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Tail Dependence among Agricultural Insurance Indices:

The Case of Iowa County-Level Rainfalls

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Abstract

Index insurance has been promoted as a cost-effective risk management alternative for agricultural producers in developing countries. In this paper, we ask whether spatially separated weather variables commonly used in index insurance design, such as rainfall at different weather stations within a defined geographical area, are more highly correlated at the tails. As a case study, we assess the degree of tail dependence exhibited by Iowa June county-level rainfalls using copulas. We search among various candidate bivariate copulas and, using goodness-of-fit for copulas, attempt to identify the copula structures that best explain the nature of dependence among rainfalls in adjacent counties. Our results provide strong evidence that lower tail dependence exists in most of adjacent county-level rainfalls in Iowa. The results also suggest that patterns of tail dependence differ across counties.

Key words: tail dependence, copulas, index insurance, weather indices

Researchers and practitioners in the field of development finance have exhibited growing interest in the use of index insurance contracts to manage the risks faced by poor agricultural producers (e.g., Miranda and Vedenov 2001; Barnett and Mahul 2007; Bryla and Syroka 2007). Unlike conventional insurance, which indemnifies the insured based on verifiable losses, index insurance indemnifies the insured based on the observed value of a specified "index". Ideally, an index is a random variable that is objectively observable, reliably measurable, and highly correlated with the losses of the insured, and which additionally cannot be influenced by the actions of the insurer or the insured. Indices that have been employed or proposed in the design of index insurance contracts for managing agricultural risk in developing countries include area-yields, rainfall, temperature, satellite-measured vegetation indices, and regional livestock mortality rates (Miranda 1991, Skees, Hartell and Hao 2006, Khalil et al. 2007).

Index insurance has been promoted as a cost-effective risk management alternative for agricultural producers in developing countries where traditional insurance is likely to fail due to high transaction costs. Index insurance is generally free of moral hazard, is less susceptible to adverse selection, and is less expensive to administer than conventional insurance (Miranda 1991, Miranda and Vedenov 2001, Barnett and Mahul 2007). However, index insurance has been criticized on the grounds that, in practice, available indices are not sufficiently correlated with losses to provide effective protection against common farm or household risks (Cummins, Lalonde and Phillips 2004; Doherty and Richter 2002; Skees 2008). The potential benefits of index insurance ultimately depend on the statistical relation between the indemnities based on the index and the losses suffered by the insured.

A question of special interest in index insurance design and analysis is whether spatially differentiated indices, such as rainfall measured at different meteorological stations, exhibit "lower tail dependence". Lower tail dependence among random variables exists if the random variables are more highly correlated at the lower tail of their distribution than in other ranges of their domains. For example, rainfall indices exhibit lower tail dependence if they are more highly correlated during times of widespread droughts.

The existence of lower tail dependence is an important question in the design of index insurance products for two reasons. First, suppose an insurer offers a range of index insurance contracts written on a variety of weather indices, say, rainfalls, at different locations in a defined geographical area. The insurer will be interested in assessing the distribution of payouts of his entire portfolio of index insurance contracts in order to calculate the maximum probable loss associated with his entire book of business. If the underlying weather variables exhibit tail dependence, then standard portfolio risk assessments based explicitly or implicitly on normal distribution theory could result in serious underestimates of the riskiness of the portfolio, leaving the insurer exposed to greater business risk than he realizes.

Second, an important task in index insurance design is to compute the expected indemnity associated with a given indemnity schedule. Indemnities, however, are paid only when the index falls below a certain threshold, an event that occurs only infrequently. As such, data available to support the calculation of this critical statistic is usually very limited. One way to address the paucity of extreme data values is to estimate the expected indemnities of multiple contracts jointly. This should lead to gains in efficiency that will depend primarily on the degree of tail dependence exhibited by the indices.

Tail dependence has been of special interest in the general finance literature in recent years. As a result of the financial crisis of 2007-9, financial analysts began to suspect that stock returns might be more highly correlated during financial crises than in normal times, thus making stock portfolios riskier than predicted by conventional asset pricing models (Durante and Jaworski 2010; Bradley and Taqqu 2004; Bradley and Taqqu 2005a; Bradley and Taqqu 2005b). The questions being addressed by financial analysts are analogous to those that must be addressed in index insurance design: in both cases, one is concerned with the degree of dependence exhibited by two or more random variables at the extremes of their distribution, or at the tail of their distribution.

