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Registration Taxes on Cars Inducing International Price Discrimination: An Optimal Tariff Approach

by

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Abstract

Pre-tax car prices are particularly low in EU countries with high registration taxes but no car production, meaning that the tax is equivalent to an import tariff and induces international price discrimination. The paper develops a theorectical model to analyse the European Commission's policy of facilitating arbitrage and thereby reducing car price differences. The effects on prices, quantities and welfare depend crucially on whether the tax is exogenous or whether it is set optimally by the importing country. The optimal tax rate depends positively on the car manufacturers' scope to price discriminate. Thus when arbitrage costs fall, tax rates are reduced.

JEL classification: F13 F15 H21

Keywords: registration tax, optimal tariff, price discrimination, car prices, European Union, tax harmonization

Outline

Non-Technical Summary

The EU car market is characterized by large price differences. Pre-tax prices are particularly low in countries with a high registration tax on cars, which do not have a car manufacturing industry, meaning that an import tariff and a specific tax are equivalent. The tax is partially born by foreign car producers who set lower prices than in other countries, and thus introducing a tax may decrease import prices and increase welfare. The European Commission aims at reducing car price differences by facilitating arbitrage, for which pre-tax prices are relevant.

The paper studies what effects it would have if this EU policy was successful. When markets become more integrated and arbitrage costs fall, the scope for price discrimination becomes smaller. As long as the tax rate is fixed, the decrease of the feasible price differences leads to a rise of the low pre-tax prices in the importing country, which reduces the volume of trade, and to a fall of the high pre-tax prices in the producing country.

When the tax rate reacts to the smaller possibilities to induce low import prices and is set optimally by the importing country, a change in arbitrage costs has additional effects by altering the tax rate. Fostering arbitrage would lead to a reduction of the tax rates which would be welcomed by the European Commission as a step towards tax harmonization. The pre-tax price increases and the tax rate falls, and either effect on tax-inclusive may dominate. Thus, the tax-inclusive price and thereby the volume of imports depends non-monotonically on arbitrage costs. Moreover, for an intermediate range of the maximum price difference, the optimal adjustment of the tax implies that the price in the producing country is unaffected when the scope for price discrimination becomes smaller. In this case, the adjustment takes place on the markets of the importing country only. Welfare effects depend on whether the tax rate is endogenous or exogenous, too. With an exogenous tax rate, price discrimination increases welfare although total quantity does not increase, which stands in contrast to the standard welfare result of Varian (1985). In contrast, welfare falls for a wide range of parameter values when the tax is endogenous, as the optimal tax rate increases when the scope to price discriminate becomes larger.

1 Introduction

Car prices within the European Union differ substantially, and the price difference for an identical new car may amount to several thousands euro. Pre-tax prices are particularly low in countries with high registration taxes on cars, because the tax is partially born by car producers themselves, which set lower prices than in other countries. For instance, in the EU27, car prices are the lowest in Denmark where the registration tax is above 100%. In Finland, which has the lowest pre-tax car prices in the eurozone, the tax amounts to around 30%.[1] In May 2006 the net price of an Opel Vectra was 21000 euros in Germany to which only VAT has to be added. In contrast, the pre-tax price was only 15800 euros in Denmark, but including the registration tax and VAT, a Danish consumer has to pay more than 40000 euros for the car (see table 1).

Firms' scope to price discriminate may be limited by the possibility of arbitrage. For arbitrage within the European internal market, it is the pre-tax prices that are relevant, as the tax rates of the country where the car will be registered apply. Thus although a Danish consumer pays a much higher tax-inclusive price, German consumers would like to buy their car in Denmark – which often means a re-import – and not vice versa. Countries with high registration taxes on cars typically do not produce cars themselves, implying that a specific tax cannot be distinguished from an import tariff. Like a tariff, the tax may lower the import price and thereby increase domestic welfare at the expense of foreign car manufacturers, and the ideas of optimal tariff theory apply. It is this aspect of registration taxes that is the focus of the paper.

	Germany	France	Netherlands	Finland	Denmark
Pre-tax price	21000	18100	18000	17200	15800
Tax-incl. price	24300	21800	27300	29800	43200

Table 1: Pre-tax and tax-inclusive prices for an Opel Vectra, May 2006. (Source: European Commission)

[1]Other countries with a substantial registration tax include the Netherlands, Ireland, Greece and Portugal. Lutz (2004) has found a significant effect of registration taxes on pre-tax car prices. Further empirical studies on European car prices include Brenkers and Verboven (2006), Ginsburgh (1985), and Goldberg and Verboven (2001). Neither of these four studies include the high-tax countries Denmark, Finland and Greece.

The European Commission wishes to harmonize taxes within the EU and has suggested to reduce registration taxes to low levels or to completely abolish them (European Commission 2002). The reasons given include that the they are an obstacle for the free movement of passenger cars[2] and that taxation of cars should be CO_2-based. The European Commission also aims to reduce the large price differences, as they run counter to the spirit of the internal market. To foster arbitrage, it has introduced new rules for distribution systems of cars in 2002, as the distribution systems were regarded to enable the car producers to segment markets[3]. Since 2005, car dealers can also open additional outlets in other EU countries. However, up to now price differences have only decreased slightly (European Commission 2006, 2007).

This paper develops a theoretical model that analyses the effects of the policy of the European Commission to reduce price differences by enhancing arbitrage. The focus is on price discrimination that is induced by registration taxes. For a fixed tax rate, a fall in arbitrage costs and thus a smaller scope for price discrimination leads to a decrease of the high price in the producing country and to an increase of the low price in the importing country. As a consequence, the volume of imports falls. Moreover, price discrimination increases aggregate welfare, although total quantity remains constant, as it partially offsets the distortion from the tax. This result stands in contrast to Varian's (1985) well-known result, that price discrimination can only improve welfare if the total quantity increases. Conversely, limiting the price discrimination by enhancing arbitrage reduces welfare.

When it is taken into account, that the tax rate may be changed in response to the changing market conditions, these results are qualified. If the government of the importing country sets the optimal tax rate to maximize domestic welfare, it will levy low taxes when arbitrage costs and therefore viable price differences are small. The aim of the European Commission that registration taxes should be reduced is then not achieved by tax harmonization directly but indirectly by limiting the margin for price discrimination by car producers and thereby constraining the importing countries' possibility to induce lower import prices by levying the tax.

Thus falling arbitrage costs that constrain price differences not only change prices directly but have additional effects by changing the optimal tax rate set. In particular, for a certain range of arbitrage costs, the adjustment of the tax completely offsets the initial effect on

[2] One of the problems occuring in this context is that if a person moves within the EU and wants to take her used car with her, this may result in double taxation.

[3] For details on the new rules for distribution systems of cars, see Brenkers and Verboven (2006).

the price in the producing country. In this case, it is optimal for the government to set the tax at a level such that the maximum possible pre-tax price difference is induced without affecting the prices in the producing country. From the point of view of the car producers, none of the prices has fallen in the new equilibrium. In the importing country, the pre-tax price increases, but as the tax rate falls, the tax-inclusive price falls and imports, total quantity and aggregate welfare increase. This stands in contrast to the above-mentioned result that for a given tax rate enhancing arbitrage reduces aggregate welfare.

If arbitrage costs fall when they are already small, the results are different. The change in the tax rate is too small to offset the effect on the price in the producing country, which therefore falls. Likewise, the fall in the tax rate does not outweigh the increase in the pre-tax price in the importing country any more, and the tax-inclusive price increases, too. Thus, the price that consumers pay in the importing country - and therefore the volume of imports - depends non-monotonically on the scope to price discriminate.

