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Christos Kotsogiannis[†] and Konstantinos Serfes[‡]

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Abstract: A rather neglected issue in the tax competition literature is the dependence of equilibrium outcomes on the presence of firms *and* shoppers (two-sided markets). Making use of a model of vertical and horizontal differentiation, within which jurisdictions compete by providing public goods and levying taxes in order to attract firms *and* shoppers, this paper characterizes the non-cooperative equilibrium. It also evaluates the welfare implications for the jurisdictions of a popular policy of tax coordination: The imposition of a minimum tax. It is shown that the interaction of the two markets affects the *intensity* of tax competition and the *degree* of optimal vertical differentiation chosen by the competing jurisdictions. Though the non-cooperative equilibrium is, as it is typically the case, inefficient such inefficiency is mitigated by the strength of the interaction in the two markets. A minimum tax policy is shown to be effective when the strength of the interaction is weak and ineffective when it is strong.

Keywords: Public goods; Tax competition; Two-Sided Market; Vertical Differentiation.
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[†]Department of Economics, University of Exeter Business School, Streatham Court, Rennes Drive, Exeter EX4 4PU, England, UK. Tel: +44 (0)1392 264500.
E-mail: c.kotsogiannis@exeter.ac.uk.

[‡]Department of Economics and International Business, Bennett S. LeBow College of Business, Drexel University, Matheson Hall, 32nd and Market Streets, Philadelphia, PA 19104, USA. E-mail: ks346@drexel.edu. Phone: (215) 895-6816. Fax: (215) 895-6975.

1 Introduction

Moorestown and Haddonfield are two U.S. towns that are in many respects similar: They are located near each other in the greater Philadelphia metropolitan area (south New Jersey), are affluent with very good school districts, and are predominantly residential. But they also differ in one important aspect: Haddonfield's main street offers a better selection of restaurants and shopping areas than Moorestown's main street does. This better selection of shopping areas in Haddonfield has two, and closely related, effects on the economic activity there. Firstly, given that shoppers value, in general, variation in shopping areas, Haddonfield is attracting shoppers from many neighboring towns (including Moorestown). Secondly, and perhaps more importantly, the higher shopper traffic into Haddonfield results in even more (and of higher quality) businesses being attracted by this town, which, in turn, attracts even more shoppers in the Haddonfield area. An explanation for the lack of shopping variety in the Moorestown area that is frequently given is that Moorestown is expensive relative to Haddonfield, in terms of all the fees/taxes, to start-up business, and relative to the public infrastructure being offered in the respective towns.[1]

Though the previous example originates from U.S towns, it is not difficult for one to be convinced that a similar tendency appears when one compares levels of taxation and public good provision across other jurisdictional units such as localities, states, or countries. As another example that nicely illustrates this point take the recent—and for most policy observers unfavorable for businesses—change in the corporate income taxation in the UK. Following the announcement of the new UK corporate tax structure a number of multinationals announced their decision to relocate to lower-tax regimes. It seems, however, that an exodus of businesses is unlikely to happen.[2] The reason, and arguably a convincing one, for this is that the benefits of relocation for tax purposes are not *evenly* spread across once one accounts for requirements of *public goods* and *market access* of different types of businesses. To put it differently: Firms pay attention not only to tax burdens but also to the benefits that accrue to these firms from market access and government spending financed by those taxes. It is conceivable, therefore, that as long as there are benefits from market access and public goods provision an increase in the fiscal burden in a given jurisdiction may not result in a dramatic decline in the number of firms in that jurisdiction.[3]

The two previous examples emphasize that, firstly, there is an important interaction between footloose firms and the number of shoppers/consumers a given jurisdiction is able to attract and, secondly, that such interaction should impinge upon how jurisdictions behave in a fiscal competition game not only in relation to the level of taxation but also the level and type of public goods being provided. It is these issues that this paper is concerned with. More specifically, central to the analysis of this paper is the idea that jurisdictions may *strategically* choose to differentiate

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[1] This view is nicely expressed in 'Moorestown works to make business boom,' (December 1, 2006, Moorestown-NEWSW*eekly*.com).

[2] This is a belief held also by policy commentators. See 'Taxation and the fiscally footllose,' (May 1, 2008, Financial Times).

[3] Indeed the empirical evidence of the 'race to the bottom' in taxation seems to be fairly weak. Stewart and Webb (2006) provide a short review on this issue and also some additional evidence of weak convergence in the tax burdens in the OECD countries.

themselves in the provision of public goods in an effort to attract firms that make use of such (loosely speaking) firm-specific public goods.[4] The consequence of this public good differentiation, since firms have a jurisdiction-specific benefit, is that it allows jurisdictions to soften the unfettered tax competition which has a detrimental impact on revenues. This mechanism has been eloquently analyzed in the recent contribution by Zissimos and Wooders (2008) who show that jurisdictions, by differentiating vertically with respect to public good provision (in the sense that one jurisdiction provides a high level of public good while the other a low one), can mitigate the downward pressure of competition on taxes. The analysis of Zissimos and Wooders (2008), though it provides us with a number of important insights, focuses, however, on the firm side of the economy, abstracting, in particular, from the consumer side (and so the market access argument emphasized in the two examples discussed earlier). It is the incorporation of the consumer side (and its interaction with the firm side) within a model of public good differentiation and its implications for equilibrium outcomes that this paper is concerned with. The consideration of such two-sided market (shoppers and firms) will be shown to have important implications for the level of taxation and the spatial distribution of firms and consumers across jurisdictions that are absent from a one-sided market.[5]

One such implication is the possibility of the 'core-periphery' outcome that makes appearance in the new economic geography (NEG) models, where agglomeration of activity emerges in equilibrium (see, for example, Kind, Knarvik and Schjelderup (2000), and Baldwin and Krugman (2004)). There is, however, a distinct difference between the NEG models and the model presented here: Agglomeration in our model does not arise because of (internal) increasing returns or sufficiently high trade costs, but as a consequence of the interaction between mobile firms and shoppers, level of taxation and public good provision.[6] The interaction between shoppers and firms arises because consumers value variety and, as a consequence, they are more willing to travel for shopping to a jurisdiction that has attracted many firms. Since firms value market size they will be, in turn, more inclined to locate in the jurisdiction that has already attracted many firms. There is, thus, a loop argument between firms and shoppers (shoppers are attracted

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[4] It has to be noted that this is not, of course, the only dimension over which jurisdictions can differentiate themselves. For an alternative view—and one that is based on 'persuasive advertising'—see the recent contribution of Konrad (2008).

[5] Though the tax competition literature is fairly sizeable it has paid no attention to two sided-markets. Wilson (1999) provides an insightful review on the tax competition literature (see also Wildasin and Wilson (2004)). Two-sided markets have only been recently the subject of analysis in Public Finance (see Kind, Kothenbuerger and Schjelderup (2008)). There is a fast-growing literature on two-sided markets with applications that mainly fall into the Industrial Organization area (see, for example, Caillaud and Julien (2003), Armstrong (2006) and Rochet and Tirole (2003, 2006)). A working definition of a two-sided market—see, for instance, Rochet and Tirole (2006)—is that the volume of transactions, in addition to the aggregate price of the 'platform', also depends on how the aggregate price is divided between the two sides. In the present framework, as will be discussed in more details shortly, the 'platform' is the jurisdiction and the two sides that transact in a given jurisdiction are the firms and the shoppers. The taxes represent prices. It has to be noted, though, that there is a slight deviation between the two-sided market used in the present paper and that of the working definition: The tax in our model will fall on the firm side only and not on both sides. However, the main elements of a two-sided market and the existence of externalities between the two sides make still appearance in the present model.

[6] Agglomeration forces can arise through other sources, Boadway, Cuff and Marceau (2004). Another feature of the NEG models is the home market-bias that drives the incentive of firms to locate in the high-demand jurisdiction. This bias creates location rents and gives rise to the incentive of jurisdictions to set in equilibrium a positive tax (as in, for instance, Haufler and Wooton (1999)). As will be shown later on, this home-bias makes appearance here, too, but it does so through a different mechanism.

by firms, firms by shoppers and so on) arising from the fact that once a firm locates in a particular jurisdiction, it benefits not only itself but also all other firms, through the demand of the existing consumers that have located (or do their shopping) there. This implies the existence of a *cross-group externality* the strength of which depends on how much consumers value variety.

Interestingly, as will be shown in the formal analysis below, the strength of the cross-group externality will crucially affect market outcomes (tax and public goods). The results show that there is vertical differentiation with respect to the public goods provision (Proposition 2). The degree of vertical differentiation is affected by the intensity of the interaction between footloose firms and shoppers. The high public good jurisdiction over-invests, relative to the social optimum, when the cross-group externality is strong and under-invests when the cross-group externality is weak. The equilibrium of the fiscal competition game results in asymmetric shares (firm and shopper/consumer) between the two jurisdictions. Agglomeration (equivalently, and in the form it will be expressed here, '*tipping*'), where all firm activity is concentrated in one jurisdiction, is also a distinct possibility.[7] For low levels of the cross-group externality the firm side is shared between the two jurisdictions and an increase in the magnitude of the externality intensifies tax competition. Despite the decline in tax revenue, as a consequence of the cross-group externality, it is shown that public good investment can be higher than in a model with no externality (one-sided model). On the other hand, when the cross-group externality is sufficiently strong *all* firms locate in the high public good jurisdiction, and further increases in the cross-group externality will lead to a higher tax rate levied by the high public good jurisdiction, 'race to the top' (Proposition 1). The first-best optimal policy always involves all firms locating in the high public good jurisdiction. The implication of this—and perhaps a surprisingly result—is that the inefficiency of the non-cooperative outcome is mitigated as the cross-group externality increases and all firms locate in the high public good jurisdiction (Proposition 3). A minimum tax is *effective* when the cross-group externality is sufficiently weak and *ineffective* when the cross-group externality is sufficiently strong (Proposition 4).

The rest of the paper is organized as follows. The model is presented in Section 2. Section 3 presents the main analysis. The social planner's problem is presented in Section 4, whereas Section 5 evaluates a popular policy proposal: The imposition of a minimum tax across the two jurisdictions. Section 6 compares the equilibrium outcomes of the present analysis to those of the one-sided model of Zissimos and Wooders (2008). Finally, Section 7 concludes. All proofs are relegated in Appendices.

2 The structure of the model

The model is that of Zissimos and Wooders (2008) but it has been appropriately modified to incorporate the demand side of the economy. It features two jurisdictions A and B, indexed by k, and two distinct groups of agents: Firms, denoted by f, and shoppers, denoted by s. Each jurisdiction provides a public good, denoted by x_k, and levies taxes to the firms, denoted by τ_{fk}.

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[7] As touched upon previously, albeit briefly, when only the firm side is considered (one-sided model) agglomeration *never* appears in equilibrium. We come back to this in Section 6.

Jurisdictions. Each jurisdiction $k = A, B$ is defined as a center of economic activity. They are both located on the $[0, 1]$ interval, with jurisdiction A being at 0 and B at 1. The cost of the public good incurred by jurisdiction k is increasing and convex in the level of the public good and is given by $x_k^2/2$.

Firms. Firms are perfectly mobile across jurisdictions and, taking the level of the public good x_k and the tax rate τ_{fk} in jurisdiction $k = A, B$ as given, make a decision upon which jurisdiction to locate. Each firm locates in one jurisdiction. A shopper that travels to jurisdiction k buys one unit of good from each firm that is located in that jurisdiction. The implication of this, as will be discussed in more details shortly, is that an additional firm in a given jurisdiction k exerts a positive externality on all other firms that have located in that jurisdiction.[8]

Firms are heterogeneous and characterized by a parameter w which is uniformly distributed on the $[0, 1]$ interval. The mass of firms is one. A firm with parameter w has a (fixed) cost function $F - wx_k$:[9] The component $F > 0$ is independent of the jurisdiction the firm operates in, whereas wx_k depends on the level of public good x_k provided by the jurisdiction $k = A, B$. The interpretation of this latter component is that a firm with a higher w receives a higher benefit from the public good provided by jurisdiction k. Each firm is able to sell each unit of output at, the exogenously set, price p which without loss of generality is set equal to one.[10]

In deciding in which jurisdiction to locate, firms care about the number of shoppers in the jurisdictions, the level of taxes levied, as well as the level of the public good provided by both jurisdictions. Denoting by n_{sk}^e the number of shoppers firms expect to make their purchases in jurisdiction k, the level of profit for firm w who locates in jurisdiction k is given by[11]

$$\pi_{wk} = n_{sk}^e - (F - wx_k) - \tau_{fk} . \tag{1}$$

Shoppers. Shoppers, the mass of which is also one, are uniformly distributed on the $[0, 1]$ interval and they incur a disutility if they do not shop in their ideal location, denoted by z. The (strictly positive) per-unit travel cost is denoted by t. Shoppers, unlike firms, are assumed to be attached to a particular jurisdiction, in the sense that each shopper prefers the closer jurisdiction from his location, all else equal.[12] The price shoppers pay, for one unit of a good they purchase, is the same across both jurisdictions and equal to one. Shoppers value *product variety* and, thus, a

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[8] This will be, for instance, the case if there is sufficient complementarity between the goods being produced.

[9] One may wonder about the lack, from the specification of the cost function, of a component that captures the cost of producing the demanded level of output. This omission is, however, without loss of generality as long as the marginal cost of production is independent of the level of demand.

[10] This is not an uncommon assumption in the literature. Kanbur and Keen (1993), and Zissimos and Wooders (2008) also assume that prices are exogenously fixed.

[11] Notice that by setting $p = 1$ there is no guarantee that all firms' profits are nonnegative. This, however, is not problematic given that p, being exogenous, can be scaled up so to ensure nonnegativity of profits. Equally, the cost $F - wx_k$, $k = A, B$, maybe negative if the public good investment is high enough. The model, however, does not place any constraints on the sign of the fixed cost. It it conceivable, for instance, that the cost is negative and so jurisdictions subsidize the firms (a possibility that can be excluded by scaling up the fixed cost F).

[12] A reasonable justification of this modeling assusmption is that shoppers have already made their decisions regarding the place of residence somewhere between the two jurisdictions. To do their shopping they then have to choose which jurisdiction to travel to.

shopper that has traveled to a given jurisdiction makes a purchase from all firms located in that jurisdiction.[13] The utility gain for a typical shopper of jurisdiction k of an additional firm located in that jurisdiction is captured parametrically by $\alpha \geq 1$. The net utility from each purchase is $\alpha - 1$ (recall that all prices have been set equal to one). The implication of this is that if a jurisdiction attracts an additional firm this will lead to more shoppers traveling to that jurisdiction, which in turn will attract even more firms as so on. For wanting a convenient label, α will be called 'cross-group externality'. It is worth mentioning at this stage, something that we turn to in Section 6, that in the limiting case in which $\alpha = 1$ the cross-group externality disappears and the model reduces to the benchmark one-sided vertical differentiation model explored by Gabszewicz and Thisse (1979), Shaked and Sutton (1982), and, in a context similar to the present, Zissimos and Wooders (2008).

