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The safe are rationed, the risky not -
an extension of the Stiglitz-Weiss model

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May 4, 2011

Abstract

Using only two risk types in the Stiglitz-Weiss model it turns out that the return function for banks has to be double hump-shaped. We derive the demand for loans and the supply of loans and find that loans are provided at two interest rates in equilibrium. The safe borrowers are rationed at the lower interest rate, whereas the risky borrowers are not rationed at all. Compared to the existing literature this suggests that the more heterogenous the risk types are, the less credit is rationed. However, credit-rationing persists in equilibrium as long as we consider a discrete number of types.

1   Introduction

Stiglitz and Weiss (1981) show that information on credit markets is asymmetric and not available to everyone. This concept is widely accepted in the literature with important implications for the supply of loans in equilibrium on credit markets. Stiglitz and Weiss (1981) find that banks cannot observe the risk types of borrowers and have to lend at an average interest rate, at which all risk types demand loans. Thus, the demand for loans exceeds the supply of loans in equilibrium and random credit rationing takes place for all risk types. Stiglitz and Weiss (1981) use a two risk type setup and a hump-shaped return function for banks. More recently, Arnold and Riley (2009) challenge the results from Stiglitz and Weiss (1981). They use a continuum of risk types and a monotonic increasing return function for banks.[1] They find that in equilibrium loans are given at, at least, two interest rates and show numerically that "...rationing is unlikely." (Arnold and Riley, 2009, p2012).[2]

The results by Stiglitz and Weiss (1981) and by Arnold and Riley (2009) describe two special cases of rationing on credit markets. Our paper shows that there exist intermediate solutions. We analyse the Stiglitz-Weiss model for two risk types, but with a double hump-shaped return function for banks. We find rationing exists in equilibrium, which is neither as much as in Stiglitz and Weiss (1981) not as little as in Arnold and Riley (2009). Certain risk types are rationed and others are not.

The actors in our model are borrowers and banks. The borrowers invest in a project that could be either safe or risky, depending on their risk type. They demand loans from banks and post collateral to secure their loans. If the project succeeds the borrowers receive the return and repay the loans and the interest. Otherwise, if the project fails, the borrowers are not able to repay the loans and loose their collateral. First, we consider the optimal behaviour of borrowers and derive the demand function for loans. The borrowers maximize their return and demand loans as long as their expected return is non-negative. In a second step, we consider the behaviour of banks. As we assume imperfect markets with asymmetric information, banks cannot observe the risk type of borrowers. They give loans at an average interest rate. We assume that the banks lend loans (loan-subgame) before they refinance (deposit-subgame).[3] The expected return function for banks, which turns out to be double hump-shaped (Coco, 1997), reflects the supply of loans. Finally, we combine the demand function and the supply function and observe different equilibria. Subsequently, we test whether these equilibria are competitive and Pareto optimal.

We find a competitive equilibrium exists and loans are provided at two different interest rates. The safe types are rationed at the lower interest rate, whereas risky types are not rationed at all. In our model we show that between the rationing of all risk types (Stiglitz and Weiss, 1981) and the complete removal of credit rationing (Arnold and Riley, 2009)

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[1]In a footnote Arnold and Riley (2009) mention the discrete two borrower case with a double hump-shaped return function for banks. See Steger and Waelde (2007) for a formal model.

[2]Arnold (2010) provides a game-theoretic foundation for the results in Arnold and Riley (2009).

[3]For a survey on the ordering of the subgames see Arnold (2010).

intermediate solutions exist, where only certain types are rationed. This suggests that the more heterogenous the types are, the less credit is rationed in equilibrium.

In the literature, the discussion of rationing on credit markets started when Stiglitz and Weiss (1981) posed the question: "Why is credit rationed?". This work has had a wide influence for the future macroeconomic modeling with credit rationing, see Bester (1985) and Greenwald and Stiglitz (1987). The bank's expected return function receives special attention. Stiglitz and Weiss (1981) recognize that the return function for banks could be double hump-shaped and a second interest rate might appear in equilibrium. Surprisingly, they conclude that this double hump-shape is very unlikely and that the return function must be hump-shaped with a global maximum at the lower interest rate. They reject the second interest rate as very unlikely. Most of the literature, even textbooks, take this assumption as given, like Blanchard and Fisher (1989), Hillier and Ibrahimo (1993), De Meza and Webb (1987, 1992), Hellmann and Stiglitz (2002) and Suominen (2004). Other economists like Hillier (1997) and Walsh (2001) assume the double hump-shaped return function for banks, but remain with the one-interest rate equilibrium. Coco (1997, 1999) and, more recently, Arnold and Riley (2009) prove that there must be more than one local maxima for the expected return function. Arnold and Riley (2009) go further and show that a monotonic increasing return function results if they assume a continuum of types. This affects the results from Stiglitz and Weiss (1981) and has important consequences for the existence of credit rationing on credit markets.

Our paper takes into account of the two risk type setup with a double hump-shaped return function for banks, an issue that was not considered by Stiglitz and Weiss (1981) and by Arnold and Riley (2009). We present this missing consideration in a game-theoretical model and show that there are more general solutions than the solutions presented in Stiglitz and Weiss (1981) and in Arnold and Riley (2009).

The paper is structured as follows. Section 2 shows the Stiglitz-Weiss model and the modification of the return function for banks. In section 3 we consider the optimal behaviour of borrowers and banks. The equilibrium, and the deviations from it, will be studied in section 4. The results are shown in section 5. A short conclusion closes the paper.

2 The model

We have risk-neutral borrowers and banks. There are two types of borrowers, the safe type denoted as $i = 1$ and the risky type denoted as $i = 2$. The number of safe investors is N_1 and the number of risky borrowers is N_2. Each of these borrowers has one project to undertake. The project is either safe or risky, which is defined by the type of borrower. We have to mention that safe project does not mean that there is no risk in the project. It only means that the probability of success for the first type is higher than the, exogenously given, probability of success for projects of the second type. We consider an one period model. At the beginning of the period the investment is made. The total amount for

the investment, B, has to be borrowed from a bank and collateral, C, is given, where $0 < C < B$. We assume the investment, B, to be indivisible. We exclude that a whole group of similar borrowers receives only a part of the amount of the demanded loan, described as credit rationing of type one by Keeton (1979). In our model only randomly chosen borrowers are denied loan. At the end of the period, the borrower receives a payoff $R_i \geq 0$. If the projects succeed, the return for type 1 borrower is R_1 and the return for type 2 is R_2. The payoff in the failure of the project is zero and the borrower loses all her collateral. The probabilities of success are p_1 and p_2 respectively, where $0 < p_2 < p_1 < 1$. With $(1 - p_i)$ the project fails. Both projects have the same expected return, $E(R_i) = p_i R_i = \overline{R}$. Thus, by assuming $p_1 > p_2$ it follows that $R_1 < R_2$. If $\overline{R} > (1 + r_i) B$ at the end of the period, the borrower has to repay the amount of loan, B, including an interest rate, r_i, to the bank.