Annual car taxes, that may have similar effects, are usually based on characteristics of the car such as horsepower or fuel efficiency. In contrast, the high registration taxes are generally levied ad valorem. However, the model cannot be solved analytically for the case of an ad valorem tax. Therefore, in this paper per unit taxation is considered first, and subsequently, it is shown by numerical simulation that the results are the same for both kinds of taxation.

The paper is organized as follows. Section 2 sets out the model. Section 3 briefly discusses the case of an exogenous per unit tax rate, whereas the main part of the paper is in section 4 where the interaction of price discrimination and optimal taxation is analysed. Section 5 summarizes the results obtained for ad valorem taxation and section 6 concludes.

2 The Model

There are two countries. In one of them, one firm produces and sells the good, whereas the other country imports it.[4] Marginal costs c are constant. The importing country

[4] The results do not hinge on the assumption of a monopoly. It can be shown that the qualitative effects are the same when there are two symmetric firms in the producing country with Bertrand competition in differentiated products as in Holmes (1989). Price differentiation may have fundamentally different effects in oligopolies when there is best response asymmetry, i.e. when the strong market of one firm is the weak market of the other one. This is not the case in the discussion of registration taxes, as all firms would like to set the lower price in the same country. A survey on price discrimination in oligopolies can

levies a consumption tax (for instance a registration tax) on the good that induces an incentive for price discrimination.[5] Demand in the producing country equals

$$X = a - bP, \tag{1}$$

where X denotes quantity and P the price. In the importing country, the quantity is denoted by x and the producer price by p. Demand depends, however, on the tax-inclusive price ρ

$$x = \alpha(a - b\rho). \tag{2}$$

Note that the parameters in the demand function are the same in both countries, apart from the parameter α, that represents the relative size of the importing country.

Thus, without the tax, the monopolist would set the same price in both countries even if he could set the prices separately. The only incentive for price discrimination comes from the tax.

The tax may be levied per unit or ad valorem, i.e.

$$\rho = p + t \qquad \text{or} \qquad \rho = \kappa p, \tag{3}$$

where $t > 0$ is the per unit tax rate and $\kappa > 1$ represents an ad valorem tax with the rate $\kappa - 1$.

Price discrimination may be limited by the possibility of arbitrage, for which the pre-tax prices are relevant. Arbitrage is possible at costs s, which thereby is the maximum feasible price difference the firm can sustain. If the price difference is higher than s, arbitrage would take place and the firm would not sell anything at the high price, which cannot be profit-maximizing behavior. Arbitrage costs s should be understood as the cost-equivalent of all barriers to arbitrage.[6]

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be found in Stole (2007).

[5] In the producing country, a registration tax always lowers welfare and it is thus assumed that no specific tax is levied. There is of course the normal VAT on cars in producing countries, but the focus of this paper is on specific taxation, issues of tax competition refering to VAT are beyond the scope of the paper. A survey on tax competition can be found in Lockwood (2001).

[6] The price difference can be higher than the actual transportation costs and reflects all obstacles for arbitrage. For instance, consumers may have preferences to buy at their local car dealer, because they might want to establish a good basis for the long-term relation of after sales services. It may also be easier to make warranty claims at the car dealer where the car was bought, although car producers are obliged to grant warranty that is valid in the whole EU).

The government of the importing country sets the tax rate in an initial stage and aims at maximizing welfare w measured as the sum of consumer surplus plus tax revenue, which equals gross consumer surplus minus the import receipt

$$\begin{aligned} w &= \int_0^x \rho(z)dz - \rho x + tx = \int_0^x \left(\frac{\alpha a - z}{\alpha b}\right) dz - px \qquad (4) \\ &= \frac{1}{2}\frac{\alpha}{b}a^2 - \frac{1}{2}\alpha b\rho^2 - p\alpha(a - b\rho). \qquad (5) \end{aligned}$$

The welfare of the producing country equals consumer surplus plus profits of the firm, and thus gross consumer surplus plus revenue from exports minus the costs of production[7]

$$W = \frac{1}{2b}a^2 - \frac{1}{2}bP^2 + p\alpha(a - b\rho) - c(a - bP + \alpha(a - b\rho)). \qquad (6)$$

Note that aggregate welfare equals the sum of gross consumer surplus in both countries minus production costs

$$w + W = \frac{1}{2}\frac{\alpha}{b}a^2 - \frac{1}{2}\alpha b\rho^2 + \frac{1}{2b}a^2 - \frac{1}{2}bP^2 - c(a - bP + \alpha(a - b\rho)). \qquad (7)$$

For the interpretation of some results, total welfare will alternatively be divided into

$$\begin{aligned} \widetilde{w} &= \frac{1}{2}\frac{\alpha}{b}a^2 - \frac{1}{2}\alpha b\rho^2 - c\alpha(a - b\rho) \qquad (8) \\ \text{and} \quad \widetilde{W} &= \frac{1}{2b}a^2 - \frac{1}{2}bP^2 - c(a - bP), \qquad (9) \end{aligned}$$

where $\widetilde{w}$ and $\widetilde{W}$ equal gross consumer surplus minus the production costs for the respective quantity in the individual countries. Note that $\widetilde{w}+\widetilde{W} = w+W$, and that $\widetilde{W}$ is the welfare of the producing country without the profits from exporting, which are instead added to the importing country's welfare. As long as the price is above marginal costs c, the welfare aggregates $\widetilde{w}$ and $\widetilde{W}$ increase unambiguously, when the respective price ρ or P falls, and vice versa.

[7] The monopolist's profits are included in the producing country's welfare, i.e. it is assumed that the firm is domestically owned.

3 Per unit taxation: exogenous tax rate

Segmented markets

When markets are segmented, the monopolist can set prices in the two countries independently.

In the producing country, the profit equals

$$\Pi = (P - c)(a - bP) \tag{10}$$

resulting in the well-known result of a profit-maximizing price of

$$P^*_{seg} = \frac{1}{2}\frac{a + bc}{b}. \tag{11}$$

The monopolist's profit in the importing country is

$$\pi = (p - c)(a - b(p + t)) \tag{12}$$

resulting in a pre-tax price of

$$p^*_{seg} = \frac{1}{2}\frac{a + bc}{b} - \frac{1}{2}t \tag{13}$$

and a tax-inclusive price of

$$\rho^*_{seg} = \frac{1}{2}\frac{a + bc}{b} + \frac{1}{2}t. \tag{14}$$

Thus half of the tax is born by the consumers in the importing country and the other half by the monopolist, resulting in a price difference of pre-tax prices of

$$P^*_{seg} - p^*_{seg} = \frac{1}{2}t. \tag{15}$$

Note that all prices are independent of the size of the importing country α and that the equilibrium price in the producing country P^*_{seg} does not depend on the tax rate t.

The quantity in the producing country is unchanged, whereas sales in the importing country fall in response to the tax. It is straightforward to see, that the welfare in the producing country W falls when a tax is introduced or increased, as $\widetilde{W}$ remains unchanged and profits from exporting fall (as both the pre-tax price and the volume of exports fall). Aggregate welfare $w + W = \widetilde{w} + \widetilde{W}$ also depends negatively on the tax, as $\widetilde{W}$ remains unchanged and $\widetilde{w}$ falls with the increase in the tax-inclusive price ρ. The impact of a tax increase on the welfare of the importing country is ambiguous, depending on whether the tax rate is below or above the optimal tax rate (see below section 4). These are the well-known considerations and results of (optimal) tariff theory.