Denoting by n_{fk}^e the number of firms that shoppers expect to locate in jurisdiction k, a shopper with location characteristic z receives utility

$$U_z = \begin{cases} V + (\alpha - 1)\, n_{fA}^e - tz, & \text{if she shops in jurisdiction } A \\ V + (\alpha - 1)\, n_{fB}^e - t\,(1 - z), & \text{if she shops in jurisdiction } B. \end{cases} \qquad (2)$$

It is assumed that V is sufficiently high so that all shoppers shop and so the market is covered.

The sequence of events in the game is as follows. In stage 1 jurisdictions choose the level of the public good. The strategic interaction between the jurisdictions can take a number of forms. In many contexts it is natural to conceive of a dominant jurisdiction having a first-mover advantage relative to the other jurisdiction. In others, however, it may be more appealing to conceive of all jurisdictions moving simultaneously (as in Zissimos and Wooders (2008)). Interestingly, however, if in the present two-sided-market context both jurisdictions move simultaneously a pure strategy equilibrium may not exist.[14] Existence is restored if jurisdictions move sequentially in the public goods provision game. This is the case that attention is confined to here.[15] In stage 2, and given the level of the public good x_k, jurisdiction $k = A, B$ chooses taxes τ_{fk} (moves

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[13] The reason for such modeling choice is tractability. An alternative modeling assumption might be that each shopper buys from some firms only but more firms are valued by shoppers. The reason for this is that a larger pool of firms increases the probability of a better match between firms and consumers. This is the approach followed, but within a different context, by Konishi (2005). In such economic environment an externality of the type emphasized shortly will still be present thereby preserving the main insights of this paper. The benefit of the modeling approach taken here is that it is simple enough to yield sharp insights into the issues.

[14] The proof of this is available upon request. Non-existence is not surprising given, as it will be demonstrated later on (and in Appendix A.2), the lack of quasi-concavity and a discontinuity of the objective functions. When the cross-group externality α is low (that is, close to 1), then an equilibrium in the simultaneous move game exists. So, the model exhibits some continuity with respect to the equilibrium outcomes, since in the one-sided model (where $\alpha = 1$) an equilibrium in the simultaneous move game does exist. The non-existence appears for relatively high values of α. Roger (2007), in a two-sided model where both sides are vertically differentiated, obtains a similar non-existence result.

[15] An alternative to the sequential moves in the public good game is to look for a mixed strategy equilibrium. It is difficult, however, for one to find convincing arguments in favor of mixed strategies in the context of public good investments (or investments in general), as these investments take place very infrequently, require long planning and are primarily irreversible. It seems, thus, more appropriate to assume that jurisdictions move sequentially. Since the jurisdiction that moves first makes (weakly) higher net revenues than the jurisdiction that follows, both jurisdictions will prefer to have the first mover advantage. Though this preemption game is not formally modeled, it is used implicitly as a justification for the approach taken by the present analysis. Tirole (1988, p.297) offers a discussion on this issue.

are simultaneous). In stage 3, firms and shoppers, after they observe the strategic choices made by the jurisdictions, choose in which jurisdiction to locate and in which jurisdiction to make their purchases, respectively. The solution concept used in the solution of this game is that of a subgame perfect equilibrium in pure strategies. We turn now to the analysis, starting from the last stage, of the game.

3   Solving for the subgame perfect equilibrium

3.1   Stage 3: Firms' and shoppers' location decision

Suppose, without loss of generality, that $x_A \geq x_B$ and thus the difference in public good provision between the two jurisdictions, denoted by Δ, is non-negative, that is, $\Delta \equiv x_A - x_B \geq 0$.

Using (1), the marginal firm makes the same profits by locating either in jurisdiction A or jurisdiction B and so

$$n_{sA}^e - (F - wx_A) - \tau_{fA} = n_{sB}^e - (F - wx_B) - \tau_{fB}, \tag{3}$$

which, upon solving for w, identifies the location of the marginal firm, denoted by $\hat{w}$ in $[0, 1]$, given by

$$\hat{w} = \frac{(\tau_{fA} - \tau_{fB}) - (n_{sA}^e - n_{sB}^e)}{\Delta}. \tag{4}$$

Firms with $w \geq \hat{w}$ locate in jurisdiction A and firms with $w \leq \hat{w}$ locate in jurisdiction B. A word of clarification is in order here. It is possible that the public good offered by the two jurisdictions is homogeneous in the sense that $\Delta \equiv x_A - x_B = 0$. Firms, in this case, respond to a Bertrand-type game by locating in the jurisdiction that offers them the lowest fiscal burden, relative to the difference in the size of shoppers in the two jurisdictions.[16] Thus, when $\Delta = 0$, equation (4) takes the value of[17]

$$\hat{w} = \begin{cases} 1, & \text{if} \quad (\tau_{fA} - \tau_{fB}) > (n_{sA}^e - n_{sB}^e) \\ 0, & \text{if} \quad (\tau_{fA} - \tau_{fB}) < (n_{sA}^e - n_{sB}^e) \\ 0, & \text{if} \quad (\tau_{fA} - \tau_{fB}) = (n_{sA}^e - n_{sB}^e). \end{cases} \tag{5}$$

The fraction of firms that locates in jurisdiction A and in jurisdiction B are given, respectively, by

$$n_{fA} = 1 - \hat{w} \quad \text{and} \quad n_{fB} = \hat{w}. \tag{6}$$

Consumers can shop in either jurisdiction. Following the utility function in (2), the marginal shopper derives the same utility by shopping in either jurisdiction A or in jurisdiction B implying that

$$V + (\alpha - 1) n_{fA}^e - tz = V + (\alpha - 1) n_{fB}^e - t(1 - z). \tag{7}$$

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[16] In the one-sided market, in which $n_{sA}^e - n_{sB}^e = 0$, the share of firms in the two jurisdictions is driven by a comparison between τ_{fA} and τ_{fB}. In the two-sided market, the share of firms is more complex because of the interconnection between shoppers, who value variety (and so a large pool of firms in their locality), and firms, who value a large pool of shoppers (in the place of location).

[17] When $\Delta = 0$ and the difference in taxes is equal to the difference in the number of shoppers (third case in condition (5)) there is complete symmetry in the policy instruments. To ensure the existence of an equilibrium in pure strategies, in this case, it is assumed that all firms locate in jurisdiction A. One could also allow—without affecting the results in any significant way—for a 50-50 split of the firms between the two jurisdictions. Breaking the tie in favor of jurisdiction A preserves some continuity, since, as it will be shown later on, when $\Delta > 0$ but low jurisdiction A (the high public good jurisdiction) attracts all the firms (and market tips).

Solving equation (7) for z, the location of the marginal shopper, denoted by $\hat{z}$, is given by

$$\hat{z} = \frac{(\alpha - 1)\left(n_{fA}^e - n_{fB}^e\right) + t}{2t}. \tag{8}$$

It is, thus, the case that shoppers with $z \geq \hat{z}$ locate in jurisdiction B and shoppers with $z \leq \hat{z}$ locate in jurisdiction A. With $\hat{z} \in [0,1]$, the fraction of shoppers that locates in jurisdiction A and in jurisdiction B are given, respectively, by

$$n_{sA} = \hat{z} \quad \text{and} \quad n_{sB} = 1 - \hat{z}. \tag{9}$$

In equilibrium it must be that expectations are confirmed, that is, $n_{fA} = n_{fA}^e$, $n_{fB} = n_{fB}^e$, $n_{sA} = n_{sA}^e$ and $n_{sB} = n_{sB}^e$.

One of the interesting features of the model, as will be shown shortly below, is that the tax-subgame equilibrium can be an interior, and so both jurisdictions set strictly positive taxes, or a corner one and so one jurisdiction sets a strictly positive tax whereas the other jurisdiction sets a zero one. In the latter equilibrium the firm market tips in the sense that all firms locate in the jurisdiction that sets a strictly positive tax rate.

An interior outcome is determined by the simultaneous solution of the system of equations (6) and (9). Solving this system one obtains the firm shares in jurisdictions A and B, given, respectively, by

$$n_{fA} = \frac{\Delta t + t\left(\tau_{fB} - \tau_{fA}\right) - (\alpha - 1)}{\Delta t - 2(\alpha - 1)} \in (0,1) \quad \text{and} \quad n_{fB} = \frac{t\left(\tau_{fA} - \tau_{fB}\right) - (\alpha - 1)}{\Delta t - 2(\alpha - 1)} \in (0,1), \tag{10}$$

and shopper shares given by

$$n_{sA} = \frac{2(\alpha - 1)\left(\tau_{fB} - \tau_{fA} - 1\right) + \Delta\left(-1 + t + \alpha\right)}{2\left[\Delta t - 2(\alpha - 1)\right]} \in (0,1), \quad \text{and} \tag{11}$$

$$n_{sB} = \frac{2(\alpha - 1)\left(\tau_{fA} - \tau_{fB} - 1\right) + \Delta\left(1 + t - \alpha\right)}{2\left[\Delta t - 2(\alpha - 1)\right]} \in (0,1).$$

When tipping occurs the derivation of the limits of these shares will be different (and will be provided in the proofs of the corresponding Propositions).

It is worth noticing, at this stage, that the cross-group externality α and the degree of horizontal differentiation (cost of travel) t convey the same meaning, but in inverse relationship. The reason for this is intuitive: Shoppers, in deciding in which jurisdiction to shop, trade off the utility received from variety α and the cost per unit of travel t. More variety implies a high α and so a higher cost per unit of travel t is required for the marginal shopper $\hat{z}$ to be indifferent, following condition (8), between shopping in jurisdiction A or B. For simplicity, in the ensuing analysis all the results will be cast in terms of the cross-group externality α.

The analysis now turns to the tax competition stage of the game.

3.2   Stage 2: Competition in taxes

The (gross) revenue function of jurisdiction $k = A, B$ is given by[18]

$$R_k(\tau_{fA}, \tau_{fB}) = n_{fk}\tau_{fk} \ . \tag{12}$$

Each jurisdiction $k = A, B$, taking the cost of the public goods in both jurisdictions as given, chooses own destination-based tax τ_{fk} to maximize revenues given by (12) holding Nash conjectures against the other jurisdiction. The following Proposition summarizes, for any degree of vertical differentiation $\Delta \equiv x_A - x_B \geq 0$, cross-group externality $\alpha \geq 1$, and cost per unit of traveling $t > \alpha - 1$,[19] the Nash equilibrium of the tax competition subgame.

Proposition 1 *(Tax competition). The tax subgame equilibrium is characterized by:*
 i. Firm taxes

$$\tau_{fA}^* = \begin{cases} \frac{2\Delta}{3} - \frac{(\alpha-1)}{t} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} \\[2mm] \frac{\alpha-1}{t} & \text{if} \quad \Delta \leq \frac{3(\alpha-1)}{t} \end{cases} \qquad \tau_{fB}^* = \begin{cases} \frac{\Delta}{3} - \frac{(\alpha-1)}{t} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} \\[2mm] 0 & \text{if} \quad \Delta \leq \frac{3(\alpha-1)}{t}. \end{cases} \tag{13}$$

 ii. Jurisdiction firm- and shopper-shares

$$n_{fA}^* = \begin{cases} \frac{2\Delta t - 3(\alpha-1)}{3[\Delta t - 2(\alpha-1)]} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} \\[2mm] 1 & \text{if} \quad \Delta \leq \frac{3(\alpha-1)}{t} \end{cases} \qquad n_{fB}^* = \begin{cases} \frac{\Delta t - 3(\alpha-1)}{3[\Delta t - 2(\alpha-1)]} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} \\[2mm] 0 & \text{if} \quad \Delta \leq \frac{3(\alpha-1)}{t}, \end{cases} \tag{14}$$

$$n_{sA}^* = \begin{cases} \frac{1}{2} + \frac{\Delta(\alpha-1)}{6[\Delta t - 2(\alpha-1)]} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} \\[2mm] \frac{1}{2} + \frac{\alpha-1}{2t} & \text{if} \quad \Delta \leq \frac{3(\alpha-1)}{t} \end{cases} \qquad n_{sB}^* = \begin{cases} \frac{1}{2} - \frac{\Delta(\alpha-1)}{6[\Delta t - 2(\alpha-1)]} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} \\[2mm] \frac{1}{2} - \frac{\alpha-1}{2t} & \text{if} \quad \Delta \leq \frac{3(\alpha-1)}{t}. \end{cases} \tag{15}$$

 iii. Revenue functions (excluding the cost of the public good)

$$R_A = \begin{cases} \frac{[2\Delta t - 3(\alpha-1)]^2}{9t[\Delta t - 2(\alpha-1)]} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} \\[2mm] \frac{\alpha-1}{t} & \text{if} \quad \Delta \leq \frac{3(\alpha-1)}{t} \end{cases} \qquad R_B = \begin{cases} \frac{[\Delta t - 3(\alpha-1)]^2}{9t[\Delta t - 2(\alpha-1)]} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} \\[2mm] 0 & \text{if} \quad \Delta \leq \frac{3(\alpha-1)}{t} \ . \end{cases} \tag{16}$$

Proposition 1 reveals that the tax competition equilibrium is affected by the extent of vertical differentiation across the two jurisdictions $\Delta \equiv x_A - x_B$ measured relative to the threshold $3(\alpha - 1)/t$. In particular, if the degree of vertical differentiation is high, in the sense that $\Delta \geq 3(\alpha - 1)/t$, then the solution is interior whereas if the degree of vertical differentiation is low, in the sense that $\Delta \leq 3(\alpha - 1)/t$, the solution is a corner one. Though the precise value of the threshold is, arguably, model specific it is worth emphasizing that it critically depends upon the cross-group externality α and the transportation cost t.

It helps the exposition if the circumstances under which these two equilibria arise are discussed. We turn to this, starting with the interior solution, next.

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[18] For expositional convenience, throughout the paper, the dependence of functions on parameters is suppressed.

[19] The assumption that $t > \alpha - 1$ is common in two-sided market models. Armstrong (2006), for example, in a model where both sides are horizontally differentiated, introduces this assumption to ensure that both sides of the market do not tip in favor of one 'platform'. In the model analyzed here, this assumption prevents the shopper side from tipping. The results are not expected to change fundamentally if one also allows the shopper side to tip.