Assessing tail dependence among agricultural indices forces us to think more broadly about how one should measure association among random variables. Association between two random variables is typically measured empirically using the Pearson correlation coefficient, a statistical measure of the degree of linear dependence that exists between a pair of random variables over their entire domain. The Pearson linear correlation coefficient, however, is not a useful measure of association in index insurance design and analysis for two reasons. First, two indices could be strongly related to each other, but in a nonlinear fashion that would go undetected by the linear correlation coefficient. Second, the Pearson linear correlation coefficient is a measure of global dependence, and could ultimately provide misleading information regarding the degree of association at the critical tails of the distribution.

Actuarial and statistical assessments of index insurance products call for the use of flexible multivariate statistical methods that can faithfully capture the potentially nonlinear distributional dependencies that exists among indices, particularly in the extremes of the distributions. Copulas, which provide a theoretical framework for capturing complex dependencies among random variables, are well-suited for this task. In this paper, we search among various candidate bivariate copulas and, using goodness-of-fit tests, attempt to identify the copula structures that best explain the nature of tail dependence among June rainfalls in adjacent Iowa counties, using 1954-2008 data obtained from National Climatic Data Center (NCDC).

Copulas

A bivariate copula is a function that describes how two univariate marginal distributions are combined to form a bivariate joint distribution (Nelsen 2006; Embrechts, Lindskog, and McNeil 2001; Trivedi and Zimmer 2007; Yan 2007). Formally, a bivariate copula $C(u,v)$ can be written as a function $C\colon [0,1]^2 \rightarrow [0,1]$ such that (Nelsen 2006)

$$C(u,0) = C(0,v) = 0, \forall u,v \in [0,1]$$

$$C(u,1) = u \quad \text{and} \quad C(1,v) = v, \forall u,v \in [0,1]$$

$$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \geq 0, \forall u_1 \leq u_2, v_1 \leq v_2.$$

In other words, a bivariate copula is a joint cumulative distribution function of two dependent random variables u and v. Both u and v, on the margin, are uniformly distributed on the unit interval. A natural choice of u and v is the cumulative distribution function (cdf) of random variables.

How copulas work to capture the dependence among jointly distributed random variables is explained by Sklar's Theorem (Nelson 2006). Sklar's Theorem for bivariate copulas states that any continuous bivariate cumulative distribution function $F: R^2 \rightarrow [0,1]$ can be uniquely written as

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)), \quad (1)$$

where C is an bivariate copula and F_i is the i^{th} marginal cumulative distribution function associated with F. Conversely, if C is an bivariate copula and $F_i : R \rightarrow [0,1]$ is a univariate cumulative distribution function, then F as defined above is a cumulative distribution function on R^2 with marginal cumulative distributions F_i. The joint probability density function associated with a differentiable cumulative distribution function F can be recovered from its copula decomposition through the relation

$$f(x_1, x_2) = c(F_1(x_1), F_2(x_2)) f_1(x_1) f_2(x_2), \quad (2)$$

where c is the joint probability density function associated with C and f_i is the univariate probability density function associated with F_i.

Copulas are useful in index insurance analysis because they provide a general, flexible framework for modeling the joint distributions of indices whose marginal distributions are unknown or members of distinct parametric families. Multivariate normal distributions are commonly used in actuarial analysis to model joint distributions. The assumption of normality, however, is not always tenable in agricultural index insurance design and analysis. Certain agricultural indices, such as rainfall, cannot be negative, and therefore are obviously not normally distributed. Moreover, agricultural indices may exhibit complex dependence structures, such as asymmetric tail dependence, that cannot be adequately captured by a joint normal distribution. As such, the use of multivariate normal distributions to model agricultural indices may lead to extremely inaccurate assessments of loss probabilities and expected indemnities.

