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Export growth and factor market competition: theory and evidence*

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Abstract

Empirical evidence suggests that sectoral export growth decreases exporters' survival probability, whereas non–exporters are unaffected. Models with firm heterogeneity in total factor productivity predict the opposite. To solve this puzzle, we develop a two–factor framework where firms differ in factor shares. In this model, export growth increases competition for the factor used intensively by exporters, eliminating some of them, while non–exporters benefit. Our empirical analysis shows that the forces highlighted in the model drive the firm selection experienced by the Chilean manufacturing sector, suggesting that heterogeneity in factor shares is crucial to understand how firms react to trade liberalization.

JEL classification numbers: F12, F14, F16, L11
Keywords: Firm dynamics, two–factor trade model, firm heterogeneity in factor shares, Chile, manufacturing industry

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1 Introduction

Ever since detailed firm level trade data has become available, many studies have focused on the effects of import growth on firm dynamics. For instance, Bernard et al. (2006a) show that imports from low–wage countries have a negative impact on plant survival and growth among US firms. Similarly, Bernard et al. (2006b) show that declining trade costs invite more foreign varieties into the domestic market and reduce domestic sales and, accordingly, the survival probability of all domestic firms. On the other hand, little systematic evidence exists on the role of *export* growth for firm dynamics. The purpose of this paper is to fill this gap in the literature.

We start by documenting the effects of export growth on Chilean manufacturing firms during the period 1990–99. Interestingly, we find that exporting firms are more likely to cease production, the larger are sector–wide exports. We do not find any relationship between sector–wide exports and the survival probability of non–exporters. This finding is remarkable, as it is at odds with the predictions of the existing theoretical literature, where the source of firm heterogeneity is total factor productivity (TFP). In fact, both the models by Melitz (2003) and Bernard et al. (2003) predict that export growth will lead the least productive non–exporting firms to exit the market. At the same time, there is abundant evidence suggesting that firm heterogeneity in factor input ratios is substantial and at least as important as firm heterogeneity in TFP.[1] Still, little is known about how differences in technology as captured by differences in factor shares affect the link between trade liberalization and firm survival.

In this paper, we develop a new theoretical model of trade in which differences in factor input ratios are the source of heterogeneity among firms. Thus, our analysis enriches the modeling of the production side, and we will argue that firm heterogeneity in factor input ratios is crucial to understand the industry–wide adjustments brought about by trade liberalization.

We cast our discussion in a general equilibrium setting with one monopolistically competitive sector in each country. Each firm produces a unique variety of a differentiated final good using capital and labor. Upon market entry, firms choose the factor share parameter characterizing their CES production function. In general, firms find it optimal to adopt different technologies to limit factor market competition. After entry, and to start production, firms have to pay a fixed cost, which depends on the capital intensity of their technology.

We start by characterizing the autarkic equilibrium. Since the coexistence of firms with different factor input ratios is ubiquitous in the real world, we restrict the technology space so that capital and labor intensive technologies coexist in equilibrium. We show that the autarkic equilibrium is unique in the sense that the mass of firms which choose a specific factor intensity in production is uniquely determined by the parameters of the model.

Next, we study the trade equilibrium arising in a completely symmetric two–country world. In a setting with fixed export costs, which implies that only the more capital intensive firms can afford

[1] This has been documented by Bernard and Jensen (1995), Alvarez and López (2005) and Leonardi (2007) among others.

to serve the foreign market, we characterize the firm selection induced by trade liberalization. We highlight three different effects. First, trade liberalization provides additional profit opportunities for exporting firms. Second, it decreases domestic market shares for both exporters and non–exporters. Third, it increases factor market competition due to the additional production for exports. In particular, since exporters are the more capital intensive firms, the increase in factor market competition increases (decreases) the relative price of capital (labor) and negatively affects exporters, while it positively affects non–exporters. We also show that this effect becomes stronger, the larger is the difference in factor intensities between the two types of firms. As a result, the burden of increased factor market competition induced by trade liberalization falls entirely on capital intensive exporters and some of the exporting firms might be forced to cease production. Extending the model to multiple trading countries strengthens this result. Thus, our theoretical framework is able to rationalize the empirical facts we have documented for the case of Chile, which are instead at odds with the predictions of Melitz's (2003) model.

Much of the existing literature has emphasized the role of productivity differences among firms. How does heterogeneity in factor shares interact with heterogeneity in TFP in shaping the firm selection process? To answer this question we extend our model and assume that within a group of firms with identical factor input ratios, firms differ with respect to TFP. Trade liberalization now leads to two distinct factor relocations between firms. On the one hand, factors move towards the more productive firms *within* a group of firms with identical factor shares. On the other hand, factors also move between capital and labor intensive firms. While the first process increases sector–wide TFP, the second has a priori an ambiguous effect. Still, under some mild assumptions, we are able to show that the larger is the difference in factor intensities between capital and labor intensive firms, the smaller is the increase in sector–wide TFP due to trade liberalization. Thus, factor market competition dampens the positive effect on sector–wide TFP, which has been highlighted by Melitz (2003). This allows our model to provide a rationale for the findings of the recent literature, which has highlighted that the effects of trade liberalization on sector–wide TFP might be only moderate (Lawless and Whelan 2008).[2]

Our model has identified an important channel through which export growth affects firm selection and sector–wide TFP improvement. First, increased factor market competition is more detrimental for exporters, the bigger is the difference in factor intensities between exporters and non–exporters. Second, the increase in TFP is smaller, the larger is the difference in factor intensities between the two types of firms. Using our Chilean dataset, we are able to show that both of these mechanisms are at work, thus highlighting the importance of modeling heterogeneity in factor shares to explain firm selection.

Our paper contributes to the literature on trade with firm heterogeneity, which has been pioneered by Bernard et al. (2003) and Melitz (2003). The two papers in this tradition that come closest to ours are Bernard et al. (2007) and Yeaple (2005). Bernard et al. (2007) extend the Melitz (2003) setup by considering two factors of production and, additionally, two monopolistically

[2]For a recent alternative explanation see Atkeson and Burstein (2010).

competitive sectors with different capital–labor ratios in production. In their model – differently from ours – within each sector firms are homogeneous with respect to the capital–labor ratios, while they still differ with respect to TFP. Bernard et al. (2007) thus are able to provide important insights into the inter–industry and intra–industry factor relocations due to trade liberalization. By construction, though, they do not analyze how firm heterogeneity in capital–labor ratios interacts with globalization. This is because, *within* sectors, a firm's export status only depends on its TFP, and not on its factor shares in production.

In Yeaple (2005), on the other hand, firms choose their technology upon market entry. Labor is the only factor of production, but workers differ with respect to their skills. The author assumes that for each technology, a higher skill level leads to higher profits per worker, and similarly a more advanced technology also leads to higher profits for any given skill level of the employee. Because of these monotone relationships, trade liberalization leads to the same type of firm selection as in Melitz (2003): the relative mass of exporters increases, whereas the relative mass of non–exporters decreases. In our setup, on the other hand, firms produce with standard CES technologies with two inputs, and, as a result, we do not have a monotone relationship between factor intensities and profits. While the paper by Yeaple (2005) provides important insights into how trade liberalization affects workers' skill–premia, it does not consider factor share heterogeneity across firms and thus it cannot explain those stylized facts about trade liberalization, which refer to factor market competition.

The remainder of the paper is organized as follows. In section 2 we present evidence on firm selection in the presence of export growth for Chile. Section 3 lies out our model, and in section 4 we solve for the autarkic equilibrium. In section 5 we consider the effects of trade liberalization in a symmetric two–country setting. Section 6 extends the model to N countries, whereas section 7 combines our setting with the standard Melitz–type heterogeneity in TFP. In section 8 we provide an empirical evaluation of our model. Section 9 concludes the paper.

2 Motivation

To introduce our analysis, we use a well–known plant–level dataset of the manufacturing sector of Chile, which has been employed by several previous studies.[3] This dataset covers all manufacturing plants with 10 or more workers and focuses on the period 1990–1999. We choose this period since in this decade the Chilean government signed several free trade agreements, which significantly reduced the trade barriers faced by Chilean exporters.[4] The data come from the Annual Survey of Manufacturing Industries carried out by the National Institute of Statistics of Chile.[5]

The data highlights an interesting pattern which is at odds with the existing literature. While

[3]This dataset has been used, among others, by Pavcnik (2002), Pavcnik (2003) and Kasahara and Rodrigue (2008).

[4]During the 1990s Chile established free trade agreements with Canada, Central America, Mercosur and Mexico. It also signed partial trade liberalization agreements with Argentina, Bolivia, Colombia, Ecuador and Venezuela.

[5]See appendix A for a more detailed description of the dataset.

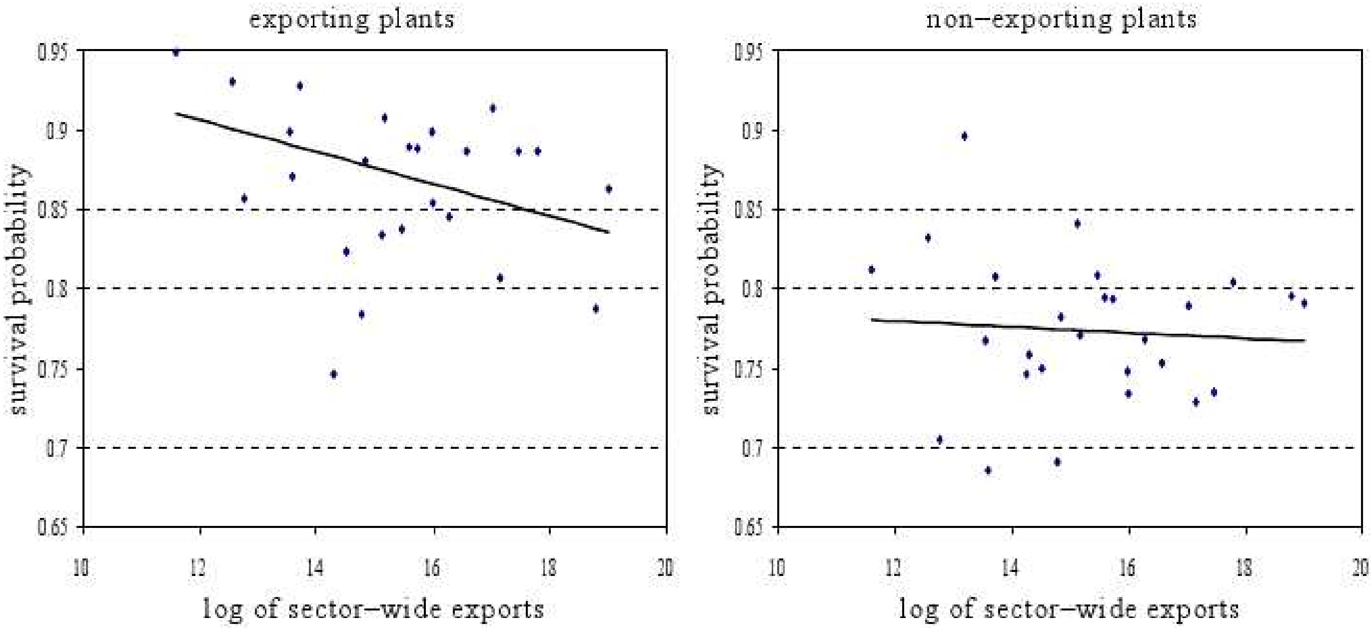

Figure 1: Three–year average survival probability and sectoral exports (1990–1999)

exporting plants are more likely to survive than non–exporting plants (see table A2 in the appendix), figure 1 shows that, on average, exporting firms are less likely to survive, the larger are the sector–wide export volumes. At the same time, non–exporters appear not to be affected.

In order to analyze how export growth affects different types of firms in a more systematic fashion, we estimate the following probit model, separately for exporters and non–exporters:

$$Pr(S_{ij,t+\tau} = 1) = \Phi\left[\beta_1 \log(Exp_{jt}) + \lambda'\Omega_{ijt} + \delta_j + \delta_t\right],$$

where $S_{ij,t+\tau}$ equals one if plant i operating in sector j survived from year t to year $t+\tau$. Φ is the standard normal distribution function, Exp_{jt} measures the exports of sector j in year t, Ω_{ijt} is a vector of plant characteristics that includes size (measured by the log of employment), total factor productivity (in logs),[6] age (in logs), skill intensity,[7] and a set of dummy variables for plants that import intermediate inputs, plants with foreign ownership, and plants that use foreign technology licenses. The variables δ_j and δ_t are respectively 3–digit sector and year fixed effects that control for unobserved heterogeneity at the sector level and over time. Estimating a regression with plant level data, but including sector time–varying variables may underestimate the standard

[6]Total factor productivity is the residual of a regression that estimates a Cobb–Douglas production function for each 3–digit industry using the method proposed by Olley and Pakes (1996) and later modified by Levinsohn and Petrin (2003), which corrects for the simultaneity bias associated with the fact that productivity is not observed by the econometrician, but it may be observed by the firm. In some cases the production functions were estimated at the 2–digit level due to the small number of observations available for some industries at the 3–digit level of disaggregation. We estimated the production function separately for exporters and non–exporters to account for the fact that these two types of firms produce with different factor intensities.

[7]Skill intensity is the ratio between skilled workers' wages and total wages.

errors (Moulton 1990). To correct for this problem, standard errors are clustered at the 3–digit sector–year level. All specifications also include a measure of multinational corporations presence, which is calculated as the fraction of value added accounted by plants with foreign ownership at the 3–digit level.[8] Some specifications also include a measure of the size of the sector (either total employment or total value added).

A positive sign for β_1 would suggest that a firm is more likely to survive τ periods ahead if sector–wide exports increase. The analysis focuses on three–year survival rates ($\tau = 3$), but we have estimated the same specification using one– and five–year survival rates obtaining similar results. Table 1 presents our findings for both exporters and non–exporters. Consistent with previous studies,[9] larger plants, older plants, plants that are more productive, and those that use imported intermediate inputs are more likely to survive. Plants with foreign ownership, on the other hand, are more likely to exit, which is consistent with the findings of Alvarez and Görg (2009). As in Bernard et al. (2006a) the share of total wages paid to skilled workers is negatively correlated with plant survival, but only for the case of non–exporters. The estimates for the dummy for plants that use foreign technology licenses are not statistically significant. Finally, the presence of multinational corporations in the sector does not have any significant effect on survival.