3.2.1 Interior equilibrium of the tax competition subgame $(\Delta \geq 3(\alpha - 1)/t)$

In an interior equilibrium the high public good jurisdiction, following (13), levies a higher tax, attracts more firms and more shoppers, following (14) and (15), respectively, and, following (16), enjoys higher revenues than the low public good jurisdiction.

It helps intuition, at this stage, to consider the comparative statics properties of the tax competition subgame of Proposition 1. Routinely differentiating (the appropriate parts of) (13) and (14) one obtains

$$\frac{\partial \tau_{fA}^*}{\partial \Delta} = \frac{2}{3} > 0, \ \frac{\partial \tau_{fB}^*}{\partial \Delta} = \frac{1}{3} > 0, \tag{17}$$

$$\frac{\partial \tau_{fA}^*}{\partial \alpha} = \frac{\partial \tau_{fB}^*}{\partial \alpha} = -\frac{1}{t} < 0, \tag{18}$$

$$\frac{\partial n_{fA}^*}{\partial \Delta} = -\frac{t(\alpha - 1)}{3\left[\Delta t - 2\left(\alpha - 1\right)\right]^2} < 0, \ \frac{\partial n_{fB}^*}{\partial \Delta} = -\frac{\partial n_{fA}^*}{\partial \Delta} > 0, \tag{19}$$

$$\frac{\partial n_{fA}^*}{\partial \alpha} = \frac{\Delta t}{3\left[\Delta t - 2\left(\alpha - 1\right)\right]^2} > 0, \ \frac{\partial n_{fB}^*}{\partial \alpha} = -\frac{\partial n_{fA}^*}{\partial \alpha} < 0. \tag{20}$$

That, following (17), an increase in vertical differentiation makes tax competition less intense thereby allowing jurisdictions to increase taxes is not new but has previously explored, in this context, by Zissimos and Wooders (2008). What is new, however, is that: i) following (18), the equilibrium tax rate of both jurisdictions decreases as the cross-group externality α increases, for given level of vertical differentiation Δ and ii) the tax bases of the two jurisdictions become more asymmetric as, following (19), the degree of vertical differentiation Δ decreases, and, following (20), the cross-group externality increases.

It is, thus, the case that tax competition becomes *more intense* with an increase in the cross-group externality. The intuition for this is the following. One can straightforwardly verify that taxes are strategic complements (and so both best response functions are upward sloping).[20] As the level of the cross-group externality α increases, for given tax rates, two effects occur. Firstly, following (10), the tax base of the high public good jurisdiction A increases while that of B decreases. This makes tax cuts more costly for the high public good jurisdiction, because its tax base has increased, and less costly for the low public good jurisdiction. Secondly, following again from (10), as the cross-group externality α increases a tax cut attracts more marginal firms. Both effects make tax cuts more profitable for the low public good jurisdiction as the cross-group externality increases, but they are opposing for the high public good jurisdiction. It turns out that the second effect is stronger and the high public good jurisdiction also becomes more aggressive in setting its taxes as the externality intensifies. Hence, with a higher cross-group externality α, each jurisdiction's best response function shifts so that it sets a lower tax for any tax levied by the rival. This in turn implies lower equilibrium taxes.[21]

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[20] This can be straightforwardly seen from the first order conditions given in Appendix A.1.

[21] The intuition behind an increase in the cost of traveling t follows similar reasoning and it is, therefore, omitted. Also the intuition behind (19) and (20) is provided shortly below.

3.2.2 Tipping equilibrium of the tax competition subgame ($\Delta \leq 3(\alpha - 1)/t$)

Proposition 1 also emphasizes the possibility of a tipping equilibrium that arises if the degree of vertical differentiation is sufficiently low, in the sense that $\Delta \leq 3(\alpha - 1)/t$. To see why this happens notice that, following (4), in every location decision a firm makes it evaluates the difference in the i) number of shoppers $n_{sA}^e - n_{sB}^e$, ii) tax burden $\tau_{fA} - \tau_{fB}$, and iii) public good provision $\Delta \equiv x_A - x_B$ that prevail in the two jurisdictions. Suppose now that the degree of vertical differentiation Δ decreases by one unit (keeping α and t fixed). The consequence of such a reduction is that the equilibrium tax differential between the two jurisdictions, given by $\tau_{fA}^* - \tau_{fB}^* = \Delta/3$, falls by a fraction of one unit that is, $\partial(\tau_{fA}^* - \tau_{fB}^*)/\partial\Delta = 1/3$.[22] This implies that, for given difference in the allocation of shoppers $n_{sA}^e - n_{sB}^e > 0$, the location of the marginal firm changes, following (4), by

$$\frac{\partial \hat{w}}{\partial \Delta} = \frac{n_{sA}^e - n_{sB}^e}{\Delta^2} > 0, \tag{21}$$

and so more firms locate in jurisdiction A. This is not, however, the end of the story, since, with now more firms locating in jurisdiction A, there are more shoppers willing to locate there thereby moving the location of the marginal firm closer to 0. This process continues until the market tips in favor of jurisdiction A, as Δ decreases further. Not all shoppers, however, would locate into this jurisdiction. The reason for this is that for some consumers of jurisdiction B, jurisdiction A is 'very far away' and so locating there is costly relative to the benefit they would derive from the existence of more varieties there. Since jurisdiction B has no tax base, it also sets a zero tax rate. Jurisdiction A, on the other hand, having the market tipped in its favor, can afford to set a strictly positive tax.

Tipping in the firm market is also likely to occur with an increase in the cross-group externality α. The reason for the dependence of tipping on this measure is intuitive. A high cross-group externality α means that shoppers care more about the number of firms in a given jurisdiction. This implies that the high public good jurisdiction, which has already attracted more firms, is now able to attract more consumers. A consequence of this is that more firms will be attracted to this jurisdiction which will attract even more consumers and so on. If the market has tipped, a further increase in the cross-group externality α will allow jurisdiction A to further increase its tax.[23]

The discussion in the last two subsections shows that, overall, the effect of the cross-group externality α on the tax of the high public good jurisdiction is non-monotonic (and in particular U-shaped): Decreasing when the firm side is shared (in which case α is low) and increasing when tipping has occurred (in which case α is high). The same effect is monotonic for the low public good jurisdiction (decreasing).

---

[22] This is a consequence of the fact that the high public good jurisdiction A responds by reducing its tax by more than jurisdiction B.

[23] This interpretation is reminiscent to the idea, that makes appearance in the NEG models, that a 'race to the top' arises because of the existence of location rents for firms. It is because of these rents that jurisdictions are capable of increasing taxes and firms accepting them. For a recent contribution, see Baldwin and Krugman (2004). See also footnote 6.

We turn now to the first stage of the game in which jurisdictions compete in the provision of the public goods.

3.3   Stage 1: Competition in public goods provision

The government of jurisdiction $k = A, B$ maximizes net revenues. Assume now that, without loss of generality, jurisdiction A moves first in the choice of public good provision whereas B moves second. Combining (16) with the cost of the public good, and allowing for the degree of vertical differentiation Δ to be positive or negative, one obtains the net revenue function of jurisdiction A. Depending on the level of public goods the two jurisdictions provide, and whether the firm market is shared or not between them, there are four possible configurations (and thus four regions in the net revenue function of a jurisdiction). These configurations imply that the net revenue function of jurisdiction A is given by

$$
R_A\left(x_A, x_B\right) = \begin{cases} \frac{[-\Delta t - 3(\alpha-1)]^2}{9t[-\Delta t - 2(\alpha-1)]} - \frac{x_A^2}{2} & \text{if} \quad \Delta \leq -\frac{3(\alpha-1)}{t} & \text{(Region 1)} \\ -\frac{x_A^2}{2} & \text{if} \quad -\frac{3(\alpha-1)}{t} \leq \Delta < 0 & \text{(Region 2)} \\ \frac{\alpha-1}{t} - \frac{x_A^2}{2} & \text{if} \quad 0 \leq \Delta \leq \frac{3(\alpha-1)}{t} & \text{(Region 3)} \\ \frac{[2\Delta t - 3(\alpha-1)]^2}{9t[\Delta t - 2(\alpha-1)]} - \frac{x_A^2}{2} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} & \text{(Region 4)}, \end{cases}
\tag{22}
$$

and depicted in Figure 1. In this figure, region 1 depicts the case in which jurisdiction A is the low public good jurisdiction and the firm market is shared. In region 2 jurisdiction A is the low public good jurisdiction and the firm market is tipped in favor of jurisdiction B. Region 3 depicts the case in which jurisdiction A is the high public good jurisdiction and the market is tipped in its favor, whereas region 4 illustrates the case in which jurisdiction A is the high public good jurisdiction and the market is shared.

The problem of jurisdiction A is then to maximize its net revenue, given by (22), by the appropriate choice of x_A, anticipating the response of jurisdiction B.[24] Such maximization problem is not trivial. The reason for this is the existence of the cross-group externality which gives rise to i) a discontinuity, and ii) different shapes of (a part of) the revenue function.

To facilitate the understanding of the characterization of the public goods equilibrium, a short discussion on the properties of the net revenue function of jurisdiction A, and, in particular, its dependence on the cross-group externality α, will prove helpful. We do so with the help of Figure 1. As depicted in Figure 1, the net revenue function is decreasing in regions 1-3, irrespective of the size of the cross-group externality α, except at the point where there is no vertical differentiation and, thus, $\Delta = 0$. At that point, the net revenue function is discontinuous. Region 4, however, does depend on the cross-group externality α, giving rise to a range of shapes (in order not to over burden the figure, Figure 1 depicts only one possible shape, the inverse U-shaped).

The magnitude of the cross-group externality α shapes the incentives of jurisdiction A, the leader, to invest in public good in the following way. When the cross-group externality α is sufficiently

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[24] Jurisdiction B has a net revenue function of the same structure as that of jurisdiction A, obtained by a simple relabeling of the variables in (22). The full details of the characterization of a jurisdiction's net revenue function can be found in Appendices A.2 and A.3.
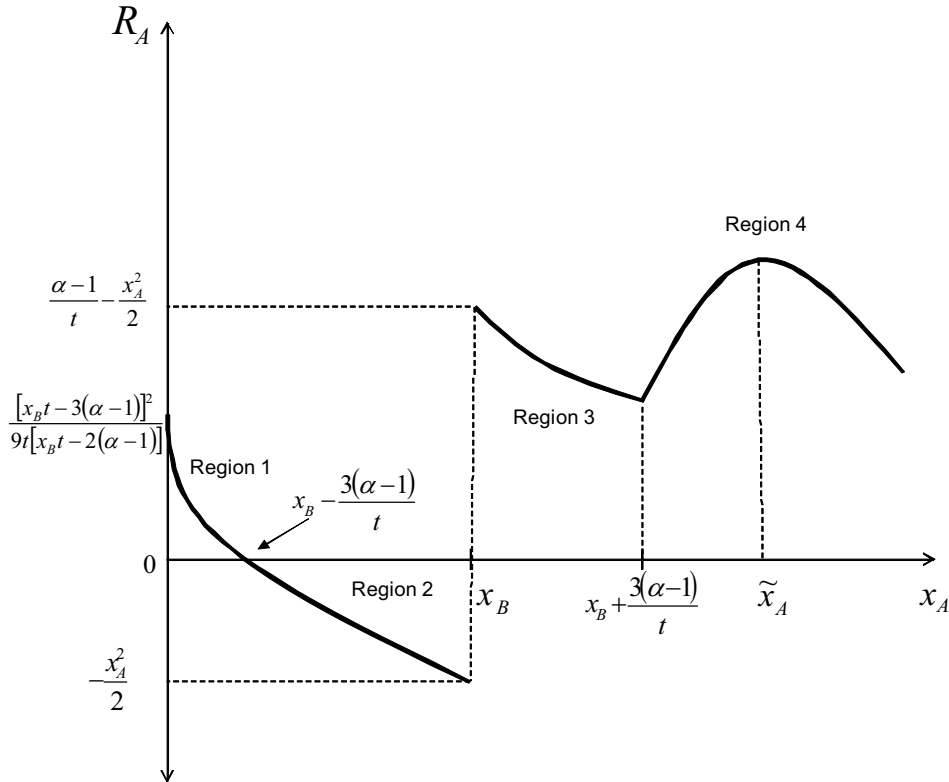
Figure 1: Net revenue function of jurisdiction A

low (that is, close to one), then the maximum of the net revenue function occurs in region 4. Jurisdiction A, then, will choose public good provision that corresponds to the point where the maximum is attained, denoted by $\tilde{x}_A$ in Figure 1. At this level of public good, jurisdiction B will have no incentive to leapfrog jurisdiction A.[25] As the cross-group externality α increases, the maximum in region 4 decreases (the net revenue may become everywhere decreasing in region 4). Because of the discontinuity of the net revenue function, leapfrogging by jurisdiction B becomes a profitable possibility (jurisdiction B can marginally choose $x_B = x_A + \varepsilon$ at $\Delta = 0$), an incentive that becomes more pronounced as the cross-group externality increases. The leader, then, and in order to prevent jurisdiction B from becoming the high public good jurisdiction, will choose a high level of the public good. Intuition suggests, then, that, in this case, the response of jurisdiction B will be to choose a sufficiently low level (zero) of the public good.

We now turn to the formal characterization of the subgame perfect Nash equilibrium.

Proposition 2 *(Public goods investments). The subgame perfect Nash equilibrium is described as follows:*[26]

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[25] Not surprisingly, this is also the case in a one-sided market, where $\alpha = 1$.

[26] The precise parameter thresholds are given in the proof of Proposition 2 in Appendix A.4. Notice also that the closed form solutions to case 2*iii*, being tediously long, are not presented in the Proposition.

i. High cross-group externality: Jurisdiction A (the first-mover) chooses

$$x_A^* = \frac{\sqrt{2(\alpha - 1)}}{\sqrt{t}},$$

where $x_A^* < 3(\alpha - 1)/t$ and jurisdiction B (the follower) chooses $x_B^* = 0$. Because $\Delta < 3(\alpha - 1)/t$, the firm side tips in favor of jurisdiction A. Both jurisdictions earn zero net revenue. As the cross-group externality decreases, jurisdiction A lowers its public good investment.

ii. Medium cross-group externality: Jurisdiction A (the first-mover) chooses

$$x_A^* \in \left(\frac{3(\alpha - 1)}{t}, \frac{\sqrt{2(\alpha - 1)}}{\sqrt{t}} \right),$$

and jurisdiction B chooses $x_B^* = 0$. Because $\Delta > 3(\alpha - 1)/t$, the firm side is shared between the two jurisdictions (no tipping). Both jurisdictions earn strictly positive net revenue, with jurisdiction A's net revenue being higher than B's. As the cross-group externality decreases, the leader lowers its public good investment.

iii. Low cross-group externality: Jurisdiction A (the first-mover) chooses $x_A^* = \max\{\tilde{x}_A, \bar{x}\}$ where $\tilde{x}_A \in [3(\alpha - 1)/t, \infty)$ satisfies

$$\frac{\partial R_A(\tilde{x}_A, 0)}{\partial x_A} = \frac{[2t\tilde{x}_A - 3(\alpha - 1)][2t\tilde{x}_A - 5(\alpha - 1)]}{9[t\tilde{x}_A - 2(\alpha - 1)]^2} - \tilde{x}_A = 0,$$

and $\bar{x} \in \left(3(\alpha - 1)/t, \sqrt{2(\alpha - 1)}/\sqrt{t} \right)$. The follower chooses $x_B^* = 0$. Because $\Delta > 3(\alpha - 1)/t$, the firm side is shared between the two jurisdictions (no tipping). Both jurisdictions earn strictly positive net revenue, with jurisdiction A's net revenue being higher than B's.

