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A strategic mediator who is biased into the same direction as the expert can improve information transmission

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A strategic mediator who is biased into the same direction as the expert can improve information transmission⁰

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Abstract

This note reconsiders communication between an informed expert and an uninformed decision maker with a strategic mediator in a discrete Crawford and Sobel (1982) setting. We show that a strategic mediator may improve communication even when he is biased into the same direction as the expert. The mediator improves communication, however, only if some information transmission is possible with unmediated communication.

JEL classifications: C72, D82, D83

Keywords: Communication, Information, Cheap talk, Mediation

1 Introduction

Recent research has shown that a strategic mediator can improve upon communication in a Crawford Sobel (1982) setting, if his and the expert's bias point into opposite directions. Ivanov (2010) demonstrates that communication between a positively biased expert and the decision maker can be

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improved by a strategic mediator with negative bias. In fact, a strategic mediator with a specific negative bias can achieve as much information transmission in equilibrium as the optimal non-strategic mediator characterized by Goltsman et al. (2009). Ivanov (2010) also shows that, when the mediator is either unbiased or biased into the same direction as the expert, but to a lesser degree, the mediator cannot improve upon direct communication. This leaves open the question, however, whether a mediator with a stronger bias in the same direction as the expert may improve communication.

We show that, even when the biases point into the *same* direction, a strategic mediator can improve communication. We give an example that is kept as simple as possible to make this point. The mediator's bias is his private information and may either be zero (so that the mediator is unbiased) or greater than the bias of the expert. This introduces an uncertainty on the decision makers side about the expert's message, even when all players play pure strategies.² As shown by Ambrus, Azevedo, and Kamada (2010), some uncertainty about the expert's message is needed to improve information transmission. Whereas in Ivanov (2010) and in Ambrus, Azevedo, and Kamada (2010) the uncertainty arises from a mixed equilibrium strategy of the mediator, in our setup it arises from the decision maker's uncertainty about the mediator's type. This allows us to focus on pure strategies and simplifies the equilibrium analysis. We find that the mediator *can* improve communication, but interestingly *only if* some information transmission is possible under direct communication.³

¹Closely related are two recent papers. Ambrus, Azevedo, and Kamada (2010) consider a chain of strategic mediators, and give a sufficient condition for when a mediator improves communication in a fairly general setup. Ambrus, Azevedo, Kamada, and Katagi (2010) consider the open versus the closed rule in congressional decision making. They argue that, if communication between a legislature and a biased lobbyist is mediated by a committee, and the open rule is used, then the legislature will prefer a committee with a bias opposite to the lobbyist's.

²Li and Madarasz (2007) show that not revealing the mediator's type can increase information transmission in mediated communication.

³The sufficient condition for when a mediator improves communication given by Ambrus, Azevedo, and Kamada (2010) applies to settings where, whithout a mediator, there is only a babbling equilibrium and no information is transmitted. Our results thus point out that a mediator may improve communication under wider circumstances.

2 Model

Nature chooses the state of the world θ from the uniform distribution over the set $\{0,1,2,3\}$. The expert learns θ and sends a message $s \in \{0,1,2,3\}$ to the mediator. The mediator learns s (but not θ) and sends a message $m \in \{0,1,2,3\}$ to the decision maker. Finally, the decision maker observes m (but neither s nor θ) and chooses an action $a \in \mathbb{R}$. The payoffs of the expert, mediator, and decision maker are, respectively,

$$u_E = -(a - (\theta + b))^2,$$

 $u_M = -(a - (\theta + b_M))^2,$
 $u_D = -(a - \theta)^2.$

The bias of the expert b is strictly positive and commonly known. The bias of the mediator b_M is his private information and takes value $b_M = 0$ with probability $p \in (0,1)$ and $b_M = \beta > 0$ with probability 1 - p. Note that the biases of the expert and the mediator point into the same direction. Without loss of generality, we focus on equilibria where different messages of the expert which are sent with positive probability in equilibrium lead to different probability distributions over actions.⁵

2.1 Unmediated communication

As a benchmark, consider the case of unmediated communication between the decision maker and the expert. For this case, it is straightforward to show the following: If b < 1/2, a fully revealing equilibrium exists. If 1/2 < b < 1, the optimal (i.e., most informative) equilibrium is a 2-action equilibrium where type $\theta = 0$ reveals his type truthfully, thereby separating himself from all the other types, and the other types pool on some other message. If b > 1, only a babbling equilibrium exists and no information is transmitted.⁶

⁴For simplicity, we consider a discrete environment. The logic of our argument, however, does not depend on this. It will become clear below that we need at least four different states of the world to make our point.