We assume at least four banks in the model denoted by $k = 1, ..., K$ where $4 \leq K \leq \infty$. Two banks offering multiple credit contracts would also be sufficient, but to assume there are four banks simplifies the consideration. Two banks offering multiple credit contracts would also be sufficient, but to assume there are four banks simplifies the consideration. We assume a bank collects deposits to the price of ρ_k and gives loan to the price of r_k. The supply of deposits on the capital market, given by $L^s(\rho)$, is assumed to be monotonic and strictly increasing in ρ. Furthermore, the banks are assumed to compete in prices (Bertrand, 1883). It is therefore necessary that at least two banks compete at each interest rate.

In order to get a competitive solution, the loan-subgame precedes the deposit-subgame (Stahl, 1988). In the loan-subgame, the loan market is characterized by asymmetric information. This means that banks have no information about the borrower's risk type. Interest rates cannot be conditioned on the borrower's risk type. The borrowers start applying for loan at the lowest interest rate. If they do not receive a loan at this interest rate, they try to get loan at the next possible, higher, interest rate in the market, given that their expected profit is non-negative. The bank k acts with $(r_k, \lambda_k) \geq (0, 0)$, where r_k is the interest rate and λ_k is the credit ceiling up to which the bank is willing to lend at interest rate r_k. If there is more than one bank offering loan at the same interest rate, r_k, the one with the highest credit ceiling, λ_k, serves the whole demand. If several banks bid the same credit ceiling at the same interest rate the bank which serves the market is chosen by a random tie-breaking rule, with the same probability for each bank. However, at each interest rate, only one bank gives capital to the borrowers. If the credit ceiling, λ_k, of the chosen bank is not high enough to satisfy the whole demand, loan is rationed at this interest rate. This rationing is random, because banks cannot observe the type of the borrower. The loan really given at an interest rate is L_{r_n}.

Subsequently, the deposit-subgame takes place. The banks act with $(\rho_k, \delta_k) \geq (0, 0)$. The deposit rate the bank is willing to pay is ρ_k and δ_k is the quantity of deposits the bank is willing to collect. Because of the timing of this game, the deposit-subgame follows the loan-subgame, the amount of deposits which the bank collects would be the same as the amount of loans it gives. The bank collects deposits in order of decreasing deposit

rates, ρ_k. If the depositor cannot place the desired amount at the bank, she has to decide either to save at the next lower rate or not at all. If supply equals demand at a deposit rate ρ_k, every bank collects the demanded amount of deposits. In the case of excess supply, the depositors are randomly rationed. If there is higher demand of deposits than supply of deposits, each bank receives a proportional amount of the quantity it demands. Additionally, we assume that there is no secondary market for deposits at which a bank has the possibility of refinancing. If a bank fails to refinance, this leads to an infinitely negative profit. The bank promises an amount of loan to the borrowers and could not collect the capital it needs to distribute the promised amount of capital. On the other hand, collecting an amount of deposits without giving loan, leads to a negative profit which is not infinite. The loss equals the amount of the deposit rates the bank has to pay to the exceeded depositors. In our model, however, we assume that a bank never fails to refinance the loan, if it offers a sufficiently high deposit rate ρ_k.

3 Optimality conditions

3.1 The optimal behaviour of borrowers

An entrepreneur could either provide capital to a bank as a depositor or demand capital from a bank as a borrower. Depositors want to save their money at the highest possible interest rate to maximize their profits. If they are unable to safe the amount of money at the highest interest rate, ρ_n, they can try to invest the money at the next lower interest rate, ρ_{n-1}. Entrepreneurs will supply capital as long as they have stored all their money or as long as they find another, more profitable, project. We can order deposit rates as $\rho_1 > \rho_2 > ... > \rho_N$ where N is the number of market deposit rates at which deposits are raised, $D_n > 0$. As those actions of the depositors are straightforward and we furthermore assume that a bank never fails to refinance, we will not deepen this consideration here.

On the other hand entrepreneurs can demand loans from a bank as borrowers. The profit for a borrower is the maximum of either the return minus the repayment to the bank, loan and interest, or the loss of the collateral as $\pi(R_i, r) = \max(R_i - (1 + r)B, -C)$ where $i = 1, 2$. The borrower applies for loan if her expected profit is non-negative. We use the "epsilon altruism" designed by Hillier (1997). He shows that if investors are indifferent about investing or not, they choose to invest into the project, as this would be welfare increasing for the economy. However, the expected profit of a borrower is given as

$$E[\pi(R_i, r)] = \overline{R} - p_i(1 + r)B - (1 - p_i)C \geq 0. \tag{1}$$

We find that the expected profit from a risky project is larger than the expected profit from a safe project only if $C < (1 + r)B$. This appears to be true for all non-negative interest rates r in our model, as we assume $0 < C < B$. From equation (1) it follows that

until

$$r' \leq r^* \equiv \frac{\overline{R} - (1 - p_1)C}{p_1 B} - 1 \tag{2}$$

safe borrowers demand capital. Risky borrower demand capital as long as

$$r' \leq r^{\max} \equiv \frac{\overline{R} - (1 - p_2)C}{p_2 B} - 1. \tag{3}$$

We find that $r^* < r^{\max}$. This is proven in appendix A.

To describe the form of the demand function of capital, we calculate which borrowers are demanding loans if the interest rate r' is appearing in different intervals. If $0 \leq r' \leq r^*$ all borrowers demand loan. If $r^* < r' \leq r^{\max}$ only risky borrower demand loan. At the point $r^{\max}$, the following equation for the expected profit of a risky investor holds

$$E[\pi(R_2, r^{\max})] = p_2[R_2 - (1 + r^{\max})B] - (1 - p_2)C = 0. \tag{4}$$

It becomes clear that $r^{\max}$ is the highest possible interest rate at which borrowers apply for loan. So if $r^{\max} < r'$ no borrower is demanding loan anymore. It follows that the demand for loan, $L^D(r)$, is constant with discrete jumps at r^* and $r^{\max}$, such that

$$L^D(r) = \begin{cases} N_1 + N_2, & \text{for } 0 \leq r' \leq r^* \\ N_2, & \text{for } r^* < r' \leq r^{\max} \\ \emptyset, & \text{for } r^{\max} < r'. \end{cases} \tag{5}$$

This formulation of the demand function for capital assumes that no loan is given yet.