The main variable of interest is the estimate of the effect of sector–wide exports. Table 1 shows that, for the case of exporters, higher export volumes at the sectoral level are negatively correlated with a plant's survival probability. Furthermore, this result is statistically significant at the conventional levels in all specifications and is robust to the inclusion of controls for the sector size (employment and value added). It is possible, however, that the number of exporters (or the respective survival rate) might influence sector–wide exports. If this is the case then the estimates in table 1 may suffer from an endogeneity bias. To address this concern, we instrument exports using a measure of the level of foreign income relevant for each 3–digit sector.[10] The exclusion restriction, in this case, requires foreign income to be correlated with exports but not correlated with any other factors that affect the exporters' survival probability. This assumption is likely to be satisfied as changes in foreign income directly affect the demand for Chilean products and, thus, exports, but do not affect the probability of survival of exporters other than through exports. The instrument, on the other hand, is likely to be correlated with the level of exports. Indeed, the estimate for the instrument in the first stage is positive and significant, and it passes the F–test for the exclusion restriction (see Staiger and Stock 1997). As shown in table A4 in the appendix, this IV procedure confirms our previous results, i.e. that an exporting plant's survival probability

[8] As a robustness check, the analysis also uses inflows of FDI at the 2–digit level. The results are not significantly affected when this alternative measure is used.

[9] See, for example, Dunne, Roberts, and Samuelson (1989), Salvanes and Tveteras (2004) and López (2006).

[10] This is computed as a weighted average of the per capita GDP of the 15 main export destination countries of each sector. The 15 main destination countries in each sector receive the majority of Chilean exports. Their share in total exports of the sector ranges from 81.2% to 99.5%. The average share across all sectors is 92%. See appendix B for details on how this variable is computed.

is negatively correlated with sector–level exports.[11]

This finding is puzzling in the light of the existing theoretical literature, which, following Melitz (2003), has focused on firm heterogeneity in total factor productivity. In fact, in a standard setting á la Melitz, an increase in exports at the sectoral level leads exporting plants to become larger, *without reducing* their number. In other words, exporting plants do not "die" due to the increase in overall export volumes. Instead, the adjustment takes place among the non–exporting plants, among which the least productive ones exit the market. These results hold also in the multi–factor extensions of the Melitz (2003) model, like the one proposed by Bernard et al. (2007), because all firms *within* a sector are assumed to use inputs in the same proportions. Importantly, in our data, also this prediction is not supported. In fact, as shown in the second panel of table 1 (and table A4) non–exporting plants are not affected if sector–wide exports grow.

To account for this remarkable pattern, we need to develop a richer model, which will focus on how competition in factor markets affects the firm selection brought about by increased exports.

3 Model setup

Home's economy is characterized by a representative consumer and a single monopolistically competitive industry. We start by describing the demand side, and proceed then to consider production, focusing first on the technologies available to the firms and then on market entry.

The preferences of the representative consumer are given by a CES utility function of the type

$$U = \left[\int_{\upsilon \in \Upsilon} q(\upsilon)^{\frac{\sigma-1}{\sigma}} d\upsilon \right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

The parameter $\sigma > 1$ is the elasticity of substitution between different varieties, and Υ is the set of available goods, indexed by υ. The representative consumer is endowed with fixed amounts of capital $\overline{K}$ (human or physical) and labor $\overline{L}$, which we assume to be perfectly mobile within a country, but immobile between countries. The consumer's overall income is given by

$$I = w\overline{L} + r\overline{K},$$

where w is the wage rate and r is the return to capital. Utility maximization subject to the budget constraint leads to the demand for each individual variety, which is given by

$$q(\upsilon) = IP^{\sigma-1} p(\upsilon)^{-\sigma}, \tag{2}$$

[11] In an additional specification available upon request, we have also controlled for the direct effect of foreign tariff changes on the survival probability of Chilean manufacturing firms. Including this measure does not affect the sign and significance of our results on export volumes. We have decided not to include tariffs in our benchmark specification as in the case of the preferential agreements signed by Chile during our sample period, reductions in non–tariff barriers have most likely played a bigger role than tariff reductions, but we cannot precisely measure them.

where $P = \left[\int_{\upsilon\in\Upsilon} p(\upsilon)^{1-\sigma} d\upsilon\right]^{\frac{1}{1-\sigma}}$ is the price index, which is dual to the utility function.

Turning to the supply side of the economy, there is a continuum of potentially active firms, each of which produces a different variety of the same good, combining capital K and labor L according to the following CES production function:

$$q(\phi) = \left[\phi^{1-\alpha} K^{\alpha} + (1-\phi)^{1-\alpha} L^{\alpha}\right]^{1/\alpha}, \quad 0 < \alpha < 1 \tag{3}$$

where $q(\phi)$ is the firm's output and $\phi \in [0, 1]$ a factor share parameter characterizing the technology. In the remainder of the paper we index firms by ϕ. The elasticity of substitution between inputs is given by $\varsigma = \frac{1}{1-\alpha}$.

If a firm decides to produce, it faces a fixed production cost and a constant marginal cost $c(\phi)$. The latter is given by

$$c(\phi) = \left[\phi r^{1-\varsigma} + (1-\phi) w^{1-\varsigma}\right]^{1/(1-\varsigma)}.$$

Clearly, as long as $r \neq w$, firms choosing different values of ϕ face different marginal costs. Production requires a fixed cost which takes the following form:

$$F(\phi) = c(\phi) f(\phi). \tag{4}$$

This structure of fixed costs is common in two–factor trade models (e.g., Markusen and Venables 2000) and implies that firms have to pay for the fixed input requirement $f(\phi)$ in terms of their final output.[12] We assume that $f(\phi_i) > f(\phi_j)$ if $\phi_i > \phi_j$, i.e. the more capital intensive is the technology, the higher is the fixed input requirement.

The market entry process takes the following form. Ex–ante, all firms are identical. Market entry is costless. After entry, firms have the choice between two different technologies: a capital intensive technology, characterized by ϕ_K, and a labor intensive technology, characterized by ϕ_L, with $\phi_K > \phi_L$. Firms maximize profits, which are given by

$$\pi(\phi_i) = \frac{I p(\phi_i)^{-\sigma}}{P^{1-\sigma}} \left[p(\phi_i) - c(\phi_i)\right] - c(\phi_i) f(\phi_i), \quad i = K, L, \tag{5}$$

and the resulting output price is given by $p(\phi_i) = \frac{\sigma}{\sigma-1} c(\phi_i)$.

[12] Alternatively, we could assume that firms have to pay for $f(\phi)$ in terms of labor, i.e. $F(\phi) = w f(\phi)$, or in terms of capital, i.e. $F(\phi) = r f(\phi)$. Our results are robust to these alternative specifications of $F(\phi)$.

4 Autarkic equilibrium

We choose labor as the numéraire and set $w = 1$. As a result, r denotes the *relative* price of capital. In equilibrium factor markets clear. Applying Shephard's Lemma, this implies:

$$\overline{L} = \sum_{i=L,K} a_{Li} \left[q(\phi_i) + f_i \right] \eta_i \tag{6}$$

$$\overline{K} = \sum_{i=L,K} a_{Ki} \left[q(\phi_i) + f_i \right] \eta_i . \tag{7}$$

η_i denotes the mass of firms of type i active in the market, whereas the terms $a_{Li} \equiv (1-\phi_i)\, c(\phi_i)^{\varsigma}$ and $a_{Ki} \equiv \phi_i r^{-\varsigma} c(\phi_i)^{\varsigma}$ are, respectively, the unit labor and capital requirements for variety i. Furthermore, let $f(\phi_i) \equiv f_i$ in order to save on notation.

Since market entry is costless, in the autarkic equilibrium *each* firm's profits are driven to zero. As firms can choose among two different technologies, a zero profit condition has to be formulated for each of them separately, and is given by

$$IP^{\sigma-1} p(\phi_i)^{-\sigma} = q(\phi_i) = (\sigma - 1)\, f_i, \quad \text{with} \quad i = L, K. \tag{8}$$

Equations 6, 7 and 8 can be used to perform some comparative statics exercises. For this purpose and for the remainder of our paper we will assume $\varsigma = \sigma$ to simplify the algebra without affecting the thrust of our results. Thus, σ will denote the elasticity of substitution between inputs in production and between varieties in consumption. We start by considering the relationship between firms' production and factor prices:

Lemma 1 *An exogenous increase in the aggregate production of the capital (labor) intensive firms increases (decreases) the relative price of capital r.*

Proof. Dividing equations 6 and 7 by each other, considering that, in general equilibrium, $\frac{q(\phi_i)}{\sigma-1} = f_i$ and some simple transformation leads to:

$$\frac{\overline{L}}{\overline{K}} = \underbrace{\frac{a_{LK} + a_{LL} \frac{q(\phi_L)\eta_L}{q(\phi_K)\eta_K}}{a_{KK} + a_{KL} \frac{q(\phi_L)\eta_L}{q(\phi_K)\eta_K}}}_{\equiv \Theta} . \tag{9}$$

The term Θ denotes relative labor demand in the economy. An increase in aggregate production of capital intensive firms ($q(\phi_K)\eta_K$) has the following impact on Θ:

$$\frac{\partial \Theta}{\partial [q(\phi_K)\eta_K]} = \frac{q(\phi_L)\eta_L (a_{LK}a_{KL} - a_{LL}a_{KK})}{[a_{KK} q(\phi_K)\eta_K + a_{KL} q(\phi_L)\eta_L]^2}. \tag{10}$$

$\frac{\partial \Theta}{\partial [q(\phi_K)\eta_K]} < 0$ since $a_{LK}a_{KL} - a_{LL}a_{KK} < 0$ due to $\phi_K > \phi_L$. Thus, r has to adjust such that Θ becomes larger again. We can show that an increase in r has two positive effects on Θ. First, it increases a_{Li}, while it decreases a_{Ki}, $i = K, L$. Second, it increases $\frac{q(\phi_L)}{q(\phi_K)} = \frac{c(\phi_L)^{-\sigma}}{c(\phi_K)^{-\sigma}}$, which has a positive impact on Θ since $a_{LK}a_{KL} - a_{LL}a_{KK} < 0$. Thus, r has to increase so that factor markets clear again after the increase in $q(\phi_K)\eta_K$. However, if we assume $\sigma = \varsigma$, equation 9 simplifies to:

$$\frac{\overline{L}}{\overline{K}} = \frac{1 - \phi_K + (1 - \phi_L)\frac{\eta_L}{\eta_K}}{\phi_K + \phi_L\frac{\eta_L}{\eta_K}} r^{\sigma}, \tag{11}$$

which illustrates the positive relationship between relative labor demand and r in a more straightforward way. ■

A second comparative static result is given by lemma 2:

Lemma 2 *An increase in the relative price of capital increases the profits of labor intensive firms, while it decreases the profits of capital intensive firms.*

Proof. Substituting the terms for I, P and $p(\phi_K)$ into equation 5 and calculating the partial derivative of $\pi(\phi_K)$ with respect to r leads to:

$$\frac{\partial\pi(\phi_K)}{\partial r} = \frac{\overline{K}(1 - \phi_K) - \overline{L}\phi_K r^{-\sigma}}{\sigma P^{1-\sigma}} + \frac{\left(\overline{L} + r\overline{K}\right)(1 - \sigma)r^{-\sigma}\eta_L\left(\phi_K - \phi_L\right)}{\sigma P^{2-2\sigma}} < 0 \tag{12}$$

$\frac{\partial\pi(\phi_K)}{\partial r}$ is negative since $\frac{\overline{K}}{\overline{L}} < \frac{\phi_K}{1-\phi_K}r^{-\sigma}$, which follows from equation 11, $\phi_K > \phi_L$ and $\sigma > 1$. It can be shown along the same lines that the profits of labor intensive firms increase with r. ■

The intuition for lemma 2 is as follows. An increase in the relative price of capital ceteris paribus increases the relative price of the capital intensive goods. This shifts demand away from capital intensive goods and towards labor intensive ones, leading to higher (lower) profits for the labor (capital) intensive firms.

Using lemma 1 and 2, we are now ready to establish our first proposition.

