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A Generalized Nash-Cournot Model for the North-Western European Natural Gas Markets with a Fuel Substitution Demand Function: The GaMMES Model.

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Abstract

This article presents a dynamic Generalized Nash-Cournot model to describe the evolution of the natural gas markets. The aim of this work is to provide a theoretical framework that would allow us to analyze future infrastructure and policy developments, while trying to answer some of the main criticisms addressed to Cournot-based models of natural gas markets. The major gas chain players are depicted including: producers, consumers, storage and pipeline operators, as well as intermediate local traders. Our economic structure description takes into account market power and the demand representation tries to capture the possible fuel substitution that can be made between the consumption of oil, coal and natural gas in the overall fossil energy consumption. We also take into account the long-term aspects inherent to some markets, in an endogenous way. This particularity of our description makes the model a Generalized Nash Equilibrium problem that needs to be solved using specialized mathematical techniques. Our model has been applied to represent the European natural gas market and forecast, until 2030, after a calibration process, consumption, prices, production and natural gas dependence. A comparison between our model, a more standard one that does not take into account energy substitution, and the European Commission natural gas forecasts is carried out to analyze our results. Finally, in order to illustrate the possible use of fuel substitution, we studied the evolution of the natural gas price as compared to the coal and oil prices. This paper mostly focuses on the model description.

keywords

Energy markets modeling, Game theory, Generalized Nash-Cournot equilibria, Quasi-Variational Inequality.

1 Introduction

Quantitative studies and mathematical models are necessary to understand the economic and strategic issues that define energy markets in the world. In that vein, the study of natural gas markets is particularly interesting because most of them, particularly in Europe, show a high dependence on a small number of producers exports. According to Mathiesen & al. [29], this market structure can be analyzed thanks to strategic interactions and market power. This

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market power can be exerted at the different stages of the gas chain: by the producers in the upstream market or the local intermediate traders in the downstream market. The European markets are also characterized by long-term contracts established between the producers and the intermediate local independent traders. These long-term contracts were initially designed as a risk-sharing measure between producers and local traders. They are usually analyzed, in particular, as a tool to mitigate the producers market power. The combination of the strategic interactions and the long-term contracts makes the study of the natural gas markets evolution particularly subtle and rich.

The economic literature provides an important panel of numerical models whose objective is to describe the natural gas trade structure. As an example, we can cite the "World Trade Gas Model" (Baker Institute) [34], the "EUGAS" model (Cologne University) [33], the "GASTALE" model (Energy Research Centre of the Netherlands) [27] or the "World Gas Model" (University of Maryland) ([7], an extension of the work developed in [12] and [13]). However, most of these models present some necessary simplifying assumptions concerning either the description of the market economic structure or the demand function. For instance, the "EUGAS" model assumes pure and perfect competition between the players and thus neglects market power to allow a detailed description of the infrastructure. The "GASTALE" and "World Gas Model" depict strategic interactions between the players via a Nash-Cournot competition and the latter model also uses exogenous long-term contracts. However, the former model does not include investments in production or in pipeline and storage infrastructure. Besides, the demand representation for all these previous models does not take explicitly into account the possible substitution between different types of fuels (natural gas, oil and coal for instance). All these drawbacks have been analyzed in detail in [36]

The model we develop, named GaMMES, Gas Market Modeling with Energy Substitution, tries to address some of the limitations proposed in [36]. It also is based on an oligopolistic approach of the natural gas markets. The interaction between all the players is a Generalized Nash-Cournot competition and we explicitly take into consideration, in an endogenous way, the long-term contractual aspects (prices and volumes) of the markets. Our representation of the demand is new and rich because it includes the possible substitution, within the overall energy consumption, between different types of fuels. Hence, in our work, we mitigate market power exerted by the strategic players: they cannot force the natural gas price up freely because some consumers would switch to other fuels.

We study both the upstream and downstream stages of the gas chain, while modeling the possible strategic interactions between all the players, through all the stages. The production side is detailed at the production field level and we choose a functional form derived from Golombek [16] for the production costs. We assume, in our representation that the producers sell their gas through long-term contracts to a set of independent traders who sell it back to end-users, where the Nash-Cournot competition is exerted. Storage and transportation aspects are taken care of by global regulated storage and transportation operators. Producers also have the possibility to directly target end-users for their sales. Both producers and independent traders share market power. The long-term contracts are endogenous to our model and this property (among others) makes our formulation a Generalized Nash-Cournot game. The introduction in our model of the independent traders, that can exert market power in the sport markets is a new feature in the description of the natural gas trade. This allows us to represent long-term contracts and mitigate the producers' market power.

The demand side is also detailed. We use a system dynamics approach [2] in order to model possible fuel substitutions within the global energy demand of a consuming country, between the consumption of coal, oil and natural gas. This approach allows us to derive a new and interesting mathematical functional form for the demand function that includes naturally the competition between these. This particular new feature of the gas markets description, we have introduced in our model, induces a flexibility in the gas demand representation. It allows us, for example, to study the sensitivity of the gas consumption and prices over the oil and coal prices.

We include all the possible investments in the gas chain (production, infrastructure, etc.) and make the long-term contracts prices and quantities endogenous to the model thanks to an MCP (mixed complementarity problem) formulation.

The remaining parts of the paper are as follows: the first part is a general description of the choosen economic structure representation. All the players are presented and are divided into two categories: the strategic and the non-strategic ones. The strategic interaction is also detailed in this part. The second part presents the notation used and a brief description of a system dynamics approach to model the consumers' behavior investment in coal, oil or natural gas so that their utility is optimized. The third part is dedicated to the mathematical representation of the markets: the optimization programs associated with all the strategic and non-strategic players are presented and discussed. We also explain in this part how we make the long-term contracts' prices and volumes endogenous to our model. The next part is an application of our model to the European natural gas trade where the calibration process and the results are discussed. A comparison between our model, a more standard one where the demand does not take into consideration fuel substitution and the European Commission natural gas forecast is carried out in order to compare between the results. The last part summarizes the work.

2 The model

2.1 Economic description

Our description of the natural gas markets divides them into two stages.

The upstream market is represented by gas producers, each with a dedicated trader (export division) to sell gas to other traders or directly to end-users. An example would be Gazexport for Gazprom. The set of producers and dedicated traders is denoted as P.

Besides the market players just mentioned, there are a number of independent traders whose activity is to buy gas from the big producers (or their traders) and to sell it to the final users in the downstream market. This type of traders includes all the firms whose production is small, compared to their sales (e.g., EDF and GDF-SUEZ[1]). The associated index for these players is I.

The different target markets (the consumers) are divided into three sectors: power generation, industrial, and residential, represented respectively as D_1, D_2 and D_3. However, it is easy to demonstrate that if the sectors do not interact with each other (i.e., the different demand curves are independent), the study of only one sector can easily be generalized to the three. We will make the assumption that the different demand curves do not interact (as an example, the gas price in the industrial sector does not depend *a priori* on the residential price), which may not be realistic for some situations. Hence, to simplify our notation and modeling, we will consider only one consumption set D to represent each country total gross natural gas consumption.

[1]GDF-SUEZ produces 4.4 % of its natual gas supplies [15]

We assume that each dedicated trader can either establish long-term contracts with the independent traders or sell his gas to the spot markets.

The first situation corresponds to a gas trade under a fixed, contracted price, not dependent on the quantities sold (in a first approximation). These quantities are also fixed by the contract. The second situation is characterized by the fact that the spot price is a consequence of the competition between all the traders in the downstream markets, via a specified inverse demand function.

All the traders compete via a Nash-Cournot interaction, during a finite number of years Num. Time will be indexed by $t \in T$ (five-year time steps) and we will take into account seasonality by distinguishing, for each year t, between the off-peak and peak seasons. The seasons will be indexed by M. They correspond basically to different demand regimes.
The main advantage of the GaMMES model is that it takes into account, in an endogenous way, long-term contracts between the independent traders and the producers. Obviously, this representation is quite realistic for the natural gas trade since the latter is still dominated by long-term selling/purchase prices and volumes. The long-term contracts imports represented, in 2004, more than 46% of the European natural gas consumption and 80% of the total European imports [8] and [22]. Another advantage inherent to our description is that the inverse demand function takes explicitly into consideration the possible substitution between consumption for natural gas and the competing fuels.

Considering the energy substitutions in the natural gas demand mitigates the market power that can be exerted by all the strategic players in the end-use markets. Indeed, this is due to the fact that the consumers have the ability to reduce the natural gas share in their energy mixes if the market price for natural gas is much higher than the substitution fuel's (such as oil and coal) price. Therefore, the producers may not have a considerable incentive to reduce their natural gas production in order to force the price up. This model property allows us to take into account the natural gas price dependence on oil and coal prices. Indeed, the Nash-Cournot interaction will link the natural gas price to the coal and oil prices because of the demand function dependence on these parameters.

In order to take into consideration the intra and extra-European physical network of the transport and distribution networks, we need to introduce a pipeline operator whose role is to minimize the transmission costs over all the arcs of the topology. We denote by N the set of all the nodes including the production fields, the consuming markets and the storage sites. Added to the transport cost minimization objective, the pipeline operator has also the possibility to make investments in order to increase the arc capacities, if necessary.

All the arc transport costs are exogenous to the model. The congestion prices are taken into consideration endogenously: they can be obtained by computing the dual variables corresponding to the infrastructure capacity constraint. The set of all these arcs is A. An arc can either be a pipeline or an LNG route.

In order to be able to meet high levels of consumption, we assume that the independent traders have access to a set of storage sites to store natural gas in the off-peak season, and withdraw it in the peak one. Obviously, they have to support a capacity reservation, storage, withdrawal and transport costs. All the storage nodes, indexed by the set S, are managed by a global storage operator player. This player can invest in order to increase the storage capacity of each site.

Both the pipeline and storage operators are assumed not to have market power. The storage and transport costs are hence exogenous to the model. The strategic players are therefore the producers/dedicated traders and the independent traders. Obviously, this assumption is an important simplification of reality, where market power can also be exerted by the storage and pipeline operators. However, it is consistent with what can be found in the literature [7], [27]. The storage cost, which is assumed to be supported by the independent traders, is represented thanks to capacity reservation and storage/withdrawal costs. We consider that the average time for the storage investments to be realized is $delay_s$ years (five years). The situation is similar for the infrastructure ($delay_i$) and production capacity investments ($delay_p$) costs supported by the pipeline operator and the producers.

We take into consideration the depreciation of the production capacity in the upstream side of the market by introducing a depreciation factor per time unit, at each production node: dep_f. To simplify the model, (and because of lack of data concerns) we decided not to take into account the transport capacity depreciations.

2.2 Notation

The units choosen for the model are the following: quantities in toe (i.e., Ton Oil Equivalent) or Bcm and unit prices in \$/toe or \$/cm.
The following table summarizes the notation chosen for the exogenous parameters and the endogenous variables.