If we look at the residual demand of capital we have to differentiate this expression from the demand function of capital. The residual demand function consists of the demand at a special interest rate after loan was given at lower interest rates. We find that the residual demand function is also changing at r^* and $r^{\max}$, even if there is no loan given at all. At interest rates larger than r^*, all safe borrowers drop out. At interest rates above $r^{\max}$ all risky borrower stop demanding capital. We have to check whether the residual demand function at higher interest rates is changing if loan is given at lower interest rates. To do so we denote residual demand at r_n, an interest rate at which loan is given, as $l^D(r_n)$. The loan given at r_n is denoted as L_{r_n}. With $r^* < r^{**} < r^{\max}$, we find if no loan is given below r^*, the residual demand at r^* satisfies

$$l^D(r^*) = L^D(r^*). \tag{6}$$

The residual demand at r^{**} is given by

$$l^D(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)}\right] L^D(r^{**}) \tag{7}$$

where L_{r^*} as loan given at r^*. Now, let loan be given at $r^* < r^{**}$ and $r' < r^{**}$. Thus, the

residual demand at r^{**} is defined by

$$l^D(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)} - \frac{L_{r'}}{L^D(r')}\right] L^D(r^{**}). \tag{8}$$

Equation (6) and equation (7) are obvious. The proof of equation (8) is shown in appendix B. We show the functional form of the residual demand of capital in figure 1.
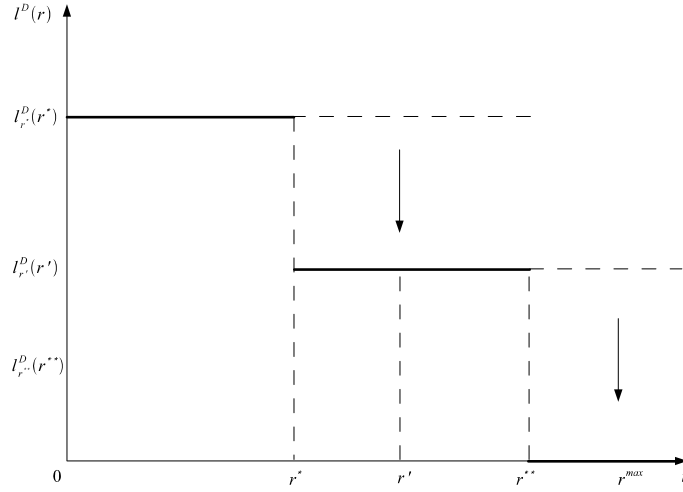

Figure 1 *Residual demand function at r^{**} in the case $r^* < r' < r^{**}$*

We can apply some simplifications at the residual demand function if we consider the residual demand for only two types of borrowers. If $r' < r^*$ the demand for loan at r' is equal to the demand for loans at r^*. Knowing this, we can write the residual demand at r^* using equation (7) as

$$l^D(r^*) = \left[1 - \frac{L_{r'}}{L^D(r')}\right] L^D(r^*) \tag{9}$$

and use $L^D(r') = L^D(r^*)$ to get

$$l^D(r^*) = L^D(r^*) - L_{r'}. \tag{10}$$

Thus, we find the residual demand at r^* is given by the demand at r^* minus the loan given at r'.

If we assume $r^* < r' < r^{**}$, i.e. loan is given at r^* and r', residual demand at r^{**} satisfies equation (8)

$$l^D(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)} - \frac{L_{r'}}{L^D(r')}\right] L^D(r^{**}). \tag{11}$$

7

We find residual demand at r' is equal to the residual demand at r^{**} before any loan is given at this interest rate. We use this fact, i.e. $L^D(r') = L^D(r^{**})$, such that

$$l^D(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)}\right] L^D(r^{**}) - L_{r'} \qquad (12)$$

$$= l^D(r') - L_{r'}. \qquad (13)$$

We therefore can see that the residual demand at r^{**} is given by the residual demand at r' minus the loan given at r' for a two borrower type setup.

3.2 The optimal behaviour of banks

The repayment the bank receives is the minimum of the repayment of the loan including interest or, if the project fails, the collateral,

$$\min\left((1+r)B, R_i + C\right). \qquad (14)$$

Thus, the return of a project can be written as the sum of the repayment a bank receives and the profit for the borrower as

$$R_i = \min\left((1+r)B, R_i + C\right) + \pi(R_i, r). \qquad (15)$$

We find, the more the bank gets, the less the investor receives and vice versa. The bank cannot distinguish between the types of borrowers, but it knows that borrowers only apply for loan if their expected profit is non-negative, $E[\pi(R_i, r)] \geq 0$ with $i = 1, 2$.

Considering $r < r^*$, we know both types of borrowers invest in their project and demand loan. Hence, the expected profit for a bank is given by

$$\begin{aligned} E(\pi_b|_{r \leq r^*}) &= \sum_{i=1}^{2} N_i[p_i(1+r)B + (1 - p_i)C] \\ &= \sum_{i=1}^{2} N_i\left(\overline{R} - E[\pi(R_i, r)]\right). \end{aligned} \qquad (16)$$

The return function for a bank gives the rate of return given the amount of given loan and the expected repayment. However, the return function of a bank, $\varrho(r)$, for $r < r^*$ is given by

$$\begin{aligned} \varrho(r|_{r \leq r^*}) &= \frac{E(\pi_b|r \leq r^*)}{(N_1 + N_2)B} - 1 \\ &= \frac{\sum_{i=1}^{2} N_i\left(\overline{R} - E[\pi(R_i, r)]\right)}{(N_1 + N_2)B} - 1. \end{aligned} \qquad (17)$$

Until $r = r^*$ all borrower types demand loan. The return function of a bank, $\varrho(r)$, is increasing in the interest rate until it reaches its local maximum value at r^* with

8

$\varrho(r^*) = \rho^*$. At values above r^*, the expected profit of a safe borrower becomes negative and she leaves the market. As of now, only risky borrowers demand loan, because they can still achieve positive expected profits. Thus, in the interval $r^* < r \le r^{\max}$ the expected repayment for a bank is given by

$$
\begin{aligned}
E(\pi_b|_{r^*<r\le r^{\max}}) &= N_2[p_2(1+r)B + (1-p_2)C] \\
&= N_2 \left(\overline{R} - E[\pi(R_2, r)] \right).
\end{aligned} \tag{18}
$$

At larger interest rates than r^*, the return function drops discontinuously to the value

$$
\begin{aligned}
\varrho(r|_{r^*<r\le r^{\max}}) &= \frac{E(\pi_b|_{r^*<r\le r^{\max}})}{N_2 B} - 1 \\
&= \frac{\overline{R} - E[\pi(R, r)]}{B} - 1.
\end{aligned} \tag{19}
$$

