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On Dynamic Compromise

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Abstract

What prevents majorities from extracting surplus from minorities in legislatures? We study an infinite horizon game where a legislative body votes to determine distributive policy each period. Proposals accepted by a simple majority are implemented, otherwise the status quo allocation prevails. We construct a symmetric Markov perfect equilibrium that exhibits compromise in the following sense: if the initial status quo allocation is "not too unequal", then the Markov process is absorbed into allocations in which more than a minimum winning majority receives a positive share of the social surplus with positive probability. The compromise is only sustainable if, starting from the "unequal" allocations, the Markov process is absorbed into allocations in which there is a complete absence of compromise. The compromise equilibrium exists when discounting is neither too small nor too large. We find that, contrary to intuition, the range of discount factors for which this equilibrium exists increases as the number of legislators increases. In this sense, compromise is easier in larger legislatures.

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1 Introduction

Many legislatures require the consent of only a simple majority to implement policies. It follows that when distributive policy is being determined, a simple majority of legislators can split the entire surplus among themselves. Yet observed outcomes in which resources are distributed beyond a minimal majority are common. Examples such as military bases, transportation and agricultural subsidies all share this property to varying degrees. This paper examines the possibility that *compromise* — agreed upon outcomes in which more than a minimum winning majority receives a positive share of the social surplus — is sustainable in legislatures.

We posit a model of legislative bargaining and construct an equilibrium in which compromise may occur. In each period, a law is proposed by a randomly selected legislator and is passed by a majority vote.[1] If the current period's proposal fails to achieve a majority vote, the previous period's allocation is implemented. In this setting, each member of today's majority is concerned that he might belong to tomorrow's minority. With concave preferences, each legislator has an incentive to smooth his allocation over time, and this motive can lead to an equilibrium in which compromise occurs.

The setup resembles the legislative process that distributes benefits under many federal spending programs in the United States. The distribution of benefits is enacted by the legislature and written into law. This distribution remains effective until new legislation is passed to alter it. When benefits are distributed by this kind of process, laws made today can potentially be overturned tomorrow, so legislators must consider the trade-offs between appropriating political spoils in the short term or more equitably sharing benefits in the long term.[2] In particular, a legislator with proposal power cannot be certain he will have proposal power in the future and may prefer to distribute benefits among more legislators than a bare majority, in order to secure benefits for himself in the future.

We construct a symmetric Markov perfect equilibrium such that if the initial status quo allocation is not too unequal, the equilibrium Markov process is absorbed

[1] This stylized legislative process is common in the literature on legislative bargaining. It was introduced by Baron and Ferejohn (1989) who argue that, with a large number of legislators, each seeking to put forward his own policy, a legislative process that does not favor a particular legislator will result in a randomly selected proposer each period.

[2] We show that this motivation can be present in the absence of other explanations for an even distribution of benefits such as reputation or institutional effects.

into a closed class of proposals that exhibit compromise.[3] Critically, the compromise is only sustainable if starting from the remaining unequal allocations, the Markov process is absorbed into a closed class of proposals in which there is *no compromise.* If initial allocations are unevenly distributed, legislators cherry-pick minimum-winning coalitions and eventually the equilibrium transitions into this class. The sensitivity to initial conditions suggests that distributive policies that have a balanced allocation have their origins in conditions that were already somewhat equitable, while the reverse is true for unevenly distributed policies.

Standard repeated game arguments could also explain compromise in such a setting. If legislators condition on the entire history of play, simple trigger strategies could yield the compromise we observe, but there are good reasons why trigger strategies would be problematic in our setting. The first is that there is little evidence of legislators being "punished" in the allocation of benefits through distributive policy, so trigger strategies do not appear to be the equilibrium strategies being played.[4] In addition, most legislatures are characterized by periodic turnover. For example, the United States House of Representatives consists of 435 members each serving a two-year term, in Israel there are 120 members of the Knesset each serving 4-year terms, and the Mexican Chamber of Deputies comprises 500 deputies each serving three-year terms with no re-election. Even with powerful incumbency advantages, Matland and Studlar (2004) estimate 10% average annual legislative turnover for 25 countries, or 32% turnover per election. This periodic turnover can result in a lack of institutional memory.

Given potential memory problems it makes sense to restrict attention to Markov perfect equilibria, since these are subgame perfect equilibria in which players choose strategies that require no memory of past play beyond that which is relevant to today's payoffs. The Markov assumption does not imply that individual legislators have no memory. However, equilibrium "memory" captured by a legislature with turnover can be appropriately modeled as depending only on the current, payoff relevant information.