Arbitrage limiting price discrimination

Assume now, that arbitrage costs s are lower than the induced price difference $\frac{1}{2}t$. In this case, the monopolist is restricted in setting its prices, and the maximum feasible price difference equals s. As the arbitrage condition is binding, the firm actually only sets one price and $P = p + s$, leading to profits of

$$\begin{aligned} \Pi^{tot} &= (p+s-c)X + (p-c)x && (16) \\ &= (p+s-c)(a-b(p+s)) + (p-c)(\alpha(a-b(p+t))) && (17) \end{aligned}$$

and resulting in the equilibrium pretax price in the importing country

$$\begin{aligned} p_s^* &= \frac{1}{2}\frac{a+bc}{b} - s\frac{1}{1+\alpha} - \frac{1}{2}t\frac{\alpha}{1+\alpha} && (18) \\ &= p_{seg}^* + \frac{1}{2}\frac{t-2s}{1+\alpha}. && (19) \end{aligned}$$

This means for the price in the producing country

$$\begin{aligned} P_s^* &= p + s = \frac{1}{2}\frac{a+bc}{b} + s\frac{\alpha}{1+\alpha} - \frac{1}{2}t\frac{\alpha}{1+\alpha} && (20) \\ &= P_{seg}^* - \frac{\alpha}{2}\frac{t-2s}{1+\alpha}, && (21) \end{aligned}$$

and for the tax-inclusive price in the importing country

$$\rho_s^* = p + t = \frac{1}{2}\frac{a+bc}{b} - s\frac{1}{1+\alpha} + \frac{1}{2}t\frac{\alpha+2}{1+\alpha}. \qquad (22)$$

Note that the effect of an exogenous change in the tax rate on the pre-tax price p_s^* is smaller than on segmented markets, but the price P_s^* in the producing country is now affected by the tax, too. With these expressions, we can derive the following proposition on prices, quantities and welfare when the scope for price discrimination changes.

Proposition 1: Assume that the tax rate is exogenous and the arbitrage condition $P - p \leq s$ is binding.
When the scope to price discriminate increases, i.e. when s rises,

(i) both the pretax and the tax inclusive price p_s^* and ρ_s^* in the importing country fall whereas the price in the producing country P_s^* rises

(ii) the sales in the importing country rises, while the quantity in the producing country falls and total quantity remains constant,

(iii) welfare in the importing country and aggregate welfare increases. The change in welfare in the producing country is ambiguous.

Proof: (i) follows directly from equation (19) to (22), and (ii) is a straightforward calculation inserting the expressions for the prices into the demand functions.
(iii) Express welfare in the importing country as a function of ρ,

$$w = \frac{1}{2}\frac{\alpha}{b}a^2 - \frac{1}{2}\alpha b\rho^2 - \alpha(\rho - t)(a - b\rho). \tag{23}$$

Thus

$$\frac{dw}{ds} = \frac{dw}{d\rho}\frac{d\rho}{ds} = -\alpha\left(a - b\rho + bt\right)\frac{d\rho}{ds} > 0, \tag{24}$$

as $a - b\rho = x \geqslant 0$ in the relevant region and $\dfrac{d\rho}{ds} < 0$ by (i).
Aggregate welfare is the sum of both countries' gross consumer surplus minus production costs,

$$W + w = \frac{1}{2}\frac{1}{b}a^2 - \frac{1}{2}bP^2 + \frac{1}{2}\frac{\alpha}{b}a^2 - \frac{1}{2}\alpha b\rho^2 - c(x + X) \tag{25}$$

and thus

$$\begin{aligned}
\frac{d(W+w)}{ds} &= -bP\frac{dP}{ds} - \alpha b\rho\frac{d\rho}{ds} - c\frac{d(x+X)}{ds} \\
&= -bP\frac{\alpha}{1+\alpha} + \alpha b\rho\frac{1}{1+\alpha} \qquad \text{(by ii)} \\
&= b\alpha\frac{\rho - P}{1+\alpha} > 0 \qquad \text{(as } \rho > P\text{)}
\end{aligned}$$

For the welfare of the producing country W, the calculation is given in the appendix.■

The results on prices and quantities correspond to the usual results on price discrimination. When the firm has more scope for price discrimination, it decreases its lower price on its weak market, and increases its higher price on its strong market, with the quantities on the individual markets reacting accordingly. Due to the assumption of linear demand, total quantity remains constant. The size of the effects depends on the size of the importing country α. The larger the relative size of a country is, the smaller is its share of price adjustment to a change in s.

In the importing country, (more) price discrimination lowers the price, and both the consumer surplus and the tax revenue - and thus welfare - increases. Consumers in the producing country lose due to the higher price, but the firm gains, as it has more scope to move its prices towards the monopoly prices of the individual markets. It is ambiguous, which effect dominates.

The result on total welfare is surprising at first glance. Aggregate welfare unambiguously increases, although total quantity is constant. This result stands in contrast to the well-known result of Varian (1985) that price discrimination can only increase welfare if total quantity increases. The puzzle is explained by the fact that the importing country has the low pre-tax price that is relevant for arbitrage and for price discrimination, but the high tax-inclusive price that is relevant for consumers. Allowing for price discrimination makes cars in the importing country cheaper and redistributes the quantity from consumers with a lower to consumers with a higher willingness to pay.

Note however that the combination of the results that price discrimination increases welfare and that the tax induces price discrimination does not mean that introducing a tax is welfare improving. Allowing for price discrimination increases welfare if the tax is exogenously given. Introducing or increasing a tax decreases total quantity

$$\begin{aligned} x_s^* + X_s^* &= a - bP_s^* + \alpha(a - b\rho_s^*) && (26) \\ &= \frac{1}{2}(1+\alpha)(a-cb) - \frac{1}{2}b\alpha t && (27) \end{aligned}$$

and also decreases total welfare, as can be shown by straightforward, albeit tedious, computation using equations (5) and (19) to (22).

4 Per unit taxation: endogenous tax rate

In this subsection, the tax rate is not considered as exogenous, but it is assumed instead that the government of the importing country chooses the tax rate in order to maximize domestic welfare, taking into account how the firm will react to the tax rate set. In particular, it is analysed how the arbitrage condition affects the optimal tax policy and what effects this has on prices, quantities and welfare. As a benchmark, the case of segmented markets is again discussed first.

Segmented markets

Using equation (13) and (14) on the equilibrium prices, the welfare of the importing country can be expressed as a function of the tax rate

$$w = \frac{1}{2}\frac{\alpha}{b}a^2 - \frac{1}{2}\alpha b\rho^2 - p\left(\alpha(a-b\rho)\right) \tag{28}$$

$$= -\frac{1}{8}\frac{\alpha}{b}\left(-a^2 + 2abc - 2abt - b^2c^2 + 2b^2ct + 3b^2t^2\right) \tag{29}$$

implying the welfare-optimizing tax rate

$$\widehat{t}_{seg} = \frac{a-bc}{3b}. \tag{30}$$

and equilibrium prices

$$\widehat{p}^*_{seg} = \frac{1}{2}\frac{a+bc}{b} - \frac{1}{2}t^{opt} = \frac{1}{3}\frac{a+2bc}{b} \tag{31}$$

$$\widehat{\rho}^*_{seg} = \frac{1}{2}\frac{a+bc}{b} + \frac{1}{2}t^{opt} = \frac{1}{3}\frac{2a+bc}{b}. \tag{32}$$

As markets are segmented, the price in the producing country is independent of the tax rate, it continues to be

$$\widehat{P}^*_{seg} = P^*_{seg} = \frac{1}{2}\frac{a+bc}{b}, \tag{33}$$

and the difference of pre-tax prices, that is relevant for arbitrage, is

$$P^*_{seg} - \widehat{p}^*_{seg} = \frac{1}{2}\frac{a+bc}{b} - \frac{1}{3}\frac{a+2bc}{b} = \frac{1}{6}\frac{a-bc}{b}. \tag{34}$$

Arbitrage limiting price discrimination

When the possibility of arbitrage limits the firm's scope to price discriminate, the above result on the optimal tax rate does not hold any more. When setting the tax, the government of the importing country has to take into account that the price difference it induces cannot exceed the arbitrage costs, i.e. $P - p \leq s$. As soon as $s < \frac{1}{6}\frac{a-bc}{b}$, the tax rate $\widehat{t}_{seg}$ will no longer be optimal, and a lower rate will be chosen, as determined in the following proposition.

