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JEL: E31, E32, E37, E52.

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Price Stickiness Asymmetry, Persistence and Volatility in a New-Keynesian Model

Alessandro Flamini*

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Preliminary

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In a two-sector New-Keynesian model, this paper shows that the dispersion in the degree of sectoral price stickiness plays a key role in the determination of the dynamics of aggregate inflation and, consequently, of the whole economy. The dispersion in price stickiness reduces the persistence of inflation and, to a smaller extent, of the interest rate. It also reduces the volatility of inflation, the interest rate and the output-gap. Thus two economies with the same average degree of price stickiness but a different variance may behave very differently, highlighting the relevance of sectoral data for economic estimations and forecasts.

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1 Introduction

A main feature of real-world economies is the presence of multi sectors. How sectoral asymmetry relates to the working of an economy is therefore an issue potentially very important for the estimations and simulations of macro models. This paper focuses on sectoral asymmetry in *price stickiness* and asks how it affects the economic dynamics. Drawing on Benigno (2004) and Woodford (2003), the current paper addresses this question presenting a two-sector New Keynesian model with the assumption of habit persistence.

*Current address: Department of Economics, University of Sheffield, 9 Mappin Street, Sheffield S1 4DT, UK. Email: a.flamini@sheffield.ac.uk. I thank for comments Mustafa Caglayan, Alpay Filiztekin and Kostas Mouratidis. I have also benefited from a useful discussion with Paul Levine and seminar partecipants at the University of Surrey. Any mistake is my responsability.

Sectoral asymmetry in price stickiness in the New-Keynesian model has been pioneered by Aoky (2001) and Benigno (2004). Both papers show that focusing the policy response on the sector with sticky or stickier price maximises welfare. Abstracting from optimal monetary policy and considering sectoral asymmetry in wage contracts Dixon and Kara (2010a) show that in economies with the same average contract length, monetary shocks are more persistent in presence of longer contracts. They also show in Dixon and Kara (2010b) that accounting for the distribution of contract length substantially improves the ability of the model to replicate the inflation persistence found in the data.

The contribution of this paper lies in showing that sectoral asymmetry in price stickiness play a key role in the determination of the aggregate inflation process and, via inflation, of the interest rate and aggregate output gap processes. The analysis shows that the dispersion in sectoral price stickiness affects negatively the *persistence* of aggregate inflation and, to a minor extent, of the interest rate. It also affects negatively the *variability* of aggregate inflation, the interest rate and the output-gap. The analysis suggests that these results are quantitatively important too. Interestingly, these findings imply that two economies sharing the mean degree of price stickiness but not the variance may respond to exogenous disturbances very differently.

The intuition for these findings is the following. Breaking the symmetry in sectoral price stickiness introduces the relative price of sectoral goods into the picture which, affecting sectoral inflations in opposite ways, changes dramatically the shock transmission mechanism of the economy.

The plan of the paper is as follows. Section 2 presents the model where consumption habits are introduced into a two-sector New-Keynesian model. It derives the non-linear optimal conditions, shows the existence and uniqueness of the steady state, the log-linearized relations used in the following analysis, and the calibration of the structural parameters. Section 3 investigates the relation between sectoral asymmetry in price stickiness and the dynamics of the economy in presence of shocks to the price level, technology and the preferences of the household. Specifically, the analysis is carried out via impulse response functions, autocorrelations, and standard deviations of the endogenous variables. Concluding remarks are in section 4.

2 The model

The economy is populated by a continuum of unit mass of identical infinite-lived households each seeking to maximize

$$U_t = E_t \sum_{T=t}^{\infty} \beta^{T-t} \left\{ \widetilde{u}\left(C_T - \eta C_{T-1}; \overline{C}_T\right) - \int_0^1 \widetilde{v}\left[H_T\left(j\right)\right] dj \right\}$$

where β is the intertemporal discount factor, C_t represents all interest-rate-sensitive expenditure including investments and is defined as a CES aggregate

$$C_t \equiv \left[(n_s)^{1/\rho} \left(C_t^s\right)^{(\rho-1)/\rho} + (n_m)^{1/\rho} \left(C_t^m\right)^{(\rho-1)/\rho} \right]^{\rho/(\rho-1)} \tag{1}$$

of the goods C_t^s and C_t^m which are produced, respectively, by the s and m-sector, with ρ defining their elasticity of substitution and n_s and n_m ($n_s \equiv 1 - n_m$) denoting the number of goods of sector s and m in C_t, respectively. Each sectoral good is, in turn, a Dixit-Stiglitz aggregate of the continuum of differentiated goods produced in the sector:

$$C_t^s \equiv \left[n^{s^{-\frac{1}{\theta_s}}} \int_0^{n_s} \left(C_t^s(i)\right)^{1-\frac{1}{\theta_s}} di \right]^{\frac{\theta_s}{\theta_s - 1}}, \qquad C_t^m \equiv \left[n^{m^{-\frac{1}{\theta_m}}} \int_{n_s}^{1} \left(C_t^m(i)\right)^{1-\frac{1}{\theta_m}} di \right]^{\frac{\theta_m}{\theta_m - 1}} \tag{2}$$

where $\theta^k > 1,\ k = s, m$, is the sectoral elasticity of substitution between any two differentiated goods. Period preferences on consumption and labour are modeled as CRRA functions

$$\widetilde{u}\left(C_t - \eta C_{t-1}; \overline{C}_t\right) = \overline{C}_t^{\frac{1}{\widetilde{\sigma}}} \frac{\left(C_t - \eta C_{t-1}\right)^{1-\frac{1}{\widetilde{\sigma}}} - 1}{1 - \frac{1}{\widetilde{\sigma}}}, \tag{3}$$

$$\widetilde{v}\left[H_t(j)\right] \equiv \frac{H_t^{1+\nu}(j)}{1+\nu}, \tag{4}$$

where $\overline{C}_t$ is an exogenous preference shock, $H_t(j)$ is the quantity supplied of labour of type j, $\widetilde{\sigma} > 0$ captures the intertemporal elasticity of substitution in consumption, $0 \leq \eta < 1$ measures the degree of habit persistence, and $\nu > 0$ is the inverse of the elasticity of goods production. The price index for the minimum cost of a unit of C_t is given by

$$P_t \equiv \left[n_s \left(P_t^s\right)^{1-\rho} + (n_m) \left(P_t^m\right)^{1-\rho} \right]^{1/(1-\rho)}, \tag{5}$$

with P^s, P^m denoting, respectively, the Dixit-Stiglitz price index for goods produced in the s and m sector

$$P_t^s \equiv \left[(n_s)^{-1} \int_0^{n_s} p^s(i)^{1-\theta_s} di \right]^{\frac{1}{1-\theta_s}}, \qquad P_t^m \equiv \left[(n_m)^{-1} \int_{n_s}^{1} p^m(i)^{1-\theta_m} di \right]^{\frac{1}{1-\theta_m}}.$$