In this game the payoff-relevant information, or the state variable, is the status quo allocation and the current period's proposer. The stage game is a game of

[3] We use the term *closed class* as defined in Norris (1997). It is similar to the notion of an absorbing state, but refers to a set of states. Once a state in the closed class has been reached, the Markov process will not transition to a state outside that class.

[4] See Krehbiel (1991).

pure conflict and, since the state variable does not embody any information that is mutually beneficial to all legislators, they have no reason to be more cooperative at certain times than others.[5] The one shot Nash equilibrium of this game is for the proposer to offer a minimum winning coalition their status quo allocation, and extract the remainder of the surplus for himself. The difficulty in finding compromise in Markov equilibria is the proposer's natural desire to extract short term gains. Yet an incentive to overcome this is present in our game, without requiring our equilibrium to use a punishment scheme. The previous literature has so far been unable to show this.

To sustain the equilibrium legislators' discount factors must lie in an intermediate range, that is, they are required to be neither too patient nor too impatient. To understand the importance of *impatience*, suppose legislators discount factors were very high, and suppose someone deviated from the compromise class. To prevent such a deviation, the process must spiral towards the no-compromise class. However, a legislator with a very high discount factor would unilaterally make an offer to return to the compromise class, that would in turn be accepted, thereby halting the spiral towards no-compromise.[6]

We find that as the number of legislators increases, the range of discount factors that sustains these equilibria increases. Since more legislators means greater uncertainty over the agenda-setter, continuations that involve no compromise become less attractive, while compromise continuations become more attractive. This is in contrast to the conventional wisdom due to Olson (1965) that suggests cooperation diminishes as group size increases.[7]

We also find that as the utility function becomes more concave the upper bound and the lower bound on the range of admissible discount factors both decrease.

[5] This is in contrast to games that are studied in Battaglini and Coate (2006) where the state variable is public debt. A high level of public debt is mutually disadvantageous to all legislators. This echoes a result by Dutta (1995), where he finds that sustaining efficient Markov prefect equilibria requires some amount of "state symmetry". This condition is clearly violated in our model.

[6] Markov equilibria of the type of game we analyze are notoriously difficult to characterize because of the infinite multi-dimensional state space. As such, we characterize one equilibrium with the feature that compromise is a possible outcome, and it obtains when discount factors are within a certain range. Other equilibria of this model with risk averse legislators have not yet been found in the current literature, but work has been done by Kalandrakis (2003), (2007) and Duggan and Kalandrakis (2006) in closely related models. These are discussed at the end of the Introduction.

[7] Other authors that have also shown a converse result are Pecorino (1999), Haag and Lagunoff (2007), and Esteban and Ray (2001).

The intuition is that as concavity increases, the compromise continuation becomes more attractive, hence less-patient legislators are willing to offer it, but at the same time, less patience is needed for there to be an incentive to spiral towards the no-compromise.

The fact that we find equilibria that exhibit compromise when restricted to Markovian strategies implies that even institutions without a mechanism for conveying history can result in a compromise outcome. Additionally, we show that compromise can be possible in a legislative setting without explicit motives for it, such as in vote-trading. With distributive policy any outcome that improves the position of one legislator is at the expense of some other legislator so there is no incentive to trade votes. The rationale for compromise in our equilibrium is based on the aversion to the possibility of being disadvantaged in the future. This rationale sustains compromise without appealing to the possibility of side payments, as in Merlo (2000), log-rolling, parliamentary procedures or any other features of legislatures that could have been included in the model.

Our work is related to that of Dixit, Grossman and Gul (2000) who investigate political compromise based on tacit cooperation in a two-party framework. They consider efficient subgame perfect equilibria for a similar model with only two players and employ trigger strategies to sustain compromise. They find that the possibility of compromise between parties diminishes with the length of time any single party retains power. The model analyzed by Dixit et.al is relevant when considering compromise between political parties in a two-party system, in contrast we consider compromise among many competing legislators, each of whom is interested solely in distribution of benefits towards their district.[8] Lagunoff (2001) looks at the question of civil liberties and the formation of legal standards through majority voting. He finds that due to imprecise signals and the possibility of making mistakes, in equilibrium, groups choose standards that are not too severe. This argument parallels what we find. The lack of severe standards can be viewed as a compromise outcome and is driven by the noise introduced by imprecise signalling. Similarly in our model, compromise is driven by the uncertainty over the identity of future agenda-setters.[9]

Our game closely resembles that analyzed by Kalandrakis (2004) who considers

[8]We do not factor in ideological bias along party lines when considering payoffs, hence coalitions form independent of party affiliation. This basis of coalition formation is supported in empirical work by Lee (2000).