Proposition 2: (Optimal tax rate)

Assume that the government of the importing country sets the tax rate in order to maximize the country's welfare.

(a) The optimal tax rate $\widehat{t}$ equals

(i) (interior solution)

if $s \leq (a - bc)\dfrac{\alpha}{2b(3\alpha+4)}$

$$\widehat{t} = \frac{a - bc}{b}\frac{\alpha\left(1+\alpha\right)}{\left(\alpha+2\right)\left(3\alpha+2\right)} + \frac{2s\alpha}{\left(\alpha+2\right)\left(3\alpha+2\right)}, \tag{35}$$

(ii) (corner solution)

if $(a - bc)\dfrac{\alpha}{2b(3\alpha+4)} < s < \dfrac{a - bc}{6b}$

$$\widehat{t} = 2s \tag{36}$$

(iii) (segmented markets)

if $s \geq \dfrac{a - bc}{6b}$

$$\widehat{t} = \widehat{t}_{seg} = \frac{1}{3}\frac{a - bc}{b} \tag{37}$$

Proof:

(i) Assume that the arbitrage condition is binding and thus $P = p + s$. To get the welfare of the importing country as a function of the tax rate, insert the equilibrium prices for a given tax rate (equation 13 and 14) into equation 5. Straightforward maximization results in the optimal tax rate $\widehat{t}$ stated. It remains to check, whether for this tax rate, the arbitrage condition limiting price discrimination is indeed binding. On segmented markets, a tax rate $\widetilde{t}$ induces a price difference of $\frac{1}{2}\widetilde{t}$ (equation 15), and thus the firm will only choose prices such that $P = p + s$ as long as $s \leq \frac{1}{2}\widetilde{t}$. Solving the inequality $s < \frac{1}{2}\widehat{t} = \frac{1}{2}\alpha\left(1+\alpha\right)\dfrac{a - bc}{b\left(\alpha+2\right)\left(3\alpha+2\right)} + \dfrac{s\alpha}{\left(\alpha+2\right)\left(3\alpha+2\right)}$ for s results in $s < (a - bc)\dfrac{\alpha}{2b(3\alpha+4)}$, giving the range of parameters for which the solution for the optimal tax rate is consistent.

(iii) When markets are segmented, the optimal tax rate is $\dfrac{1}{3}\dfrac{a - bc}{b}$ and it induces a price difference of $\dfrac{a - bc}{6b}$. As long as the maximum feasible price difference s is at least as large (i.e. as long as $s \geq \dfrac{a - bc}{6b}$), the arbitrage condition is irrelevant and the solution for segmented markets continues to hold.

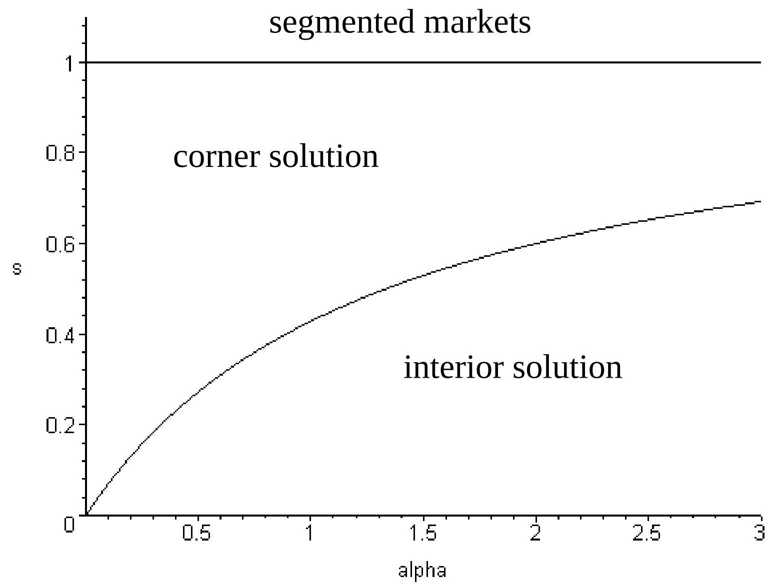

Figure 1: Regions for different solutions for the optimal tax

(ii) Note that $(a-bc)\frac{\alpha}{2b(3\alpha+4)} < \frac{a-bc}{6b}$, thus for s between these two boundaries, neither of the two solutions applies. It is then optimal for the government to set the tax rate such that it is just not binding. If the firm is unrestricted in its price setting, a tax of $\widehat{t} = 2s$ induces a price difference of s, the maximum feasible one. A lower tax rate is not optimal, as s would not be binding for the firm and thus increasing the tax rate would increase welfare (as it moves in the direction of the optimal tax rate in case of unrestricted pricing). On the other hand, a tax rate higher than $\widehat{t} = 2s$ also cannot be optimal, as in this case s would be binding and lowering t marginally would increase the welfare of the importing country (as t would move in the direction of the optimal tax rate under the condition that $P = p + s$). ■

The regions corresponding to the three cases of the theorem are illustrated in figure 1 showing the threshold for s as a function of the size of the importing country α. Figure 2 shows the optimal tax as a function of s, the maximum feasible price difference[8].

When arbitrage costs s are large, the maximum price difference is not binding and the same result as for segmented markets holds (case iii). For s below the threshold $\frac{a-bc}{6b}$, the arbitrage condition becomes binding when the optimal tax for segmented markets is

[8]The parameters a, b and c only alter the scale. For the plots, the parameters $a = 10$, $b = 1$ and $c = 4$ where chosen. In figures 2, 3 and 4, $\alpha = 0.6$ is assumed in addition

set, and the government of the importing country has to take this fact into account when it determines the optimal tax rate. For small s (as defined in case i), there is an interior solution for the optimal tax rate under the restriction of a binding arbitrage condition. In this case, the prices in the two countries are linked by the condition $P = p + s$, and the tax does not only have an impact on the prices in the importing country, but on the price in the producing country, too (see proposition 2 below). However, when the size of the importing country α is too small or s is too large, the tax rate derived in this way may be so low that the price difference induced if markets were segmented would be smaller than s and the condition $P = p + s$ would not hold. Facing this tax rate, the firm would not fully make use of its scope to price differentiate - but it would do so if the higher optimal tax rate for segmented markets was set. In this case, the corner solution for the optimal tax rate applies. The government sets the tax such that the arbitrage condition $P - p \leq s$ is just binding for the firm, i.e. the price difference induced if markets were segmented is exactly s.

Note that when markets become more integrated and thus the arbitrage condition binds as s falls, there is no direct transition from the case of segmented markets to the case of an interior solution for the optimal tax rate. There is always an intermediate interval where the corner solution applies. The smaller the size of the importing country α, the larger is the range of parameter values for s for which the corner solution, in which the tax of the importing country does not affect the price in the producing country, and the interior solution for the optimal tax may apply only for a small range of parameter values s.

The following corollary summarizes how the optimal tax rate depends on the maximum feasible price difference s and on the size of the importing country α. In particular, part (i) points out that in the corner solution, the optimal tax rate reacts much more strongly to changes in s than in the interior solution (see figure 2).

Corollary:

(i) For $s \leq \frac{a - bc}{6b}$, the optimal tax rate $\widehat{t}$ depends positively on arbitrage costs s. In the corner solution, the optimal tax rate reacts more strongly to a change in s than in the interior solution, as in the corner solution $\frac{d\widehat{t}}{ds} = 2$ holds, whereas in the region of the interior solution, $\frac{d\widehat{t}}{ds} < 0.14$.

(ii) In the interior solution, the optimal tax rate $\widehat{t}$ depends positively on the size of the importing country α, whereas in the corner solution, $\widehat{t}$ is independent of α.

