

Enterprise-level risk assessment of geographically diversified commercial farms: a copula approach

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Abstract

As agriculture becomes more industrialized, the role of risk measures such as VaR will become more utilized. In this case it was applied to geographical diversification and also modifying the traditional VaR estimation by incorporating a copula dependence parameter into the VaR estimation. In addition, an alternative risk measure was also calculated, CVaR. The CVaR, unlike VaR, is a coherent risk measure. Thus it does not suffer from many of the shortcomings of the VaR. The land portfolio consisted of Dryland wheat production acres in Texas, Colorado, and Montana. Three series of net returns were calculated for each region. Based on the VaR and the CVaR, the portfolio was optimized based on minimizing the expected loss based on historical net revenues. The results showed that diversification could be reduced by producing in all three areas.

Introduction

Transitions that have occurred in agriculture have provided agribusinesses with the opportunity to diversify risk in new ways (Boehlje and Lins, 1998; Vedenov and Barnett, 2004; Zhu, Ghosh et al., 2008). One of those risk management techniques is that of geographical diversification. Geographical diversification research has provided no clear answers to its effectiveness as a risk management tool (Nartea and Barry, 1994; Krueger, Salin et al., 2002). This study will develop a framework for analyzing geographical diversification at the firm level.

Risk management in agriculture has been extensively researched (Pope and Prescott, 1980; Meyer, 1987; Turvey and Diver, 1987; Tomek and Peterson, 2001; Just, 2003; Just and Pope, 2003; Mishra and Lence, 2005; Peterson, Schurle et al., 2005; Pennings, Isengildina-Massa et al., 2008). Innovations in statistics and finance have opened the door to new approaches to look at old problems (Alexander and Baptista, 2004; Vaz de Melo Mendes and Martins de Souza, 2004; Patton, 2006; Acerbi, 2007; Alexander, Baptista et al., 2007; Bai and Sun, 2007). Coherent risk measures and dependency structure are current innovations that are allowing researchers to further evaluate risk management issues (Accioly and Chiyoshi, 2004; Acerbi, 2007). The applications of these innovations in the agricultural literature are still recent (Zylstra, Kilmer et al., 2003; Vedenov, 2008; Zhu, Ghosh et al., 2008).

The topic of geographical diversification provides an opportunity to evaluate both the topic of dependency and risk criterion. The problem faced when analyzing geographical diversification is accurately measuring the risk involved with the investment. The finance industry has embraced and utilized the value-at-risk (VaR) measure (Jorion, 1996). The research pertaining to VaR and agricultural economics has been very limited (Manfredo and Leuthold, 2001). VaR is a convenient way of assessing the probability of a certain level of losses given a certain confidence level. Many practitioners rely on the 95% confidence level. In other words, a

firm could be 95% confident that returns would not drop below a given level. Recent research has shown that VaR does not have the properties of a coherent risk measure (Artzner, Delbaen et al., 1999; Acerbi, 2007). To overcome the shortfalls of VaR, alternatives have been developed and are shown to be coherent. One of these is the expected shortfall or Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev, 2000; Alexander and Baptista, 2003). CVaR gives the probability that losses are equal to or greater than VaR given a certain confidence level.

Another problem encountered with this type of problem is that of properly specifying the joint distribution of returns. Traditional methods have relied heavily on the multivariate normal distribution. Research has shown that the assumption of normality for prices and yields is incorrect (Just and Weninger, 1999; Goodwin and Ker, 2002). Flexibility in specifying the distributions of both prices and yields are needed to properly specify the dependency that exists between them. A method that has been used to provide flexibility and proper specification of dependence is that of copulas. Copulas provide a link between the marginal distributions and the multivariate distribution. It allows flexibility in properly specifying the form of the individual marginal distributions and thus allowing the dependency structure to be estimated efficiently.

The purpose of this research is to implement a copula-CVaR in an agricultural setting. Annual Dryland wheat returns are generated for three regions, Texas, Colorado, and Montana. A portfolio optimization based on copula-CVaR will be calculated to evaluate the ideal dispersion of production activities. This research furthers previous research in two ways. First, copula-CVaR has not been applied in an agricultural setting. This hypothetical scenario provides an opportunity to illustrate the method of using copulas to specify dependency and CVaR as a risk management tool. The second is that geographical diversification has not addressed the possibility of producing in more than one state.