Preferences captured by equation (1) imply that the optimal sectoral consumption levels are given by

$$C_t^s = n_s C_t \left(\frac{P_t^s}{P_t}\right)^{-\rho}, \tag{6}$$

$$C_t^m = n_m C_t \left(\frac{P_t^m}{P_t}\right)^{-\rho}. \tag{7}$$

Financial markets are assumed to be complete so that at any date all households face the same budget constraint and consume the same amount. Then, utility maximization subject to the budget constraint and the no-Ponzi scheme requirement yields the condition for optimal consumption

$$\lambda_t = \beta E_t \left\{ \frac{\left[\widetilde{u}_c\left(C_{t+1} - \eta C_t; \overline{C}_{t+1}\right) - \beta\eta E_t \widetilde{u}_c\left(C_{t+2} - \eta C_{t+1}; \overline{C}_{t+2}\right)\right]}{\widetilde{u}_c\left(C_t - \eta C_{t-1}; \overline{C}_t\right) - \beta\eta E_t \widetilde{u}_c\left(C_{t+1} - \eta C_t; \overline{C}_{t+1}\right)} \frac{P_t}{P_{t+1}} \right\}, \tag{8}$$

where $\lambda_t \equiv \frac{1}{1+i_t}$ is the price of a one-period nominal bond. Finally, utility maximization requires that the optimal supply of labour of type j is given by

$$\Omega_t(j) = \Psi_t \frac{\widetilde{v}_h\left[H_t(j)\right]}{\left[\widetilde{u}_c\left(C_t - \eta C_{t-1}; \overline{C}_t\right) - \eta\beta E_t \widetilde{u}_c\left(C_{t+1} - \eta C_t; \overline{C}_{t+1}\right)\right]}, \tag{9}$$

where $\Omega_t(j)$ is the real wage demanded for labour of type j and $\Psi_t \geq 1$ is an exogenous markup factor in the labor market assuming that firms are wage-takers. Given (2), sectoral aggregate demands are

$$Y_t^s \equiv \left[\frac{1}{n_s}\int_0^{n_s} \left[y_t^s(j)\right]^{\frac{\theta^s-1}{\theta^s}} dj\right]^{\frac{\theta^s}{\theta^s-1}}, \qquad Y_t^m \equiv \left[\frac{1}{n_m}\int_n^1 \left[y_t^m(j)\right]^{\frac{\theta^m-1}{\theta^m}} dj\right]^{\frac{\theta^m}{\theta^m-1}}.$$

Turning to production, each household i is assumed to supply all type of labour and is a monopolistically competitive producer of one differentiated good, either $y^m(i)$ or $y^s(i)$. In this economy any firm i belongs to an industry j which, in turn, belongs either to sector s or m. Furthermore, there is a unit interval continuum of industries indexed by j and in each industry there is a unit interval continuum of good indexed by i so that the total number of goods is one. Since in equilibrium all the firms belonging to an industry will supply the same amount, they will also demand the same amount of labour. As a result the total demand of labour in an industry is equal to demand of labor of any differentiated firm in the industry. Next, we assume industry-specific labor as the only variable input and a sector-specific technology

$$y_t^s(i) = A_t\left[H_t^s(i)\right]^{\frac{1}{\phi^s}},$$
$$y_t^m(i) = A_t\left[H_t^m(i)\right]^{\frac{1}{\phi^m}},$$

where A_t is a technology shock, $H_t^s(i)$, $H_t^m(i)$ are the quantities of labour used by the representative firm i in the s and m-sector to produce good i respectively, and $\phi^k > 1$, $k = s, m$ is the elasticity of sectoral output with respect to hours worked. Thus the input requirement functions are

$$H_t^s = \left[\frac{y_t^s(i)}{A_t}\right]^{\phi^s}, \tag{10}$$

$$H_t^m = \left[\frac{y_t^m(i)}{A_t}\right]^{\phi^m}, \tag{11}$$

then, accounting for the preferences (1-2) the quantity demanded for each individual good in the manufacturing and services sector are, respectively,

$$\begin{aligned} y_t^s(i) &= C_t^s(i) \\ &= C_t \left(\frac{p_t^s(i)}{P_t^s}\right)^{-\theta_s} \left(\frac{P_t^s}{P_t}\right)^{-\rho}, \end{aligned} \tag{12}$$

and

$$\begin{aligned} y_t^m(i) &= C_t^m(i) \\ &= C_t \left(\frac{p_t^m(i)}{P_t^m}\right)^{-\theta_m} \left(\frac{P_t^m}{P_t}\right)^{-\rho}. \end{aligned} \tag{13}$$

In equilibrium, market clearing in the goods market requires

$$Y_t^m = C_t^m, \tag{14}$$
$$Y_t^s = C_t^s, \tag{15}$$
$$Y_t = C_t, \tag{16}$$

Then, combining (3), (8), and (16) we obtain the nonlinear version of the aggregate demand. Turning to the producers' pricing behaviour, firms in both sectors fix their prices at random intervals following the Calvo (1983) staggered price model and have the opportunity to change their prices with probability $(1-\alpha)$. Thus, a producer i in the $h = m, s$ sector that is allowed to set its price in period t chooses its new price for the random period starting in t, $\widetilde{p}_t^h$, to maximize the flow of expected profits:

$$\max_{\widetilde{p}_t^h} E_t \sum_{T=t}^{\infty} \alpha^{T-t} \lambda_{t,T} \left\{ \widetilde{p}_t^h y_T^h(i) - \left[\frac{y_T^h(i)}{A_T}\right]^{\phi_h} \Psi_T^h \frac{\left[y_T^h(j)/A_T\right]^{\nu\phi_h}}{\overline{C}_t^{\frac{1}{\sigma}} \left(C_t - \eta C_{t-1}\right)^{-\frac{1}{\sigma}} - \eta\beta \overline{C}_{t+1}^{\frac{1}{\sigma}} \left(C_{t+1} - \eta C_t\right)^{-\frac{1}{\sigma}}} P_T \right\},$$

where $\lambda_{t,T}$ is the stochastic discount factor by which financial markets discount random nominal income in period T. Accounting for firm i demand function in sector h, and considering that the firm's pricing decision cannot change the real wage, the f.o.c. is

$$\begin{aligned} 0 = E_t \sum_{T=t}^{\infty} \alpha^{T-t} \lambda_{t,T} \Bigg\{ & C_T \left(\frac{\widetilde{p}_t^h}{P_T^h}\right)^{-\theta_h} \left(\frac{P_T^h}{P_T}\right)^{-\rho} - \theta_h C_T \left(\frac{\widetilde{p}_t^h}{P_T^h}\right)^{-\theta_h - 1} \frac{\widetilde{p}_t^h}{P_T^h} \left(\frac{P_T^h}{P_T}\right)^{-\rho} - \\ & \left[-\phi_h \theta_h \left(\frac{C_T}{A_T}\right)^{\phi_h} \left(\frac{\widetilde{p}_t^h}{P_T^h}\right)^{-\phi_h\theta_h - 1} \frac{1}{P_T^h} \left(\frac{P_T^h}{P_T}\right)^{-\phi_h\rho}\right] \frac{\Psi_T^h \left[C_t \left(\frac{p_t^h(j)}{P_t^h}\right)^{-\theta_h} \left(\frac{P_t^h}{P_t}\right)^{-\rho} \frac{1}{A_T}\right]^{\nu\phi_h}}{\overline{C}_T^{\frac{1}{\sigma}} \left(C_T - \eta C_{T-1}\right)^{-\frac{1}{\sigma}} - \eta\beta \overline{C}_{T+1}^{\frac{1}{\sigma}} \left(C_{T+1} - \eta C_T\right)^{-\frac{1}{\sigma}}} P_T \Bigg\} \end{aligned} \tag{17}$$

2.1 Existence and uniqueness of the steady state equilibrium

In presence of flexible prices, the monopolistic competitive representative firm i in sector m sets the optimal price $\widetilde{p}_t^m$ in any period to maximize the period profit

$$\max_{\widetilde{p}_t^m} \left\{ \widetilde{p}_t^m y_t^m (i) - \left[\frac{y_t^m (i)}{A_t^m} \right]^{\phi_m} \left[\Psi_t^m \frac{\widetilde{v}_h [H_t (j)]}{\left[\widetilde{u}_c \left(C_t - \eta C_{t-1}; \overline{C}_t \right) - \eta\beta E_t \widetilde{u}_c \left(C_{t+1} - \eta C_t; \overline{C}_{t+1} \right) \right]} P_t \right] \right\},$$

and the f.o.c. consists of setting the price as a mark-up on marginal costs

$$\widetilde{p}_t^m = \frac{\theta}{(\theta - 1)} \phi_m \left[\frac{y_t^m (i)}{A_t^m} \right]^{\phi_m - 1} \left[\Psi_t^m \frac{\widetilde{v}_h [H_t (j)]}{\left[\widetilde{u}_c \left(C_t - \eta C_{t-1}; \overline{C}_t \right) - \eta\beta \widetilde{u}_c \left(C_{t+1} - \eta C_t; \overline{C}_{t+1} \right) \right]} P_t \right].$$

