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Collusion in repeated auctions: a simple dynamic mechanism

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Abstract

We analyze collusion in an infinitely repeated version of a standard auction with a continuum of types. Because of the lack of efficiency results in this setting the literature has focused on determining and comparing benchmarks on how well bidders can collude. Aoyagi (2003) has shown that the bidders can improve upon static bid rotation, making use of a dynamic bid rotation scheme, but this scheme does not allow to determine how much bidders can improve upon bid rotation. In this paper we design a very simple dynamic mechanism that improves upon static bid rotation and in the limit recovers one third of the gap between static bid rotation and efficiency, independently of the underlying distribution of values.

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1 Introduction

The possibility that bidders collude in auctions by lowering their bids in a coordinated fashion at the expense of the seller has received ample amount of empirical attention (e.g. Hendricks and Porter (1989), Baldwin et. al. (1997), Pesendorfer (2000), Cramton and Schwartz (2002)). From a theoretical point of view, the main question addressed is whether the best collusive agreement is achievable in equilibrium. This optimal way in which bidders can collude is to allocate the good to the bidder who values it the most without leaving any rent to the seller. The latter is accomplished if the bidder who has the highest valuation bids the reservation price while the other bidders do not participate in the auction. Since the valuation of each bidder is private information it is not hard to understand that such a collusive agreement has little chance of success if there are no enforceable side-payments available or if there are no future auctions in which the bidders participate. In the absence of the latter two features bidders have an incentive to lie about their valuation.

How well bidders manage to collude thus depends on how they can best strike a balance between incentive compatibility and allocating the good efficiently. In a static setting this can be achieved at zero cost if binding side-payments are available. However, as these side-payments leave a 'paper trail', which make them easily visible to antitrust authorities, the literature has emphasized that collusion arises in an ongoing relationship (e.g. McAfee and McMillan (1992), Athey and Bagwell (2001), Johnson and Robert (1998), Aoyagi (2003 and 2007), Skrzypacz and Hopenhayn (2004)). Can bidders, in a repeated environment, obtain an efficient allocation of the good in every period leaving zero rent to the seller if monetary payments are not available? In this case incentive compatibility must be obtained through transfers of future utility. These transfers are restricted to lie in the set of equilibrium continuation values. Athey and Bagwell (2001) show that when the typespace is binary (finite) and satisfies a specific distributional assumption, the first best can be achieved in equilibrium by trading favors intertemporally. More generally, when the typespace is finite and types are distributed identically and independently (iid), then only asymptotic efficiency can be guaranteed (Fudenberg et al. (1994)). In a repeated auction setting, Aoyagi (2007) confirmed this result and extended it to affiliated types.

Auctions are mostly studied assuming a continuum of types. Unfortunatley, in this case no (asymptotic) folk theorem is available. But then, what does the best collusive scheme look like? Aoyagi (2003) builds on dynamic mechanism design to demonstrate the existence of a dynamic bid rotation scheme that outperforms the static 'bidding ring' proposed by McAfee and McMillan (1992). Nonetheless, his mechanism does not allow to pin down exactly how much better bidders can do in equilibrium.

The purpose of the present note is to propose a very simple dynamic mechanism that also improves upon static bid rotation and allows us to exactly pin down how much better bidders can do. We take the two-bidder environment presented in Aoyagi (2003)[1] and introduce a mechanism in which claims fulfill two roles: on the one hand they serve to allocate the good in an efficient way and on the other they induce incentive compatibility. At any point in time, bidders are either in a punishment state or in a reward state. Being punished means that there is some probability, $(1-\varphi)$, that one is not to participate and the good is allocated to the other bidder, who is in the reward state, at the reservation

[1] We do so for the case without affiliated types as this would unnecessarily complicate the main message we wish to convey.

price. Higher claims transfer future utility to the other player, and the simplicity of the mechanism is to be found here: the transfer is done in only one period and these expected transfers do not depend on the current state. We show that this collusive scheme can only be supported as an equilibrium of the repeated auction if $\varphi < \frac{1}{3}$. In the limit, this mechanism recovers one third of the gap between static bid rotation and efficiency and, interestingly, this is independent of the distribution of types as long as it satisfies a common hazard rate assumption.

The rest of the paper is organized as follows : Section 2 discusses the static setup and provides basic notation. In section 3, the repeated auction is introduced together with the collusive mechanism. Section 4 contains the main result of the paper. Section 5 concludes.