[9]Battaglini and Coate (2006) also include a bargaining game similar to ours as part of their model, but the status quo is not endogenous.

three risk neutral legislators. He finds that equilibrium outcomes are absorbed in a closed class in which the proposer takes all. Kalandrakis (2007) extends this result to the case of five or more risk-neutral legislators and with arbitrary recognition probabilities. He again shows that outcomes are absorbed in the no-compromise closed class.[10] Duggan and Kalandrakis (2006) prove a general existence result for this class of games, but the result does not extend to equilibria with more than a minimum winning majority obtaining a positive allocation. The equilibria found in the papers discussed above result in the proposer being able to extract the entire surplus. This provides a counterpoint to our paper, as we show that a sharing outcome is possible in equilibrium.

Earlier work on the dynamics of distributive policy was done by Baron and Ferejohn (1989), Baron (1996) and Gerber and Ortuno-Ortin (1998). Baron and Ferejohn (1989) was one of the earliest works to look at the dynamics of political compromise. Baron (1996) and Gerber and Ortuno-Ortin (1998) considered the case of a public good and a single dimensional policy space. Baron (1996) looked at a dynamic policy setting game, whereas Gerber and Ortuno-Ortin (1998) looked at a static game and considered the endogenous formation of coalitions. Both these papers obtained a version of the Median Voter Theorem, with some form of compromise occurring in equilibrium, but these are not games of direct conflict.

This paper adds to a growing literature emphasizing the constraints on proposal making power in dynamic distributive policy (Fong (2006), Battaglini and Palfrey (2007), Diermeier and Fong (2007)). Notably, Diermeier and Fong (2007) investigate endogenous limits on proposal power. They show that in an institutional setting in which one legislator is always the proposer and can make repeated proposals, the other legislators have an incentive to reject proposals allocating a zero share to other non-proposing legislators, hence reducing the ability of the proposer to extract the entire surplus. This is complimentary to the result in our equilibrium starting from the "well distributed" status quo allocations. We also show that starting from "uneven" status quo allocations the proposal power is essentially unconstrained as in

[10]Kalandrakis (2007) also shows that with equal recognition probabilities, and for a subset of state variables where a half or more legislators have a zero status quo allocation, the equilibrium strategies for the linear utility game satisfy incentive constraints for utility functions that exhibit some concavity. He provides sufficient conditions on the utility functions to ensure this incentive compatibility. The equilibrium we construct is complementary in the sense that we have similar strategies leading to the no-compromise class for the same set of state variables. The derived restrictions on concavity in Kalandrakis (2007) are satisfied by a more restrictive concavity condition that we assume.

earlier models. We show this in an environment where proposal power is randomly allocated to each legislator in each period thereby reducing the incentive to "protect" other non-proposing legislators' rights.

The remainder of the paper is organized as follows. In Section 2 we present the general model with $n+1$ legislators. In Section 3 we give a brief illustration of the equilibrium we characterize, and some intuition for the strategies. In Section 4 we formally characterize the equilibrium. Section 5 provides some comparative statics results and Section 6 concludes.

2 The Model

We present here a stylized version of the legislative process. Let $I = \{1, \ldots, n+1\}$ be a set of $n+1$ symmetric legislators, where $n \geq 6$ and n even.[11] They play a policy setting game over an infinite number of periods $t = 1, 2, \ldots$. Each period a surplus of unit size is divided among the $n+1$ legislators' districts, and each split of the surplus is an element of the n-dimensional simplex, Δ^n. Let the vector $s^t \in \Delta^n$ denote the division of the surplus in period t, where $s^t = (s_1^t, s_2^t, \ldots, s_{n+1}^t)$ and s_i^t is the share to legislator i's district. Legislator i is concerned only about the welfare of his district. The payoff to legislator i in period t, is given by $u(s_i^t)$, where $u(\cdot)$ is increasing and strictly concave allowing for inter-temporal gains from smoothing. All legislators discount the future with a common discount factor, δ, and legislator i seeks to maximize average expected lifetime utility for his own district,[12]

$$E\left[\sum_{t=1}^{\infty} \delta^{t-1}(1-\delta)u(s_i^t)\right].$$

At the beginning of each period a legislator, $x^t \in I$, is randomly recognized to make a proposal for the division of the surplus for that period. Legislators are recognized with equal probability in each period. This proposer selection process was first motivated by Baron and Ferejohn (1989) as discussed in the introduction. The recognized legislator, x^t, then makes a proposal, $\mathbf{p} \in \Delta^n$, which is voted on by all

[11]In many legislatures district representation is allocated by population size hence the assumption of symmetry seems appropriate.