Henceforward, the return function is continuous increasing in the interest rate until $r = r^{\max}$. The expected profit for the residual risky borrowers is decreasing in r and at $r = r^{\max}$ it equals zero. The return for the banks at $r = r^{\max}$ equals $\varrho(r^{\max}) = \overline{R}/B - 1$. This return for the bank is larger than the return at $r \le r^*$, shown in equation (17). Looking at equation (15), we see that the bank receives the whole return from the project. At interest rates larger than $r^{\max}$, even risky borrowers do not demand loan anymore. Since the bank's return function achieves its highest level at the maximum interest rate $r^{\max}$, there is an interest rate, r', in the interval $r^* < r' \le r^{\max}$ at which the return has to equal ρ^*. Hence, $\varrho(r^*) = \varrho(r^{**}) = \rho^*$. At interest rates $r^{**} < r' \le r^{\max}$ banks get higher returns than in the interval $r' \le r^*$. We illustrate the double hump-shaped return function for a bank in the following figure 2.
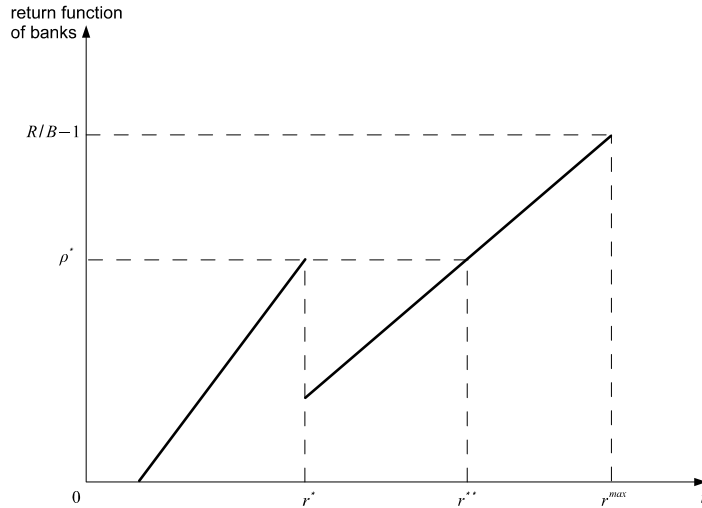

Figure 2 *Double hump-shaped return function for banks*

9

4 Which equilibrium will appear?

Taking the demand function for capital shown in figure (1) and the return function for a bank, i.e. the supply of capital, in figure (2) we can consider three different allocations and the resulting equilibria.

The first equilibrium is described by a supply of capital at the interest rate ρ^* that is higher than the demand at interest rate r^*, so that $L^S(\rho^*) > L^D(r^*)$. Here, we result in a market clearing equilibrium for $r' < r^*$ with $\rho < \rho^*$. No borrower is rationed. This allocation is shown in figure 3.
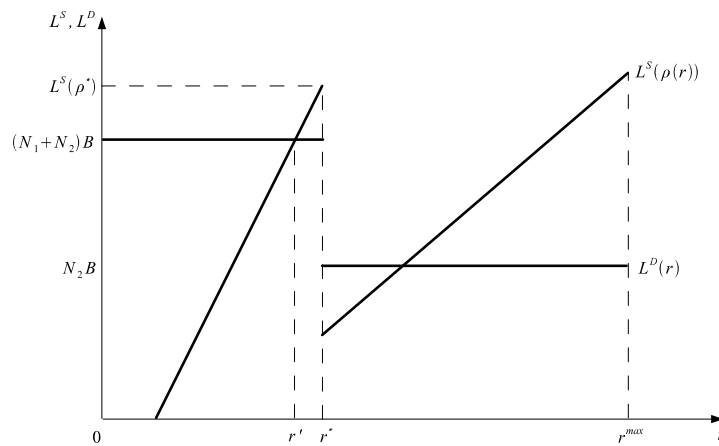

Figure 3 *Market clearing equilibrium with an equilibrium interest rate $r' < r^*$*

The second possible equilibrium is characterized by an intersection of demand and supply in the higher part of the demand function, $0 < L^S(\rho^*) < L^D(r^{\max})$. Also, in this case, we consider a market clearing equilibrium. However, the equilibrium is now located at interest rates $r' > r^*$ with $\rho > \rho^*$. Only risky borrowers demand and receive loan. We show this equilibrium graphically in figure 4.
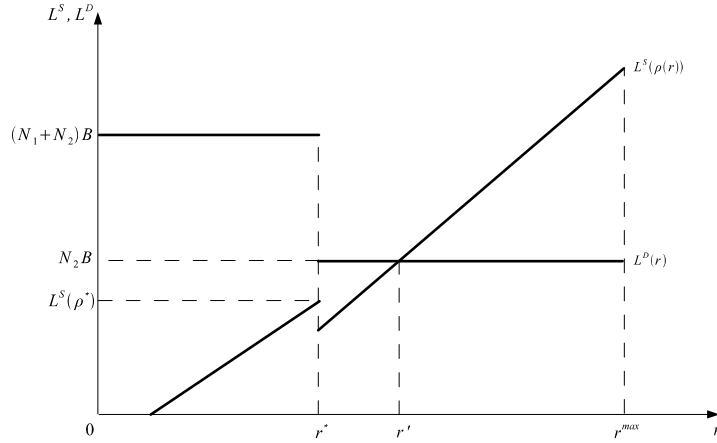
Figure 4 *Market clearing equilibrium with $r^* < r' < r^{\max}$*

As we focus on an equilibrium with credit rationing in this model, we assume a supply of loan that lies between the above described equilibria with $L^D(r^{\max}) < L^S(\rho^*) < L^D(r^*)$ (Stiglitz and Weiss, 1981) as shown in figure 5.
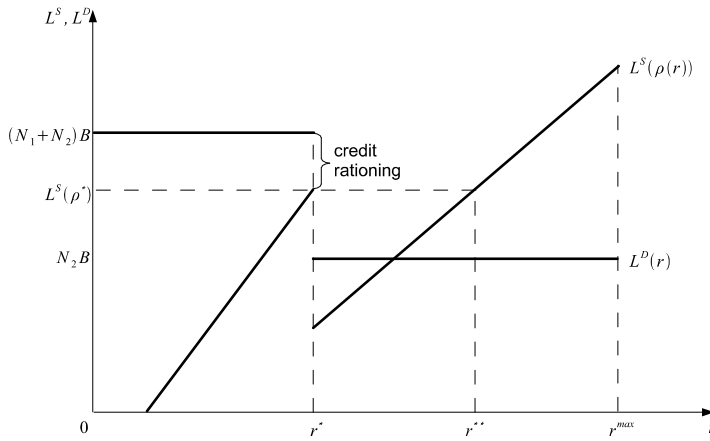

Figure 5 *The two-interest rate equilibrium*

First, we have to explain why a one interest allocation is no equilibrium in this case. As we know a bank receives at least the same return at interest rates larger r^{**} than at r^*. It follows therefore that it is not possible to serve borrowers only at r^*, where they

are randomly rationed. Due to positive residual demand at interest rates above r^*, the bank could achieve positive profits by offering small amounts of loan close to the return maximizing interest rate $r^{\max}$. That is why an allocation with a single equilibrium interest rate at r^* could not be an equilibrium. Another possible equilibrium could be one, where loans are only given at $r^{\max}$. At this point, the supply of loan is higher than the demand. But also this allocation is not an equilibrium. Another bank could offer loan at an interest rate slightly lower than $r^{\max}$ skim the whole demand and realize positive profits.