Proposition 1 *There exists a unique and stable autarkic equilibrium.*

Proof. The proof proceeds in two steps. First, equation 11 shows that the relationship between r and $\frac{\eta_L}{\eta_K}$ as it results from the factor market clearing condition is negative. Second, taking the ratio of the zero profit conditions for capital and labor intensive firms (equation 8) we have

$$\frac{q\left(\phi_K\right)}{q\left(\phi_L\right)} = \frac{\left(\phi_K r^{1-\sigma} + 1 - \phi_K\right)^{-\sigma/(1-\sigma)}}{\left(\phi_L r^{1-\sigma} + 1 - \phi_L\right)^{-\sigma/(1-\sigma)}} = \frac{f_K}{f_L}. \tag{13}$$

Equation 13 can be solved to determine the relative price of capital r in the autarkic equilibrium (subscript a):

$$r_a = \left[\frac{\left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma}(1 - \phi_L) - (1 - \phi_K)}{\phi_K - \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma}\phi_L}\right]^{1/(1-\sigma)}. \tag{14}$$

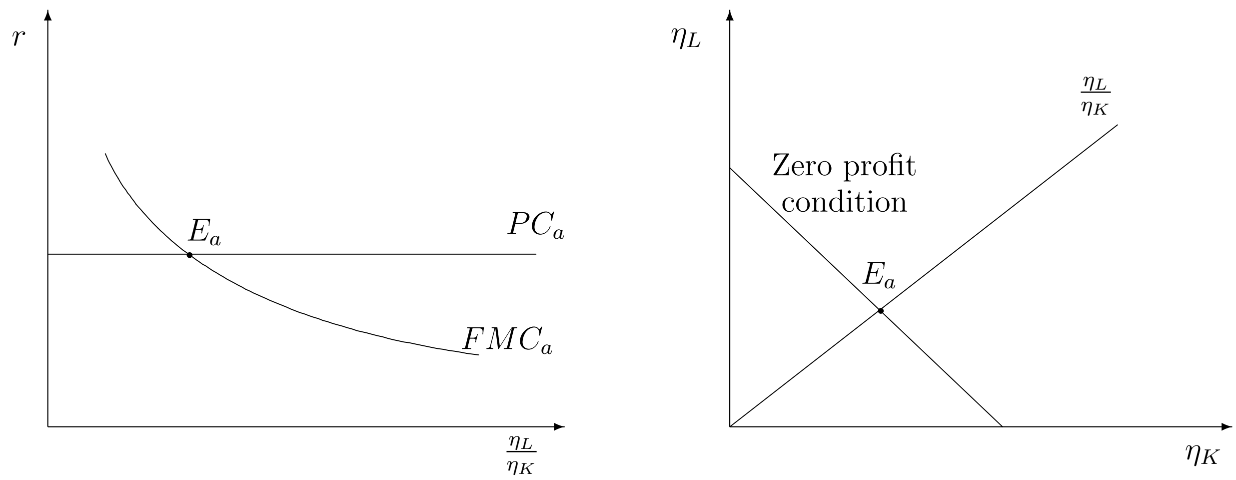

Figure 2: Autarkic equilibrium

Notice that r_a is defined only if $\phi_K - \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma} \phi_L > 0$, i.e. if $\frac{\phi_K}{\phi_L} > \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma}$. If, in contrast, $\frac{\phi_K}{\phi_L} < \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma}$, firms only choose the labor intensive technology. Since we focus on a general equilibrium with both types of firms active, we will consider only the case of $\frac{\phi_K}{\phi_L} > \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma}$ in the following. Furthermore, notice that $r_a < 1$ since $f_K > f_L$. Thus, the capital intensive firms realize higher revenues in general equilibrium, which are used to pay for the higher fixed input requirement f_K. Finally, equation 14 also shows that r_a does not depend on $\frac{\eta_L}{\eta_K}$. Thus, substituting equation 14 into equation 11 we can solve for $\frac{\eta_L}{\eta_K}$. Once r_a and $\frac{\eta_L}{\eta_K}$ are known, we can determine all the remaining variables of the model. ■

In the remainder of the paper, we will refer to equation 11 as the *relative factor market clearing* condition (FMC). Equation 14 determines instead the relative price of capital, given that both types of firms are active, and we will refer to it as the *price of capital* condition (PC). In the left panel of Figure 2, we depict the two curves. Their intersection establishes the relative price of capital r and the relative mass of labor intensive firms $\frac{\eta_L}{\eta_K}$ in the autarkic equilibrium. Once $\frac{\eta_L}{\eta_K}$ has been determined, we can also obtain the absolute number of active firms by using one of the two zero profit conditions. This is done in the right panel of the figure.

Finally, it is useful to determine calculate the average capital share parameter over all active firms $\widetilde{\phi}$:

$$\widetilde{\phi} = \frac{\phi_K \eta_K + \phi_L \eta_L}{\eta_K + \eta_L}$$

Notice that, as shown in appendix C, an industry with $\eta_L + \eta_K$ homogenous firms producing with the capital share parameter $\widetilde{\phi}$ leads to the same aggregate outcomes as an industry with η_L and η_K heterogeneous firms, each producing with the capital share parameters ϕ_L and ϕ_K, respectively.

5 Free trade equilibrium

In this section, we extend our analysis to a two–country setting to study the effect of a bilateral trade liberalization. To keep our analysis simple, we compare the autarkic and the free trade equilibria, and assume zero transport costs.[13]

We study the firm selection in each country, which is due to increased competition on goods and factor markets. The former is induced by the inflow of foreign varieties. The latter is instead the result of increased production by exporters. To provide intuition for our results, we consider first the impact of increased competition on goods markets, and then turn to increased competition on factor markets. Thus, we first focus on how the inflow of foreign varieties influences the mass of the two types of firms, holding factor prices fixed. We then consider the full general equilibrium effects with endogenously determined factor prices. Throughout our discussion we assume that production factors are immobile across countries.

Home and Foreign are assumed to be completely symmetric. Utility maximization in Foreign results in the following demand function for a variety produced in Home:

$$q_F(\phi) = I_F P_F^{\sigma-1} p(\phi)^{-\sigma},$$

where the subscript F denotes variables of Foreign. In order to export, we assume that a domestic firm needs to set up a distribution network, which leads to a fixed export cost given by:

$$F_X = c(\phi) f_X. \tag{15}$$

We make the following assumption on the magnitude of the export cost parameter f_X:

$$I_F P_F^{\sigma-1} p\left(\phi_L\right)^{-\sigma} < f_X(\sigma-1) \qquad \text{and} \qquad I_F P_F^{\sigma-1} p\left(\phi_K\right)^{-\sigma} \geq f_X(\sigma-1). \tag{16}$$

This assumption implies that only capital intensive firms will earn non–negative profits by serving the foreign market, whereas no labor intensive firm will find it optimal to export.[14] Total demand for a domestically produced capital intensive variety increases to

$$q\left(\phi_K\right) + q_F\left(\phi_K\right) = 2IP^{\sigma-1} p\left(\phi_K\right)^{-\sigma} \tag{17}$$

[13]This setting describes well the experience of Chile in the 1990s, since the implementation of an array of free trade agreements can be considered as a non–marginal shock to the economy. Still, the mechanisms we highlight in this section would still be at work if we included strictly positive iceberg transport costs or tariffs which fall marginally.

[14]Remember that profits from exporting are given by $\pi(\phi_i) = I_F P_F^{\sigma-1} p(\phi_i)^{1-\sigma}\sigma^{-1} - c(\phi_i) f_X$ for $i \in \{K, L\}$. Substituting $p(\phi_i) = \frac{\sigma}{\sigma-1} c(\phi_i)$ into this equation leads to the conditions in equation 16. Furthermore, remember that $r < 1$ in equilibrium, which implies $p(\phi_L) > p(\phi_K)$.

and the aggregate price index decreases to

$$P = \left[2\eta_K p\left(\phi_K\right)^{1-\sigma} + \eta_L p\left(\phi_L\right)^{1-\sigma}\right]^{1/(1-\sigma)} \tag{18}$$

following trade liberalization. For labor intensive firms, trade liberalization ceteris paribus does not affect the supply decision and the zero profit condition is still given by 8. On the other hand, trade liberalization affects the supply decision of capital intensive firms and their zero profit condition becomes:

$$2q\left(\phi_K\right) \;=\; (\sigma-1)(f_K+f_X). \tag{19}$$

Dividing equations 19 and 8 by each other and remembering that $q(\phi_i) = IP^{\sigma-1}p(\phi_i)^{-\sigma}$, we can solve for r in the free trade equilibrium (subscript *ft*):

$$r_{ft} \;=\; \left[\frac{\Psi(1-\phi_L)-(1-\phi_K)}{\phi_K-\Psi\phi_L}\right]^{1/(1-\sigma)}, \tag{20}$$

with $\Psi = \left(\frac{f_K+f_X}{2f_L}\right)^{(\sigma-1)/\sigma}$. We will refer to equation 20 as the PC–equation in the free trade equilibrium. Finally, considering also the additional factor demand due to production for exports leads to the following FMC condition with free trade:

$$\frac{\overline{L}}{\overline{\overline{K}}}r^{-\sigma} \;=\; \frac{2(1-\phi_K)+(1-\phi_L)\frac{\eta_L}{\eta_K}}{2\phi_K+\phi_L\frac{\eta_L}{\eta_K}}. \tag{21}$$

Summarizing our results so far we obtain:

Lemma 3 *Compared to autarky, a bilateral trade liberalization has the following consequences:*

i) the aggregate price index P decreases in each country due to the availability of additional varieties from abroad; the decrease in P ceteris paribus decreases the profits of exporting and non–exporting firms and reflects an increase in goods market competition;

ii) capital intensive firms increase their production due to additional profit opportunities abroad;

iii) the relative price of capital r increases due to additional production by capital intensive exporters; the increase in r ceteris paribus decreases the profits of capital intensive firms and increases the profits of labor intensive firms.

Proof. Parts *i)* and *ii)* follow from equations 17 and 18. Part *iii)* follows from lemma 1 and lemma 2. ■

Notice that it is a priori ambiguous whether trade liberalization leads to a firm selection in favor of or against either type of firms, i.e. whether $\frac{\eta_L}{\eta_K}$ increases or decreases. The additional availability of foreign varieties affects both capital and labor intensive firms negatively, and ceteris paribus

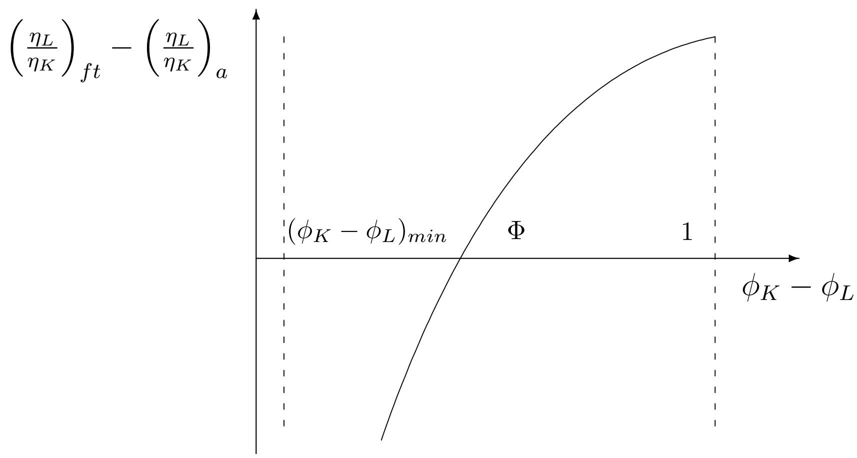

Figure 3: The role of the factor intensity gap

drives both types of firms out of the market. At the same time, the increased profit opportunities abroad affect capital intensive firms positively, ceteris paribus leading to additional entry of this type of firms. Finally, the increased competition on factor markets, which is reflected by the increase in r, affects capital intensive firms negatively and labor intensive firms positively, ceteris paribus leading to exit of capital intensive firms and entry of labor intensive firms.

The net effect of trade liberalization on the two types of firms crucially depends on the difference in capital share parameters $\phi_K - \phi_L$. In the following, we will refer to $\phi_K - \phi_L$ as the *factor intensity gap* between exporters and non–exporters. The factor intensity gap determines (i) the extent to which r increases with trade liberalization and (ii) the extent to which firms are affected by the increase in r, and its role is characterized in the following:

Proposition 2 *There exists a threshold value Φ for the factor intensity gap such that bilateral trade liberalization leads to the following pattern of firm selection:*

i) if $\phi_K - \phi_L > \Phi$, $\frac{\eta_L}{\eta_K}$ increases, i.e. the relative mass of non–exporters increases;

ii) if $\phi_K - \phi_L < \Phi$, $\frac{\eta_L}{\eta_K}$ decreases, i.e. the relative mass of non–exporters decreases.

In general, the larger is $\phi_K - \phi_L$, the more detrimental (beneficial) is trade liberalization for a single exporting (non–exporting) firm.

Proof. See appendix D. ■

Figure 3 illustrates the firm selection with trade liberalization. $(\frac{\eta_L}{\eta_K})_{ft}$ stands for the relative mass of labor intensive firms under free trade, while $(\frac{\eta_L}{\eta_K})_a$ stands for the relative mass of labor intensive firms under autarky. The minimum technological difference, which is denoted by $(\phi_K - \phi_L)_{min}$, is defined as that difference $\phi_K - \phi_L$, which leads to $(\eta_K)_a = 0$. In appendix D we prove that the relationship between $(\frac{\eta_L}{\eta_K})_{ft} - (\frac{\eta_L}{\eta_K})_a$ and $\phi_K - \phi_L$ is as illustrated by figure 3.

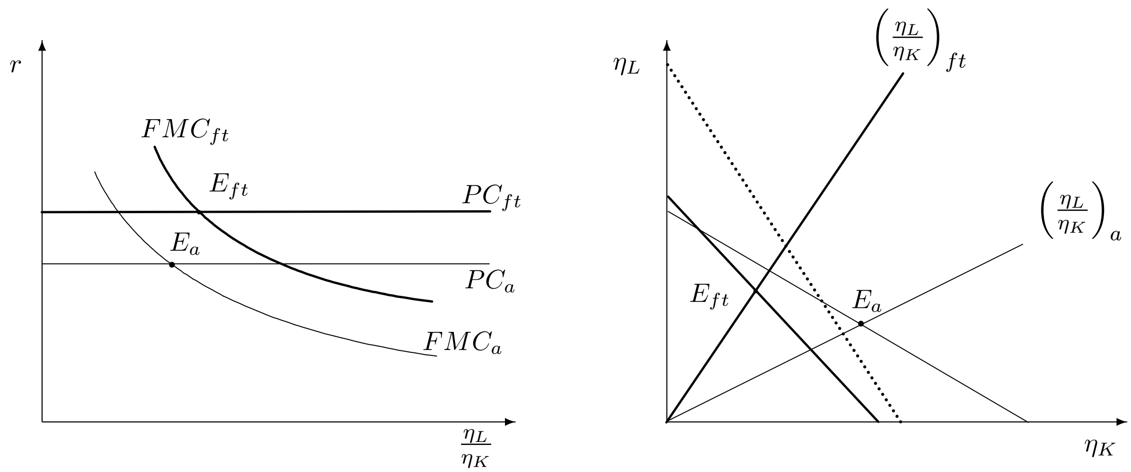

Figure 4: Trade liberalization

The intuition behind proposition 2 is as follows. First, the increase in r brought about by trade liberalization is larger, the larger is the difference $\phi_K - \phi_L$. Second, for a given increase in the relative price of capital r the losses (gains) for the capital (labor) intensive firms are larger, the larger is $\phi_K - \phi_L$. Thus, we can conclude that labor (capital) intensive firms will unambiguously gain (lose) from trade liberalization and firms of this type will enter (exit) the market if the factor intensity gap is sufficiently large.