[12]The discount factor can be thought of as not only the usual discounting of future payoffs, but also including some small probability of leaving office.

legislators, each legislator having a single vote. A simple majority of votes is required for a proposal to be implemented, hence the proposer requires $\frac{n}{2}$ legislators besides himself to be in agreement. If the proposal fails to achieve $\frac{n}{2}$ other legislators' vote, the status quo allocation, s^{t-1}, prevails. The persistence of the status quo allocation reflects the fact that the allocation schemes of the policies we consider remain intact if no new legislation is passed to alter it.

We ask whether or not the physical payoff-relevant information is enough to allow coordination on an equitable outcome as is reflected in the data. As argued in the introduction, legislatures that are characterized by a large number of members with periodic turnover may embody little institutional memory. We therefore focus on the class of subgame perfect equilibria consisting of Markovian strategies, i.e. Markov perfect equilibria. Before formally defining our notion of a Markov perfect equilibrium it is useful to give a brief description of the equilibrium strategies we characterize. We do this in the next section, and then formally characterize the equilibrium.

3 Equilibrium Illustration

Following Kalandrakis (2007), let Δ_θ, $\theta = 0, \ldots, n$ be a collection of subsets of Δ^n such that a number, θ, of the legislators receive zero allocation. That is

$$\Delta_\theta \equiv \left\{ s^{t-1} \in \Delta^n : |\{i : s_i^{t-1} = 0\}| = \theta \right\}$$

For example, letting $\mathcal{P}$ denote the set of all permutations, Φ, of a vector, we have

$$\Delta_n = \mathcal{P}(1, \underbrace{0, \ldots, 0}_{n}).$$

The set Δ_n is the set of proposals in which one legislator receives the entire share, hence we will call Δ_n our *no-compromise* class. Define also the set $\overline{\Delta}_1$ where one legislator receives zero, and the remaining legislators receive an equal share. That is, $\overline{\Delta}_1 \equiv \mathcal{P}(\frac{1}{n}, \ldots, \frac{1}{n}, 0)$. We refer to the set $\overline{\Delta}_1$ as our *compromise* class of proposals. Note that $\overline{\Delta}_1 \subset \Delta_1$.

The model is specified for seven or more legislators, but we can illustrate the essential elements of the equilibrium for the three-legislator case. The equilibrium strategies are inconsistent with this case, but the intuition for the results remain intact. [13] With three legislators, all policy proposals lie in the two dimensional simplex. The set of proposals where a single legislator receives a zero share and the remaining legislators split the surplus evenly are the points that lie half-way along each face of the simplex as illustrated in simplex on the left in Figure 1. Using the the $n = 6$ case as an example of the true equilibrium, this represents the compromise class $\mathcal{P}\left(\frac{1}{6}, \dots, \frac{1}{6}, 0\right)$. The vertices of the simplex on the right in Figure 1, illustrate the no-compromise class, or all allocations in $\mathcal{P}\left(1, 0, \dots, 0\right)$.

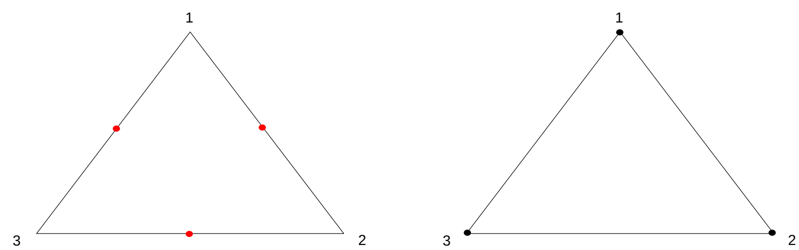

Figure 1: Closed Classes

The main proposition of the paper states that we characterize a symmetric Markov perfect equilibrium, MPE, in which the compromise class is reached in equilibrium. Clearly the set $\overline{\Delta}_1$ is not the most obvious compromise class to sustain in equilibrium, the obvious one being perfect sharing or $\left(\frac{1}{n+1}, \dots, \frac{1}{n+1}\right)$. We find that this class cannot be sustained as an equilibrium with the strategies specified because there needs to be some asymmetry in payoffs within the compromise class. This is to allow a modest punishment if there is a deviation from the compromise to allocations that lead back to compromise. We will discuss in section 4.3 how the equilibrium extends to other compromise classes that display some asymmetry.

