



BANCA D'ITALIA
EUROSISTEMA

Temi di Discussione

(Working Papers)

Heterogeneity and learning with complete markets

by Sergio Santoro

April 2011

Number

806



BANCA D'ITALIA
EUROSISTEMA

Temi di discussione

(Working papers)

Heterogeneity and learning with complete markets

by Sergio Santoro

Number 806 - April 2011

The purpose of the Temi di discussione series is to promote the circulation of working papers prepared within the Bank of Italy or presented in Bank seminars by outside economists with the aim of stimulating comments and suggestions.

The views expressed in the articles are those of the authors and do not involve the responsibility of the Bank.

Editorial Board: MARCELLO PERICOLI, SILVIA MAGRI, LUISA CARPINELLI, EMANUELA CIAPANNA, DANIELA MARCONI, ANDREA NERI, MARZIA ROMANELLI, CONCETTA RONDINELLI, TIZIANO ROPELE, ANDREA SILVESTRINI.

Editorial Assistants: ROBERTO MARANO, NICOLETTA OLIVANTI.

HETEROGENEITY AND LEARNING WITH COMPLETE MARKETS

by Sergio Santoro*

Abstract

We study an endowment economy with complete markets and heterogeneous agents who do not have rational expectations, but form their beliefs using adaptive learning algorithms that may differ from one individual to another. We show that market completeness allows agents to smooth consumption across states of nature, but not across time, and that the initial wealth distribution is not enough to pin down the long-run equilibrium. Consequently, initial differences in beliefs create persistent consumption imbalances that are not grounded in fundamentals. In some cases these imbalances are eventually unsustainable: the debt of one of the agents would grow without bounds, and binding borrowing limits are necessary to prevent Ponzi schemes. Finally, we find that our slight departure from rational expectations affects efficiency properties of the competitive equilibrium: if the social welfare function attaches fixed Pareto weights to the different individuals, there are configurations of individual expectations under which society is better off with financial autarky than with complete markets. The first best can be restored by introducing a distortionary tax on borrowing, which transfers consumption from the more optimistic agent to the other.

JEL Classification: C62, D83, D84.

Keywords: learning, heterogeneous agents, complete markets.

Contents

1. Introduction.....	5
2. The model	8
2.1 Financial autarky	9
2.2 Complete markets	9
3. Equilibrium allocations with complete markets	10
3.1 Allocations when the debt limits are not binding	11
3.2 When are the debt limits binding?.....	15
4. Welfare	19
4.1 Pitfalls of financial integration when agents are learning	20
4.2 Efficiency.....	21
5. Conclusions.....	22
Appendix	24
References	27

* Bank of Italy, Economic Outlook and Monetary Policy Department. Email: sergio.santoro@bancaditalia.it.

1 Introduction¹

In general equilibrium models where agents receive idiosyncratic income shocks, the possibility to trade a complete set of state-contingent bonds works as an effective insurance device, allowing each individual to smooth consumption across states and across time in a way that eliminates the impact of agent-specific shocks. This “risk sharing” property implies that, for any initial wealth distribution, individual consumption is a well-defined, stationary function of aggregate shocks only, and that the competitive equilibrium achieves the first best, hence providing a rationale for financial innovation in the form of creating new instruments and opening up new markets.

These findings critically hinge on the assumption of rational expectations: agents have perfect knowledge of their own optimization problem, are capable of solving it, and know that all the other agents in the economy share the same degree of rationality. As a consequence, all subjective beliefs coincide with the objective expectations. However, a growing body of empirical evidence questions the assumption that expectations are rational and homogeneous among agents. Early results in this spirit can be found in Roberts (1998), where the forecasts of US agents collected in the Michigan and Livingston surveys are shown to be inconsistent with the hypothesis of purely rational expectations; instead, they provide evidence in favor of an intermediate degree of rationality, with beliefs partly explained by a simple form of backward-looking expectations. More recently, Branch (2004) and Branch (2007) show evidence suggesting that models featuring heterogeneous beliefs, and which allow the degree of heterogeneity to change over time, provide a better fit of survey data. Pfajfar and Santoro (2010) analyze the Michigan survey micro data on inflation expectations, documenting the fact that agents in different percentiles of the survey seem to be associated with different forecasting schemes. Moreover, several papers have shown that in many setups such departures from rational expectations are not innocuous. To recall only few examples, in the context of monetary policy it has been argued that interest rate rules that are optimal or guarantee determinacy under RE, can lead to instability if private expectations follow adaptive learning (see Bullard and Mitra (2002), Evans and Honkapohja (2003a), Evans and Honkapohja (2003b) and Evans and Honkapohja (2006)). In an asset pricing model, Adam, Marcet, and Nicolini (2009) show how a small departure from rational expectations significantly helps in matching numerous empirical facts.

¹I thank Francesco Caprioli and Giuseppe Ferrero for useful conversations. A previous version of this paper has been presented at the conference on “Expectations, Asset Bubbles and Financial Crises”, held in the Erasmus University of Rotterdam; I would like to thank the audience for the comments. All remaining errors are my own. The views expressed herein are those of the author, and do not necessarily reflect those of the Eurosystem or of the Banca d’Italia.

In this paper we take a normative point of view, and assess the robustness of typical properties of complete markets when we slightly depart from rational expectations. In particular, we endow each agent with a full understanding of her own optimization problem, of the asset market structure, and of the probability distribution of the exogenous shocks, while she does not know if other individuals have the same degree of rationality: hence, she is not sure if her subjective beliefs turn out to coincide with objective expectations. Instead, she computes forecasts of next period's contingent consumption according to an estimated model which is correctly specified, in the sense that nests rational expectations as a particular case, and updates the estimates using an algorithm commonly used in the adaptive learning literature (for an extensive monograph on adaptive learning, see Evans and Honkapohja (2001)). To prevent Ponzi schemes, we also impose debt limits that would never bind under rational expectations, close in spirit to the natural debt limits introduced in Aiyagari (1994).

We find that in our framework consumption is smoothed across states but not across time: in each period agents consume fractions of the aggregate endowment that are independent of the realizations of idiosyncratic shocks, but are not constant over time. In particular, they depend on past beliefs in a persistent way: heterogeneity in initial beliefs does not fade away asymptotically, and agents' expectations (and the consumption distribution) can converge to different equilibria, depending on initial beliefs and on aggregate shocks. The fact that consumption is not smoothed across time does not mean that anything can happen: in order to be able to repay her debt, if in some periods an household consumes more than in the rational expectations benchmark, then in other periods she must consume less.

We show that this type of dynamics would not be guaranteed by the evolution of beliefs for a relevant set of initial conditions of the learning process. In these cases the expectations-driven consumption imbalances are not sustainable forever, and the debt limit of one of the agents becomes binding in finite time, forcing her to cut consumption.

Moreover, our slight departure from rational expectations also affects efficiency properties of the competitive equilibrium, and suggests that differences in individual beliefs should be taken into account by policymakers when designing financial innovation. In fact, if we consider a social planner that attaches to the different agents fixed Pareto weights², we show that there exist configurations of individual expectations such that social welfare is lower with complete markets than with financial autarky. This result reflects the existence of a distortion associated to the presence of learning, which generates inefficient consumption imbalances, char-

²For example, the weights that would implement the competitive equilibrium under rational expectations.

acterized by more consumption going to the more optimistic agent.³ We also show a time and state dependent tax on borrowing that restores the first best.

Several papers study the consequences of introducing some kind of heterogeneity in the learning process followed by different groups of agents. Most of them consider linear (or linearized) forward looking models, where endogenous variables depend (among other things) on expectations that different individuals compute in different ways, and derive the implications of this heterogeneity for the long run behavior of the model; examples in this vein are Honkapohja and Mitra (2006) and Berardi (2007). We show that taking into account the existence of nonlinearities might have relevant consequences when analyzing the interaction of heterogeneous learning with the complete markets assumption. Caprioli (2008) derive similar results in an optimal fiscal policy framework: when markets are complete and private sector is learning, initial beliefs matter also in the long run, conditioning the optimal allocations obtained in the limit. Closer to our work is the paper by Assenza and Berardi (2009), which analyzes a credit economy à la Kiyotaki and Moore (1997) when expectations are not rational. They find that, when the learning mechanism used to update beliefs differs between borrowers and lenders, eventually the former *voluntarily* decide to default on their debt. There are two main differences between our approach and the one adopted in Assenza and Berardi (2009): (i) in our paper we do not introduce the financial frictions that characterize the Kiyotaki and Moore model, and show that even in this plain vanilla setup heterogeneous learning can lead one of the agents to accumulate an unsustainable debt⁴; (ii) we focus on an *involuntarily* overborrowing, which is explicitly ruled out by Assenza and Berardi (2009).

Another related strand of literature studies endowment economies with complete markets when agents preferences differ from the standard von Neumann-Morgenstern expected utility paradigm: they do not have full knowledge of the probability assessment of the possible states of the world, and try to guard against this form of ambiguity, also referred to as *Knightian uncertainty*. Liu (1998) shows that, if ambiguity is modeled using maxmin expected utility, and agents differ in their degree of Knightian uncertainty, the equilibrium prices and allocations are indeterminate in absence of aggregate uncertainty, while they are determinate but history dependent when aggregate uncertainty is introduced. Rigotti and Shannon (2005) show instead that indeterminacy arise robustly also in presence of aggregate uncertainty, if individual preferences over state contingent consumption bundles are incomplete as in Bewley (2002).

The rest of the paper is organized as follows. Section 2 describes the model,

³See the next section for a definition of what we mean by “optimistic” in this paper.

⁴However, in our setup agents never default in equilibrium, because the existence of borrowing limits stops the exploding path of debt before it becomes too high to be repaid.

and derives the individual optimality conditions under the two polar settings of financial autarky and complete markets. Section 3 characterizes the competitive equilibrium when markets are complete, in the benchmark case of rational expectations and under learning, shows how the learning allocations evolve over time, and provides sufficient conditions ensuring that debt limits bind in equilibrium. Section 4 is devoted to the impact of learning on the efficiency properties of the model. Section 5 concludes.

2 The Model

There are two agents in the economy, denoted by $i = 1, 2$.⁵ In each period $t \geq 0$ there is a realization of a stochastic event $s_t \in S$, which is publicly observable; we assume that the stochastic process $\{s_t\}$ is exogenous and Markov, and has a continuous support. Let the history of events up and until time t be denoted by $s^t \equiv [s_0, s_1, \dots, s_t]$, and the probability density function of s^t by $f(s^t)$. Agent i receives a stochastic endowment $\omega_t^i(s^t)$ that depends on the history s^t , and purchases a history-dependent consumption bundle $c_t^i(s^t)$; his utility function is given by:

$$U_t^i = \widehat{E}_t^i \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau^i) \quad (1)$$

where $\beta \in (0, 1)$, c_τ^i is shorthand notation for $c_\tau^i(s^\tau)$ ⁶, and $u(\cdot)$ is an increasing, twice continuously differentiable, strictly concave function, that satisfies standard Inada conditions; moreover, we assume that this one-period utility function represents homothetic preferences. Note that the operator \widehat{E}_t^i represents the conditional expectations of agent i , which are not necessarily rational. We study the model under two hypothesis for financial markets: (i) in the first case we assume financial autarky, so that each agent has no possibility to transfer wealth across time and states; (ii) alternatively, we assume that at each date $t \geq 0$ households have access to spot markets for bonds contingent on all possible realizations of s_{t+1} (*Arrow securities*); we denote by $b_t^i(\bar{s}, s^t)$ an Arrow security that pays one unit of consumption at $t + 1$ if and only if $s_{t+1} = \bar{s}$, and it is competitively traded at price $p_t^b(\bar{s})$.

