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On the Magic Behind David Ricardo's Four Mystical Numbers

by

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Abstract

Building on Ruffin's recent interpretation of Ricardo's four numbers as labour embodied in trade rather than domestic labour coefficients, I identify the magic behind these four mystical numbers. I show that the magic lies in Ricardo's underlying logic in predicting the pattern of international trade. Using a single graphical framework, I show that Ricardo's logic provides a device for a unified treatment of the commodity and the factor content predictions of international trade with an arbitrary number of goods, factors and countries.

JEL classification: F11

Keywords: Ricardo's magic numbers, pattern of trade predictions, higher dimensional issues in trade theory.

Outline

"The basis reason Ricardo's theory is often misinterpreted is that it was often misinterpreted in the past. If a theory once acquires an established meaning, each generation of economists bequeaths this meaning to the next, and it is almost impossible for a famous theory to get a fresh rehearing. Perhaps one hearing is all a theory is entitled to, but one may plead that Ricardo deserves at least a rehearing - his theory is relatively more widely misunderstood today than it was in his lifetime. One can build a strong case that the modern economist need not be acquainted with Ricardo's work, but there is no case for his being acquainted with an imposter." (George Stigler, 1958, p. 367)

1. Introduction

The genesis of the theory of comparative advantage is found in the following passage from chapter VII of Ricardo's (1817) *Principles of Political Economy and Taxation*:

"The quantity of wine which she [Portugal] shall give in exchange for the cloth of England, is not determined by the respective quantities of labour devoted to the production of each, as it would be, if both commodities were manufactured in England, or both in Portugal.

England may be so circumstanced, that to produce the cloth may require the labour of **100** men if she attempted to make the wine, it might require the labour of **120** men for the same time. England would therefore find it her interest to import wine, and to purchase it by the exportation of cloth.

To produce the wine in Portugal, might require only the labour of **80** men for one year, and to produce the cloth in the same country, might require the labour of **90** men for the same time, It would therefore be advantageous for her to export wine in exchange for cloth." (Ricardo, 1817, p.82)

Following the lead of John Stuart Mill, Ricardo's four numbers have been interpreted as the country-specific unit labour coefficients of producing cloth and wine.[1] Given this interpretation, England's comparative advantage in cloth results from its lower relative cost of producing cloth, i.e. 100/120<90/80. However a disturbing fact of this interpretation is that "the principle (which) is of the very heart and soul of our field" (Ethier, 1984, p. 132) had an illogical beginning. Ricardo draws a conclusion about

[1] See Maneschi (2004, p. 441) for a brief history of the input coefficient interpretation of Ricardo's four numbers.

England's pattern of trade based on the first two numbers; however, a pattern of trade prediction based on a relative cost comparison requires information on all four numbers.

In an insightful paper, Roy Ruffin (2002) has rescued Ricardo from the accusation of 'illogical conclusion' by suggesting that the numbers represent the labour needed to produce the wine and cloth that is actually traded.[2] In a follow-up piece, Manechi (2004) has elaborated on Ricardo's labour measure of the gains from trade and pointed out the consistency of Ricardo's measure with the 'eighteenth-century rule' of the gains from trade.

Building on Ruffin and Maneschi, I discuss Ricardo's numbers in a graphical framework that reveals the amazing generality of Ricardo's pattern of trade prediction.[3] In fact, the "magic" behind Ricardo's numbers is that the underlying logic provides a device for a unified treatment of the commodity and the factor content predictions of international trade with an arbitrary number of countries, goods and factors and no specific assumptions about functional forms.

2. Ricardo's labour content formulation

Ricardo's development of comparative advantage was tightly linked to his labour theory of value. Hence, I will discuss Ricardo's logic in a graphical framework that is akin to his labour value formulation. In Ricardo's formulation, the value of a commodity is measured by the quantity of labour embodied in it. By contrast, the modern transformation curve formulation of the law of comparative advantage is based on Gottfried Haberler's opportunity cost reformulation of the law where the value of good X is measured in terms of forgone units of good Y.[4] A straightjacket of the opportunity cost formulation is that the underlying logic is not extendable to higher dimensions.

