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# AD VALOREM TAXES AND THE FISCAL GAP IN FEDERATIONS

*by*

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# 1 Introduction

A common feature of federal fiscal systems is the fiscal gap: transfer of funds between levels of governments. Boadway and Keen (1996), in a framework of labor specific taxation, show the possibility of the fiscal gap to be negative requiring a transfers of resources from the states to the federal government. This result is intuitive. In federal systems, tax base co-occupancy by levels of government create negative vertical (between levels) fiscal externalities.<sup>1</sup> To undo such externalities, and achieve second-best efficiency in public good provision, the federal government sets a negative (specific) labor tax. This creates a need of resources that, in the absence of other revenues, must come from other levels of government.

This paper extends the contribution of Boadway and Keen (1996) by considering *ad valorem* taxation. As shown by Dahlby and Wilson (2003), and reconfirmed here, under *ad valorem* taxation the vertical externality can be of any sign. It is the implication of this, not explored by Dahlby and Wilson (2003), for the optimal federal tax and the sign of the fiscal gap that we are concerned with here. The analytics show that the federal government can always replicate the second-best unitary outcome. Interestingly, the sign of the federal optimal tax, in contrast to the case of specific taxation analyzed by Boadway and Keen (1996), crucially depends on the elasticity of the demand for labor. It is also shown that the direction of intergovernmental transfers can be towards either level of government, and so is, in general, ambiguous. The consequence of this is that a precise evaluation of the fiscal gap requires an explicit consideration of the underlying fundamentals of the federal economy.

## 2 The background of the model

The model is one of federal fiscal interactions, familiar from Boadway and Keen (1996), appropriately modified to deal with issues of *ad valorem* taxation. The model features a federal economy with  $k$  (symmetric) states, populated by  $nk$  identical, but immobile, households. The representative household has utility of the form  $u(x, l) + b(g) + B(G)$ , where  $x$  is a private good (and numeraire),  $l$  is labor, and  $g$  and  $G$  are state and federal public goods, respectively. The sub-utility  $u(x, l)$  is quasi-concave, increasing in  $x$  and decreasing in  $l$ . Both  $b(g)$  and  $B(G)$  are increasing and concave.

The local public good  $g$  provided by each state government is financed by taxing, at the rate  $t$ , labor income  $wl$ , where  $w$  denotes the gross wage rate. The federal government

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<sup>1</sup>See, for instance, Keen and Kotsogiannis (2002).

provides the federal public good  $G$ , financed by taxing labor income at the rate  $T$ . Consolidated taxation is denoted by  $\tau \equiv t + T$ .

The representative consumer maximizes  $u(x, l) + b(g) + B(G)$  subject to the constraint  $x = \bar{w}l$ , where  $\bar{w} = (1 - \tau)w$  is net wage. Labor supply, denoted by  $l(\bar{w})$ , is implicitly defined by  $u_x(\cdot)\bar{w} + u_l(\cdot) = 0$ .<sup>2</sup> It is assumed that  $l'(\bar{w}) > 0$ . Indirect utility is then given by  $v(\bar{w}) = u(\bar{w}l(\bar{w}), l(\bar{w}))$  with, as an envelope property,  $v' = u_x l$ . Output in each state is produced by technology  $f(nl)$ , which has the usual properties  $f' > 0 > f''$ . Output can be costlessly used for  $x$ ,  $g$  and  $G$ .

The private sector maximizes profits, given by  $\pi = f(nl) - wnl$ , and thus chooses labor demand that satisfies  $f'(nl) = w$ . This latter condition, since  $l(\bar{w})$ , implicitly defines the equilibrium gross wage rate  $w((1 - \tau), n)$  with, after denoting by  $z \equiv l'\bar{w}/l > 0$  the elasticity of labor supply and  $\epsilon \equiv f'/(nl f'') < 0$  the elasticity of demand for labor,

$$w_\tau = \frac{w}{1 - \tau} \frac{z}{z - \epsilon} > 0, \quad (1)$$

(with the inequality following from  $w > 0$  and  $0 < \tau < 1$ ). Net wage  $\bar{w} = (1 - \tau)w((1 - \tau), n)$ , following (1), gives

$$\bar{w}_\tau = (1 - \tau)w_\tau - w = \frac{w\epsilon}{z - \epsilon} < 0. \quad (2)$$