⁵We interpret the agents as different households; alternatively, they could represent countries, as in Kim and Kim (2003).

⁶We suppress the explicit dependence on s^τ when it does not cause any ambiguity; otherwise, we switch back to the original notation $c_\tau^i(s^\tau)$

2.1 Financial Autarky

In the polar case of financial autarky, agent i chooses the consumption plan that maximizes equation (1) subject to the sequence of budget constraints:

$$c_t^i \leq \omega_t^i \quad (2)$$

where ω_t^i is shorthand notation for $\omega_t^i(s^t)$, and we assume that it can be observed by agent i . The solution of this problem is trivial, and requires that:

$$c_t^i = \omega_t^i \quad (3)$$

for each i and t . Note that under financial autarky the degree of rationality of the agents plays no role.

2.2 Complete Markets

Under this alternative market structure, each household faces a sequence of budget constraints of the form:

$$c_t^i + \int_{\bar{s} \in S} p_t^b(\bar{s}) b_t^i(\bar{s}, s^t) d\bar{s} \leq \omega_t^i + b_{t-1}^i(s^t) \quad (4)$$

Moreover, we have to impose some restrictions on asset trades to prevent Ponzi scheme. We choose to impose state-by-state debt limits that, for any history s^t , coincide with the maximum amount that agent i can repay starting from t , in case her consumption is forever equal to a fixed proportion ε of aggregate endowment; we call them *ε -natural debt limits*.⁷ Finally, we assume that consumption of agent i cannot fall below a proportion ε of aggregate endowment. We write these two additional set of constraints as follows:

$$-b_t^i(\bar{s}, s^t) \leq B_{\varepsilon,t}^i(\bar{s}, s^t) \quad (5)$$

and

$$c_t^i \geq \varepsilon \omega_t \quad (6)$$

where $\omega_t \equiv \omega_t^1 + \omega_t^2$ is aggregate endowment. Under rational expectations (RE hereafter), constraints (5) and (6) are usually imposed with $\varepsilon = 0$, and preferences alone are sufficient to guarantee that they are never binding in equilibrium; since, as we argue below, this is not the case when RE are replaced with learning, CRRA

⁷For a discussion on natural debt limits and complete markets under RE, see Ljungqvist and Sargent (2004), Ch. 8.

preferences could result in utility equal to $-\infty$. To prevent this unrealistic outcome, throughout the rest of the paper we assume that $\varepsilon > 0$.⁸ At time t , agent i chooses $c_t^i(s^t)$ and $b_t^i(\bar{s}, s^t)$ for any $\bar{s} \in S$ to maximize (1) subject to (4), (5) and (6). Let's denote with λ , ζ and μ the Lagrange multipliers on constraints (4), (5) and (6), respectively; hence, for all \bar{s} and i the following first-order conditions (FOCs) must hold:

$$0 = u'(c_t^i) f^i(s^t) - \lambda_t^i(s^t) + \mu_{\varepsilon,t}^i(s^t) \quad (7)$$

$$0 = -\lambda_t^i(s^t) p_t^b(\bar{s}) + \beta \widehat{\lambda}_{t+1}^i(\bar{s}, s^t) + \zeta_{\varepsilon,t}^i(s^t; \bar{s}) \quad (8)$$

where $\widehat{\lambda}_{t+1}^i(\bar{s}, s^t)$ is the value of the Lagrange multiplier on the budget constraint that, at time t , agent i expects to hold in period $t+1$ if $s_{t+1} = \bar{s}$; this belief can deviate from the true value if either the consumption in $t+1$, when $s_{t+1} = \bar{s}$, is different from the expected one (i.e., if $\widehat{c}_{t+1}^i(\bar{s}, s^t) \neq c_{t+1}^i(\bar{s}, s^t)$), or if the conditional probability density function is not correctly anticipated (i.e., if $\widehat{f}^i(s_{t+1} = \bar{s} | s^t) \neq f(s_{t+1} = \bar{s} | s^t)$), or if both of these sources of non-rationality materialize. For given beliefs and state-contingent bond prices $\{p_t^b(\bar{s})\}_{\bar{s} \in S}$, the optimum for household i is characterized by the FOCs (7) and (8), the sequence of budget constraints (4) with the sign of equality, the sequences of additional constraints (5) and (6) plus the appropriate complementary slackness conditions. Note that we assume that agents take beliefs as given: in Kreps (1998) terminology, they are anticipated utility maximizers.

3 Equilibrium Allocations with Complete Markets

Starting from the individual optimality conditions derived above, we now define the complete markets equilibrium.⁹

Definition 1. *A competitive equilibrium is a set of beliefs $\widehat{c}_{t+1}^i(\bar{s}, s^t)$ and $\widehat{f}^i(s_{t+1} = \bar{s} | s^t)$ for all \bar{s} , i and t , a stochastic process for consumption plans $c^i \equiv \{c_t^i(s^t)\}_{t=0}^{\infty}$ for $i = 1, 2$, and a stochastic process for state-contingent bond prices $\{p_t^b\}_{t=0}^{\infty}$, such that:*

1. *for $i = 1, 2$, given prices and beliefs, the consumption allocations satisfy agent i 's optimality conditions;*

⁸Nevertheless, it is easy to show that, for ε sufficiently small, constraints (5) and (6) would still be non-binding under RE.

⁹In the following definition, we implicitly assume that initial wealth of agents is zero ($b_{-1}^i(s_0) = 0$ for all i).

2. consumptions allocations and the implied bond portfolios $\{b_t^i(\cdot, s^t)\}_{t=0}^\infty$ satisfy the goods market clearing condition $c_t^1 + c_t^2 = \omega_t^1 + \omega_t^2$ for all t and all realizations of s^t , and the bond market clearing conditions $b_t^1(\bar{s}, s^t) + b_t^2(\bar{s}, s^t) = 0$ for all t , all realizations of s^t and \bar{s} .

This definition leaves open the determination and evolution of beliefs: allocations and prices depend not only on the realization of the exogenous state s^t , but also on some parameters characterizing the specific assumptions on beliefs' determination.¹⁰ Let's start considering an economy where all the agents have RE, namely $\hat{c}_{t+1}^i(\bar{s}, s^t) = c_{t+1}^i(\bar{s}, s^t)$ and $\hat{f}^i(s_{t+1} = \bar{s}|s^t) = f(s_{t+1} = \bar{s}|s^t)$ for any i , \bar{s} and t . Under this hypothesis, the competitive equilibrium of the model is well known, namely each household consumes a fraction of aggregate endowment $\omega_t \equiv \omega_t^1 + \omega_t^2$ that is constant across time and states:

$$c_t^1 = \psi^{RE} \omega_t, \quad c_t^2 = (1 - \psi^{RE}) \omega_t$$

Without loss of generality we can assume for simplicity that $\omega_0^1 = \omega_0^2$, and that $E_0 \omega_t^1 = E_0 \omega_t^2$ for any t ; in this case, it is easy to show that $\psi^{RE} = 1 - \psi^{RE} = \frac{1}{2}$.

In the rest of this section we characterize the competitive equilibrium under adaptive learning. In order to departure only slightly from RE, we retain the assumption that the conditional probability density function of the exogenous shocks is perfectly known by the agents; hence, the unique change with respect to the baseline framework is that we do not impose that $\hat{c}_{t+1}^i(\bar{s}, s^t) = c_{t+1}^i(\bar{s}, s^t)$.

Moreover, in the rest of the paper we suppose that the one-period utility function is of the standard CRRA form:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma}, & \text{if } \sigma \neq 1 \\ \log(c), & \text{if } \sigma = 1 \end{cases} \quad (9)$$

where $\sigma > 0$ denotes risk aversion.

3.1 Allocations when the debt limits are not binding

We start by assuming that, analogously to the RE case, the constraints (5) and (6) are not binding; this step will allow us to analyze under which conditions such an outcome can be sustained in equilibrium.

If (5) and (6) are not binding, then the relations (7) and (8) can be rewritten equivalently as:

$$u'(c_t^i) p_t^b(\bar{s}) = \beta u'(\hat{c}_{t+1}^i(\bar{s}, s^t)) f(s_{t+1} = \bar{s}|s^t) \quad (10)$$

¹⁰In case of the learning algorithm considered below, endogenous variables depend on the initial conditions of beliefs and of tracking parameters; in order to simplify notation, we do not indicate explicitly this dependence.

Note that the fact that $\widehat{c}_{t+1}^i(\bar{s}, s^t)$ can be different from $c_{t+1}^i(\bar{s}, s^t)$ does not mean that anything goes, and that any kind of beliefs can be supported in a competitive equilibrium. In fact, dividing equation (10) for agent 1 by the analogous relation for agent 2, we get:

$$\frac{u'(c_t^1)}{u'(c_t^2)} = \frac{u'(\widehat{c}_{t+1}^1(\bar{s}, s^t))}{u'(\widehat{c}_{t+1}^2(\bar{s}, s^t))} \quad (11)$$

for any \bar{s} . In other words, at time t the ratio of the one-step-ahead marginal utility of consumption forecasted by the two households has to be independent of the realization of s_{t+1} . When markets are complete and debt limits are not binding, even with non-rational expectations relative consumption is smoothed *across states*; as we will see below, departing from RE implies that typically it cannot be smoothed *across time*, and consumption allocations become *history dependent*. Intuitively, when agents are learning, equilibrium consumption depends not only on the history of exogenous states s^t , but also on the initial conditions of beliefs. Arrow securities are not sufficient to equalize consumption: there is a *missing market*, because agents cannot diversify away differences in initial beliefs.

Note that, if beliefs do not satisfy equation (11) for any \bar{s} , then trade in contingent assets would be endogenously restricted: debt limits for some Arrow securities should bind in equilibrium, while those for some others would not. In what follows we choose beliefs that rule out this possibility, in order to minimize the departure from RE.