Exogenous factors

P	set of producers-dedicated traders
I	set of independent traders
D	set of gas consuming countries in the downstream market (no distinction between the sectors) $D \subset N$
T	time $T = \{0, 1, 2, ..., Num\}$
M	set of seasons. Off-peak (low-consumption) and peak (high-consumption) regimes
F	set of all the gas production fields. $F \subset N$
N	set of the nodes
S	set of the storage sites $S \subset N$
A	set of the arcs (topology)
Rf_f	field f's total gas resources (endowment)
Kf_f	field f's initial capacity of production, year 0
Ic_s	injection marginal cost at storage site s (constant)
Wc_s	withdrawal marginal cost at storage site s (constant)
Rc_s	reservation marginal cost at storage site s (constant)
Pc_f	production cost function, field f
Tc_a	transport marginal cost through arc a (constant)
Tk_a	pipeline initial capacity through arc a, year 0
Ks_s	initial storage capacity at site s, year 0
Is_s	investment marginal costs in storage (constant)
Ip_f	investment marginal costs in production (constant)
Ik_a	investment marginal costs in pipeline capacity through arc a (constant)
O	incidence matrix $\in \mathrm{M}_{F \times P}$. $O_{fp} = 1$ if and only if producer p owns field f
B	incidence matrix $\in \mathrm{M}_{I \times D}$. $B_{id} = 1$ if and only if trader i is located at the consumption node d
$M1$	incidence matrix $\in \mathrm{M}_{F \times N}$. $M1_{fn} = 1$ if and only if node n has field f
$M2$	incidence matrix $\in \mathrm{M}_{I \times N}$. $M2_{in} = 1$ if and only if trader i is located at node n
$M3$	incidence matrix $\in \mathrm{M}_{D \times N}$. $M3_{dn} = 1$ if and only if node n has market d
$M4$	incidence matrix $\in \mathrm{M}_{S \times N}$. $M4_{sn} = 1$ if and only if node n has storage site s
$M5$	incidence matrix $\in \mathrm{M}_{A \times N}$. $M5_{an} = 1$ if and only if arc a starts at node n
$M6$	incidence matrix $\in \mathrm{M}_{A \times N}$. $M6_{an} = 1$ if and only if arc a ends at node n
H	maximum value for the quantities produced and consumed

We could have used different upper bounds for the different variables. However, to simplify the notation, we will use the same value H.

fl_f	field f's flexibility: the maximum modulation between production during off-peak and peak seasons
min_{pi}	percentage of the minimum quantity that has to be exchanged on the long-term contract trade between i and p
δ	discount factor
$delay_{s,i,p}$	period of time necessary to undertake the technical investments
$loss_a$	loss factor through arc a
dep_f	depreciation factor of the production capacity at field f

Endogenous variables

x^t_{mfpd}	quantity of gas produced by p from field f for the end-use market d, year t, season m in Bcm
zp^t_{mfpi}	quantity of gas produced by p from field f dedicated to the long-term contract with trader i, year t, season m in Bcm
zi^t_{mpi}	quantity of gas bought by trader i from producer p with a long-term contract year t, season m in Bcm
up_{pi}	quantity of gas sold by producer p to trader i with a long-term contract, each year in Bcm
ui_{pi}	quantity of gas bought by trader i from producer p on the long-term contract, each year in Bcm
y^t_{mid}	quantity of gas sold by i to the market d, year t, season m in Bcm
ip^t_{fp}	producer p's increase of field f's production capacity, due to investments in production, year t in Bcm/time unit
q^t_{mfp}	production of producer p from field f, year t, season m in Bcm
p^t_{md}	market d's gas price, result of the Cournot competition between all the traders, year t, season m in \$/cm
η_{pi}	long-term contract price contracted between producer p and trader i in \$/cm
r^t_{is}	amount of storage capacity reserved by trader i at site s, year t in Bcm
in^t_{is}	volume injected by trader i at site s, year t in Bcm
is^t_s	increase of storage capacity at site s, year t due to the storage operator investments in Bcm/time unit
ik^t_a	increase of the pipeline capacity through arc a, year t, due to the TSO investments in Bcm/time unit
fp^t_{mpa}	gas quantity that flows through arc a from producer p year t, season m in Bcm
fi^t_{mia}	gas quantity that flows through arc a from trader i year t, season m in Bcm
τ^t_{ma}	the dual variable associated with arc a capacity constraint year t, season m in \$/cm. It represents the congestion transportation cost over arc a

The table is divided into two parts. The upper half represents the exogenous parameters or functions whereas the lower half represents the different decision variables and the inherent retail prices.

The indices p, d, i, f, n, s, a, m and t are such that $p \in P$, $d \in D$, $i \in I$ $f \in F$, $n \in N$, $s \in S$, $a \in A$, $m \in M$ and $t \in T$.

The long-term contract between producer p and trader i fixes both a unit selling price and an amount to be purchased by the independent trader i each year from producer p. Both price and quantity will be specified endogenously by the model.

Matrix O is such that $O_{fp} = 1$ if producer p owns field f and $O_{fp} = 0$ otherwise.

Figure 1 represents a schematic overview of GaMMES.

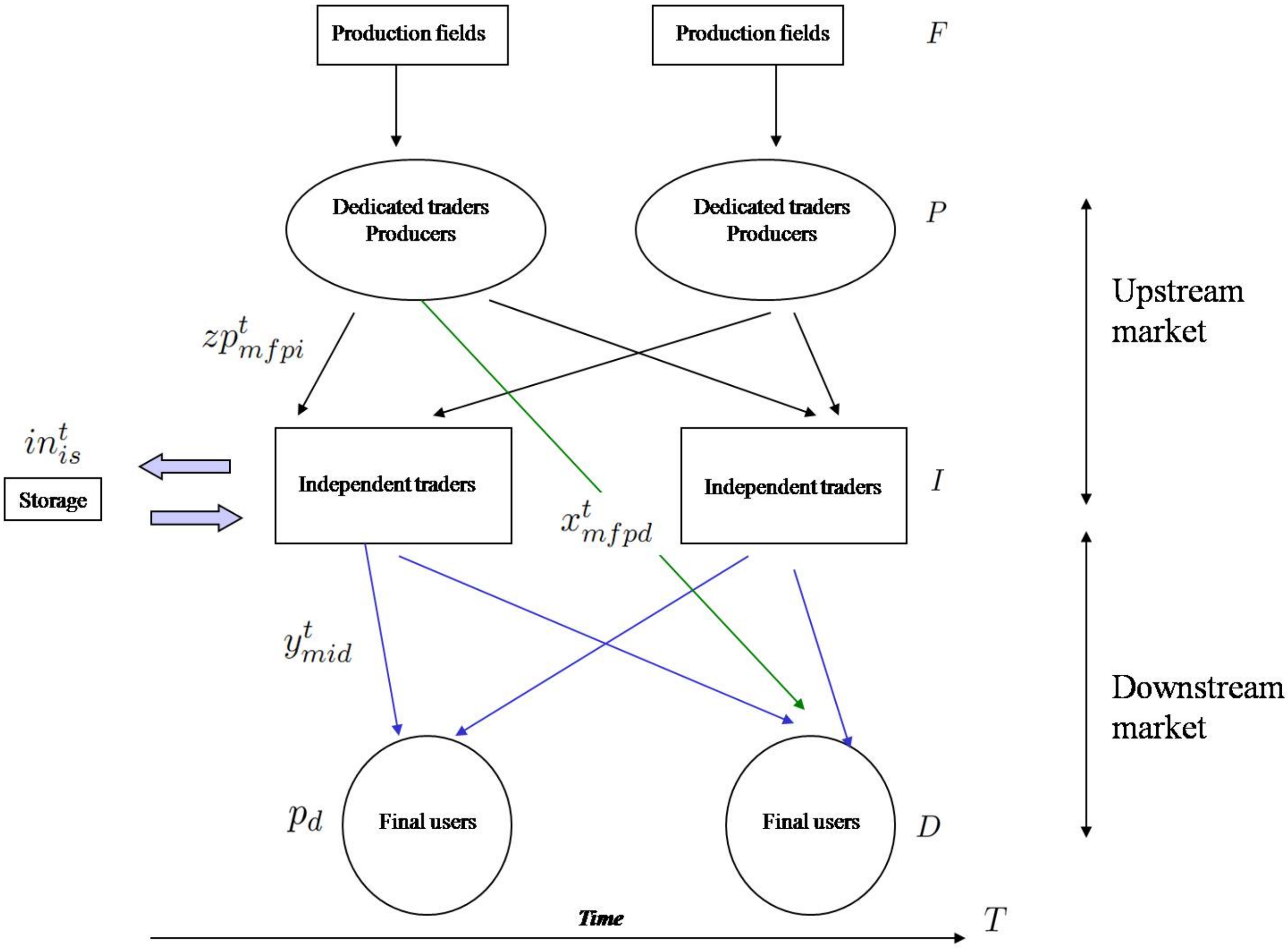

Figure 1:
The market representation in GaMMES

2.3 The inverse demand function

We need to specify a functional form for the inverse demand function, which links the price p_d at market d to the quantity brought to the market. Most of the natural gas models [34], [33], [27], [7] do not take into account fuel substitution. Let h be the specific inverse demand function. We assume that the long-term contract quantities do not directly influence the market competition price, which is to say that $p^t_{md} = h(\sum_i y^t_{mid} + \sum_f \sum_p x^t_{mfpd})$. (Actually, this assumption is necessary to guarantee the concavity of the objective functions of each strategic player's maximization problem, regardless of the quantities decided by the other competitors. Otherwise, this assumption can be dropped if linear functions are used).
As mentioned in the introduction, we want to capture the inter-fuel substitution in the global energy consumption. To be able to do so, we used a system dynamics approach that models

the behavior of the consumers who have to decide whether they invest in new burners that use either oil, coal or natural gas. Our putty-clay model, based on the work presented in [32], is fully developed in [2]. If we denote by Q^t_{md} the quantity $\sum_i y^t_{mid} + \sum_f \sum_p x^t_{mfpd}$, our gas demand study [2] provides the following inverse demand function:

$$p^t_{md} = \begin{array}{ll} pc^t_{md} + \frac{1}{\gamma^t_{md}} \text{atanh}\left(\frac{\alpha^t_{md}+\beta^t_{md}-Q^t_{md}}{\alpha^t_{md}}\right) & \text{if } Q^t_{md} \geq \beta^t_{md} + \frac{\alpha^t_{md}\beta^t_{md}}{\alpha^t_{md}+\beta^t_{md}} \\ p'c^t_{md} + \frac{1}{\gamma'^t_{md}} \text{atanh}\left(\frac{\alpha'^t_{md}+\beta'^t_{md}-Q^t_{md}}{\alpha'^t_{md}}\right) & \text{if } Q^t_{md} \leq \beta^t_{md} + \frac{\alpha^t_{md}\beta^t_{md}}{\alpha^t_{md}+\beta^t_{md}} \end{array} \quad (1)$$

where the parameters α, β, γ and pc, which are time and season-dependent must be calibrated.

The distinction between the domains $Q^t_{md} \geq \beta^t_{md} + \frac{\alpha^t_{md}\beta^t_{md}}{\alpha^t_{md}+\beta^t_{md}}$ and $Q^t_{md} \leq \beta^t_{md} + \frac{\alpha^t_{md}\beta^t_{md}}{\alpha^t_{md}+\beta^t_{md}}$ is needed to take into account the anticipated scrapping of burners and avoid absurd situations where the price rises towards $+\infty$ (and also to guarantee the concavity of the objective functions). The parameters α', β', γ' and $p'c$ are calculated to guarantee the continuity of h and its derivative h'. To make the price converge toward 0 when the quantity goes to $+\infty$, we need to force $\beta' = 0$

The function atanh is such that:

$$\forall x \in (-1, 1) \ \text{atanh}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

The following table gives the values of the inverse demand function parameters, for the global natural gas consumption in year 2003 in France, Germany, Italy, the UK, Belgium and the Netherlands. The natural gas volumes in 2002 are exogenous.

Parameters	France	Germany	Italy	UK	Belgium	The Netherlands
$\beta(\times 10^3$ktoe)	22.87	43.70	41.28	41.88	22.89	23.49
$\alpha(\times 10^3$ktoe)	2.76	4.00	3.60	2.80	2.76	1.05
p_c(\$/toe)	172.5	242.9	268.3	175.8	230.4	217.5
$\gamma(\times 10^{-2}(\$/\text{toe})^{-1})$	0.72	0.98	0.96	1.00	1.48	0.88
$\beta'(\times 10^3$ktoe)	0.00	0.00	0.00	0.00	0.00	0.00
$\alpha'(\times 10^3$ktoe)	13.20	24.67	23.23	23.18	13.20	12.81
p'_c(\$/toe)	350.8	404.1	441.2	379.5	316.6	549.1
$\gamma'(\times 10^{-2}(\$/\text{toe})^{-1})$	0.96	1.03	0.96	0.79	1.99	0.48

Figure 2 gives the demand function shape (i.e., the variation of the quantity Q_d over the price p_d in a given market).

Actually, as described in the economic description of the markets, we need to distinguish between the off-peak/peak season parameters of the inverse demand function.

To calibrate the demand function for the future, we need to specify a scenario for the global fossil energy demand and the oil and coal market prices. Our system dynamics approach [2] will allow us to understand how the global demand is going to be shared between the consumption of the three fuels.