Notice, for later use, that the effect of taxation, state and/or federal, on the (gross) value of labor, denoted by  $r((1 - \tau), n) = w((1 - \tau), n)l(\bar{w})$ , is given by

$$r_\tau = w_\tau l + w l' \bar{w}_\tau = \frac{w}{1 - \tau} \frac{z}{z - \epsilon} l (1 + \epsilon), \quad (3)$$

and so its sign depends upon the elasticity of labor demand  $\epsilon$ . We turn to this shortly below.

Profits (rents)  $\pi$  are taxable by the federal government, at a fixed rate  $\theta$ , and by the state governments at the rate of  $(1 - \theta)$ .<sup>3</sup> Notice, for later use, that differentiation of  $\pi$ , after using (1), gives

$$\pi_\tau = -nlw_\tau = -nl \frac{w}{1 - \tau} \frac{z}{z - \epsilon} < 0. \quad (4)$$

Denoting by  $S$  the vertical transfer, the state public good is given by

$$g(t, T, \tau, S, n, \theta) = tnr((1 - \tau), n) + (1 - \theta)\pi((1 - \tau), n) + S, \quad (5)$$

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<sup>2</sup>A subscript denotes the derivative of a function of several variables whereas a prime denotes the derivative of a function of one variable.

<sup>3</sup>The allocation of rents for the level of taxation is of course important. See, for instance, Kotsogiannis and Makris (2002). We return to this, briefly, in Section 3.

with, after using (3),

$$g_t = nr + tnr_\tau + (1 - \theta)\pi_\tau, \quad (6)$$

$$g_T = tnr_\tau + (1 - \theta)\pi_\tau, \quad (7)$$

$$g_S = 1. \quad (8)$$

From (6) and (7), it can be readily seen that

$$g_t = g_T + wnl. \quad (9)$$

Federal public good provision is given by

$$G(t, T, \tau, S, n, \theta) = Tknr((1 - \tau), n) + k\theta\pi((1 - \tau), n) - kS, \quad (10)$$

with

$$G_T = knr + Tknr_\tau + k\theta\pi_\tau, \quad (11)$$

$$G_t = Tknr_\tau + k\theta\pi_\tau, \quad (12)$$

$$G_S = -k < 0. \quad (13)$$

Notice, from (11) and (12), that

$$G_T = knwl + G_t. \quad (14)$$

Equation (12), central to the present analysis, gives the vertical externality caused by the tax setting behavior of the state governments. Making use of (3) and (4), (12) can be written as<sup>4</sup>

$$G_t = knl \frac{w}{1 - \tau} \frac{z}{z - \epsilon} [T(1 + \epsilon) - \theta], \quad (15)$$

which takes the sign of  $T(1 + \epsilon) - \theta$ . This, thus, shows that—contrary to the case of specific taxation considered by Boadway and Keen (1996) in which this externality is unambiguously negative<sup>5</sup>—under *ad valorem* taxation the vertical fiscal externalitty can be positive. It is the implication of this for the level of federal taxation and the sign of the fiscal gap that is our focus here.

The analysis now proceeds by exploring the equilibrium outcome pursued by a unitary country. This will serve as a benchmark so the equilibrium alternative, that of when fiscal policies are pursued by both levels of government, can be compared with.

Equilibrium in a unitary country involves maximization of  $v(\bar{w}) + b(g) + B(G)$ , choosing  $\tau, G, g$ , subject to the consolidated budget constraint  $G + kg = \tau knr((1 - \tau), n) +$

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<sup>4</sup>Equation (15) confirms, for the case in which  $k = n = 1$ , the result in Dahlby and Wilson (2003).

<sup>5</sup>This implies that state specific-taxes are *too high* from an equilibrium point of view.