Figure 4 illustrates the effect of trade liberalization on the mass of firms active in equilibrium. The left panel shows that, starting from the autarkic equilibrium E_a, trade liberalization shifts the PC curve upwards. This results from new profit opportunities abroad for capital intensive firms, which requires an increase in the relative price of capital r for the free entry conditions to hold again. Trade liberalization also leads to increased competition in factor markets, which shifts the FMC curve upwards. In fact, if relative demand for capital increases, the relative price of this factor must also increase to re–establish factor market clearing. The free trade equilibrium is illustrated by point E_{ft}, which, consistently with the empirical evidence discussed in section 2, is drawn such that the relative mass of capital intensive firms decreases.

The right panel of the same figure captures also the role played by the increased availability of foreign varieties. We keep factor prices constant for the moment in order to separate the effects of increased factor market competition from those of increased goods market competition. Starting from the autarkic equilibrium E_a, increased availability of foreign varieties and new profit opportunities abroad make the line illustrating the zero profit conditions for capital intensive firms shift inwards and become steeper (dotted line). Allowing factor prices to adjust (r increases) flattens the curve and makes it shift inwards. The new equilibrium point is indicated by E_{ft}. In general, the mass of capital intensive firms η_K decreases, whereas η_L can increase or decrease.[15]

Importantly, notice that in our model a capital intensive firm will never react to the increase

[15] Appendix E formally derives the shifts of the zero profit condition in the right panel.

in the relative price of capital by exiting the foreign market, while still serving the domestic one. If factor market competition is sufficiently strong so that some exporting firms leave the foreign market, these firms will cease production completely, i.e. they will also exit the domestic market. The reason for this result lies in our market entry procedure. Since firms do not have any uncertainty about their technology and market entry is free, each firm realizes zero profits on the domestic market in the autarkic equilibrium, i.e. $\pi(\phi) = 0$ (see equation 8). Thus, an increase in the relative price of capital due to trade liberalization negatively affects the profits of capital intensive firms in the domestic market, and only if the firm is able to make positive profits from exporting, it might be able to survive.

This finding is in contrast with the standard results in the literature (see Melitz 2003, among others). In these models an increase in sector–wide exports increases the wage rate, which decreases profits of all firms proportionately and leads the least productive firms to exit the market, whereas the marginal exporting firms become non–exporters.

It is interesting to determine the effect of trade liberalization on the industry–wide average capital share parameter $\widetilde{\phi}$. This is done in the following:

Proposition 3 *Compared to autarky, trade liberalization leads to an increase in the average industry–wide capital share parameter.*

Proof. See appendix F . ■

6 The N country case

We now extend our analysis to the case of $N \geq 2$ symmetric countries, which are freely trading among each other.[16] We focus on a trade liberalization experiment that involves all countries simultaneously.

Compared to the two–country case, the aggregate output of a capital intensive firm now increases to

$$Nq(\phi_K) = NIP^{\sigma-1} \left[p(\phi_K)\right]^{-\sigma}, \quad N \geq 2,$$

with trade liberalization. The zero profit condition for a capital intensive firm is now given by

$$q(\phi_K) = \frac{(\sigma-1)[f_K + (N-1)f_X]}{N}, \tag{22}$$

whereas the corresponding condition for labor intensive firms is still given by equation 8. Dividing equation 22 by equation 8 and solving for the relative price of capital, we obtain the N country version of the free trade PC–curve:

$$r_{ft} = \left[\frac{\Xi(1-\phi_L) - (1-\phi_K)}{\phi_K - \Xi\phi_L}\right]^{1/(1-\sigma)},$$

[16] Notice that increasing the size of the single trading partner, which we have considered before, would lead to similar results.

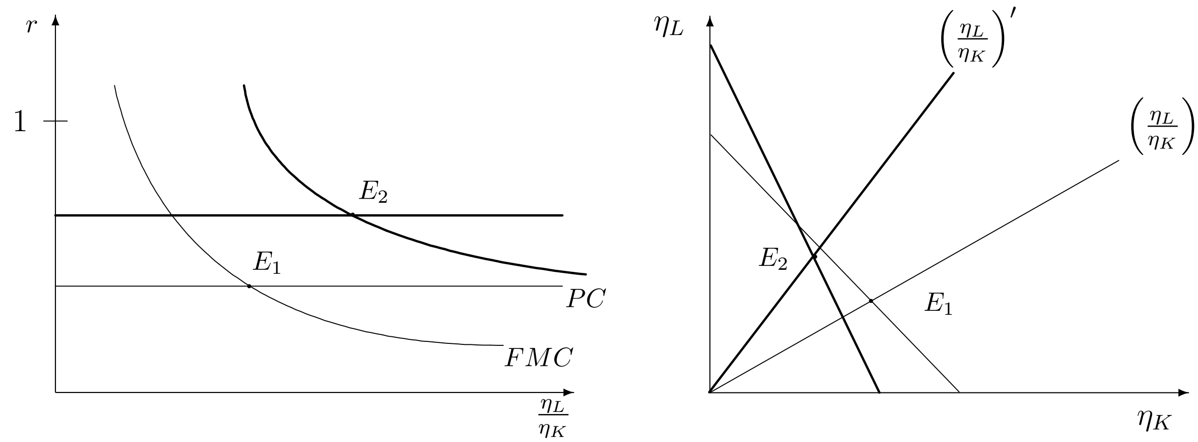

Figure 5: The effect of an increase in N

with $\Xi = \left[\frac{f_K+(N-1)f_X}{f_L}\frac{1}{N}\right]^{(\sigma-1)/\sigma}$. The relative factor market clearing condition (FMC) for the N country case can be solved directly for $\frac{\eta_L}{\eta_K}$:

$$\frac{\eta_L}{\eta_K} = \frac{1-\phi_K - r^{-\sigma}\frac{\overline{L}}{\overline{K}}\phi_K}{r^{-\sigma}\frac{\overline{L}}{\overline{K}}\phi_L - (1-\phi_L)} N. \tag{23}$$

We can now study the effect of an increase in N on the firm selection induced by trade liberalization, starting from the initial equilibrium E_1 (see figure 5). Consider the PC–curve. It is straightforward to show that as N increases it shifts upwards (the thicker black line). Intuitively, since N ceteris paribus increases the profits of capital intensive firms, r has to increase as well for the zero profit condition of capital intensive firms to hold. Remember from lemma 2 that the capital intensive firms' profits decrease as r increases. Furthermore, in the limit, as N approaches infinity, r converges to the following value

$$\overline{r}_{ft} = \left[\frac{\left(\frac{f_X}{f_L}\right)^{(\sigma-1)/\sigma}(1-\phi_L)-(1-\phi_K)}{\phi_K - \left(\frac{f_X}{f_L}\right)^{(\sigma-1)/\sigma}\phi_L}\right]^{1/(1-\sigma)},$$

and $\overline{r}_{ft} < 1$ since $f_X > f_L$.

Turning now to the FMC–curve, as N becomes larger, the curve shifts rightward, i.e. $\frac{\eta_L}{\eta_K}$ increases for a given r (see equation 23). This is because as the number of trading partners becomes larger, aggregate relative capital demand ceteris paribus increases. Thus, the new equilibrium is given by E_2. Importantly, in equilibrium the relationship between $\frac{\eta_L}{\eta_K}$ and N is linear. Thus, if N goes to infinity, $\frac{\eta_L}{\eta_K}$ goes to infinity as well.[17]

[17] Notice that r is bounded from above by 1. Thus, if N is sufficiently large, trade liberalization always increases

Consider now the right panel of figure 5. An increase in the number of trading partners N shifts the zero profit condition further to the left and the curve becomes steeper. Thus, we can summarize our main finding for the N country case in the following proposition.

Proposition 4 *As the number N of trading partners becomes sufficiently large, trade liberalization always leads to an increase in the relative mass of non–exporting firms $\frac{\eta_L}{\eta_K}$.*

Proof. See appendix G. ■

7 Adding heterogeneity in TFP

A large empirical literature has documented the existence of substantial firm heterogeneity in TFP within a narrowly defined sector (Bernard and Jensen 1995 and Alvarez and López 2005, among others), and thus it is important to study how heterogeneity in factor shares interacts with heterogeneity in TFP in shaping firm selection with trade liberalization. To keep the analysis general, we focus on the case of $N \geq 2$ symmetric trading partners in the free trade situation.

To incorporate firm heterogeneity in TFP we follow Melitz (2003) and modify the market entry procedure. In particular, after having chosen the technology parameter ϕ_L or ϕ_K and to actually enter the market, firms have to pay a sunk market entry fee f_E. Payment of this allows firms to draw their TFP parameter A from a common and exogenously given probability distribution with support $[1, \infty)$, density $g(A)$ and cumulative density $G(A)$.[18] Since the random TFP parameter reflects a firm's uncertainty about, e.g., how well workers perform or how consumers evaluate a variety, it is reasonable to assume that a firm learns its TFP after it has chosen its capital share parameter. The production function of a firm with capital share parameter ϕ is now given by:

$$q(\phi, A) \;=\; A\left[\phi^{1-\alpha}K^{\alpha} + (1-\phi)^{1-\alpha}L^{\alpha}\right]^{1/\alpha}, \quad 0 \,<\, \alpha \,<\, 1.$$

The corresponding marginal cost function $c(\phi, A)$ results as:

$$c\left(\phi, A\right) = \frac{1}{A}\left(\phi r^{1-\sigma} + 1 - \phi\right)^{1/(1-\sigma)}, \quad \sigma \,>\, 1.$$

As in Melitz (2003), we assume that TFP does not influence the fixed cost. We therefore choose the following specification

$$F(\phi) \;=\; Ac(\phi_i, A)f_i \;=\; \left(\phi_i r^{1-\sigma} + 1 - \phi_i\right)^{1/(1-\sigma)} f_i, \quad i \,=\, K,\, L.$$

$\frac{\eta_L}{\eta_K}$, irrespective of the magnitude of the factor intensity gap $\phi_K - \phi_L$. In other words, an increase in N shifts the concave curve in figure 3 downwards.

[18] Notice that without a sunk entry fee firms could enter and exit the market costlessly, and thus draw their productivity parameter repeatedly until they obtain the highest possible productivity level.

A firm's current period profits can then be expressed as

$$\pi\left(\phi_i, A\right) = \frac{Ip\left(\phi_i, A\right)^{1-\sigma}}{P^{1-\sigma}\sigma} - Ac\left(\phi_i, A\right) f_i.$$

Again, we assume that firms pay for f_E with their final output, so that the sunk market entry costs for a firm with capital share parameter ϕ_i, $i = L, K$, are given by:

$$F_E\left(\phi_i\right) \;\; = \;\; Ac\left(\phi_i, A\right) f_E \; = \; \left(\phi_i r^{1-\sigma} + 1 - \phi_i\right)^{1/(1-\sigma)} f_E.$$

Notice that TFP does not affect the sunk entry cost either. We can now define the minimum productivity level A_i^*, such that a firm starts producing. A_i^* is determined by the following zero cutoff profit condition:

$$IP^{\sigma-1}p\left(\phi_i, A_i^*\right)^{-\sigma} = q\left(A_i^*, \phi_i\right) \quad = \quad A_i^*(\sigma-1)f_i, \quad i = K, L. \tag{24}$$

Given the threshold TFP parameter A_i^*, free entry implies that the ex–ante expected profits from market entry are equal to zero. Thus, the free entry condition can be written as follows:

$$[1 - G(A_i^*)] \int_{A_i^*}^{\infty} \pi(\phi_i, A)\mu(A)dA = F_E(\phi_i), \quad i = L, K, \tag{25}$$

where $\mu(A) = \frac{g(A)}{1-G(A_i^*)}$. The first term on the left hand side of equation 25 represents the probability that a firm of type i starts producing after entry. The second term describes the average profits of active firms. The term on the right hand side represents the sunk entry cost.

The following lemma characterizes the threshold TFP parameter in the autarkic equilibrium:

Lemma 4 *The threshold TFP parameter $A_{a,i}^*$ in the autarkic equilibrium is given by the solution to the following equation*

$$\left[1 - G\left(A_{a,i}^*\right)\right] \left[\left(\frac{\widetilde{A}_{a,i}}{A_{a,i}^*}\right)^{\sigma-1} - 1\right] \;\; = \;\; \frac{f_E}{f_i}, \qquad i \; = \; L, \; K, \tag{26}$$

where $\left(\frac{1}{\widetilde{A}_{a,i}}\right)^{1-\sigma} \equiv \int_{A_{a,i}^*}^{\infty} \left(\frac{1}{A}\right)^{1-\sigma} \mu(A)dA.$

Proof. See appendix H. ■

Notice that $A_{a,i}^*$ depends only on σ, f_E, f_i and $g(A)$. Thus, $A_{a,i}^*$ is independent from $A_{a,j}^*$, $i \neq j$. To determine the autarkic equilibrium, we proceed as in section 4, and construct the modified version of the price of capital curve (PC) and the factor market clearing condition (FMC). To derive the autarkic PC–curve under the presence of firm heterogeneity in TFP, we take the ratio of the zero cutoff profit conditions for the two types of firms (equation 24), and solve this for r_a:

$$r_a = \left[\frac{\left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma} \left(\frac{A^*_{a,K}}{A^*_{a,L}}\right)^{(1-\sigma)^2/-\sigma} (1-\phi_L) - (1-\phi_K)}{\phi_K - \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma} \left(\frac{A^*_{a,K}}{A^*_{a,L}}\right)^{(1-\sigma)^2/-\sigma} \phi_L} \right]^{1/(1-\sigma)}. \tag{27}$$

Notice that if $A^*_{a,K} = A^*_{a,L}$ equation 27 simplifies to equation 14, i.e. we are back to our standard case. To derive the FMC condition, we need to consider that, compared to the baseline model, an increase in productivity decreases the unit factor requirements, whereas it increases aggregate output since the price of each variety declines. The modified FMC condition becomes:

$$\frac{\overline{L}}{\overline{K}} = \frac{(1-\phi_K)\,\widetilde{A}^{\sigma-1}_{a,K} + (1-\phi_L)\,\widetilde{A}^{\sigma-1}_{a,L}\frac{\eta_L}{\eta_K}}{\phi_K r^{-\sigma}\widetilde{A}^{\sigma-1}_{a,K} + \phi_L r^{-\sigma}\widetilde{A}^{\sigma-1}_{a,L}\frac{\eta_L}{\eta_K}}. \tag{28}$$

and we refer the reader to appendix I for the derivation. Combining the PC and the FMC conditions we can determine the autarkic equilibrium, which is characterized in the following

Proposition 5 *There exists a unique, stable autarkic equilibrium with firm heterogeneity in factor shares and TFP.*

Proof. See appendix J. ■

We are now ready to determine the industry–wide average capital share parameter $\widetilde{\phi}$ and the industry–wide average TFP parameter $\widetilde{A}$ in the autarkic equilibrium (subscript a):

$$\widetilde{\phi}_a = \frac{\phi_K \widetilde{A}^{\sigma-1}_{a,K} + \phi_L \widetilde{A}^{\sigma-1}_{a,L}(\frac{\eta_L}{\eta_K})_a}{\widetilde{A}^{\sigma-1}_{a,K} + \widetilde{A}^{\sigma-1}_{a,L}(\frac{\eta_L}{\eta_K})_a}, \qquad \widetilde{A}_a = \left[\frac{\widetilde{A}^{\sigma-1}_{a,K} + \widetilde{A}^{\sigma-1}_{a,K}(\frac{\eta_L}{\eta_K})_a}{1+(\frac{\eta_L}{\eta_K})_a}\right]^{1/(\sigma-1)}.$$

Notice that an industry with $\eta_L + \eta_K$ homogeneous firms, each producing with technology parameters $\widetilde{\phi}_a$ and $\widetilde{A}_a$, leads to the same aggregate outcome as an industry with η_L and η_K heterogeneous firms, each producing with parameters ϕ_L, $\widetilde{A}_{a,L}$, ϕ_K and $\widetilde{A}_{a,K}$, respectively (see appendix K).