2 Stage Game Auction

We assume that there are two bidders. Generically, we denote one bidder i and the other bidder j. We focus on the independent private value case (IPV) which assumes the bidders are ex-ante symmetric and draw an independent private value for the good from a common continuous distribution F with strictly positive continuously differentiable density f and support $\Theta = [0,1]$. We assume that F satisfies the following hazard rate condition: $h'(\theta) < 0$ where $h(\theta) = \frac{1-F(\theta)}{f(\theta)}$. We allow for the fact that one or all bidders do not participate in the auction. Hence the bidders choose a bid from the set $B = \{\varnothing \cup R^{+}\}$. We assume for simplicity that the seller's reservation price equals zero. In what will follow we will focus on a first price sealed bid auction but it will be straightforward to see that our reasoning holds for any auctioning rule used by the auctioneer such that:

- The highest bidder obtains good. The other bidders does not pay a transfer to the seller. When there is a tie, the good is allocated randomly with equal probability to any of the two bidders.
- If nobody bids, the good remains in the hands of the seller.

The expected payoff of efficient collusion, v^*, is defined as $v^* = \int_0^1 \theta F(\theta) f(\theta) d\theta$. The expected payoff of a static bid rotation (McAfee and McMillan (1992)), is equal to $\frac{\bar{v}}{2}$ where $\bar{v} = E(\theta) = \int_0^1 \theta f(\theta) d\theta$: each bidder obtains the good with equal probability. There exists a symmetric Bayesian-Nash equilibrium for this game with expected payoff v^N. Given the assumption on h we have that $v^N < \frac{\bar{v}}{2}$. Since[2] $v^* > \frac{\bar{v}}{2}$ we have that $v^* > \frac{\bar{v}}{2} > v^N$.

3 The Repeated Auction

3.1 Setup

In the repeated game we assume that the bidders' private values are iid over time and we allow for pre-play communication in each period. Communication is introduced by assuming that the players have access to a communication device: the *center*. The task

[2]By integration by parts we get that $v^* = \frac{1}{2} - \frac{1}{2}\int_0^1 F(\theta)^2 d\theta$ and $\frac{\bar{v}}{2} = \frac{1}{2} - \frac{1}{2}\int_0^1 F(\theta) d\theta$. Hence $v^* - \frac{\bar{v}}{2} = \frac{1}{2}\int_0^1 F(\theta)(1-F(\theta)) d\theta > 0$.

of the latter is to collect the bidders' claims, and on the basis of these to recommend each bidder how much to bid[3].

Coordination through communication is then modelled as follows. In each period[4] t the bidders play the following stage game:

1. Each bidder $i = 1, 2$ observes her type θ_i^t.

2. Each bidder $i = 1, 2$ makes an announcement to the mechanism denoted by $\hat{\theta}_i(\theta_i)$, according to an **announcement rule** $\hat{\theta}_i$:

$$\hat{\theta}_i(.) : \Theta \to \Theta.$$

3. In the collusive stage of the mechanism there are two possible states: R_1 and R_2, where R_i is the state where bidder i is rewarded and bidder j is penalized. Given the current state $R_i \in \{R_1, R_2\}$ and announcements $\left(\hat{\theta}_1, \hat{\theta}_2\right) \in \Theta \times \Theta = \Theta^2$, the mechanism instructs each bidder how much to bid using the **instruction rule** $m : \{R_1, R_2\} \times \Theta^2 \to B^2$ where m is defined by

   - with probability φ the bidder with the highest claim obtains the good,
   - with probability $1-\varphi$ the bidder in the reward state obtains the good, regardless of his claim.

4. Given the claims of both bidders there is a **transition rule,** $\pi^i(.,.)$**,** to tomorrow's state which is *independent of today's state*: $\pi^i : \Theta^2 \to [0, 1]$ is the probability that bidder i's state next period will be R_j.