Let s^m be the *real* marginal cost in the $m-$sector

$$s^m \left(y_t^m (i), C_t, \frac{P_t^s}{P_t^m}; \xi_t \right) \equiv \phi_m \left[\frac{y_t^m (i)}{A_t^m} \right]^{\phi_m - 1} \left[\frac{\Psi_t^m \widetilde{v}_h [y_t^m (i)]}{\left[\widetilde{u}_c \left(C_t - \eta C_{t-1}; \overline{C}_t \right) - \eta\beta \widetilde{u}_c \left(C_{t+1} - \eta C_t; \overline{C}_{t+1} \right) \right]} \frac{P_t}{P_t^m} \right], \tag{18}$$

where "real" is with respect to the price of the composite good in the m sector. Notice that accounting for (5) we obtain

$$\frac{P_t}{P_t^m} = \left[n_s \left(Q_t^{1-\rho} - 1 \right) + 1 \right]^{\frac{1}{1-\rho}}, \tag{19}$$

and

$$\frac{P_t}{P_t^s} = \left[n_m \left(Q_t^{\rho-1} - 1 \right) + 1 \right]^{\frac{1}{1-\rho}}, \tag{20}$$

where $Q_t \equiv \frac{P_t^s}{P_t^m}$, so that s^m turns out to be a function only of $(y_t^m (i), C_t, Q_t; \xi_t^m)$ where $\xi_t^m \equiv \left(A_t^m, \Psi_t^m, \overline{C}_t \right)'$ is a vector of shocks. Then the f.o.c. can be rewritten as

$$\frac{\widetilde{p}_t^m}{P_t^m} = \frac{\theta_m}{(\theta_m - 1)} s^m \left(y_t^m (i), C_t, Q_t; \xi_t^m \right). \tag{21}$$

Now, rearranging the demand for good i in sector m given by (13) we obtain

$$\frac{p_t^m (i)}{P_t^m} = \frac{[y_t^m (i)]^{-\frac{1}{\theta_m}}}{C_t^{-\frac{1}{\theta_m}}} \left(\frac{P_t^m}{P_t} \right)^{-\frac{\rho}{\theta_m}}.$$

Then, accounting for (21) the supply of good i must satisfy

$$\frac{[y_t^m (i)]^{-\frac{1}{\theta_m}}}{C_t^{-\frac{1}{\theta_m}}} \left(\frac{P_t^m}{P_t} \right)^{-\frac{\rho}{\theta_m}} = \frac{\theta_m}{(\theta_m - 1)} s^m \left(y_t^m (i), C_t, Q_t; \xi_t^m \right).$$

Now notice that the LHS and the RHS are, respectively, decreasing and increasing in $y_t^m(i)$. Thus there is only one value of $y_t^m(i)$ that satisfies the previous equation given (C_t, Y_t^s, Q_t). In equilibrium all the firms in the m-sector produce the same quantity so that it must be that $y_t^m(i) = Y_t^m$. Hence

$$\frac{[Y_t^m]^{-\frac{1}{\theta_m}}}{C_t^{-\frac{1}{\theta_m}}}\left(\frac{P_t^m}{P_t}\right)^{-\frac{\rho}{\theta_m}} = \frac{\theta_m}{(\theta_m - 1)} s^m\left(Y_t^m, C_t, Q_t; \xi_t^m\right),$$

and accounting for (18) and (19) we obtain

$$\frac{[Y_t^m]^{-\frac{1}{\theta_m}}}{C_t^{-\frac{1}{\theta_m}}}\left(\frac{P_t^m}{P_t}\right)^{-\frac{\rho}{\theta_m}} = \frac{\theta_m}{(\theta_m - 1)}\phi_m\left[\frac{Y_t^m}{A_t^m}\right]^{\phi_m - 1}\frac{\left[y_t^m(j)/A_t^m\right]^{\nu\phi_m}\left[n_s\left(Q^{1-\rho} - 1\right) + 1\right]^{\frac{1}{1-\rho}}}{\left[\overline{C}_t^{\frac{1}{\widetilde{\sigma}}}\left(C_t - \eta C_{t-1}\right)^{-\frac{1}{\widetilde{\sigma}}} - \eta\beta\overline{C}_{t+1|t}^{\frac{1}{\widetilde{\sigma}}}\left(C_{t+1|t} - \eta C_t\right)^{-\frac{1}{\widetilde{\sigma}}}\right]},$$

which, assuming no shocks and accounting for (7) and (14-16) boils down to

$$\frac{(\theta_m - 1)}{\theta_m\phi_m} = \frac{[Y^m]^{\nu\phi_m + \phi - 1}}{(1 - \eta\beta)\left[(1 - \eta)Y\right]^{-\frac{1}{\widetilde{\sigma}}}}\left[n_s Q^{1-\rho} + n_m\right]^{\frac{1}{1-\rho}}. \quad (22)$$

Similarly for the other sector

$$\frac{(\theta_s - 1)}{\theta_s\phi_s} = \frac{[Y^s]^{\nu\phi_s + \phi_s - 1}}{(1 - \eta\beta)\left[(1 - \eta)Y\right]^{-\frac{1}{\widetilde{\sigma}}}}\left[n_s + n_m(Q)^{\rho - 1}\right]^{\frac{1}{1-\rho}}. \quad (23)$$

Now accounting for (6-7) and the sectoral market clearing conditions (14-16) and (19-20) we obtain

$$Y^m = n_m Y\left[n_s(Q)^{1-\rho} + n_m\right]^{\frac{\rho}{1-\rho}}, \quad (24)$$

$$Y^s = n_s Y\left[n_s + n_m(Q)^{\rho-1}\right]^{\frac{\rho}{1-\rho}}, \quad (25)$$

thus we have to solve a system of four equations (22-25) in four unknowns (Y^m, Y^s, Y, Q).

2.2 Log-linearized relations

I now log-linearize the equilibrium conditions around the steady state where the variables $\left(Y_t^m, Y_t^s, Y_t, Q_t, \frac{P_{t+1}}{P_t}, \frac{P_{t+1}^s}{P_t^s}, \frac{P_{t+1}^m}{P_t^m}\right)$ are equal to $(Y^m, Y^s, Y, Q, 1, 1, 1)$ and all the shocks are equal to one. Loglinearizing the Euler equation account being taken of the market clearing condition leads to the aggregate demand

$$\begin{aligned} y_t \;=\; & \frac{\eta}{1 + \eta(1 + \beta\eta)}y_{t-1} + \frac{1 + \eta\beta(1 + \eta)}{1 + \eta(1 + \beta\eta)}y_{t+1|t} - \frac{\eta\beta}{1 + \eta(1 + \beta\eta)}y_{t+2|t} \\ & - \frac{\widetilde{\sigma}(1 - \eta)(1 - \eta\beta)}{(1 + \eta + \beta\eta^2)}\left(\widehat{i}_t - \pi_{t+1|t}\right) + \frac{1 - \eta}{1 + \eta(1 + \beta\eta)}\left[\overline{c}_t - (\eta\beta + 1)\overline{c}_{t+1|t} + \eta\beta\overline{c}_{t+2|t}\right] \end{aligned} \quad (26)$$

where $\overline{c}_t \equiv \log \overline{C}_t$ and loglinearizing the f.o.c. for the firm's problem (17) with respect to sector m and s we obtain

$$\pi_t^m = \kappa^m \left[\omega_m + \varphi\left(1+\eta^2\beta\right)\right] y_t - \kappa^m\varphi\eta y_{t-1} - \kappa^m\varphi\eta\beta y_{t+1|t} + \kappa^m\overline{Q}^s\left(\rho\omega_m+1\right) q_t$$
$$- \kappa^m\left[\left(1+\omega_m\right) a_t + \varphi\left(1-\eta\right)\left(\overline{c}_t - \eta\beta\overline{c}_{t+1|t}\right) - \psi_t^m\right] + \beta\pi_{t+1|t}^m \tag{27}$$