⁵For any equilibrium where different messages of the expert lead to the same probability distribution over actions there exists an outcome equivalent equilibrium where these messages are replaced by one and the same message.

⁶See Appendix A.1 for a proof of these statements.

2.2 Mediated communication

Ivanov (2010) shows that a strategic mediator whose bias points into the opposite direction as the expert's bias can ensure as much information transmission as an optimally chosen unstrategic mediator even if the expert's bias is so large that, in direct communication, only the babbling equilibrium would exist. In our setting, however, in which the mediator's and the expert's biases point into the same direction, introducing the mediator cannot improve upon the babbling equilibrium. This is especially interesting since, as we will show below, the mediator in our setting can improve upon equilibria in which there is *some* information transmitted in direct communication.

Proposition 1 If no information can be transmitted by direct communication between the decision maker and the expert, then no information can be transmitted by mediated communication either.

The intuition behind this result is the following. Suppose that some information can be transmitted with mediated communication. Then there exists an equilibrium of the mediated communication game where the expert of type $\theta = 0$ separates himself, while the higher types $\theta = 1, 2, 3$ pool. In this equilibrium, the mediator will send only two messages with positive probability. Similarly the decision maker takes only two different actions with positive probability. Hence we call the equilibrium a 2-action equilibrium. (See Figure 1 for an illustration of this and the subsequent argument.)

In order to improve communication if direct talk amounts only to babbling (i.e. if b > 1), the mediator has to make sure that the lowest type prefers revealing his type over pooling with the others. Thus, the mediator must reduce the lowest type's incentive to exaggerate θ .

Consider the 2-action equilibrium. After receiving the message that reveals type $\theta=0$, the mediator will either pass on this information, inducing a low action of zero (if he is unbiased), or else will distort the message upwards, inducing a higher action (if he is biased). Thus, if the expert of the lowest type reports truthfully to the mediator, he thereby chooses a lottery combined of the low and the high action. If, instead, he exaggerates his type, he gets the high action with certainty. Thus, he reveals his type to the mediator only if he prefers the low action over the high action.

⁷See Appendix A.2 for a proof based on this intuition.

Note that the low action is exactly the same as the one that the decision maker would choose in an equilibrium of the direct communication game in which the lowest type of the expert is revealed - namely zero. The high action, however, is lower than the one that the decision maker would choose in a 2-action equilibrium of the direct communication game. This is due to the fact that in the 2-action equilibrium of the mediated communication game, the high message is less credible than in the 2-action equilibrium of the direct communication game. This, in its turn, is because in the mediated communication game, the high message could come from a biased mediator who exaggerates the type of the expert when in truth $\theta = 0$.

Given the concavity of the expert's preferences, the above considerations imply that if a 2-action equilibrium of the mediated communication game exists, then there is also a 2-action equilibrium of the direct communication game. Thus, the mediator cannot improve communication whenever b>1 so that direct talk amounts only to babbling.

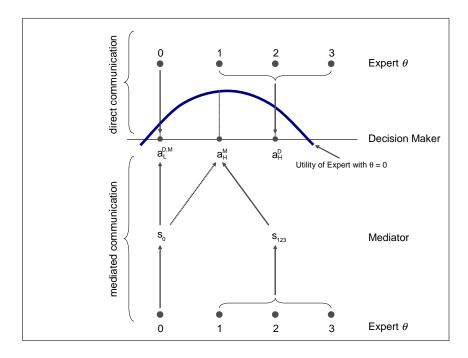


Figure 1: Direct communication versus mediated communication

However, when there exists an equilibrium of the direct communication game in which at least *some* information is transmitted, having the mediator may improve communication:

Proposition 2 Suppose that $1/2 < b < \bar{b} \approx 0.656$. Then there exists a positive measure set of parameters (β, p) with $\beta > b$ and $p \in (0, 1)$ such that mediation improves information transmission, compared to direct communication.

Note that for $1/2 < b < \bar{b}$, where $\bar{b} \gtrsim 0.65$, the 2-action equilibrium of the direct communication game exists and is the most informative equilibrium. To prove that strategic mediation can improve information transmission, we show that there exists a 3-action equilibrium of the mediated communication game.

The equilibrium strategies are as follows (see Figure 2): Type $\theta = 0$ of the expert sends a message s_0 , $\theta = 1$ sends a different message s_1 , and 2 and 3 pool on a third message s_{23} . The unbiased type of the mediator truthfully reports what he has received. The biased mediator distorts communication upwards: after seeing s_0 , he reports s_1 , and after seeing s_1 or s_{23} , he reports s_{23} .