2.4 The mathematical description

This section details the mathematical description of our model. It presents the optimization problems of all the supply chain players. Note that the dual variables are written in parentheses

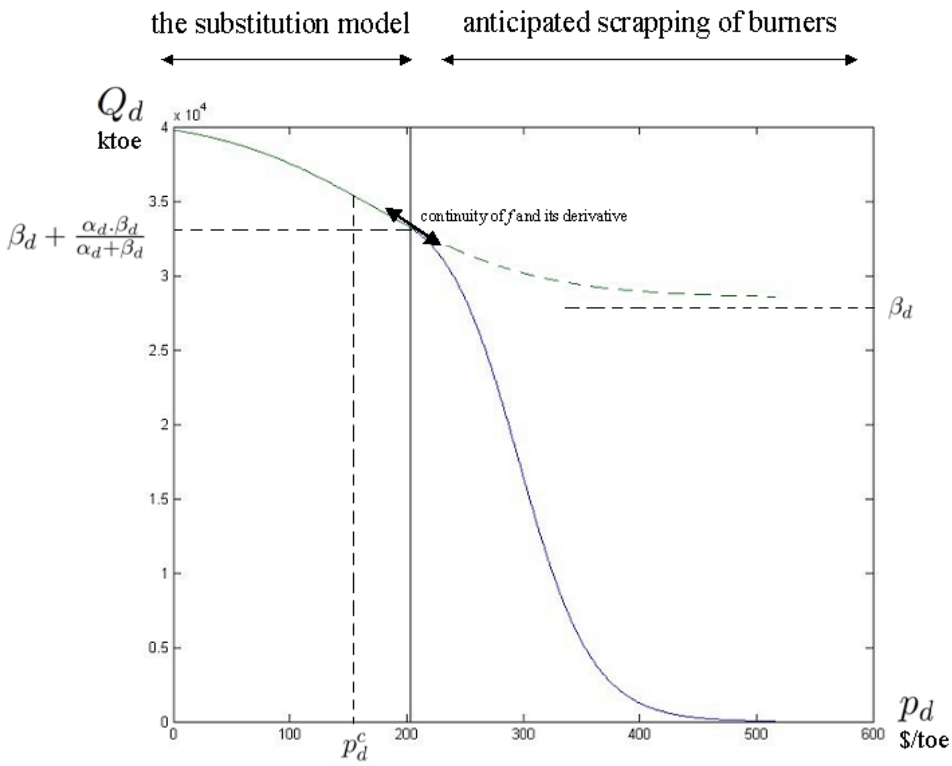

Figure 2:
The demand function

by their associated constraints.

Producer p's maximization program is given below. The corresponding decision variables are zp^t_{mfpi}, x^t_{mfpd}, ip^t_{fp}, q^t_{mfp} and up_{pi}.

Max

$$
\begin{aligned}
&\sum_{t,m,f,i} \delta^t \eta_{pi}(zp^t_{mfpi}) \\
&+ \sum_{t,m,f,d} \delta^t \left(p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}}) \right) x^t_{mfpd} \\
&- \sum_{t,f} \delta^t Pc_f \left(\sum_{t' \leq t} \sum_{m} q^{t'}_{mfp}, Rf_f \right) \\
&+ \sum_{t,f} \delta^t Pc_f \left(\sum_{t' < t} \sum_{m} q^{t'}_{mfp}, Rf_f \right) \\
&- \sum_{t,f} \delta^t Ip_f ip^t_{fp} \\
&- \sum_{t,m,p,a} \delta^t ((Tc_a + \tau^t_{ma}) fp^t_{mpa})
\end{aligned}
$$

such that:

$$
\begin{array}{rlrll}
\forall t,\ f, & \sum_p \sum_{t' \leq t} \sum_m q^{t'}_{mfp} - Rf_f & \leq 0 & (\phi^t_f) & \text{(2a)} \\
\forall t,\ f,\ m, & \sum_p q^t_{mfp} - Kf_f(1-dep_f)^t & & & \\
 & -\sum_p \sum_{t' \leq t - delay_p} ip^{t'}_{fp}(1-dep_f)^{t-t'} & \leq 0 & (\chi^t_{mf}) & \text{(2b)} \\
\forall t,\ m,\ f, & -q^t_{mfp} + \left(\sum_i zp^t_{mfpi} + \sum_d x^t_{mfpd} \right) & \leq 0 & (\gamma^t_{mfp}) & \text{(2c)} \\
\forall t,\ f & \sum_m \sum_p ((-1)^m q^t_{mfp}) - fl_f & \leq 0 & (\vartheta 1^t_f) & \text{(2d)} \\
\forall t,\ f, & -\sum_m \sum_p ((-1)^m q^t_{mfp}) - fl_f & \leq 0 & (\vartheta 2^t_f) & \text{(2e)} \\
\forall t,\ f,\ d,\ m, & x^t_{mfpd} - O_{fp}H & \leq 0 & (\epsilon 1^t_{mfpd}) & \text{(2f)} \\
\forall t,\ f,\ i,\ m, & zp^t_{mfpi} - O_{fp}H & \leq 0 & (\epsilon 2^t_{mfpi}) & \text{(2g)} \\
\forall t,\ f,\ m, & q^t_{mfp} - O_{fp}H & \leq 0 & (\epsilon 3^t_{mfp}) & \text{(2h)} \\
\forall t,\ f, & ip^t_{fp} - O_{fp}H & \leq 0 & (\epsilon 4^t_{fp}) & \text{(2i)} \\
\forall t,\ m,\ n, & \sum_a M6_{an} fp^t_{mpa}(1-loss_a) - \sum_a M5_{an} fp^t_{mpa} & & & \\
 & +\sum_f M1_{fn} q^t_{mpf} - \sum_d \sum_f M3_{dn} x^t_{mfpd} & & & \\
 & -\sum_i \sum_f M2_{in} zp^t_{mfpi} & = 0 & (\alpha p^t_{mpn}) & \text{(2j)} \\
\forall t,\ i, & up_{pi} - \sum_{f,m} zp^t_{mfpi} & = 0 & (\eta p^t_{pi}) & \text{(2k)} \\
\forall\ i, & ui_{pi} - up_{pi} & = 0 & (\eta_{pi}) & \text{(2l)} \\
\forall t,\ m,\ d,\ i,\ f, & zp^t_{mfpi},\ x^t_{mfpd},\ ip^t_{fp},\ q^t_{mfp},\ up_{pi} & \geq 0 & &
\end{array}
$$

We denote by $\overline{x^t_{mfpd}}$ the total amount of gas brought in year t, season m to the market d by all the players different from producer p.
The term

$$\sum_{t,m,f,i} \delta^t \eta_{pi}(zp^t_{mfpi}) + \sum_{t,m,f,d} \delta^t \left(p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}}) \right) x^t_{mfpd}$$

is the revenue, which is obtained from the sales on the long-term contracts sales to the independent traders or directly from the retail markets.
The term

$$\sum_{t,m,p,a} \delta^t((Tc_a + \tau^t_{m,a}) fp^t_{mpa})$$

is the transport and congestion costs charged by the pipeline operator to producer p. The dual variable τ^t_{ma} is associated with the pipeline capacity constraint through the arc a. It represents the congestion price on the corresponding pipeline (see the transport operator optimization problem for more explanation).
The term

$$\sum_{t,f} \delta^t Ip_f ip^t_{fp}$$

is the investment cost in production at the different production fields.
The term

$$\sum_{t,f} \delta^t \left(Pc_f \left(\sum_{t' \leq t} \sum_m q^{t'}_{mfp}, Rf_f \right) - P\_c_f \left(\sum_{t' < t} \sum_m q^{t'}_{mfp}, Rf_f \right) \right)$$

is the actualized production cost. This term's explanation is as follows:
The production cost (at field f) Pc_f depends on two variables, the total quantity produced, which will be denoted q and the natural gas resources Rf_f. The Golombek production cost function we used is as follows:

$$\forall q \in [0, Rf_f),\ Pc_f(q, Rf_f) = a_f q + b_f \frac{q^2}{2} - \ Rf_f c_f \left(\frac{Rf_f - q}{Rf_f} \ln \left(\frac{Rf_f - q}{Rf_f} \right) + \frac{q}{Rf_f} \right) \tag{3}$$

or if written for the marginal production cost

$$\forall q \in [0, Rf_f),\ \frac{\mathrm{d}Pc_f}{\mathrm{d}q} = a_f + b_f q \ + \ c_f \ln \left(\frac{Rf_f - q}{Rf_f} \right) \tag{4}$$

In our model, the production cost function is dynamic. The gas volume available to be extracted is dynamically reduced at each period, taking into account the exhaustivity of the resource.
If at year 1, the production is $q1$ and at year 2 $q2$, the total cost is hence:

$$cost = Pc_f(q1, RES_f) + \delta(Pc_f(q1 + q2, RES_f) - Pc_f(q1, RES_f))$$

Hence, to estimate that cost at year t, we need to calculate the production cost of the sum over all the extracted volumes until year t and subtract the cost we have at year $t - 1$.

The explanation of the constraints is straightforward:
The constraint (2a) bounds each field's production by its reserves.
The constraint (2b) bounds the seasonal quantities produced by each field's production capacity, taking explicitly into account the different dynamic investments, that decrease with time because of the production depreciation factor.

The constraint (2c) states that the total production must be greater than the sales (to the long-term and spot markets). The constraints (2d) and (2e) can be rewritten as follows:

$$\forall t,\ f \mid \sum_m ((-1)^m \sum_p q^t_{mfp})| \leq fl_f$$

This fixes a maximum spread between the off-peak/peak production at each field. $(-1)^m$ is equal to 1 in the off-peak season and -1 in the peak season.
The consraint (2j) is a market-clearing condition at each node, regarding the flows from producer p depending on whether this node is a production field, an independent trader location or a demand market.
The constraint (2k) equates the sales of producer p for the long-term contracts to the contracted volume up_{pi}, each year.
The constraint (2l) describes the following: For each pair of producer/independent trader (p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contracts prices and volumes endogenous to the description so that they become an output of the model.
The constraint (and the similar other ones) (2f) allows producer p to use only the fields he owns (for production, investments, sales etc.). We recall that the incidence matrix O is such as $O_{fp} = 1$ if and only if producer p owns field f.

Independent trader i's maximization program is given below. The corresponding decision variables are zi^t_{mpi}, y^t_{mid}, r^t_{is}, in^t_{is} and ui_{pi}.

Max

$$
\begin{aligned}
&\sum_{t,m,d} \delta^t \left(p^t_{md}(y^t_{mid} + \overline{y^t_{mid}}) y^t_{mid} \right) \\
&- \sum_{t,p,m} \delta^t \left(\eta_{pi} zi^t_{mpi} \right) \\
&- \sum_{t,s} \delta^t \left(Rc_s r^t_{is} \right) \\
&- \sum_{t,s} \delta^t \left((Ic_s + Wc_s) in^t_{is} \right) \\
&- \sum_{t,m,i,a} \delta^t \left(Tc_a + \tau^t_{ma} \right) fi^t_{mia}
\end{aligned}
$$

such that:

$$
\begin{aligned}
&\forall t,\ m, && \sum_p zi^t_{mfpi} - \left(\sum_d y^t_{mid} + (-1)^m \sum_s in^t_{is} \right) && = 0 && (\psi^t_{mi}) && \text{(5a)} \\
&\forall t,\ s, && in^t_{is} - r^t_{is} && \leq 0 && (\mu^t_{is}) && \text{(5b)} \\
&\forall t,\ m,\ n, && \sum_a M6_{an} fi^t_{mia}(1 - loss_a) - \sum_a M5_{an} fi^t_{mia} && && && \\
& && - \sum_d M3_{dn} y^t_{mid} + \sum_p M2_{in} zi^t_{mpi} && && && \\
& && - (-1)^m \sum_s M4_{sn} in^t_{is} && = 0 && (\alpha i^t_{min}) && \text{(5c)} \\
&\forall t,\ p, && ui_{pi} - \sum_m zi^t_{mpi} && = 0 && (\eta i^t_{pi}) && \text{(5d)} \\
&\forall\ p, && ui_{pi} - up_{pi} && = 0 && (\eta_{pi}) && \text{(5e)} \\
&\forall t,\ m,\ p, && - zi^t_{mpi} + min_{pi} \sum_m zi^t_{mpi} && \leq 0 && (\upsilon^t_{mpi}) && \text{(5f)} \\
&\forall t,\ s, && \sum_i r^t_{is} - Ks_s - \sum_{t' \leq t - delay_s} is^{t'}_s && \leq 0 && (\beta s^t_s) && \text{(5g)} \\
&\forall t,\ m,\ s,\ d, && zi^t_{mpi},\ y^t_{mid},\ r^t_{is},\ in^t_{is},\ ui_{pi} && \geq 0 && &&
\end{aligned}
$$

The term

$$
\sum_{t,m,d} \delta^t \left(p^t_{md}(y^t_{mid} + \overline{y^t_{mid}}) y^t_{mid} \right) - \sum_{t,p,m} \delta^t \left(\eta_{pi} zi^t_{mpi} \right)
$$

is the net profit.
The term

$$
\sum_{t,s} \delta^t \left(Rc_s r^t_{is} \right)
$$

is the storage capacity reservation cost.
The term

$$
\sum_{t,s} \delta^t \left((Ic_s + Wc_s) in^t_{is} \right)
$$

are the storage/withdrawal costs. [2]
The term

$$\sum_{t,m,i,a} \delta^t \left(Tc_a + \tau^t_{ma}\right) fi^t_{mia}$$

is the transport and congestion costs charged by the pipeline operator from the independent trader i.