Given (11), a natural way to model individual expectations is to posit a perceived law of motion (PLM):

$$c_t^i = \psi_t^i \omega_t \quad (12)$$

for $i = 1, 2$. Combining (11) with (12), and using the definition of the one-period utility function (9), we get:

$$c_t^2 = \frac{\psi_t^2}{\psi_t^1} c_t^1 \quad (13)$$

Using this equation to substitute out c_t^2 from the goods market clearing condition, we obtain the actual law of motion (ALM) for c_t^1 :

$$c_t^1 = \frac{\psi_t^1}{\psi_t^1 + \psi_t^2} \omega_t \quad (14)$$

and, analogously:

$$c_t^2 = \frac{\psi_t^2}{\psi_t^1 + \psi_t^2} \omega_t \quad (15)$$

Hence, the T -mapping from PLM to ALM is given by:

$$T \left(\begin{array}{c} \psi_t^1 \\ \psi_t^2 \end{array} \right) = \left(\begin{array}{c} \frac{\psi_t^1}{\psi_t^1 + \psi_t^2} \\ \frac{\psi_t^2}{\psi_t^1 + \psi_t^2} \end{array} \right) \quad (16)$$

We assume that agents update their beliefs using a (generalized) recursive least squares (RLS) learning algorithm of the form:

$$\psi_t^i = \psi_{t-1}^i + \gamma_t^i (R_t^i)^{-1} \omega_{t-1} (c_{t-1}^i - \psi_{t-1}^i \omega_{t-1}) \quad (17)$$

$$R_t^i = R_{t-1}^i + \gamma_t^i ((\omega_{t-1})^2 - R_{t-1}^i) \quad (18)$$

for $i = 1, 2$. The values γ_t^i are positive, deterministic, and non-increasing gain sequences that represents how much agent i 's expectations are updated in the direction of the last forecast error; in other words, the higher is γ_t^i , the less “confident” household i is in her own beliefs. We assume that these gains have a simple recursive structure:

$$\begin{aligned} \frac{1}{\gamma_t^i} &= \frac{1}{\gamma_{t-1}^i} + 1 \quad \text{for } t \geq 2 \\ \frac{1}{\gamma_1^i} &\geq 1 \quad \text{given} \end{aligned} \quad (19)$$

Note that we allow heterogeneity in agents' learning schemes along three dimensions: (i) in the initial beliefs ψ_0^i , which captures differences in *relative optimism*¹¹, (ii) in the initial gain γ_1^i , which captures differences in *relative confidence*¹², and (iii) in the initial estimates of the unconditional second moment of the aggregate endowment process, R_0^i . As a result, the variable $\gamma_t^i (R_t^i)^{-1}$ may differ between the agents; it implies that the same forecast error would yield different revisions of the estimated ψ^i 's: if agent 1 is more confident and/or believes the aggregate endowment to be more noisy, she pays less attention to the forecast error. We define this heterogeneity as a difference in *relative volatility-adjusted confidence*. We do not restrict the ψ_t^i 's to add up to one, as would be the case under RE: since decisions are decentralized, and households have no knowledge about how the rest of the economy forms its beliefs and decides its actions, it seems natural to posit that there is no mechanism that makes individual forecasts mutually consistent.¹³ When $\psi_t^1 + \psi_t^2$ is larger (smaller) than one we say that *market is optimistic (pessimistic)*.

To study the asymptotic behavior of agents' beliefs, we proceed along the lines of Honkapohja and Mitra (2006) and stack together the learning algorithms (17)-(18) for the two households. Let's define $\theta_t \equiv (\psi_t^1, Q_t^1, \psi_t^2, Q_t^2)'$, where $Q_{t-1}^i \equiv R_{t-1}^i$. Then we can write:

$$\theta_t = \theta_{t-1} + \bar{\gamma}_t H(\theta_{t-1}, X_t) + \bar{\gamma}_t^2 \rho_t(\theta_{t-1}, X_t) \quad (20)$$

¹¹If $\psi_0^1 > \psi_0^2$, we say that agent 1 is more optimistic than agent 2, and viceversa.

¹²The interpretation of the initial gain as an indicator of confidence is adopted in Adam, Marcet, and Nicolini (2009).

¹³Instead, market forces, that operate through bond prices, guarantee that actual individual consumption allocations are mutually consistent.

where $X_t \equiv (\omega_t, \omega_{t-1})'$ is the state vector, and $\bar{\gamma}_t \equiv \max\{\gamma_t^1, \gamma_t^2\}$; moreover:

$$H(\cdot) \equiv \begin{pmatrix} (Q_{t-1}^1)^{-1} (\omega_{t-1})^2 (T(\psi_{t-1}^1, \psi_{t-1}^2) - \psi_{t-1}^1) \\ (\omega_t)^2 - Q_{t-1}^1 \\ (Q_{t-1}^2)^{-1} (\omega_{t-1})^2 (T(\psi_{t-1}^1, \psi_{t-1}^2) - \psi_{t-1}^2) \\ (\omega_t)^2 - Q_{t-1}^2 \end{pmatrix} \quad (21)$$

and:

$$\rho_t(\cdot) \equiv \begin{pmatrix} \frac{\gamma_t^1 - \bar{\gamma}_t}{\bar{\gamma}_t^2} H_1(\cdot) \\ \frac{\gamma_{t+1}^1 - \bar{\gamma}_t}{\bar{\gamma}_t^2} H_2(\cdot) \\ \frac{\gamma_t^2 - \bar{\gamma}_t}{\bar{\gamma}_t^2} H_3(\cdot) \\ \frac{\gamma_{t+1}^2 - \bar{\gamma}_t}{\bar{\gamma}_t^2} H_4(\cdot) \end{pmatrix} \quad (22)$$

Equation (20) is a recursive stochastic algorithm of the standard form studied in stochastic approximation literature, and we analyze its asymptotic properties using the tools presented in Evans and Honkapohja (2001). We define an *associated ordinary differential equation* (ODE):

$$\frac{d\theta}{d\tau} = h(\theta), \quad \text{where } h(\theta) = \lim_{t \rightarrow \infty} EH(\theta, X_t) \quad (23)$$

and observe that the set of its locally stable fixed points coincides with the set Ψ of locally stable fixed points of the simpler ODE:

$$\frac{d\psi}{d\tau} = T(\psi) - \psi \quad (24)$$

where $\psi \equiv (\psi^1, \psi^2)'$. If $\bar{\psi} \in \Psi$, we say that $\bar{\psi}$ is *E-stable*. Under suitable regularity conditions, learning converges locally to $\bar{\psi}$ if and only if $\bar{\psi}$ is E-stable. One of these regularity condition is the assumption that all the rest points of (24) are locally isolated; instead, if there is a non-trivial connected set of rest points of (24) (as is the case when the derivative of $T(\psi) - \psi$, evaluated at $\bar{\psi}$ has at least a zero eigenvalue), Evans and McGough (2005a) and Evans and McGough (2005b) propose an alternative definition of E-stability: a continuum Ψ of rest points of (24) is E-stable if, for any $\bar{\psi} \in \Psi$, the derivative of $T(\psi) - \psi$ evaluated at $\bar{\psi}$ has eigenvalues with negative real part, apart from zero eigenvalues arising from the connectedness of Ψ . We are now ready to state and prove the following Proposition.

Proposition 1. *There are no locally isolated rest points of (24). Moreover, the set*

$$\Psi = \left\{ (\bar{\psi}^1, \bar{\psi}^2)' \in \mathbb{R}^2 : \bar{\psi}^1 = k, \bar{\psi}^2 = 1 - k, \forall k \in [0, 1] \right\}$$

is E-stable in the sense of Evans and McGough (2005a) and Evans and McGough (2005b).

Proof. See the Appendix. □

This result implies that also the long-term fraction of total endowment consumed by agent i can take any value, depending on where the beliefs converge. In turn, this is determined not only by the realization of the shock process $\{\omega_t\}_{t=0}^{\infty}$, but also by time 0 relative optimism and relative confidence of the households: initial conditions matter, and they influence the behavior of the economy also in the limit. In presence of heterogeneous learning, differences in initial beliefs are not damped over time, and expectations-driven consumption imbalances are not orderly unwind.¹⁴

In other words, consumption allocations given by (14) and (15) are non-ergodic. This feature is not novel in a complete markets model with sequential trading: under RE, it is well known that the steady-state wealth distribution is undetermined, and the long-run wealth distribution actually reached by the economy is pinned down by initial conditions on wealth. Our results extends this non-ergodicity property to individual beliefs: when they are not pinned down by the RE hypothesis, initial differences in optimism and confidence exerts a persistent influence on allocations.

3.2 When are the debt limits binding?

This non-ergodicity raises the natural question of whether *any* consumption allocations given by (14) and (15) (together with initial conditions for the ψ^i 's, the γ^i 's and the R^i 's) can be sustained in an equilibrium where debt limits never bind; since these allocations satisfy by construction the Euler equations (10), for $i = 1, 2$, all we have to do is to check under which conditions the budget constraints hold for any t and s^t , if consumption is determined by the candidate equilibrium (14)-(15). It turns out to be useful to rewrite the sequence of period-by-period budget constraints in terms of the intertemporal constraint that must hold in period 0; adapting the results obtained in Caprioli (2008), it is possible to show that the competitive equilibrium (if debt limits never bind) can equivalently be characterized by the conditions:

$$0 = E_0 \sum_{t=0}^{\infty} \beta^t A_t^i u'(c_t^i) (c_t^i - \omega_t^i) \quad (25)$$

for $i = 1, 2$, where A_t^i is defined by the recursion:

$$A_t^i = A_{t-1}^i \left(\frac{1}{\psi_t^1 + \psi_t^2} \frac{\psi_t^i}{\psi_{t-1}^i} \right)^\sigma \quad (26)$$

¹⁴Note that, in the case of a continuum of possible equilibria, no theoretical connection between E-stability and real-time learning is known; however, below we analyze in greater details the dynamics induced by learning, providing support to our focus on E-stability.

with initial condition $A_0^i = 1$. Equation (25) can be decomposed in the following way:

$$0 = E_0 \sum_{t=0}^{\infty} \beta^t A_t^i u'(c_t^i) \left(\frac{\psi_t^i}{\psi_t^1 + \psi_t^2} - \psi^{RE} \right) \omega_t + E_0 \sum_{t=0}^{\infty} \beta^t A_t^i u'(c_t^i) (\psi^{RE} \omega_t - \omega_t^i)$$

where the second term is equal to zero; moreover, if $\frac{\psi_t^i}{\psi_t^1 + \psi_t^2} > \psi^{RE}$ for any t and s^t , simple inspection shows that the first term in the above condition turns out to be strictly positive. We can summarize these results as follows.

Lemma 1. *Let I be equal to either 1 or 2, and $\frac{\psi_t^I}{\psi_t^1 + \psi_t^2} \geq \psi^{RE}$ for any t and s^t , with the strict inequality holding for at least one $T < \infty$; then, agent I 's debt grows unboundedly.*

This Lemma provides sufficient conditions under which *debt limits must bind in equilibrium*, to prevent Ponzi schemes: if one agent consumes more than in the benchmark RE case for some periods, then she must consume less in other periods. We now turn to analyze in greater details the dynamics induced by learning, and show that the condition of Lemma 1 is satisfied for a large set of possible initial beliefs so that, at some point in time, the debt limit (5) must become binding.

A few preliminary observations are worth mentioning. If $\psi_0^1 + \psi_0^2 = 1$, then $\frac{\psi_0^i}{\psi_0^1 + \psi_0^2} = \psi_0^i$, and beliefs remain constant: the economy starts from a self-confirming equilibrium, where incoming data never contradicts agents' beliefs. Hence, the long-run distribution of consumption under the candidate equilibrium (14)-(15) is trivially given by (ψ_0^1, ψ_0^2) .

In the general case of $\psi_0^1 + \psi_0^2 \neq 1$ dynamics are more interesting, and depend on relative volatility-adjusted confidence in the following way.

Proposition 2. *Without loss of generality, let's assume that $\psi_0^1 + \psi_0^2 > 1$.¹⁵ If consumption allocations are given by (14)-(15), then for any realization of the aggregate endowment process $\{\omega_t(s^t)\}_{t=0}^{\infty}$, the corresponding consumption ratio $\left\{ \frac{c_t^1(s^t)}{c_t^2(s^t)} \right\}_{t=0}^{\infty}$ is a monotonic increasing (decreasing) sequence if and only if $\gamma_1^1 (R_1^1)^{-1} < (>) \gamma_1^2 (R_1^2)^{-1}$; if $\gamma_1^1 (R_1^1)^{-1} = \gamma_1^2 (R_1^2)^{-1}$, then the consumption ratio is constant over time.*

Proof. See the Appendix. □

¹⁵When $\psi_0^1 + \psi_0^2 < 1$, symmetric results hold.