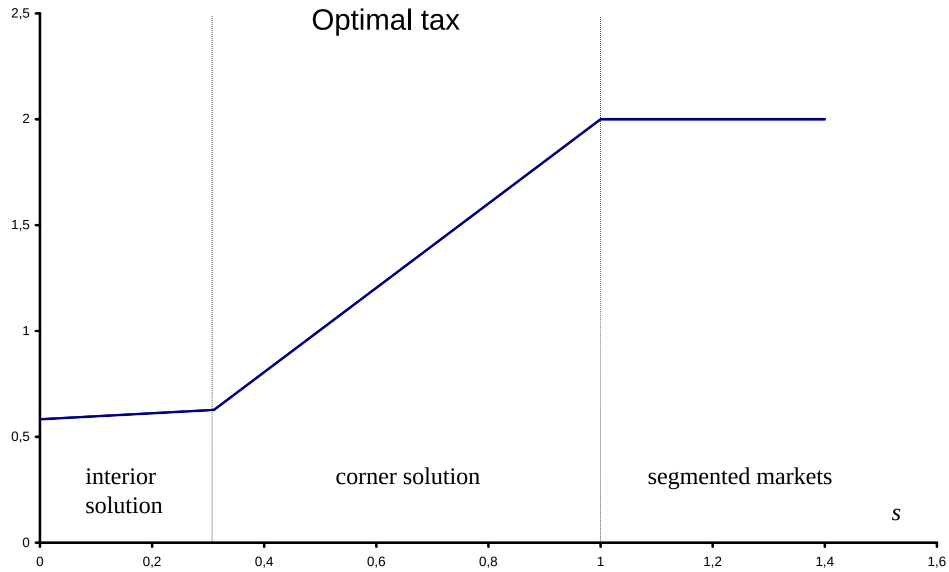

Figure 2: Optimal tax rate

(iii) As long as $s > 0$ or $\alpha > 0$, the optimal tax rate is positive.

Proof: Most of the corollary follows directly from proposition 2. (i) In the interior solution, $\frac{d\widehat{t}}{ds} = \frac{2s\alpha}{(\alpha+2)(3\alpha+2)}$. It is straightforward to show, that the maximum of $\frac{d\widehat{t}}{ds}$ for $\alpha \geq 0$ is smaller than 0.14.

(ii)In the interior solution,

$$\frac{d\widehat{t}}{d\alpha} = \frac{(4+8\alpha+5\alpha^2)(a-bc)+2bs(4-3\alpha^2)}{b(\alpha+2)^2(3\alpha+2)^2}$$

The first term of the numerator is positive and the second term is positive as long as $s < \sqrt{\frac{4}{3}}$. When $s > \sqrt{\frac{4}{3}}$, the second term is negative, but using $s < \frac{a-bc}{6b}$, a positive lower boundary can be found. ■

As a next step, the equilibrium prices are considered when the arbitrage condition is binding and the government of the importing country sets the optimal tax rate $\widehat{t}$.

In the case of an *interior solution* for the optimal tax rate (as defined in proposition 2(i)), the monopolist will set the following prices in the importing and the producing country

$$\widehat{p}_s^* = \frac{1}{2}\frac{a+bc}{b} - \frac{1}{2}\frac{(a-bc)\,\alpha^2}{b\,(\alpha+2)\,(3\alpha+2)} - s\frac{4\,(1+\alpha)}{(\alpha+2)\,(3\alpha+2)} \tag{38}$$

$$\widehat{P}_s^* = \widehat{p}_s^* + s = \frac{1}{2}\frac{a+bc}{b} - \frac{1}{2}\frac{(a-bc)\,\alpha^2}{b\,(\alpha+2)\,(3\alpha+2)} + \alpha s\frac{(4+3\alpha)}{(\alpha+2)\,(3\alpha+2)}, \tag{39}$$

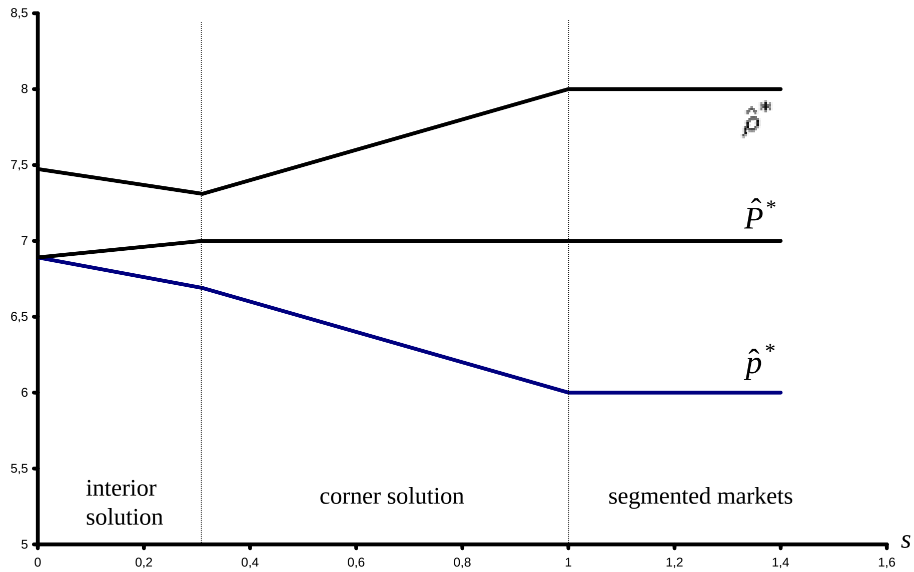

Figure 3: Prices when the tax is set optimally

and the tax-inclusive price in the importing country is

$$\widehat{\rho}_s^* = \widehat{p}_s^* + t = \frac{1}{2}\frac{a+bc}{b} + \frac{1}{2}\frac{(a-bc)\,\alpha}{(3\alpha+2)\,b} - s\frac{2}{(3\alpha+2)}. \tag{40}$$

These expressions are derived by inserting $\widehat{t}$ into equations 19 to 22.

In the *corner solution*, (as defined in proposition 1 iii), the equilibrium prices equal the respective prices for segmented markets (equations 13 and 14) for the tax rate $\widehat{t} = 2s$. In the producing country, the price is unaffected by the possibility of arbitrage,

$$\widehat{P}_s^* = P_{seg}^* = \frac{1}{2}\frac{a+bc}{b} \tag{41}$$

whereas the pre-tax and the tax-inclusive price in the importing country equal

$$\widehat{p}_s^* = \frac{1}{2}\frac{a+bc}{b} - \frac{1}{2}\widehat{t} = \frac{1}{2}\frac{a+bc}{b} - s \tag{42}$$

$$\widehat{\rho}_s^* = \frac{1}{2}\frac{a+bc}{b} + \frac{1}{2}\widehat{t} = \frac{1}{2}\frac{a+bc}{b} + s. \tag{43}$$

From these expressions, the following proposition on the effects of a change in arbitrage possibilities on prices, quantities and welfare can be derived. Figure 3 shows the equilibrium prices as a function of arbitrage costs s, and figure 4 plots the aggregate welfare. The cases of an interior solution for the optimal tax (corresponding to "small values of s") and of the corner solution (corresponding to "intermediate values of s") are defined as in proposition 2.

Proposition 3: Assume that arbitrage limits price discrimination in the sense that the maximum feasible price difference s is binding if the optimal tax for segmented markets is set (i.e. $s < \dfrac{a - bc}{6b}$). If the tax is set optimally, the following holds.

(i) The pre-tax price in the importing country $\widehat{p}_s^*$ depends negatively on s. The effect of s on $\widehat{p}_s^*$ is larger in the corner solution than in case of an interior solution for the optimal tax rate.