$k\pi((1 - \tau), n)$ . It is straightforward to verify that the necessary conditions yield the familiar optimality rule for the provision of public goods in a second-best environment, given by

$$\frac{nb'(g)}{u_x} = \frac{1}{1 - \frac{\tau wl'}{l}} = \frac{nkB'(G)}{u_x}, \quad (16)$$

which simply states that at the unitary optimum the *ad valorem* tax  $\tau$  is set such that the sum of the marginal rate of substitution between both the federal and state public goods and the private good  $x$  must be equal to the marginal cost of public funds (MCPF), given by  $1/(1 - (\tau wl'/l))$ . Equations (16) together with the unitary budget constraint characterize the unitary second-best optimum.

We turn now to the characterization of the equilibrium in which fiscal policy is pursued by both levels of government. This equilibrium focuses on the case in which the federal government has a first mover advantage *vis à vis* the state governments. Each state government holds Nash conjectures relative to the federal and all other state governments.

### 3 Characterization of the equilibrium

The typical state chooses  $(t, g)$  to maximize  $v(\bar{w}) + b(g) + B(G)$  subject to (5), taking as given the decision variables of the federal government,  $(T, S, G, n, \theta)$ . The necessary condition of this problem is given by

$$v'\bar{w}_\tau + b'g_t = 0 \equiv \Omega(t, T, S, n, \theta), \quad (17)$$

which implicitly defines  $t(T, S; \theta, n)$  with, in particular,

$$t_T = -1 + \frac{n(b')^2 r_\tau - b''v'nwl\bar{w}_\tau}{b'\Omega_t}, \quad (18)$$

$$t_S = -b''g_t/\Omega_t \leq 0, \quad (19)$$

where  $\Omega_t < 0$  is the second order condition of the state government maximization problem.

The federal government maximizes  $v(\bar{w}) + b(g(t, T, \tau, S, \theta, n)) + B(G(t, T, \tau, S, n, \theta))$  subject to  $t(T, S, n, \theta)$  choosing appropriately  $T, S$  (and residually  $G$ ). The necessary conditions for  $T$  and  $S$  being, respectively,

$$v'\bar{w}_\tau(1 + t_T) + b'(g_T + g_t t_T) + B'(G_T + G_t t_T) = 0, \quad (20)$$

$$v'\bar{w}_\tau t_S + b'(g_S + g_t t_S) + B'(G_S + G_t t_S) = 0. \quad (21)$$

It can be shown that (20) can be written—after making use of (8), (9), (13), (14), and (17)—as

$$\frac{nb'(g)}{u_x} \left[ \frac{1}{1 + G_t(1 + t_T)/(nkw)} \right] = \frac{nkB'(G)}{u_x}. \quad (22)$$

Equation (22) is central to this analysis. It shows how the MCPF for the federal government relates to that of the state governments.<sup>6</sup> If it happens to be that  $G_t(1 + t_T) < 0$  then the MCPF for the federal government exceeds that of the state governments. But if it is the case that  $G_t(1 + t_T) > 0$  then the state MCPF exceeds that of the federal. Though, as already noted after (15), the sign of  $G_t$  can be determined, close inspection of (18) reveals that the sign of  $1 + t_T$  (and so  $G_t(1 + t_T)$ ) is in general ambiguous.<sup>7</sup> This has an important implication for the direction of intergovernmental transfer  $S$  to which we now turn.

The choice of the transfers  $S$  satisfies (21). Evaluating (21), using (17) and multiplying through by  $n/u_x$ , one obtains

$$\left( \frac{nb'(g)}{u_x} - \frac{nkB'(G)}{u_x} \right) + \frac{nB'(G)}{u_x} G_t t_S = 0. \quad (23)$$

Equation (23) determines the direction of the intergovernmental transfer  $S$  in the presence of the vertical externality. The terms within the parentheses capture the difference in MCPF between the state and federal public goods. They simply say that the transfer should go from the government with the lower MCPF to the one with the higher MCPF. The last term in (23), that points to the opposite direction, captures the effect of the transfer on the extent of the vertical externality. To see this suppose that the state MCPF is greater than the federal MCPF (and so the federal public good is too high because of a positive vertical externality  $G_t$ ) then the transfer should go from the federal government to the state governments. But the transfer will affect state taxation too and, therefore, the extent of the vertical externality. So, following (19), the transfer, since  $G_t t_S < 0$ , will reduce the vertical externality. At the optimum, of course, (19) will hold with equality. Analogous reasoning applies to the case in which  $G_t < 0$ .