Copula Families

A number of parametric families of copulas are commonly used in statistical analysis of dependence. The two most frequently used parametric copula families are elliptical copulas, which include the Gaussian and Student-t copulas, and Archimedean copulas. The two-dimensional Gaussian copula distribution is (Freez and Valdez 1998):

$$C(u,v) = \Phi_{\rho}(\Phi^{-1}(u), \Phi^{-1}(v)) \text{, for any } u, v \in [0,1], \tag{3}$$

where, Φ^{-1} is the inverse of standard normal cumulative distribution function, and Φ_{ρ} represents the standard bivariate normal distribution with correlation ρ. Two-dimensional Student-t copula is defined analogously to the Gaussian copula by using a multivariate extension of the *t* distribution with parameter α:

$$C(u,v) = F_{t_{\alpha}}(F_t^{-1}(u), F_t^{-1}(v)). \tag{4}$$

Another widely studied parametric family of copulas is the Archimedean copulas. An Archimedean copula is constructed through a generator φ:

$$C(u,v) = \varphi^{-1}[\varphi(u) + \varphi(v)], \tag{5}$$

where $\varphi : [0,1] \to [0,\infty)$ is a continuous, strictly decreasing, convex function with $\varphi(1) = 0$ (Nelsen 2006). Three widely used one-parameter Archimedean copulas are

Frank copula, Clayton copula and Gumbel copula whose generator functions are shown in table 1.

Copula Functions and Tail Dependence

Interests in describing asymmetric dependence at extreme values lead to the introduction of tail dependence. Tail dependence measures the dependence between two random variables in the upper-right and lower-left quadrants of their domains (Nelsen 2006). In other words, tail dependence measures how large the association among random variables is when one random variable or all the variables has/have large (or small) values. According to Nelson (2006), the parameter of asymptotic lower tail dependence, noted by λ_L, is the conditional probability in the limit that one variable takes a very low value, given that the other also takes a very low value. Similarly, the parameter of asymptotic upper tail dependence, noted by λ_U, is the conditional probability in the limit that one variable takes a very high value, given that the other also takes a very high value. The asymptotic tail dependence parameters for copula function are shown as following (Nelsen 2006)

$$\lambda_L = \lim_{t \to 0^+} \frac{C(t,t)}{t}, \tag{6}$$

$$\lambda_U = 2 - \lim_{t \to 1^-} \frac{1 - C(t,t)}{1-t}. \tag{7}$$

The asymptotic tail dependence parameters λ_L and λ_U cannot be solved analytically for all the families of copulas. For some of the families, λ_L and λ_U can be easily evaluated, while for others, they can only be solved numerically. In the case of Gaussian copula and Student-t copula, the copula functions are symmetric, which implies that the asymptotic upper and lower tail dependences are identical. For an Archimedean copula with generator φ, the tail dependence parameters can be written as (Nelsen 2006)

$$\lambda_L = \lim_{t \to 0^+} \frac{\varphi^{-1}(2\varphi(t))}{t} = \lim_{x \to \infty} \frac{\varphi^{-1}(2x)}{\varphi^{-1}(x)} \quad \text{and} \tag{8}$$

$$\lambda_U = 2 - \lim_{t \to 1^-} \frac{1-\varphi^{-1}(2\varphi(t))}{1-t} = 2 - \lim_{x \to 0^+} \frac{1-\varphi^{-1}(2x)}{1-\varphi^{-1}(x)}. \tag{9}$$

The asymptotic parameters for Frank copula, Clayton copula and Gumbel copula are summarized in table 2. As is shown in table 2, the Clayton copula can describe the asymmetric lower tail dependence but cannot capture the upper tail dependence, and the Gumbel copula can model the asymmetric upper tail dependence but cannot capture the lower tail dependence. The Frank copula cannot characterize either the lower tail dependence or the upper tail dependence.

One approach to detecting and measuring lower or upper tail dependence is to fit different copulas and to compare their performance using the goodness-of-fit statistics for copulas. If the Clayton copula provides a better fit than other copulas, the existence of lower tail dependence can be concluded; if a Gumbel copula provides a better fit than other copulas, the existence of upper tail dependence can be concluded. Based on the evidence of the degree of lower tail dependence, upper tail dependence or both lower tail and upper tail dependence, the related index insurance products, for drought, flood or both, could be designed and actuarially analyzed.

Empirical Estimation Methods

We now examine tail dependencies among rainfalls in adjacent Iowa counties using five distinct copulas: Gaussian, Student-t, Frank, Clayton and Gumbel.

Data

County-level June rainfalls from 1954 to 2008 (55 years) for all 99 Iowa counties were obtained from National Climatic Data Center (NCDC)[1]. In this paper, we examine tail dependence between rainfalls in adjacent counties. Among the 99 counties, there are 297 pairs of adjacent counties. Based on a visual assessment of the histograms of rainfall data for each county, we selected the lognormal distribution to model the marginal distributions of rainfalls. The parameters of lognormal distribution are estimated for rainfall separately for each county. Table 3 reports the summary of the descriptive statistics for the pooled June rainfall data of all 99 counties.