5. We define a **dynamic mechanism** M to be a collection of the assignment rule m and transition rule $\pi^i(.,.)$. In short

$$M = \{m, \pi^i(.,.)\}.$$

After observing the recommendation of the mechanism and his true valuation for the good, each bidder places her bid according to a **bidding rule** $\hat{b}_i$, $i = 1, 2$ where

$$\hat{b}_i(.) : B \times \Theta \to B$$

Moreover, let θ_i be the **honest reporting rule** for bidder i: $\theta_i(x) = x$, for all $x \in \Theta$, $i = 1, 2$. Let b_i be the **obedient bidding rule** (bidders follow the mechanism's instructions) so that $b_i(m(\hat{\theta}_i, \hat{\theta}_j, R_i), \theta_i) = m_i(\hat{\theta}_i, \hat{\theta}_j, R_i)$ where $j \neq i$.

We assume that the bidders decide on the rules of the mechanism at time zero. The mechanism is assumed to begin in a "collusive phase": at time zero the state is chosen at random after which it is determined by the claims of the bidders. After any observable deviation the mechanism reverts to a "non-collusive phase" which is characterized by playing the Bayesian Nash equilibrium forever, in which bidders obtain v^N per period.

[3]We introduce the idea of a communication center for ease of exposition and to work with a comparable set-up to that of Aoyagi (2003). We could do without the center by letting the players, when announcing their types, also propose: a) a bidding rule based on the announcements and the outcome of the randomization device, and; b) an adjustment rule governing the probabilities used for randomization as a function of announcements. The bidders would then effectively assume the role of the communication device. Such a set-up would be similar to that of Athey and Bagwell (2001).

[4]For our exposition we do not need the time superscripts and hence omit them below.

3.2 The Mechanism as a Perfect Public Equilibrium

Let $U_i(\hat{\theta}, \hat{b}, M)$ denote bidder $i's$ expected payoff (bidder $j's$ is defined analogously) from the stage game as a function of the announcement, bidding and instruction rules. Communication history for a bidder in period t in the repeated game is the sequence of his announcements and instructions in periods 1, 2, ..., $t-1$. Private history is the sequence of private signals θ_{ik} in periods $k = 1, 2, ..., t-1$. Finally, public history in period t is a sequence of outcomes of the assignment rule used by the mechanism, the actual bids and communication history. Bidder $i's$ strategy $\hat{\sigma}_i$ is a pair of announcement and bidding rules $(\hat{\theta}_i, \hat{b}_i)$ for each period defined as a function of his public and private histories. Define σ to be the honest and obedient strategy which selects the pair (θ, b) for all histories. Bidders aim to maximize their expected discounted payoff given a common discount factor $\delta < 1$. The collusive mechanism $M = \{m(\varphi), \pi^i(.,.)\}$ is an equilibrium if the pair $\Sigma = (\sigma_i, \sigma_j)$ of honest and obedient strategies is a perfect public equilibrium (PPE) of the repeated game, i.e., if σ_i is optimal against (σ_j, M) after any public history of the game. That is, what is required is that bidders are truthful and obedient.

4 The Main Result

Given a mechanism M, *interim welfare* for bidder i after observing his valuation and given a truthful and obedient strategy of bidder j is equal, in states R_i and R_j, to

$$\begin{aligned} W_i^{R_i}(\theta_i, \hat{\theta}_i) &= (1-\delta)(\varphi\theta_i F(\hat{\theta}_i) + (1-\varphi)\theta_i) + \\ &\quad \delta \int_0^1 [\pi(\hat{\theta}_i, \theta_j) W_i^{R_j} + (1-\pi(\hat{\theta}_i, \theta_j)) W_i^{R_i}] f(\theta_j) d\theta_j, \quad (1) \\ W_i^{R_j}(\theta_i, \hat{\theta}_i) &= (1-\delta)\varphi\theta_i F(\hat{\theta}_i) + \\ &\quad \delta \int_0^1 [\pi(\hat{\theta}_i, \theta_j) W_i^{R_j} + (1-\pi(\hat{\theta}_i, \theta_j)) W_i^{R_i}] f(\theta_j) d\theta_j. \quad (2) \end{aligned}$$

where $W_i^{R_i}$ $\left(W_i^{R_j}\right)$ is the ex ante expected payoff for bidder i in state R_i (R_j): $W_i^{R_i} = E_{\theta_i}\left[W_i^{R_i}(\theta_i, \theta_i)\right]$ and $W_i^{R_j} = E_{\theta_i}\left[W_i^{R_j}(\theta_i, \theta_i)\right]$. If there exists an incentive compatible transition mapping $\pi^i(\theta) = \pi^i(\hat{\theta}_i, \hat{\theta}_j)$, then expected payoffs in each state can be written recursively as:

$$\begin{aligned} W_i^{R_i} &= (1-\delta)(\varphi v^* + (1-\varphi)E\theta) + \delta(\pi^i W_i^{R_j} + (1-\pi^i) W_i^{R_i}), \quad (3) \\ W_i^{R_j} &= (1-\delta)\varphi v^* + \delta(\pi^i W_i^{R_j} + (1-\pi^i) W_i^{R_i}), \quad (4) \\ W_j^{R_j} &= (1-\delta)(\varphi v^* + (1-\varphi)E\theta) + \delta((1-\pi^i) W_j^{R_i} + \pi^i W_j^{R_j}), \quad (5) \\ W_j^{R_i} &= (1-\delta)\varphi v^* + \delta((1-\pi^i) W_j^{R_i} + \pi^i W_j^{R_j}), \quad (6) \end{aligned}$$

where $\pi^i = E\pi^i(\theta) = \int_0^1 \int_0^1 \pi^i(\theta) f(\theta_i) f(\theta_j) d\theta_i d\theta_j$. From the above we have that:

$$W_i^{R_i} - W_i^{R_j} = W_j^{R_j} - W_j^{R_i} = (1-\delta)(1-\varphi)E\theta = (1-\delta)(1-\varphi)\bar{v} \quad (7)$$

We now provide conditions under which the transition rule $\pi^i(\theta)$ induces local incentive compatibility. We need, for bidder i that:

$$\frac{W_i^{R_i}(\theta_i, \hat{\theta}_i)}{\partial \hat{\theta}_i} |_{\hat{\theta}_i = \theta_i} = 0 \text{ and } \frac{W_i^{R_j}(\theta_i, \hat{\theta}_i)}{\partial \hat{\theta}_i} |_{\hat{\theta}_i = \theta_i} = 0. \quad (8)$$

Similar conditions hold true for bidder j. Now define

$$\pi^i(\theta_i, \theta_j) = \pi^i_i(\theta_i) + \pi^i_j(\theta_j) \text{ where:} \tag{9}$$

$$\pi^i_i(\theta_i) = \frac{1}{2} - \frac{\varphi}{\delta(1-\varphi)\bar{v}} \int_0^{\theta_i} \theta f(\theta) d\theta, \tag{10}$$

$$\pi^i_j(\theta_j) = \frac{\varphi}{\delta(1-\varphi)\bar{v}} \int_0^{\theta_j} \theta f(\theta) d\theta. \tag{11}$$

Then $\pi^i(\theta_i, \theta_j)$ induces local incentive compatibility since the latter implies (from (8)):

$$\text{for bidder } i \ : \ (1-\delta)\varphi\theta_i f(\theta_i) + \delta\pi'_i(\theta_i)(W_i^{R_i} - W_i^{R_j}) = 0, \tag{12}$$

$$\text{for bidder } j \ : \ (1-\delta)\varphi\theta_j f(\theta_j) - \delta\pi'_j(\theta_j)(W_j^{R_j} - W_j^{R_i}) = 0, \tag{13}$$

$$\pi'_i(\theta_i) = -\frac{(1-\delta)\varphi\theta_i f(\theta_i)}{\delta(W_i^{R_i} - W_i^{R_j})} = -\frac{\varphi\theta_i f(\theta_i)}{\delta(1-\varphi)\bar{v}}, \tag{14}$$

$$\pi'_j(\theta_j) = \frac{(1-\delta)\varphi\theta_j f(\theta_j)}{\delta(W_j^{R_j} - W_j^{R_i})} = \frac{\varphi\theta_j f(\theta_j)}{\delta(1-\varphi)\bar{v}}. \tag{15}$$

Moreover, since the payoffs satisfy the single crossing property, local incentive compatibility implies global incentive compatibility. Given our definition of $\pi^i(\theta_i, \theta_j)$ above we obtain that $\pi^i = \frac{1}{2}$. Now let $\varphi = \frac{\delta}{2+\delta}$ and observe that this implies that $\pi^i(1, 0) = 1$ and hence $\pi^i(\theta_i, \theta_j) \in [0, 1]$ for all $(\theta_i, \theta_j) \in \Theta^2$. Since $\delta < 1$ we have that

$$\varphi < \frac{1}{3} \tag{16}$$

and for all $\delta < 1$ *on schedule* incentive compatibility is satisfied.