This result has a simple intuition. If market is initially optimistic, the variable $\psi_0^1 + \psi_0^2$ has to gradually diminish towards one. The cost of the adjustment is not necessarily borne by the two agents in the same proportion; the household which has a higher volatility-adjusted confidence (the one with smaller $\gamma_1^i (R_1^i)^{-1}$) is going to revise downwards her beliefs at a smaller pace, hence putting the heaviest burden on the other household's shoulders. In turn, this means that her consumption, relative to the other agent consumption, grows over time, converging to a higher level than the one it started from. If the household initially more optimistic happens to be the one which is also more confident, Proposition 2 states that her consumption should be larger than the other households consumption at any t . Hence, combining Lemma 1 with Proposition 2, we get the following Corollary.

Corollary 1. *Let's assume that $\psi_0^1 + \psi_0^2 > 1$, and that $\psi_0^1 > \psi_0^2$. If $\gamma_1^1 (R_1^1)^{-1} < \gamma_1^2 (R_1^2)^{-1}$, then there exists a time period $T < \infty$ such that agent 1's debt limit (5) binds at T .*

The fact that the debt limit becomes binding, together with the existence of the constraint (6), means that household 1 consumption drops to ε and remains constant at this level. In other words, if the interaction between heterogeneous non-rational beliefs and complete markets allows an agent to consume in the first periods more than what she would be entitled to under RE, she accumulates debt that has to repay contracting consumption in the rest of her life. Notice that only slight departures from RE are necessary to deliver this outcome: even if the ψ_0^i 's are arbitrarily close to (but different from) ψ^{RE} , the debt limit will eventually be reached. This is a consequence of the fact that, as argued above, Arrow securities are enough to smooth consumption across states, but not across time; hence, non-ergodicity of the allocations implies that even small initial differences in beliefs are sufficient to induce persistent consumption imbalances that can violate the no-Ponzi condition.¹⁶

If instead the household initially more optimistic happens to be the one which is less confident, Proposition 2 leaves open the possibility that in the long run she consumes less than the other household, hence potentially satisfying the intertemporal budget constraint (25). Numerical simulations shows that, for some initial conditions for ψ^i 's and γ^i 's, this is actually the case.

¹⁶This result hinges on the hypothesis that, when debt limits are not binding, consumption allocations depend on forecasts computed only one step ahead. Assuming that agents use long-horizon forecasts to decide today's consumption, along the lines put forward by Preston (2005), might help to pin down a long run equilibrium that, independently of the initial beliefs, satisfies the no-Ponzi condition. This is left as a relevant topic for future research.

3.2.1 Special case: no aggregate uncertainty

In the rest of this section we simplify the model assuming that there is no aggregate uncertainty; this simplification allows us to get even sharper results on the conditions ensuring that debt limits never bind in equilibrium.

We normalize to one the constant aggregate endowment. As is common in the literature on two types, the shock is assumed to be perfectly negatively correlated across the agents; hence, we can write $\omega_t^1(s^t) = s_t$ and $\omega_t^2(s^t) = 1 - s_t$, where s_t can take values between zero and one. Consumption in agents' PLM is now a constant; we modify the agent i 's learning algorithm accordingly:

$$\psi_t^i = \psi_{t-1}^i + \gamma_t^i (c_{t-1}^i - \psi_{t-1}^i)$$

Using the ALM when debt limits are not binding to substitute out c_{t-1}^i , we get:

$$\psi_t^i = \psi_{t-1}^i + \gamma_t^i \left(\frac{\psi_{t-1}^i}{\psi_{t-1}^1 + \psi_{t-1}^2} - \psi_{t-1}^i \right) \quad (27)$$

where the γ_t^i 's are still given by (19). Equation (27) makes clear that, when debt limits are not binding, ψ^i 's are deterministic sequences in absence of aggregate uncertainty. This implies that also consumption allocations and marginal utilities are independent of stochastic shocks. We can use this result to rewrite the intertemporal budget constraint (25) as follows:

$$0 = \sum_{t=0}^{\infty} \beta^t A_t^i u' (c_t^i) (c_t^i - E_0 \omega_t^i) \quad (28)$$

Given our assumptions on the endowment process, we know that $E_0 \omega_t^1 = E_0 \omega_t^2 = \frac{1}{2}$. Let's consider the case of an economy characterized by initial beliefs such that $\psi_0^1 + \psi_0^2 > 1$, and $\psi_0^1 > \psi_0^2$; by Proposition 2 we know that equation (28) can hold only if $\gamma_1^1 > \gamma_1^2$. Hence, for any value of ψ_0^1 , ψ_0^2 and γ_1^2 such that $\psi_0^1 + \psi_0^2 > 1$, and $\psi_0^1 > \psi_0^2$, we define the function F^1 of γ_1^1 as:

$$F^1 (\gamma_1^1; \psi_0^1, \psi_0^2, \gamma_1^2) \equiv \sum_{t=0}^{\infty} \beta^t A_t^1 u' (c_t^1) \left(c_t^1 - \frac{1}{2} \right) \quad (29)$$

As argued above, for $\gamma_1^1 \leq \gamma_1^2$ we have that $F^1(\cdot) > 0$; moreover, $F^1(\cdot)$ is continuous. Hence, if it were also monotonic decreasing, we could conclude by the Intermediate Value Theorem that there is at most one value of γ_1^1 such that $F^1(\cdot) = 0$; numerical simulations confirm that $F^1(\cdot)$ is indeed monotonic decreasing.¹⁷ We

¹⁷The results are available from the author upon request. We conjecture that it should be possible to prove this finding analytically, but the corresponding Proposition is not completely worked out yet.

denote by $\gamma_1^{1*}(\psi_0^1, \psi_0^2, \gamma_1^2)$ this value, if it exists. Next, we define the set of all the possible initial conditions of beliefs and gains as $\Xi \equiv [0, 1]^4$, and the subset of Ξ such that $\psi_0^1 + \psi_0^2 > 1$, $\psi_0^1 > \psi_0^2$ and the intertemporal budget constraint (28) holds as:

$$\Xi_+^{NRE,1} \equiv \{(\psi_0^1, \psi_0^2, \gamma_1^1, \gamma_1^2) \in \Xi : \psi_0^1 + \psi_0^2 > 1, \psi_0^1 > \psi_0^2, \gamma_1^1 = \gamma_1^{1*}(\psi_0^1, \psi_0^2, \gamma_1^2)\}$$

By the same token, if $\psi_0^1 + \psi_0^2 > 1$ and $\psi_0^1 < \psi_0^2$, for $\gamma_1^1 \geq \gamma_1^2$ we have that $F^1(\cdot) < 0$; moreover, also in this case numerical simulations suggest that $F^1(\cdot)$ is monotonic decreasing, which implies that $\gamma_1^{1*}(\psi_0^1, \psi_0^2, \gamma_1^2)$ is unique, if it exists. Let's define the subset of Ξ such that $\psi_0^1 + \psi_0^2 > 1$, $\psi_0^1 < \psi_0^2$ and the intertemporal budget constraint (28) holds as:

$$\Xi_+^{NRE,2} \equiv \{(\psi_0^1, \psi_0^2, \gamma_1^1, \gamma_1^2) \in \Xi : \psi_0^1 + \psi_0^2 > 1, \psi_0^1 < \psi_0^2, \gamma_1^1 = \gamma_1^{1*}(\psi_0^1, \psi_0^2, \gamma_1^2)\}$$

We can apply similar arguments to construct the sets $\Xi_-^{NRE,1}$ and $\Xi_-^{NRE,2}$ of all the initial conditions such that $\psi_0^1 + \psi_0^2 < 1$ and the intertemporal budget constraint (28) holds. Finally, when $\psi_0^1 + \psi_0^2 = 1$ we know that the economy starts from a self-confirming equilibrium; in this case, the intertemporal budget constraint holds if and only if the economy starts from the RE equilibrium:

$$\Xi^{RE} \equiv \left\{ (\psi_0^1, \psi_0^2, \gamma_1^1, \gamma_1^2) \in \Xi : \psi_0^1 = \psi_0^2 = \frac{1}{2} \right\}$$

To sum up, the set of all the initial conditions of beliefs and gains such that the intertemporal budget constraint holds is given by:

$$\Xi^* = \Xi_+^{NRE,1} \cup \Xi_+^{NRE,2} \cup \Xi_-^{NRE,1} \cup \Xi_-^{NRE,2} \cup \Xi^{RE}$$

Note all the sets composing Ξ^* are subsets of \mathbb{R}^4 whose dimension is strictly smaller than 4; hence, their Lebesgue measure μ is zero. By subadditivity, we conclude that $\mu(\Xi^*) = 0$.

Remark 1. *Numerical investigations show that the set of all the initial conditions of beliefs and gains such that the intertemporal budget constraint (28) holds has Lebesgue measure zero; hence, there exists typically a time $T < \infty$ starting from which the debt limit of one of the agents starts binding.*

4 Welfare

The assumption that agents have non-rational expectations, updated according to a learning algorithm, introduces a distortion in the complete markets setup.

Hence, in this section we study the implications of this distortion for the efficiency properties of the competitive equilibrium. We first consider how a full financial integration, namely a shift from financial autarky to complete markets, affects social welfare, and then we turn to analyze to what extent the First and Second Welfare Theorems hold in our framework.

4.1 Pitfalls of Financial Integration when Agents are Learning

Consider a social welfare function of the form:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\alpha^1 \frac{(c_t^1)^{1-\sigma} - 1}{1-\sigma} + \alpha^2 \frac{(c_t^2)^{1-\sigma} - 1}{1-\sigma} \right) \quad (30)$$

where α^i is the (non-negative) Pareto weight attached on individual i 's utility. Notice that the expectations in (30) are rational: the social planner fully knows how the agents behave. We can now evaluate U_0 under financial autarky (denoted by U_0^A) and under complete markets (denoted by U_0^C). It is well known that, when expectations are rational, the former is smaller than the latter; instead, in our setting this is not always the case. To show formally this finding, we again concentrate on the economy with no aggregate uncertainty; in this setting the rational planner can exactly compute at time 0 the whole sequence of future agents' beliefs, since they are non-stochastic. In the general case this is not possible anymore, and the calculation of $E_0 \psi_t^i \omega_t$ would become pretty much involved; however, we conjecture that it should not affect the main conclusion.

Proposition 3. *Let $\sigma \geq 1$; then, for any α^i , with $i = 1, 2$, there exist initial conditions for individual beliefs ψ_0^i and gain parameters γ_1^i , and a set of values of ε such that $U_0^C < U_0^A$.*

Proof. See the Appendix. □

Hence, for empirically plausible values of risk aversion, financial integration can be detrimental for social welfare. If agents' utility function is sufficiently concave, when markets are complete the increase in *across-time* average consumption differential with respect to RE, due to expectations-driven consumption imbalances and, possibly, binding debt limits, has the potential to more than offset the reduction in volatility stemming from the *across-states* smoothing allowed by Arrow securities. This result shows to what extent the consumption imbalances created by the interaction between learning and market completeness can drive the economy far from an equilibrium grounded in fundamentals.

Our Proposition assumes exogenous weights α^i ; in particular, they are independent of ψ_0 . To overturn the result stated in Proposition 3, the social welfare function should attach a higher weight to the household that is bounded to consume more in the long run, with respect to what would be optimal under RE. It is worth noting that also under RE, if we consider α^i that do not depend on initial conditions (in particular, on the initial wealth distribution), a result analogous to Proposition 3 could be obtained; the novel feature of our setting is that also initial beliefs of the households are now crucial to assess the desirability of financial innovation.