Proposition 2 shows that in the subgame perfect Nash equilibrium, irrespective of the significance of the cross-group externality, the follower always chooses zero public good provision. This results in a high degree of vertical differentiation. Jurisdiction A, having a first mover advantage, surprisingly, does not have any strict net revenue advantage over the follower, when the externality is sufficiently high. Both jurisdictions, in this case, earn zero (net) revenues.[27] The reason for this is the following. Given that the externality is high the equilibrium involves tipping. This implies that the jurisdiction with low public good (jurisdiction B), having no tax base, must earn zero net revenue. This, in turn, implies that the high public good jurisdiction, too, earns zero net revenue. The reason for this is that if jurisdiction A enjoyed strictly positive revenue then the follower would have a profitable deviation by marginally leapfrogging the leader, inducing market tipping in its favor and earning strictly positive net revenue.

The first mover (jurisdiction A) chooses its public good investment to ensure that it will not be leapfrogged by the follower and, given that this does not happen, it maximizes its own net revenue function. When the externality is high and medium the first consideration is binding. Jurisdiction A provides high levels of the public good in order to prevent the follower from becoming the high public good jurisdiction. When the externality is low, either consideration may bind (depending

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[27] It can also be straightforwardly verified that the endogenously chosen degree of vertical differentiation is non-monotonic (U-shaped) in the cross-group externality α.

on the strength of the externality). In this case, Proposition 2 shows that there is an $\tilde{x}_A$ that maximizes the net revenue function (given that $x_B = 0$) and an $\bar{x}$ that prevents leapfrogging.[28]

Having described the sub-game perfect equilibrium of the game, naturally, one might wonder how (in)efficient the equilibrium in the public good stage of the game is. To facilitate such comparison one needs to solve for the social planner's problem and compare the solution with the non-cooperative outcome of Proposition 2. We turn to this next.

4 Welfare analysis (first-best)

A social planner maximizes social surplus (denoted by ω and defined to be equal to the sum of aggregate profits and aggregate utility) given by

$$\omega(x_A, x_B, z, w) =$$

$$\int_0^z \left[V + (\alpha - 1)(1 - w) - t\ell\right] d\ell + \int_z^1 \left[V + (\alpha - 1) w - t(1 - \ell)\right] d\ell$$
$$+ \int_0^w \left[(1 - z) - (F - \ell x_B)\right] d\ell - \frac{x_B^2}{2} + \int_w^1 \left[z - (F - \ell x_A)\right] d\ell - \frac{x_A^2}{2}, \qquad (23)$$

by choosing public goods investments x_A and x_B, the location of the marginal shopper z, and the location of the marginal firm w.[29] It is easy to verify that (23) reduces to

$$\omega(x_A, x_B, z, w) = V - F + \frac{x_A(1 - w^2)}{2} + \frac{x_B w^2}{2} + \alpha w(1 - z) + \alpha z(1 - w)$$
$$- \left(\frac{t}{2} + tz^2 - tz\right) - \left(\frac{x_A^2}{2} + \frac{x_B^2}{2}\right), \qquad (24)$$

and so it is the components of (24) that the social planner seeks to maximize. Leaving aside the term $V - F$ which is fixed, the terms $x_A(1 - w^2)/2 + x_B w^2/2$ give the total cost reduction for the firms in both jurisdictions. Such terms are maximized at an asymmetric solution, where only one jurisdiction invests and attracts all the firms, due to the complementarity between public good investment and the number of firms in a given jurisdiction. The terms in $\alpha w(1 - z) + \alpha z(1 - w)$ give the total cross-group externality which is maximized at an asymmetric solution where one jurisdiction attracts all firms and shoppers and, thus, requires either $w = 0$ and $z = 1$, or $w = 1$ and $z = 0$. This is due to the complementarity between the number of firms and shoppers in a given jurisdiction. The terms $t/2 + tz^2 - tz$ give the total travel cost which, as it is usual in Hotelling-type models, is minimized at a symmetric solution, that is when $z = 1/2$. Finally, the terms in $x_A^2/2 + x_B^2/2$ give the cost of the public goods, minimized at a symmetric solution. This is because the cost functions are convex and the social planner would like to spread the investment cost across the two jurisdictions (as long as a jurisdiction has attracted some firms). The first-best solution will balance optimally the above trade-offs.

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[28] It is worth noting that as the cross group externality approaches 1, the optimal choice of jurisdiction A converges to 4/9, which is the public good investment in the one-sided model (see also Section 6).

[29] Notice that taxes do not enter the specification in (23) since they are simply transfers from firms to jurisdictions.

The first order conditions of (24) with respect to x_A, x_B, z and w give

$$\frac{\partial \omega}{\partial x_A} = \frac{1-w^2}{2} - x_A, \tag{25}$$

$$\frac{\partial \omega}{\partial x_B} = \frac{w^2}{2} - x_B, \tag{26}$$

$$\frac{\partial \omega}{\partial z} = \alpha\left(1-2w\right) + t - 2tz, \tag{27}$$

$$\frac{\partial \omega}{\partial w} = \alpha\left(1-2z\right) - w\left(x_A - x_B\right). \tag{28}$$

If an interior maximum exists then it will satisfy (25)-(28) with equality. It can be shown that such solution does not exist.[30] Denoting optimality by sp, the optimal solution is given by $x_A^{sp} = 1/2$, $x_B^{sp} = 0$, $w^{sp} = 0$ and $z^{sp} = 1/2 + \alpha/(2t)$.

It is thus the case that, since $w^{sp} = 0$, the social planner always wants the firm side to be tipped in favor of the high public good jurisdiction. Summarizing:

Proposition 3 *(Efficiency). The social planner's solution is described as follows:*

i. Jurisdiction A is the high public good jurisdiction, with $x_A^{sp} = 1/2$, and jurisdiction B makes the minimum public good investment that is, $x_B^{sp} = 0$.

ii. All firms are located in the high public good jurisdiction, jurisdiction A, that is, $n_{fA}^{sp} = 1$.

iii. Jurisdiction A (the high public good jurisdiction) attracts more shoppers ($z^{sp} = 1/2 + \alpha/(2t)$) than jurisdiction B, the low public good jurisdiction.

A simple comparison of Propositions 2 and 3 reveals that the low public good jurisdiction always chooses the first-best level of public good investment. The efficient level of firm location (tipping) is achieved, too, when the cross-group externality is sufficiently high (Proposition 2*i*). Even in this case, however, the high public good jurisdiction over-invests as $x_A^* = \sqrt{2\left(\alpha - 1\right)}/\sqrt{t} > x_A^{sp} = 1/2$. If the cross-group externality is medium or low (in the sense of Proposition 2*ii* and *iii*) then the market is shared, and the low public good jurisdiction attracts some firms. In this case, the high public good jurisdiction selects a level of investment that is above the first-best level when the externality is medium (in the sense that $x_A^* > x_A^{sp} = 1/2$), and under-invests (in the sense that $x_A^* < x_A^{sp} = 1/2$) when the externality is low. To emphasize:

Corollary 1 *At the non-cooperative equilibrium, the low public good jurisdiction (jurisdiction B) makes the efficient level of investment. The level of the high public good jurisdiction (jurisdiction A) is inefficient. Depending on the level of the cross-group externality, this investment can be above or below the efficient level.*

---

[30] Solving (25)-(27) and substituting into (28) yields a third degree polynomial in w, given by $\partial \omega/\partial w = [2tw^3 - \left(t - 4\alpha^2\right)w - 2\alpha^2]/(2t) = 0$. To solve for the w we calculate the slope of w at the two boundaries. It can be easily shown that $\partial \omega(w = 0)/\partial w = -2\alpha^2/(2t) < 0$ and $\partial \omega(w = 1)/\partial w = (t + 2\alpha^2)/(2t) > 0$. Because the first order condition is strictly negative at $w = 0$ the first intersection with the zero axis would correspond to a local minimum (there will be at least one intersection since the first order condition at $w = 1$ is positive). Using the *Descartes' rule of signs* it can be shown that the polynomial has only one real positive root (because it changes sign only once). Thus, this root must correspond to an interior local minimum. Based on these arguments, an interior solution does not exist.

Proposition 2i has also shown that a strong cross-group externality makes the firm market tip in favor of the high public good jurisdiction. It is so the case that:

Corollary 2 *When the cross-group externality is sufficiently high (as in Proposition 2i) the firm market tips and the non-cooperative outcome is the efficient one, in terms of the allocation of firms.*

The social planner prefers more shoppers in the high public good jurisdiction than the number of shoppers that prevails in the non-cooperative equilibrium. First, assume that the non-cooperative outcome exhibits tipping. Now compare z^{sp} of Proposition 3iii with $\hat{z} = 1/2 + (\alpha - 1)/(2t) < z^{sp}$.[31] Second, when there is no tipping $\hat{z}$ is even lower. Shoppers, when they choose where to go, do not internalize the positive externality their action has on the firms and as a result fewer shoppers (than the first-best number) decide to shop in jurisdiction A. The reason for this is that shoppers care only about the part of the externality that they receive, which is $\alpha - 1$, as opposed to the whole externality α. This externality is, however, internalized by the social planner who may also prefer the shopper side to tip in favor of A (if $t < \alpha$).

The preceding discussion has pointed out that the non-cooperative outcome may be inefficient. The issue that arises, then, is whether tax coordination policies can improve upon this inefficiency. In the next section we look at a policy coordination proposal: A minimum tax.[32]

5   Tax policy coordination: A minimum tax

Suppose now that the two jurisdictions—for levels of the public goods investments given in Proposition 2—negotiate in order to reach an agreement on a minimum tax. For a minimum tax to influence policy, and so to be binding, it must be that it is strictly higher than the equilibrium tax of jurisdiction B (as it is given in Proposition 1). For an agreement to be feasible the minimum tax, denoted by μ, must be i) constrained Pareto efficient, in the sense that no jurisdiction $k = A, B$ can increase its tax revenues by deviating from this minimum tax without reducing the tax revenues of the other jurisdiction[33] and ii) individually rational in the sense that both jurisdictions earn *strictly* higher revenues than before the imposition of the minimum tax (strict individual rationality). We thus search, starting from the tax rates of Proposition 1, for the set of all feasible minimum taxes. To this end, let $M = \{\mu \in \mathbb{R}_+: \text{conditions } i) \text{ and } ii) \text{ are satisfied}\}$. There are two cases to consider: The case in which the market is shared and the case in which the market is tipped.

Recall that in the case in which the market is shared (which occurs if $\Delta > 3(\alpha - 1)/t$, the equilibrium taxes, following Proposition 1, are given by $\tau_{fA}^* = 2\Delta/3 - (\alpha - 1)/t$ and $\tau_{fB}^* =$

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[31] $\hat{z}$ follows from (8) after setting $n_{fA}^e - n_{fB}^e = 1$.

[32] An earlier version of this paper also considered the tax policy harmonization considered by Zissimos and Wooders (2008). Under this policy proposal the two jurisdictions attempt to reach an agreement on bringing their taxes closer together in the sense that the high public good jurisdiction will lower its tax by μ_A, while the low public good jurisdiction will raise its tax by μ_B, relative to the equilibrium taxes of Proposition 1. This policy proposal, not surprisingly, cannot make both jurisdiction better off.

[33] This implies that that a minimum tax agreement is on the constrained Pareto frontier. It is constrained because after the imposition of the minimum tax the jurisdictions act non-cooperatively.

$\Delta/3 - (\alpha - 1)/t$. Since $\tau_{fA}^* > \tau_{fB}^*$, the minimum tax, denoted by $\tau_k^{\min}$, should bind jurisdiction $k = B$ and should take the form

$$\tau_B^{\min} = \tau_{fB}^* + \mu = \frac{\Delta}{3} - \frac{(\alpha - 1)}{t} + \mu. \tag{29}$$

Given $\tau_B^{\min}$, jurisdiction A's best response is to set[34]

$$\tau_A^{\min} = \frac{2\Delta}{3} - \frac{(\alpha - 1)}{t} + \frac{\mu}{2}. \tag{30}$$

Given the taxes defined in (29) and (30), the revenue functions of the two jurisdictions A and B—making use of (10) into (12)—are given, respectively, by

$$R_A(\mu) = \frac{[4\Delta t - 6(\alpha - 1) + 3\mu t]^2}{36t[\Delta t - 2(\alpha - 1)]}, \tag{31}$$

$$R_B(\mu) = \frac{[\Delta t - 3(\alpha - 1) + 3\mu t][2\Delta t - 6(\alpha - 1) - 3\mu t]}{18t[\Delta t - 2(\alpha - 1)]}. \tag{32}$$

It can be verified that the revenue function of jurisdiction A in (31) is increasing in μ, whereas the revenue function of jurisdiction B in (32) is concave in μ and attains a maximum at

$$\mu^* = \frac{\Delta}{6} - \frac{\alpha - 1}{2t}. \tag{33}$$

It is intuitive that any minimum tax that is Pareto constrained cannot be less than the μ^* defined by (33). For if it is, following the argument regarding the shape of the revenue functions of the two jurisdictions, both jurisdictions can increase their revenues. It follows, then, that if a set of minimum taxes M exists then it must contain minimum taxes with the property that $\mu > \mu^*$. To state this differently: Any agreed upon minimum tax must involve $\mu > \mu^*$, as is defined by (33), as this ensures that a minimum tax agreement is on the *constrained* Pareto frontier.