Given these strategies, the decision maker infers from s_0 that $\theta = 0$ and thus chooses action $a_1 = 0$. After receiving s_1 , the decision maker believes that $\theta = 0$ with probability 1 - p and $\theta = 1$ with probability p, so he chooses $a_2 = p$. After observing s_{23} , the decision maker believes that $\theta = 1$ with probability (1-p)/(3-p), and that $\theta = 2$ and $\theta = 3$ with probability 1/(3-p) each. His choice of action in this case is $a_3 = (6-p)/(3-p)$.

It is straightforward to show that for any $b \in \left(\frac{1}{2}, \bar{b}\right)$, these strategies are an equilibrium for a set of parameters (β, p) with positive measure.⁸ (One example is $p = \beta = 0.9$ and b = 0.6.) The expected payoff of the decision maker in this equilibrium is

$$-\frac{1}{4}\left(2(1-p)p^2 + (1-p)\left(\frac{6-p}{3-p} - 1\right)^2 + \left(\frac{6-p}{3-p} - 2\right)^2 + \left(\frac{6-p}{3-p} - 3\right)^2\right)$$

$$\geq -\frac{1}{2}$$
(1)

⁸See Appendix A.3.

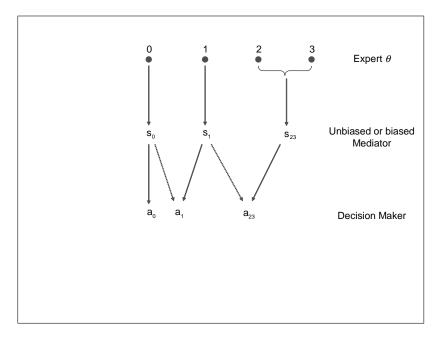


Figure 2: Best equilibrium of the mediated communication game

Thus, the decision maker gets a higher payoff in this equilibrium than in the 2-action equilibrium of the direct communication game, in which his payoff amounts to

$$-\frac{1}{4}\left(0+(1-2)^2+0+(3-2)^2\right)=-\frac{1}{2}.$$

We conclude that a strategic mediator with a positive bias - a bias pointing into the same direction as the expert's - can improve information transmission, but only if some information can already be transmitted via direct communication between the expert and the decision maker. The reason for this is the following: In order to give the lowest type of the expert an incentive to report honestly, the mediator has to make it more attractive for this type to reveal himself to the mediator (revelation) than to mimic the next-highest type (deviation). To achieve this, the mediator must make sure that (1) he moves to the right the action that the expert of type 0 expects in response to a revelation, and (2) that the decision maker's expected reactions to the revelation and to the deviation, respectively, are not two linear combinations of the same actions. Instead, the lottery of actions that the expert of type $\theta = 0$ triggers by deviation must include some action a' that is higher than

any possible action that the decision maker would choose after a revelation. If a' is sufficiently high, this guarantees that mimicking the next-highest type becomes less attractive to the expert with $\theta = 0$ than revealing his true type.

Thus, the biased mediator has to distort *both* messages upwards: the one that reveals the lowest type of the expert and the next highest one. This, however, can only be done if there are at least three different messages sent by the expert in equilibrium, i.e. only if the mediated communication game has an equilibrium with at least three actions.

If the expert has a bias b > 1, however, this is not possible. Thus, a strategic mediator with a positive bias can improve information transmission if the starting-point is a 2-action equilibrium of the direct communication game; but he cannot improve things if the starting point is a situation in which only the babbling equilibrium exists under direct communication.

3 Conclusion

Although our setting is extremely simple, it provides two important insights. The first insight is that a strategic mediator with a positive bias can improve communication - although for this to hold, at least some information transmission must already be possible via direct communication. The second insight concerns the relative intensities of the conflicts between the decision maker, the strategic mediator, and the expert: For the mediator to improve information transmission in Ivanov (2010), the conflict between the mediator and the expert must be more intensive than the conflict between the expert and the decision maker. In our setting, by contrast, the conflict between the mediator and the expert must be less intensive than the conflict between the mediator and the decision maker. This suggests that for any strategic mediator to improve communication, this mediator must be somewhat more extreme than the two parties whose conflict he is to alleviate: Either he must be more extreme than the decision maker (as in Ivanov (2010)), or he must be more extreme than the expert (as in our setting). We leave it to future research to explore this in a more general setting.

A Appendix

A.1 Unmediated communication

We start by showing two standard properties in our context.