Now, we consider the effects of a multilateral trade liberalization (i.e. a movement from autarky to free trade) among N countries. We assume that the fixed exporting cost f_X is such that $f_X \geq f_K$. We can now determine a second threshold value A^*_{Xi}, which represents the minimum productivity level that enables a firm to serve the N foreign markets after liberalization. This threshold is determined by the following zero cutoff profit condition:

$$I_F P_F^{\sigma-1} p(\phi_i, A^*_{Xi})^{-\sigma} = q(\phi_i, A^*_{Xi}) = A^*_{Xi}(\sigma-1)f_X. \tag{29}$$

Equation 29 implies the following. First, $A^*_{Xi} \geq A^*_i$ since $f_X \geq f_i$. Thus, not all firms which serve the domestic market export as well. Second, $A^*_{XL} > A^*_{XK}$, i.e. labor intensive firms need a higher productivity level in order to be able to export, compared to capital intensive firms. This follows from the fact that $p(\phi_K, A) < p(\phi_L, A)$ for any given TFP parameter A. Thus, a higher

TFP parameter has to compensate for the otherwise higher marginal costs of labor intensive firms. Finally, dividing equations 24 and 29 by each other and solving for A^*_{Xi} leads to:

$$A^*_{Xi} \;=\; A^*_i \left(\frac{f_i}{f_X}\right)^{1/(1-\sigma)} . \tag{30}$$

The free entry condition has to be modified to account for the additional ex–ante expected export profits, and becomes

$$[1-G(A^*_i)] \int_{A^*_i}^{\infty} \pi(\phi_i, A)\mu(A)dA + (N-1)\left[1-G(A^*_{Xi})\right] \int_{A^*_{Xi}}^{\infty} \pi_X(\phi_i, A)\mu_X(A)dA = F_E(\phi_i),$$

where $\mu_X(A) = \frac{g(A)}{1-G(A^*_{Xi})}$. The term $1-G(A^*_{Xi})$ stands for the probability that a firm of type i will be exporting after market entry. The term $\int_{A^*_{Xi}}^{\infty} \pi(\phi_i, A)\mu_X(A)dA$ stands for the average export profits over all exporting firms. Notice that $1-G(A^*_{XK}) > 1-G(A^*_{XL})$ since $A^*_{XL} > A^*_{XL}$. Thus, if a firm has chosen a capital intensive technology, it is more likely to become an exporter, compared to having chosen a labor intensive technology.

Lemma 5 characterizes the threshold TFP parameter for firms of type i, $i = L, K$, in the free trade equilibrium and the impact of trade liberalization on the threshold TFP parameter:

Lemma 5 *The threshold TFP parameter $A^*_{ft,i}$, $i = L, K$, in the free trade equilibrium is given by the solution to the following equation*

$$\left[1-G\left(A^*_{ft,i}\right)\right]\left[\left(\frac{\widetilde{A}_{ft,i}}{A^*_{ft,i}}\right)^{\sigma-1} - 1\right] + (N-1)\left[1-G\left(A^*_{Xi}\right)\right]\left[\left(\frac{\widetilde{A}_{Xi}}{A^*_{Xi}}\right)^{\sigma-1} - 1\right]\frac{f_X}{f_i} \;=\; \frac{f_E}{f_i}, \tag{31}$$

where $\left(\frac{1}{\widetilde{A}_{Xi}}\right)^{1-\sigma} \equiv \int_{A^*_{Xi}}^{\infty} \left(\frac{1}{A}\right)^{1-\sigma} \mu_X(A)dA$. *Trade liberalization increases $A^*_{ft,i}$.*

Proof. See appendix L. ■

Furthermore, in the following we will assume that the TFP parameter follows a Pareto distribution with density $g(A) = \frac{k}{A^{k+1}}$ and shape parameter $k \geq \sigma - 1$.[19] Thus, we can formulate lemma 6:

Lemma 6 *The increase in A^*_K due to trade liberalization is larger than the increase in A^*_L. Furthermore, the increase in $\widetilde{A}_K$ due to trade liberalization is larger than the increases in $\widetilde{A}_L$.*

Proof. See appendix M. ■

To understand the intuition behind lemma 5 and 6, notice that trade liberalization increases ex–ante expected profits and thus triggers additional entry of both types of firms. Competition becomes stronger, which implies that only the more productive firms of either type will survive.

[19]Notice that the assumption $k \geq \sigma - 1$ guarantees that the average TFP parameters $\widetilde{A}_i^{\sigma-1}$ and $\widetilde{A}_{Xi}^{\sigma-1}$ are defined. Axtell (2001) and Luttmer (2007), among others, have shown that a Pareto–distribution describes appropriately the distribution of TFP across firms in manufacturing.

Since the share of exporters among capital intensive firms is larger, new entry of capital intensive firms exceeds new entry of labor intensive firms. Thus, the TFP improvement among capital intensive firms is larger than the TFP improvement among labor intensive firms.

Notice though that the PC–curve is only indirectly affected by trade liberalization since it has been derived from the zero cutoff profit condition for the supply to the *domestic* market. The increase in the ratio of TFP thresholds $\frac{A_K^*}{A_L^*}$ due to trade liberalization will shift the PC–curve upwards.[20] As for the relative factor market clearing condition, following trade liberalization it takes the following form:

$$\frac{\overline{L}}{\overline{K}} r^{-\sigma} = \frac{(1-\phi_K)\widetilde{A}_{ft,K}^{\sigma-1} + (1-\phi_L)\frac{1+s_L}{1+s_K}\widetilde{A}_{ft,L}^{\sigma-1}\frac{\eta_L}{\eta_K}}{\phi_K \widetilde{A}_{ft,K}^{\sigma-1} + \phi_L \frac{1+s_L}{1+s_K}\widetilde{A}_{ft,L}^{\sigma-1}\frac{\eta_L}{\eta_K}}, \tag{32}$$

with $s_i \equiv \frac{1-G(A_{Xi}^*)}{1-G(A_i^*)}$ denoting the share of exporters among firms of type i. Notice that trade liberalization increases $\widetilde{A}_K^{\sigma-1}$ relative to $\widetilde{A}_L^{\sigma-1}$, while $\frac{1+s_L}{1+s_K} < 1$. Thus, relative capital demand ceteris paribus increases, which shifts the FMC–curve to the right.

The extent of the factor relocation between capital and labor intensive firms depends, as in section 5, on the factor intensity gap $\phi_K - \phi_L$ between exporters and non–exporters. We can show that $\frac{\eta_L}{\eta_K}$ increases (decreases) with trade liberalization if $\phi_K - \phi_L$ is at its maximum (minimum) level. Furthermore, trade liberalization is more detrimental (beneficial) for the capital intensive (labor intensive) firms, the larger is $\phi_K - \phi_L$ (see appendix N).

Finally, the industry–wide average technology parameters in the free trade equilibrium are given by (see appendix O):

$$\widetilde{\phi}_{ft} = \frac{\phi_K \widetilde{A}_{ft,K}^{\sigma-1} + \phi_L \frac{1+s_L}{1+s_K}\widetilde{A}_{ft,L}^{\sigma-1}(\frac{\eta_L}{\eta_K})_{ft}}{\widetilde{A}_{ft,K}^{\sigma-1} + \frac{1+s_L}{1+s_K}\widetilde{A}_{ft,L}^{\sigma-1}(\frac{\eta_L}{\eta_K})_{ft}}, \qquad \widetilde{A}_{ft} = \left[\frac{\widetilde{A}_{ft,K}^{\sigma-1} + \frac{1+s_L}{1+s_K}\widetilde{A}_{ft,L}^{\sigma-1}(\frac{\eta_L}{\eta_K})_{ft}}{1+\frac{1+s_L}{1+s_K}(\frac{\eta_L}{\eta_K})_{ft}}\right]^{1/(\sigma-1)}.$$

Comparing $\widetilde{A}_a$ with $\widetilde{A}_{ft}$ leads to proposition 6

Proposition 6 *Trade liberalization increases the sector–wide average TFP parameter $\widetilde{A}$. The increase in $\widetilde{A}$ is larger, the smaller is the factor intensity gap $\phi_K - \phi_L$.*

Proof. See appendix P. ■

The intuition for proposition 6 is as follows: on the one hand, as shown in lemma 5, the increase in A_i^* and $\widetilde{A}_i$, $i = L, K$, does not depend on the factor intensity gap $\phi_K - \phi_L$. On the other hand, proposition 2 has shown that the factor relocation between capital and labor intensive firms depends on the factor intensity gap. Since the increase in $\widetilde{A}_L$ with trade liberalization is smaller than the increase in $\widetilde{A}_K$, an increase (a decrease) in $\frac{\eta_L}{\eta_K}$ moderates (strengthens) the positive TFP effect of trade liberalization.

[20] This follows from equation 27.

The theoretical models that have built upon Melitz's (2003) pioneering contribution, have emphasized the positive TFP effect of trade liberalization. At the same time, recent empirical evidence (Lawless and Whelan 2008) has suggested that these effects might be only moderate. Our analysis suggests that, in the presence of heterogeneity in factor shares, the increase in factor market competition might actually dampen the increase in average TFP brought about by trade liberalization, by forcing some of the capital intensive firms out of the market. Looking at factor markets is thus crucial to gain a more nuanced understanding of the firm selection process.

8 Additional evidence

Having highlighted the role of heterogeneity in factor input shares for firm selection, we now return to the data to determine whether the channels we have identified in the theoretical analysis do indeed play a role. In particular, we will focus on Propositions 2 and 6, which summarize the core of our findings. Thus, we will study how export growth and the factor intensity gap between exporters and non–exporters interact in shaping firm selection. In our empirical implementation we focus on differences in skill (human capital) intensities across firms.[21]

Proposition 2 suggests that, the larger is the factor intensity gap between exporters and non–exporters, the more adverse is the effect of an increase in sector–wide exports on the probability of survival for exporters. Non–exporters, on the other hand, should not be affected significantly. In order to assess this prediction, we first compute a measure of skill intensity for each plant as the share of skilled wages in the total wage bill.[22] Since in our theoretical model we have only two types of firms, we construct the empirical counterpart to it by calculating the difference between the skill intensity of the median exporter and the skill intensity of the median non–exporter in each 3–digit ISIC sector and year. We call this difference the sector skill gap. Next, we divide the 3–digit sectors into two groups: those that have a sector skill gap above the median across all sectors and those that fall instead below the median. We then define a dummy variable equal to one for sectors whose skill gap is above the median, and interact this variable with the aggregate exports of that sector. A negative and statistically significant estimate for the interaction term in the regression for exporters would support the predictions of our model.

The first three columns of Table 2 present the results of including the interaction term on the 3–year survival probability of exporting plants.[23] In all specifications, the impact of exports on survival probability is still negative and significant. The dummy for high sector skill gap is positive and significant, whereas the estimate for the interaction term is negative and statistically significant in all cases. This implies that an increase in exports reduces the exporters' survival

[21] Using physical capital intensities instead does not affect the direction of our results.

[22] This measure has been used, among others, by Pavcnik (2003), Bernard, Jensen, and Schott (2006a), and Alvarez and López (2009).

[23] To calculate the marginal effect of interaction variables, we have used the *inteff* command in Stata, as suggested by Ai, Norton, and Wang (2004). Looking at 1– and 5–year survival probabilities leads to similar results, which are available upon request.

probability, and the effect is larger in sectors in which the skill intensity gap between exporters and non–exporters is larger. The same effect is, however, not found among non–exporters, as shown in columns 4–6. In this case, the sign of the interaction term is either positive or negative. Still, it is either similar or smaller in magnitude and of the opposite sign than the estimate for the direct effect of exports, which implies that the negative effect of the interaction term and the positive estimate for exports cancel out. In other words, this confirms that export volumes do not affect the probability of survival of non–exporters.

A second important prediction of our model follows from our analysis of the interaction between heterogeneity in TFP and heterogeneity in factor shares. In particular, proposition 6 has shown that an increase in exports should increase sector–wide average productivity by less if the skill intensity gap between capital and labor intensive firms is large. To assess this hypothesis, we estimate the effect of exports on productivity at the sector level, by including an interaction term between exports and the sector skill gap dummy defined above. Our measure of sector j average productivity at time t, TFP_{jt} is a weighted average of plant–level productivity, where weights are the share of the plant in industry output:[24]

$$TFP_{jt} = \sum_{i=1}^{\eta_{jt}} s_{ijt} TFP_{ijt},$$

The s_{ijt} term represents plant i's share in total output at time t, TFP_{ijt} is total factor productivity of plant i at time t, and η_{jt} is the number of plants in industry j at time t. To assess the importance of factor relocation between firms on sector–wide TFP we follow Olley and Pakes (1996) and Pavcnik (2002) and decompose TFP into two elements: the unweighted mean of productivity and a covariance term between productivity and output:

$$TFP_{jt} = \overline{TFP}_{jt} + \sum_{i=1}^{\eta_{jt}} \Delta s_{ijt} \Delta TFP_{ijt},$$

where $\Delta s_{ijt} = s_{ijt} - \overline{s}_{jt}$, and $\Delta TFP_{ijt} = TFP_{ijt} - \overline{TFP}_{jt}$, with $\overline{s}_{jt}$ and $\overline{TFP}_{jt}$ representing unweighted mean market share and unweighted mean productivity respectively. The covariance term represents the contribution to the aggregate weighted productivity resulting from the reallocation of market shares and resources across plants of different productivity levels.