The equilibrium we characterize is subject to initial conditions. If initial allocations are "well distributed" among non-proposing legislators, the proposer finds the compromise proposal to be most attractive in equilibrium. What we mean by "well distributed" will be made precise in the characterization below. If the initial allocations are closer to the vertices, the no-compromise becomes more attractive. Assuming legislator 1 is the proposer, the allocations that lead to the compromise class lie within the boundary illustrated by the shaded area in Figure 2.

[13]One reason the prescribed strategies are not an equilibrium in the case of three legislators is that the compromise, generally speaking, involves one legislator receiving zero and the remaining legislators splitting the surplus evenly. In the case of three legislators, this proposal takes the form $(\frac{1}{2}, \frac{1}{2}, 0)$. However a cherry-picking proposal which can lead to the no-compromise, also takes this form in the case of three legislators, hence the strategies are inconsistent.

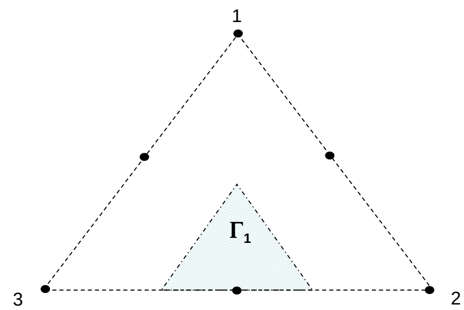

Figure 2: Equilibrium Illustration

Impatient legislators whose smoothing incentives are not too extreme always have an incentive to capture as much of the surplus as possible for their district at the expense of the other legislative districts. This is reflected in the equilibrium strategies. Consider allocations that are not on the face of the simplex. We will call these *interior allocations* and their defining characteristic is that the proposing legislator cannot identify $\frac{n}{2}$ non-proposing legislators with a zero status quo allocation.[14] With the $n = 6$ case this corresponds to allocations where at least 5 legislators have a positive share, and if the proposer has a zero status quo allocation, at least 4 legislators can have a positive share and it is still considered interior.

Now consider interior allocations that are outside the shaded area. Under the equilibrium strategies a proposing legislator identifies a minimum winning coalition of legislators that demand the least, and includes them in a coalition by offering them just enough to make them indifferent between the status quo allocation and the current proposal. In the $n = 6$ case, the coalition consists of at least 3 other legislators in addition to the proposer. These legislators will be "cherry-picked" to form a minimum winning coalition and the other three legislators will be frozen out and given no allocation. In the very next period if the proposing legislator was one of those in the original coalition, hence has a positive status quo allocation, he has the opportunity to extract the entire surplus. This leads to a sustained no-compromise outcome where each legislator grabs the entire surplus when he is the proposer. If the proposer in this period has a zero status quo allocation (hence was not in the original coalition), he will cherry-pick the other two legislators who were frozen out, and one member of the original coalition who now has a positive status quo allocation. In the next period the no-compromise class is reached.

[14]Notice that by this definition if the proposer has a zero status quo, and $\frac{n}{2} - 1$ other legislators have a zero status quo, this is also an interior allocation.

Notice from the above discussion that a legislator with a zero status quo allocation will strictly prefer receiving zero when the proposal is in $\mathcal{P}(1, 0, \ldots, 0)$, to receiving zero when the proposal is in $\mathcal{P}(s_1, \ldots, s_{\frac{n}{2}+1}, 0, \ldots, 0)$. This is because under both proposals the current period payoff is 0, but under the first proposal if he becomes the proposer in the next period he receives a payoff of 1, whereas under the second proposal if he becomes the proposer in the second period he must share the surplus with at least one other legislator.

What prevents the spiral towards no-compromise from happening in the shaded region? These allocations are well-distributed making the demands of the minimum winning coalition relatively high, so the proposer makes an offer to split the surplus between himself and one other legislator, knowing that in the next period the proposer will have an incentive to sustain this sharing. Once legislators have the expectation that sharing will occur, it is too costly to buy them off with a cherry-picking proposal, so the compromise is maintained. In the next sections we formalize this intuition.