As for the feasibility set, it is also easy to specify :
The constraint (5a) is a gas quantity balance for each trader. The term $(-1)^m$ is equal to 1 in the off-peak season and -1 otherwise. An implicit assumption we use in our description is that all the storage sites must be "empty" (regardless of the working gas quantitities) at the end of each year.
The equation (5b) implies that each independent trader has to pay for a storage reservation quantity, each year and at each storage site s, to be able to store his gas.
The constraint (5d) forces each trader to purchase the same quantity, in long-term contracts from each producer and year.
The constraint (5e) is similar to the constraint (2l) of the producers' optimization program. For each pair of producer/independent trader (p, i), the gas quantity sold by p in the long-term contract market must be equal to the gas quantity purchased by i. Therefore, this is a supply/demand equation in the long-term contracts market. The associated dual variable η_{pi} is the corresponding contract unit selling/purchase price, because we do not assume the existence of market power in the long-term contract trade. Using this technique, it is possible to make the long-term contracts prices and volumes endogenous to the description so that they become an output of the model.
The constraint (5f) fixes a minimum percentage of the annual contracted volume min_{pi} that has to be exchanged between p and i each season of each year.
The constraint (5g) is a storage constraint expressed at each storage node, taking into account the investments decided by the storage operator.

On the transportation side of our model, we will assume that the producers pay the transport costs to bring natural gas from the production fields to the independent traders' locations and the end-use markets. The traders support the transport costs to store/withdraw gas or bring it to the end-users for their sales.

The pipeline operator optimization (cost minimization) program is given below. The corresponding decision variables are fp^t_{mpa}, fi^t_{mia} and ik^t_a.

[2] There are no storage losses in the model. They can easily be taken into account by increasing the transportation losses of the arcs that start at the storage nodes.

Min

$$\sum_{t,m,a} \delta^t \left(Tc_a + \tau^t_{ma}\right) \sum_p fp^t_{mpa}$$
$$+ \sum_{t,m,a} \delta^t \left(Tc_a + \tau^t_{ma}\right) \sum_i fi^t_{mia}$$
$$+ \sum_{t,a} \delta^t Ik_a ik^t_a$$

such that:

$$\forall t,\ m,\ a, \quad \sum_p fp^t_{mpa} + \sum_i fi^t_{mia} - \left(Tk_a + \sum_{t' \le t - delay_i} ik^{t'}_a\right) \le 0 \quad (\tau^t_{ma}) \tag{6a}$$

$$\begin{aligned}\forall t,\ m,\ p,\ n, \quad & \sum_a M6_{an} fp^t_{mpa}(1 - loss_a) - \sum_a M5_{an} fp^t_{mpa} \\ & + \sum_f M1_{fn} q^t_{mpf} - \sum_d \sum_f M3_{dn} x^t_{mfpd} \\ & - \sum_i \sum_f M2_{in} zp^t_{mfpi} = 0 \quad (\alpha p^t_{mpn})\end{aligned} \tag{6b}$$

$$\begin{aligned}\forall t,\ m,\ i,\ n, \quad & \sum_a M6_{an} fi^t_{mia}(1 - loss_a) - \sum_a M5_{an} fi^t_{mia} \\ & - \sum_d M3_{dn} y^t_{mid} + \sum_p M2_{in} zi^t_{mpi} \\ & - (-1)^m \sum_s M4_{sn} in^t_{is} = 0 \quad (\alpha i^t_{min})\end{aligned} \tag{6c}$$

$$\forall t,\ m,\ a,\ p,\ i, \quad fp^t_{mpa},\ fi^t_{mia},\ ik^t_a \ge 0$$

The objective function contains both the transport/congestion and invesment costs. The congestion cost through arc a, τ^t_{ma}, is the dual variable associated with the constraint (6a). This constraint concerns the physical seasonal capacity of arc a, including the possible time-dependent investments.
The other consraints are market-clearing conditions at each node, depending on whether this node is a production field, an independent trader location, a demand market or a storage site and depending on whether the transportation costs are supported by the producers or the independent traders.
We consider both pipeline and LNG routes for transport. The liquefaction and regasification costs are included in the transportation cost on the LNG arcs. We assume, in our representation that the physical losses occur at the end nodes of the arcs.

The storage operator optimization (cost minimization) program is given below. The corresponding decision variable is is^t_s.

Min

$$\sum_{t,s} \delta^t Is_s is_s^t$$

such that:

$$\forall t,\ s, \quad \sum_i r_{is}^t - Ks_s - \sum_{t' \le t-delay_s} is_s^{t'} \quad \le 0 \quad (\beta s_s^t) \tag{7a}$$

$$\forall t,\ s, \quad is_s^t \quad \ge 0$$

The storage operator only controls the different investments that dynamically increase the storage capacity of each storage node. The incentive this player has to invest is due to the constraint he must satisfy: the capacity available at each storage site must be sufficient to meet the volumes the independent traders have to store each year in the off-peak season.

If we take a closer look at the optimization program of a producer, we will notice that his feasibility set depends on the decision variables of the independent traders. Also, the feasibility set of any independent trader's optimization program depends on the producers decision variables. The situation is similar for the pipeline and storage operators. This particularity makes our formulation (the KKT conditions) a **Generalized Nash-Cournot problem**. Similarly, the Gerneralized Nash-Cournot problem can also be formulated as a Quasi Variational Inequality problem (QVI). In order to solve our problem, we look for the particular solution that makes our problem a VI formulation [18]. More details about the VI solution search are given in Section 2.6.

When the KKT conditions are written, we obtain the Mixed Complementarity Problem given in Section 2.7.

2.5 The concavity of the players' objective functions

This section demonstrates the concavity of all the players' objective functions.
We will demonstrate that the production cost is convex with respect to the quantity produced. The storage/withdrawal/investments costs are convex functions because they are linear.

Let's consider a producer p. First we demonstrate the convexity of the Golombek production cost function. We consider a production field f. To simplify the notation, let us denote by q the produced volume (a variable) and by Rf_f the reserve (a constant). We recall that the cost function Pc_f is as follows:

$$\frac{d\ Pc_f}{d\ q} : \quad [0, Rf_f) \longrightarrow R^+$$
$$q \longrightarrow a_f + b_f q + \ c_f \ \ln\left(\frac{Rf_f - q}{Rf_f}\right)$$

where $c_f \le 0$ and $b_f \ge 0$.
Pc_f is a $C^2([0, Rf_f))$ function (twice continuously differentiable) and we have :

$$\forall q \in [0, Rf_f)\ \frac{d^2 Pc_f}{d^2 q} = b_f - \frac{c_f}{Rf_f - q} \ge 0$$

Thus, Pc_f is convex.

Producer p's objective function is:
$+\sum_{t,m,f,i} \delta^t \eta_{pi}(zp^t_{mfpi})$
$+\sum_{t,m,f,d} \delta^t \left(p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}})\right) x^t_{mfpd}$
$-\sum_{t,f} \delta^t \left(Pc_f\left(\sum_{t'\leq t}\sum_m q^{t'}_{mfp}, Rf_f\right) - Pc_f\left(\sum_{t'<t}\sum_m q^{t'}_{mfp}, Rf_f\right)\right)$
$-\sum_{t,f} \delta^t Ip_f ip^t_{fp}$
$-\sum_{t,m,p,a} \delta^t((Tc_a + \tau^t_{ma}) fp^t_{mpa})$

As mentioned before, the inverse demand function has been linearized. Let's write the natural gas price in market d as follows:

$$p^t_{md} = a^t_{md} - b^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}})$$

where $b^t_{md} > 0$. The function $\sum_{t,m,f,d} \delta^t \left(p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}})\right) x^t_{mfpd}$ is therefore a concave function of the variables x^t_{mfpd}. Indeed the Hessian matrix H^t_{md} associated with the spot market profit is diagonal and such that the diagonal terms are $H^t_{md} = -2b^t_{md} < 0$. Hence, the Hessian matrix is negative definite.

Let us consider the global cost function GP:
$q^t_{mfp} \longrightarrow GP(q^t_{mfp}) = -\sum_{t,f} \delta^t \left(Pc_f\left(\sum_{t'\leq t}\sum_m q^{t'}_{mfp}, Rf_f\right) - Pc_f\left(\sum_{t'<t}\sum_m q^{t'}_{mfp}, Rf_f\right)\right)$.
And let's demonstrate that GP is concave. Let's consider two variable vectors $q1^t_{md}$ and $q2^t_{md}$ and $\lambda \in [0,1]$.

$GP(\lambda q1^t_{md} + (1-\lambda) q2^t_{md})$
$=$
$-\sum_{t,f} \delta^t \left(Pc_f\left(\sum_{t'\leq t}\sum_m (\lambda q1^{t'}_{md} + (1-\lambda) q2^{t'}_{md}), Rf_f\right)\right)$
$+\sum_{t,f} \delta^t \left(Pc_f\left(\sum_{t'<t}\sum_m (\lambda q1^{t'}_{md} + (1-\lambda) q2^{t'}_{md}), Rf_f\right)\right)$
$=$
$-\sum_f \sum_{t=0}^{Num} \delta^t \left(Pc_f\left(\sum_{t'\leq t}\sum_m (\lambda q1^{t'}_{md} + (1-\lambda) q2^{t'}_{md}), Rf_f\right)\right)$
$+\sum_f \sum_{t=0}^{Num-1} \delta^{t+1} \left(Pc_f\left(\sum_{t'\leq t}\sum_m (\lambda q1^{t'}_{md} + (1-\lambda) q2^{t'}_{md}), Rf_f\right)\right)$
$=$
$-\sum_f \sum_{t=0}^{Num-1} (\delta^t - \delta^{t+1}) \left(Pc_f\left(\sum_{t'\leq t}\sum_m (\lambda q1^{t'}_{md} + (1-\lambda) q2^{t'}_{md}), Rf_f\right)\right)$
$-\sum_f \delta^{Num} \left(Pc_f\left(\sum_{t'\leq Num}\sum_m (\lambda q1^{t'}_{md} + (1-\lambda) q2^{t'}_{md}), Rf_f\right)\right)$
$=$
$-\sum_f \sum_{t=0}^{Num-1} \delta^t (1-\delta) \left(Pc_f\left(\sum_{t'\leq t}\sum_m (\lambda q1^{t'}_{md} + (1-\lambda) q2^{t'}_{md}), Rf_f\right)\right)$
$-\sum_f \delta^{Num} \left(Pc_f\left(\sum_{t'\leq Num}\sum_m (\lambda q1^{t'}_{md} + (1-\lambda) q2^{t'}_{md}), Rf_f\right)\right)$

Since $0 \leq \delta \leq 1$ and Pc_f is convex, we can write:

$-\sum_f \sum_{t=0}^{Num-1} \delta^t(1-\delta) \left(Pc_f \left(\sum_{t'\leq t}\sum_m (\lambda q1_{md}^{t'} + (1-\lambda)q2_{md}^{t'}), Rf_f\right)\right)$
$-\sum_f \delta^{Num} \left(Pc_f \left(\sum_{t'\leq Num}\sum_m (\lambda q1_{md}^{t'} + (1-\lambda)q2_{md}^{t'}), Rf_f\right)\right)$
$\geq$
$-\lambda\sum_f \sum_{t=0}^{Num-1} \delta^t(1-\delta) \left(Pc_f \left(\sum_{t'\leq t}\sum_m q1_{md}^{t'}, Rf_f\right)\right)$
$-(1-\lambda)\sum_f \sum_{t=0}^{Num-1} \delta^t(1-\delta) \left(Pc_f \left(\sum_{t'\leq t}\sum_m q2_{md}^{t'}, Rf_f\right)\right)$
$-\lambda\sum_f \delta^{Num} \left(Pc_f \left(\sum_{t'\leq Num}\sum_m q1_{md}^{t'}, Rf_f\right)\right)$
$-(1-\lambda)\sum_f \delta^{Num} \left(Pc_f \left(\sum_{t'\leq Num}\sum_m q2_{md}^{t'}, Rf_f\right)\right)$
$=$
$\lambda GP(q1_{md}^t) + (1-\lambda)GP(q2_{md}^t)$

Hence, the cost function is concave. The rest of the profit is made of linear functions of the decision variables. The concavity of the producers objective function is thus demonstrated.

The independent traders' objective function can be demonstrated in a similar way. Like for the producers, the spot maket benefit is also concave.

The pipeline and storage operators objective functions are concave because they are linear. The feasibility sets are all convex due to linearity of the constraint functions.

2.6 The (Quasi)-Variational Inequality and Generalized Nash-Cournot games

In this section, we recall Harker's result [18] in order to understand how to theoretically solve a Generalized Nash-Cournot problem.

A standard Nash-Cournot problem is a set of optimization programs where some of the players can influence other players' payoff via the objective functions. In a Generalized Nash-Cournot formulation, some players can also change the feasibility sets of other players, via their decision variables. In our particular model, if we consider an independent trader i, the constraint

$$\forall\ p,\ i,\ ui_{pi} = up_{pi}$$

contains the producers decision variables up_{pi}. These decision variables influence trader i's feasibility set. This influence on the feasible region of one player by another is also the case for the pipeline operator. Indeed, each node's market-clearing condition mixes the decision variables of the producers, the independent traders and the pipeline operator. The storage operator feasibility set contains constraints that mix its decision variables with those of the independent traders, at each storage site.