In order to control for unobserved heterogeneity at the industry level and for common shocks that may have affected all sectors, we include 3–digit level sector and year dummy variables.[25] To avoid potential simultaneity problems, exports are included lagged one period. The results are presented in table 3. The first column suggests that an increase in exports increases TFP. By looking at column 3, we see though that over a third of this increase is driven by the reallocation

[24] Remember that we calculate TFP separately for exporters and non–exporters.

[25] Including additional control variables, such as the share of MNC in total output, the size of the sector, and the skill intensity of the sector does not affect the results in any significant way.

of resources towards the more productive firms. The effect, however, varies across sectors. To see this, notice how the estimate for the interaction term between exports and the dummy for sectors with high skill gap is negative and significant in column 1. This finding is completely explained by the negative effect on the covariance term in column 3, which suggests that an increase in the volume of exports generates a smaller reallocation of resources toward the more productive firms in sectors in which the skill gap between capital and labor intensive firms is high. This result is consistent with our theoretical model and highlights the importance of the channels we have identified.

9 Conclusions

In this paper, we have began our analysis by documenting how Chilean exporters are less likely to survive than non–exporters in the presence of export growth. We have argued that this stylized fact is a puzzle from the point of view of the existing theoretical literature, and to address it we have developed a new theoretical framework, in which the main driver of firm heterogeneity is given by differences in factor input ratios across firms.

We have obtained several results. First, in a setting in which capital intensive firms have higher fixed production costs and exporting leads to fixed costs, only the more capital intensive firms can afford to serve the foreign market after trade liberalization. Second, an increase in sector–wide exports increases competition for capital, and its relative price. This reduces the profits of capital intensive exporters, and increases those of labor intensive non–exporters. As a result, some of the exporters will have to cease production. This effect is stronger, the bigger is the difference in factor intensities between exporters and non–exporters.

Next, we have extended our analysis to include heterogeneity in TFP á la Melitz (2003), and have studied how the two sources of heterogeneity interact in shaping firm selection. We have shown that trade liberalization always increases sector–wide TFP, but that the size of the effect is negatively related to the difference in factor input ratios between capital and labor intensive firms.

Last, we have assessed two important predictions of our model using our Chilean firm–level dataset. Not only have we found broad support for theoretical analysis, but we have also been able to verify that the main channels we have identified do play a key role in explaining the observed firm dynamics in Chile. Thus, our paper highlights the importance of taking into account heterogeneity in factor input ratios to explain firm dynamics.

Appendix

A Description of the dataset

The dataset includes variables such as sales, value added, employment, wages, exports, imports of intermediate inputs, industry affiliation (ISIC Rev. 2),[26] and other plants' characteristics.[27] Each plant has a unique identification code which allows the researcher to follow it over time. Table $A1$ shows the number of plants according to their export status. There is an average of 4911 plants during the period. About 21 percent of them are exporters, while the rest only produces for the domestic market. Table $A2$ presents one year, three year and five year survival rates. Exporters are systematically more likely to survive than non–exporters, especially over long periods of time. For instance, out of the total number of exporters in 1990, 85 percent continue operating five years later. The corresponding figure for non–exporters is only 77 percent. Table $A3$ shows the unconditional mean values for several characteristics of both exporters and non–exporters. Exporters are larger, more productive, more skill intensive, and are more likely to be foreign owned compared to non–exporters. Plants that export are also more likely to use imported intermediate inputs and purchase foreign technologies through licenses.

B Average foreign income

The level of foreign income is measured as a weighted average of the level of per capita GDP of the 15 main destination countries of Chilean exports for each industry. We divide the manufacturing sector into 28 sub–sectors according to the 3–digit ISIC code. For each of these sectors we use data from customs to calculate the main destinations of Chilean exports. The averages of the shares of each country are used as weights. Thus, we define the foreign income relevant for sector j at time t as:

$$GDP_{jt} = \sum_{c=1}^{15} GDP_{ct}\, s_{cj},$$

where GDP_{ct} is the real per capita GDP of country c in year t (the per capita GDPs are in constant U.S. dollars and come from the PennWorld Table v. 6.1). We keep the weights s_{cj} constant for the entire period and compute them as:

$$s_{cj} = \sum_{t=1}^{T} \frac{1}{T} \frac{Exports_{cjt}}{Exports_{jt}},$$

where $Exports_{cjt}$ is the value of exports from sector j to country c at time t, and $Exports_{jt}$ is the value of exports from sector j to all countries c at time t. T is the number of years.

[26] There are 29 manufacturing sectors at the 3–digit level. They include sectors such as food processing, textiles, paper products, chemicals and metal products.

[27] All monetary variables are in constant 1985 pesos (annual price deflators are available in the case of Chile at the 4–digit ISIC level).

C Average capital share parameter in autarky

The zero profit conditions (equation 8) imply that, in general equilibrium, $q(\phi_i) + f_i = q(\phi_i)\frac{\sigma}{\sigma-1}$ for $i = L, K$. Furthermore, if we define the average capital share parameter in autarky as $\widetilde{\phi}_a \equiv \frac{\phi_K + \phi_L\left(\frac{\eta_L}{\eta_K}\right)_a}{1+\left(\frac{\eta_L}{\eta_K}\right)_a}$ equations 6 and 7 can be rewritten as:

$$\overline{L} = I\,\frac{1-\widetilde{\phi}_a}{\widetilde{\phi}_a r^{1-\sigma} + 1 - \widetilde{\phi}_a}, \qquad \text{and} \qquad \overline{K} = I\,\frac{\widetilde{\phi}_a}{\widetilde{\phi}_a r^{1-\sigma} + 1 - \widetilde{\phi}_a}$$

Notice that these are the factor market equilibrium conditions that would result with $\eta_K + \eta_L$ average firms, each of which producing with the capital share parameter $\widetilde{\phi}_a$. Finally, using the definition of $\widetilde{\phi}_a$ it follows immediately that: $P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(\widetilde{\phi}_a r^{1-\sigma} + 1 - \widetilde{\phi}_a\right)(\eta_K + \eta_L)$.

D Proof of proposition 2

The proof proceeds in four steps. *First*, we show that $r_{ft} \geq r_a$. Let $\Psi_a \equiv \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma}$ and $\Psi_{ft} \equiv \left(\frac{f_K+f_X}{2f_L}\right)^{(\sigma-1)/\sigma}$. The ratio $\frac{r_{ft}}{r_a}$ is then given by:

$$\frac{r_{ft}}{r_a} = \left\{\frac{[\Psi_{ft}(1-\phi_L)-(1-\phi_K)]\,[\phi_K - \Psi_a\phi_L]}{[\Psi_a(1-\phi_L)-(1-\phi_K)]\,[\phi_K - \Psi_{ft}\phi_L]}\right\}^{1/(1-\sigma)}.$$

$\frac{r_{ft}}{r_a} \geq 1$ since $\Psi_{ft} \leq \Psi_a$, which follows from our assumption that $f_K \geq f_X$.

Second, we show that $\frac{r_{ft}}{r_a}$ is smaller, the larger is ϕ_K and the smaller is ϕ_L, i.e. the larger is the factor intensity gap between capital and labor intensive firms. In fact:

$$\frac{\partial(\frac{r_{ft}}{r_a})}{\partial\phi_K} = \frac{(\phi_K - \Psi_{ft}\phi_L)(\phi_K - \Psi_a\phi_L)\left(\phi_L r_{ft}^{1-\sigma} r_a^{1-\sigma} + 1 - \phi_L\right)}{(\frac{r_a}{r_{ft}})^\sigma \frac{1-\sigma}{\Psi_a - \Psi_{ft}}\left\{[\Psi_a(1-\phi_L)-(1-\phi_K)]\,[\phi_K - \Psi_{ft}\phi_L]\right\}^2} < 0$$

since $\Psi_a \geq \Psi_{ft}$ and $\phi_K - \Psi_m\phi_L > 0$, $m = a, ft$, if the two types of firms are active in general equilibrium. Furthermore:

$$\frac{\partial(\frac{r_{ft}}{r_a})}{\partial\phi_L} = \frac{(\phi_K - \Psi_{ft}\phi_L)(\phi_K - \Psi_a\phi_L)\left(\phi_K r_{ft}^{1-\sigma} r_a^{1-\sigma} + 1 - \phi_K\right)}{(\frac{r_a}{r_{ft}})^\sigma \frac{1-\sigma}{\Psi_{ft} - \Psi_a}\left\{[\Psi_a(1-\phi_L)-(1-\phi_K)]\,[\phi_K - \Psi_{ft}\phi_L]\right\}^2} > 0$$

since, again, $\Psi_a \geq \Psi_{ft}$ and $\phi_K - \Psi_m\phi_L > 0$, $m = a, ft$, if the two types of firms are active in general equilibrium. Since the relationship between $\frac{r_{ft}}{r_a}$ and ϕ_i, $i = K, L$ is monotonic, we assume for the remainder of appendix D that $\phi_K = 1 - \phi_L$ and, thus, $\phi_K > 0.5 > \phi_L$.

Third, we can show that the rightward shift of the *FMC*–condition with trade liberalization does not depend on the factor intensity gap $\phi_K - \phi_L$. Solving the *FMC*–conditions under autarky and free trade (equations 11 and 21) for $(\frac{\eta_L}{\eta_K})_a$ and $(\frac{\eta_L}{\eta_K})_{ft}$, and taking their ratio results in:

$$\frac{(\eta_L/\eta_K)_{ft}}{(\eta_L/\eta_K)_a} = \frac{\frac{2(1-\phi_K)-2\frac{\overline{L}}{\overline{K}}r_{ft}^{-\sigma}\phi_K}{\frac{\overline{L}}{\overline{K}}r_{ft}^{-\sigma}(1-\phi_K)-\phi_K}}{\frac{1-\phi_K-\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}\phi_K}{\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}(1-\phi_K)-\phi_K}}.$$

Thus, for *each constant* level of $r = r_{ft} = r_a$ we get $\frac{(\eta_L/\eta_K)_{ft}}{(\eta_L/\eta_K)_a} = 2$, i.e. the relative mass of labor intensive firms ceteris paribus doubles with trade liberalization. Therefore, the rightward shift of the *FMC*–curve does not depend on the factor intensity gap.

Fourth, we can show that $\frac{\eta_L}{\eta_K}$ decreases (increases) with trade liberalization if the factor intensity gap is at its minimum (maximum) level. The *maximum* value of the factor intensity gap is 1 since ϕ_K and ϕ_L are restricted by the interval $[0, 1]$. We define the *minimum* level of the factor intensity gap as that value which leads to $(\frac{\eta_K}{\eta_L})_a = 0$. $(\frac{\eta_K}{\eta_L})_a$ is given by (remember that we assume $\phi_K = 1 - \phi_L$):

$$\left(\frac{\eta_K}{\eta_L}\right)_a = \frac{\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}(1-\phi_K) - \phi_K}{1 - \phi_K - \frac{\overline{L}}{\overline{K}}r_a^{-\sigma}\phi_K} \tag{33}$$

Thus, $(\frac{\eta_K}{\eta_L})_a = 0$ if $\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}(1-\phi_K^{min}) - \phi_K^{min} = 0$ and the minimum factor intensity gap results as the solution to the following equation: $\phi_K^{min} - (1-\phi_K^{min}) = 2\phi_K^{min} - 1$. In order to prove that ϕ_K^{min} is uniquely defined, we substitute the term for r_a into $\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}(1-\phi_K^{min}) - \phi_K^{min} = 0$. Rearranging terms leads to:

$$\left(\frac{\eta_K}{\eta_L}\right)_a = 0 \quad \Longleftrightarrow \quad \underbrace{\left[\frac{\phi_K(\Psi_a+1)-1}{\phi_K(\Psi_a+1)-\Psi_a}\right]^{\sigma/(\sigma-1)}\frac{1-\phi_K}{\phi_K}}_{\equiv\Pi} = \frac{\overline{K}}{\overline{L}}.$$

Notice that the squared bracket is defined since we have restricted the technology space so that capital and labor intensive technologies coexist in equilibrium (see also the proof to proposition 1). We are now able to determine the following partial derivative:

$$\frac{\partial\Pi}{\partial\phi_K} = \frac{\sigma}{\sigma-1}\left[\frac{\phi_K(\Psi_a+1)-1}{\phi_K(\Psi_a+1)-\Psi_a}\right]^{\frac{1}{1-\sigma}}\frac{(\Psi_a+1)(1-\Psi_a)}{[\phi_K(\Psi_a+1)-\Psi_a]^2} - \left(\frac{\phi_K(\Psi_a+1)-1}{\phi_K(\Psi_a+1)-\Psi_a}\right)^{\frac{\sigma}{\sigma-1}}\frac{1}{\phi_K^2}. \tag{34}$$

Equation 34 shows that $\frac{\partial\Pi}{\partial\phi_K} < 0$ for all values of ϕ_K since $\Psi_a > 1$. Thus, $(\frac{\eta_K}{\eta_L})_a = 0$ *only if* $\phi_K = \phi_K^{min}$. Furthermore, notice that $(\frac{\eta_K}{\eta_L})_a > 0$ if the numerator in the term for $\frac{\eta_K}{\eta_L}$ (equation 33) is negative since the denominator is already negative due to $\frac{1-\phi_K}{r^{-\sigma}\phi_K} < \frac{\overline{L}}{\overline{K}}$. Thus, if $\phi_K > \phi_K^{min}$ we get $\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}(1-\phi_K) - \phi_K < 0$ and $(\frac{\eta_K}{\eta_L})_a > 0$.