and

$$\pi_t^s = \kappa^s \left[\omega_s + \varphi\left(1+\eta^2\beta\right)\right] y_t - \kappa^s\varphi\eta y_{t-1} - \kappa^s\varphi\eta\beta y_{t+1|t} - \kappa^s\overline{Q}^m\left(\rho\omega_s+1\right) q_t$$
$$- \kappa^s\left[\left(1+\omega_s\right) a_t + \varphi\left(1-\eta\right)\left(\overline{c}_t - \eta\beta\overline{c}_{t+1|t}\right) - \psi_t^s\right] + \beta\pi_{t+1|t}^s \tag{28}$$

where $a_t \equiv \log A_t,\ \psi_t^h \equiv \Psi_t^h,\ h = s, m$ and

$$\overline{Q}^s \equiv \frac{n_s Q^{1-\rho}}{n_s\left(Q^{1-\rho}-1\right)+1}, \qquad \overline{Q}^m = 1-\overline{Q}^s, \qquad \omega_h \equiv \phi_h\left(v+1\right)-1,$$

$$\kappa^h \equiv \frac{\left(1-\alpha^h\right)\left(1-\alpha^h\beta\right)}{\alpha^h\left(1+\omega_h\theta_h\right)}, \qquad h = m, s, \qquad \varphi \equiv \frac{1}{\left(1-\eta\right)\widetilde{\sigma}\left(1-\eta\beta\right)},$$

and the exogenous shocks follow

$$\begin{aligned} a_{t+1} &= \gamma_a a_t + \varepsilon_{t+1}^a, \\ \overline{c}_{t+1} &= \gamma_c \overline{c}_t + \varepsilon_{t+1}^c, \\ \psi_{t+1}^s &= \gamma_\psi \psi_t^s + \varepsilon_{t+1}^{\psi s}, \\ \psi_{t+1}^m &= \gamma_\psi \psi_t^m + \varepsilon_{t+1}^{\psi m}, \end{aligned}$$

where $E_t\left(\varepsilon_{t+1}^h\right) = 0$, $h = a,\ c,\ \psi s,\ \psi m$. Log-linearizing the price index (5) we obtain aggregate inflation

$$\pi_t = \left(1-\widetilde{n}\right)\pi_t^s + \widetilde{n}\pi_t^m, \qquad \widetilde{n} \equiv \frac{n_m}{n_s\left(Q^{1-\rho}-1\right)+1}, \tag{29}$$

and substituting the sectoral inflations we obtain aggregate inflation in terms of lagged, current, and expected output gap, the relative price, expected inflation, and the exogenous shocks

$$\begin{aligned} \pi_t = {} & \left\{\left(1-\widetilde{n}\right)\kappa^s\left[\omega_s + \varphi\left(1+\eta^2\beta\right)\right] + \widetilde{n}\kappa^m\left[\omega_m + \varphi\left(1+\eta^2\beta\right)\right]\right\} y_t - \varphi\eta\left[\left(1-\widetilde{n}\right)\kappa^s + \widetilde{n}\kappa^m\right] y_{t-1} \\ & -\varphi\eta\beta\left[\left(1-\widetilde{n}\right)\kappa^s + \widetilde{n}\kappa^m\right] y_{t+1|t} - \left[\left(1-\widetilde{n}\right)\kappa^s\overline{Q}^m\left(\rho\omega_s+1\right) - \widetilde{n}\kappa^m\overline{Q}^s\left(\rho\omega_m+1\right)\right] q_t \\ & - \left(1-\widetilde{n}\right)\kappa^s\left[\left(1+\omega_s\right)a_t - \psi_t^s\right] - \widetilde{n}\kappa^m\left[\left(1+\omega_m\right)a_t - \psi_t^m\right] \\ & -\varphi\left(1-\eta\right)\left[\left(1-\widetilde{n}\right)\kappa^s + \widetilde{n}\kappa^m\right]\left(\overline{c}_t - \eta\beta\overline{c}_{t+1|t}\right) + \beta\pi_{t+1|t} \end{aligned}$$

Finally, defining $q_t \equiv \log \frac{Q_t}{Q}$, the law of motion for the log deviation of the relative price from its steady state value is given by

$$q_t = q_{t-1} + \pi_t^s - \pi_t^m. \tag{30}$$

The model is closed with a Taylor rule describing the behaviour of the central bank

$$i_t = \delta_0 i_{t-1} + (1 - \delta_0)\, \delta_1 \pi_t + (1 - \delta_0)\, \delta_2 y_t.$$

2.3 Calibration

The degree of habits persistence is $\eta = 0.8$; the elasticity of intertemporal substitution in consumption is $\widetilde{\sigma} = 0.638$; the elasticity of substitution between C_t^s and C_t^m in the CES consumption aggregate is $\rho = 3$; the number of firms in the s-sector is $n_s = 0.5$; and in the m-sector is $n_m = 1 - n_s$; the elasticity of sectoral output with respect to hours worked is $\phi_h = 1.333,\ h = s, m$; the inverse of the elasticity of goods production is $\nu = 1$; the sectoral elasticity of substitution between any two differentiated goods is $\theta_h = 7.88,\ h = s, m$; the intertemporal discount factor is $\beta = 0.99$; the coefficients of the Taylor rule are $\delta_0 = 0.8;\ \delta_1 = 1.5;\ \delta_2 = 0.5/4$; the AR coefficients of the exogenous processes are $\gamma_a = \gamma_\psi = \gamma_c = 0.95$ and for any shock the variance is $\sigma_\varepsilon^2 = 0.009^2$. Finally, the sectoral degree of price stickiness $\alpha_h,\ h = s, m$ is let free to vary in the range $\{0.5, 0.6, 0.7, 0.8, 0.9\}$ as described in the analysis below.

3 Sectoral asymmetry in price stickiness and economic dynamics

This section studies the relation between sectoral asymmetry in price stickiness and the dynamics of the economy in presence of exogenous positive shocks to the price level, technology and the preferences of the household. All the shocks are supposed to hit symmetrically both sectors. Through impulse response functions, autocorrelations, and variances of the endogenous variables, the analysis combines three perspectives to investigate qualitatively and quantitatively the relation at issue.

3.1 IRFs analysis

Figures 1-3 show the impulse response functions to a preference shock and to symmetric cost-push and technology shocks in presence of sectoral symmetry and asymmetry in the degree of price stickiness. Starting with Panels 1a-3a, each figure reports the symmetric case, first row ($\alpha_s = \alpha_m = 0.5$), and two asymmetric cases differing in the degree of sectoral asymmetry, second and third row ($\alpha_s = 0.7,\ \ \alpha_m = 0.5$ and $\alpha_s = 0.9,\ \ \alpha_m = 0.5$). What is interesting here is that each panel reveals a common behaviour of aggregate inflation in response to an increasing degree of price stickiness asymmetry. This behaviour can be stated as follows:

Result 1: Price Stickiness Asymmetries and Aggregate Inflation. For any shock considered, the larger the asymmetry in price stickiness, the lower the deviation of aggregate inflation from its steady state value, the lower the initial impact of the shock and the lower the persistence of the response to the shock.

This result has significant implications on the behaviour of the interest rate. To describe them, it is important to note that the cost-push shock and the technology shock affect the economy only from the supply side, while the preference shock also from the demand side[1]. Furthermore, the impact of the preference shock on the demand and supply side goes in opposite directions[2]. This, as explained below, implies that sectoral asymmetry, via inflation, play a different role on the dynamics of the interest rate with a preference shock and with a cost-push or technology shock.

Starting with the preference shock, panel 1a shows a positive and negative impact of the shock on the output gap and inflation, respectively. Now, under symmetry, first row, the fall in inflation affects the interest rate via the Taylor rule more than the increase in the output gap. As a result the interest rate falls considerably in the initial periods. Yet, breaking the symmetry, second and third row, Result 1 implies that inflation is less affected by the shock and, therefore the negative impact on the interest rate is attenuated. This leaves the interest rate more exposed to the positive impact of the output gap and consequently the monetary policy turns out to be more active in the subsequent periods.