A VI (Variational Inequality) problem can be formulated as follows: given a set $K \in \mathbb{R}^n$ and a mapping $F : K \longrightarrow \mathbb{R}^n$, find $x^* \in K$ s.t.

$$\forall y \in K,\ F(x^*)^t(y - x^*) \geq 0$$

It is straightforward that a standard Nash-Cournot problem can be expressed as a VI formulation if the objective functions are differentiable (is suffices to write the necessary and sufficient conditions on the gradient of the objective functions that characterize the optimum).

A QVI (Quasi-Variational Inequality) problem adds mixed constraints [10]. Given n point-to-set mappings $K_i : \mathbb{R}^n \longrightarrow \mathbb{R},\ i \in \{1, 2...n\}$ and $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, find $x^* \in \mathbb{R}^n$ s.t. $\forall i \in$

$\{1, 2...n\}$ $x_i^* \in K_i(x^*)$ and

$$\forall y \in \mathbb{R}^n \ s.t. \ \forall i \in \{1, 2...n\} \ y_i \in K_i(x^*), \quad F(x^*)^t(y - x^*) \geq 0$$

A generalized Nash-Cournot problem can be expressed as a QVI formulation. Unlike VI problems, a QVI formulation often has an infinite set of equilibria. In some particular cases, a QVI problem can be slightly changed into a VI formulation. This is possible, in particular if the QVI is issued from a Generalized Nash-Cournot problem, which is our case. The idea is quite simple: we want to make the mappings K_i independent of the variables x_i. To do so, we make all the constraints that mix different players decision variables common to all these players. From the KKT conditions point of view, Harker [18] demonstrated that the "VI solution" is obtained by giving the same dual variables to the common constraints. As an example, in our problem, this leads to the fact that the producers and independent traders, see the same dual variables η_{pi}. Economically speaking, this means that they have the same appreciation of the long-term contracts prices.

Using this technique, we make sure we end up with a VI solution [18].

2.7 The KKT conditions and MCP formulation

This section presents the KKT conditions derived from our model. Once the KKT conditions written, we get the Mixed Complementarity Problem given below.

The producers KKT conditions

$$\forall t,\ m,\ f,\ p,\ i, \qquad 0 \le zp^t_{mfpi} \quad \perp \quad \delta^t \eta_{pi} - \gamma^t_{mfp} - \epsilon 2^t_{mfpi} - \eta p^t_{pi} - \sum_n M2_{in} \alpha p^t_{mpn} \le 0 \tag{8a}$$

$$\forall t,\ m,\ f,\ p,\ d, \qquad 0 \le x^t_{mfpd} \quad \perp \quad \delta^t p^t_{md}(x^t_{mfpd} + \overline{x^t_{mfpd}}) + \delta^t \frac{\partial p^t_{md}}{\partial x^t_{mfpd}}(x^t_{mfpd} + \overline{x^t_{mfpd}}) x^t_{mfpd} - \gamma^t_{mfp} - \epsilon 1^t_{mfpd} - \sum_n M3_{dn} \alpha p^t_{mpn} \le 0 \tag{8b}$$

$$\begin{aligned}\forall t,\ m,\ f,\ p, \qquad 0 \le q^t_{mfp} \quad \perp \quad & - \sum_{t' \ge t} \delta^{t'} \frac{\partial Pc_f}{\partial q}\Big(\sum_{t'' \le t'} \sum_m q^{t''}_{mfp}, Rf_f\Big) \\ & + \sum_{t' > t} \delta^{t'} \frac{\partial Pc_f}{\partial q}\Big(\sum_{t'' < t'} \sum_m q^{t''}_{mfp}, Rf_f\Big) \\ & - \sum_{t' \ge t} \phi^{t'}_f - \chi^t_{mf} + \gamma^t_{mfp} \\ & - (-1)^m (\vartheta 1^t_f - \vartheta 2^t_f) - \epsilon 3^t_{mfp} \\ & + \sum_n M1_{fn} \alpha p^t_{mpn} \le 0 \end{aligned} \tag{8c}$$

$$\forall t,\ f,\ p, \qquad 0 \le ip^t_{fp} \quad \perp \quad - \delta^t Ip_f - \epsilon 4^t_{fp} + \sum_m \sum_{t' \ge t + delay_p} \chi^{t'}_{mf} (1 - dep_f)^{t'-t} \le 0 \tag{8d}$$

$$\forall t,\ p,\ i, \qquad 0 \le up_{pi} \quad \perp \quad \sum_t \eta p^t_{pi} - \eta_{pi} \le 0 \tag{8e}$$

$$\forall t,\ f, \qquad 0 \le \phi^t_f \quad \perp \quad \sum_p \sum_{t' \le t} \sum_m q^{t'}_{mfp} - Rf_f \le 0 \tag{8f}$$

$$\forall t,\ m,\ f, \qquad 0 \le \chi^t_{mf} \quad \perp \quad \sum_p q^t_{mfp} - Kf_f(1 - dep_f)^t - \sum_p \sum_{t' \le t - delay_p} ip^{t'}_{fp} (1 - dep_f)^{t-t'} \le 0 \tag{8g}$$

$$\forall t,\ m,\ f,\ p, \qquad 0 \le \gamma^t_{mfp} \quad \perp \quad - q^t_{mfp} + \sum_i zp^t_{mfpi} + \sum_d x^t_{mfpd} \le 0 \tag{8h}$$

$$\forall t,\ f, \qquad 0 \le \vartheta 1^t_f \quad \perp \quad \sum_m \sum_p (-1)^m q^t_{mfp} - fl_f \le 0 \tag{8i}$$

$$\forall t,\ f, \qquad 0 \leq \vartheta 2^t_f \quad \perp \quad -\sum_m \sum_p (-1)^m q^t_{mfp} - fl_f \leq 0 \tag{9a}$$

$$\forall t,\ f, m,\ p,\ d, \qquad 0 \leq \epsilon 1^t_{mfpd} \quad \perp \quad x^t_{mfpd} - O_{fp} H \leq 0 \tag{9b}$$

$$\forall t,\ m, f,\ p,\ i, \qquad 0 \leq \epsilon 2^t_{mfpi} \quad \perp \quad zp^t_{mfpi} - O_{fp} H \leq 0 \tag{9c}$$

$$\forall t,\ m, f,\ p, \qquad 0 \leq \epsilon 3^t_{mfp} \quad \perp \quad q^t_{mfp} - O_{fp} H \leq 0 \tag{9d}$$

$$\forall t,\ f,\ p, \qquad 0 \leq \epsilon 4^t_{fp} \quad \perp \quad ip^t_{fp} - O_{fp} H \leq 0 \tag{9e}$$

$$\begin{aligned} \forall t,\ m,\ p,\ n, \qquad \text{free} \quad \alpha p^t_{mpn} \qquad & \sum_a M_6(a,n) fp^t_{mpa}(1 - loss_a) \\ & - \sum_a M5_{an} fp^t_{mpa} + \sum_f M1_{fn} q^t_{mpf} \\ & - \sum_d \sum_f M3_{dn} x^t_{mfpd} \\ & - \sum_i \sum_f M2_{in} zp^t_{mfpi} = 0 \end{aligned} \tag{9f}$$

$$\forall t,\ p,\ i, \qquad \text{free} \quad \eta p^t_{pi} \qquad up_{pi} - \sum_{f,m} zp^t_{mfpi} = 0 \tag{9g}$$

$$\forall\ p,\ i, \qquad \text{free} \quad \eta_{pi} \qquad ui_{pi} - up_{pi} = 0 \tag{9h}$$

The independent traders' KKT conditions

$$\begin{aligned} \forall t,\ m,\ p,\ i, \qquad 0 \leq zi^t_{mpi} \quad \perp \quad & - \delta^t \eta_{pi} - \eta i^t_{pi} \\ & + \psi^t_{mi} \\ & + \sum_n M2_{in} \alpha i^t_{min} \\ & + (1 - min_{pi}) \upsilon^t_{mpi} \leq 0 \end{aligned} \tag{10a}$$

$$\begin{aligned} \forall t,\ m,\ i,\ d, \qquad 0 \leq y^t_{mid} \quad \perp \quad & \delta^t p^t_{md}(y^t_{mfpd} + \overline{y^t_{mfpd}}) \\ & \delta^t \frac{\partial p^t_{md}}{\partial y^t_{mid}} (y^t_{mfpd} + \overline{y^t_{mfpd}}) y^t_{mid} \\ & - \psi^t_{mi} - \sum_n M3_{dn} \alpha i^t_{min} \leq 0 \end{aligned} \tag{10b}$$

$$\forall t,\ i,\ s, \qquad 0 \leq r^t_{is} \quad \perp \quad - \delta^t Rc_s + \mu^t_{is} - \beta s^t_s \leq 0 \tag{10c}$$

$$
\begin{array}{llllll}
\forall t,\ i,\ s, & 0 \le in_{is}^t & \perp & -\delta^t(Ic_s + Wc_s) & \le 0 & (11a) \\
 & & & -\mu_{is}^t - \sum_m (-1)^m \psi_{mi}^t & & \\
 & & & -\sum_n M4_{sn} \alpha i_{min}^t (-1)^m & & \\
\forall t,\ p,\ i, & 0 \le ui_{pi} & \perp & \sum_t \eta i_{pi}^t + \eta_{pi} & \le 0 & (11b) \\
\forall t,\ m,\ i, & \text{free} \quad \psi_{mi}^t & & \sum_p zi_{mpi}^t - \sum_d y_{mid}^t + (-1)^m \sum_s in_{is}^t & = 0 & (11c) \\
\forall t,\ i,\ s, & 0 \le \mu_{is}^t & \perp & in_{is}^t - r_{is}^t & \le 0 & (11d) \\
\forall t,\ m,\ i,\ n, & \text{free} \quad \alpha i_{min}^t & & \sum_a M6_{an} fi_{mia}^t (1 - loss_a) & = 0 & (11e) \\
 & & & -\sum_a M5_{an} fi_{mia}^t - \sum_d M3_{dn} y_{mid}^t & & \\
 & & & +\sum_p M2_{in} zi_{mpi}^t & & \\
 & & & -(-1)^m \sum_s M4_{sn} in_{is}^t & & \\
\forall t,\ p,\ i, & \text{free} \quad \eta i_{pi}^t & & ui_{pi} - \sum_m zi_{mpi}^t & = 0 & (11f) \\
\forall\, p,\ i, & \text{free} \quad \eta_{pi} & & ui_{pi} - up_{pi} & = 0 & (11g) \\
\forall t,\ m,\ p,\ i, & 0 \le \upsilon_{mpi}^t & & -zi_{mpi}^t + min_{pi} \sum_m zi_{mpi}^t & \le 0 & (11h) \\
\forall t,\ s, & 0 \le \beta s_s^t & \perp & \sum_i r_{is}^t - Ks_s - \sum_{t' \le t - delay_s} is_s^{t'} & \le 0 & (11i)
\end{array}
$$

The pipeline operator KKT conditions

$$
\begin{array}{llllll}
\forall t,\ m,\ p,\ a, & 0 \le fp_{mpa}^t & \perp & -\delta^t(Tc_a + \tau_{ma}^t) - \tau_{ma}^t & \le 0 & (12a) \\
 & & & +\sum_n M6_{an} \alpha p_{mpn}^t (1 - loss_a) & & \\
 & & & -\sum_n M5_{an} \alpha p_{mpn}^t & &
\end{array}
$$

$$\forall t,\ m,\ i,\ a, \qquad 0 \le fi^t_{mia} \quad \perp \quad -\delta^t(Tc_a + \tau^t_{ma}) - \tau^t_{ma} + \sum_n M6_{an}\alpha i^t_{min}(1 - loss_a) - \sum_n M5_{an}\alpha i^t_{min} \le 0 \tag{13a}$$

$$\forall t,\ a, \qquad 0 \le ik^t_a \quad \perp \quad -\delta^t Ik_a + \sum_{t' \ge t + delay_i} \tau^{t'}_{ma} \le 0 \tag{13b}$$

$$\forall t,\ m,\ a, \qquad 0 \le \tau^t_{ma} \quad \perp \quad \sum_p fp^t_{mpa} + \sum_i fi^t_{mia} - Tk_a - \sum_{t' \le t - delay_i} ik^t_a \le 0 \tag{13c}$$

$$\forall t,\ m,\ p,\ n, \qquad \text{free} \quad \alpha p^t_{mpn} \qquad \sum_a M_6(a,n) fp^t_{mpa}(1 - loss_a) - \sum_a M5_{an} fp^t_{mpa} + \sum_f M1_{fn} q^t_{mpf} - \sum_d \sum_f M3_{dn} x^t_{mfpd} - \sum_i \sum_f M2_{in} zp^t_{mfpi} = 0 \tag{13d}$$

$$\forall t,\ m,\ i,\ n, \qquad \text{free} \quad \alpha i^t_{min} \qquad \sum_a M6_{an} fi^t_{mia}(1 - loss_a) - \sum_a M5_{an} fi^t_{mia} - \sum_d M3_{dn} y^t_{mid} + \sum_p M2_{in} zi^t_{mpi} - (-1)^m \sum_s M4_{sn} in^t_{is} = 0 \tag{13e}$$

The storage operator KKT conditions

$$\forall t,\ s, \qquad 0 \le is^t_s \quad \perp \quad -\delta^t Is_s + \sum_{t' \ge t + delay_s} \beta s^{t'}_s \le 0 \tag{14a}$$

$$\forall t,\ s, \qquad 0 \le \beta s^t_s \quad \perp \quad \sum_i r^t_{is} - Ks_s - \sum_{t' \le t - delay_s} is^{t'}_s \le 0 \tag{14b}$$

3 The European natural gas markets model

This section puts the model at work and presents our numerical results.