Geographical Diversification

Agriculture

On the agricultural side, the results of geographical diversification have provided conflicting answers. On the one hand, it was shown that by diversifying geographically, a grape grower could increase profits by producing in the US and in Chile (Kreuger, Salin et al., 1999). Another study looked at diversification within Central Illinois (Nartea and Barry, 1994). The authors analyzed the costs and returns of geographical diversification in Central Illinois. The question being answered was whether or not geographical diversification was a legitimate risk management strategy for individual grain growers in Illinois. The costs that the authors analyzed were the increased transportation costs, monitoring costs, and the loss due to poor machinery coordination. They then compare this to the increased returns received. The authors concluded that there was no realizable gain from diversifying geographically in Central Illinois.

Davis et al. (Davis, Price et al., 1997) examined the impact of geographical diversification on peach orchards in Georgia. Using a stochastic production function, the authors estimated the yield variability that could be reduced by geographically scattering peach orchards. They found that for every mile increase in distance between orchards, correlation between yields dropped by 2%.

Banking

Liang and Rhoades (Liang and Rhoades, 1988) studied the impact of geographical diversification in the banking industry. The study was motivated by the changes in banking regulations that were taking place in the late 80's. Many banks had begun to expand beyond state borders because regulations had changed and now banks were able to expand into different regions. The authors argue that geographical diversification will reduce insolvency risk but in

turn may increase operating risk through increased management costs and the issues surrounding the acquisition of a new firm. To test this hypothesis, the authors looked at 5,500 banks over the time period of 1976 to 1985. They tested the impact of geographic diversification on overall diversification. They found that indeed geographic diversification reduces insolvency risk but caution must be taken because of the potential increase in operating risk which could offset any potential gains from geographic diversification.

In another study, the impact of geographic diversification was specifically applied to small banks that were acquired by larger banks (Rose and Wolken, 1990). The results of the mergers provided no long run advantage for the small banks. But, in the short run, the merger provided some opportunities for entry into new markets.

Sector versus Geographic Diversification

Industry in general has researched this problem as well. One study examined the differences between sector diversification and geographic diversification (Ehling and Ramos, 2006). The authors examined industries that were within the Eurozone. The authors argued that with the implementation of the Euro, the gains of geographic diversification would be diminished. Using a mean-variance efficiency test (Basak, Jagannathan et al., 2002), the authors tested whether companies were better off by diversifying in the two different manners. The authors found that the results depended on the constraints imposed on the models. If short-selling constraints were imposed, then geographic diversification outperforms the sector diversification. If the problem is unconstrained, then the two strategies are statistically equivalent.

In another article, Kim and Mathur examined the impact of geographical diversification on firm performance (Kim and Mathur, 2007). The author's results suggested that geographical diversification increased operating costs but also increased return on equity and return on assets when compared to industrially diversified firms. These results suggest that there are some possible gains from geographic diversification. Other research has also looked at the impacts of geographic diversification (Qian, 2002; Carrieri, Errunza et al., 2004).

Methodology and Data

Properly allocating resources is essential for effective decision making. In this case, the resource being allocated is the farmland used for production. Resource allocation problems have relied on traditional portfolio methods to specify how to best diversify resources (Crisostomo and Featherstone, 1990; Harwood, Heifner et al., 1999). Portfolio theory relies on correlation as the measure of dependence. When the underlying distributions are normal or elliptical, this correlation measure is appropriate. If returns are not normally distributed, estimates of the efficient portfolio using the mean-variance approach could be erroneous (Hatherley and Alcock, 2007). Some of the issues with using linear correlation as a dependence measure are:

1. If two random variables are independent, the correlation coefficient is zero. The opposite of this cannot be true, if the two random variables are uncorrelated, that does not imply independence.
2. Correlation is invariant under strictly linear transformations. This is not true under nonlinear strictly increasing transformations.
3. Linear correlation is only defined for finite variances.

The task at hand is to incorporate alternative structures of dependence¹ into the portfolio model. The impact of using alternative dependence measures has recently begun to be researched heavily in the finance and insurance fields (Patton, 2002). This has opened the door to re-examine portfolio problems using these alternative dependence structures. One method that has been recommended is the use of multivariate copulas (Hennessy and Lapan, 2002).