Therefore, since $\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}(1-\phi_K^{min}) - \phi_K^{min} = 0$ and $r_{ft} > r_a$ it follows immediately that $\frac{\eta_K}{\eta_L}$ increases with trade liberalization if the factor intensity gap is at its minimum level. Finally, if the factor intensity gap between exporters and non–exporters is at its maximum, i.e. $\phi_K = 1$ and $\phi_L = 0$, we get $(\frac{\eta_K}{\eta_L})_a = \frac{Kf_L}{Lf_K} > (\frac{\eta_K}{\eta_L})_{ft} = \frac{Kf_L}{L(f_X+f_K)}$, i.e. $\frac{\eta_K}{\eta_L}$ decreases with trade liberalization.

E The zero profit condition in the right panel of figure 4

In this appendix the subscript a denotes variables in the autarkic equilibrium, $ft1$ variables in the free trade equilibrium *before* any adjustment of relative factor prices and $ft2$ variables in the free trade equilibrium *after* the adjustment of relative factor prices. Considering equations 2, 8 and 19, we can derive the axis–intercepts of the capital intensive firms' zero profit condition in the autarkic equilibrium and after trade liberalization. In the autarkic equilibrium they are given by:

$$\eta_{K,a} = \left[\frac{I}{p(\phi_K)}\right]_a \frac{1}{(\sigma-1)f_K} \quad \text{and} \quad \eta_{L,a} = \left[\frac{Ip(\phi_K)^{-\sigma}}{p(\phi_L)^{1-\sigma}}\right]_a \frac{1}{(\sigma-1)f_K}.$$

After trade liberalization and *before* any adjustment of relative factor prices, the axis intercepts are given by:

$$\eta_{K,ft1} = \left[\frac{I}{p(\phi_K)}\right]_{ft1} \frac{1}{(\sigma-1)(f_K+f_X)} \quad \text{and} \quad \eta_{L,ft1} = \left[\frac{Ip(\phi_K)^{-\sigma}}{p(\phi_L)^{1-\sigma}}\right]_{ft1} \frac{2}{(\sigma-1)(f_K+f_X)}.$$

Since $\left[\frac{I}{p(\phi_K)}\right]_a = \left[\frac{I}{p(\phi_K)}\right]_{ft1}$ and $\left[\frac{Ip(\phi_K)^{-\sigma}}{p(\phi_L)^{1-\sigma}}\right]_a = \left[\frac{Ip(\phi_K)^{-\sigma}}{p(\phi_L)^{1-\sigma}}\right]_{ft1}$, we get the following result: $\frac{\eta_{K,ft1}}{\eta_{K,a}} = \frac{f_K}{f_K+f_X} < 1$ and $\frac{\eta_{L,ft1}}{\eta_{L,a}} = \frac{2f_K}{f_K+f_X} \geq 1$. Thus, the capital intensive firms' zero profit condition shifts inwards and becomes steeper.

In order to determine how the increase in r affects the η_K–axis intercept, we have to consider the following partial derivative:

$$\frac{\partial[I/p(\phi_K)]}{\partial r} = -r^{-\sigma}c(\phi_K)^{\sigma-2}K\phi_K\left(\frac{\overline{L}}{\overline{\overline{K}}} - \frac{1-\phi_K}{\phi_K r^{-\sigma}}\right) < 0.$$

Thus, $\left[\frac{I}{p(\phi_K)}\right]_{ft2} < \left[\frac{I}{p(\phi_K)}\right]_{ft1}$ and $\eta_{K,ft2} < \eta_{K,ft1}$.

In order to determine how the increase in r affects the η_L–axis intercept, we *first* have to consider that the increase in r makes the capital intensive firms' zero profit condition ceteris paribus flatter: the slope of the zero profit condition after trade liberalization is given by $\frac{d\eta_L}{d\eta_K} = -2\left[\frac{p(\phi_K)}{p(\phi_L)}\right]^{1-\sigma}$ and $\frac{\partial[p(\phi_K)/p(\phi_L)]}{\partial r} > 0$. *Second*, we have to consider that a division of the zero profit conditions of the two types of firms leads to $\left[\frac{p(\phi_K)}{p(\phi_L)}\right]^{-\sigma} = \frac{f_K}{f_L}$ in the autarkic equilibrium (see equation 8) and to $\left[\frac{p(\phi_K)}{p(\phi_L)}\right]^{-\sigma} = \frac{f_K+f_X}{2f_L}$ in the free trade equilibrium (see equation 19). Thus:

$$\eta_{L,a} = \left[\frac{I}{p(\phi_L)}\right]_a \frac{1}{(\sigma-1)f_L} < \eta_{L,ft2} = \left[\frac{I}{p(\phi_L)}\right]_{ft2} \frac{1}{(\sigma-1)f_L}$$

since $\frac{\partial[I/p(\phi_L)]}{\partial(r/w)} > 0$. *Third*, since the capital intensive firms' zero profit condition becomes flatter with the increase in r and since $\eta_{K,ft2} < \eta_{K,ft1}$, we can conclude that $\eta_{L,ft2} < \eta_{L,ft1}$.

F Proof of proposition 3

Remember that $\widetilde{\phi}_a = \frac{\phi_K + \phi_L \left(\frac{\eta_L}{\eta_K}\right)_a}{1 + \left(\frac{\eta_L}{\eta_K}\right)_a}$. Since the production of each individual capital intensive firm ceteris paribus doubles, the average sector–wide capital share parameter is given by $\widetilde{\phi}_{ft} = \frac{2\phi_K + \phi_L \left(\frac{\eta_L}{\eta_K}\right)_{ft}}{2 + \left(\frac{\eta_L}{\eta_K}\right)_{ft}}$. Thus, $\widetilde{\phi}_{ft} > \widetilde{\phi}_a$ if and only if $(\phi_K - \phi_L)\left[2\left(\frac{\eta_L}{\eta_K}\right)_a - \left(\frac{\eta_L}{\eta_K}\right)_{ft}\right] \geq 0$. This condition always holds since $2\left(\frac{\eta_L}{\eta_K}\right)_a = \left(\frac{\eta_L}{\eta_K}\right)_{ft}$ if $f_K = f_X$ and $2\left(\frac{\eta_L}{\eta_K}\right)_a > \left(\frac{\eta_L}{\eta_K}\right)_{ft}$ if $f_K > f_X$, for any factor intensity gap $\phi_K - \phi_L$.

G Proof of proposition 4

If we define $\Psi_N \equiv \left[\frac{f_K + (N-1)f_X}{N f_L}\right]^{\frac{\sigma-1}{\sigma}}$, the PC–condition can be written as follows:

$$r_{ft} = \left[\frac{\Psi_N(1-\phi_L) - (1-\phi_K)}{\phi_K - \Psi_N \phi_L}\right]^{\frac{1}{1-\sigma}}.$$

The partial derivative of r_{ft} with respect to the number of trading partners N results as follows:

$$\frac{\partial r_{ft}}{\partial N} = \frac{1}{1-\sigma} r_{ft}^{\sigma} \frac{\phi_K - \phi_L}{(\phi_K - \Psi\phi_L)^2} \frac{\partial \Psi}{\partial N}, \qquad \text{with} \quad \frac{\partial \Psi}{\partial N} = \frac{\sigma - 1}{\sigma} \Psi^{1/(1-\sigma)} \frac{f_L(f_X - f_K)}{(N f_L)^2}.$$

Since $f_K \geq f_X$ and $\phi_K > \phi_L$, it follows that $\frac{\partial r_{ft}}{\partial N}$, i.e. the PC–curve shifts upwards with an increase in N. Notice also that $\lim_{N\to\infty} \Psi = \left(\frac{f_X}{f_L}\right)^{\frac{\sigma-1}{\sigma}}$. Thus, if $N \to \infty$, we get $r_{ft} > r_a$ if $f_X < f$ and $r_{ft} = r_a$ if $f_K = f_X$. Furthermore, it is straightforward to check that $\lim_{N\to\infty} r_{ft} < 1$ since $\left(\frac{f_X}{f_L}\right)^{(\sigma-1)/\sigma} > 1$, i.e. r_{ft} is always strictly smaller than 1, even if $N \to \infty$. The *FMC*–curve, instead, establishes a *linear* relationship between $\frac{\eta_L}{\eta_K}$ and N. Thus, if N is sufficiently large, $\frac{\eta_L}{\eta_K}$ always increases with trade liberalization, irrespective of the magnitude of the factor intensity gap $\phi_K - \phi_L$.

Turning now to the panel on the right of figure 5, the η_L–axis intercept of the zero profit condition for capital intensive firms is given by:

$$\eta_{L,ft} = \frac{N I p(\phi_K)^{-\sigma}}{p(\phi_L)^{1-\sigma}} \frac{1}{[f_K + (N-1)f_X](\sigma - 1)}.$$

The partial derivative with respect to N results as:

$$\frac{\partial \eta_{L,ft}}{\partial N} = \frac{I p(\phi_K)^{-\sigma}}{p(\phi_L)^{1-\sigma}} \frac{f_K - f_X}{[f_K + (N-1) f_X]^2 (\sigma - 1)} \geq 0.$$

Furthermore, $\lim_{N\to\infty} \eta_{L,ft} = \frac{I p(\phi_K)^{-\sigma}}{f_X p(\phi_L)^{1-\sigma}(\sigma-1)} \geq \eta_{L,a} = \frac{I p(\phi_K)^{-\sigma}}{f_K p(\phi_L)^{1-\sigma}(\sigma-1)}$. Finally, the η_K–axis intercept of the zero profit condition for capital intensive firms is given by:

$$\eta_{K,ft} = \frac{I}{p(\phi_K)} \frac{1}{[f_K + (N-1) f_X](\sigma-1)}.$$

Thus, if $N \to \infty$ we get $\eta_{K,ft} \to 0$ and $\eta_{L,ft} > 0$.

H Proof of lemma 4

Using our definition of $\widetilde{A}_K$, equation 25 can be rewritten as follows:

$$\left[\frac{1}{\sigma-1}\frac{1}{\widetilde{A}_K} q\left(\widetilde{A}_K, \phi_K\right) - f_K\right] = \frac{f_E}{[1 - G(A_K^*)]}. \tag{35}$$

Since $\frac{q\left(A_K^*, \phi_K\right)}{q\left(\widetilde{A}_K, \phi_K\right)} = \left(\frac{\widetilde{A}_K}{A_K^*}\right)^{-\sigma}$, the zero cutoff profit condition (equation 24) can be transformed to:

$$q\left(\widetilde{A}_i, \phi_i\right) = \left(\frac{\widetilde{A}_i}{A_i^*}\right)^{\sigma} A_i^*(\sigma-1) f_i. \tag{36}$$

Substituting equation 36 into equation 35 and simplification leads to equation 26 in the main part.

I *FMC* condition under TFP heterogeneity

With heterogeneity in TFP we get $a_{KK} = A_K^{\sigma-1}\phi_K r^{-\sigma} c(A_K, \phi_K)^{\sigma}$ and $a_{LK} = A_K^{\sigma-1}(1-\phi_K) c(A_K, \phi_K)^{\sigma}$. Notice that $\frac{\partial a_{KK}}{\partial A_K} < 0$ and $\frac{\partial a_{LK}}{\partial A_K} < 0$, since $c(A_K, \phi_K)^{\sigma} = \left[\frac{1}{A_K}\left(\phi_K r^{1-\sigma} + 1 - \phi_K\right)^{1/(1-\sigma)}\right]^{\sigma}$. Fixed costs are not influenced by the TFP parameter, and are still given by equation 4. Let $\widetilde{f}_K \equiv \frac{f_{EK}}{1-G\left(A_K^*\right)} + f_K$, i.e. $\widetilde{f}_K$ stands for the total fixed costs in general equilibrium. The *FMC* condition can be rewritten as:

$$\frac{\overline{L}}{\overline{K}} = \frac{\sum_{i=L,K}(1-\phi_i)\left[\left(\phi_i r^{1-\sigma} + 1 - \phi_i\right)^{1/(1-\sigma)}\right]^{\sigma}\left[\frac{1}{\widetilde{A}_i} q\left(\widetilde{A}_i, \phi_i\right) + \widetilde{f}_i\right]\eta_i}{\sum_{i=L,K}\phi_i r^{-\sigma}\left[\left(\phi_i r^{1-\sigma} + 1 - \phi_i\right)^{1/(1-\sigma)}\right]^{\sigma}\left[\frac{1}{\widetilde{A}_i} q\left(\widetilde{A}_i, \phi_i\right) + \widetilde{f}_i\right]\eta_i}. \tag{37}$$

Finally, remembering that $q\left(\widetilde{A}_i, \phi_i\right) = IP^{\sigma-1}\left[\frac{\sigma}{\sigma-1}\frac{1}{\widetilde{A}_i}\left(\phi_i r^{1-\sigma} + 1 - \phi_i\right)^{1/(1-\sigma)}\right]^{-\sigma}$ and that free entry implies $\frac{q\left(\widetilde{A}_i, \phi_i\right)}{\widetilde{A}_i(\sigma-1)} = \widetilde{f}_i = \frac{f_{Ei}}{1-G\left(A_i^*\right)} + f_i$, equation 37 can be simplified to:

$$\frac{\overline{L}}{\overline{K}} = \frac{(1-\phi_K)\widetilde{A}_K^{\sigma-1} + (1-\phi_L)\widetilde{A}_L^{\sigma-1}\frac{\eta_L}{\eta_K}}{\phi_K r^{-\sigma}\widetilde{A}_K^{\sigma-1} + \phi_L r^{-\sigma}\widetilde{A}_L^{\sigma-1}\frac{\eta_L}{\eta_K}} \tag{38}$$

J Proof of proposition 5

To establish proposition 5, notice that from lemma 4 we know that $A_{a,K}^*$ and $A_{a,L}^*$ only depend on the parameters f_E and f_i and the distribution of A. Therefore, $\widetilde{A}_{a,K}^{\sigma-1}$ and $\widetilde{A}_{a,L}^{\sigma-1}$ are determined from equation 26 alone. Thus, equations 27 and 28 can be solved for r and $\frac{\eta_L}{\eta_K}$ like in the autarkic equilibrium *without* firm heterogeneity in TFP. Finally, notice that the right hand side of equation 38 still depends positively on $\frac{\eta_L}{\eta_K}$, i.e. equation 38 is still represented by a negatively sloping curve.