4.2 Efficiency

The first best allocations can be found maximizing equation (30) subject to technological constraints and initial conditions. It is easy to show¹⁸ that the solution is characterized by the optimality condition:

$$\frac{c_t^1}{c_t^2} = \left(\frac{\alpha^1}{\alpha^2} \right)^{\frac{1}{\sigma}} \quad (31)$$

To gauge the source of the distortion introduced by learning, we can compare it with the analogous optimality condition that holds in a competitive equilibrium, equation (11), that we rewrite here in a more convenient form:

$$\frac{c_t^1}{c_t^2} = \begin{cases} \frac{\psi_t^1}{\psi_t^2}, & \text{if debt limits are not binding} \\ \frac{\varepsilon}{1-\varepsilon}, & \text{if debt limit for agent 1 is binding} \end{cases} \quad (32)$$

It is clear that in both instances relative consumption is independent of the realizations of individual endowments, due to market completeness; however, in the first best relative consumption is also independent of time and of aggregate endowment, while it is not the case in the competitive equilibrium (unless the following condition hold: $\psi_0^1 = \psi_0^2 = \frac{1}{2}$). The equivalence between Pareto optima and competitive equilibria breaks down in this environment.

Our decentralized equilibrium can be seen as the outcome of a command economy in which the planner has time-varying Pareto weights that evolve in a suitable manner. If the α^i 's change over time, the optimality condition in the first best now reads:

$$\frac{c_t^1}{c_t^2} = \left(\frac{\alpha_t^1}{\alpha_t^2} \right)^{\frac{1}{\sigma}} \quad (33)$$

which makes possible the equivalence with the allocation implied by equation (32), as long as the time-varying weights track the evolution of beliefs: the planner delivers more and more consumption to the agent who (in the competitive equilibrium)

¹⁸See, among others, Ljungqvist and Sargent (2004), Ch. 8.

is more and more optimistic. If a debt limit binds in the competitive equilibrium at some $T < \infty$, from T on the planner should deliver less consumption to the over-indebted household.

This result can be related to the literature on insurance with limited commitment; when households are free to renege their debt, Marcat and Marimon (1998) show that the optimal allocations can be interpreted as the solution to a standard planner problem with Pareto weights that shift over time in such a way that consumers never want to default: when the participation constraint of a given household is binding, her consumption is increased for many periods.

If we want to achieve the first best in the more general case of fixed Pareto weights, we should introduce some other distortion in the individual problem designed in such a way to offset the one stemming from learning. The most straightforward way to do so is to modify the budget constraint of the households, introducing a distortionary tax on borrowing τ_t^i . In order to guarantee that the fiscal authority budget is balanced in every period, and that the first best consumption can be sustained in equilibrium, the proceeds of this tax are rebated as a lump-sum transfer $T_t^i = \tau_t^i \int_{\bar{s} \in S} p_t^b(\bar{s}) b_t^i(\bar{s}, s^t) d\bar{s}$. Agent i budget constraint becomes:

$$c_t^i + (1 - \tau_t^i) \int_{\bar{s} \in S} p_t^b(\bar{s}) b_t^i(\bar{s}, s^t) d\bar{s} \leq \omega_t^i + b_{t-1}^i(s^t) + T_t^i \quad (34)$$

which implies that the optimality condition (32) becomes:

$$\frac{c_t^1}{c_t^2} = \begin{cases} \left(\frac{1-\tau_t^1}{1-\tau_t^2} \right)^{\frac{1}{\sigma}} \frac{\psi_t^1}{\psi_t^2}, & \text{if debt limits are not binding} \\ \frac{\varepsilon}{1-\varepsilon}, & \text{if debt limit for agent 1 is binding} \end{cases} \quad (35)$$

If the planner set taxes as a function of beliefs in such a way that the right hand side of (35) is equal to $\left(\frac{\alpha^1}{\alpha^2} \right)^{\frac{1}{\sigma}}$ every period, the competitive equilibrium allocations replicate the Pareto optimum associated with the weights α^1 and α^2 , and debt limits never bind. In particular, it should tax more the agent who is “too” optimistic with respect to the ratio of Pareto weights, in order to contain her consumption.

5 Conclusions

In this paper we have developed a simple endowment economy inhabited by two agents, which receive idiosyncratic shocks to their income, can trade in a full array of Arrow securities, and face debt limits to prevent Ponzi schemes. We depart from the standard full-insurance model by assuming that individuals are not rational, but form their beliefs in a way consistent with the adaptive learning literature;

the algorithm they use to update their one-step-ahead consumption expectations might display heterogeneity in terms of initial conditions.

We found that also in our setting market completeness is enough to smooth consumption across states of nature; however, the learning process hampers the possibility to smooth consumption across time, and imparts to the system dynamics that are not present under rational expectations. Asymptotically, the impact of initial beliefs does not fade away, and it contributes to determine the consumption allocations characterizing the self-confirming equilibrium where the economy settles down: when market is initially optimistic, the consumption of the more confident household, relative to the other agent's consumption, tends to grow over time. Hence, it either converges to a higher level than the one it started from, if this consumption plan does not entail the accumulation of an infinitely large debt, or face a sudden drop, when the debt limit starts binding. The opposite holds when market is initially pessimistic.

Moreover, the introduction of learning also affects efficiency properties of the competitive equilibrium, and suggests that differences in individual beliefs should be taken into account by policymakers when designing financial innovation. In fact, if we consider a social planner that attaches to the different agents fixed Pareto weights, we show that there exists a configuration of individual expectations such that social welfare is lower with complete markets than with financial autarky. This result reflects the existence of a distortion associated to the presence of learning, which generates inefficient consumption imbalances, characterized by more consumption going to the more optimistic agent. We also show a time and state dependent tax on borrowing that restores the first best.

We want to emphasize that this paper features only a slight departure from rational expectations: each agent understands her individual maximization problem and how to solve it, and knows the probability distribution of the exogenous shocks. The only source of bounded rationality is the lack of knowledge of how the other household forms her decisions, which is a relevant piece of information to correctly forecast own future consumption in a general equilibrium model where prices, that might diverge from their rational expectations value, guarantee market clearing. Moreover, in the long run the economy settles down in a self-confirming equilibrium, so that expectations become asymptotically model consistent. Even so, our results show that easing the rational expectations hypothesis might have significant implications, hence strengthening the case for a careful consideration of private sector beliefs when designing financial innovation policies.

Appendix

Proof of Proposition 1. We start noting that rest points of equation (24) must satisfy the following condition:

$$\left(\begin{array}{c} \frac{\psi^1}{\psi^1+\psi^2} - \psi^1 \\ \frac{\psi^2}{\psi^1+\psi^2} - \psi^2 \end{array} \right) = 0 \quad (\text{A.1})$$

Simple inspection confirms that, when ψ^i 's are between 0 and 1, the above equation is satisfied for any point in the set Ψ defined in the statement of the Proposition.

To check E-stability, we compute the Jacobian $DT - I$:

$$DT(\psi) - I = \left(\begin{array}{cc} \frac{\psi^2}{(\psi^1+\psi^2)^2} - 1 & \frac{-\psi^1}{(\psi^1+\psi^2)^2} \\ \frac{-\psi^2}{(\psi^1+\psi^2)^2} & \frac{\psi^1}{(\psi^1+\psi^2)^2} - 1 \end{array} \right) \quad (\text{A.2})$$

When evaluated at any of the points in Ψ , it is easy to show that the eigenvalues of the above matrix are -1 and 0 ¹⁹, hence completing the proof. \square

We now state and prove a short technical Lemma, which will be useful in the proof of Proposition 2

Lemma 2. *Let $\gamma_1^1 (R_1^1)^{-1} \geq \gamma_1^2 (R_1^2)^{-1}$. Then, $\gamma_t^1 (R_t^1)^{-1} \geq \gamma_t^2 (R_t^2)^{-1}$ for any $t > 1$.*

Proof. Let's consider the random variable defined by:

$$G_t \equiv \gamma_t^1 (R_t^1)^{-1} - \gamma_t^2 (R_t^2)^{-1}$$

Given the recursive law of motion of R_t^i as described in equation (18), it can be rewritten as:

$$\frac{\gamma_t^1 R_{t-1}^2 + \gamma_t^1 \gamma_t^2 ((\omega_{t-1})^2 - R_{t-1}^2) - \gamma_t^2 R_{t-1}^1 - \gamma_t^1 \gamma_t^2 ((\omega_{t-1})^2 - R_{t-1}^1)}{R_t^1 R_t^2}$$

The above equation, together with some simple algebra, allows us to write:

$$G_t \geq 0 \Leftrightarrow \gamma_t^1 R_{t-1}^2 (1 - \gamma_t^2) - \gamma_t^2 R_{t-1}^1 (1 - \gamma_t^1) \geq 0$$

or, equivalently:

$$G_t \geq 0 \Leftrightarrow \frac{1 - \gamma_t^2}{1 - \gamma_t^1} \geq \frac{\gamma_t^2 (R_{t-1}^2)^{-1}}{\gamma_t^1 (R_{t-1}^1)^{-1}} \quad (\text{A.3})$$

¹⁹In fact, the trace of the matrix is -1 , while the determinant is equal to 0 .

Observe that the definition of the γ^i 's, equation (19), implies that $1 - \gamma_t^i = \frac{\gamma_t^i}{\gamma_{t-1}^i}$, for $i = 1, 2$. Plugging this equivalent representation into equation (A.3), we get:

$$G_t \geq 0 \Leftrightarrow \frac{\frac{\gamma_t^2}{\gamma_{t-1}^2}}{\frac{\gamma_t^1}{\gamma_{t-1}^1}} \geq \frac{\gamma_t^2 (R_{t-1}^2)^{-1}}{\gamma_t^1 (R_{t-1}^1)^{-1}}$$

which can be manipulated to conclude that:

$$G_t \geq 0 \Leftrightarrow G_{t-1} \geq 0$$

A trivial inductive argument completes the proof. \square

Proof of Proposition 2. First of all, note that $\frac{c_t^1}{c_t^2} = \frac{\psi_t^1}{\psi_t^2}$; moreover, using equation (17) the ratio $\frac{\psi_t^1}{\psi_t^2}$ can be written down recursively:

$$\begin{aligned} \frac{\psi_t^1}{\psi_t^2} &= \frac{\psi_{t-1}^1 + \gamma_t^1 (R_t^1)^{-1} \omega_{t-1} \left(\frac{\psi_{t-1}^1}{\psi_{t-1}^1 + \psi_{t-1}^2} \omega_{t-1} - \psi_{t-1}^1 \omega_{t-1} \right)}{\psi_{t-1}^2 + \gamma_t^2 (R_t^2)^{-1} \omega_{t-1} \left(\frac{\psi_{t-1}^2}{\psi_{t-1}^1 + \psi_{t-1}^2} \omega_{t-1} - \psi_{t-1}^2 \omega_{t-1} \right)} \\ &= \frac{\psi_{t-1}^1 \frac{(\psi_{t-1}^1 + \psi_{t-1}^2)}{(\omega_{t-1})^2} + \gamma_t^1 (R_t^1)^{-1} (1 - (\psi_{t-1}^1 + \psi_{t-1}^2))}{\psi_{t-1}^2 \frac{(\psi_{t-1}^1 + \psi_{t-1}^2)}{(\omega_{t-1})^2} + \gamma_t^2 (R_t^2)^{-1} (1 - (\psi_{t-1}^1 + \psi_{t-1}^2))} \end{aligned}$$

which implies that, for any $t > 0$:

$$\frac{\psi_t^1}{\psi_t^2} \geq \frac{\psi_{t-1}^1}{\psi_{t-1}^2} \Leftrightarrow \frac{\frac{(\psi_{t-1}^1 + \psi_{t-1}^2)}{(\omega_{t-1})^2} + \gamma_t^1 (R_t^1)^{-1} (1 - (\psi_{t-1}^1 + \psi_{t-1}^2))}{\frac{(\psi_{t-1}^1 + \psi_{t-1}^2)}{(\omega_{t-1})^2} + \gamma_t^2 (R_t^2)^{-1} (1 - (\psi_{t-1}^1 + \psi_{t-1}^2))} \geq 1$$