The logic inherent in Ricardo's labour value formulation is captured in Figure 1. The horizontal axes pertain to the labour content of cloth; it is positive if cloth is imported

[2] Ruffin has also brought to light the neglected paper by Sraffa (1930) in which Sraffa provides a similar interpretation.

[3] Neither Maneschi nor Ruffin discuss how Ricardo's logic extends to higher dimensional predictions in commodity and factor space. Although Ruffin (2004, p. 730/31) notes that Ricardo's logic is extendable to multiple goods and countries, he focuses only on the single factor model.

[4] The transformation curve diagram wasn't introduced into trade theory until the 1930s. See Bernhofen (2005) for a discussion of Gottfried Haberler's 1930 reformulation of comparative advantage in terms of opportunity costs.

and negative if it is exported. The vertical axes pertain to the labour content of wine; it is positive if wine is imported and negative if it is exported.

[insert Figure 1 here]

The first component of Ricardo's pattern of trade prediction can be found in the following two sentences in chapter VII:

"*The same rule which regulates the relative value of commodities in one country does not regulate the relative value of the commodities exchanged between two or more countr*ies….*The labour of 100 Englishmen cannot be given for that of 80 Englishmen, but the produce of the labour of 100 Englishmen may be given for the produce of the labour of 80 Portugese, 60 Russians, or 120 East Indians*. (Ricardo, 1817, p.81ff).

The dashed line in Figure 1 depicts the rule governing domestic exchange: the labour of 100 workers embodied in domestic cloth production must always be exchanged for the labour of 100 workers embodied in domestic wine production. Ricardo postulated that in international trade the labour exchange rate will be different. Ricardo's first two numbers pertain to the international terms of trade faced by England and measured in units of English workers. He assumed that if cloth is traded for wine on the international market the labour of 100 English workers embodied in cloth could be exchanged for 120 English workers embodied in wine.

In the case of two goods, England faces two trading possibilities: (i) export cloth and import wine or (ii) import cloth and export wine. Formally, Ricardo's data for England defines a set **T** of trading possibilities, $\mathbf{T} = \{\mathbf{T^1},\mathbf{T^2}\}$, where $\mathbf{T^1}=(L_c^{exp},L_w^{imp})=(-100, 120)$ and $\mathbf{T^2}=(L_c^{imp},L_w^{exp}) = (100,-120)$. [5]

The second component of Ricardo's comparative advantage formulation is that by *assuming* gains from trade he uses the gains from trade as a criterion to predict which of the two possible trading patterns should be observed. This is illustrated in Figure 1. Since $\mathbf{T^1}$ yields a gain of 20 workers and $\mathbf{T^2}$ yields a loss of 20 workers, Ricardo predicts that England will export cloth and import wine.

3. Predicting the factor content of trade

The amazing point is that Ricardo's logic is extendable to a full fledged general equilibrium pattern of trade prediction with an arbitrary number of goods and factors. Figure 2 illustrates this for the 2-factor, multi-commodity case. The dashed line in Figure 2 depicts the rule governing domestic exchange in the absence of international trade: factor 1

[5] Since the argument is the same for Portugal, I just consider England.

will be exchanged for factor 2 at the domestic autarky factor price ratio w^a_1/w^a_2. International trade permits the economy to exchange factor services at an exchange ratio which will be different from the autarky exchange ratio. Ricardo's gains from trade criterion provides a device to decide which trading pattern should occur in equilibrium. Assume, for instance, that $\mathbf{F^1}$ and $\mathbf{F^2}$ are two "candidate vectors" for an equilibrium factor content of trade. As can be seen in Figure 2, since $\mathbf{F^1}$ yields a gain, i.e. $\mathbf{w^aF^1}>0$, $\mathbf{F^1}$ could be a possible equilibrium configuration; on the other hand, $\mathbf{F^2}$ will not be observed in equilibrium as it leads to a welfare loss, i.e. $\mathbf{w^aF^2}<0$.