Off schedule deviations are deterred by Nash Reversion. The highest incentive to deviate is when a bidder is told not to bid while having the highest valuation, $\theta = 1$. Deviating is then deterred when $\delta > \delta^{NR}$ where[5] $\delta^{NR} = \frac{1}{\varphi v^* + (1-\varphi)\frac{\bar{v}}{2} - v^N + 1} < 1$.

The expected payoff of $M = \{m(\varphi), \pi^i(.,.)\}$ for i, in each state, becomes:

$$W_i^{R_i} = (1 - \frac{2\varphi}{(1-\varphi)})(\varphi v^* + (1-\varphi)E\theta) + \frac{2\varphi}{(1-\varphi)}(\frac{W_i^{R_i} + W_i^{R_j}}{2}), \tag{17}$$

$$W_i^{R_j} = (1 - \frac{2\varphi}{(1-\varphi)})\varphi v^* + \frac{2\varphi}{(1-\varphi)}(\frac{W_i^{R_i} + W_i^{R_j}}{2}). \tag{18}$$

Before the auction one randomizes (50/50) over who will start in the punishment and reward phases. Because of symmetry, the expected payoff of the mechanism becomes:

$$\frac{W_i^{R_i} + W_i^{R_j}}{2} = \frac{W_j^{R_i} + W_j^{R_j}}{2} = \varphi v^* + (1-\varphi)\frac{\bar{v}}{2}. \tag{19}$$

Hence when bidders become very patient ($\delta \to 1$) the expected payoff approaches

$$\frac{1}{3}v^* + \frac{2}{3} \cdot \frac{\bar{v}}{2} \tag{20}$$

We thus have the the following proposition:

[5]The monotone hazard condition guarantees us that the expected payoff at any state of the collusive phase of the mechanism is always higher than that of the non-collusive phase, v^N, and hence observable deviations can be deterred by Nash reversion.

Proposition 1 *Let* $\varphi = \frac{2\delta}{1+\delta}$*, then for any* $\delta > \delta^{NR}(M)$ *the mechanism* M *defined as above is an equilibrium of the repeated auctions game. Moreover, a bidder's expected payoff converges to* $\frac{1}{3}v^* + \frac{2}{3}\frac{\bar{v}}{2}$ *as* $\delta \to 1$.

The above defined probability mapping guarantees that the bidders will always announce their valuation in a truthful manner locally. The single crossing property then guarantees that incentive compatibility is also satisfied globally. In particular, the whole transfer needed to obtain incentive compatibility is obtained in the next period only. The cost is that the good is allocated in an efficient way only with probability φ. We would like to stress that our mechanism can, with patient enough bidders, recover one third of the gap between bid rotation and efficiency, independent of the underlying distribution. In order to gain some intuition, note that the transfer needed to guarantee incentive compatibility occurs through having higher announcements lead to a higher probability of being punished in the next period. Adding all the incentives for all $\theta_i \in [0,1]$ one obtains $\varphi\bar{v}$, the exact amount by which the expected utility of announcing the highest value, $\theta = 1$, must be reduced in the next period, compared to announcing the lowest value. Assume away discounting then $(\pi^i(1,\theta_j) - \pi^i(0,\theta_j))\,(1-\varphi)\bar{v}$ is the expected decrease in utility next period. In order for $\pi^i(\theta_i,\theta_j)$ to be a probability we imposed that $\pi^i(1,0) = 1$, or that $\pi^i(1,0) - \pi^i(0,0) = \frac{1}{2}$. In the limit scenario, $\delta = 1$, we see immediately that φ is independent of $\bar{v}$ and $\varphi = \frac{1}{3}$.

5 Concluding Remarks

We have constructed a very simple dynamic mechanism that outperforms the bid rotation scheme proposed by McAfee and McMillan (1992). It is similar in nature to the mechanism of Aoyagi (2003) but it displays some noteworthy differences. First, the mechanism reduces the gap between the equilibrium static bid rotation payoff and the efficient payoff with one third. Second, the mechanism achieves truthtelling in every period but requires that the good is not always allocated to the bidder with the highest valuation, although the announced valuations are known to be correct. Third, the mechanism is perhaps surprisingly simple, but once one attempts to generalize, things quickly become much more involved. Making the transition probabilities depend on the current state, for instance, makes the model intractable.

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