K Aggregation under TFP heterogeneity — autarky

Adding the TFP–terms $\left(\frac{1}{\widetilde{A}_L}\right)^{1-\sigma}$ and $\left(\frac{1}{\widetilde{A}_K}\right)^{1-\sigma}$ to the factor market clearing conditions of appendix C and defining $\widetilde{A}_a^{\sigma-1} \equiv \frac{\widetilde{A}_{a,K}^{\sigma-1}+\widetilde{A}_{a,L}^{\sigma-1}\left(\frac{\eta_L}{\eta_K}\right)_a}{1+\left(\frac{\eta_L}{\eta_K}\right)_a}$ and $\widetilde{\phi}_a \equiv \frac{\phi_K\widetilde{A}_{a,K}^{\sigma-1}+\phi_L\widetilde{A}_{a,L}^{\sigma-1}\left(\frac{\eta_L}{\eta_K}\right)_a}{\widetilde{A}_{a,K}^{\sigma-1}+\widetilde{A}_{a,L}^{\sigma-1}\left(\frac{\eta_L}{\eta_K}\right)_a}$, the factor market equilibrium conditions can be rewritten as follows:

$$\begin{aligned}
\overline{L} &= I\frac{\left[1+\left(\frac{\eta_L}{\eta_K}\right)_a\right]\widetilde{A}_a^{\sigma-1}\left(1-\widetilde{\phi}_a\right)}{\left[1+\left(\frac{\eta_L}{\eta_K}\right)_a\right]\widetilde{A}_a^{\sigma-1}\left(\widetilde{\phi}_a r_a^{1-\sigma}+1-\widetilde{\phi}_a\right)} &=& \; I\frac{1-\widetilde{\phi}_a}{\widetilde{\phi}_a r_a^{1-\sigma}+1-\widetilde{\phi}_a} \\
\overline{K} &= I\frac{\left[1+\left(\frac{\eta_L}{\eta_K}\right)_a\right]\widetilde{A}_a^{\sigma-1}\widetilde{\phi}_a}{\left[1+\left(\frac{\eta_L}{\eta_K}\right)_a\right]\widetilde{A}_a^{\sigma-1}\left(\widetilde{\phi}_a r_a^{1-\sigma}+1-\widetilde{\phi}_a\right)} &=& \; I\frac{\widetilde{\phi}_a}{\widetilde{\phi}_a r_a^{1-\sigma}+1-\widetilde{\phi}_a}.
\end{aligned}$$

These are the same conditions which would result in an economy with $\eta_a = \eta_{a,K} + \eta_{a,L}$ average firms, each of which producing with the technology parameters $\widetilde{A}_a$ and $\widetilde{\phi}_a$.

L Proof of lemma 5

Since A_{Xi}^* is a function of A_i^* (see equation 30), equation 31 can be solved for $A_{ft,i}^*$. In order to prove that A_i^* increases with trade liberalization, we show that the term $\left(1-G\left(A_i^*\right)\right)\left[\left(\frac{\widetilde{A}_i}{A_i^*}\right)^{\sigma-1}-1\right] \equiv \Lambda$ depends negatively on A_i^*. Remember that $\widetilde{A}_i = \left[\int_{A_i^*}^{\infty} A^{\sigma-1}\mu(A)dA\right]^{1/(\sigma-1)}$ is also a function of A_i^*. Using Leibniz's rule to calculate $\frac{\partial \widetilde{A}_i}{\partial A_i^*}$, we obtain:

$$\frac{\partial \Lambda}{\partial A_i^*} = -\frac{1-G\left(A_i^*\right)}{A_i^*}(\sigma-1)\left(\frac{\widetilde{A}_i}{A_i^*}\right)^{\sigma-1} < 0.$$

Since trade liberalization adds the ex–ante expected profits from serving $N-1$ foreign markets to the left hand side of the free entry condition (see equation 31), the threshold TFP–parameter A_i^* has to increase so that Λ decreases and the free entry condition in the free trade situation holds again.

M Proof of lemma 6

Assuming that A is distributed on $[1,\infty)$ according to a Pareto distribution with density $g(A) = \frac{k}{A^{k+1}}$ and $k > \sigma-1$, we get the following:

$$1-G(A) = \left(\frac{1}{A}\right)^{K} \quad \text{and} \quad \frac{\widetilde{A}_i}{A_i^*} = \left(\frac{k}{1+k-\sigma}\right)^{1/(\sigma-1)}.$$

Thus, in the autarkic equilibrium the free entry condition is given by:

$$\left(\frac{1}{A^*_{a,i}}\right)^k f_i \left(\frac{k}{1+k-\sigma} - 1\right) \;=\; f_E.$$

Solving for $A^*_{a,i}$ yields:

$$A^*_{a,i} \;=\; \left(\frac{f_i}{f_E}\,\frac{\sigma-1}{1+k-\sigma}\right)^{1/k}.$$

In the free trade equilibrium the free entry condition results as:

$$\left(\frac{1}{A^*_{ft,i}}\right)^k f_i \frac{\sigma-1}{1+k-\sigma} + \left\{\frac{1}{A^*_{ft,i}\left[\frac{f_i}{(N-1)f_X}\right]^{1/(1-\sigma)}}\right\}^k f_X \frac{\sigma-1}{1+k-\sigma} \;=\; f_E.$$

Solving for $A^*_{ft,i}$ leads us to:

$$A^*_{ft,i} \;=\; \left\{\frac{f_i + \left[\frac{f_i}{(N-1)f_X}\right]^{k/(\sigma-1)}}{f_E}\;\frac{\sigma-1}{1+k-\sigma}\right\}^{1/k}.$$

Thus, we can determine the following ratio:

$$\frac{A^*_{ft,i}}{A^*_{a,i}} \;=\; \left[1 + f_i^{(1+k-\sigma)/(\sigma-1)}(Nf_X)^{k/(1-\sigma)}\right]^{1/k}. \tag{39}$$

Equation 39 shows that a larger value for f_i leads to a larger increase in A^*_i with trade liberalization. Thus, since $f_K > f_L$, the increase in A^*_K due to trade liberalization exceeds the increase in A^*_L. Since $\frac{\widetilde{A}_i}{A^*_i} = \left(\frac{k}{1+k-\sigma}\right)^{1/(\sigma-1)}$, it follows immediately that the increase in $\widetilde{A}_K$ also exceeds the increase in $\widetilde{A}_L$ due to trade liberalization.

N Firm selection with trade liberalization and TFP heterogeneity

We can illustrate the relationship between the factor intensity gap and firm selection with trade liberalization again by the upward shift of the PC–curve and the rightward shift of the FMC–curve. The upward shift of the PC–curve is reflected by the ratio $\frac{r_{ft}}{r_a}$, whereas the rightward shift of the FMC–curve is reflected by the ratio $\frac{(\eta_L/\eta_K)_{ft}}{(\eta_L/\eta_K)_a}$.

If we define $\Psi_a \equiv \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma}\left(\frac{A^*_{a,K}}{A^*_{a,L}}\right)^{(1-\sigma)^2/-\sigma}$ and $\Psi_{ft} \equiv \left(\frac{f_K}{f_L}\right)^{(\sigma-1)/\sigma}\left(\frac{A^*_{ft,K}}{A^*_{ft,L}}\right)^{(1-\sigma)^2/-\sigma}$, the ratio $\frac{r_{ft}}{r_a}$ is given by:

$$\frac{r_{ft}}{r_a} \;=\; \left\{\frac{\left[\Psi_{ft}(1-\phi_L)-(1-\phi_K)\right]\left[\phi_K-\Psi_a\phi_L\right]}{\left[\Psi_a(1-\phi_L)-(1-\phi_K)\right]\left[\phi_K-\Psi_{ft}\phi_L\right]}\right\}^{1/(1-\sigma)}.$$

Notice that $\Psi_{ft} \leq \Psi_a$ since A^*_K increases with trade liberalization. Thus, $\frac{r_{ft}}{r_a} > 1$.

Furthermore, the ratio $\frac{(\eta_L/\eta_K)_{ft}}{(\eta_L/\eta_K)_a}$ results as follows:

$$\frac{(\eta_L/\eta_K)_{ft}}{(\eta_L/\eta_K)_a} = \frac{\frac{(1-\phi_K)-\frac{\overline{L}}{\overline{K}}r_{ft}^{-\sigma}\phi_K}{\frac{\overline{L}}{\overline{K}}r_{ft}^{-\sigma}(1-\phi_K)-\phi_K}}{\frac{1-\phi_K-\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}\phi_K}{\frac{\overline{L}}{\overline{K}}r_a^{-\sigma}(1-\phi_K)-\phi_K}} \frac{\left(\frac{\widetilde{A}_{a,L}}{\widetilde{A}_{a,K}}\right)^{\sigma-1}}{\left(\frac{\widetilde{A}_{ft,L}}{\widetilde{A}_{ft,K}}\right)^{\sigma-1}}. \tag{40}$$

Compared to the case *without* firm heterogeneity in TFP, the term with the average TFP parameters adds to the right hand side of equation 40. Since the average TFP parameters do *not* depend on the factor intensity gap $\phi_K - \phi_L$, both $\frac{r_{ft}}{r_a}$ and $\frac{(\eta_L/\eta_K)_{ft}}{(\eta_L/\eta_K)_a}$ react to a change in the factor intensity gap the same way as we described in appendix D for the case *without* firm heterogeneity in TFP.

O Aggregation under TFP heterogeneity — free trade

Using the terms for $\widetilde{A}_{ft}^{\sigma-1}$ and $\widetilde{\phi}_{ft}$ from the main part, equation 32 can be rewritten as follows:

$$\frac{\overline{L}}{\overline{K}}r^{-\sigma} = \frac{1-\widetilde{\phi}_{ft}}{\widetilde{\phi}_{ft}}. \tag{41}$$

Equation 41 is the relative *FMC* condition that would result in an economy with $\eta_{ft} = \eta_{ft,K} + \eta_{ft,L}$ average firms, each of which producing with the technology parameters $\widetilde{\phi}_{ft}$ and $\widetilde{A}_{ft}$.

P Proof of proposition 6

First, we will show that $\widetilde{A}_{ft}^{\sigma-1} > \widetilde{A}_a^{\sigma-1}$ even if trade liberalization leads to the maximum possible increase of $\frac{\eta_L}{\eta_K}$. Remember that labor intensive firms experience a smaller productivity–enhancing firm selection with trade liberalization. We have shown previously that a factor intensity gap $\phi_K - \phi_L = 1$ leads to the maximum possible increase of $\frac{\eta_L}{\eta_K}$ with trade liberalization. If $\phi_K = 1$ and $\phi_L = 0$, the *FMC* and the *PC*–conditions reduce, respectively, to:

$$\frac{\overline{L}}{\overline{K}}r_{ft}^{-\sigma} = \frac{1+s_L}{1+s_K}\frac{\widetilde{A}_{ft,L}^{\sigma-1}}{\widetilde{A}_{ft,K}^{\sigma-1}}\left(\frac{\eta_L}{\eta_K}\right)_{ft} \quad \text{and} \quad r_{ft} = \left(\frac{f_K}{f_L}\right)^{-1/\sigma}\left(\frac{A^*_{ft,L}}{A^*_{ft,K}}\right)^{(1-\sigma)/\sigma}.$$

Notice that the limiting case of $s_L = s_K = 0$ would represent the autarkic equilibrium. Combining these two conditions, we obtain:

$$\frac{\overline{L}}{\overline{K}}\frac{f_K}{f_L} = \frac{1+s_L}{1+s_K}\left(\frac{\frac{\widetilde{A}_{ft,L}}{A^*_{ft,L}}}{\frac{\widetilde{A}_{ft,K}}{A^*_{ft,K}}}\right)^{\sigma-1}\left(\frac{\eta_L}{\eta_K}\right)_{ft}. \tag{42}$$

Remember that $\frac{\widetilde{A}_i}{A_i^*} = \left(\frac{k}{1+k-\sigma}\right)^{1/(\sigma-1)}$, $i = K, L$, under the assumption of a Pareto distribution for the TFP parameter. Therefore, the term $\left(\frac{\widetilde{A}_{ft,L}/A^*_{ft,L}}{\widetilde{A}_{ft,K}/A^*_{ft,K}}\right)^{\sigma-1}$ on the right hand side of equation 42 is constant across equilibria. This implies that trade liberalization, i.e. the term $\frac{1+s_L}{1+s_K}$ falls below 1, increases $\frac{\eta_L}{\eta_K}$, so that the product $\frac{1+s_L}{1+s_K}\frac{\eta_L}{\eta_K}$ remains constant. However, the product $\frac{1+s_L}{1+s_K}\frac{\eta_L}{\eta_K}$ is the weighting factor for $\widetilde{A}_L$ in the term for $\widetilde{A}$. Thus, in the case of a maximum possible increase of $\frac{\eta_L}{\eta_K}$ with trade liberalization, the *relative* weights for $\widetilde{A}_K$ and $\widetilde{A}_L$ in the term for $\widetilde{A}$ do not change. Since both $\widetilde{A}_K$ and $\widetilde{A}_L$ increase with trade liberalization, it follows immediately that the sector–wide average TFP parameter $\widetilde{A}$ increases with trade liberalization, even if the factor intensity gap is at its maximum value.

Second, trade liberalization definitely leads to an increase of $\widetilde{A}$ if the factor intensity gap is smaller than 1. The increase in $\frac{\eta_L}{\eta_K}$ with trade liberalization then becomes smaller, implying that $\widetilde{A}_K$ gets a larger *relative* weight, while $\widetilde{A}_L$ gets a *smaller* relative weight in the term for $\widetilde{A}$. Thus, any smaller factor intensity gap definitely leads to an increase in the sector–wide average TFP parameter $\widetilde{A}$.

Finally, since the increase in $\widetilde{A}_K$ with trade liberalization exceeds the increase in $\widetilde{A}_L$, an increase (decrease) in $\frac{\eta_L}{\eta_K}$ implies that the increase in $\widetilde{A}$ with trade liberalization becomes ceteris paribus smaller.

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