The minimum tax agreement must also be individually rational for both jurisdictions. For jurisdiction A things are simple: As its revenues are increasing in $\mu > 0$, it is individually rational to accept any minimum tax. This is not, however, true—given the concavity of the revenue function under a minimum tax—for jurisdiction B. Individual rationality for this jurisdiction dictates that the minimum tax agreeable should not yield lower revenues than in the no minimum tax equilibrium of Proposition 1. A simple comparison of the revenue function of jurisdiction B under no minimum tax, given by Proposition 1, and the revenue function in (32) reveals that this is the case if and only if

$$\mu < \frac{\Delta}{3} - \frac{\alpha - 1}{t}. \tag{34}$$

From (33) and (34) it, then, follows that any

$$\mu \in \left(\frac{\Delta}{6} - \frac{\alpha - 1}{2t}, \frac{\Delta}{3} - \frac{\alpha - 1}{t} \right), \tag{35}$$

is (strictly) individually rational and lies on the constrained Pareto frontier.

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[34] This follows from the first order condition of jurisdiction A, given in Appendix A1.

It is, thus, the case that coordination, in the form of a minimum tax, is beneficial for the governments of the jurisdiction. This is an implication of the fact that minimum tax coordination improves efficiency in the sense that more firms now locate in the high public good jurisdiction A, relative to the no minimum tax case, thereby resulting in a more asymmetric outcome in terms of firm shares across the two jurisdictions.[35] This, as emphasized in Proposition 3, improves efficiency as the social planner's solution involves tipping in favor of the high public good jurisdiction.[36]

Interestingly, as the cross-group externality increases, a minimum tax agreement is *less* likely to exist. The reason for this is that—as demonstrated in the previous section—as α increases the non-cooperative equilibrium, holding the levels of the public goods fixed, becomes more efficient. The implication of this is that there is less room for a Pareto improvement. This is being shown formally next.

Suppose that the firm side has tipped that is, $\Delta \leq 3(\alpha - 1)/t$. In this case the equilibrium taxes, following Proposition 1, are given by, $\tau_{fA}^* = (\alpha - 1)/t$ and $\tau_{fB}^* = 0$. The minimum tax, in this case, is

$$\tau_B^{\min} = \tau_{fB}^* + \mu = \mu. \tag{36}$$

Given $\tau_B^{\min}$, jurisdiction A's best response is to set

$$\tau_A^{\min} = \frac{(\alpha - 1)}{t} + \mu. \tag{37}$$

In this case, the high public good jurisdiction responds by increasing its tax by the same amount the low public good jurisdiction increased its tax. There will be no effect on the firm shares and the revenue of the low public good jurisdiction will not increase as a result of a minimum tax imposition. Therefore, there does not exist a (strictly) individually rational agreement that is different from the equilibrium under no minimum tax.

Summarizing the preceding discussion:

Proposition 4 *(Minimum tax). Fix $\Delta \equiv x_A - x_B$ as it is given in Proposition 2.*

    i. Firm side is shared and so $\Delta > 3(\alpha - 1)/t$. Any minimum tax $\mu > 0$ in the set $M = (\Delta/6 - (\alpha - 1)/(2t), \Delta/3 - (\alpha - 1)/t)$ is strictly individually rational for both jurisdictions and lies on the constrained Pareto frontier.

    ii. Firm side has tipped and so $\Delta \leq 3(\alpha - 1)/t$. There does not exist a minimum tax that makes both jurisdictions strictly better off and lies on the constrained Pareto frontier, that is $M = \varnothing$.

A recurring feature of the analysis is the importance of the cross-group externality. One, then, might wonder how the equilibrium of the fiscal competition game in a two-sided market compares to equilibrium in the one-sided market. We turn to this next.

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[35] To see this first notice that, following (29) and (30), minimum tax coordination, for given vertical differentiation $\Delta > 0$, implies $(\tau_A^{\min} - \tau_B^{\min}) = \Delta/3 - \mu/2$ and so a reduction in the tax differential relative to the no minimum tax coordination case. Following (10), then more firms locate in the high public good jurisdiction A.

[36] This is in the spirit of Kanbur and Keen (1993), and Zissimos and Wooders (2008).

6   Two-sided *versus* one-sided market

Notice that if the cross-group externality is equal to one then Proposition 1 reduces to the equilibrium of the one-sided market analyzed by Zissimos and Wooders (2008). In this case the outcome is always in the interior and the equilibrium taxes are given by

$$\tau_{fA}^{*}\left(x_{A}, x_{B}, \alpha = 1\right) = \frac{2\Delta}{3} \quad \text{and} \quad \tau_{fB}^{*}\left(x_{A}, x_{B}, \alpha = 1\right) = \frac{\Delta}{3}. \tag{38}$$

In the non-cooperative equilibrium analyzed by Zissimos and Wooders (2008) the equilibrium levels of the public good are given by[37]

$$x_{A}^{*} = \frac{4}{9} \quad \text{and} \quad x_{B}^{*} = 0, \tag{39}$$

whereas the market shares are given by

$$n_{fA}^{*} = \frac{2}{3} \quad \text{and} \quad n_{fB}^{*} = \frac{1}{3}. \tag{40}$$

In such a one-sided market, efficiency, denoted by s, dictates that

$$x_{A}^{s} = \frac{1}{2} \quad \text{and} \quad x_{B}^{s} = 0, \tag{41}$$

and

$$n_{fA}^{s} = 1 \quad \text{and} \quad n_{fB}^{s} = 0. \tag{42}$$

Efficiency. It is, thus, the case that in a one-sided model, the high public good jurisdiction *always* under-invests relative to the social optimum since, following from (39) and (41), $x_{A}^{*} = 4/9 < x_{A}^{s} = 1/2$. In a two-sided model over-investment is also possible (Corollary 1). It is also the case that in a one-sided market, following (40) and (42), there is *always* a divergence between the social optimum and the non-cooperative outcome in terms of firm shares. The (one-sided market) first-best always entails one jurisdiction (the high public good one) attracting all firms, while in the non-cooperative solution both jurisdictions have strictly positive firm shares. In contrast, in a two-sided market, when the externality is strong, the social optimum coincides with the non-cooperative outcome, in the sense that all firms are located in the high public good jurisdiction (Corollary 2).

Firm shares/agglomeration. The one-sided model, following (40), always yields an interior equilibrium with both jurisdictions attracting firms. The analysis developed here is flexible enough to generate, depending on the strength of the cross-group externality, an interior as well as, following Proposition 1, the 'core-periphery' equilibrium outcome in which the firm market tips in favor of the high public good jurisdiction.

Public good provision. Strong and medium cross-group externality (Proposition 2i and ii) lead to a higher public good provision, on part of the high public good jurisdiction, in the two-sided model than in the one-sided, that is, $x_{A}^{*} > 4/9$. Despite the fact that the presence of

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[37]Note that even if jurisdictions move sequentially when they choose public goods investments, in the one-sided model, the equilibrium does not change. So, the difference in the results with our model is not attributed to the assumption about the timing of the moves in the public goods game. The proof of this is available upon request.

the cross-group externality intensifies tax competition, when the firm side is shared (medium externality), and hence tax revenues decline, public goods investment is higher than when the externalities are ignored. When the externality is low, public good provision in the two-sided market x_A^* can be less or greater than 4/9. Perhaps not surprisingly, it can be be shown that as $\alpha \to 1$ (or $t \to \infty$), the level of public good provision (of Proposition 2iii) x_A^* converges to 4/9, the outcome of the one-sided market.[38]

Taxes. When tipping occurs, equilibrium taxes are more asymmetric across the two jurisdictions than in a one-sided model. The high public good jurisdiction levies a higher tax (when the cross-group externality α is high and the firm side tips) than the one-sided tax, $\tau_{fA}^* = (\alpha - 1)/t > 2\Delta/3 = 8/27$ (since $\Delta = x_A^* - x_B^* = 4/9$). The low public good jurisdiction levies a lower tax than the one-sided tax, $\tau_{fB}^* = 0 < \Delta/3 = 4/27$. When the firm side is shared, equilibrium taxes for both jurisdictions are lower than in the benchmark one-sided model.

Minimum tax. Finally, a minimum tax is *always* effective in a one-sided model, whereas in a two-sided model this is the case *only* when the externality is weak. A strong externality renders such a policy proposal ineffective.

7   Concluding remarks

A neglected issue in the tax competition literature is the dependence of equilibrium outcomes on the presence of firms *and* shoppers (two-sided markets). Making use of a model of vertical and horizontal differentiation within which jurisdictions compete, by providing public goods and levying taxes to attract firms *and* shoppers, the paper has explored such dependence. An attractive feature of the model is that it is flexible enough to yield firm and shopper shares that are very asymmetric across jurisdictions, depending on the degree of the interaction in the two markets. This allows us to obtain agglomeration of firm activity in one jurisdiction, which also appears in the new economic geography (NEG) models.

The interaction of the two-sides in the tax competition game has been shown to give rise to a cross-group externality, which has profound implications for the equilibrium outcomes. For instance, as shown in Proposition 1, this cross-group externality intensifies the asymmetry of the shares (firm and shopper/consumer) between the jurisdictions (an observation that is absent in the one-sided model). Within such framework, it has been shown that jurisdictions choose different public goods investments (vertical differentiation). The low public good jurisdiction chooses the minimum possible level (zero) and the high public good jurisdiction chooses a strictly positive investment. The degree of vertical differentiation, relative to that in the one-sided model, is: *i*) higher if the cross-group externality is strong and *ii*) lower if the externality is weak (Proposition 2). For low levels of the cross-group externality the firm side is shared between the two jurisdictions and an increase in the magnitude of the externality intensifies tax competition. On the other hand, when the externality is strong the firm side tips in favor of the high public

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[38] The proof of the comparison of the public good investment between one-sided and two-sided markets can be found in the proof of Proposition 2 in Appendix A.4.

good jurisdiction, and further externality increases will lead to a higher tax rate levied by the high public good jurisdiction (Proposition 1). The high public good jurisdiction over-invests, relative to the social optimum, when the externality is strong and under-invests when the externality is weak. The first-best outcome always involves firm tipping. In that sense, the inefficiency of the non-cooperative outcome is mitigated as the cross-group externality increases and the firm side tips (Proposition 3). A minimum tax is effective when the externality is weak and *ineffective* when the externality is strong (Proposition 4).

The results of the model also cast some light on the empirical determinants of the incentives of multinationals to locate in a given jurisdiction. There is considerable evidence which, generally, confirms that corporate taxation has a significantly negative effect on the investment decision of multinational firms. The magnitude of such effect, however, seems to depend, crucially, upon the specific aspects of the investment being undertaken by the multinationals as well as upon sector-specific characteristics of the firms that locate their businesses.[39] What the model and the analysis suggest here is that to proper assess the effect of taxation on the investment decision of multinationals, one requires in the empirical estimation not only the level of taxation as an explanatory variable but, as noted in the introductory section, also firm-specific public goods and market access.

The limitations of this paper suggest avenues for future research. One avenue that is, arguably, worth exploring is the dependence of the equilibrium outcomes upon endogenously determined prices. Following the work of Trandel (1994), the introduction of an additional stage for the determination of prices is not expected to change qualitatively the results if the equilibrium is in the interior. What is difficult to conjecture, however, is the extent to which such economy can support an equilibrium with tipping in the firm market. The reason for this is that if all firms locate in a high public good jurisdiction the marginal firm might want to re-locate to the low public good jurisdiction and charge a higher price to the remaining shoppers. This response by the marginal firm indicates the possibility that a tipping equilibrium may be 'more difficult' to emerge.

Firms have also been assumed to be footloose. It would be interesting to investigate the role of the cross-group externality on public goods investments and tax competition when firms have a certain degree of attachment to a particular jurisdiction.

Public goods in the model benefit only the firms. In reality, however, public goods also confer utility to consumers. A fruitful extension to the model analyzed here might, therefore, be a situation where the public goods investments benefit both sides of the market and not only firms. Investments in infrastructure is an example that fits well here. An interesting case to explore in such an environment, is the extent to which the *principle of maximum differentiation* holds. Intuition, however, suggests that it might not since, in this case, the utility derived from the public good provision might mitigate any strategic incentives arising from vertical differentiation.

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[39] And, of course, on the tax measures being used. On this see, for instance, Desai and Hines (2004). See also Herger, Kotsogiannis and McCorriston (2008).

There remains much scope for the analysis of tax competition in richer models of two-sided markets. We hope to have shown that the task is worthwhile and that the conclusions can be instructive.

Appendices

A.1 Proof of Proposition 1

The first order conditions of the revenue functions for jurisdiction A and B with respect to taxes are given, respectively, by

$$\frac{\partial R_A}{\partial \tau_{fA}} = \frac{\Delta t - (\alpha - 1) + t\tau_{fB} - 2t\tau_{fA}}{\Delta t - 2(\alpha - 1)} = 0 \text{ and } \frac{\partial R_B}{\partial \tau_{fB}} = \frac{-(\alpha - 1) + t\tau_{fA} - 2t\tau_{fB}}{\Delta t - 2(\alpha - 1)} = 0. \quad (A.1.1)$$

Sufficiency requires that $\partial^2 R_k / \partial \tau_{fk}^2 = -(2t)/[\Delta t - 2(\alpha - 1)] < 0, \ k = A, B$, which is satisfied if and only if

$$\Delta > \frac{2(\alpha - 1)}{t}. \quad (A.1.2)$$

The proof now explores the dependency of optimality on the values of Δ. There are two cases to consider.

Interior equilibrium. In such an equilibrium no jurisdiction has an incentive to unilaterally deviate from the solution to the system of the first order conditions in (A.1.1) given by $\tau_{fA} = 2\Delta/3 - (\alpha - 1)/t$ and $\tau_{fB} = \Delta/3 - (\alpha - 1)/t$. The proof now verifies that there exists an interval $\Delta \geq 3(\alpha - 1)/t$ such that $\hat{w}, \hat{z} \in [0, 1]$.