Copulas are used to model multivariate distributions. An extensive treatment of copulas can be found in numerous books and research articles (Patton, 2002). For the purpose of this paper, a basic treatment of copulas will be sufficient to lay the foundation for future applications. A copula function is formally defined as (Patton, 2002):

a **copula** is a multivariate cumulative distribution function defined on the n -dimensional unit cube $[0, 1]^n$ with the following properties:

1. The range of $C(u_1, u_2, \dots, u_n)$ is the unit interval $[0, 1]$;
2. $C(u_1, u_2, \dots, u_n) = 0$ if any $u_i = 0$, for $i = 1, 2, \dots, n$.
3. $C(1, \dots, 1, u_i, 1, \dots, 1) = u_i$, for all $u_i \in [0, 1]$

By definition there are an infinite number of copulas that may be generated. In the field of risk management, one family of copulas that has been used extensively is that of the Archimedean copula (Hennessy and Lapan, 2002). The Archimedean has become used extensively for these applications because of the relative ease of calculating the copula.

¹ Embrechts et al formally defined dependence as: “summarizes the dependence structure between two random variables in one number.”

Sklar's Theorem

Although the application of copulas to statistical problems is relatively recent, the theory behind copulas was developed in 1959 (Sklar 1959). Sklar's Theorem states (Nelsen, 2006):

Let H be a joint distribution function with margins F and G . Then there exists a copula C such that for all x, y in \overline{R} ,

$$H(x,y) = C(F(x),G(y))$$

If F and G are continuous, then the copula function C is unique. If F and G are not continuous, then C is uniquely determined on $\text{Ran}F \times \text{Ran}G$. In addition, if C is a copula and F and G are distribution functions, then the function H is a joint distribution function with margins F and G .

Previous research has focused on some families of copula functions (Patton, 2002). The purpose of this research is to not elaborate exclusively on these functions². The Gaussian Copula will be used for this research.

Gaussian Copula

An extension of the multivariate normal distribution is that of a Gaussian Copula. The convenience of the Gaussian copula is that it can be used to model multivariate data that may exhibit non-normal dependencies and fat tails. The Gaussian Copula is formally defined as:

$$C(u_1, \dots, u_n; \Sigma) = \Phi^K(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n); \Sigma) \quad (1)$$

² For a complete review of copula theory refer to Joe, H. *Multivariate Models and Dependence Concepts* (London, Chapman & Hall, 1997), Nelsen, R.B. *An Introduction to Copulas* (New York, Springer, 2006).

The copula function $C(u_n)$ is defined by the standard multivariate normal distribution (Φ^K) and the linear correlation matrix (Σ). When $n = 2$, equation 1 can be rewritten in the following manner.

$$C(u_1, u_2) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(u)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)}\right\} dr ds \quad (2)$$

ρ is the linear correlation between the two variables.

The copula density function is now derived in the following manner.

$$c(\Phi(x_1), \dots, \Phi(x_n)) = \frac{f^{Gaussian}(x_1, \dots, x_n)}{\prod_{i=1}^n f^{Gaussian}(x_i)} \quad (3)$$

Equation (3) can then be rewritten using the definitions of the Gaussian functions.

$$= \frac{\frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} x' \Sigma^{-1} x\right)}{\prod_{i=1}^n \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} x_i^2\right)} \quad (4)$$

Simplifying equation (4) results in the final equation for the copula density function.

$$= \frac{1}{|\Sigma|^{1/2}} \exp\left(-\frac{1}{2} \zeta' (\Sigma^{-1} - I) \zeta\right) \quad (5)$$

Once the dependence between the returns has been estimated, a joint distribution function can be estimated. The returns generated through the joint distribution function will be used to calculate the CVaR. CVaR builds upon the original theory of VaR. The foundations of VaR can be traced back to 1952 (Roy, 1952). Roy defined in his seminal article the safety first theory. The safety theory included the use of a shortfall constraint that specified a given probability level of disaster. Some forty years later, Philippe Jorion built on that theory and called it VaR (Jorion, 1996). Value at risk can formally be defined as (McNeil, 2005):

Given some confidence level $\alpha \in (0, 1)$. The VaR of a given portfolio is given at the confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $(1 - \alpha)$.

$$VaR_{\alpha} = \inf \{l \in \mathfrak{R} : P(L > l) \leq 1 - \alpha\} = \inf \{l \in \mathfrak{R} : F_L(l) \geq \alpha\}$$

One of the major drawbacks of using VaR is that it is not coherent. VaR has been shown to be not sub-additive which means the VaR of a portfolio of two securities may be greater than the VaR of each individual security (Alexander and Baptista, 2004). VaR has also shown to estimate erroneous results when the data is not normally distributed (Stoica, 2006).