(ii) The tax inclusive price $\widehat{\rho}_s^*$ depends non-monotonically on s, as it depends negatively on s in case of an interior solution, but positively in case of the corner solution.

Accordingly, the volume of imports depends non-monotonically on s.

However, for $\widehat{\rho}_{s=0}^* < \widehat{\rho}_{seg}^*$, i.e. if a uniform price has to be set, the tax inclusive price is lower and thus the quantity imported is higher than on segmented markets.

(iii) $\widehat{P}_s^*$ depends positively on s in case of an interior solution and is unaffected by s in the corner solution. Thus in case of an interior solution, the quantity sold in the producing country falls when s increases, and it is independent of s in the corner solution.

(iv) As s rises, total quantity falls.

(v) As s rises, welfare in the importing country rises and welfare in the producing country falls. In the corner solution, total welfare unambiguously falls. In contrast, the change in welfare is ambiguous in case of an interior solution. It rises, when s is close to 0, and it decreases, when s is close to the threshold $(a - bc)\dfrac{\alpha}{2b(3\alpha + 4)}$ between the cases of interior and corner solutions.

(vi) The effect of a change in s on aggregate welfare is much smaller in the interior solution than in the corner solution. More precisely, for a given α the minimum of the absolute slope $\left|\dfrac{d(w + W)}{ds}\right|$ in the region of the corner solution is more than 25 times as large as the maximum of the absolute value $\left|\dfrac{d(w + W)}{ds}\right|$ on the interval of the interior solution.

In the following, these results will interpreted and discussed. The formal proof is given in the appendix.

With an endogenous tax rate, a change in the scope to price discriminate has two effects – the direct effect of an increase in s and the indirect effect through raising the optimal tax rate. For a given tax rate, an increase in s lowers both the pre-tax price p and the

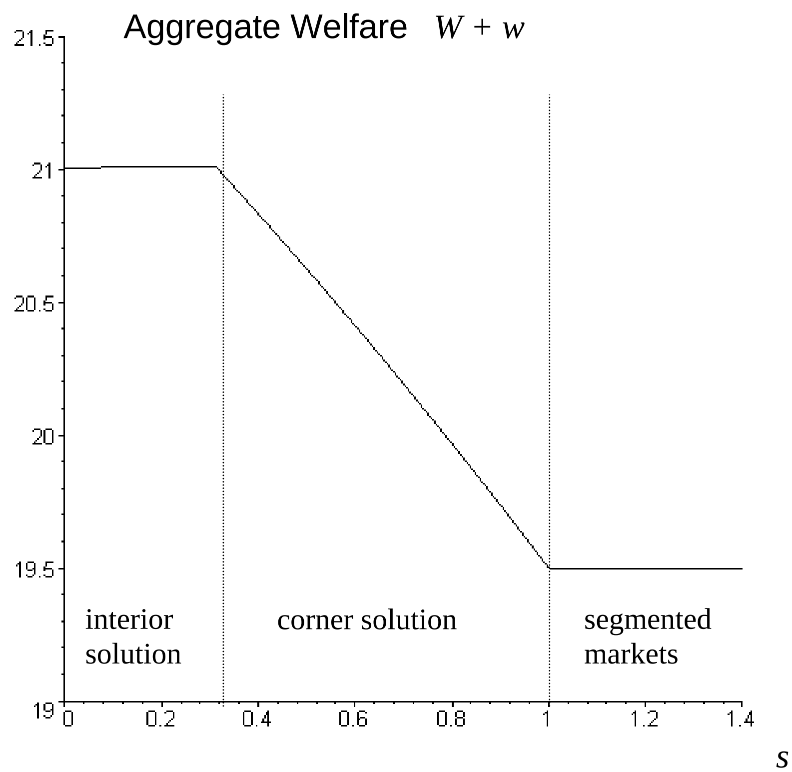

Figure 4: Aggregate welfare when the tax is set optimally

tax inclusive price ρ, whereas the rise in the tax rate itself lowers p and increases ρ. For p these two effects reinforce each other, while for ρ they act in opposite directions. Note that the tax rate reacts much less strongly to changes in s in the interior solution than in the corner solution. This explains, why in the former case, the direct effect dominates and ρ falls in response to an increase of s, and in the latter case, the indirect effect of the tax increase dominates and ρ increases. In particular, the volume of imports x depends non-monotonically on the scope to price discriminate (parts i and ii).

The direct effect of an increase in s increases the price P in the producing country, whereas the indirect effect of the tax increase lowers it. In the corner solution for the optimal tax rate, these two effects exactly offset each other and P is independent of s (part iii). In fact, in the corner solution, the tax rate is chosen as the highest tax rate that does not affect P. In case of an interior solution, the direct effect of the increase in s dominates, thus P rises and the quantity X sold in the producing country falls. There is no direct effect of a change in s on total quantity $x + X$, but it falls due to the tax increase (part iv).

Finally, welfare is considered in part (v) and (vi) of the proposition. As the binding arbitrage condition does not only restrict the monopolist in setting its prices, but also constitutes a restriction for the government of the importing country on setting its tax to influence the price setting of the monopolist, the welfare of the importing country increases when s increases. In contrast, welfare in the producing country falls. In the corner solution, the price P is unchanged, and the result of falling welfare is due to the decrease in profits from exports. In the interior solution, the price P increases and thus $\widetilde{W}$, the welfare minus the profit from exports, also falls.

In case of a corner solution, the aggregation leads to an unambiguous result: A larger scope to price discriminate decreases total welfare. This can be easily seen by considering, that P and therefore $\widetilde{W}$ remain unchanged, whereas ρ rises and thus $\widetilde{w}$ falls. In the case of an interior solution, the change in total welfare is ambiguous. For $s = 0$, a marginal increase in s increases total welfare. The intuition is that there nevertheless is a positive optimal tax, and – similar to the case of an exogenous tax in proposition 1 – allowing for some price discrimination redistributes the good to the market with the higher tax inclusive consumer price and thus the higher (marginal) willingness to pay for the good. When s is larger, the effect of a falling total quantity dominates and total welfare falls, when arbitrage costs s – and therefore the scope for price discrimination – rise. However, these welfare effects are small compared to the effects in the corner solution (see figure 4).

5 Ad valorem taxation

When the tax is levied ad valorem, the model is no longer analytically tractable , as the condition for the optimal tax rate is a polynomial of third degree in the case of segmented markets and of fourth degree in the case of a binding arbitrage condition.

However, the numerical simulation using various parameter combinations show that the qualitative results derived for per unit taxation also hold in the case of ad valorem taxation In this section, these results are summarised.

5.1 Exogenous tax rate

Segmented markets

Facing an ad valorem tax in the importing country, the monopolist will set the following prices in the producing and in the importing country, respectively,

$$P^{\#}_{seg} = \frac{a}{2b} + \frac{1}{2}c \qquad \text{and} \qquad p^{\#}_{seg} = \frac{a}{2b\kappa} + \frac{1}{2}c \tag{44}$$

resulting in a price difference that is relevant for arbitrage of

$$P^{\#}_{seg} - p^{\#}_{seg} = \frac{1}{2} a \frac{\kappa - 1}{b\kappa}. \tag{45}$$

The tax inclusive equilibrium price

$$\rho = \frac{a}{2b} + \frac{1}{2}\kappa c \tag{46}$$

depends on the tax rate κ only via marginal costs c, and if $c = 0$, it would be independent of the tax κ.