Computing Goodness-of-Fit

[1] http://www.ncdc.noaa.gov/oa/ncdc.html

After the estimation of marginal distribution for each county, we estimate parameters for each of the five copulas and for each pair of adjacent counties using the fitted marginal distributions by maximum likelihood estimation (MLE). To compare the performance of these copula functions, the Goodness-of-fit statistic for copulas is calculated for each fitted copula function and for each pair of adjacent counties.

Goodness-of-fit is a measure of how well a statistical model fits a set of observations. Genest, Quessy and Remillard (2006) develop a Goodness-of-fit statistic and apply parametric bootstrapping to compare the fit provided by copulas. Suppose $F(x_1, x_2)$ is the joint distribution based on specific copula function as is shown in (1). Let $K(\theta, t) = P\{F(x_1, x_2) \leq t\}$ with the copula parameter θ. The empirical version of $K(\theta, t)$ is defined as

$$K_n(t) = \frac{1}{n}\sum_{j=1}^{n} 1(V_j \leq t), \quad t \in [0,1], \tag{10}$$

where n represents the size of sample, V_j are pseudo-observations defined by $V_j = \frac{1}{n}\sum_{k=1}^{n} 1(X_{1k} \leq X_{1j}, X_{2k} \leq X_{2j})$, and $1(V_j \leq t)$ refers to the indicator function that has the value of 1 when $V_j \leq t$ and the value of 0 when $V_j > t$. The Goodness-of-fit statistic for copulas is given by

$$S = \int_0^1 |\gamma(t)|^2 \, k(\theta, t) dt \tag{11}$$

where $\gamma(t) = \sqrt{n}[K_n(t) - K(\theta, t)]$ and $k(\theta, t)$ represents the density function of

$K(\theta, t)$.

In order to compute the Goodness-of-fit statistic based on the empirical process γ, we generate a large number of independent samples of size *n* from the fitted copulas, and compute the corresponding values of the statistic *S* for each copula and for each pair of adjacent counties. In this paper, we use Gaussian kernel density function to fit the empirical distribution. Specifically, the procedure using bootstrap follows three steps. First, we fit a bivariate kernel density of the observations and calculate the cdf's of bivariate kernel function at $N \times N$ grids in the area $[0,1]^2$. Here, we use N = 50. Second, we generate 1000 random samples of size *n*, in our case *n* = 55, from the fitted copula function $\hat{C}$ with the estimated parameter $\hat{\theta}$. For each of these samples, we fit a bivariate kernel function and obtain the cdf's of bivariate kernel at the same grids as in step one. Third, for each of the 1000 samples, a Goodness-of-fit statistic for copulas, *S*, is computed based on the cdf's in the first step and the second step and the kernel density function in the second step. We repeat the procedure for each of the five copulas. The means of the *S* statistic in the 1000 samples generated from the five copulas are used to compare the performance of the five copulas.

Empirical Estimation Results

The comparison of the five copulas is conducted for each of the 297 adjacent pairs of counties using the Goodness-of-fit statistic for copulas. The performance of copulas is evaluated by the rankings of Goodness-of-fit statistic for each pair of counties. Table 4 shows the percentage of rankings for each of the five copulas. In all the 297 pairs of adjacent rainfalls, 43%, or 128 pairs, are best fitted by the Clayton copula, 18%, or 53 pairs, are best fitted by the Gumbel copula, 16%, or 48 pairs, are best fitted by the Student-t copula, 13%, or 39 pairs, are fitted best by the Gaussian copula, and only 10%, or 29 pairs, are best fitted by the Frank copula. Considering the second best fit, 33% of the 297 pairs select the Clayton copula, and 39% of the pairs select Gumbel copula. When it comes to the worst fit with respect to the Goodness-of-fit statistic, 39% of the pairs list the Frank copula as the worst fit, and 29% of the pairs list it as the second worst fit. It is obvious that the Clayton copula performs best in fitting the rainfall data of adjacent counties, the Gumbel copula is the second best one, and the Frank copula performs worst. Gaussian copula and Student-t copula perform better than the Frank copula but worse than the Clayton copula and the Gumbel copula.