Lemma 1 Suppose that types θ and $\theta' > \theta$ send the same message s. Then any θ'' with $\theta < \theta'' < \theta'$ also send s.

Proof. Suppose to the contrary that θ'' sends a message $s'' \neq s$. Denote the action take by the decision maker after observing s by a, and the action taken after observing s'' by a''. Suppose that $a < \theta'' + b$. Then a'' > a since type θ'' prefers a''. Moreover, a'' cannot be too far to the right of $\theta'' + b$. But then θ' prefers a'' over a, a contradiction. The remaining cases can be ruled out similarly.

Lemma 1 implies that, in any equilibrium, the set of expert's types is partitioned into cells, and types in the same cell send the same message. We call the difference between the highest and the lowest type belonging to the same cell the length of the cell.

Lemma 2 In the most informative equilibrium, the length of the cells of the partition is weakly increasing.

Proof. To prove this, it is sufficient to rule out the following three possibilities. (i) Types 0 and 1 pool, while types 2 and 3 send separate messages. Then we must have $b \le 1/2$, for otherwise 2 would have an incentive to imitate 3. But then a fully separating equilibrium exists. (ii) Types 1 and 2 pool, while types 0 and 3 send separate messages each. Then $b \le 1/4$, otherwise 2 prefers to imitate 3, and again a fully separating equilibrium exists. (iii) Types 0, 1, and 2 pool while type 3 separates himself. This cannot be an equilibrium since for any b > 0, type 2 prefers to imitate type 3.

We are now in a position to consider the most informative equilibrium. It is straightforward that, whenever b < 1/2, there is a fully revealing equilibrium. If 1/2 < b < 1, there is no equilibrium where types 0 and 1 send separate messages which fully reveal their respective types, since then type 0 would prefer to imitate type 1. In particular, there is no fully separating equilibrium. Moreover, there is no equilibrium where 0 and 1 pool on some message s while 2 and 3 pool on a different message $s' \neq s$, since type 1 would prefer to send s'. However, there is an equilibrium where type 0 separates

himself and types 1, 2, and 3 pool. By Lemma 2, it follows that this is the most informative equilibrium. Finally, if b > 1, this equilibrium breaks down since type 0 would prefer to imitate the others. Hence there is no information transmission in equilibrium.

A.2 Mediation does not improve communication if b > 1

Call an equilibrium a *n-action equilibrium* if the decision makers takes n different actions with positive probability on the equilibrium path. The following lemma is the key step in our argument.

Lemma 3 Suppose that without mediation there is no 2-action equilibrium. Then with mediation there is no 2-action equilibrium either.

Proof. Suppose that with mediation there is a 2-action equilibrium. There is a critical type θ' such that in the supposed equilibrium, all $\theta \leq \theta'$ send the same "low" message, and all $\theta > \theta'$ send the same "high" message. (This can be shown by essentially the same argument as in the proof of Lemma 1, replacing actions by certainty equivalents of the lotteries over actions induced by messages of the expert.) Furthermore, the mediator cannot report truthfully always because then a 2-action equilibrium would exist without mediation as well. The unbiased mediator never has an incentive to distort the message. The biased mediator will distort the message, if at all, only upwards. Thus, in the supposed equilibrium the mediator reports truthfully if he is unbiased, and always sends the high message when he is biased.

When the decision maker receives the low message from the mediator, he knows that $\theta \leq \theta'$ and chooses his action equal to the average of these types, i.e.

$$a_l = \frac{1}{\theta' + 1} \sum_{\theta = 0}^{\theta'} \theta.$$

When the decision maker receives the high message from the mediator, he chooses some action a_h which is strictly in between a_l and the average of the types of the expert that send the high message:

$$a_l < a_h < \frac{1}{4 - \theta'} \sum_{\rho = \rho' + 1}^4 \theta$$

Consider the critical type θ' in this equilibrium. By assumption, without mediation there is no 2-action equilibrium where types $\theta \leq \theta'$ send a low message and all others a high message. This implies that type θ' strictly prefers $\frac{1}{4-\theta'}\sum_{\theta=\theta'+1}^4 \theta$ over a_l . By concavity of the payoff functions, θ' strictly prefers a_h over a_l . But with mediation, θ' can induce a_h by sending the high message, contradicting equilibrium.

Whenever an n-action equilibrium exists where n > 2, there exists also a 2-action equilibrium. Thus Lemma 3 implies that, whenever without mediation there is only a babbling equilibrium, mediation cannot improve upon communication.