Arbitrage limiting price discrimination

When the arbitrage condition is binding, i.e. when the maximum feasible price difference $s < \frac{1}{2} a \frac{\kappa - 1}{b\kappa}$, the two prices the firm sets are linked by $P = p + s$ and the firm actually only sets one price maximizing its profits

$$\Pi = (p + s - c)(a - b(p + s)) + (p - c)(\alpha(a - bp\kappa)), \tag{47}$$

resulting in equilibrium prices

$$p^{\#}_{s} = \frac{a + \alpha a - 2bs}{2b\,(1 + \alpha\kappa)} + \frac{1}{2}c \tag{48}$$

$$\rho^{\#}_{s} = \kappa\, p^{\#}_{s} = \kappa \frac{a + \alpha a - 2bs}{2b\,(1 + \alpha\kappa)} + \frac{1}{2}c\kappa \tag{49}$$

$$P^{\#}_{s} = p^{\#}_{seg} + s = \frac{a + \alpha a + 2bs\alpha\kappa}{2b\,(1 + \alpha\kappa)} + \frac{1}{2}c. \tag{50}$$

From these expressions for the prices, quantities and welfare can be derived. All results of section 3 as summarised in proposition 1 continue to hold for an ad valorem tax. The price in the producing country and the pre-tax price in the importing country depend negatively on κ, whereas the tax-inclusive price rises with the tax. A larger scope to price discriminate, i.e. a larger s, lowers both the pretax and the tax-inclusive price in the importing country, but increases the price in the producing country. Aggregate output is independent of s, but depends negatively on the tax rate. Welfare in the importing country and aggregate welfare increase, whereas the change in welfare in the producing country is ambiguous (see appendix).

5.2 Endogenous tax rate

Segmented markets

When markets are segmented and the government of the importing country maximizes welfare w (using the expression for the prices in equation (48) and (49), the first order condition for the optimal tax rate is

$$\frac{d}{d\kappa}w = \frac{1}{4}\alpha\frac{-ab\kappa^2 c - b^2\kappa^3 c^2 + b^2\kappa^2 c^2 + a^2}{b\kappa^2} = 0 \tag{51}$$

which is equivalent to the equation of 3rd degree

$$f(\kappa) = -\kappa^2 cb\,(a - cb) - b^2\kappa^3 c^2 + a^2 = 0. \tag{52}$$

As $\frac{df}{d\kappa} < 0$ for all $\kappa > 0$ and $f(1) = a\,(a - bc) > 0$, this equation has a unique solution $\widehat{\kappa}_{seg}$ with $\widehat{\kappa}_{seg} > 1$, corresponding to a positive optimal tax rate, which is independent of α.

Arbitrage limiting price discrimination

When the possibility of arbitrage limits the firm's scope to price discriminate, the government of the importing country has to take into account that it cannot induce difference larger than s by setting its tax, i.e. that $P - p \leq s$. Deriving the first order condition for the optimal tax $\widehat{\kappa}$ by considering the first derivative of $w = \frac{1}{2}\frac{\alpha a^2}{b} - \frac{1}{2}\alpha b\rho^2 - p\,(\alpha(a - b\rho))$ using equations (48) and (49) leads to an equation of fourth degree with tedious expressions of the parameters a, b, c, α and s as coefficients and numerical examples have to be used to derive further results. Details on the parameter values chosen for these simulations are given in the appendix. As in the case of per unit taxation, an interior solution and a corner solution have to be distinguished with the threshold value for s depending on the size of the importing country α. The boundary between these two cases continues to be concave as in figure 1.

In case of the corner solution, the importing country's tax does not affect the price in the producing country and the optimal tax set induces a price difference such that the arbitrage condition $P - p \leq s$ is just binding. Using $P - p = s = \frac{1}{2}a\frac{\kappa - 1}{b\kappa}$ (equation 45), it follows that the optimal tax rate in the corner solution equals

$$\widehat{\kappa}^{opt} = \frac{a}{a - 2bs}. \tag{53}$$

Using this expression, prices are determined by equations (44) and (46), and quantities and welfare can be derived. In the range of the interior solution, i.e. for small s, the optimal tax rate $\widehat{\kappa}$ as a function of the maximum feasible price difference s, has to be derived numerically - implying respective functions of prices, quantities and welfare. The simulation shows that all qualitative results of section 4, in particular those summarised in the corollary of proposition 2 and in proposition 3, also hold for the case of an ad valorem tax. The plots of the optimal tax rate, prices and welfare continue are similar to figure 2 to 4.

In particular, the optimal tax rate $\widehat{\kappa}$ depends positively on s, and the slope of $\widehat{\kappa}$ as a function on s is much larger in the corner solution than in the interior solution. This differing reaction of $\widehat{\kappa}$ on s explains the non-monotonic behaviour of the tax-inclusive price ρ, which moves in the same direction as the pre-tax price p in the interval of the interior solution and depends negatively on s in the interval of the interior solution, but moves in the opposite direction and depends positively on s in the corner solution. Aggregate welfare continues to depend non-monotonically on s in the interior solution, but falls with s in the corner solution. However, as in the case of per unit taxation, the effect on aggregate welfare is much larger in the corner solution and the positive welfare effect of price discrimination for small s seems to be of limited relevance.

6 Conclusion

High registration taxes in countries without a car manufacturing sector are one reason for the large pre-tax price differences observed on the European car market. These taxes not only lead to high tax-inclusive prices but also induce foreign car producers to set low pre-tax prices, resulting in welfare gains for the importing country at the expense of producing countries. The European Commission aims at further market integration by facilitating arbitrage and thereby making price differences smaller. The paper discusses what effects would occur if this policy was successful using a model in which the tax is the only incentive for price discrimination.

When the government of the importing country chooses the tax rate optimally, a change in arbitrage costs not only has direct effects on prices by limiting the scope for price discrimination, but it has additional indirect effects by lowering the optimal tax rate.

From the optimization problem of the government to set the tax rate such that it maximizes domestic welfare, two types of solutions emerge – an interior solution that applies for small arbitrage costs and a corner solution that is relevant for an intermediate range of arbitrage costs (before arbitrage costs are so high that markets can be considered as segmented). In the corner solution, the tax is set at a rate such that the condition that price differences cannot exceed arbitrage costs is just binding and the price in the producing country is not yet affected. Hence the whole adjustment to falling arbitrage costs takes place on the market of the tax-imposing country. Only when the markets are highly integrated already and arbitrage costs are so small that the interior solution applies, a change in arbitrage costs also affects the price in the producing country.

In any case, the low pre-tax price in the importing country increases when arbitrage costs fall. However, in the corner solution the effect that the optimal tax rate falls is so large, that the tax-inclusive price falls in spite of the increase of the pre-tax price. In contrast, in the interior solution the effect on the optimal tax rate is small and thus both the pre-tax and the tax-inclusive price move in the same direction, i.e. both increase, when arbitrage is further facilitated. Hence, the tax-inclusive price – and thus the volume of imports – depends non-monotonically on arbitrage costs.

Accordingly, the welfare result also differ in the two cases. Due to the large fall in the optimal tax rate in response to falling arbitrage costs, aggregate welfare always increases when arbitrage costs are in range of the corner solution. In contrast, in the interior solution, the welfare effect is ambiguous, but relatively small. However, welfare in the importing country always falls, as its scope to set an optimal tax to lower its import price is further restricted. It is in the interest of the importing countries that high price differences are feasible.

Thus the effects of the policy of the European Union that aims at lowering the price differences on the car market, substantially depend on whether the tax rate should be considered as exogenous or whether (and to what extent) the high registration taxes will be lowered in reaction to import price increases when markets become more integrated and the firms' scope to price discriminate becomes smaller.