4 Markov Perfect Equilibrium

With Markov perfect equilibria players' strategies condition only on information that is relevant to current period payoffs. The payoff relevant variables in this model are the status quo allocation and the identity of the proposing legislator, $(s^{t-1}, x^t) \in \Delta^n \times I$. A complete history of the state is therefore defined as $h^t = (s^0, x^1, \ldots, s^{t-1}, x^t)$.

Each legislator's strategy is a pair (α_i, σ_i) such that α_i is legislator i's acceptance strategy and σ_i is legislator i's mixed proposal strategy. A mixed proposal strategy for legislator i, is a probability function $\sigma_i(\cdot; h^t)$. Given a history of the state h^t and a proposal $\mathbf{p}$, $\sigma_i(\mathbf{p}; h^t)$ will be the probability legislator i assigns to proposal $\mathbf{p}$. An acceptance strategy for legislator i is a binary function $\alpha_i(\cdot; h^t)$ such that

$$\alpha_i(\mathbf{p}; h^t) = \begin{cases} 1 & \text{if legislator } i \text{ accepts proposal } \mathbf{p}, \\ 0 & \text{if legislator } i \text{ rejects proposal } \mathbf{p}. \end{cases}$$

A strategy profile is given by (α, σ) where α is a vector of acceptance strategies for all legislators, and σ is a vector of proposal strategies. Note that these acceptance and proposal strategies can potentially condition on the entire history of the state, h^t. We restrict our attention to Markovian strategies for the reasons explain before hence we consider only proposal and acceptance strategies that condition on (s^{t-1}, x^t). That is, we focus on a strategy pair $[\alpha_i(\cdot; s^{t-1}, x^t), \sigma_i(\cdot; s^{t-1}, x^t)]$.[15]

We seek a notion of symmetry for the legislators' strategies reflecting the fact that any legislator i will be expected to behave in the same manner as legislator j if he was in legislator j's position. More concretely, define the one-to-one operator, $\Phi : I \rightarrow I$ that represents any permutation of the identity of the legislators. Given a proposal vector, $\mathbf{p} = (p_1, \ldots, p_{n+1})$, and permutation $\Phi(\cdot)$, we denote the resulting permuted proposal as $\mathbf{p}_\Phi = (p_{\Phi(1)}, \ldots, p_{\Phi(n+1)})$. A permutation of the state variable (s^{t-1}, x^t) is therefore denoted $(s_\Phi^{t-1}, \Phi(x^t))$, and a symmetric strategy profile is given by the following definition.

Definition 1. A strategy profile (α, σ) is *symmetric* if for any permutation of the identities of legislators, Φ,

$$\begin{aligned}
\alpha_i(\mathbf{p}; s^{t-1}, x^t) &= \alpha_{\Phi(i)}(\mathbf{p}_\Phi; s_\Phi^{t-1}, \Phi(x^t)), \text{ and} \\
\sigma_i(\mathbf{p}; s^{t-1}, x^t) &= \sigma_{\Phi(i)}(\mathbf{p}_\Phi; s_\Phi^{t-1}, \Phi(x^t)).
\end{aligned}$$

Given a proposal, $\mathbf{p}$, and an acceptance strategy profile α the law of motion for the period's allocation is given by

$$s^t = \begin{cases} \mathbf{p} & \text{if } \sum\limits_{i \neq x^t} \alpha_i(\mathbf{p}; s^{t-1}, x^t) \geq \frac{n}{2}, \\ s^{t-1} & \text{otherwise.} \end{cases}$$

This simply says that if the proposal receives the required majority of votes it is implemented, otherwise the policy reverts to the status quo. The expected dynamic payoff for any legislator i, given a strategy profile, (α, σ), and a state (s^{t-1}, x^t), $V_i(\alpha, \sigma; s^{t-1}, x^t)$ is therefore

$$V_i(\alpha, \sigma; s^{t-1}, x^t) = \int_{\Delta^n} \left\{(1-\delta)u(s_i^t) + \delta E_{x^{t+1}}[V_i(\alpha, \sigma; s^t, x^{t+1})]\right\} \sigma_{x^t}[\mathbf{p}; s^{t-1}, x^t] d\mathbf{p}.$$