On the other hand, in presence of a cost-push or technology shock (panel 2a and 3a respectively), the shock hits the economy only through the supply side. Thus Result 1 implies that the larger the asymmetry, the less the shock passes through to the rest of the economy, specifically, the lower the deviations of the output-gap from its steady state value, and the less active the monetary policy.

Summing up, Result 1 implies that sectoral asymmetry plays a key role in amplifying or attenuating the deviations of the interest rate from its long run equilibrium according to the type of the shock that hits the economy.

The analysis based on Panels 1a-3a provides a useful starting point to illustrate how sectoral asymmetry affect the behaviour of aggregate inflation and Result 1 helps to fix the ideas. Yet, it does not disentangle the impact of the *mean* and the *variance* in the degree of sectoral price stickiness on aggregate inflation. Indeed,

[1] The latter has this peculiarity as shocks to the utility function affect on the one hand the Euler equation and therefore the aggregate demand and, on the other hand, they affect the marginal rate of substitution between labour and consumption entering in the optimal supply of labour, and therefore the aggregate supply.

[2] Indeed in equation (**??**) the coefficient of the preference shock is negative and in equation (26) it is positive.

both moments increases when α_s increases. This issue leads to ask to what extent, if any, two economies sharing the same mean degree of price stickiness but differing in terms of variance respond differently to exogenous disturbances. Panels 1b-3b, address this question reporting on the first row an economy with symmetric sectors where price stickiness is the same in both sectors, specifically $\alpha_s = \alpha_m = 0.7$, and in the second row an economy with asymmetric sectors with the same mean of the symmetric economy, i.e. $\alpha = 0.7$, but different sectoral price stickiness, specifically $\alpha_s = 0.9,\ \alpha_m = 0.5$. Interestingly, the analysis reveals an important relation between the variance of sectoral price stickiness and the behavior of aggregate inflation that can be stated as follows

Corollary 1: Variance of Sectoral Price Stickiness and Aggregate Inflation. For any shock considered, for a given mean value of price stickiness, the larger the variance (i.e. the larger the asymmetry), the lower the deviation of inflation from its steady state value, the lower the initial impact of the shock and the lower the persistence of the response to the shock.

What are the implications of Corollary 1 on the behaviour of the two economies? Panel 1b refers to the response of the two economies to a preference shock. In the asymmetric case the initial and subsequent response of the interest rate is, respectively, lower and higher than in the symmetric case. This is due to the fact that breaking the symmetry the impact of the shock on inflation is attenuated. Consequently, a minor initial decrease of the interest rate is required to stabilize inflation and thus monetary policy can focus more on the stabilization of the output gap by increasing the interest rate. In term of the output gap, this policy results in a better stabilization for the asymmetric economy.

Panel 2b and 2c refer to the response of the two economies to a cost-push and a technology shock respectively. They show that the policy response in the asymmetric economy is half as active as in the symmetric one and, similarly, that the deviations of output-gap and inflation from their steady state values in the asymmetric economy tend to be half as large as in the symmetric economy.

Summing up, the economy featuring higher dispersion in sectoral price stickiness is less perturbed by exogenous disturbances and tends to exhibit a less active policy. These findings will be assessed quantitatively in the following analysis yet, before deepening the investigation with the study of the autocorelation and standard deviation of the endogenous variables, it is worth stopping for a natural question: what drives these findings? to explain the mechanism at work we first notice that breaking the symmetry introduces a new variable and a new relation into the working of the economy. The new variable is the relative price of the sectoral goods which appears, as a log-deviation, in the second and third row of Figures 1a-3a and in the second row of Figures 1b-3b. The new relation is a relation between the relative price and sectoral inflations. With symmetry, sectoral inflations coincide and thus the relative price Q_t is constant and equal to its steady state value Q so that $q_t = 0$. Breaking the symmetry, a *symmetric* shock hits sectoral inflations differently: the stickier the sectoral price, the smaller the impact of the shock[3]. Thus sectoral inflations start to differ and q_t enters the picture driven by its low of motion (30). But the relation between q_t and sectoral inflations works in the other direction too. Indeed, the presence of the relative price activates a switching demand mechanism that, in presence of a symmetric shock, acts *asymmetrically* on the sectoral inflations: it offsets the shock in the sector hit more and strengthens it in the sector hit less[4]. Since the offsetting action outpaces the magnifying action, this mechanism reduces the difference between sectoral inflations caused by the shock. Yet, given the law of motion of q_t, as long as the sign of the difference between sectoral inflations generated by the shock does not change, the relative price continues to increase its deviation from the steady state amplifying the switching demand mechanism. This process inexorably leads to a turning point for q_t where the impact of the switching demand mechanism exceeds the impact of the shock, the difference in sectoral inflations changes sign, and q_t starts to go in the opposite direction converging to its steady state value. This is illustrated by the hump-shaped path of q_t shown in Figure 1-3. To conclude, the relation between q_t and sectoral inflations results in different paths for the latter which, in turn, affect the behaviour of aggregate inflation and the rest of the economy.

3.2 Autocorrelation analysis: sectoral asymmetry in price stickiness and the persistence of y, i, and π

The Impulse Response analysis has signalled an important impact of sectoral asymmetry in the degree of price stickiness on the dynamics of the economy. In order to characterize this impact further, it is useful to focus on the persistence of y, i, and π. Adopting as persistence measure the sum of the first five autocorrelation coefficients, Figure 4 illustrates the case of *sectoral symmetry* and plots the persistence of the endogenous variables against price stickiness. The graphic shows that when the average degree of price stickiness increases from 0.5 to 0.9, the persistence of the inflation variables (which is the same in the symmetric case as sectoral and aggregate inflation coincide) increases too. This is due to the price stickiness mechanism itself that buffering changes in marginal costs caused by exogenous disturbances tends to insulate inflation generating more persistence[5]. The increase in inflation persistence leads in turn to an increase in the interest rate persistence. Interestingly, these findings starkly contrast with the ones provided by the *sectoral asymmetry* case illustrated in Figure 5. Here the graphic plots the persistence of the endogenous variables against

[3] This can be seen in the sectoral AS, equations (27-28) noting that κ in the coefficient of the shocks is inversely related to the degree of price stickiness α.

[4] This can be seen in the equations for sectoral inflations (27-28) where the coefficients for q_t have opposite signs.

[5] This can be observed in the inflation equations noticing that the composite coefficient κ is inversely related to the degree of price stickiness α.

the degree of price stickiness in the s-sector keeping constant the degree of price stickiness in the m-sector at $\alpha_m = 0.5$. Due to the mechanism that generates Results 1 explained above, the persistence of the m-sector inflation falls, and falls more than the increase in the persistence in the s-sector. Thus, the persistence of aggregate inflation up to $\alpha_s = 0.7$ rises faintly and then falls more and more markedly. As in the symmetric case, the interest rate persistence tends to mirror the aggregate inflation persistence.

This analysis has shown that sectoral asymmetry in price stickiness is a key factor determining the persistence of inflation and the interest rate. Disentangling the variance effect from the mean effect we consider two economies that share the same mean degree of sectoral asymmetry but a different variance. Setting the mean degree of sectoral asymmetry equal to 0.7, Figure 4, for $\alpha_s = \alpha_m = 0.7$, provides the persistence of the endogenous variables for an economy with *zero* variance and Figure 5 for, $\alpha_s = 0.9$ and $\alpha_m = 0.5$, provides the persistence of the endogenous variables for an economy with *positive* variance equal to 0.04. Then the persistence measures reveal that increasing the dispersion in sectoral price stickiness, but keeping the same mean value, leads to a significant reduction in the persistence of aggregate inflation and to a modest reduction in the persistence of the interest rate. Specifically, inflation and interest rate persistence falls, respectively, of 23.3% and 5.5%.

Summing up, the autocorrelation analysis suggests that the relation between the dispersion in price stickiness asymmetry and the persistence of inflation and the interest rate, which was revealed by the impulse response function analysis, is quantitatively important.