An alternative to the VaR that is shown to be coherent is the CVaR (Rockafellar and Uryasev, 2000; Alexander, Coleman et al., 2006; Acerbi, 2007). The CVaR is defined as:

$$CVaR = \inf_{\alpha} (\alpha + (1 - \beta)^{-1} \mathbb{E}([F_L(l) - \alpha]^+))$$

When the CVaR is continuous, it can be written as the conditional expectation of the loss:

$$CVaR = (1 - \beta)^{-1} \int_{F(l) \geq \alpha} F_L(l) p(S) dS$$

Where $p(S)$ represents the probability density for S .

Data

Three geographically distinct areas were chosen for this application. The locations were chosen based on harvesting windows and distance criteria. Pampa, Texas, Akron, Colorado, and Big Sandy, Montana were chosen for the analysis. All three areas grow Dryland wheat. For this analysis, yields for both spring and fall plantings were used. County level yields and prices were collected from 1973 until 2006 (USDA, NASS). These were used to calculate gross annual returns for each location.

The feasibility of the harvesting windows and the costs associated with each individual enterprise are based off of previously estimated work (Wolfley, 2008). Wolfley estimated farm-level operating costs for each area. These costs estimates were then used to calculate net-annual returns for each farm. These net returns form the basis of the CVaR measure. In this case, the VaR measure can be thought of as the probability of a dollar loss per acre.

Results

Using both the traditional VaR and the copula CVaR, a portfolio optimization was performed minimizing the respective risk measures. The results represent the VaR measures if the portfolio is equally weighted between the three production regions, the results of the copula CVaR are more conservative than both the traditional VaR and the copula based VaR. This represents the methodology of the CVaR in emphasizing expected shortfall or the lower tail. The copula VaR varies more at the 99% level than the traditional VaR. This is as expected as the Gaussian Copula will capture the tails more than the traditional mean-variance approach. The VaR measures can be interpreted as there is a 5% probability that the per/acre returns for the farm

portfolio will be \$4.00 under both the copula-VaR and the traditional VaR. Using the copula-CVaR, there is a 5% probability that losses will exceed \$8.84 per/acre.

Optimizing the portfolio based on minimizing CVaR, the optimal portfolio did not include any production within Texas. The optimal portfolio consisted of 47% in Colorado and 53% in Montana. Given these results, there is a 1% chance that there will be \$15.35 loss per acre. At 5% level, there is no expected loss. This can be compared to the VaR measures for all three areas with no diversification. At the 5% level for all individual areas, there is an expected loss. For Texas, there is an expected loss of \$34.42 per/acre, Colorado an expected loss of \$8.94, and for Montana an expected loss of \$4.95.

Conclusions

The role of VaR in corporate finance theory and research has been well established. The role of VaR in agricultural finance and agribusiness applications has seen only a few applications. This paper provides an application for implementing VaR into an agribusiness decision making. As agriculture becomes more industrialized, the role of risk measures such as VaR will become more utilized. In this case it was applied to geographical diversification and also modifying the traditional VaR estimation by incorporating a copula dependence parameter into the VaR estimation. In addition, an alternative risk measure was also calculated, CVaR. The CVaR, unlike VaR, is a coherent risk measure. Thus it does not suffer from many of the shortcomings of the VaR. In this report, geographical diversification relates to producing crops in three different regions. The land portfolio consisted of Dryland wheat production acres in Texas, Colorado, and Montana. Based on the VaR and the CVaR, the portfolio was optimized based on minimizing the expected loss based on historical gross revenues and estimated

production costs. The results showed that diversification could be reduced by producing in all three areas. The CVaR optimization consisted of only producing in Montana and Colorado.

The results of this report do not take into consideration the costs that could be involved with geographical diversification. Transportation and management issues are some of the main costs involved with geographical diversification. This could be a topic for further research. In addition, future estimation could incorporate alternative copulas into the model could provide better risk estimates.

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