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SUBSTITUTION IN CONSUMPTION; AN APPLICATION TO THE ALLOCATION OF TIME

A Comment

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1. Introduction

The paper by S. Késenne (1983) focuses on the relation between the commodity-price effect and the good-price effect within the Becker framework of the allocation of time. Assuming input separability in linear homogeneous production functions, the author derives the matrix of price elasticities of the demand for market goods for a 2-commodity-4-goods case. It turns out that these elasticities can be expressed as simple functions of the elasticity of factor substitution (which only depends on technology) and the shadow price elasticity of the demand for commodities (which only depends on preferences). In section 3 of the paper these elasticities appear as parameters in a double logarithmic equation, which is estimated using Belgian time series data on the allocation of time. Through this approach, the author claims to be able to disentangle the effects of tastes and technology on consumer allocation of time.

In this note, I will argue that on the basis of the data available and the assumptions made, only the parameters of the production function of leisure activity can be estimated. Moreover, this is possible only because the assumptions are so special that there is no room left for tastes to influence behavior.

2. The model

The empirical model starts from the (unspecified) utility function $u(a_1,a_2)$, where a_1 is a leisure activity and a_2 is a so-called semi-leisure activity. Both a_1 and a_2 are unobservable. The linear homogeneous production functions

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$a_1(q_1, t_1)$ and $a_2(q_2, t_2)$ are assumed to be of the CES type, i.e.,

$$a_i(q_i, t_i) = [\alpha_i q_i^{-\rho_i} + (1-\alpha_i) t_i^{-\rho_i}]^{-1/\rho_i}, \qquad i=1,2, \tag{1}$$

where q_i is good input and t_i is time input, for the ith activity and $\rho_i = (1-\sigma_i)/\sigma_i$, with σ_i the elasticity of factor substitution between q_i and t_i. The parameters α_i measure the degree to which technology is good intensive.

The time-income constraint is

$$p_1 q_1 + p_2 q_2 + w(t_1 + t_2) = m, \tag{2}$$

where p_1 and p_2 are prices of q_1 and q_2 respectively, w is the wage rate and m is full income.

Késenne assumes that semi-leisure good q_2 and semi-leisure time t_2 are constant over the sample period, since they only represent the most essential consumption goods and the time needed to consume them. As a consequence, a_2 is also constant. Hence, maximizing $u(a_1, a_2)$ subject to (1) and (2) is equivalent to maximizing $a_1(q_1, t_1)$ subject to,

$$p_1 q_1 + w t_1 = m^* \equiv m - p_2 q_2 - w t_2. \tag{3}$$

Note that by these assumptions tastes have become irrelevant for observed behavior.

Solving the maximization problem yields:

$$t_1 = m^* \{(1-\alpha_1)/w\}^{\sigma_1} / [\{(1-\alpha_1)/w\}^{\sigma_1} \cdot w + \{\alpha_1/p_1\}^{\sigma} \cdot p_1]. \tag{4}$$

The parameters α_1 and σ_1 describe the production of a_1 but do not provide any information on the structure of tastes. There is no relation between the shadow price elasticity of the demand for commodities (which only depends on tastes) and α_1 and σ_1.

Even if q_2 and t_2 are not constant and observations on these goods were available, there are some difficulties with Késenne's approach, as will be explained below. Késenne estimates the following equation by OLS, after taking first differences:

$$\begin{aligned}\ln t_1 = {} & \alpha + \beta \ln m + \eta_{11}[k_1 \ln p_1 + (1-k_1)\ln w] \\ & + \eta_{12}[k_2 \ln p_2 + (1-k_2)\ln w] + \sigma_1[k_1(\ln p_1 - \ln w)],\end{aligned} \tag{5}$$

where η_{1i} is the elasticity of the demand for a_1 with respect to the shadow price p_{ai} of commodity i and k_i is the share of the cost of one good in the cost of producing commodity i, which Késenne calculates as

$$k_1 = \frac{p_1 q_1}{p_1 q_1 + w t_1}, \qquad k_2 = \frac{p_2 q_2}{p_2 q_2 + w t_2}. \tag{6}$$

Eq. (5) is obtained as follows. First, the author shows that

$$\begin{aligned} \gamma &\equiv \frac{\partial t_1}{\partial p_1} \cdot \frac{p_1}{t_1} = k_1(\eta_{11} + \sigma_1), \\ \delta &\equiv \frac{\partial t_1}{\partial p_2} \cdot \frac{p_2}{t_1} = k_2 \eta_{12}, \\ \lambda &\equiv \frac{\partial t_1}{\partial w} \cdot \frac{w}{t_1} = (1 - k_1)\eta_{11} - k_1 \sigma_1 + (1 - k_2)\eta_{12}. \end{aligned} \tag{7}$$

Next, he assumes γ, δ and λ to be constant, so that he specifies a double logarithmic relation

$$\ln t_1 = \alpha + \beta \ln m + \gamma \ln p_1 + \delta \ln p_2 + \lambda \ln w. \tag{8}$$

Substitution of (7) into (8) and rearranging terms yields (5).

There are two problems with eq. (5). The first one concerns the estimation method. Since k_1 and k_2 depend on the endogenous variables q_1, q_2, t_1 and t_2, OLS will generally yield inconsistent estimates, whereas the small sample properties of the OLS estimator in this case are unknown. The second problem concerns the double logarithmic specification. One might argue that this specification serves as a reasonable approximation to a generally unknown relation. However, in the household production model the demand for goods equations represent the influence of both preferences and technology. Consequently, by explicitly assuming CES production functions and double logarithmic demand for goods equations, the author implicitly assumes a certain functional form for the utility function. It is at least questionable whether this utility function has reasonable properties, especially in view of the fact that double logarithmic demand equations satisfy the theoretical restrictions of demand theory if and only if all income elasticities are unity, all own prices elasticities are minus one and all cross price elasticities are zero.

A better procedure would be to specify a utility function and to derive the demand for goods equations by maximizing the utility function subject to (1) and (2). However, as has been argued convincingly by Pollak and Wachter (1975) such an approach has no empirically testable implications, unless outputs of household production are measurable. In Késenne's example outputs cannot be measured, as is usually the case in empirical work.[1] In the absence of direct measures of the commodities produced by the household, it is basically impossible to distinguish between a production function and a utility function interpretation of the household's behavior. Consequently, any conclusions on the separation of tastes and technology are entirely dependent on non-testable assumptions such as input separability and linear homogeneity of production functions. Indeed, as argued above with respect to (4), Késenne's assumptions effectively eliminate any possible role for tastes.

In view of these facts, we must conclude that Késenne's approach is not (and cannot be) capable of disentangling the effects of tastes and technology on consumer allocation of time.

References

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[1]An exception is the paper by Rosenzweig and Schultz (1983) who use birth weight as an indicator of the output of the household health production function.