To show the two-interest rate equilibrium, we assume banks raise deposits $L^S(\rho^*)$ by paying the deposit rate ρ^* to depositors and loan is given at two interest rates, r^* and r^{**}. Furthermore, we think of a demand that is exceeding supply at the lower interest rate, but not at the higher so that $L^D(r^{**}) < L^S(\rho^*) < L^D(r^*)$. Finally, we assume $L_{r^*} + L_{r^{**}} = L^S(\rho^*)$ where $L_{r^{**}} = l^D(r^{**})$. Thus, we know

$$L_{r^{**}} = l^D(r^{**}) = L^S(\rho^*) - L_{r^*}. \tag{20}$$

At r^{**}, residual demand equals residual supply so

$$L_{r^*} + l^D(r^{**}) = L^S(\rho^*). \tag{21}$$

where $l^D(r^{**}) = \left[1 - \left(L_{r^*}/L^D(r^*)\right)\right] L^D(r^{**})$. Banks give loan L_{r^*} at interest rate r^* and satisfy the whole demand $L^S(\rho^*) - L_{r^*}$ at interest rate r^{**}. They receive the return $\varrho(r^*) = \varrho(r^{**}) = \rho^*$ from borrowers at both interest rates. Recall, we assume that the loan-subgame precedes the deposit-subgame and that at least two banks at each equilibrium interest rate offer loan. The following theorem states that the two-interest rate allocation is equilibrium in the model.

Theorem 1 *In the loan-subgame, at least two banks set $(r_k, \lambda_k) = (r^*, L_{r^*})$, while at least two other banks set $(r_k, \lambda_k) = (r^{**}, L^S(\rho^*) - L_{r^*})$. In the deposit-subgame, the bank that serves the loan demand at r^* sets $(\rho_k, \delta_k) = (\rho^*, L_{r^*})$ and the bank that gives loan at r^{**} sets $(\rho_k, \delta_k) = (\rho^*, L^S(\rho^*) - L_{r^*})$. For all other banks which are not giving loan choosing $(\rho_k, \delta_k) = (0, 0)$ is optimal.*

We prove the theorem by showing that it is not profitable for any bank to deviate from these strategies. First, we consider the deposit-subgame. We notice that the amount of deposits a bank has to raise to refinance itself is already determined in the loan-subgame. We assume a deposit rate, $\tilde{\rho}$, exists such that the amount of deposits raised at this rate is sufficient to refinance aggregate loan given. Note that $\tilde{\rho}$ does not necessarily have to equal ρ^*. It is an optimal strategy for all banks to set $(\rho_k, \delta_k) = (\tilde{\rho}, l_k)$. A bid of $\rho_k < \tilde{\rho}$ or $\delta_k < l_k$ would lead to a default as banks are not able to refinance completely. On the other hand, by bidding $\rho_k > \tilde{\rho}$ or $\delta_k > l_k$, banks would be able to refinance. However, this would raise the cost of refinancing to $(1 + \rho_k)d_k$. Since the loan given by the bank and the revenue is predetermined in the loan-subgame, this strategy is also not profitable. Thus, what happens to the banks that do not bid. The strategy $(\rho_k, \delta_k) = (0, 0)$ is optimal

for them, i.e. to raise no deposits. For collected deposits $\delta_k > 0$, the bank would have to pay $\rho_k > 0$, without getting any revenue from the loan side. A deviation from this strategy would therefore end up in losses for the bank. It follows that the deposit-subgame strategies described in the theorem 1 represent a Nash equilibrium.

Now we consider the loan-subgame. For that we take the optimal strategies from theorem 1 for the deposit-subgame as given. We show that it does not yield positive expected profits if banks deviate from the given strategies. The highest credit ceiling initially set by at least one bank at interest rate r_n is denoted as $\overline{\lambda}_{r_n}$. We know a bank gives loan L_{r^*} at r^* and another bank gives $L^S(\rho^*) - L_{r^*}$ at r^{**}. Suppose that the corresponding highest credit ceilings, $\overline{\lambda}_{r^*} = L_{r^*}$ and $\overline{\lambda}_{r^{**}} = L^S(\rho^*) - L_{r^*}$, are initially offered by at least two banks at each interest rate. As we assumed loan is only given at r^* and r^{**}, no bank offers a credit ceiling at another interest rate. In the following, we use the symbol Δ to denote a change in the variable.

We consider the following two strategies corresponding to aggregate loan given. We assume (i) a bank makes an additional loan supply at an interest rate r', at which no loan is given before, whereas the previous given credit ceilings persist and (ii) a bank that offers a higher credit ceiling $\Delta\overline{\lambda} > 0$ than each existing credit ceiling at interest rate r^* or r^{**}. The question is how the aggregate loan given $\Delta(\sum_n L_{r_n})$ is effected by these strategies. We find that at new interest rates below r^* or in the situation at which more loan is supplied at r^*, additional loan given is created. At all other interest rates above r^*, no additional loan given is created. We prove this in appendix C.

Knowing where additional credit is created and where not, we assume that the strategies named in theorem 1 mean zero profits for banks. If we could show that there is no profitable derivation, theorem 1 is fulfilled. We therefore look at the deposit rate the bank has to pay to its depositors. We have to bear in mind that the bank k is chosen by a random tie-breaking rule if more than one bank offers the same credit ceiling at one interest rate. Thus, we form a function that shows the probability p_{r_k} for a bank k to be chosen, which is given by

$$p_{r_k} = \begin{cases} 1, & \text{if } \lambda_k > \lambda_{k-1} \\ \frac{1}{k}, & \text{if } \lambda_k = \lambda_{k-1} \\ 0, & \text{if } \lambda_k < \lambda_{k-1}. \end{cases} \tag{22}$$

If bank k is bidding more loan at interest rate r_n than any other bank, it can give loan with $p_{r_k} = 1$. If the bid is equal to other bank's bid the probability to be chosen is $1/k$. If the bank's bid is smaller than all other bids, it will definitely not be chosen. By using this function we can write the expected profit or a bank k as

$$E(\pi_k) = p_{r_k}\varrho(r_k)l_k - \rho_k d_k \tag{23}$$

where l_k is loan given by a bank and d_k as the deposits collected by bank. To maximize their expected profits, the banks would only collect the amount of deposits they need to refinance its given loans, $d_k = l_k$. Hence, we can see that the aggregate deposits collected from banks can reach the maximum amount of aggregate loan given in the market $\sum_n L_{r_n}$.

We assume that there is enough supply to satisfy the entire demand at banks. However, this does not necessarily mean that all borrowers receive loan. With the assumption, we can conclude that a bank is always able to refinance its loan given completely by setting a sufficiently high deposit rate as shown before. Considering five deviating strategies, shown in appendix D, we find that setting $(r_k, \lambda_k) = (r^*, L^*)$ or $(r_k, \lambda_k) = (r^{**}, L^S(\rho^*) - L^*)$ in the loan-subgame is a Nash equilibrium. We have to repeat here that it is necessary that at each interest rate at least two banks set the credit ceiling as described in the equilibrium tuple. If one bank deviates from r^* or r^{**}, there has to be another bank that continues to offer loan at that interest rate. Otherwise deviating would possibly be profitable.

This consideration shows that the two-interest equilibrium given in figure 5 and theorem 1 must be the resulting equilibrium for a two borrower case in the Stiglitz-Weiss model.