Appendix:

Proof of proposition 1, (iii)

It remains to show, that the sign of $\frac{dW}{ds}$ is ambiguous.

$$W = \frac{1}{2}\frac{1}{b}a^2 - \frac{1}{2}bP^2 - c(a - bP) + (p - c)\left(\alpha(a - b\rho\right)$$

and

$$\frac{dW}{ds} = \frac{1}{2}\alpha\frac{-\left(1+\alpha\right)\left(a - bc\right) + b\left(\alpha + 2\right)\left(t - 2s\right)}{\left(1+\alpha\right)^2}$$

The first term of the numerator is negative, whereas the second term is positive (as $s < \frac{1}{2}t$). If $2s - t \cong 0$ (which is possible), then $\frac{dW}{ds} < 0$.
For $s = 0$ and a high (exogenous) tax rate, the expression is positive. It can be checked that no inconsistencies occur (as e.g. negative expressions for prices or quantities) for a tax rate that makes the numerator marginally positive.

Proof of proposition 3

(i) It follows directly from the expressions for $\widehat{p}_s$ that in the interior solution $\frac{d\widehat{p}_s^*}{ds} = -\frac{4\left(1+\alpha\right)}{\left(\alpha+2\right)\left(3\alpha+2\right)}$ and in the corner solution $\frac{d\widehat{p}_s^*}{ds} = -1$. Moreover, $\frac{4\left(1+\alpha\right)}{\left(\alpha+2\right)\left(3\alpha+2\right)} = \frac{4+4\alpha}{4+8\alpha+3\alpha^2} < 1$ for $\alpha > 0$, thus in the corner solution, the absolute value of the derivative is larger.

(ii) The directions of change of $\widehat{\rho}_s^*$ follows directly from the solutions given in the text. Moreover

$$\begin{aligned}
\widehat{\rho}_{s=0}^* - \widehat{\rho}_{seg}^* &= \frac{1}{2}\frac{a+bc}{b} - \frac{1}{2}\left(a - bc\right)\frac{\alpha^2}{b\left(\alpha+2\right)\left(3\alpha+2\right)} - \left(\frac{1}{3}\frac{2a+bc}{b}\right) \\
&= -\frac{1}{3}(a - bc)\frac{3\alpha^2+4\alpha+2}{b\left(\alpha+2\right)\left(3\alpha+2\right)} < 0
\end{aligned}$$

(iii) is obvious from from the expressions for $\widehat{P}_s^*$.

(iv) The reaction of total quantity $x_s^* + X_s^*$ can be derived by inserting the expressions for the prices into the demand functions. It can also be shown by the consideration, that the direct effect of s on total quantity is 0 and the effect of the tax increase is negative, as can be seen from

$$\begin{aligned} x_s^* + X_s^* &= a - bP_s^* + \alpha(a - b\rho_s^*) \\ &= \frac{1}{2}\left(1+\alpha\right)\left(a - cb\right) - \frac{1}{2}b\alpha t \end{aligned}$$

(v) The effects on welfare are derived by inserting the respective equilibrium prices into the expressions for the welfare and diffentiating.

a) interior solution

Welfare in the importing country:

$$\frac{dw}{ds} = 2\left(\left(1+\alpha\right)\left(a-bc\right) + 2bs\right)\frac{\alpha}{\left(\alpha+2\right)\left(3\alpha+2\right)} > 0$$

Welfare in the producing country:

$$\frac{dW}{ds} = -\alpha\frac{\left(8+22\alpha+20\alpha^2+5\alpha^3\right)\left(a-bc\right) + bs\left(32+64\alpha+40\alpha^2+9\alpha^3\right)}{\left(\alpha+2\right)^2\left(3\alpha+2\right)^2} < 0$$

Total welfare:

$$\frac{d(w+W)}{ds} = \alpha\frac{\alpha\left(2+2\alpha+\alpha^2\right)\left(a-bc\right) - bs\left(16+32\alpha+28\alpha^2+9\alpha^3\right)}{\left(\alpha+2\right)^2\left(3\alpha+2\right)^2} \gtreqless 0 \qquad (54)$$

For $s = 0$,

$$A = \frac{d(w+W)}{ds} = \alpha\frac{\alpha\left(2+2\alpha+\alpha^2\right)\left(a-bc\right) - bs\left(16+32\alpha+28\alpha^2+9\alpha^3\right)}{\left(\alpha+2\right)^2\left(3\alpha+2\right)^2} > 0. \quad (55)$$

For the upper boundary of the range of the interior solution $s = (a-bc)\dfrac{\alpha}{2b(3\alpha+4)}$

$$B = \frac{d(w+W)}{ds} = -\frac{1}{2}\frac{\left(a-bc\right)\alpha^3}{\left(3\alpha+2\right)\left(\alpha+2\right)\left(3\alpha+4\right)} < 0. \qquad (56)$$

b) For the corner solution:

Welfare in the importing country:

$$\frac{dw}{ds} = \frac{1}{2}\alpha(a - bc - 6bs) > 0$$

because $s < \dfrac{a-bc}{6b}$.

Welfare in the producing country:

$$\frac{dW}{ds} = -\alpha\left(a - bc - 2bs\right) < 0$$

Total welfare:

$$\frac{d(w+W)}{ds} = -\frac{1}{2}\alpha\left(a - bc + 2bs\right) < 0. \tag{57}$$

(vi) This part of the proposition compares the slopes $\frac{d(w+W)}{ds}$ of the aggregate welfare functions in the interior and in the corner solution.

For the interior solution, $\frac{d(w+W)}{ds}$ depends negatively on s (equation 54) and it can easily be shown that the largest absolute value in this range is in $s = 0$, (i.e. it equals A as defined in equation 55) by showing that $-A/B > 1$.

For the corner solution, the absolute value $\left|\frac{d(w+W)}{ds}\right|$ is positive and depends positively on s, hence the smallest value for the corner solution is in the lower endpoint of the relevant interval. Inserting the boundary of the two cases, $s = (a - bc)\frac{\alpha}{2b(3\alpha+4)}$ into equation (57) gives

$$C = 2\alpha\left(1+\alpha\right)\frac{a - bc}{4 + 3\alpha}. \tag{58}$$

The function

$$f(\alpha) = \frac{C}{A} = \frac{2(1+\alpha)\left(\alpha+2\right)^2\left(3\alpha+2\right)^2}{\alpha\left(4+3\alpha\right)\left(2+2\alpha+\alpha^2\right)}$$

has for $\alpha \geqq 0$ a unique minimum in $\alpha_0 = 0,70241$ with $f(\alpha_0) = 25,084$, which completes the proof.

Proof of welfare results for exogenous ad valorem tax (Section 5.1)

Using equations (44)-(46), it follows that for the welfare of the importing

$$\frac{d}{ds}w = \frac{1}{2}\alpha\frac{a\left(\kappa-1\right)\left(\kappa^2\alpha+2\kappa-2\right) - (a-cb)\kappa\left(\kappa-2\right)\left(1+\alpha\kappa\right) - 2\kappa bs\left(\kappa-2\right)}{\left(1+\alpha\kappa\right)^2}$$

For $\kappa \leqq 2$ and thus $\kappa - 2 \leqq 0$, the numerator is positive (note that $\kappa > 1$).

For $\kappa > 2$, the derivative depends negatively on s and inserting the upper boundary of $s = \frac{1}{2}a\frac{\kappa-1}{b\kappa}$ leads to a positive lower boundary of the derivative $\frac{d}{ds}w$. (If s is larger, the arbitrage condition is not binding and markets are segmented).

Parameter values for the numerical simulation for an endogenous ad valorem tax (Section 5.2)

The parameters a and b can be normalised to $a = 10$ and $b = 1$ by rescaling the quantity units and the "monetary units" or the numeraire, in which prices are measured.

The model was then solved numerically for the six combinations of the parameter values $c = 4$ and $c = 8$, and $\alpha = 0.2$, $\alpha = 0.6$ and $\alpha = 1$.

In each case, the optimal tax rate is numerically determined as a function of s (for a grid on the whole range of s where markets are not segmented and including the endogenously determined threshold between the interior and the corner solution), and the prices, quantities and welfare functions determined therefrom.

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