Simple algebra shows that:

$$\frac{\frac{(\psi_{t-1}^1 + \psi_{t-1}^2)}{(\omega_{t-1})^2} + \gamma_t^1 (R_t^1)^{-1} (1 - (\psi_{t-1}^1 + \psi_{t-1}^2))}{\frac{(\psi_{t-1}^1 + \psi_{t-1}^2)}{(\omega_{t-1})^2} + \gamma_t^2 (R_t^2)^{-1} (1 - (\psi_{t-1}^1 + \psi_{t-1}^2))} \geq 1 \quad (\text{A.4})$$

if and only if:

$$\left(\gamma_t^1 (R_t^1)^{-1} - \gamma_t^2 (R_t^2)^{-1} \right) [1 - (\psi_{t-1}^1 + \psi_{t-1}^2)] \geq 0 \quad (\text{A.5})$$

The condition (A.5) is written in terms of γ^i 's, R^i 's and ψ^i 's at time t , while the statement of the Proposition is in terms of initial confidence and beliefs. However,

we know from Lemma 2 that if $\gamma_1^1 (R_1^1)^{-1} > (<)\gamma_1^2 (R_1^2)^{-1}$, then $\gamma_t^1 (R_t^1)^{-1} > (<)\gamma_t^2 (R_t^2)^{-1}$ for any $t < \infty$; hence, all remains to show is that, if $\psi_0^1 + \psi_0^2 > (<)1$, then $\psi_t^1 + \psi_t^2 \geq (>)1$ for any $t < \infty$. To do this, we use equation (17) to obtain:

$$\begin{aligned} \psi_t^1 + \psi_t^2 &= \psi_{t-1}^1 + \psi_{t-1}^2 + \gamma_t^1 (R_t^1)^{-1} (\omega_{t-1})^2 \left(\frac{\psi_{t-1}^1}{\psi_{t-1}^1 + \psi_{t-1}^2} - \psi_{t-1}^1 \right) + \\ &\quad \gamma_t^2 (R_t^2)^{-1} (\omega_{t-1})^2 \left(\frac{\psi_{t-1}^2}{\psi_{t-1}^1 + \psi_{t-1}^2} - \psi_{t-1}^2 \right) \end{aligned}$$

Without loss of generality, let's assume that $\gamma_t^1 (R_t^1)^{-1} > \gamma_t^2 (R_t^2)^{-1}$.²⁰ Starting from the above equation, simple algebra shows that:

$$\begin{aligned} \psi_t^1 + \psi_t^2 \geq 1 \Leftrightarrow \psi_{t-1}^1 + \psi_{t-1}^2 - 1 \geq &\left(\psi_{t-1}^2 - \frac{\psi_{t-1}^2}{\psi_{t-1}^1 + \psi_{t-1}^2} \right) \left(\gamma_t^2 (R_t^2)^{-1} - \gamma_t^1 (R_t^1)^{-1} \right) (\omega_{t-1})^2 \\ &+ (\psi_{t-1}^1 + \psi_{t-1}^2 - 1) \gamma_t^1 (R_t^1)^{-1} (\omega_{t-1})^2 \end{aligned} \quad (\text{A.6})$$

Since $\left(\gamma_t^2 (R_t^2)^{-1} - \gamma_t^1 (R_t^1)^{-1} \right) < 0$ by Lemma 2, and $\gamma_t^1 (R_t^1)^{-1} (\omega_{t-1})^2 < 1$, equation (A.6) shows that $\psi_{t-1}^1 + \psi_{t-1}^2 > 1$ implies that $\psi_t^1 + \psi_t^2 > 1$. A trivial inductive argument completes the proof. \square

Proof of Proposition 3. Here we sketch the proof, which is based on the fact that $u(0) = -\infty$ when $\sigma \geq 1$, while $u(1)$ is always finite. Hence, the term:

$$\alpha^1 \frac{(c_t^1)^{1-\sigma} - 1}{1-\sigma} + \alpha^2 \frac{(c_t^2)^{1-\sigma} - 1}{1-\sigma} \quad (\text{A.7})$$

in the social welfare function gets arbitrarily close to $-\infty$ when consumption of one of the two agents approaches zero. Given Proposition 2, if $\gamma_1^1 = \gamma_1^2$ the consumption allocations remain at their initial level, as long as debt limits don't bind. Therefore, we can pick an initial distribution of beliefs ψ_0 such that c^1 is arbitrarily close to 0 as long as debt limits don't bind. By Lemma 1, the debt limit for agent 2 starts binding from some $T < \infty$, and we can set ε so that c^2 is arbitrarily close to 0 from T on. This equilibrium is such that the term (A.7) is arbitrarily close to $-\infty$ for any t , which completes the proof. \square

²⁰In case $\gamma_t^1 (R_t^1)^{-1} < \gamma_t^2 (R_t^2)^{-1}$, we could use an analogous procedure.

References

- ADAM, K., A. MARCET, AND J. P. NICOLINI (2009): “Stock Market Volatility and Learning,” Discussion paper.
- AIYAGARI, S. R. (1994): “Uninsured Idiosyncratic Risk and Aggregate Saving,” *Quarterly Journal of Economics*, 109, 659–684.
- ASSENZA, T., AND M. BERARDI (2009): “Learning in a Credit Economy,” *Journal of Economic Dynamics and Control*, 33, 1159–1169.
- BERARDI, M. (2007): “Heterogeneity and Misspecification in Learning,” *Journal of Economic Dynamics and Control*, 31, 3203–3227.
- BEWLEY, T. F. (2002): “Knightian Decision Theory. Part I,” *Decisions in Economics and Finance*, 25, 79–110.
- BRANCH, W. A. (2004): “The Theory of Rationally Heterogeneous Expectations: Evidence from Survey Data on Inflation Expectations,” *The Economic Journal*, 114(497), 592–621.
- (2007): “Sticky Information and Model Uncertainty in Survey Data on Inflation Expectations,” *forthcoming Journal of Economic Dynamics and Control*, 31, 245–276.
- BULLARD, J., AND K. MITRA (2002): “Learning about Monetary Policy Rules,” *Journal of Monetary Economics*, 49, 1105–1129.
- CAPRIOLI, F. (2008): “Tax Smoothing, Learning and Debt Volatility,” mimeo.
- EVANS, G. W., AND S. HONKAPOHJA (2001): *Learning and Expectations in Macroeconomics*. Princeton: Princeton University Press.
- (2003a): “Adaptive Learning and Monetary Policy Design,” *The Journal of Money, Credit and Banking*, 35(6), 1045–1072.
- (2003b): “Expectations and the Stability Problem for Optimal Monetary Policies,” *Review of Economic Studies*, 70(4), 807–824.
- (2006): “Monetary Policy, Expectations and Commitment,” *Scandinavian Journal of Economics*, 108(1), 15–38.
- EVANS, G. W., AND B. MCGOUGH (2005a): “Monetary Policy, Indeterminacy and Learning,” *Journal of Economic Dynamics and Control*, 29, 1809–1840.

- (2005b): “Stable Sunspots Solutions in Models with Predetermined Variables,” *Journal of Economic Dynamics and Control*, 29, 601–625.
- HONKAPOHJA, S., AND K. MITRA (2006): “Learning Stability in Economies with Heterogeneous Agents,” *Review of Economic Dynamics*, 9, 284–309.
- KIM, J., AND S. H. KIM (2003): “Spurious Welfare Reversals in International Business Cycle Models,” *Journal of International Economics*, 60, 471–500.
- KIYOTAKI, N., AND J. MOORE (1997): “Credit Cycles,” *Journal of Political Economy*, 105, 211–248.
- KREPS, D. (1998): “Anticipated Utility and Dynamic Choice,” in *Frontiers of Research in Economic Theory*, ed. by D. P. Jacobs, E. Kalai, and M. Kamien. Cambridge: Cambridge University Press.
- LIU, W. F. (1998): “Heterogeneous Agent Economies with Knightian Uncertainty,” Discussion Papers in Economics 53, University of Washington.
- LJUNGQVIST, L., AND T. J. SARGENT (2004): *Recursive Macroeconomic Theory*. Cambridge: The MIT Press.
- MARCET, A., AND R. MARIMON (1998): “Recursive Contracts,” Economics Working Papers 337, Universitat Pompeu Fabra.
- PFAJFAR, D., AND E. SANTORO (2010): “Heterogeneity, Learning and Information Stickiness in Inflation Expectations,” *Journal of Economic Behavior And Organization*, 75, 426–444.
- PRESTON, B. (2005): “Learning about monetary policy rules when long-horizon expectations matter,” *International Journal of Central Banking*, 1(2), 81–126.
- RIGOTTI, L., AND C. SHANNON (2005): “Uncertainty and Risk in Financial Markets,” *Econometrica*, 73, 203–243.
- ROBERTS, J. M. (1998): “Inflation expectations and the transmission of monetary policy,” Federal Reserve Board.

RECENTLY PUBLISHED “TEMI” (*)

- N. 782 – *Central bank's macroeconomic projections and learning*, by Giuseppe Ferrero and Alessandro Secchi (December 2010).
- N. 783 – *(Non)persistent effects of fertility on female labour supply*, by Concetta Rondinelli and Roberta Zizza (December 2010).
- N. 784 – *Stars and comets: an exploration of the patent universe*, by Carlo Menon (January 2011).
- N. 785 – *Sectoral money demand and the great disinflation in the US*, by Alessandro Calza and Andrea Zaghini (January 2011).
- N. 786 – *Public sector efficiency and political culture*, by Raffaella Giordano and Pietro Tommasino (January 2011).
- N. 787 – *Monetary incentives vs. monitoring in addressing absenteeism: experimental evidence*, by Francesco D'Amuri (January 2011).
- N. 788 – *FaMIDAS: a mixed frequency factor model with MIDAS structure*, by Cecilia Frale and Libero Monteforte (January 2011).
- N. 789 – *Policies for local development: an evaluation of Italy's "Patti Territoriali"*, by Antonio Accetturo and Guido de Blasio (January 2011).
- N. 790 – *Testing for east-west contagion in the European banking sector during the financial crisis*, by Emidio Coccozza and Paolo Piselli (February 2011).
- N. 791 – *Are incentives for R&D effective? Evidence from a regression discontinuity approach*, by Raffaello Bronzini and Eleonora Iachini (February 2011).
- N. 792 – *Evaluating the impact of innovation incentives: evidence from an unexpected shortage of funds*, by Guido de Blasio, Davide Fantino and Guido Pellegrini (February 2011).
- N. 793 – *Credit availability and investment in Italy: lessons from the "Great Recession"*, by Eugenio Gaiotti (February 2011).
- N. 794 – *Bridging the gap between migrants and the banking system*, by Giorgio Albareto and Paolo Emilio Mistrulli (February 2011).
- N. 795 – *I fondi comuni aperti in Italia: performance delle società di gestione del risparmio*, by Michele Leonardo Bianchi and Maria Grazia Miele (February 2011).
- N. 796 – *Securitization is not that evil after all*, by Ugo Albertazzi, Ginette Eramo, Leonardo Gambacorta and Carmelo Salleo (February 2011).
- N. 797 – *Reserve management and sovereign debt cost in a world with liquidity crises*, by Flavia Corneli and Emanuele Tarantino (March 2011).
- N. 798 – *Managerial incentives, financial constraints and ownership concentration*, by Marco Protopapa (March 2011).
- N. 799 – *Bootstrap LR tests of stationarity, common trends and cointegration*, by Fabio Busetti and Silvestro di Sanzo (March 2011).
- N. 800 – *Performance pay and shifts in macroeconomic correlations*, by Francesco Nucci and Marianna Riggi (March 2011).
- N. 801 – *Monetary and macroprudential policies*, by Paolo Angelini, Stefano Neri and Fabio Panetta (March 2011).
- N. 802 – *Imperfect information, real-time data and monetary policy in the euro area*, by Stefano Neri and Tiziano Ropele (March 2011).