5   The safe are rationed, the risky are not

As a first result we find, contrary to Stiglitz and Weiss (1981), that loans are supplied at two interest rates in equilibrium. This has important implications on the credit rationing. At the lower equilibrium interest rate, safe types are rationed and at the higher interest rate all risky types receive loans. The credit rationing is reduced compared to Stiglitz and Weiss (1981).

Furthermore, we suggest that credit rationing is decreasing as risk types of borrowers become more heterogeneous. To prove this statement we look at the Stiglitz-Weiss model at the one end, the Arnold-Riley model (Arnold and Riley, 2009) at the other end, and our model in between. Stiglitz and Weiss (1981) assume two borrower types and a hump-shaped return function for banks. A single interest rate in equilibrium appears, at which safe types pay larger interest rates than risk adjusted interest rates would suggest. Finally, all types are randomly rationed. The result still holds, when Stiglitz and Weiss (1981) relax their assumption of only two borrower types and replace it by a continuum of risk types. Arnold and Riley (2009) reconsider the assumption of a continuum of risk types to prove that the shape of the return function for banks has to be monotonic. They describe an equilibrium where loans are supplied at, at least, two interest rates. They argue that "...rationing is hard to explain using this model." (Arnold and Riley, 2009, p2013). In our model, we consider two risk types and are therefore faced to a double hump-shaped return function for banks. We observe an equilibrium in which loan is supplied at two interest rates. Only safe borrowers are rationed at the lower interest rate, risky borrowers are not rationed at all. Coming from Stiglitz and Weiss (1981) with credit rationing for all risk types via our model, where only certain types are rationed, to Arnold and Riley (2009) with not rationing at all, we suggest that credit rationing is declining as risk types become more heterogenous. However, credit rationing persists in equilibrium as long as we consider a discrete number of types.

6   Concluding remarks

Our paper presents an extension of the Stiglitz-Weiss model. We consider two borrower types and as a consequence, the return function for banks becomes double hump-shaped. We find that credit rationing exists in equilibrium only for certain risk types. To classify our findings, we view our paper as a generalization of the Stiglitz-Weiss model. We show that between the credit rationing for all types in Stiglitz and Weiss (1981) and no rationing at all in Arnold and Riley (2009) intermediate solutions exist. This suggests that the more heterogenous the borrower types are, the less credit is rationed in equilibrium.

Further research will include an analytical proof for a model with n borrower types, where $n > 2$ and n goes to infinity, as well as a numerical solution of the discrete risk type model.

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A Proof of $r^* < r^{\max}$

For $r^* < r^{\max}$ we get

$$\frac{\overline{R} - (1-p_1)C}{p_1 B} - 1 < \frac{\overline{R} - (1-p_2)C}{p_2 B} - 1. \tag{24}$$

Rewriting equation (24) gives

$$C < \overline{R}. \tag{25}$$

As we assume $\overline{R} > B > C$, this condition is fulfilled for all C and $\overline{R}$.

B Proof of residual demand at r^{**}

We have to consider two cases. First we assume $r^* < r' < r^{**}$. Using equation (7) the residual demand is given by

$$l^D(r') = \left[1 - \frac{L_{r^*}}{L^D(r^*)}\right] L^D(r'). \tag{26}$$

The residual demand at r^{**} is given by

$$l^D(r^{**}) = \left[1 - \frac{L_{r'}}{l^D(r')}\right] l^D(r^{**}). \tag{27}$$

with $l^D_{r'}(r^{**})$ denoting the residual demand at r^{**} when all applications for loan at r' are withdrawn, such that

$$l^D_{r'}(r^{**}) = \left[1 - \frac{L_{r^*}}{L^D(r^*)}\right] L^D(r^{**}). \tag{28}$$

Now inserting equation (28) and equation (26) into equation (27) gives

$$l^D(r^{**}) = \left[1 - \frac{L_{r'}}{\left[1 - \frac{L_{r^*}}{L^D(r^*)}\right] L^D(r')}\right] \left[1 - \frac{L_{r^*}}{L^D(r^*)}\right] L^D(r^{**}). \tag{29}$$

Rearranging implies equation (8), such that

$$l^D(r^{**}) = \left[1 - \frac{L_{r'}}{\left[1 - \frac{L_{r*}}{L^D(r^*)}\right]L^D(r')}\right]\left[1 - \frac{L_{r*}}{L^D(r^*)}\right]L^D(r^{**})$$

$$= \left[1 - \frac{L_{r*}}{L^D(r^*)} - \frac{L_{r'}}{\left[1 - \frac{L_{r*}}{L^D(r^*)}\right]L^D(r')} + \frac{L_{r'}L_{r*}}{\left[1 - \frac{L_{r*}}{L^D(r^*)}\right]L^D(r')L^D(r^*)}\right]L^D(r^{**})$$

$$= \left[1 - \frac{L_{r*}}{L^D(r^*)} - \frac{L_{r'}\left[L^D(r^*) - L_{r*}\right]}{\left[\frac{L^D(r^*) - L_{r*}}{L^D(r^*)}\right]L^D(r')L^D(r^*)}\right]L^D(r^{**})$$

$$= \left[1 - \frac{L_{r*}}{L^D(r^*)} - \frac{L_{r'}}{L^D(r')}\right]L^D(r^{**}). \tag{30}$$

which is the same expression as equation (8).

Secondly assuming $r' < r^* < r^{**}$ we have the residual demand at r^* given a loan given at r' from equation (7) as

$$l^D(r^*) = \left[1 - \frac{L_{r'}}{L^D(r')}\right]L^D(r^*). \tag{31}$$

With $l^D_{r*}(r^{**})$ denoting the residual demand at r^{**} if all demand at r^* is withdrawn, then we have at r^{**} the following residual demand

$$l^D(r^{**}) = \left[1 - \frac{L_{r*}}{l^D(r^*)}\right]l^D_{r*}(r^{**}), \tag{32}$$

with

$$l^D_{r*}(r^{**}) = \left[1 - \frac{L_{r'}}{L^D(r')}\right]L^D(r^{**}). \tag{33}$$

18

Now we use equation (31) and equation (33) in equation (32) and find

$$
\begin{aligned}
l^D(r^{**}) &= \left[1 - \frac{L_{r^*}}{\left[1 - \frac{L_{r'}}{L^D(r')}\right] L^D(r^*)}\right] \left[1 - \frac{L_{r'}}{L^D(r')}\right] L^D(r^{**}) \\
&= \left[1 - \frac{L_{r'}}{L^D(r')} - \frac{L_{r^*}}{\left[1 - \frac{L_{r'}}{L^D(r')}\right] L^D(r^*)} + \frac{L_{r^*} L_{r'}}{\left[1 - \frac{L_{r'}}{L^D(r')}\right] L^D(r^*) L^D(r')}\right] L^D(r^{**}) \\
&= \left[1 - \frac{L_{r'}}{L^D(r')} - \frac{L_{r^*}}{L^D(r^*)} \frac{L^D(r') - L_{r'}}{\left[1 - \frac{L_{r'}}{L^D(r')}\right] L^D(r')}\right] L^D(r^{**}) \\
&= \left[1 - \frac{L_{r^*}}{L^D(r^*)} - \frac{L_{r'}}{L^D(r')}\right] L^D(r^{**}).
\end{aligned}
\tag{34}
$$

which is equal to equation (8).