Making use of the fact that $\tau_{fA} - \tau_{fB} = \Delta/3$ into (10) and (11) and these into (4) and (8) one obtains, in an interior equilibrium, that

$$\hat{w} = \frac{\Delta t - 3(\alpha - 1)}{3[\Delta t - 2(\alpha - 1)]} \quad \text{and} \quad \hat{z} = \frac{1}{2} + \frac{\Delta(\alpha - 1)}{6[\Delta t - 2(\alpha - 1)]}. \quad (A.1.3)$$

The denominators of both $\hat{w}$ and $\hat{z}$ in (A.1.3) are strictly positive if and only if $\Delta > 2(\alpha - 1)/t$, a condition that is satisfied from the second order condition in (A.1.2). Moreover, $\hat{w} \geq 0$ if and only if $\Delta \geq 3(\alpha - 1)/t$, $\hat{w} \leq 1$ if and only if $\Delta \geq 3(\alpha - 1)/(2t)$ and $\hat{z} \leq 1$ if and only if $\Delta \geq 6(\alpha - 1)/[3t - (\alpha - 1)]$. A simple comparison between $\Delta \geq 3(\alpha - 1)/t$, $\Delta \geq 3(\alpha - 1)/(2t)$ and $\Delta \geq 6(\alpha - 1)/(3t) - (\alpha - 1)$ reveals that the critical Δ that satisfies $\hat{w}, \hat{z} \in [0, 1]$ (under the assumption that $t > \alpha - 1$) is given by $\Delta = 3(\alpha - 1)/t$. It is, thus, the case that the tax rates, given in (13), $\tau_{fA}^* = 2\Delta/3 - (\alpha - 1)/t > 0$ and $\tau_{fB}^* = \Delta/3 - (\alpha - 1)/t > 0$, are the equilibrium ones if $\Delta \geq 3(\alpha - 1)/t$. In such an equilibrium, the revenue functions, excluding the cost of the public good, are given by

$$R_A(x_A, x_B) = \frac{[2\Delta t - 3(\alpha - 1)]^2}{9t[\Delta t - 2(\alpha - 1)]} \quad \text{and} \quad R_B(x_A, x_B) = \frac{[\Delta t - 3(\alpha - 1)]^2}{9t[\Delta t - 2(\alpha - 1)]}. \quad (A.1.4)$$

Firm side tips in favor of jurisdiction A. Consider now the case $0 \leq \Delta \leq 3(\alpha - 1)/t$. In this case there is an equilibrium under which

$$\tau_{fA}^* = \frac{\alpha - 1}{t} \text{ and } \tau_{fB}^* = 0. \quad (A.1.5)$$

To show that this is an equilibrium we proceed as follows. Given that $\Delta \leq 3(\alpha - 1)/t$ the firm side tips in favor of A ($\hat{w}$ from (A.1.3) becomes zero). Suppose that $\tau_{fB}^* = 0$. The level of

jurisdiction A's tax rate under which tipping happens is the one that sets $n_{fA} - n_{fB} = 1$, and is given by $\tau_{fA}^* = (\alpha - 1)/t$. In this equilibrium, following from (8), $n_{sA} - n_{sB} = (\alpha - 1)/t < 1$ (given the assumption $t > (\alpha - 1)$), implying that jurisdiction B's shopper share will never be zero. Now, for (A.1.5) to be an equilibrium, jurisdiction B must have no incentive to deviate from $\tau_{fB}^* = 0$. Indeed this is the case since the share of firms located in B is zero. Jurisdiction A, too, has no incentive to deviate from $\tau_{fA}^* = (\alpha - 1)/t$ by lowering the level of the tax, since by doing so it cannot increase the number of firms located there. What remains to be checked is whether jurisdiction A has the incentive to raise its tax.

To show that it does not, suppose, first, that $\Delta \in (2(\alpha - 1)/t, 3(\alpha - 1)/t]$. Under this range of Δ jurisdiction A's tax rate response will be continuous and given by the derivatives of n_{fA} and n_{sA} from (10) and (11) with respect to τ_{fA}. Following from (A.1.1), the first order condition of jurisdiction A evaluated at $\tau_{fA} = (\alpha - 1)/t$ and assuming that $\tau_{fB} = 0$, gives

$$\left. \frac{\partial R_A}{\partial \tau_{fA}} \right|_{\tau_{fA} = \frac{\alpha - 1}{t} \text{ and } \tau_{fB} = 0} = \frac{\Delta t - 3(\alpha - 1)}{\Delta t - 2(\alpha - 1)}, \tag{A.1.6}$$

which is strictly negative when $\Delta \in (2(\alpha - 1)/t, 3(\alpha - 1)/t]$. Since under this Δ the revenue function is concave, jurisdiction A cannot profit by increasing the level of the tax.

Suppose now that $\Delta \leq 2(\alpha - 1)/t$. In this case a small increase in τ_{fA} will result in jurisdiction A loosing all firms. The reason behind this is the following. At the candidate equilibrium (A.1.5), we have $\hat{w} = 0$ (all firms locate in jurisdiction A). Suppose τ_{fA} increases. From (4), jurisdiction A will, initially, loose $\frac{1}{\Delta}$ firms to jurisdiction B. Given this, the firm difference between the two jurisdictions increases by $\frac{2}{\Delta}$. This, in turn, implies, following from (8), that jurisdiction A will loose $\frac{(\alpha - 1)}{2t} \frac{2}{\Delta}$ shoppers to jurisdiction B. This implies that the shopper difference between the two jurisdictions will increase by $\frac{(\alpha - 1)}{t} \frac{2}{\Delta}$. As a consequence, in addition to the initial $\frac{1}{\Delta}$, $\frac{2(\alpha - 1)}{\Delta^2 t}$ more firms will be lost. It is straightforward to show that his process continues to infinity and is described by the series

$$\sum_{k=1}^{\infty} \frac{1}{\Delta^k} \left(\frac{2(\alpha - 1)}{t} \right)^{k-1}, \tag{A.1.7}$$

This series gives the effect of a tax change on the number of firms that join jurisdiction A. Using the ratio test, this series is convergent only if $\Delta > 2(\alpha - 1)/t$.[40] If this condition is satisfied, then it can be shown that the series converges to $\frac{t}{\Delta t - 2(\alpha - 1)}$. Notice that this is the absolute of the derivative of n_{fA}, from (10), with respect to τ_{fA}.

If now $\Delta \leq 2(\alpha - 1)/t$ then the series diverges. In this case, a small tax increase results in jurisdiction A loosing all the firms. Hence, such a deviation is not profitable. This proves that

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[40]According to the ratio test, if $\left| \frac{a_{k+1}}{a_k} \right|$ approaches a number less than one as k approaches infinity, then $\sum_k a_k$ converges. Otherwise, the series diverges. In the case we analyze we have that

$$L = \lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| = \frac{2(\alpha - 1)}{t\Delta}, \tag{A.1.8}$$

and so $L < 1$ if and only if $\Delta > 2(\alpha - 1)/t$.

(A.1.5) is indeed an equilibrium. The revenue functions, excluding the cost of the public good, in this equilibrium are given by

$$R_A = \frac{\alpha - 1}{t} \text{ and } R_B = 0. \tag{A.1.9}$$

Collecting the intervals of Δ and the corresponding optimal responses one arrives at the tax rates, shares and revenue functions given in Proposition 1. $\qquad\square$

A.2 Characterization of jurisdiction B's net revenue function (A.2.10)

For the proof of proposition 2 (see Appendix A.4), we first need to determine the best-response of B (the follower) for any x_A. For this reason, in this and the next Appendix, we characterize the net revenue function of jurisdiction B, given by

$$R_B\left(x_A, x_B\right) = \begin{cases} \frac{[\Delta t - 3(\alpha-1)]^2}{9t[\Delta t - 2(\alpha-1)]} - \frac{x_B^2}{2} & \text{if} \quad \Delta \geq \frac{3(\alpha-1)}{t} & \text{(Region 1)} \\ -\frac{x_B^2}{2} & \text{if} \quad 0 \leq \Delta \leq \frac{3(\alpha-1)}{t} & \text{(Region 2)} \\ \frac{\alpha-1}{t} - \frac{x_B^2}{2} & \text{if} \quad -\frac{3(\alpha-1)}{t} \leq \Delta < 0 & \text{(Region 3)} \\ \frac{[-2\Delta t - 3(\alpha-1)]^2}{9t[-\Delta t - 2(\alpha-1)]} - \frac{x_B^2}{2} & \text{if} \quad \Delta \leq -\frac{3(\alpha-1)}{t} & \text{(Region 4)}. \end{cases} \tag{A.2.10}$$

As noted in text, (A.2.10) is simply a relabeling of (22). A word of caution is in order here. In this and the Appendices that follow, to avoid clustering the exposition, the discussion regarding B's net revenue function utilizes Figure 1 which refers to the net revenue function of A. This is feasible because the two revenue functions are identical after a relabeling of the variables. We use the properties of (A.2.10) in the proof of Proposition 2 in Appendix A.4.

In region 1, jurisdiction B is the low public good jurisdiction and the firm side is shared (no tipping). In this case, the net revenue function is decreasing. The reason is that a reduction in x_B, given that the degree of vertical differentiation is non-negative, has three (positive) effects on the revenues of this jurisdiction. Firstly, a higher degree of vertical differentiation implies that, following (17), tax competition is less intense and, consequently, jurisdiction B can levy a higher tax (*tax competition effect*). Secondly, the tax base of jurisdiction B, following (19), expands as vertical differentiation increases (*market share effect*). Despite the fact that jurisdiction B provides a lower level of the public good and its tax goes up, it is still able to attract a higher share of firms. This is because, following (17), the rival jurisdiction raises its tax by more (than jurisdiction B) as the degree of differentiation increases, which, in the presence of cross-group externalities, results in some firms leaving jurisdiction A and locating in the low public good jurisdiction B. Thirdly, a reduction in x_B reduces the total cost of the public good (*public good cost effect*). Region 1 gives the standard result of the one-sided model of Zissimos and Wooders (2008) that, in order to soften the harmful effect of tax competition, the low public good jurisdiction opts for a high degree of vertical differentiation. The present specification, however, introduces an additional effect, the *market share effect*, that is not present in one-sided frameworks.

In region 2, jurisdiction B is the low public good jurisdiction and the firm side has tipped in favor of jurisdiction A. Jurisdiction B, in this case, has no tax revenues and its net revenue function is given by $R_B(x_A, 0) = -x_B^2/2$, and is decreasing in x_B.

As shown in Figure 1 (and verified by equation (A.2.10)) moving from positive values of vertical differentiation to negative ones (and so jurisdiction B from low public good jurisdiction becomes the high public good one) involves a discontinuity in the net revenue function of jurisdiction B at the point where vertical differentiation is zero, that is $\Delta = 0$. To see this, recall from Proposition 1 that with zero vertical differentiation, and as long as the cross-group externality exists, and thus $\alpha > 1$, jurisdiction B sets a zero tax whereas jurisdiction A is able to sustain a strictly positive one. In this case, the market is tipped in favor of jurisdiction A, the jurisdiction with the largest pool of shoppers, and, consequently, firms. The presence of the shopper side introduces a second source of differentiation on top of the differentiation in public goods provision. Therefore, even when $\Delta = 0$, jurisdiction A can sustain a strictly positive tax because it attracts more shoppers. With the firm market tipped in favor of jurisdiction A, jurisdiction B's net revenues, following (A.2.10), are given by $R_B(x_A, x_B) = -x_B^2/2$. If now jurisdiction B changes its public good provision marginally by, say, $dx_B > 0$, then, with $x_A = x_B$ and $\Delta = -dx_B < 0$, jurisdiction B becomes the high public good jurisdiction and the firm side tips in its favor (jurisdiction B moves into region 3 in Figure 1). The net revenue function for jurisdiction B, in this case, jumps to $R_B(x_A, x_B) = (\alpha - 1)/t - x_B^2/2$ and, interestingly, its magnitude depends positively on the level of the cross-group externality α (and negatively on the cost of traveling t). This increase in revenues reflects the fact that, as noted earlier, the jurisdiction who has attracted all firms increases its tax rate as α increases. Such benefit of course disappears (and so does the discontinuity) if the cross-group externality is not present in the sense that $\alpha = 1$.

In region 3, jurisdiction B is the high public good jurisdiction, and so $\Delta \equiv x_A - x_B < 0$, and the firm side has tipped in its favor. As we mentioned above, the net revenue function jumps to $R_B(x_A, x_B) = (\alpha - 1)/t - x_B^2/2$. After the jump, the revenue function decreases up to $x_B = x_A + 3(\alpha - 1)/t$.

In region 4, jurisdiction B is the high public good jurisdiction and the firm side is shared (no tipping). Though the precise details are presented in the next Section, it suffices to say that, in this case, the net revenue function of jurisdiction B admits a number of shapes, ranging from inverse U-shaped to everywhere decreasing. (In order not to over burden the figure, Figure 1 depicts only one possible shape, the inverse U-shape). There is a simple intuition for why this is the case. Consider two extreme cases, depending on the threshold $x_A + 3(\alpha - 1)/t$, that x_B must exceed for this region to arise: One in which the threshold is sufficiently high and one in which it is sufficiently low. In the former case, which implies that x_A and/or α are high, public good expenditure on part of jurisdiction B is high. In this case the *public good cost effect* will dominate the other two effects identified in the discussion of region 1 above, generating a net revenue function that is everywhere decreasing. In the latter case, however, which arises when x_A and α are low, this will not be the case. For low levels of x_B the *tax competition effect* dominates the *public good cost effect* and the *market share effect* and, consequently, the net revenue function

has an increasing part up to the point where the public good cost effect dominates, as Figure 1 illustrates. There are, of course, since the threshold for which this region arises depends on x_A and the indirect externality α, intermediate cases which give rise to other shapes of the net revenues function.[41]

The next Appendix characterizes the shape of the net revenue function of jurisdiction B in region 4. $\qquad\square$

A.3 Shape of jurisdiction B's net revenue function in region 4 of (A.2.10)

This Appendix characterizes the shape of the net revenue function of jurisdiction B in region 4 of (A.2.10), where jurisdiction B is the high public good jurisdiction and the firm market is shared. This occurs when $x_B \geq x_A + 3(\alpha - 1)/t$ or, equivalently, $-\Delta \geq 3(\alpha - 1)/t$. It simplifies matters if it is assumed that $x_A = 0$ and so $x_B \geq 3(\alpha - 1)/t$. Once the properties of the net revenue function are established for $x_A = 0$ one, then, can easily allow for $x_A > 0$ and complete the characterization. This is done as follows. The net revenue function is equal to gross revenue minus cost. Following (16), the gross revenue depends only on the degree of vertical differentiation Δ and not on the individual values of the x_A and x_B. This implies that the restriction $x_A = 0$ is without any loss of generality, as far as the gross revenue is concerned. In addition, the threshold above which the firm side is shared only depends on Δ as well. The cost side of jurisdiction B does, of course, depend on the value of the x_A, because x_B must exceed a higher threshold, namely $x_A + 3(\alpha - 1)/t$, instead of $3(\alpha - 1)/t$. This implies that if $x_A > 0$ the cost is higher and thus it will be 'less likely' for the net revenue function to have an increasing part. If it does, the value of revenue will be lower than the value when $x_A = 0$. The proof of Proposition 2 makes use of this argument.