(*) Requests for copies should be sent to:

Banca d'Italia – Servizio Studi di struttura economica e finanziaria – Divisione Biblioteca e Archivio storico – Via Nazionale, 91 – 00184 Rome – (fax 0039 06 47922059). They are available on the Internet www.bancaditalia.it.

2008

- P. ANGELINI, *Liquidity and announcement effects in the euro area*, *Giornale degli Economisti e Annali di Economia*, v. 67, 1, pp. 1-20, **TD No. 451 (October 2002)**.
- P. ANGELINI, P. DEL GIOVANE, S. SIVIERO and D. TERLIZZESE, *Monetary policy in a monetary union: What role for regional information?*, *International Journal of Central Banking*, v. 4, 3, pp. 1-28, **TD No. 457 (December 2002)**.
- F. SCHIVARDI and R. TORRINI, *Identifying the effects of firing restrictions through size-contingent Differences in regulation*, *Labour Economics*, v. 15, 3, pp. 482-511, **TD No. 504 (June 2004)**.
- L. GUIISO and M. PAIELLA., *Risk aversion, wealth and background risk*, *Journal of the European Economic Association*, v. 6, 6, pp. 1109-1150, **TD No. 483 (September 2003)**.
- C. BIANCOTTI, G. D'ALESSIO and A. NERI, *Measurement errors in the Bank of Italy's survey of household income and wealth*, *Review of Income and Wealth*, v. 54, 3, pp. 466-493, **TD No. 520 (October 2004)**.
- S. MOMIGLIANO, J. HENRY and P. HERNÁNDEZ DE COS, *The impact of government budget on prices: Evidence from macroeconomic models*, *Journal of Policy Modelling*, v. 30, 1, pp. 123-143 **TD No. 523 (October 2004)**.
- L. GAMBACORTA, *How do banks set interest rates?*, *European Economic Review*, v. 52, 5, pp. 792-819, **TD No. 542 (February 2005)**.
- P. ANGELINI and A. GENERALE, *On the evolution of firm size distributions*, *American Economic Review*, v. 98, 1, pp. 426-438, **TD No. 549 (June 2005)**.
- R. FELICI and M. PAGNINI, *Distance, bank heterogeneity and entry in local banking markets*, *The Journal of Industrial Economics*, v. 56, 3, pp. 500-534, **No. 557 (June 2005)**.
- S. DI ADDARIO and E. PATACCHINI, *Wages and the city. Evidence from Italy*, *Labour Economics*, v.15, 5, pp. 1040-1061, **TD No. 570 (January 2006)**.
- S. SCALIA, *Is foreign exchange intervention effective?*, *Journal of International Money and Finance*, v. 27, 4, pp. 529-546, **TD No. 579 (February 2006)**.
- M. PERICOLI and M. TABOGA, *Canonical term-structure models with observable factors and the dynamics of bond risk premia*, *Journal of Money, Credit and Banking*, v. 40, 7, pp. 1471-88, **TD No. 580 (February 2006)**.
- E. VIVIANO, *Entry regulations and labour market outcomes. Evidence from the Italian retail trade sector*, *Labour Economics*, v. 15, 6, pp. 1200-1222, **TD No. 594 (May 2006)**.
- S. FEDERICO and G. A. MINERVA, *Outward FDI and local employment growth in Italy*, *Review of World Economics*, *Journal of Money, Credit and Banking*, v. 144, 2, pp. 295-324, **TD No. 613 (February 2007)**.
- F. Busetti and A. HARVEY, *Testing for trend*, *Econometric Theory*, v. 24, 1, pp. 72-87, **TD No. 614 (February 2007)**.
- V. CESTARI, P. DEL GIOVANE and C. ROSSI-ARNAUD, *Memory for prices and the Euro cash changeover: an analysis for cinema prices in Italy*, In P. Del Giovane e R. Sabbatini (eds.), *The Euro Inflation and Consumers' Perceptions. Lessons from Italy*, Berlin-Heidelberg, Springer, **TD No. 619 (February 2007)**.
- B. H. HALL, F. LOTTI and J. MAIRESSE, *Employment, innovation and productivity: evidence from Italian manufacturing microdata*, *Industrial and Corporate Change*, v. 17, 4, pp. 813-839, **TD No. 622 (April 2007)**.
- J. SOUSA and A. ZAGHINI, *Monetary policy shocks in the Euro Area and global liquidity spillovers*, *International Journal of Finance and Economics*, v.13, 3, pp. 205-218, **TD No. 629 (June 2007)**.
- M. DEL GATTO, GIANMARCO I. P. OTTAVIANO and M. PAGNINI, *Openness to trade and industry cost dispersion: Evidence from a panel of Italian firms*, *Journal of Regional Science*, v. 48, 1, pp. 97-129, **TD No. 635 (June 2007)**.
- P. DEL GIOVANE, S. FABIANI and R. SABBATINI, *What's behind "inflation perceptions"? A survey-based analysis of Italian consumers*, in P. Del Giovane e R. Sabbatini (eds.), *The Euro Inflation and Consumers' Perceptions. Lessons from Italy*, Berlin-Heidelberg, Springer, **TD No. 655 (January 2008)**.
- R. BRONZINI, G. DE BLASIO, G. PELLEGRINI and A. SCOGNAMIGLIO, *La valutazione del credito d'imposta per gli investimenti*, *Rivista di politica economica*, v. 98, 4, pp. 79-112, **TD No. 661 (April 2008)**.

- B. BORTOLOTTI, and P. PINOTTI, *Delayed privatization*, Public Choice, v. 136, 3-4, pp. 331-351, **TD No. 663 (April 2008)**.
- R. BONCI and F. COLUMBA, *Monetary policy effects: New evidence from the Italian flow of funds*, Applied Economics, v. 40, 21, pp. 2803-2818, **TD No. 678 (June 2008)**.
- M. CUCCULELLI, and G. MICUCCI, *Family Succession and firm performance: evidence from Italian family firms*, Journal of Corporate Finance, v. 14, 1, pp. 17-31, **TD No. 680 (June 2008)**.
- A. SILVESTRINI and D. VEREDAS, *Temporal aggregation of univariate and multivariate time series models: a survey*, Journal of Economic Surveys, v. 22, 3, pp. 458-497, **TD No. 685 (August 2008)**.

2009

- F. PANETTA, F. SCHIVARDI and M. SHUM, *Do mergers improve information? Evidence from the loan market*, Journal of Money, Credit, and Banking, v. 41, 4, pp. 673-709, **TD No. 521 (October 2004)**.
- M. BUGAMELLI and F. PATERNÒ, *Do workers' remittances reduce the probability of current account reversals?*, World Development, v. 37, 12, pp. 1821-1838, **TD No. 573 (January 2006)**.
- P. PAGANO and M. PISANI, *Risk-adjusted forecasts of oil prices*, The B.E. Journal of Macroeconomics, v. 9, 1, Article 24, **TD No. 585 (March 2006)**.
- M. PERICOLI and M. SBRACIA, *The CAPM and the risk appetite index: theoretical differences, empirical similarities, and implementation problems*, International Finance, v. 12, 2, pp. 123-150, **TD No. 586 (March 2006)**.
- U. ALBERTAZZI and L. GAMBACORTA, *Bank profitability and the business cycle*, Journal of Financial Stability, v. 5, 4, pp. 393-409, **TD No. 601 (September 2006)**.
- S. MAGRI, *The financing of small innovative firms: the Italian case*, Economics of Innovation and New Technology, v. 18, 2, pp. 181-204, **TD No. 640 (September 2007)**.
- V. DI GIACINTO and G. MICUCCI, *The producer service sector in Italy: long-term growth and its local determinants*, Spatial Economic Analysis, Vol. 4, No. 4, pp. 391-425, **TD No. 643 (September 2007)**.
- F. LORENZO, L. MONTEFORTE and L. SESSA, *The general equilibrium effects of fiscal policy: estimates for the euro area*, Journal of Public Economics, v. 93, 3-4, pp. 559-585, **TD No. 652 (November 2007)**.
- R. GOLINELLI and S. MOMIGLIANO, *The Cyclical Reaction of Fiscal Policies in the Euro Area. A Critical Survey of Empirical Research*, Fiscal Studies, v. 30, 1, pp. 39-72, **TD No. 654 (January 2008)**.
- P. DEL GIOVANE, S. FABIANI and R. SABBATINI, *What's behind "Inflation Perceptions"? A survey-based analysis of Italian consumers*, Giornale degli Economisti e Annali di Economia, v. 68, 1, pp. 25-52, **TD No. 655 (January 2008)**.
- F. MACCHERONI, M. MARINACCI, A. RUSTICHINI and M. TABOGA, *Portfolio selection with monotone mean-variance preferences*, Mathematical Finance, v. 19, 3, pp. 487-521, **TD No. 664 (April 2008)**.
- M. AFFINITO and M. PIAZZA, *What are borders made of? An analysis of barriers to European banking integration*, in P. Alessandrini, M. Fratianni and A. Zazzaro (eds.): The Changing Geography of Banking and Finance, Dordrecht Heidelberg London New York, Springer, **TD No. 666 (April 2008)**.
- A. BRANDOLINI, *On applying synthetic indices of multidimensional well-being: health and income inequalities in France, Germany, Italy, and the United Kingdom*, in R. Gotoh and P. Dumouchel (eds.), Against Injustice. The New Economics of Amartya Sen, Cambridge, Cambridge University Press, **TD No. 668 (April 2008)**.
- G. FERRERO and A. NOBILI, *Futures contract rates as monetary policy forecasts*, International Journal of Central Banking, v. 5, 2, pp. 109-145, **TD No. 681 (June 2008)**.
- P. CASADIO, M. LO CONTE and A. NERI, *Balancing work and family in Italy: the new mothers' employment decisions around childbearing*, in T. Addabbo and G. Solinas (eds.), Non-Standard Employment and Quality of Work, Physica-Verlag. A Springer Company, **TD No. 684 (August 2008)**.
- L. ARCIERO, C. BIANCOTTI, L. D'AURIZIO and C. IMPENNA, *Exploring agent-based methods for the analysis of payment systems: A crisis model for StarLogo TNG*, Journal of Artificial Societies and Social Simulation, v. 12, 1, **TD No. 686 (August 2008)**.
- A. CALZA and A. ZAGHINI, *Nonlinearities in the dynamics of the euro area demand for M1*, Macroeconomic Dynamics, v. 13, 1, pp. 1-19, **TD No. 690 (September 2008)**.
- L. FRANCESCO and A. SECCHI, *Technological change and the households' demand for currency*, Journal of Monetary Economics, v. 56, 2, pp. 222-230, **TD No. 697 (December 2008)**.
- G. ASCARI and T. ROPELE, *Trend inflation, taylor principle, and indeterminacy*, Journal of Money, Credit and Banking, v. 41, 8, pp. 1557-1584, **TD No. 708 (May 2007)**.

