Poster Title: Pólya's Urn Model for Crop Yield Expectation Stochastic Process

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Introduction

FINANCIAL ASSET PRICE MODELING is a large and sophisticated field of activity. The approach and findings in the field have proven to be extraordinary useful for risk managers when pricing derivatives, investing and hedging. This success is due primarily to the insights and mechanical approaches enabled by working with a specific stochastic price process, even if the process is not quite right. Managers will develop rules of thumb to 'fix' perceived problems, as has been the case with Black-Scholes and related models.

One might expect that asset price risk management techniques would have been adapted for use in crop yield/revenue insurance markets. Many of the techniques most useful in price modeling have not been adapted. This is due largely to the absence of a plausible crop yield expectation stochastic process to work with. We present, and discuss uses for, an expected yield stochastic process as a crop matures between planting and harvest. This is the Pólya urn process.

Model

THE EXPECTED YIELD STOCHASTIC process is based on the Pólya Urn model (Mahmoud, 2009). We model the growing year as having $T + 1$ time points at which new information becomes available. Time $t = 0$ is planting while $t = T$ is harvest. We are interested in yield expectations at each time $t\in\{0, 1, \dots, T\}$. With Y_T as actual harvest yield, W_t is the information set available at t. Expected harvest yield given W_t is written as $\mu_t = \mathbb{E}[Y_T \mid W_t]$. Without loss of generality, the yield distribution is assigned support only on [0, 1] . We also assume that the expectation has logistic form, a specification widely used to model plant production processes (e.g., Tschirhart, 2000).

Specifically, let $\mu_0 = g(x) / [f(x) + g(x)]$ where x is an input choice vector while $f(x)$ and $g(x)$ are increasing functions. Writing optimal choices as $x = x^*$, abbreviate $f^* = f(x^*)$ and $g^* = g(x^*)$.

The model is one of information-conditioned updating of yield expectations as relevant events and the yield consequences are processed. At $t = 1$, new information arrives and expected yield evolves as follows (Mahmoud, 2009):

$$\mu_1 = \begin{cases} \mu_1^+ = \dfrac{g^* + c}{f^* + g^* + c} & \text{with probability } \mu_0; \\ \mu_1^- \equiv \dfrac{g^*}{f^* + g^* + c} & \text{with probability } 1 - \mu_0; \end{cases} \tag{1}$$

for $c > 0$. Here $c > 0$ recognizes good weather over the first growing period. It might be viewed as the benefit from good weather. Iterate the algorithm in (1) over $t = 2$ and further to identify the general expression

$$\mu_t = \begin{cases} \mu_t^+ = \dfrac{\mu_{t-1} + m(f^*, g^*, t, c)}{1 + m(f^*, g^*, t, c)} & \text{with probability } \mu_{t-1}; \\ \mu_t^- = \dfrac{\mu_{t-1}}{1 + m(f^*, g^*, t, c)} & \text{with probability } 1 - \mu_{t-1}; \end{cases} \tag{2}$$

$$m(f^*, g^*, t, c) = \frac{c}{f^* + g^* + (t-1)c}.$$

This is the expected yield stochastic process we posit over $t\in\{0, 1, \dots, T\}$. Figure 1 illustrates the process as a binomial tree when $T = 2$. Figure 2 presents the literal stochastic algorithm. Table 1 summarizes some properties.

Figure 1. Binomial tree for the three time point Pólya urn process, probabilities under arrows

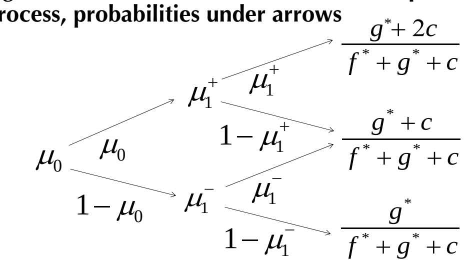

Figure 2. Pólya's urn algorithm to generate yield expectation stochastic process

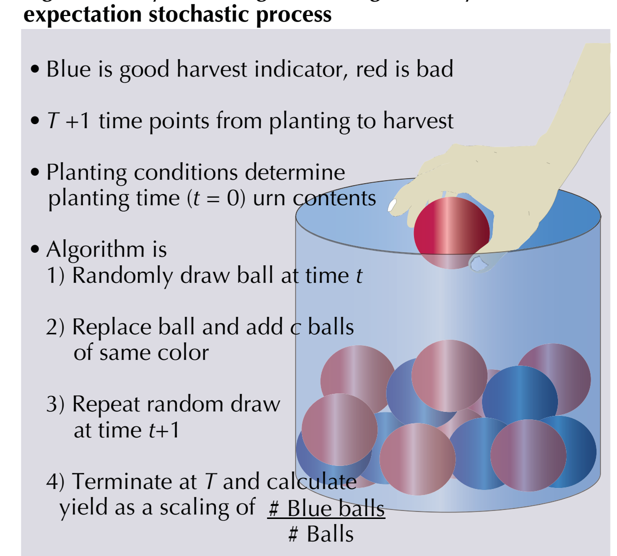

Table 1. Pólya's urn yield expectation process properties

Property	Explanation
A) Bounded Support	Values confined to an interval
B) Martingale (internal consistency)	Today's expectation of tomorrow's expectation of harvest yield equals today's expectation of harvest yield
C) Information Resilience	Yield expectations for crops with extreme yield expectations are least sensitive to new information
D) Hardening	Yield expectations become less variable as harvest approaches
E) Beta Convergence	Process converges to the beta distribution, a popular stochastic yield model (e.g., Nelson and Preckel 1989)

Three Possible Uses

1) The process allows for dynamic hedging of crop insurance contracts when financial instruments correlated with determinants of yield expectations are available. Weather derivatives could be one such class of instruments.

2) With further development, the process could be used to model the co-evolution of yield expectations and harvest price expectations in order to assess revenue insurance liability.

3) Antle (1983) and others have pointed to the importance of intra-season crop input decisions (e.g., pesticides, nitrogen, abandonment). The binomial tree approach (Hull 2009), in Figure 1, readily adapts to allow for state-conditioned decisions as the process evolves. In short, the process could be used to include grower expectations in a discrete-time real options analysis of crop production decision-making.

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