Given that $x_A = 0$, jurisdiction B's first order condition, following (A.2.10), is given by

$$\frac{\partial R_B(0, x_B)}{\partial x_B} = \frac{[2tx_B - 3(\alpha - 1)][2tx_B - 5(\alpha - 1)]}{9[tx_B - 2(\alpha - 1)]^2} - x_B. \qquad (A.3.11)$$

(A.3.11) has two components, the marginal revenue, given by

$$MR(0, x_B) = \frac{[2tx_B - 3(\alpha - 1)][2tx_B - 5(\alpha - 1)]}{9[tx_B - 2(\alpha - 1)]^2}, \qquad (A.3.12)$$

and the marginal cost given by

$$MC(0, x_B) = x_B. \qquad (A.3.13)$$

The characterization proceeds by establishing the curvature, and the relative positions, of the marginal revenue and marginal cost functions in, respectively, (A.3.12) and (A.3.13).

It is straightforward to verify that

$$\frac{\partial MR(0, x_B)}{\partial x_B} = \frac{2t(\alpha - 1)^2}{9[tx_B - 2(\alpha - 1)]^3} > 0 > -\frac{2[t(\alpha - 1)]^2}{3[tx_B - 2(\alpha - 1)]^4} = \frac{\partial^2 MR(0, x_B)}{\partial x_B^2}, \qquad (A.3.14)$$

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[41] The characterization of the net revenue function of jurisdiction B, used also in the proof of Proposition 2, is given in Appendix A.3.

and so $MR(0, x_B)$ is increasing and concave in x_B.

Evaluating—the left hand side of—(A.3.14) and the derivative of (A.3.13) at $x_B = 3(\alpha - 1)/t$ gives

$$\frac{\partial MR(0, 3(\alpha - 1)/t)}{\partial x_B} = \frac{2t}{9(\alpha - 1)} \quad \text{and} \quad \frac{\partial MC(0, 3(\alpha - 1)/t)}{\partial x_B} = 1, \tag{A.3.15}$$

and so

$$\frac{\partial MR(0, 3(\alpha - 1)/t)}{\partial x_B} = \frac{2t}{9(\alpha - 1)} \gtreqless \frac{\partial MC(0, 3(\alpha - 1)/t)}{\partial x_B} = 1 \ \text{ if and only if } \ \alpha \lesseqgtr 1 + 2t/9. \tag{A.3.16}$$

Notice also that evaluating (A.3.12) and (A.3.13) at $x_B = 3(\alpha - 1)/t$ gives, respectively,

$$MR\left(0, \frac{3(\alpha - 1)}{t}\right) = \frac{3}{9} \quad \text{and} \quad MC\left(0, \frac{3(\alpha - 1)}{t}\right) = \frac{3(\alpha - 1)}{t}, \tag{A.3.17}$$
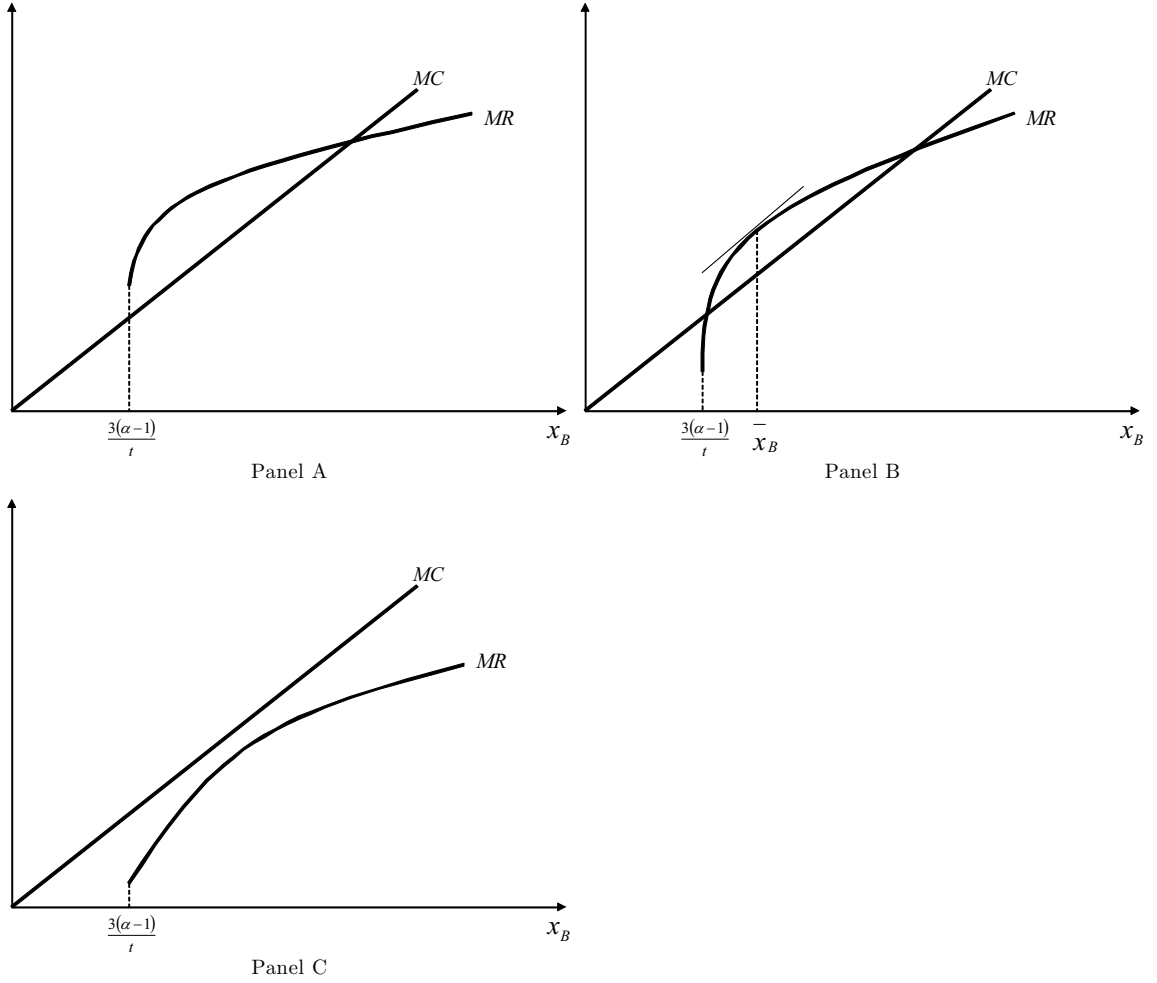
and so

$$MR\left(0, \frac{3(\alpha - 1)}{t}\right) = \frac{3}{9} \gtreqless MC\left(0, \frac{3(\alpha - 1)}{t}\right) = \frac{3(\alpha - 1)}{t} \ \text{ if and only if } \ \alpha \lesseqgtr 1 + t/9. \tag{A.3.18}$$

(A.3.18), in conjunction with (A.3.16), shows that the construction of the revenue function $R_B(\cdot)$ crucially depends on α and t. It will be shown that there are three cases to be considered, depending on the strength of the cross-group externality α. In case 1, the marginal revenue function starts above the marginal cost function and the two functions intersect once and so there is a unique maximum. In case 2, the marginal revenue function starts below, and intersects twice, the marginal cost function implying that the net revenue function exhibits first a local minimum and then a maximum. In case 3, the two functions never intersect implying that the net revenue function is everywhere decreasing. We now explore these three cases.

Case 1 $(\alpha \leq 1 + t/9)$. Notice first that following (A.3.16)—when $\alpha < 1 + 2t/9$ and at $x_B = 3(\alpha - 1)/t$—the function $MR(\cdot)$ is steeper than the function $MC(\cdot)$. It is also the case that, following (A.3.18), the function $MR(\cdot)$ starts out above $MC(\cdot)$. Given now that, following (A.3.14), $MR(\cdot)$ is increasing and strictly concave in x_B, and that $MC(\cdot)$ is increasing and linear in x_B, it must be the case that $MR(\cdot)$ and $MC(\cdot)$ intersect only once in the interval $[3(\alpha - 1)/t, \infty)$. At the point of intersection it must be that the first order condition is zero, that is $\partial R_B/\partial x_B = 0$. To the left of the intersection point, since $MR(\cdot) > MC(\cdot)$, it must be the case that $\partial R_B/\partial x_B > 0$, whereas to the right, since $MR(\cdot) < MC(\cdot)$, it must be that $\partial R_B/\partial x_B < 0$. Panel A of Figure 2 depicts this case.

Figure 2

Panel A

Panel B

Panel C

Case 2 $(1 + t/9 < \alpha < 1 + \zeta t)$.[42] When $\alpha > 1 + t/9$, following (A.3.18), the function $MR(\cdot)$ starts out below $MC(\cdot)$. Suppose also that $\alpha < 1 + 2t/9$ and so, following (A.3.16), $MR(\cdot)$ is steeper than the function $MC(\cdot)$. In this case the function $MR(\cdot)$ may intersect $MC(\cdot)$, at least for x_B's close enough to $3(\alpha - 1)/t$. This case establishes such possibility.

The function $MR(\cdot)$, given that it is increasing and strictly concave, intersects $MC(\cdot)$ if there exists a uniquely admissible level of public good, denoted by $\bar{x}_B$, such that the slope of $MR(\cdot)$ is equal to that of $MC(\cdot)$, when evaluated at $\bar{x}_B$, and at this $\bar{x}_B$ it is the case that $MR(\cdot)$ lies above $MC(\cdot)$. Using, the left hand side of, (A.3.14) and the fact that the slope of $MC(\cdot)$ is unity, it is the case that such $\bar{x}_B$ exists and is given by

$$\bar{x}_B = \frac{\sqrt[3]{\left[6t(\alpha - 1)^2\right]} + 6(\alpha - 1)}{3t}. \tag{A.3.19}$$

What remains to be shown now is that the function $MR(\cdot)$ lies above $MC(\cdot)$ when both are evaluated at the $\bar{x}_B$ given by (A.3.19). Substituting (A.3.19) into (A.3.12) and (A.3.13) reveals that this is the case as long as

$$\alpha < 1 + \zeta t, \tag{A.3.20}$$

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[42] The value of ζ is given below.

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where

$$\zeta = \frac{3\left(-1499 + 512\sqrt{15}\right)^{\frac{2}{3}} - 357 + 55\sqrt[3]{\left(-1499 + 512\sqrt{15}\right)}}{288\sqrt[3]{\left(-1499 + 512\sqrt{15}\right)}} \approx 0.114\,87.$$

The implication, if $\alpha \geq 1 + \zeta t$, is that $MR(\cdot)$ and $MC(\cdot)$ never intersect. In this case the $MR(\cdot)$ function lies below the $MC(\cdot)$ and so $\partial R_B/\partial x_B < 0$ for all $x_B > 3\left(\alpha - 1\right)/t$.

Collecting the intervals of α, it is the case that as long as $1 + t/9 < \alpha < 1 + \zeta t < 1 + 2t/9$, MR and MC intersect twice and, therefore, the first order condition holds with equality, in the sense that $\partial R_A/\partial x_B = 0$, twice in the interval $[3\left(\alpha - 1\right)/t, \infty)$. The first intersection gives a local minimum whereas the second gives a local maximum. Panel B of Figure 2 illustrates this case.

Case 3 $(\alpha \geq 1 + \zeta t)$. Now suppose that $\alpha \geq 1 + \zeta t$. Given the analysis of case 2, $MR(\cdot)$ starts below $MC(\cdot)$ and they never intersect. This implies that the net revenue function is decreasing in the relevant interval. Panel C of Figure 2 illustrates this possibility.

Summarizing the above discussion, under the assumption that $x_A = 0$, we have that (under case 1) if $\alpha \leq 1 + t/9$ the net revenue function is inverse U-shaped for $x_B \geq 3\left(\alpha - 1\right)/t$, where it attains a unique local maximum. If (under case 2) $1 + t/9 < \alpha < 1 + \zeta t$ the net revenue function exhibits a sideways S shape for $x_B \geq 3\left(\alpha - 1\right)/t$, where it attains a unique local maximum. If (under case 3) $\alpha \geq 1 + \zeta t$, the net revenue function is everywhere decreasing. $\qquad\square$

A.4   Proof of Proposition 2

The proof of this proposition proceeds by determining first, for given x_A, the best response of jurisdiction B. This, however, requires the determination of the shape of the net revenue function of jurisdiction B, for any x_A. The net revenue function of jurisdiction B has four regions. It is straightforward to establish the shape of the net revenue function in the first three regions, as illustrated in Figure 1 and discussed in Appendix A.2. Region 4, however, is a bit more complicated since, as shown in Appendix A.3, it exhibits a number of different shapes. The rest of the proof of this Proposition focuses more—relative to the other regions—on region 4 and thus utilizes Appendix A.3. Then, given the best response of jurisdiction B, the proof proceeds by determining the optimal choice of jurisdiction A.

High cross-group externality $(\alpha \geq 1 + 2t/9)$. We fix x_A and we find the best response of jurisdiction B. Following from Appendix A.3, and given that $\alpha \geq 1 + 2t/9$ (and so case 3 applies), the net revenue function of jurisdiction B in region 4 of (A.2.10) is decreasing, given that $x_A = 0$. Now we allow x_A to take any value. When $x_A > 0$ the net revenue of jurisdiction B must still be decreasing in region 4. The reason for this is that in order for jurisdiction B to be in that region it must expend more than $x_A + 3(\alpha - 1)/t$, which is higher than the threshold when $x_A = 0$. This implies that the cost is higher, while on the other hand the gross revenue only depends on the degree of vertical differentiation $\Delta = x_A - x_B$ in that region. The implication of this is that if the net revenue function is decreasing in region 4 when $x_A = 0$ it must also be decreasing when $x_A > 0$. To put it differently, in panel C of Figure 2, the function MR horizontally shifts to the

right by x_A, which implies that if there is no intersection between MR and MC when $x_A = 0$, there must not be one when $x_A > 0$. Combining all four regions of (A.2.10) (see also Figure 1, with the difference that the net revenue function in region 4 instead of inverse U-shaped is decreasing) the best response of jurisdiction x_B is to set either $x_B = 0$ or to marginally leapfrog jurisdiction A by setting $x_B = x_A + \varepsilon$.[43] If it sets $x_B = x_A + \varepsilon$, and so it becomes the high public good jurisdiction, its revenue is arbitrarily close to $R_B(x_A, x_A + \varepsilon) = (\alpha - 1)/t - (x_A)^2/2$. If it sets $x_B = 0$ its revenue depends on where x_A is located.