3.1 The representation

The model we presented in Section 2.4 has been used in order to study the North-Western European natural gas trade. The following array summarizes the representation we have studied.

Producers		**Fields**		**Consuming markets**		**independent traders**
Russia		Russia_f		France		France_{tr}
Algeria		Algeria_f		Germany		Germany_{tr}
Norway		Norway_f		The Netherlands		The Netherlands_{tr}
The Nethherlands_f		NL_f		UK		UK_{tr}
UK		UK_f		Belgium		Belgium_{tr}

Storage sites		**Seasons**		**Time**
France_{st}		off-peak		$2000 - 2040$
Germany_{st}		peak		
The Netherlands_{st}				
UK_{st}				
Belgium_{st}				

We aggregate all the production fields of each producer into one production node. We assume that each consuming market is associated with one independent local trader (indexed by tr). As an example, France_{tr} would be GDF-SUEZ and Germany_{tr} would be E-On Ruhrgas. All the storage sites are also aggregated so that there is one storage node per consuming country. As for the transport, the different gas routes given in Figure 3 were considered.
The local production in the different consuming countries is also taken into consideration (the imports from non-represented producers, which are small, are also considered). We assume that these locally consumed volumes are exogenous to the model.

3.2 The calibration

The calibration process has been carried out in order to best meet:

- the global natural gas consumption,
- the industrial sector gas price and
- the volumes produced by each gas producer,

between 2000 and 2004 (the first time period).

The model has been solved using the solver PATH [11] from GAMS. In order to shorten the running time, we used a five-year time-step resolution. We chose five years because it is the typical length of time needed to construct investments in production, infrastructure or storage. Also, the demand function has been linearized.

The data for the market prices, consumed volumes and imports is the publicly available set from IEA [23]. We define a new variable $exch^t_{mpd}$ that represents the exported volume from producer p to market d. More precisely :

$$\forall t,\ m,\ p,\ d,\ exch^t_{mpd} = \sum_i B_{id}\ zp^t_{mpi} + x^t_{mpd}$$

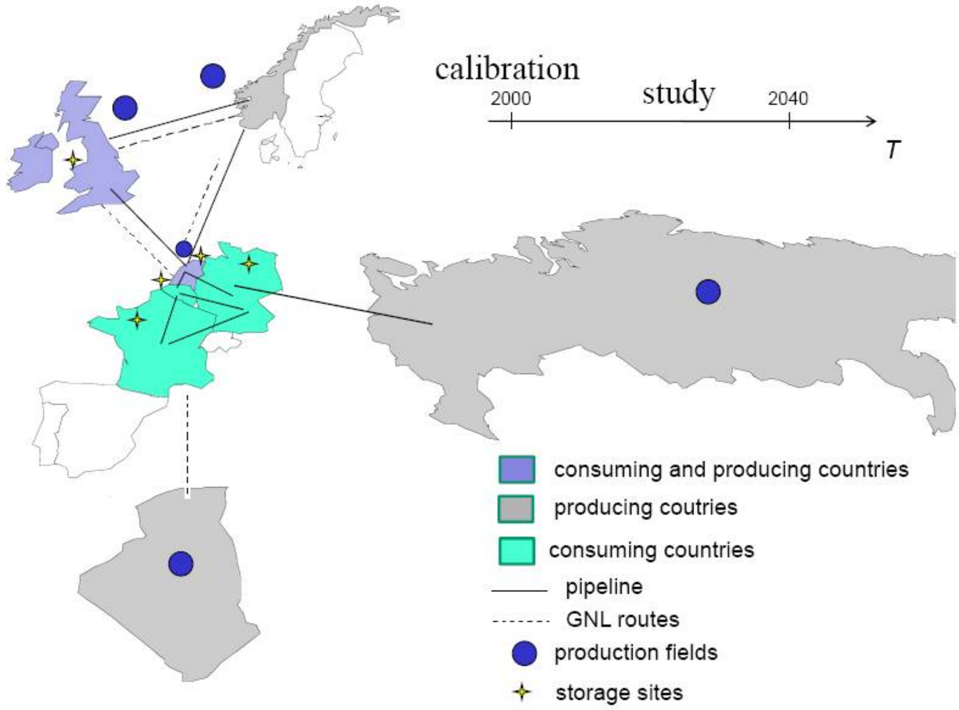

Figure 3:
The North-Western European natural gas routes, production and storage sites.

The matrix B is such that $B_{id} = 1$ if the independent trader i is located in market d (e.g., GDF-SUEZ in France, E-On Ruhrgas in Germany) and $B_{id} = 0$ otherwise. Hence, one can notice that the exchanged volumes include both the spot and long-term contract trades.

The calibration elements we used are the inverse demand function parameters α^t_{md}, γ^t_{md}, pc^t_{md} and β^t_{md}. The idea is that the system dynamics [2] model is run in order to calculate all the inverse demand function parameters, for all the markets and at each year and season of our study. The calibration technique slightly adjusts these values to make the model correctly describe the historical data (between 2000 and 2004).

In order to calibrate the produced volumes properly, we introduced security of supply parameters that link each pair of producer/consuming countries (p, d). A security of supply measure forces each country not to import from any producer, more than a fixed percentage (denoted by SSP) of the overall imports.This property can be rewritten as follows:

$$\forall t,\ m,\ p,\ d,\ exch^t_{mpd} \leq SSP_{pd} \sum_p exch^t_{mpd}$$

The security of supply parameters are also an output of the calibration process. As mentioned before, the calibration concerned only the first time period.
The calibration tolerates a maximum error of 5% for the prices and consumed quantities and 10% for the imported/exported volumes. The tolerated error is higher for the exchanged volumes because they depend on the exports decided by the producers for all the targeted consumers, even those that are not in the scope of the model. As an example, the exported volumes from Russia to CIS (CEI) countries are exogenous to our model.

3.3 Numerical results

In order to estimate the demand function parameters, our model requests exogenous inputs: the global energy demand and the evolution of the oil and coal prices. For that purpose, we used a scenario provided by the European Commision [9]. The annual gross consumption and prices growth per year we used are given in the following chart (starting from 2000) :

annual growth	*Total gross consumption (in %)*	*Oil price (in %)*	*Coal price (in %)*
France	0.46	3.71	2.61
Germany	0.06	3.71	2.61
United Kingdom	0.02	3.71	2.61
Belgium	0.06	3.71	2.61
TheNetherlands	0.11	3.71	2.61

Figure 4 gives the evolution of the natural gas consumption between 2000 and 2030 provided by our model for the countries represented. The consumption is given in Bcm/year.

The average annual growth between 2000 and 2030 is given in the following chart :

Country	*annual consumtpion growth (in %)*
France	0.61
Germany	0.23
UK	−1.35
Belgium	0.23
Netherlands	−0.94

According to our simulation, France shows the highest annual consumption growth, averaging 0,61%, between 2005 and 2030. Both the UK and the Netherlands experience a significant decrease in their natural gas consumption, as their domestic supplies are replaced by more expensive foreign imports. This effect is magnified in our model by the fact that only existing reserves are taken into account, which are depleted relatively quickly due to high installed capacities.
The consumption of all the countries shown flattens out or decreases in 2030, compared to 2000, despite the increase of the global gross demand. This is mainly due to the fact that competition in the upstream becomes less and less important with time. Indeed, in 2025, the continental Europe gas production (the UK and the Netherlands) is expected to be around 25 Bcm. This will increase the exercice of market power and the consumption growth will therefore be reduced.

Figure 5 shows the evolution of the natural gas prices, in the industrial sector, for the represented countries. We recall that the industrial sector prices are taken as a proxy for natural gas prices.

The average annual growth between 2000 and 2030 is given in the following chart:

Country	*annual price growth (in %)*
France	2.47
Germany	2.19
UK	1.28
Belgium	1.92
Netherlands	2.14

As expected, the natural gas prices increase continuously in all the countries. The prices values are driven, as a resut of the Nash-Cournot interaction by the combination of two effects:

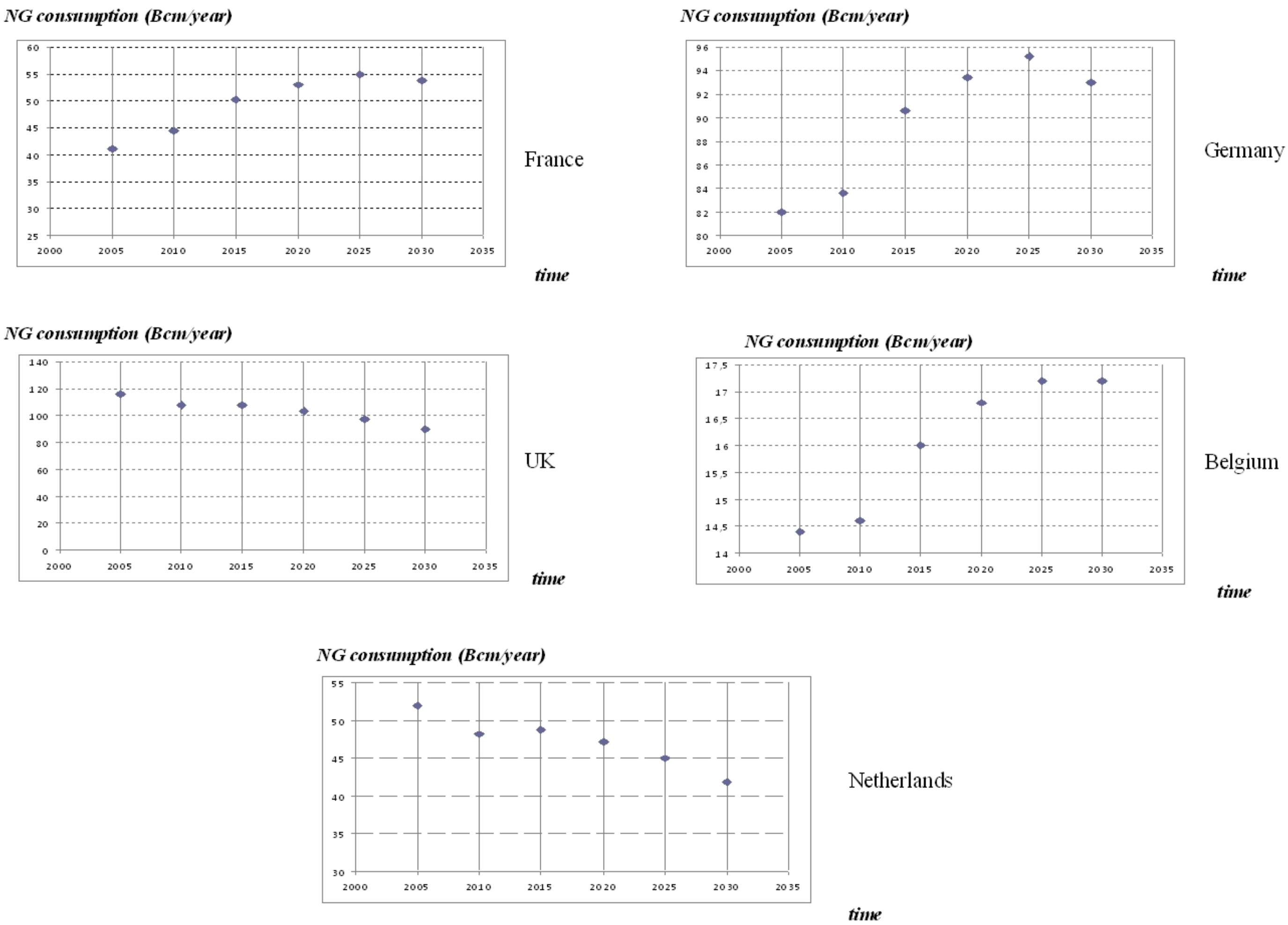

Figure 4:
The North-Western European natural gas consumption between 2005 and 2030.