A.3 Mediated communication in the case 1/2 < b < 1

Here we give the conditions under which the equilibrium described in the full text exists.

The decision maker Given the strategies of the other players and updating of believes by Bayes' rule, the decision maker's actions are optimal by construction.

The mediator First, consider the mediator with bias $b_M = \beta$. If he receives message s_0 , and his utility from reporting m_1 must be greater than his utility from either reporting m_0 or m_{23} :

$$-(p-\beta)^2 \ge -\beta^2$$

$$-(p-\beta)^2 \ge -\left(\frac{6-p}{3-p}-\beta\right)^2$$

Solving these inequalities for β , we have $\beta \geq p/2$ and

$$\beta \le \frac{1}{2} \left(p + \frac{6-p}{3-p} \right). \tag{2}$$

If the mediator with bias β receives message m_1 , he should prefer sending m_{23} over sending m_1 :

$$-\left(\frac{6-p}{3-p} - (1+\beta)\right)^2 \ge -(p - (1+\beta))^2$$

or equivalently

$$\beta \ge \frac{1}{2} \left(p + \frac{6-p}{3-p} \right) - 1. \tag{3}$$

If inequality (3) holds, then he also has no incentive to send m_0 . Note (3) implies that $\beta \geq p/2$.

If the mediator with bias β receives message m_{23} , he should prefer sending m_{23} over sending m_1 :

$$-\left(\frac{6-p}{3-p}-\left(\frac{5}{2}+\beta\right)\right)^2 \ge -\left(p-\left(\frac{5}{2}+\beta\right)\right)^2$$

or

$$\beta \ge \frac{1}{2} \left(p + \frac{6-p}{3-p} \right) - \frac{5}{2}$$

which is implied by (3). A fortiori, the mediator has no incentive to send m_0 . Second, consider the unbiased mediator. After receiving message s_0 , he gets his ideal point by truthful reporting. After receiving message s_1 , he clearly has no incentive to send m_0 , and he also has no incentive to send m_{23} (after all, in this situation, even the biased mediator prefers m_1 over m_{23}). If he receives s_{23} , he should prefer sending m_{23} over sending m_0 or m_1 . This

is clearly true since 6-p 5

0

To summarize, if (2) and (3) hold, the mediator has no incentive to deviate.

The expert Consider type $\theta = 0$ of the expert. He has no incentive to send s_1 whenever

$$-pb^{2} - (1-p)(p-b)^{2} \ge -p(p-b)^{2} - (1-p)\left(\frac{6-p}{3-p} - b\right)^{2}$$

or equivalently

$$b \le \frac{1}{4} \frac{36 - 48p + 4p^2 + 23p^3 - 13p^4 + 2p^5}{9 - 18p + 17p^2 - 7p^3 + p^4} \tag{4}$$

Clearly, deviating s_{23} is worse than deviating to s_1 .

Consider type $\theta = 1$ of the expert. Sending s_1 is better than sending s_0 . Moreover, he has no incentive to report s_{23} whenever

$$-(p-(1+b))^{2} \ge -\left(\frac{6-p}{3-p}-(1+b)\right)^{2}$$

or

$$b \le \frac{1}{2} \left(p + \frac{6 - p}{3 - p} \right) - 1. \tag{5}$$

From (5) and (3), we have $\beta \geq b$.

Consider type $\theta=2$ of the expert. He has no incentive to send s_1 (and a fortiori s_0) whenever

$$-\left(\frac{6-p}{3-p} - (2+b)\right)^2 \ge -\left(p - (2+b)\right)^2$$

or

$$b \ge \frac{1}{2} \left(p + \frac{6-p}{3-p} \right) - 2$$

which is true for any b > 0.

Similarly, type $\theta = 3$ has no incentive to deviate either.

Figure 3 below plots the right hand sides of (4) and (5) as a function of p. Assuming b > 0.5, the inequalities hold in the area under the two curves and above the straight line.

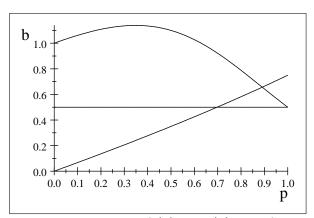
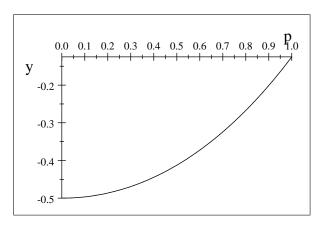


Figure 3: Right hand side of (4) and (5) as a function of p.

A.4 On inequality (1)

Here we plot the left hand side of inequality (1):



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