3.3 Unconditional Variances analysis: sectoral asymmetry in price stickiness and economic stability

As shown in the impulse response analysis, the introduction of sectoral asymmetry in price stickiness tends to reduce the deviations of π, i, and y from their long run values. A convenient way to measure these deviations consists of computing the unconditional standard deviations of the endogenous variables. Starting with the symmetric case, Figure 6 plots the standard deviation of π, i, and y against the degree of price stickiness shared by the sectors. The graphic shows that the stickier the price level, the more stable the endogenous variables. As explained above, the sticky price mechanism itself tends to filter out the exogenous disturbances delivering more stability. Moving to the asymmetric case, Figure 7 plots standard deviations against the degree of price stickiness in the s-sector keeping the degree of price stickiness in the other sector constant. Comparing the figures, two economies that in terms of price stickiness share the average but not the variance exhibit very different variability of the endogenous variables. Indeed, keeping the average price stickiness equal to 0.7 and breaking the sectoral symmetry the variability of y, i, and π decrease of the 32.6%, 48.9% and 48.9%, respectively. Thus sectoral asymmetry in price stickiness plays a key role in determining the variability of the endogenous variables.

4 Concluding remarks

This paper investigates how sectoral asymmetry in price stickiness affect the dynamics of the economy in a two-sector New Keynesian model with habits in consumption.

When sectoral symmetry is broken, the relative price between sectoral inflations appears into the New Keynesian economy and significantly alters its response to exogenous shocks. As a result, asymmetry in sectoral price stickiness leads to an important fall in the persistence of inflation and in the volatility of inflation, the interest rate and the output gap. It also leads to a moderate fall in the persistence of the interest rate. Thus, two economies differing in the dispersion of sectoral asymmetry but not in the mean may exhibit very unlike volatility and persistence.

Further analysis will take the model to the data and investigate how sectoral size and strategic complementarities affect the relation between sectoral asymmetry in price stickiness and the dynamics of the economy.

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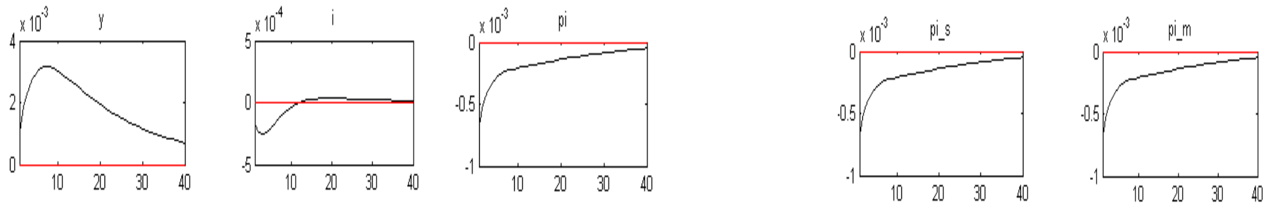
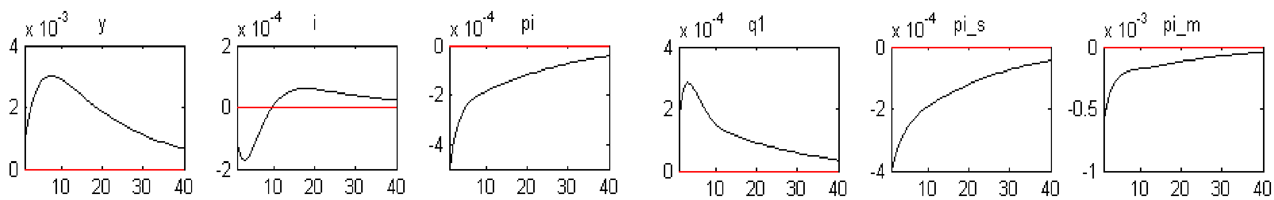
Figure 1a. IRFs to a preferences shock under symmetry and asymmetry in sectoral price stickiness.

Symmetry: $\alpha_s = \alpha_m = 0.5$



Asymmetry: $\alpha_s = 0.7$, $\alpha_m = 0.5$



Asymmetry: $\alpha_s = 0.9$, $\alpha_m = 0.5$

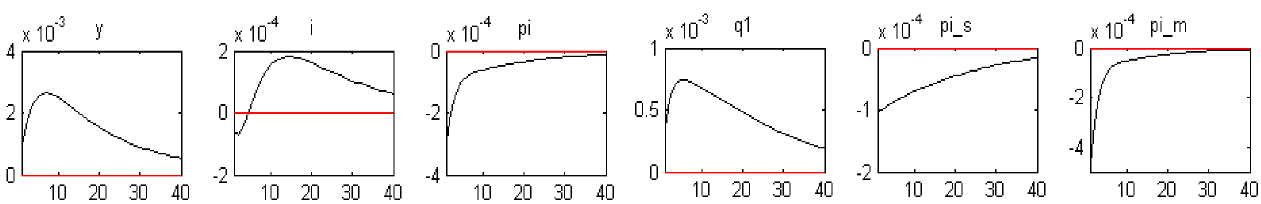
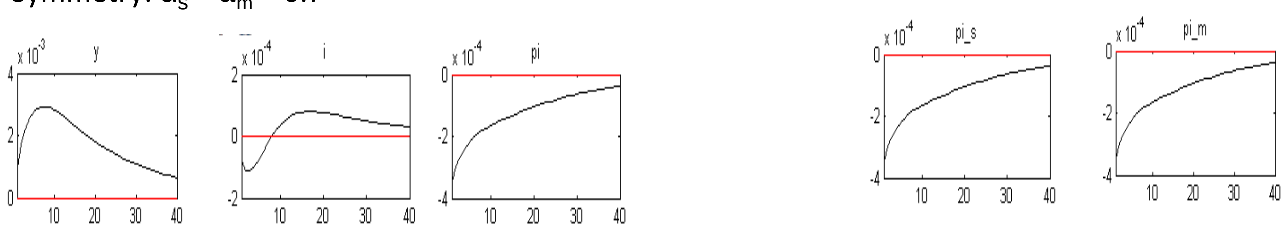


Figure 1b. IRFs to a preference shock in two economies with the same mean degree of sectoral price stickiness but a different variance.

Symmetry: $\alpha_s = \alpha_m = 0.7$



Asymmetry: $\alpha_s = 0.9$, $\alpha_m = 0.5$

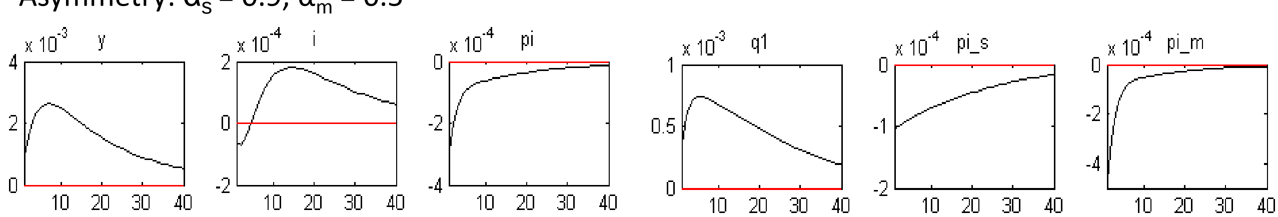


Figure2a. IRFs to a cost-push shock under symmetry and asymmetry in sectoral price stickiness