First, suppose that $x_A \geq 3(\alpha - 1)/t$. In this case jurisdiction B does not have an incentive to deviate from $x_B = 0$—with strictly positive revenues (since the firm side is shared), following region 1 in (A.2.10)—to $x_B = x_A + \varepsilon$ and so become a high public good jurisdiction (as in region 3 of (A.2.10)). To see this, suppose that $x_A = 3(\alpha - 1)/t$. In this case the net revenue of jurisdiction B at $x_B = x_A + \varepsilon$—following region 3 in (A.2.10)—is arbitrarily close to $R_B(3(\alpha - 1)/t, 0)) = (\alpha - 1)/(t) - [9(\alpha - 1)^2/(2t^2)]$, which is negative if and only if $\alpha \geq 1 + 2t/9$. Clearly, since, in this case, net revenues are decreasing in x_A, higher x_A than $3(\alpha - 1)/t$ will reduce the net revenue at $x_B = x_A + \varepsilon$ even further. These arguments suggest that if $x_A \geq 3(\alpha - 1)/t$ jurisdiction B's best response is to set $x_B = 0$.

Second, suppose that $x_A \leq 3(\alpha - 1)/t$ and so the market has tipped in favor of jurisdiction A, provided that A is the high public good jurisdiction, (region 2 in (A.2.10)). If jurisdiction B sets $x_B = 0$, its net revenue is $R_B(x_A, 0) = 0$, a consequence of the firm side tipping in favor of jurisdiction A. If it sets $x_B = x_A + \varepsilon$, and so it becomes the high public good jurisdiction, its revenue is arbitrarily close to $R_B(x_A, x_A + \varepsilon) = (\alpha - 1)/t - (x_A)^2/2$. Clearly, such a deviation is profitable if and only if $x_A < \sqrt{2(\alpha - 1)}/\sqrt{t}$. If $x_A \geq \sqrt{2(\alpha - 1)}/\sqrt{t}$, jurisdiction B does not want to deviate from $x_B = 0$. Moreover, $\sqrt{2(\alpha - 1)}/\sqrt{t} \leq 3(\alpha - 1)/t$ if and only if $\alpha \geq 1 + 2t/9$.

These arguments suggest that if $x_A \geq \sqrt{2(\alpha - 1)}/\sqrt{t}$, then jurisdiction B's best response is to set $x_B = 0$. If $x_A < \sqrt{2(\alpha - 1)}/\sqrt{t}$, then jurisdiction B's best response is to set $x_B = x_A + \varepsilon$. Given jurisdiction B's best response, jurisdiction A's best response is to set either $x_A = \sqrt{2(\alpha - 1)}/\sqrt{t}$ or $x_A = 0$. The reason for this is that any $x_A > \sqrt{2(\alpha - 1)}/\sqrt{t}$ is suboptimal because in such a case the net revenue function of jurisdiction A is decreasing, given that $x_B = 0$, following from the fact that the net revenue function of A is decreasing. This can be seen by looking at regions 3 and 4 in (A.2.10) after we switch the identities of the two jurisdictions, that is jurisdiction B becomes A. On the other hand, if $x_A < \sqrt{2(\alpha - 1)}/\sqrt{t}$ the best response of A, given that it will become the low public good jurisdiction, is to set $x_A = 0$. Both choices yield the same net revenue (zero) for the leader, suggesting that there are two candidate equilibria that involve firm tipping, since in both $\Delta < 3(\alpha - 1)/t$: $i)$ $x_A^* = \sqrt{2(\alpha - 1)}/\sqrt{t}$ and $x_B^* = 0$ and $ii)$ $x_A^* = 0$ and $x_B^* = \varepsilon$. The second one, however, is not an equilibrium given the discontinuity at $\Delta = 0$. The follower would always want to deviate by lowering x_B and when $x_B = 0$ it will want to raise x_B. The first

[43] The term 'best response' is used loosely in this proof. The reason for this is that, technically speaking, the best response function of jurisdiction B is not well-defined given the discontinuity at $\Delta = 0$ and the fact that x_B is a continuous variable. This is because a maximum in the neighborhood of $\Delta = 0$ does not exist. Nevertheless, this does not affect the proof in any significant way. The approach taken here identifies a candidate equilibrium and then shows that it is indeed an equilibrium and it is unique.

candidate is an equilibrium. It is easy to verify, given the analysis, that neither jurisdiction wishes to deviate. Furthermore, the equilibrium is unique. Notice that $x_A^* = \sqrt{2(\alpha - 1)}/\sqrt{t} > 4/9$ and so public good supply is higher than in the one-sided market.

Medium cross-group externality $(1 + \zeta t \leqslant \alpha < 1 + 2t/9)$.[44] In this case, too, the net revenue function of jurisdiction B is decreasing in $x_B \geq 3(\alpha - 1)/t$ when $x_A = 0$ (we are still in case 3 of A.3)) and thus, as argued above, it will still be decreasing with $x_A > 0$.

As in the case of the high cross-group externality examined above, the best response of jurisdiction x_B is to set either $x_B = 0$ or to marginally leapfrog jurisdiction A by setting $x_B = x_A + \varepsilon$. If it sets $x_B = x_A + \varepsilon$, and so it becomes the high public good jurisdiction, its net revenue is arbitrarily close to $R_B(x_A, x_A + \varepsilon) = (\alpha - 1)/t - (x_A)^2/2$. If it sets $x_B = 0$ its net revenue depends on where x_A is located.

First, we assume that $x_A \geq 3(\alpha - 1)/t$. If jurisdiction B sets $x_B = 0$, its net revenue, given that the firm side is shared, is $R_B(x_A, 0) = [x_A t - 3(\alpha - 1)]^2 /[9t(x_A t - 2(\alpha - 1))]$. If jurisdiction B leapfrogs jurisdiction A it must choose $x_B = x_A + \varepsilon$ and the net revenue is arbitrarily close to $R_B(x_B = x_A) = (\alpha - 1)/t - (x_A)^2/2$. It can be easily verified that $R_B(x_A, 0)$ is increasing in x_A (provided that $x_A \geq 3(\alpha - 1)/t$) and $R_B(x_A, x_B = x_A)$ is decreasing in x_A. Moreover, $R_B(x_A, 0) = 0$ at $x_A = 3(\alpha - 1)/t$ and $R_B(x_B = x_A) = 0$ at $x_A = \sqrt{2(\alpha - 1)}/\sqrt{t}$. Finally, $\sqrt{2(\alpha - 1)}/\sqrt{t} > 3(\alpha - 1)/t$, since $\alpha < 1 + 2t/9$. Thus, there exists a unique threshold, denoted by $\bar{x} \in \left(3(\alpha - 1)/t, \sqrt{2(\alpha - 1)}/\sqrt{t}\right)$, such that a deviation from $x_B = 0$ to $x_B = x_A + \varepsilon$ is unprofitable, for any $x_A \geq \bar{x}$. In addition, the threshold $\bar{x}$ increases as α increases (holding t fixed). (This can be seen by differentiating $R_B(x_A, x_B = 0)$ and $R_B(x_B = x_B)$ with respect to α). These arguments suggest that jurisdiction B's best response, when $x_A \geq 3(\alpha - 1)/t$, is to set $x_B = 0$ if $x_A \geq \bar{x}$ and to set $x_B = x_A + \varepsilon$ if $x_A < \bar{x}$.

Second, we assume that $x_A \leq 3(\alpha - 1)/t$. If jurisdiction B sets $x_B = 0$, its net revenue, given that the firm side tips in favor of A, is $R_B(x_A, 0) = 0$. If jurisdiction B leapfrogs jurisdiction A it must choose $x_B = x_A + \varepsilon$ and the net revenue is arbitrarily close to $R_B(x_B = x_A) = (\alpha - 1)/t - (x_A)^2/2$, which is strictly positive given that $\alpha < 1 + 2t/9$. Hence, jurisdiction B's best response when $x_A \leq 3(\alpha - 1)/t$ is to set $x_B = x_A + \varepsilon$.

Jurisdiction A's optimal response is either to set $x_A = \bar{x}$ (with $R_A(x_A = \bar{x}, x_B = 0) > 0$) in which case it will be the high public good jurisdiction, or to set $x_A = 0$ (with $R_A(x_A = 0, x_B = 0) = 0$), in which case it will be the low public good jurisdiction. (As we discussed in the high externality case above, it is suboptimal for jurisdiction A to set $x_A > \bar{x}$ or any $x_A < \bar{x}$ other than zero). We can show that $R_A(x_A = \bar{x}, x_B = 0) > 0$ as follows. The difference in the net revenues between the high and the low public good jurisdictions when $x_B = 0$ and $x_A \geq 3(\alpha - 1)/t$ is equal to $x_A/3 - x_A^2/2$. This difference is greater than zero if and only if $x_A \leq 2/3$. This inequality holds since $\sqrt{2(\alpha - 1)}/\sqrt{t} \leq 2/3$ when $\alpha < 1 + 2t/9$. Since the net revenue of the low public good jurisdiction is strictly positive, when $x_B = 0$ and $x_A \geq 3(\alpha - 1)/t$, it must be that the net

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[44] Where the value of ζ is given in Appendix A.3.

revenue of the high public good jurisdiction is also strictly positive. Therefore, jurisdiction A (the leader) chooses $x_A^* = \bar{x} \in \left(3\left(\alpha - 1\right)/t, \sqrt{2\left(\alpha - 1\right)}/\sqrt{t}\right)$ and jurisdiction B (the follower) chooses $x_B^* = 0$. Based on the above analysis, neither jurisdiction has an incentive to deviate.

In this equilibrium the firm side does not tip and, it is straightforward to verify that, $\bar{x} > 4/9$, which implies that the degree of vertical differentiation is higher than in the one-sided model.

Low cross-group externality $(1 \leq \alpha < 1 + \zeta t)$. In Appendix A.3 we showed that, when $\alpha < 1 + \zeta t$, the net revenue function of jurisdiction B attains a local maximum in $[3\left(\alpha - 1\right)/t, \infty)$, assuming that $x_A = 0$ (cases 1 and 2 in A.3). The same will be true for jurisdiction A if we switch the labels of the two jurisdictions. So, the net revenue function of jurisdiction A attains a local maximum $\tilde{x}_A$ in $[3\left(\alpha - 1\right)/t, \infty)$, assuming that $x_B = 0$, where $\tilde{x}_A$ satisfies the first order condition for a sharing equilibrium, given by

$$\frac{\partial R_A(x_A, 0)}{\partial x_A} = \frac{[2tx_A - 3\left(\alpha - 1\right)][2tx_A - 5\left(\alpha - 1\right)]}{9\left[tx_A - 2\left(\alpha - 1\right)\right]^2} - x_A = 0. \tag{A.4.21}$$

In this case it is possible that an investment on part of jurisdiction A in $[3\left(\alpha - 1\right)/t, \infty)$ maximizes its net revenue function (given that jurisdiction B chooses $x_B = 0$) *and* prevents jurisdiction B from leapfrogging and becoming the high public good jurisdiction. This was not the case in the previous two cases, where a public good investment on part of the leader was made solely for the purpose of preventing the follower from leapfrogging and becoming the high public good jurisdiction (since, due to high levels of the cross-group externality, the net revenue function of the leader was decreasing in $[3\left(\alpha - 1\right)/t, \infty))$.

From the discussion in the medium externality case, there exists an $\bar{x} \in (3\left(\alpha - 1\right)/t, \sqrt{2\left(\alpha - 1\right)}/\sqrt{t})$, such that jurisdiction B does not want to set $x_B = x_A + \varepsilon$, for all $x_A \geq \bar{x}$. Also, $R_B(x_A, x_B)$ is decreasing for $x_B > \tilde{x}_A$. This can be seen as follows. When $x_B = 0$, $R_A(x_A, x_B)$ decreases when $x_A > \tilde{x}_A$ (since the local maximum is attained at $\tilde{x}_A$ and beyond that point the cost dominates). Now jurisdiction B is the high public good jurisdiction (since $x_B > \tilde{x}_A$), but its net revenue function must be decreasing beyond $\tilde{x}_A$ because the cost is now even higher and Δ has not changed (everything shifts up by $\tilde{x}_A$). These arguments suggest that the best response of jurisdiction B is to set $x_B = 0$ if $x_A \geq \max\{\tilde{x}_A, \bar{x}\}$ and if $x_A < \max\{\tilde{x}_A, \bar{x}\}$ to set either $x_B = x_A + \varepsilon$ or $x_B = x_B^*$, where x_B^* satisfies the first order condition in $[3\left(\alpha - 1\right)/t + x_A, \infty)$.

Jurisdiction A, in turn, will either set $x_A = 0$ or $x_A = \max\{\tilde{x}_A, \bar{x}\}$. This can be explained as follows. If $x_A < \max\{\tilde{x}_A, \bar{x}\}$, and given B's best response, A will become the low public good jurisdiction and the best it can do is to set $x_A = 0$. On the other hand, any x_A strictly greater than $\max\{\tilde{x}_A, \bar{x}\}$ is suboptimal. In this case, B's best response is $x_B = 0$. If $\max\{\tilde{x}_A, \bar{x}\} = \tilde{x}_A$, the sub-optimality follows from the fact that $\tilde{x}_A$ is a local maximum and the fact, from A.3, that the net revenue function of A is either a sideways S or inverse U-shaped in this region. Hence, any $x_A > \tilde{x}_A$ will lower the net revenue. If $\max\{\tilde{x}_A, \bar{x}\} = \bar{x}$, then, given the above argument, we are already on the decreasing part of A's net revenue function, but such an over-investment is needed to prevent the rival from becoming the high public good jurisdiction. Nevertheless, $x_A > \bar{x}$ is excessive and consequently $x_A = \bar{x}$.

Jurisdiction A is better off following the latter strategy, that is, $x_A^* = \max\{\tilde{x}_A, \bar{x}\}$, and becoming the high public good jurisdiction, while $x_B^* = 0$. When the firm side does not tip (which is the case) the difference in net revenue between the high and low public good jurisdictions is $x_A/3 - x_A^2/2$, when $x_B = 0$. Jurisdiction A is better off becoming the high public good jurisdiction if and only if $x_A/3 \geq x_A^2/2$ which implies $x_A \leq 2/3$.

Moreover, it can be shown using (A.4.21) that the maximum $\tilde{x}_A$ is equal to 4/9 (when $\alpha \to 1$) and, as showed in the medium externality above, the maximum $\bar{x}$ is when α is as high as possible, which in the present case means arbitrarily close to $1 + \zeta t$. In such a case, $\bar{x}$ is arbitrarily close to .465. Both are less than 2/3, so jurisdiction A is better off becoming the high public good jurisdiction by setting $x_A^* = \max\{\tilde{x}_A, \bar{x}\}$. Jurisdiction B chooses $x_B^* = 0$. Based on the above analysis, neither jurisdiction has an incentive to deviate. Finally, for high α, $x_A = \bar{x} > 4/9$ and for low α, $x_A = \tilde{x}_A < 4/9$. $\qquad\qquad\square$

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