Combining (22) with (23) it is straightforward to show that the optimal federal tax is characterized by  $G_t = 0$  which, following from (22), replicates the unitary optimum in (16). Following from (15), the optimal federal tax is given by

$$T^* = \theta / (1 + \epsilon), \quad (24)$$

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<sup>6</sup>Though the optimality condition in (22) is expressed in terms of the marginal rate of substitution between public (federal and state) and private consumption its interpretation is in terms of the MCPF (federal and state). The benefit from doing this is that a simple comparison of (16) and (22) reveals the source of inefficiency at the level of state taxation. This interpretation is also followed after (23).

<sup>7</sup>The elasticity of labor demand  $\epsilon$  being a critical factor. To see this notice that, as noted earlier, if labor demand is elastic then, following (3),  $r_\tau < 0$  and so, with  $b', \bar{w}_\tau, \Omega_t < 0$ ,  $1 + t_T > 0$ . But if labor demand is inelastic then  $r_\tau > 0$  and  $1 + t_T$  may take a negative sign (a possibility that does not make appearance in the case of specific taxation of Bodaway and Keen (1996)). Such ambiguity appears also in capital taxation models: see, for instance, Keen and Kotsogiannis (2003).



and so, with  $\theta > 0$ , its sign depends on the elasticity of the demand for labor  $\epsilon$ . Comparison of (15) and (24) reveals that if the demand for labor is elastic then  $G_t < 0$  and so, since  $T^* < 0$ , the federal government subsidizes labor. If, on the other hand, it is inelastic, a necessary condition for  $G_t > 0$ , then the federal government sets  $T^* > 0$  and so taxes labor. To summarize:

**Proposition 1** *The federal government always replicates the second-best optimum with the appropriate choice of the federal ad valorem tax  $T^* = \theta/(1 + \epsilon)$ . More specifically,*

(a) *if the demand for labor is elastic, and thus the vertical externality is negative, then the federal government subsidizes labor income.*

(b) *If the demand for labor is inelastic, a necessary condition for the vertical externality to be positive, then the federal government taxes labor income.*

The result behind Proposition 1 has an important implication for the sign of the intergovernmental transfer. It is intuitive that such transfer will depend on a number of factors, including the size of rents and the intensity of preferences for the public good. To see this suppose that  $\theta = 0$  and thus the federal government has no access to revenues from rents. In this case  $T^* = 0$  and so, following from (10), the federal public expenditure should be financed by transfers from the state, that is the fiscal gap is negative. If now  $G^* = 0$ , and so public expenditure is worthless, then, it is clear—following again from (10)—that the sign of the fiscal gap will depend upon the two federal revenue sources: the revenues from labor tax, given by  $\theta f\omega/(1 + \epsilon)$  (where  $\omega \equiv f'nl/f > 0$  is the elasticity of production with respect to employment,  $nl$ ), and the revenues from rents, given by  $\theta\pi = \theta f(1 - \omega)$ . Clearly, with positive equilibrium profits (and so  $1 - \omega > 0$ ), one can easily identify conditions under which the sign of  $S$  can be positive or negative. That the fiscal gap is, in general, ambiguous might not be very surprising. What is surprising though is that this ambiguity does not arise, in the case considered here, from a property of the production function (as it does in the case of specific taxation of Boadway and Keen (1996)), but merely from the elasticity of the demand for labor.

## 4 Concluding remarks

In a framework of federal fiscal interactions where taxation is of *ad valorem* form, this paper has derived the optimal federal tax that allows the government to internalize the fiscal externalities (positive or negative) that arise at the state level of government and achieve the second-best level of public good provision. The analysis has also emphasized that the fiscal gap is, in general, ambiguous. A precise evaluation of the fiscal gap requires an explicit consideration of the underlying fundamentals of the federal economy.