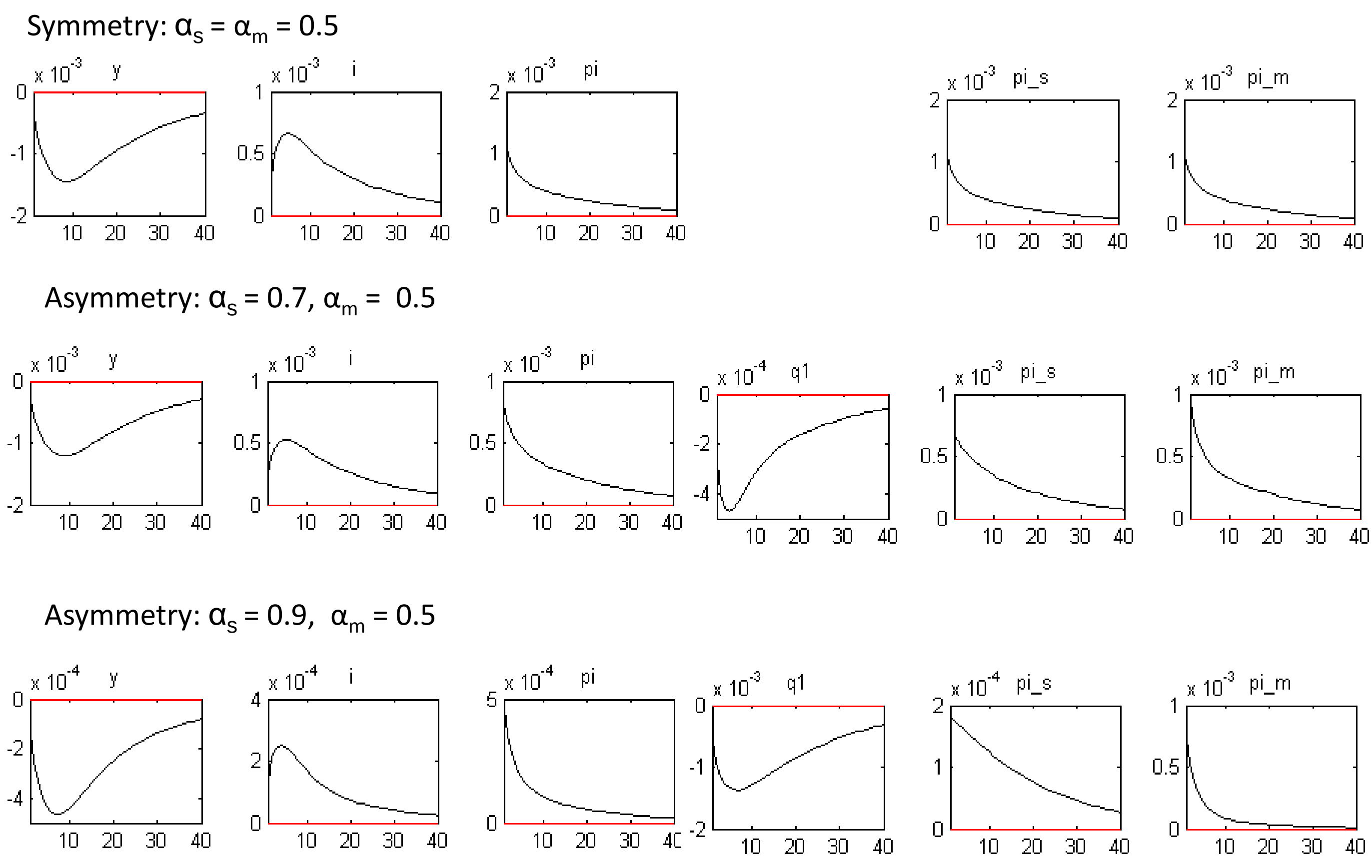


Figure 2b. IRFs to a cost-push shock in two economies with the same mean degree of sectoral price stickiness but a different variance.

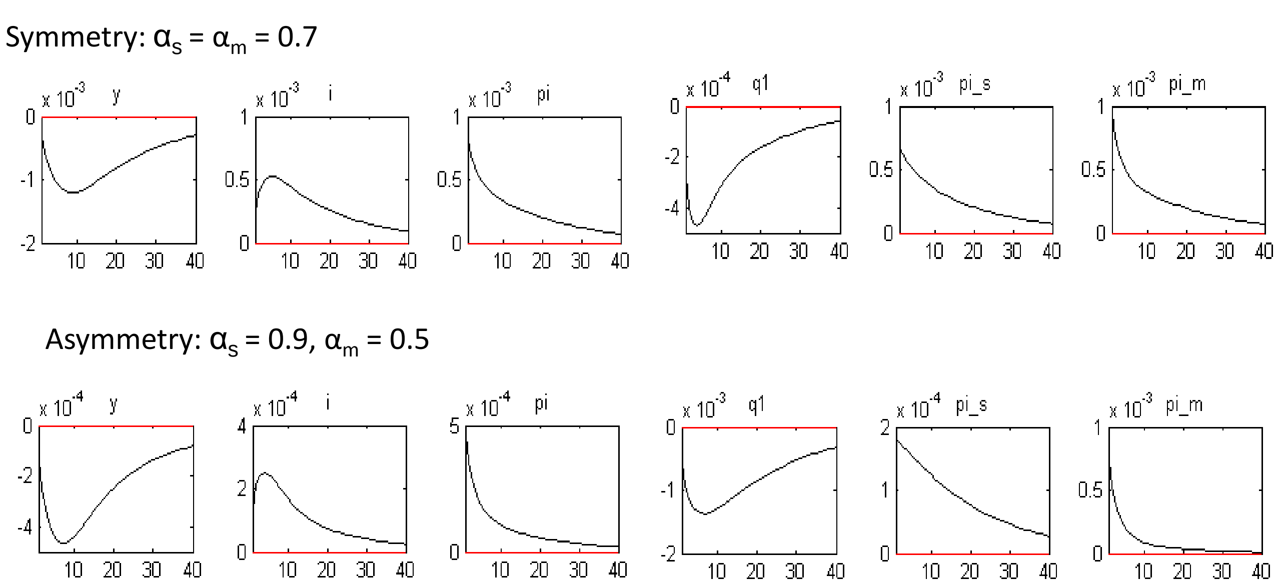
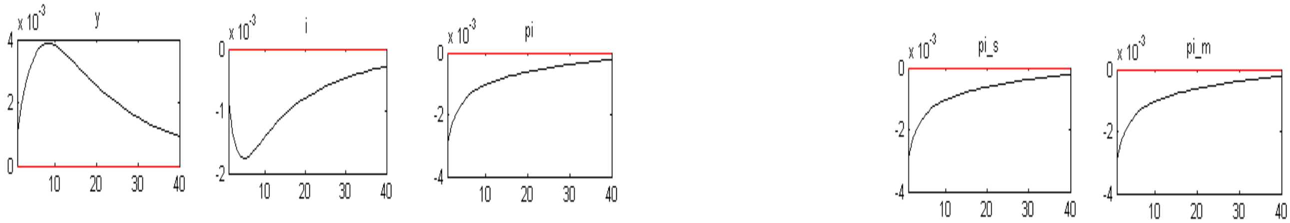
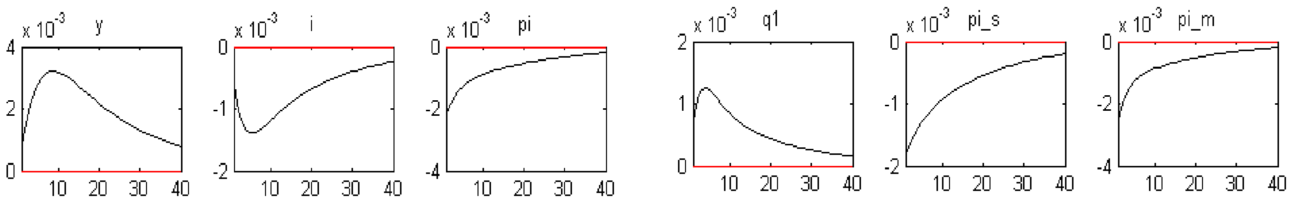


Figure 3a. IRFs to a technology shock under symmetry and asymmetry in sectoral price stickiness.