C Consideration of the possibility of creating additional loan in the loan-subgame

Generally, we can conclude, that when we assume the existence of a residual demand, a supply of loan at new interest rates, $\Delta \overline{\lambda}_{r'} > 0$ results in $\Delta L_{r'} > 0$. This leads either to a rise in the aggregates loan given over all interest rates, $\Delta(\sum_n L_{r_n}) > 0$, or to no change in the aggregate loan given, such that $\Delta(\sum_n L_{r_n}) = 0$. This will be explained below. If there is no residual demand, an increase of the credit ceiling at existing interest rates, $\Delta \overline{\lambda}_{r_n} > 0$ implies no new loan is given at new interest rates, $\Delta L_{r'} = 0$, and thus there is no change in the aggregate loan given, $\Delta(\sum_n L_{r_n}) = 0$.

Now we go deeper into the matter. A bank offers loan at an interest rate, at which no loan is given before. The following figure 6 provides a graphical illustration of five possible cases we have to look at.
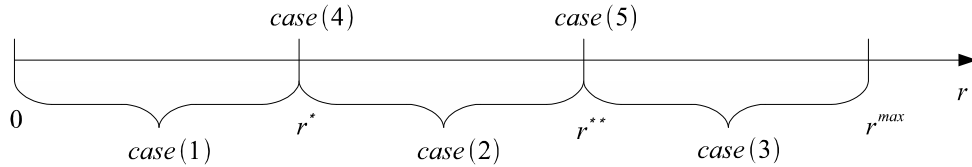

Figure 6 *Illustration of the five possible cases in the first strategy*

Cases one, two and three correspond to the first strategy, which means offering additional loan at new interest rates. Cases four and five are connected to the offer of a higher

credit ceiling at existing equilibrium interest rates. Table 1 shows the first three possible cases that occur using the first strategies mentioned above.

$\overline{\lambda}_{r_n}/r_n$	$r' < r^*$	$r^* < r' < r^{**}$	$r' > r^{**}$	$\Delta(\sum_n L_{r_n}) > 0$
Initial case	0	0	0	-
Case (1)	$\overline{\lambda}_{r'} > 0$	0	0	yes
Case (2)	0	$\overline{\lambda}_{r'} > 0$	0	no
Case (3)	0	0	$\overline{\lambda}_{r'} > 0$	no

Table 1 *the credit ceiling corresponding to offering additional loan at new interest rates*

In case (1), we look at interest rates r' that are lower than r^*. We assume a constant demand for loan as the demand function is a stepwise one, such that $L^D(r') = L^D(r^*)$. In the following we will consider three subcases depending on the amount of new loan offered. In case (a), we assume a persistent residual demand at r^{**}, in case (b), we show the situation in which there is no residual demand above r^*, and in case (c), we describe the case in which there is no residual demand even above r'. We will keep this notation also in the other cases, in order to give the consideration more structure. Looking at case (1a) we find that the additional supply at r' is offered to such an amount that there is still residual demand at r^{**}. This means that the credit ceiling at the new interest rate is smaller than the loan demand at r^* minus the loan given at r^*, so that $\lambda_{r'} < L^D(r^*) - L_{r^*}$. Hence, loan is given at three interest rates, r', r^*, and r^{**}. loan given at these three interest rates is then $L_{r'} = \lambda_{r'}$ at r', L_{r^*} at r^*, and at r^{**} loan given has to equal residual demand. In the beginning this amount was defined as $\overline{\lambda}_{r^{**}} = L^S(\rho^*) - L_{r^*}$. But now the amount of loan given at r^{**} has to be adjusted, so that

$$L_{r^{**}}^{new} < L_{r^{**}}^{old} \tag{35}$$

with

$$L_{r^{**}}^{new} = l_{new}^D(r^{**}) = \left[1 - \frac{L_{r'} + L_{r^*}}{L^D(r^*)}\right] L^D(r^{**}) \tag{36}$$

and

$$L_{r^{**}}^{old} = L^S(\rho^*) - L_{r^*} = \left[1 - \frac{L_{r^*}}{L^D(r^*)}\right] L^D(r^{**}) = \overline{\lambda}_{r^{**}} \tag{37}$$

Comparing new and old loan given at all interest rates yields an increase in aggregate loan given as

$$\Delta \sum_n L_{r_n} = L_{r'} \left[1 - \frac{L^D(r^{**})}{L^D(r^*)}\right] > 0. \tag{38}$$

The additional supply at r' leads to additional aggregate loan. Looking at case (1b), no excess demand at r^*, we find the credit ceiling offered at r', $\lambda_{r'}$, has to satisfy the following condition

$$L^D(r^*) - L_{r^*} \leq \lambda_{r'} < L^D(r^*). \tag{39}$$

Hence, all the residual demand is satisfied at r^*. At interest rates above r^*, residual demand does not exist anymore and so loan is not given there, as shown in equation (37). In this case, loan is given at two interest rates, r' and r^*. Aggregate loan given equals $L^D(r^*)$ which is higher than aggregate loan given before, $L^S(\rho^*)$. Adverse selection does not take place. All borrowers receive the amount of capital they demand. Hence, loan given rises when compared to the initial situation with credit rationing, so that

$$\Delta L_{r'} > 0 \Rightarrow \Delta(\sum_n L_{r_n}) > 0. \tag{40}$$

Case (1c) describes the situation if the credit ceiling at the new interest rate r' equals or exceeds demand at r'. This could be the case if $\lambda_{r'} \geq L^D(r^*)$. Then there is only one interest rate at which loan is given, i.e. r'. Clearly, this has exactly the same consequences for aggregate loan given as shown above: $\Delta(\sum_n L_{r_n}) > 0$. Summarizing the first case yields the result that a bank increases aggregate loan given if it offers an additional supply at interest rates below r^*, i.e. $\Delta L_{r'} > 0 \Rightarrow \Delta(\sum_n L_{r_n}) > 0$. This is due to the positive residual demand that initially existed at this interval and the persistent loan offers at r^* and r^{**}. With the new lower interest rate, the bank attracts all borrowers and credit rationing is reduced.