[insert Figure 2 here]

More generally, consider a small open economy that faces an *n*-vector of world prices p^w at which it can trade internationally. The world price vector p^w defines the set of possible commodity trading configurations $\mathbf{T^C}=\{T\in R^n|\ p^wT=0\}$ that are feasible to this economy under the assumption of balanced trade. Although the trading outcomes are the result of decentralized decision-making, under perfect competition the decision problem is equivalent to that of a welfare maximizing social planner. Given a domestic *m* by *n* technology matrix *A*, the social planner faces the corresponding set of trading configurations in factor content space: $\mathbf{T^F}=\{F=AT\ |\ p^wT=0\}$. The social planner's gains from trade criterion provides a prediction on the country's factor content of trade: $\mathbf{F}\in\mathbf{T^F}$ and $\mathbf{w^aF}>0$ (see Deardorff, 1982). Note that no assumption is necessary with regard to factor price equalization.

Under the assumption of factor price equalization, the logic implies the two-factor many-goods Heckscher-Ohlin prediction, using Ohlin's definition of relative factor scarcity. In this case $\mathbf{T^F}=\{(F_1,F_2)|\ w_1^f F_1+w_2^f F_2=0\}$, where w_i^f denotes the free trade price of factor i (i=1,2). The inequality $w_1^a F_1+w_2^a F_2>0$ implies that if the economy is relatively abundant in factor 1, i.e. $w_1^a/w_2^a<w_1^f/w_2^f$, then factor 1 must be exported ($F_1<0$) and factor 2 must be imported ($F_2>0$) to the rest of the world.

4. Predicting the commodity content of trade

Analogously, we can consider predictions on the commodity content of trade, where the social planner has to choose from $\mathbf{T^C}=\{T\in R^n|\ p^wT=0\}$, where T denotes the economy's net imports. In the case of two commodities, which is illustrated in Figure 3, $\mathbf{T^C}$ is the terms of trade line. The economy's *n*-vector of autarky prices p^a, i.e. the domestic exchange values of these goods in the absence of international trade, defines the reference point at which to evaluate the feasible trading patterns. The dashed line in Figure 3, given by $p^aT=0$, is equivalent to the dashed line in Figure 1; it separates the set of feasible trading patterns

according to the gains from trade criterion. For example, consider the two trading vectors $\mathbf{T^1}$ and $\mathbf{T^2}$ in Figure 2. With the trading vector $\mathbf{T^1}$, the ratio at which good 1 is exchanged for good 2 is advantageous relative to autarky, i.e. $p^a\mathbf{T}^1>0$.[6] In contrast, with the trading vector $\mathbf{T^2}$, the ratio at which good 2 is exchanged for good 1 is disadvantageous relative to autarky, i.e. $p^a\mathbf{T}^2<0$. As a result, only trading patterns for which $p^aT>0$ will be advantageous to the economy and this provides a pattern of trade prediction (Deardorff, 1980).

[insert Figure 3 here]

5. Concluding remarks

Comparative advantage is usually formulated in terms of countries' relative autarky prices with the underlying logic that differences in autarky prices predict the pattern of international trade. An advantage of this formulation is that it provides a common framework for thinking of the different "causes" of comparative advantage: technological differences (i.e. the Ricardian model), factor endowment differences (i.e. the Heckscher-Ohlin model) or tastes. A shortcoming of this formulation is the apparent discontinuity of the nature of the pattern of trade prediction in higher dimensions (see Ethier (1984)).

Ricardo's logic, however, suggests an alternative formulation which starts out with a set of trading opportunities that are available to the economy and uses the gains from trade as a criterion to predict in which trading activities the economy is expected to engage. The advantage of this formulation is that the basic argument can be applied both to the commodity and the factor content of trade and that it is invariant to dimensionality.

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[6] Formally, the inner product p^aT provides an upper bound for the Slutsky welfare gain from international trade. See Bernhofen (2005) for the theoretical derivation and an empirical application.

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Fig.1: Ricardo's labour content formulation