By looking at table 2, the Clayton copula is characterized by strong lower-tail dependence. The good performance of the Clayton copula, therefore, implies that for many pairs of adjacent counties, rainfalls are more strongly related when precipitation is abnormally low, which is strong evidence that lower tail dependence exists in many adjacent counties. For some pairs of adjacent counties, the Gumbel copula provides a better fit, suggesting that upper tail dependence also exists. It is possible that some pairs of adjacent rainfalls have both lower tail dependence and upper tail dependence. The Gaussian copula and the Student-t copula can also capture some degree of tail dependence. However, since they are symmetric copulas, they tend to underestimate lower tail dependence when, as is in the case of Iowa rainfall, the correlation of rainfall in adjacent counties rises asymmetrically in drought years, but may only slightly rises in years of high precipitation.

The tail dependence parameters for each pair of counties can be computed by the functions shown in table 2 using the estimated copula parameters. Table 5 reports the mean and standard deviation of estimated copula parameters for all the 297 pairs of adjacent counties and the related tail dependence. The estimated parameter of the Clayton copula has relatively highest variation across the adjacent counties. The average lower tail dependence of the 297 pairs of adjacent counties is 0.46 and the average upper tail dependence is 0.62, with standard deviation 0.17 and 0.06, respectively. The upper tail dependence tends to be more stable than lower tail dependence across all the pairs of adjacent counties. Therefore, contract design that focuses on the correlation between the indemnity and losses caused by drought may require further investigation in the change of lower tail dependence among adjacent counties.

Conclusion

The existence of tail dependence between spatially separated agricultural indices such as rainfall is important for insurers who are interested in assessing the maximum probable loss associated with his portfolio, and who must estimate expected indemnities using limited extreme value data. The Pearson linear correlation coefficient, which is commonly used in measuring dependence, is generally inadequate for the task because it cannot describe nonlinear association and cannot distinguish between lower-tail and global dependence.

In order to test for and measure tail dependence among county-level June Iowa rainfalls, we estimated a variety of copula functions, including Archimedean copulas (Clayton, Gumbel, and Frank) and elliptical copulas (Gaussian and Student-t) for adjacent county pairs. The performance of the five copulas was compared by the Goodness-of-fit statistic for copulas based on a nonparametric bootstrap procedure. Our results indicate that the Clayton copula fits the data best, which implies that lower tail dependence exists in most of adjacent county-level rainfalls in Iowa. The results suggest that accounting for tail dependence in the contexts where extreme events could substantially enhance the accuracy of loss assessment for agricultural index insurance portfolios. The results also suggest that patterns of tail dependence differ across counties. Some of the adjacent counties tend to have higher correlation when drought occurs, while some tend to have higher correlation in normal or abnormally wet years.

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Tables

Table 1. Archimedean Copula Generator Functions

Family	Parameter	$\varphi_\theta(t)$
Clayton	$\theta \geq -1, \theta \neq 0$	$\frac{1}{\theta}(t^{-\theta}-1)$
Frank	$\theta \neq 0$	$-\ln \frac{e^{\theta t}-1}{e^{\theta}-1}$
Gumbel	$\theta \geq 1$	$(-\ln t)^{\theta}$

Table 2. Lower and Upper Asymptotic Tail Dependence for Archimedean Copulas

Family	λ_L	λ_U
Clayton, $\theta \geq 0$	$2^{-1/\theta}$	0
Frank	0	0
Gumbel	0	$2-2^{1/\theta}$

Table 3. Summary of the descriptive statistics for the pooled data

Mean	Stand deviation	Maximum	Minimum	1st quarter	3rd quarter
458	250	2218	0	275	596

Table 4. Percentage of copulas' rankings in adjacent counties

Rankings	1	2	3	4	5
Gaussian copula	13%	15%	23%	25%	24%
Student-t copula	16%	18%	28%	23%	15%
Frank copula	10%	13%	9%	29%	39%
Clayton copula	43%	33%	11%	6%	7%
Gumbel copula	18%	21%	29%	17%	15%

Table 5. Summary of estimates of parameters for three Archimedean copulas and the average tail dependence of adjacent counties

Copulas	Mean of $\hat{\theta}$	Std. of $\hat{\theta}$	Mean of λ_L	Std. of λ_L	Mean of λ_U	Std. of λ_U
Frank	6.69	1.64	0	-	0	-
Clayton	1.01	0.44	0.46	0.17	0	-
Gumbel	2.20	0.35	0	-	0.62	0.06