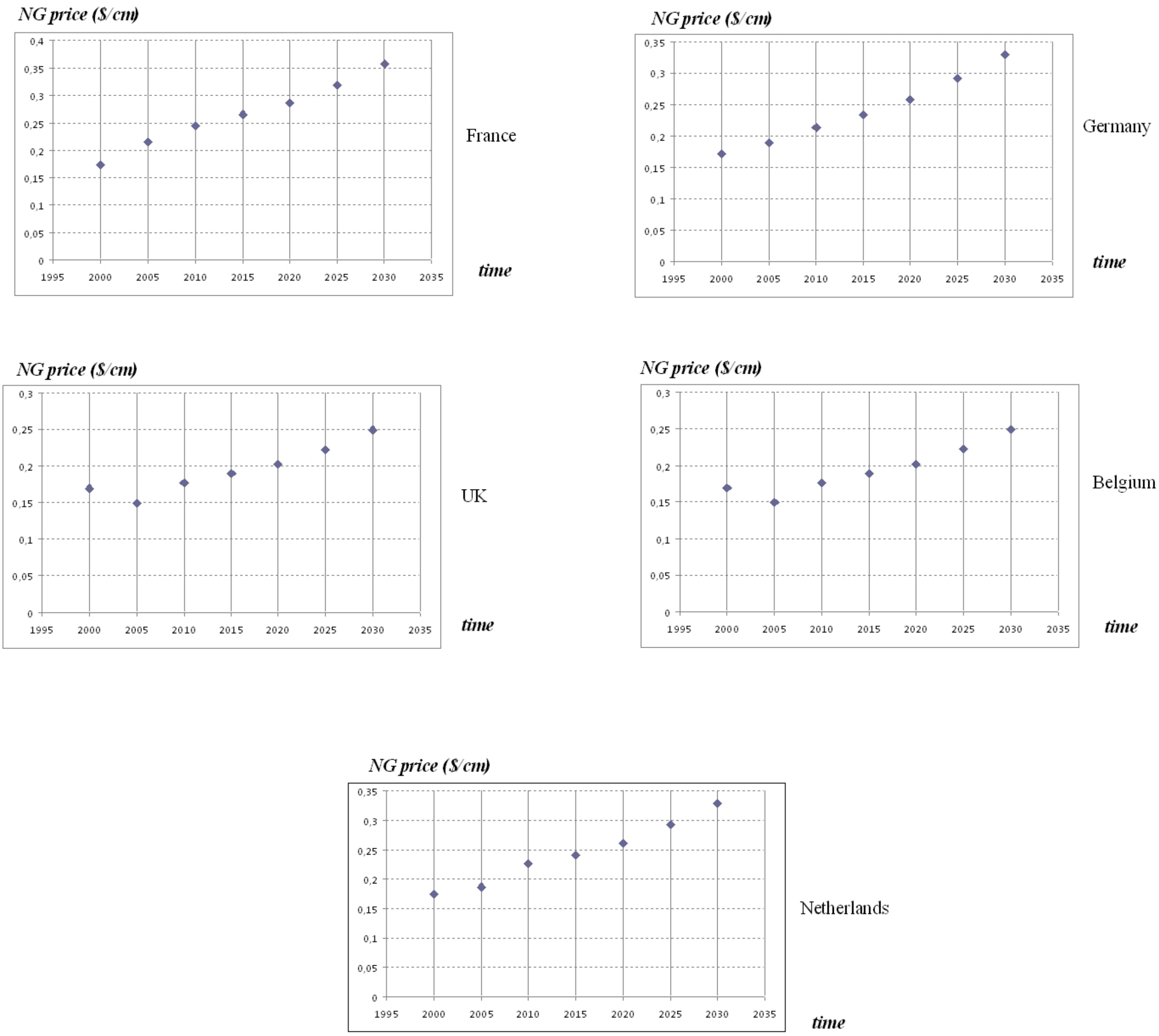

Figure 5:
The North-Western European natural gas prices in the industrial sector between 2000 and 2030.

the global energy demand and the competition between fuels (see equation 1). Since the global energy demand and the coal and oil prices increase with time, they force the gas price up. This combination explains why the natural gas price annual growth in all the countries is less important than the growth in both oil and coal. Indeed, this is due to the fact that the global energy consumption does not increase as quickly as the coal and oil prices with time.
Now it is interesting to study the evolution of the production over time. Figure 6 gives the evolution of the producing countries sales between 2000 and 2040, in Bcm/year.

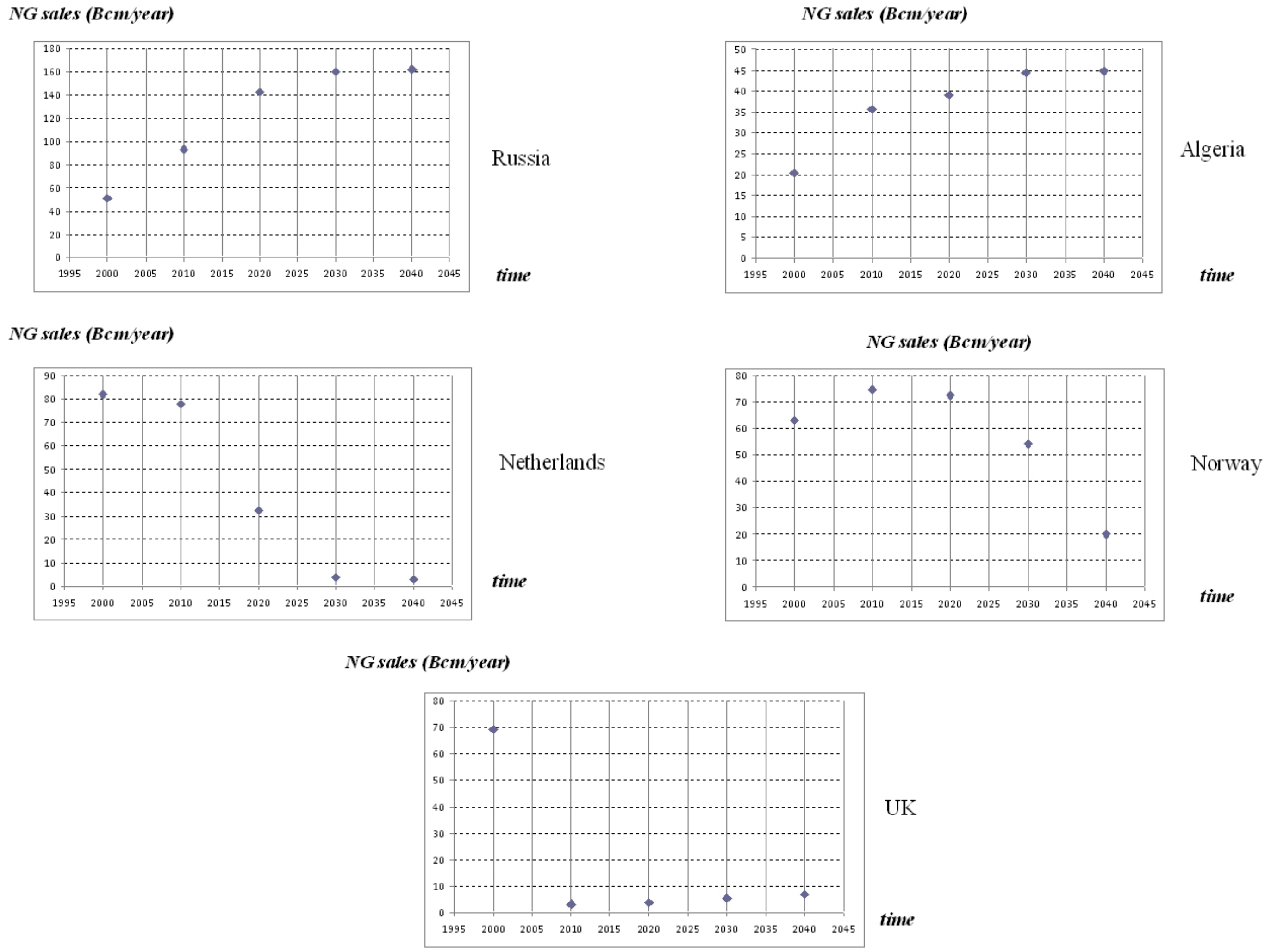

Figure 6:
The natural gas sales between 2000 and 2040.

The production in continental Europe is expected to greatly decrease in the forthcoming decades. The Norwegian production is expected to increase until 2012 before starting to decrease. The Dutch decrease is smooth (-4.5 % per year between 2000 and 2020) whereas the UK one is very sharp. The model indicates that the United Kingdom will use up more than 75% of its natural gas reserves (starting from 2000) until 2015. This may seem surprising but can be understood by the fact that we take into account only the proven reserves in 2000 [4]. Thus, we do not consider the reserves discoveries that may occur till 2045.

On the other hand, the Russian and Algerian shares in the European natural gas consumption is expected to grow in the coming decades: in 2020, the foreign imports will represent 47% of the North-Western European consumption.

In order to test the strength of the model, we compare its output versus historical values. For that purpose, we consider the consumption and prices in the European countries between 2005 and 2010 (second time-step) and compare them to what actually happened in that period. Let us recall that the second time-step has not been used in the calibration. Figure 7 gives the natural gas consumption between 2005 and 2010 in Bcm/year and prices in \$/cm in the countries represented. The left bars represent the model's output whereas the right bars represent the real historical data.

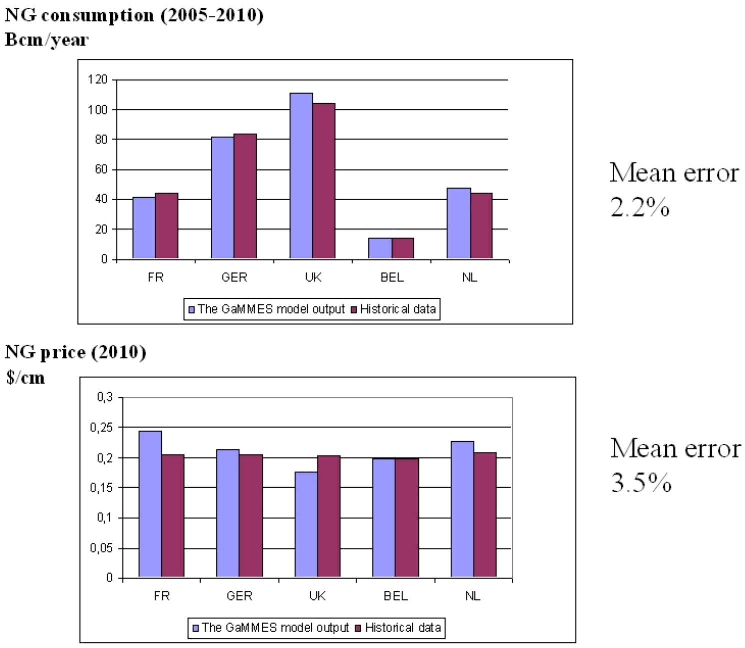

Figure 7:
Comparison between the model's output and historical data.

The average model estimation errors are 2.2% for the consumption and 3.5% for the prices. They are in the same range as the ones tolerated when calibrating the model (period 2000-2005).