A Markov perfect equilibrium strategy profile must maximize this dynamic payoff

[15] Although we restrict attention to Markov strategies as our equilibrium concept, in Definition 2 below we ensure that equilibrium strategies are robust to any history contingent strategy, $[\alpha_i(\cdot; h^t), \sigma_i(\cdot; h^t)]$.

for all legislators, for all possible states and must be a best response among *any* history contingent strategy. This leads to Definition 2.[16]

Definition 2. A symmetric *Markov Perfect Equilibrium (MPE)* is a symmetric strategy profile, (α^*, σ^*), such that for all $(s^{t-1}, x^t) \in \Delta^n \times I$, for all $[\alpha_i(\cdot; h^t), \sigma_i(\cdot; h^t)]$, for all h^t, and for all i,

$$V_i(\alpha^*, \sigma^*; s^{t-1}, x^t) \geq V_i(\alpha_i(\cdot; h^t), \alpha^*_{-i}, \sigma_i(\cdot; h^t), \sigma^*_{-i}; s^{t-1}, x^t).$$

We seek to identify a symmetric MPE in which more than a minimum winning majority of legislators receive a positive allocation in each period. We define a compromise outcome as one is which this occurs. Formally

Definition 3. An allocation, s^t, exhibits *compromise* if $|\{i : s_i^t > 0\}| > \frac{n}{2} + 1$.

Notice that our compromise class, $\overline{\Delta}_1$, satisfies this definition. To ensure that when incentives to maintain compromise do not hold, there is an incentive to cherry-pick a minimum winning coalition before obtaining the no-compromise class we have assumption 1 that essentially says that the utility function is not too concave.

Assumption 1. *The utility function $u(\cdot)$ satisfies the concavity restriction*

$$\frac{n+1-\delta}{n+1-\delta n} u(s_i^{t-1}) \leq u(2s_i^{t-1}).$$

The main proposition of the paper is proposition 1. It states that given assumption 1, an MPE exists in which starting from some initial allocations, a compromise allocation is implemented from the first period onward. This is true as long as discount factors lie within an intermediate range. In the statement of the proposition, and in the equilibrium characterization the set $\Delta_{\theta<\beta}$ indicates the union of all sets $\cup_{\theta=0}^{\beta-1}\Delta_\theta$ and, similarly the set $\Delta_{\theta\leq\beta}$ indicates the union of all sets $\cup_{\theta=0}^{\beta}\Delta_\theta$.

Proposition 1. *There exists a non-degenerate interval $[\underline{\delta}, \overline{\delta}]$, and a set of allocations, $\Gamma_i \subset \Delta_{\theta<\frac{n}{2}}$ for every $i \in I$, such that if $u(\cdot)$ satisfies assumption 1, then for every*

[16] In the case of two legislators it is easy to see that there is no payoff relevant state. Since there are only two legislators, the proposer is automatically a majority, hence, effectively a dictator. There is no inter-temporal decision so the unique solution to the single period maximization results in the proposing legislator extracting the entire surplus. This is in contrast to the results of Dixit et al. (2000).

$\delta \in [\underline{\delta}, \overline{\delta}]$ *a symmetric MPE exists where if* $(s^0, x^1) \in \cup_i(\Gamma_i \times \{i\})$ *then* $(s^{t-1}, x^t) \in \overline{\Delta}_1 \times I$ *for all* $t \geq 1$.

The proof is constructive. We characterize the equilibrium in the next section.

4.1 Equilibrium Characterization

Given an allocation $s^t = (s_1^t, \ldots, s_{n+1}^t)$ and strategies (α, σ) define the dynamic payoff to player i as

$$U_i(s^t; \alpha, \sigma) = (1 - \delta)u(s_i^t) + \delta E_{x^{t+1}}[V_i(\alpha, \sigma; s^t, x^{t+1})]$$

The equilibrium acceptance strategy for any legislator i is α_i^* such that he accepts proposals that give a dynamic payoff that is at least as great as the payoff to the status quo. That is, given proposal $\mathbf{p}$,

$$\alpha_i^*(\mathbf{p}; s^{t-1}, x^t) = \begin{cases} 1 & \text{if } U_i(\mathbf{p}; \alpha^*, \sigma^*) \geq U_i(s^{t-1}; \alpha^*; \sigma^*) \\ 0 & \text{otherwise.} \end{cases}$$

Under the equilibrium proposal strategies, $\sigma^*(\cdot; s^{t-1}, x^t)$, if more than $\frac{n}{2}$ non-proposing legislators have a zero status quo allocation, the proposing legislator will extract the entire surplus and offer a *no-compromise* proposal. If less than $\frac{n}{2}$ non-proposing legislators have a zero status quo allocation, and the status quo is not one of the compromise allocations, (what we call *interior allocations*), the proposer will either offer a *compromise* proposal or extract as much surplus as possible by using a *cherry-picking* strategy. Once a compromise proposal has been implemented, the compromise is sustained. We will consider proposal strategies for specific status quo allocations.