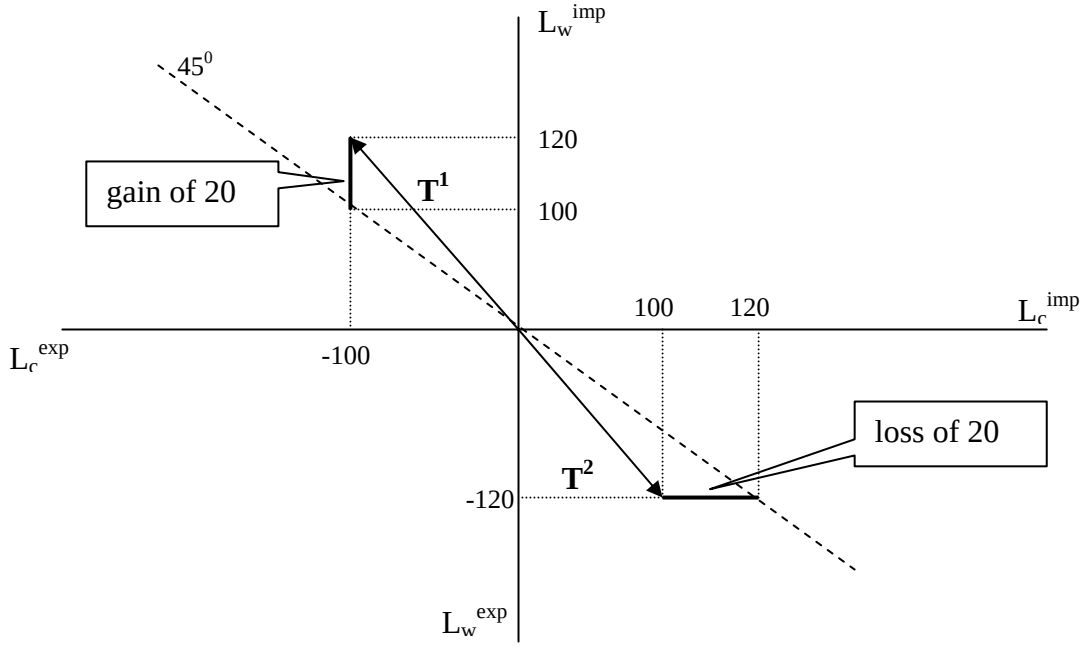

Fig. 2: Factor content prediction of international trade

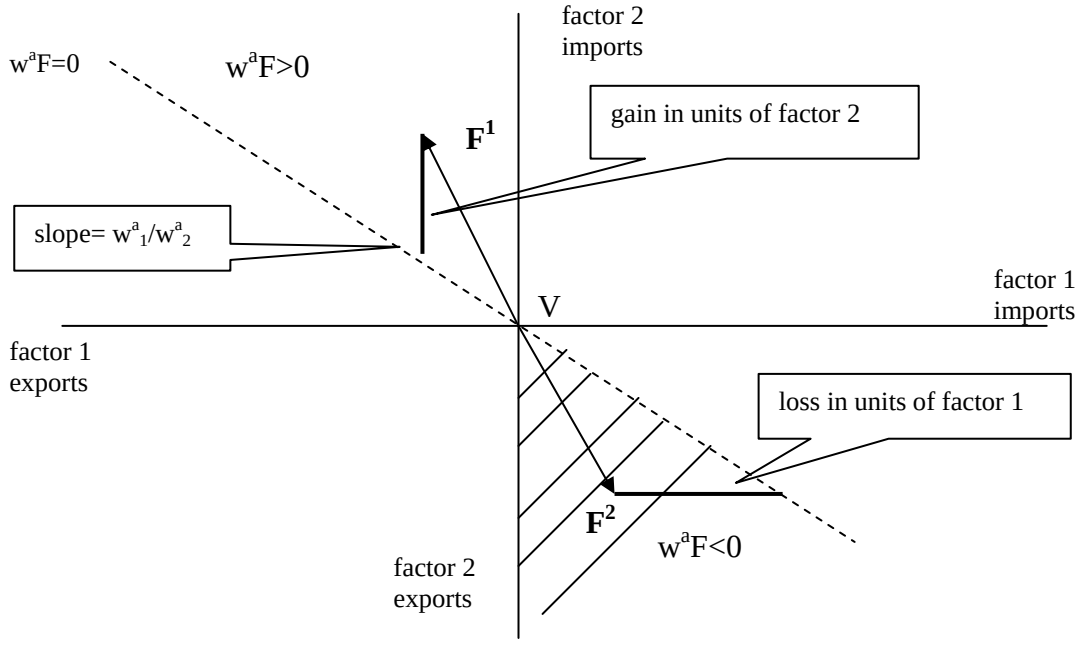

Fig. 3: Commodity prediction of international trade