- S. COLAROSSO and A. ZAGHINI, *Gradualism, transparency and the improved operational framework: a look at overnight volatility transmission*, *International Finance*, v. 12, 2, pp. 151-170, **TD No. 710 (May 2009)**.
- M. BUGAMELLI, F. SCHIVARDI and R. ZIZZA, *The euro and firm restructuring*, in A. Alesina e F. Giavazzi (eds): *Europe and the Euro*, Chicago, University of Chicago Press, **TD No. 716 (June 2009)**.
- B. HALL, F. LOTTI and J. MAIRESSE, *Innovation and productivity in SMEs: empirical evidence for Italy*, *Small Business Economics*, v. 33, 1, pp. 13-33, **TD No. 718 (June 2009)**.

2010

- A. PRATI and M. SBRACIA, *Uncertainty and currency crises: evidence from survey data*, *Journal of Monetary Economics*, v. 57, 6, pp. 668-681, **TD No. 446 (July 2002)**.
- S. MAGRI, *Debt maturity choice of nonpublic Italian firms*, *Journal of Money, Credit, and Banking*, v.42, 2-3, pp. 443-463, **TD No. 574 (January 2006)**.
- R. BRONZINI and P. PISELLI, *Determinants of long-run regional productivity with geographical spillovers: the role of R&D, human capital and public infrastructure*, *Regional Science and Urban Economics*, v. 39, 2, pp.187-199, **TD No. 597 (September 2006)**.
- E. IOSSA and G. PALUMBO, *Over-optimism and lender liability in the consumer credit market*, *Oxford Economic Papers*, v. 62, 2, pp. 374-394, **TD No. 598 (September 2006)**.
- S. NERI and A. NOBILI, *The transmission of US monetary policy to the euro area*, *International Finance*, v. 13, 1, pp. 55-78, **TD No. 606 (December 2006)**.
- F. ALTISSIMO, R. CRISTADORO, M. FORNI, M. LIPPI and G. VERONESE, *New Eurocoin: Tracking Economic Growth in Real Time*, *Review of Economics and Statistics*, v. 92, 4, pp. 1024-1034, **TD No. 631 (June 2007)**.
- A. CIARLONE, P. PISELLI and G. TREBESCHI, *Emerging Markets' Spreads and Global Financial Conditions*, *Journal of International Financial Markets, Institutions & Money*, v. 19, 2, pp. 222-239, **TD No. 637 (June 2007)**.
- U. ALBERTAZZI and L. GAMBACORTA, *Bank profitability and taxation*, *Journal of Banking and Finance*, v. 34, 11, pp. 2801-2810, **TD No. 649 (November 2007)**.
- M. IACOVIELLO and S. NERI, *Housing market spillovers: evidence from an estimated DSGE model*, *American Economic Journal: Macroeconomics*, v. 2, 2, pp. 125-164, **TD No. 659 (January 2008)**.
- F. BALASSONE, F. MAURA and S. ZOTTERI, *Cyclical asymmetry in fiscal variables in the EU*, *Empirica*, **TD No. 671**, v. 37, 4, pp. 381-402 **(June 2008)**.
- F. D'AMURI, O. GIANMARCO I.P. and P. GIOVANNI, *The labor market impact of immigration on the western german labor market in the 1990s*, *European Economic Review*, v. 54, 4, pp. 550-570, **TD No. 687 (August 2008)**.
- A. ACCETTURO, *Agglomeration and growth: the effects of commuting costs*, *Papers in Regional Science*, v. 89, 1, pp. 173-190, **TD No. 688 (September 2008)**.
- S. NOBILI and G. PALAZZO, *Explaining and forecasting bond risk premiums*, *Financial Analysts Journal*, v. 66, 4, pp. 67-82, **TD No. 689 (September 2008)**.
- A. B. ATKINSON and A. BRANDOLINI, *On analysing the world distribution of income*, *World Bank Economic Review*, v. 24, 1, pp. 1-37, **TD No. 701 (January 2009)**.
- R. CAPPARIELLO and R. ZIZZA, *Dropping the Books and Working Off the Books*, *Labour*, v. 24, 2, pp. 139-162, **TD No. 702 (January 2009)**.
- C. NICOLETTI and C. RONDINELLI, *The (mis)specification of discrete duration models with unobserved heterogeneity: a Monte Carlo study*, *Journal of Econometrics*, v. 159, 1, pp. 1-13, **TD No. 705 (March 2009)**.
- V. DI GIACINTO, G. MICUCCI and P. MONTANARO, *Dynamic macroeconomic effects of public capital: evidence from regional Italian data*, *Giornale degli economisti e annali di economia*, v. 69, 1, pp. 29-66, **TD No. 733 (November 2009)**.
- F. COLUMBA, L. GAMBACORTA and P. E. MISTRULLI, *Mutual Guarantee institutions and small business finance*, *Journal of Financial Stability*, v. 6, 1, pp. 45-54, **TD No. 735 (November 2009)**.
- A. GERALI, S. NERI, L. SESSA and F. M. SIGNORETTI, *Credit and banking in a DSGE model of the Euro Area*, *Journal of Money, Credit and Banking*, v. 42, 6, pp. 107-141, **TD No. 740 (January 2010)**.
- M. AFFINITO and E. TAGLIAFERRI, *Why do (or did?) banks securitize their loans? Evidence from Italy*, *Journal of Financial Stability*, v. 6, 4, pp. 189-202, **TD No. 741 (January 2010)**.

- S. FEDERICO, *Outsourcing versus integration at home or abroad and firm heterogeneity*, *Empirica*, v. 37, 1, pp. 47-63, **TD No. 742 (February 2010)**.
- V. DI GIACINTO, *On vector autoregressive modeling in space and time*, *Journal of Geographical Systems*, v. 12, 2, pp. 125-154, **TD No. 746 (February 2010)**.
- S. MOCETTI and C. PORELLO, *How does immigration affect native internal mobility? new evidence from Italy*, *Regional Science and Urban Economics*, v. 40, 6, pp. 427-439, **TD No. 748 (March 2010)**.
- A. DI CESARE and G. GUAZZAROTTI, *An analysis of the determinants of credit default swap spread changes before and during the subprime financial turmoil*, *Journal of Current Issues in Finance, Business and Economics*, v. 3, 4, pp., **TD No. 749 (March 2010)**.
- A. BRANDOLINI, S. MAGRI and T. M. SMEEDING, *Asset-based measurement of poverty*, *Journal of Policy Analysis and Management*, v. 29, 2, pp. 267-284, **TD No. 755 (March 2010)**.
- G. CAPPELLETTI, *A Note on rationalizability and restrictions on beliefs*, *The B.E. Journal of Theoretical Economics*, v. 10, 1, pp. 1-11, **TD No. 757 (April 2010)**.
- S. DI ADDARIO and D. VURI, *Entrepreneurship and market size. the case of young college graduates in Italy*, *Labour Economics*, v. 17, 5, pp. 848-858, **TD No. 775 (September 2010)**.

FORTHCOMING

- L. MONTEFORTE and S. SIVIERO, *The Economic Consequences of Euro Area Modelling Shortcuts*, *Applied Economics*, **TD No. 458 (December 2002)**.
- M. BUGAMELLI and A. ROSOLIA, *Produttività e concorrenza estera*, *Rivista di politica economica*, **TD No. 578 (February 2006)**.
- G. DE BLASIO and G. NUZZO, *Historical traditions of civicness and local economic development*, *Journal of Regional Science*, **TD No. 591 (May 2006)**.
- S. DI ADDARIO, *Job search in thick markets*, *Journal of Urban Economics*, **TD No. 605 (December 2006)**.
- F. SCHIVARDI and E. VIVIANO, *Entry barriers in retail trade*, *Economic Journal*, **TD No. 616 (February 2007)**.
- G. FERRERO, A. NOBILI and P. PASSIGLIA, *Assessing excess liquidity in the Euro Area: the role of sectoral distribution of money*, *Applied Economics*, **TD No. 627 (April 2007)**.
- P. E. MISTRULLI, *Assessing financial contagion in the interbank market: maximum entropy versus observed interbank lending patterns*, *Journal of Banking & Finance*, **TD No. 641 (September 2007)**.
- Y. ALTUNBAS, L. GAMBACORTA and D. MARQUÉS, *Securitisation and the bank lending channel*, *European Economic Review*, **TD No. 653 (November 2007)**.
- E. CIAPANNA, *Directed matching with endogenous markov probability: clients or competitors?*, *The RAND Journal of Economics*, **TD No. 665 (April 2008)**.
- M. BUGAMELLI and F. PATERNÒ, *Output growth volatility and remittances*, *Economica*, **TD No. 673 (June 2008)**.
- V. DI GIACINTO e M. PAGNINI, *Local and global agglomeration patterns: two econometrics-based indicators*, *Regional Science and Urban Economics*, **TD No. 674 (June 2008)**.
- G. BARONE and F. CINGANO, *Service regulation and growth: evidence from OECD countries*, *Economic Journal*, **TD No. 675 (June 2008)**.
- S. MOCETTI, *Educational choices and the selection process before and after compulsory school*, *Education Economics*, **TD No. 691 (September 2008)**.
- P. SESTITO and E. VIVIANO, *Reservation wages: explaining some puzzling regional patterns*, *Labour*, **TD No. 696 (December 2008)**.
- P. PINOTTI, M. BIANCHI and P. BUONANNO, *Do immigrants cause crime?*, *Journal of the European Economic Association*, **TD No. 698 (December 2008)**.
- R. GIORDANO and P. TOMMASINO, *What determines debt intolerance? The role of political and monetary institutions*, *European Journal of Political Economy*, **TD No. 700 (January 2009)**.
- L. FORNI, A. GERALI and M. PISANI, *Macroeconomic effects of greater competition in the service sector: the case of Italy*, *Macroeconomic Dynamics*, **TD No. 706 (March 2009)**.
- Y. ALTUNBAS, L. GAMBACORTA, and D. MARQUÉS-IBÁÑEZ, *Bank risk and monetary policy*, *Journal of Financial Stability*, **TD No. 712 (May 2009)**.
- P. ANGELINI, A. NOBILI e C. PICILLO, *The interbank market after August 2007: What has changed, and why?*, *Journal of Money, Credit and Banking*, **TD No. 731 (ottobre 2009)**.

- L. FORNI, A. GERALI and M. PISANI, *The macroeconomics of fiscal consolidations in euro area countries*, Journal of Economic Dynamics and Control, **TD No. 747 (March 2010)**.
- A. DI CESARE and G. GUAZZAROTTI, *An analysis of the determinants of credit default swap spread changes before and during the subprime financial turmoil*, in C. V. Karsone (eds.), Finance and Banking Developments, Nova Publishers, New York., **TD No. 749 (March 2010)**.
- G. GRANDE and I. VISCO, *A public guarantee of a minimum return to defined contribution pension scheme members*, Journal of Risk, **TD No. 762 (June 2010)**.
- P. DEL GIOVANE, G. ERAMO and A. NOBILI, *Disentangling demand and supply in credit developments: a survey-based analysis for Italy*, Journal of Banking and Finance, **TD No. 764 (June 2010)**.
- G. BARONE and S. MOCETTI, *With a little help from abroad: the effect of low-skilled immigration on the female labour supply*, Labour Economics, **TD No. 766 (July 2010)**.
- S. MAGRI and R. PICO, *The rise of risk-based pricing of mortgage interest rates in Italy*, Journal of Banking and Finance, **TD No. 778 (October 2010)**.