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## Appendices

### Appendix A

#### Derivation of equation (16) in text.

In this Appendix we derive the optimal rule for public good provision in a second-best environment in a unitary country.

The maximization problem for the unitary government is to maximize  $v(\bar{w})+b(g)+B(G)$ , choosing  $\tau$ ,  $G$  and  $g$ , and subject to the consolidated budget constraint as given by

$$G + kg = \tau kw((1 - \tau), n)l((1 - \tau)w((1 - \tau), n)) + k\pi((1 - \tau), n). \quad (\text{A.1})$$

Denoting by  $\mu$  the Lagrange multiplier associated with the budget constraint, necessary conditions of this maximization problem are given by

$$(\tau) : \quad v'((1 - \tau)w_\tau - w) + \mu A = 0, \quad (\text{A.2})$$

$$(G) : \quad B'(G) - \mu = 0, \quad (\text{A.3})$$

$$(g) : \quad b'(g) - \mu k = 0, \quad (\text{A.4})$$

where

$$A \equiv kwnl + \tau kw_\tau nl + \tau kwnl'((1 - \tau)w_\tau - w) + k\pi_\tau, \quad (\text{A.5})$$

with  $\pi_\tau$  conveniently written as

$$\pi_\tau = \frac{f''n^2l'wl}{1 - f''nl(1 - \tau)} < 0. \quad (\text{A.6})$$

Notice now that (1) can be written as

$$w_\tau = \frac{-f''nl'w}{1 - f''nl(1 - \tau)} > 0. \quad (\text{A.7})$$

Notice also, following from the firm's first order condition  $f'(nl) = w$ , that

$$l'(w) = 1/(nf''(nl)). \quad (\text{A.8})$$

Substituting (A.8) into (A.7) and that into (A.5) and simplifying, one arrives at

$$A \equiv kn((1 - \tau)w_\tau - w)(\tau wl' - l). \quad (\text{A.9})$$

Making use now of the fact that  $v' = u_x l$  and (A.9), straightforward manipulation of the first order conditions (A.2)-(A.4) gives the second-best tax rule in (16).  $\square$

## Appendix B

### Derivation of equations (18) and (19) in text.

Application of the implicit function theorem to (17) gives

$$\Omega_t = v''((1-\tau)w_\tau - w)^2 + v'((1-\tau)w_{\tau\tau} - 2w_\tau) + b''g_t^2 + b'g_{tt}, \quad (\text{B.1})$$

$$\Omega_T = v''((1-\tau)w_\tau - w)^2 + v'((1-\tau)w_{\tau\tau} - 2w_\tau) + b''g_Tg_t + b'g_{tT}. \quad (\text{B.2})$$

Comparison of (B.1) and (B.2) gives

$$\Omega_T = \Omega_t + b''g_Tg_t + b'g_{tT} - b''(g_t)^2 - b'g_{tt}. \quad (\text{B.3})$$

Hence

$$t_T = -\frac{\Omega_T}{\Omega_t}, \quad (\text{B.4})$$

$$= -\frac{\Omega_t + b''g_Tg_t + b'g_{tT} - b''(g_t)^2 - b'g_{tt}}{\Omega_t}, \quad (\text{B.5})$$

$$= -1 - \frac{b''(g_Tg_t - (g_t)^2) + b'(g_{tT} - g_{tt})}{\Omega_t}. \quad (\text{B.6})$$

where the second inequality follows from substituting (B.3) into (B.4).

Notice that, following (9), the term in (B.6)

$$b''(g_Tg_t - (g_t)^2) = b''((g_t - nlw)g_t - (g_t)^2), \quad (\text{B.7})$$

$$= b''(-nlwg_t), \quad (\text{B.8})$$

which, upon using (17), becomes

$$b''(g_Tg_t - (g_t)^2) = b''v'(-w + (1-\tau)w_\tau)nlw / b'. \quad (\text{B.9})$$

We determine next the derivatives  $g_{tT}$  and  $g_{tt}$  in (B.6). Differentiating (6) with respect to  $t$  and  $T$ , and comparing gives

$$g_{tT} - g_{tt} = -n(lw_\tau + l'((1-\tau)w_\tau - w)w). \quad (\text{B.10})$$

Using (B.9) and (B.10) into (B.6) one arrives at (18) in text.