In case (2), we look at an additional rate at the interval $r^* < r' < r^{**}$. Demand remains constant in this interval. Again, we differentiate if supply is smaller (case (2a)), equal (case (2b) or larger (case (2c)) than residual demand. In case (2a), the additional supply at r' is smaller than the residual demand at this interest rate, i.e. if $\lambda_{r'} < L^S(\rho^*) - L_{r^*}$, loan is given at three interest rates. The residual demand at the higher rates decreases at exactly the same amount as additional loan is supplied, i.e. $\Delta l^D(r) = -L_{r'}$ for r'. With the additional loan supply, a bank can only attract borrowers of the risky type, because safe borrowers do not demand capital at this interest rate interval. So no rationed safe borrower has an additional chance of getting capital as was the case in case (1). At r^{**}, no excess demand exists and no selection process is at work. Hence, no additional aggregate loan is created. loan given is simply redistributed. Some of the risky borrowers simply pay a lower interest rate than their initial rate, while nothing changes for the safe borrowers. Hence, $\lambda_{r'} > 0$ implies $\Delta L_{r'} > 0$ and it follows that $\Delta(\sum_n L_{r_n}) = 0$. In case (2b), supply is equal, and in case (2c), supply is higher than the residual demand at r', the whole residual demand is satisfied at the new interest rate and no residual demand exists at the higher interest rates. loan is then given at two interest rates, at r^* and r'. No additional loan given is created, here either.

If we look at interest rates with $r^{**} < r'$, case (3), the same result as in case (2) prevails. No additional loan is created, but here for another reason. Above r^{**}, no residual demand exists. Credit ceilings offered above r^{**} cannot cause additional loan given, i.e. $\lambda_{r'} > 0 \Rightarrow \Delta L_{r'} = 0 \Rightarrow \Delta(\sum_n L_{r_n}) = 0$.

Now consider the second strategy where we assume that additional loan is offered at the existing equilibrium interest rate r^* or r^{**}. The results are given in table 2.

$\overline{\lambda}_{r_n}/r_n$	r^*	r^{**}	$\Delta(\sum_n L_{r_n}) > 0$
Initial case	L_{r^*}	$L^S(\rho^*) - L_{r^*}$	-
Case (4)	$\overline{\lambda}_{r^*} > L_{r^*}$	$L^S(\rho^*) - L_{r^*}$	yes
Case (5)	L_{r^*}	$\overline{\lambda}_{r^{**}} > L^S(\rho^*) - L_{r^*}$	no

Table 2 *The two possible cases corresponding to an increase in the credit ceiling at existing equilibrium interest rates*

First of all we consider case (4) where a bank is offering additional loan at r^*. Suppose in case (4a) that the increased credit ceiling the bank offers is still smaller than the residual demand at r^*, so that $(1 + \Delta)\overline{\lambda}_{r^*} < l^D(r^*) = L^D(r^*)$. Clearly at r^* more loan is given due to of excess demand. But to see what happens with aggregate loan given, we have to consider if and how loan changes at r^{**}. loan is still given at two interest rates, $L_{r^*}^{new} = (1 + \Delta)L_{r^*}$ at r^*, and at r^{**} loan given is

$$L_{r^{**}} = l^D(r^{**}) = \left[1 - \frac{(1 + \Delta)L_{r^*}}{L^D(r^*)}\right] L^D(r^{**}). \qquad (41)$$

The value in brackets is larger than zero and smaller than one. So, if the bank increases the credit ceiling at r^* such that loan given at r^* rises by ΔL_{r^*}, the residual demand decreases by less than ΔL_{r^*}. This is again a hint towards the fact that only safe borrowers are rationed. So there is an increase in total loan given on the market, i.e. $\Delta L_{r^*} > 0 \Rightarrow \Delta(\sum_n L_{r_n}) > 0$. If the new highest credit ceiling at r^* equals (case (4b)) or exceeds (case (4c)) demand, loan is only given at that interest rate and no borrower is rationed, i.e. $L_{r*} = L^D(r^*)$ and thus $\Delta L_{r^*} > 0 \Rightarrow \Delta(\sum_n L_{r_n}) > 0$.

In case (5), a bank offers a higher credit ceiling at r^{**}. At r^{**} by assumption, no excess demand exists. So no additional loan given can be created, as

$$\Delta\overline{\lambda}_{r^{**}} > 0 \Rightarrow \Delta L_{r^{**}} = 0 \Rightarrow \Delta\left(\sum_n L_{r_n}\right) = 0. \qquad (42)$$

The bank with the increased credit ceiling simply adopts the role of the bank which was giving loan at r^{**}.

D Five deviating strategies for a bank

In the following, we have to compare the interest rate r_n, deposit rate ρ_k and the expected profit $E(\pi_k)$ for each case (1) to (5) to find out if the deviating strategy is optimal or not. We illustrated this comparison in table 3.

	Case (1)	Case (2)	Case (3)	Case (4)	Case (5)
$\sum_n L_{r_n}$	$\uparrow$	$=$	$=$	$=$	$\uparrow$
r_n	$< \rho^*$	$< \rho^*$	not possible	$= \rho^*$	$= \rho^*$
ρ_k	$> \rho^*$	$= \rho^*$	not possible	$= \rho^*$	$> \rho^*$
$E(\pi_k)$	< 0	< 0	$= 0$	$= 0$	< 0
Optimal	no	no	no	no	no

Table 3 *Profit in the five cases*

Looking at case (1) we find that at new interest rates below r^*, a bank can attract firms by offering loan. This leads to higher aggregate loan given as shown before. Now we have to examine how the increase in aggregate loan given affects the expected profit of the deviating bank. In the deposit-subgame, the aggregate demand for deposits increases and the bank would therefore have to pay a higher deposit rate than ρ^* to prevent default. Thus in order to achieve zero profits the bank has to attain a higher return than $\varrho(r_k) = \rho^*$ in the loan-subgame. However, this is not possible for the interval $r' < r^*$ if we consider the assumed shape of the bank's return function. A deviation in the form of case (1) is not optimal for a bank.

In case (2), loan is offered at an interest rate $r^* < r' < r^{**}$. We show that aggregate loan given would remain unchanged because an additional offer only affects borrowers of the risky type that are not rationed in equilibrium anyway. However, this case (2) does not yield positive expected profits for a bank because its return in this interval $\varrho(r')$ is strictly smaller than ρ^*, while it would have to raise funds in the deposit-subgame by paying exactly ρ^*.

Now consider case (3) where loan is offered at interest rates $r' > r^{**}$. From the definition of L_{r^*} in equation(21), it follows that residual demand equals supply at the interest rate r^{**}. Thus above r^{**} there is no residual demand and there is consequently no possibility to create positive returns.

Setting a higher credit ceiling at r^* (case 4) leads to an increase in aggregate loan given and hence it is possible to create positive returns, $\varrho(r^*) = \rho^*$ in the loan-subgame. However, expected profit in contrast decreases because the bank would again have to pay a higher deposit rate ($> \rho^*$) to refinance loan given. Hence, by deviating as in case (4), the only possibility is to serve some loan demand and not achieve negative expected profits. However, even this does not yield positive expected profits and thus there is no incentive to deviate from the proposed equilibrium strategy.

Another possibility to deviate from the initial case is shown in case (5). The strategy consists of setting a higher credit ceiling than the bank that served loan demand at r^{**}, capturing all the loan demand at this interest rate, and achieving the return rate $\varrho(r^{**}) = \rho^*$. This also does not yield positive profits, instead yielding zero profits, as there was no excess demand at r^{**}.