Figure 8 gives the evolution of the North-Western European natural gas dependence on foreign imports (those considered in the model). The dependence is the ratio between the foreign exports to North-Western Europe and the domestic consumption [3].
The natural gas dependence is expected to reach 70% around 2030, which will bring about important security of supply concerns [1]. However, these conclusions are to be considered cautiously because they are based on strong assumptions. Indeed, in our study, we assume that no more natural gas reserves will be found in the future and no shale gas will be produced in Europe. [4]

[3]The Norwegian sales are not taken into account in the foreign supplies, for security of supply reasons.
[4]shale gas production is expected to be negligeable in Europe, due to environmental concerns for instance. As of now, few credible assumptions exist concerning the development of European domestic shale reserves [5].

$$dependence = \frac{foreign\ exports}{total\ consumption} \tag{15}$$

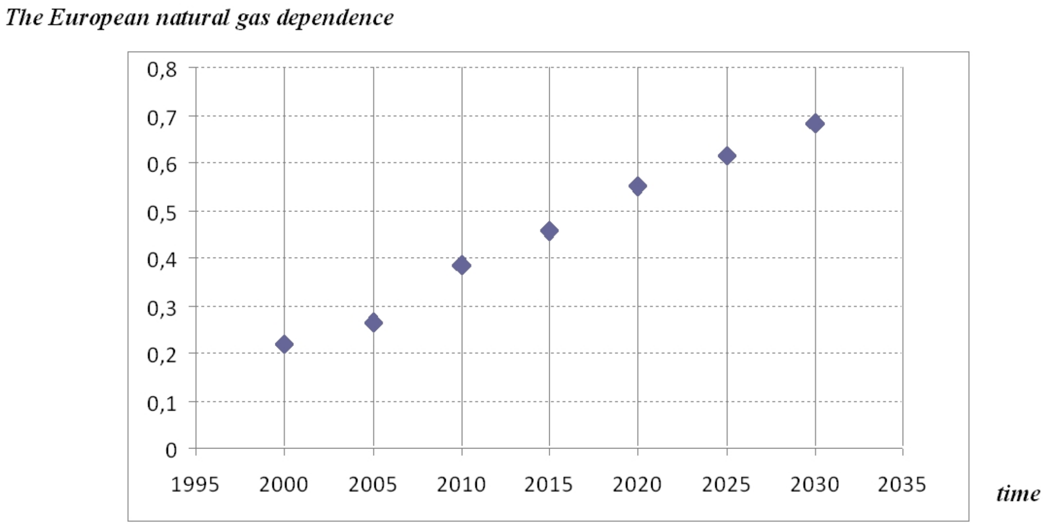

Figure 8:
The North-Western European natural gas dependence over time.

The purpose of the next comparison is to show the effects of the our fuel substitution-based demand function. To that end, we consider an alternative linear demand function of the following form:

$$q_t = a_t - bp_t \tag{16}$$

where the slope b should remain constant over time and the interceipt a_t changes as a function of the global energy demand. In our study, we made a_t evolve with the global energy demand annual growth. The slope b is a result of the calibration process. This description of the markets will be refered to as the *standard model* whereas the model we proposed in this article will be refered to as the *GaMMES model*.

Figure 9 provides the consumption and prices levels for both models considered.

We notice that the standard model provides a lower consumption than the GaMMES results. The average difference in consumption is 13%. The standard model provides lower prices than the GaMMES results. The average difference between the two models is 23% which is quite large.

Now, let's compare between the results provided by the GaMMES model, the standard model and some official forecast. For that purpose, we choose the forecast of the European Commission [9].

Figure 10 shows the evolution of the global European energy consumption between 2000 and 2030 and the average European price, forecasted in three scenarios. Ths first one is issued from the European Commission report (baseline scenario) [9]. The second one is our model forecast and the third one is the standard model forecast.

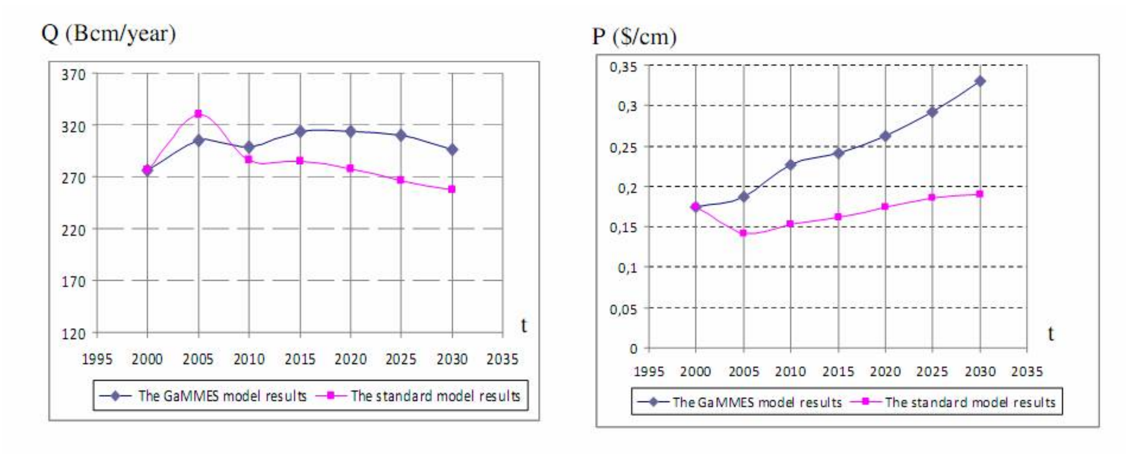

Figure 9:
Comparison between the standard and the GaMMES model: consumption and prices.

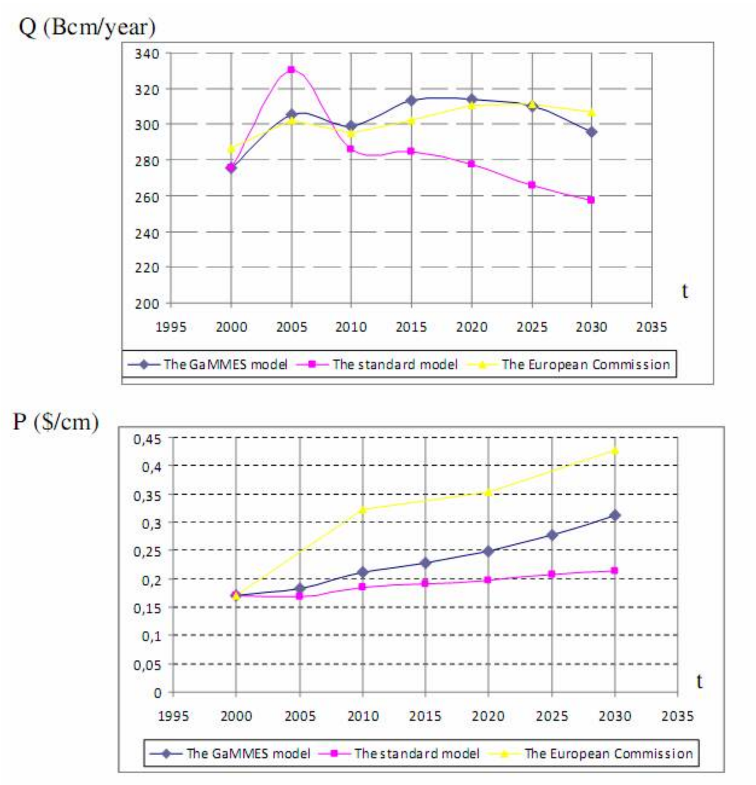

Figure 10:
The European commission, the GaMMES model and the standard model forecasts.

Comparing the results of both the GaMMES model and the standard model with the 2007 European Commission forecasts [9] gives strong support to the need to take into account fuel substitution, especially in the long run. The standard model output shows a very fast decrease of natural gas consumption in the long run. This seems at odds with the perspective of the market, since as global primary Energy consumption is exogenous, the remaining energy consumption has to be met thanks to oil and coal. This view clearly contradicts the global envolution of the different energy shares in the recent past as well as the strong support for cleaner fuels given by the european policy framework. On the contrary, the GaMMES model output gives a better outcome. The demand for gas slowly increases in the medium term, due to both higher energy gross domestic consumption and a higher share for natural gas in the energy mix [26]. The trend is compensated in the long run by the increased exercice of market power. The 2010 kink is mostly explained by the quick depletion of domestic reserves.

These previous results and those of figure 7 show that consummed quantities provided by the model are in line of the European Commission forecasts. This gives confidence in the GaMMES results, for the European Commission forecasts are subject to countries review and acceptance. Regarding the prices, GaMMES is closer to the European Commission scenario than the standard model, even if both of these scenarios underestimates the prices.

In conclusion, compared to a standard description, the GaMMES model captures correctly the evolution of the natural gas prices and consumption. It is necessary to take into consideration the fuel substitution in the natural gas markets modeling because they allow a better understanding of the consumers' behavior.

Now, it may be interesting to take advantage of the energy substitution description allowed by the GaMMES model in order to study the dependence, in a particular period, of the natural gas price in the different countries over the oil and coal prices. Starting from the baseline scenario of our model, we vary the oil and coal prices in the third period (2010-2014) and run the model each time in order to study the effect of the substitution on the natural gas prices. The coal and oil prices change the parameter p_c of the inverse demand function. Figure 11 gives the evolution of the (average) European natural gas price in 2015 over the oil and coal prices. We make these latter evolve linearly, in a similar way (starting from the baseline scenario: (-40%, -20%, +10%, +30%, +50%)). For more clarity in the presentation, we prefered showing the evolution of the natural gas price over the competition price p_c. [5]

Obviously, this evolution is an increasing function of the substitution fuels' prices. The higher the oil and coal prices are, the greater the natural gas demand will be and therefore, the higher the natural gas price will be. This property concerns also the long-term contracts prices between the producers and the independent traders η_{pi}. Hence, our model allows us to capture part of the indexation (on coal and oil prices) effects via the substitution in the inverse demand function.

4 Conclusions

This paper presents a Generalized Nash-Cournot model in order to describe the natural gas markets evolution. The demand representation is rich because it takes into account the possible energy substitution that can be made between oil, coal and natural gas. This appears in the introduction of a competition price, in the demand function. The exhaustibility of the resourse

[5]We recall that p_c represents an aggregated price of the coal and oil prices. It is an output of our system dynamics model used to define de inverse demand function, see Section 2.3.

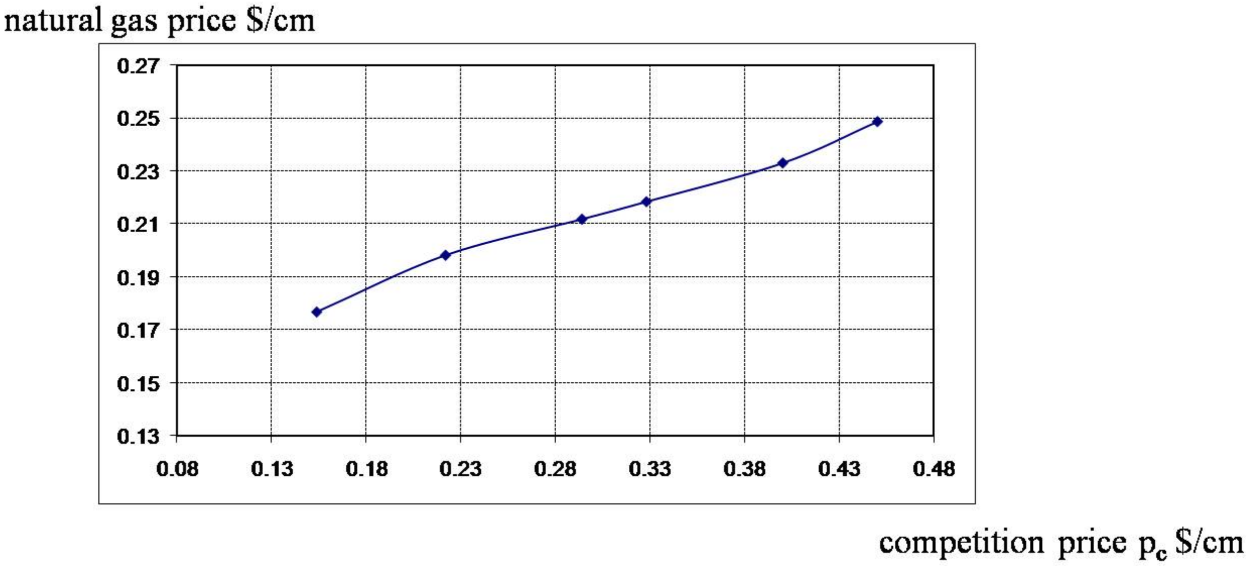

Figure 11:
Evolution of the natural gas price over the competition price in 2015.

is taken care of by the use of Golombek production cost functions.

The long-term contract prices and volumes are endogenously taken into account via dual variables. This aspect makes our formulation a Generalized Nash-Cournot model, or similarly a QVI formulation. In order to solve it, we derived the corresponding VI formulation.

The model is dynamic (2000-2040) and has been solved using the solver PATH with GAMS. After the calibration process, it has been applied to understand the European natural gas trade between 2000 and 2040 in terms of consumption, prices, production and natural gas dependence. The consumption and prices forecast carried out are consistent with those found in the literature. A study of the evolution of the natural gas dependence has been carried out. It shows that North-Western Europe will become more and more dependent on the foreign supplies in the future.

Our results have been compared with other forecasts: one provided by the European Commission and another one issued from a standard model where the energy substitution is not present. The results show that it is important to capture, while studying the natural gas demand function, the possible energy substitution regarding other possible usable fuels market prices.

In order to illustrate the possible use of fuel substitution, we studied the evolution of the natural gas price over the coal and oil prices. The coal-oil prices indexation of the natural gas price in the spot markets or in the long-term contracts can be understood thanks to these studies.

Future work could address: stochasticity when representing the impact of risk on the market or the seasonality of the demand, more policy focused analysises such as the impact of environmental policies on the gas trade evolution and the development of majors infrastructures toward Europe.

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