We turn now to equation (19). Making use of the implicit function theorem on (17) one obtains

$$t_S = -\frac{\Omega_S}{\Omega_t}, \quad (\text{B.11})$$

$$= -\frac{b''g_Sg_t + b'g_{tS}}{\Omega_t}, \quad (\text{B.12})$$

$$= -\frac{b''}{\Omega_t}g_t, \quad (\text{B.13})$$

where the last equality follows from (8) and the fact that  $g_t$ , in equation (6), is independent of  $S$  implying  $g_{tS} = 0$ . Equation (B.13) is then equation (19) in the text.  $\square$

## Appendix C

### Derivation of equations (22) in text.

Substituting (8), (9), (13), (14) and (17) into (20) one obtains

$$B' = \frac{nlwb'}{knlw + G_t(1 + t_T)}. \quad (\text{C.1})$$

Multiplying through (C.1) by  $n/u_x$ , dividing the l.h.s by  $knlw$  and rearranging gives (22).  $\square$

## Appendix D

### Derivation of equation (15).

Denote the net wage by  $\bar{w} = (1 - \tau)w$ . Define now the elasticity of supply for labor, denoted by  $z$ , as in the text (that is,  $z \equiv l'\bar{w}/l > 0$ ) and the elasticity of demand for labor, denoted by  $\epsilon$ , as also in the text (that is,  $\epsilon \equiv f'/(nl f'') < 0$ ). Then, after using the fact that in equilibrium  $f'(nl) = w$ , expression (1) can be written as

$$w_\tau = \frac{w}{1 - \tau} \frac{z}{z - \epsilon} > 0, \quad (\text{D.1})$$

where the inequality follows from  $w, z > 0$ ,  $\tau < 1$  and  $\epsilon < 0$ .

Substituting (D.1) into (2) gives

$$(1 - \tau)w_\tau - w = \frac{w\epsilon}{z - \epsilon} < 0. \quad (\text{D.2})$$

Similarly, (4) becomes

$$\pi_\tau = -nl \frac{w}{1 - \tau} \frac{z}{z - \epsilon} < 0. \quad (\text{D.3})$$

Substituting (D.1) and (D.2) into the effect of state tax on the federal tax revenue only (with  $T > 0$ ) gives

$$Tkn(w_\tau l + wl'((1 - \tau)w_\tau - w)) = Tknl \frac{w}{1 - \tau} \frac{z}{z - \epsilon} (1 + \epsilon), \quad (\text{D.4})$$

which can be negative or positive depending on the elasticity of demand for labor,  $\epsilon$ . If the demand for labor is inelastic (elastic) then this externality (ignoring revenues from profits for the moment) is strictly positive (negative).

Adding now (D.3) and (D.4), one obtains

$$G_t = knl \frac{w}{1 - \tau} \frac{z}{z - \epsilon} [T(1 + \epsilon) - \theta]. \quad (\text{D.5})$$

Expression (D.5) confirms the result in Dahlby and Wilson (2003) in the case in which  $k = n = 1$ . □

## Appendix E

**Proof of the statement that the optimal tax is characterized by  $G_t = 0$ .**

Substituting (22) into (23) gives

$$\frac{nb'}{u_x} \left( 1 - \frac{1}{1 + G_t(1 + t_T)/(knlw)} + \frac{G_t t_S}{1 + G_t(1 + t_T)/(knlw)} \right) = 0, \quad (\text{E.1})$$

which simplifies to

$$\frac{nb'}{u_x} \frac{1}{1 + G_t(1 + t_T)/(knlw)} G_t \left( \frac{1 + t_T}{nlw} + t_S \right) = 0. \quad (\text{E.2})$$

With  $1 + G_t(1 + t_T)/(knlw) \neq 0$  and also (assumed to be the case that)  $(1 + t_T)/(nlw) + t_S \neq 0$ , it follows from (E.2) that  $G_t = 0$  as claimed. □



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