}\}$. ■

Proof of Proposition 2. Consider equations 1 and 2. When agents are homogeneous, the right hand sides of both expressions are independent of an agent's identity. (Note that if F_i is the same for all agents, then π_i and Ω_{ik} will be as well.) Therefore, the equilibrium payment functions T

will also be identical across all agents. Since T is identical for all agents, the one-to-one mapping between agent qualifications and payments is also the same across agents. This means that in equilibrium agent i submits a higher payment than agent j if and only if $q_i > q_j$. Lemma 3 implies that the all-pay auction mechanism is a full revelation mechanism in this setting. ■

Proof of Proposition 4. The proof to Proposition 4 follows the same method as the proof to Proposition 1. Therefore, we do not provide as many details or discussion here as we do in the earlier proof. Here, we assume an arbitrary vector of function Z that meet the conditions in Proposition 1, then using the same method as in the proof to Proposition 1, we solve for $\tilde{\theta}$ and $\tilde{T}$. Such $\tilde{\theta}$ and $\tilde{T}$ may always be found (and therefore they always exist). It is straightforward to show that for each i, $\tilde{T}_i$ is strictly increasing in q_i; therefore the conditions of Lemma 3 are met and $\tilde{\Gamma} = \{k, \tilde{\theta}\}$ is a full revelation mechanism.

To derive the equilibrium payment function $\tilde{T}_i$, the analysis considers the payment of agent i taking as given that all other agents make payments according to $\tilde{T}_{-i}$. Agent i chooses t_i to maximizes his expected utility:

$$\begin{aligned}\int_{\hat{q}_{-i}} \pi_i(\hat{q}_{-i})\Big[&\Omega_{ik}(\tilde{\theta}_i(t_i); \{Z_j(\hat{q}_j)\}_{\forall j\neq i}) \times V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) \\ &+ \Big(1 - \Omega_{ik}(\tilde{\theta}_i(t_i); \{Z_j(\hat{q}_j)\}_{\forall j\neq i})\Big) \times V_i(p_i^*(\tilde{Q}_i(t_i), \hat{q}_{-i}))\Big] d\hat{q}_{-i} - c_i\, t_i.\end{aligned}$$

This is a more general version of Eq. 5 in the earlier proof. First order conditions for the agent's problem are given by

$$\begin{aligned}\int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right)\Bigg[&\Big(1 - \Omega_{ik}(\tilde{\theta}_i(t_i); \{Z_j(\hat{q}_j)\}_{\forall j\neq i})\Big) \times V_i'\left(p_i^*(\tilde{Q}_i(t_i), \hat{q}_{-i})\right) \times \frac{\partial p_i^*(q_i,\hat{q}_{-i})}{\partial q_i}\Big|_{q_i=\tilde{Q}_i(t_i)} \times \frac{\partial \tilde{Q}_i(t_i)}{\partial t_i} \\ &+ \frac{\partial \Omega_{ik}(q_i; \{Z_j(\hat{q}_j)\}_{\forall j\neq i})}{\partial q_i}\Big|_{q_i=\tilde{\theta}_i(t_i)} \times \frac{\partial \tilde{Q}_i(t_i)}{\partial t_i} \times \Big[V_i\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) - V_i(p_i^*(\tilde{Q}_i(t_i), \hat{q}_{-i}))\Big]\Bigg] d\hat{q}_{-i} - c_i = 0.\end{aligned}$$

The strict monotonicity of $\tilde{T}_i$ means that in equilibrium, $1/\tilde{Q}_i'(t_i) = \tilde{T}_i'(q_i)$. Also in equilibrium all agents including i contribute according to their equilibrium payment functions, therefore $\tilde{Q}_i(t_i) = q_i$. Additionally, $\tilde{\theta}_i(\hat{T}_i(q_i)) = Z_i(q_i)$. The first order conditions simplify to

$$\tilde{T}_i'(q_i) = \frac{1}{c_i}\int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right)\left[\left(1 - \Omega_{ik}(Z_i(q_i); \{Z_j(\hat{q}_j)\}_{\forall j\neq i})\right) \times V_i'\left(p_i^*\left(q_i, \hat{q}_{-i}\right)\right) \times \frac{\partial p_i^*\left(q_i, \hat{q}_{-i}\right)}{\partial q_i}\right] d\hat{q}_{-i}.$$

Given the conditions imposed by the proposition on Z, this expression is strictly positive. Therefore, the requirements of Lemma 3 are met, and $\tilde{\Gamma}$ is a full revelation mechanism. It is straightforward to integrate with respect to q_i in order to solve for $\tilde{T}_i$.

$$\tilde{T}_i(q_i) = \frac{1}{c_i}\int_0^{q_i}\int_{\hat{q}_{-i}} \pi_i\left(\hat{q}_{-i}\right)\left[\left(1 - \Omega_{ik}(y; \hat{q}_{-i})\right) \times V_i'\left(p_i^*\left(Z_i(y), \{Z_j(\hat{q}_j)\}_{\forall j\neq i}\right)\right) \times \frac{\partial p_i^*\left(q_i, \hat{q}_{-i}\right)}{\partial q_i}\Big|_{q_i=y}\right] d\hat{q}_{-i} dy.$$

■

Proof of Proposition 5. The proof to Proposition 5 follows the same method as the proof to Proposition 1. Therefore, we do not provide as many details or discussion here as we do in the earlier proof. Consider the expected payoff function for agent i:

$$EU_i = \begin{cases} \Phi_{i,k}(q_i)v_i - c_i t_i & \text{if } \breve{Q}_i(t_i) \geq q_i \\ \Phi_{i,k}(\breve{Q}_i(t_i))v_i - c_i t_i & \text{if } \breve{Q}_i(t_i) \leq q_i. \end{cases}$$

If agent i over represents its qualifications, it increases the probability it receives access, but does not increase the probability it receives a prize. This is because when an agent receives access it discloses its evidence, and the decision maker will award it the prize only if fewer than k other agents disclosed or signaled higher qualifications than q_i. If agent i under represents its qualifications, it only wins access and can disclose its true qualifications if fewer than k other agents have actual qualifications above $\breve{Q}_i(t_i)$. If it doesn't win access, it will not win a prize. If it does win access, it will win a prize (i.e., if $Q_i(t_i) < q_i$ is one of the k highest qualifications, so will be q_i).

It should be clear from the expected payoff function that an agent will never pay $t_i > \breve{T}_i(q_i)$. The analysis can therefore solve for the equilibrium payment function by focusing on the case when $\breve{Q}_i(t_i) \leq q_i$. First order conditions are $\Phi'_{i,k}(\breve{Q}_i(t_i))\breve{Q}'_i(t_i)v_i - c_i = 0$. In equilibrium, $\breve{Q}_i(t_i) = q_i$, and given the strict monotonicity of $\breve{T}_i$, $\breve{Q}'_i(t_i) = 1/\breve{T}'_i(q_i)$. Therefore, the first order conditions may be rewritten as Eq. 4, which gives the slope of the equilibrium payment function. Inverting Eq. 4 with respect to q_i gives Eq. 3, the equilibrium payment function. Note that the slope of the payment function is strictly increasing in q_i, satisfying the requirement that for all agents, $\breve{T}_i$ is strictly increasing in q_i. ■

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