Symmetry: $\alpha_s = \alpha_m = 0.5$



Asymmetry: $\alpha_s = 0.7$, $\alpha_m = 0.5$



Asymmetry: $\alpha_s = 0.9$, $\alpha_m = 0.5$

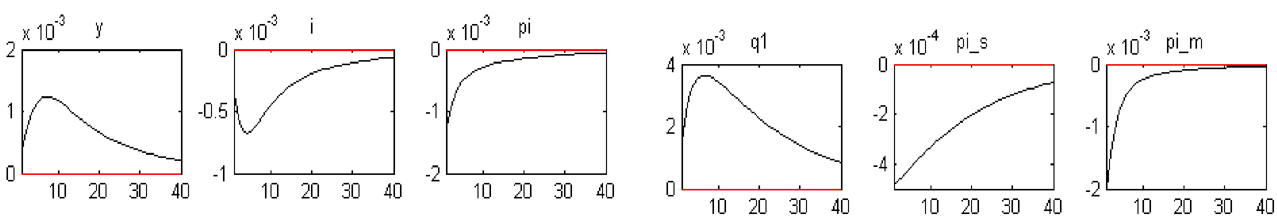
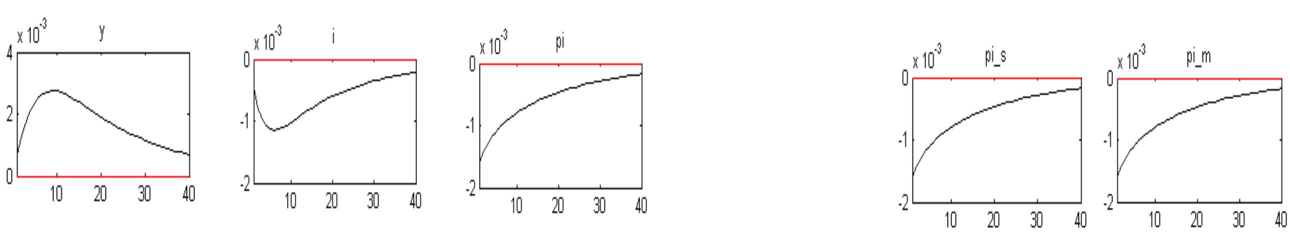


Figure 3b. IRFs to a technology shock in two economies with the same mean degree of sectoral price stickiness but a different variance.

Symmetry: $\alpha_s = \alpha_m = 0.7$



Asymmetry: $\alpha_s = 0.9$, $\alpha_m = 0.5$

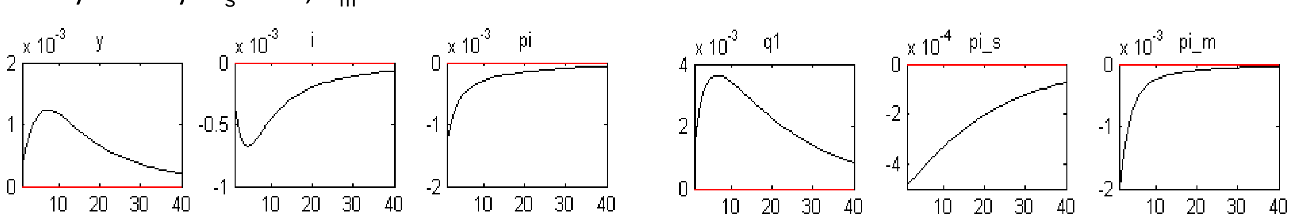


Figure 4. Persistence in terms of the sum of the first five autocorrelation coefficients and symmetric price stickiness.

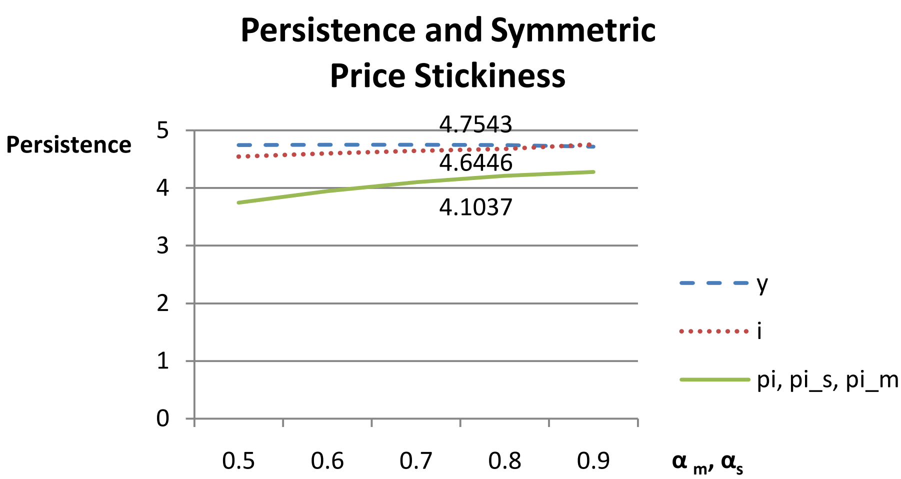


Figure 5. Persistence in terms of the sum of the first five autocorrelation coefficients and asymmetric price stickiness under $\alpha_m = 0.5$ and $0.5 \leq \alpha_s \leq 0.9$.

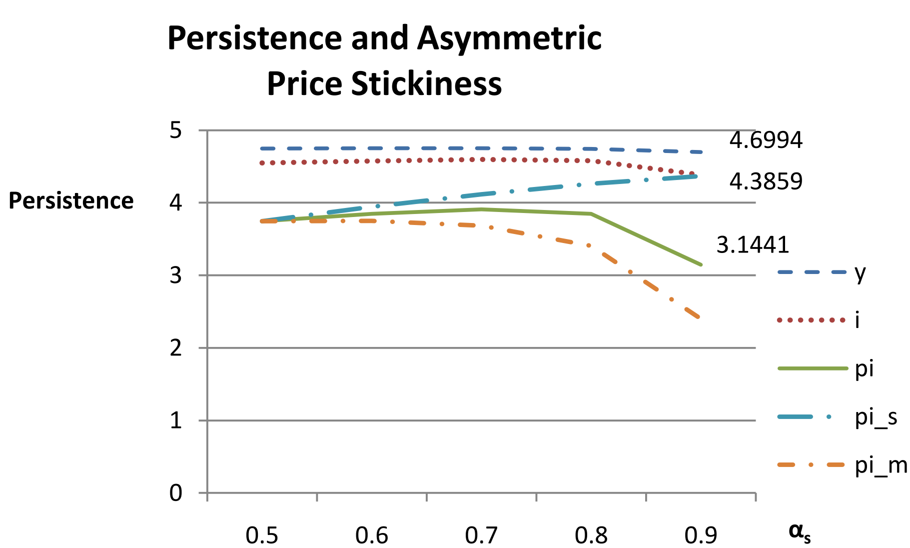


Figure 6. Standard deviation and symmetric price stickiness.

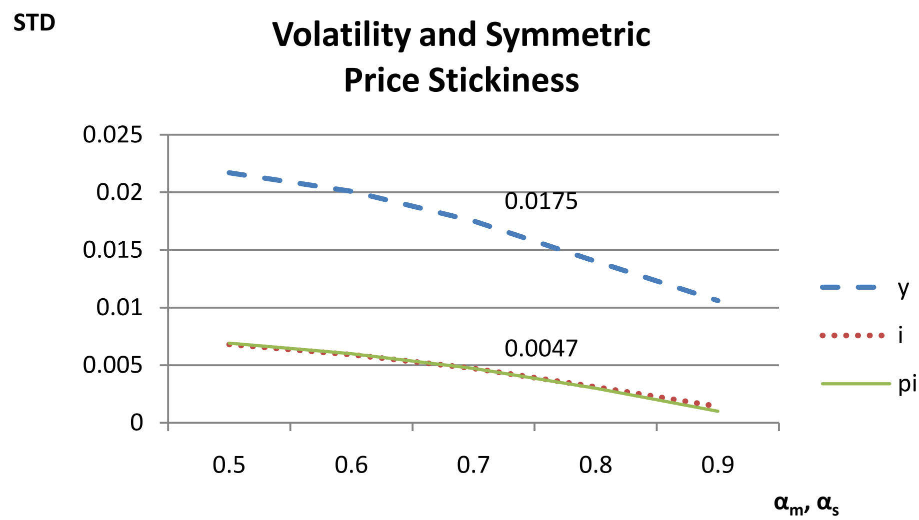


Figure 7. Standard deviation and asymmetric price stickiness under α_m = 0.5 and 0.5 ≤ α_s ≤ 0.9.