4.1.1 Status Quo Allocations in the No-Compromise and Compromise Classes

First consider all (s^{t-1}, x^t), such that $s^{t-1} \in \Delta_n$. These are status quo allocations in the no-compromise class. The equilibrium proposal strategy is to assign probability one to the no-compromise proposal, $\underline{\mathbf{p}}$, where

$$\underline{p}_i = \begin{cases} 1 & \text{for } i = x^t \\ 0 & \text{for } i \neq x^t \end{cases}.$$

Notice that this proposal is also an element of the set Δ_n this means that the no-compromise class, Δ_n, represents a closed class of proposals. Now consider $s^{t-1} \in \Delta_{\theta > \frac{n}{2}}$, or $s^{t-1} \in \Delta_{\frac{n}{2}}$ and $s^{t-1}_{x^t} \neq 0$. These are allocations where more than $\frac{n}{2}$ non-proposing legislators have a zero status quo allocation. The equilibrium proposal is again the no-compromise proposal with probability 1, so $\sigma_i^*(\underline{\mathbf{p}}; s^{t-1}, x^{t-1}) = 1$.

Given these equilibrium strategies we can write down the dynamic payoff to the no-compromise proposal, $\underline{\mathbf{p}}$, which is

$$U_i(\underline{\mathbf{p}}; \alpha^*, \sigma^*) = \begin{cases} (1-\delta)u(1) + \delta E_{x^{t+1}}[V_i(\alpha^*, \sigma^*; \underline{\mathbf{p}}, x^{t+1})] & \text{if } i = x^t, \\ (1-\delta)u(0) + \delta E_{x^{t+1}}[V_i(\alpha^*, \sigma^*; \underline{\mathbf{p}}, x^{t+1})] & \text{if } i \neq x^t. \end{cases}$$

In the next period, since the status quo is an element of the no-compromise class, the equilibrium strategy is the no-compromise proposal again so we can define the recursive payoffs when the status quo is in the no compromise class as $\underline{V}_x$ for the proposer and $\underline{V}_z$ for everyone else. With probability $\frac{1}{n+1}$ each legislator is the proposer in the next period, hence any legislator's continuation value is $\underline{V}_x$ with probability $\frac{1}{n+1}$ and $\underline{V}_z$ with probability $\frac{n}{n+1}$. This gives

$$\underline{V}_x = (1-\delta)u(1) + \tfrac{\delta}{n+1}\left(\underline{V}_x + n\underline{V}_z\right),$$

$$\underline{V}_z = (1-\delta)u(0) + \tfrac{\delta}{n+1}\left(\underline{V}_x + n\underline{V}_z\right).$$

Normalizing $u(0) = 0$ and $u(1) = 1$, and solving gives

$$\underline{V}_x = \tfrac{n+1-\delta n}{n+1} \tag{1}$$

and

$$\underline{V}_z = \tfrac{\delta}{n+1}. \tag{2}$$

Now consider all (s^{t-1}, x^t), such that $s^{t-1} \in \overline{\Delta}_1$. These are status quo allocations in the compromise class. Define the compromise proposal generally as, $\overline{\mathbf{p}}^j$, such that legislator j receives zero and the remaining legislators split the surplus evenly. So

$$\overline{p}_i^j = \begin{cases} 0 & \text{if } i = j, \\ \frac{1}{n} & \text{otherwise.} \end{cases}$$

If $s^{t-1} \in \overline{\Delta}_1$ and $s^{t-1}_{x^t} = \frac{1}{n}$ then one legislator (excluding the proposer) has a zero status quo allocation and the remaining n legislators split the surplus evenly. The equilibrium strategy assigns probability 1 to the compromise proposal $\overline{\mathbf{p}}^j$, for j such that $s^{t-1}_j = 0$. So the legislator that had a zero status quo allocation is given zero again and all other legislators split the surplus evenly.

If $s^{t-1} \in \overline{\Delta}_1$ and $s^{t-1}_{x^t} = 0$, so the proposer's status quo allocation is zero, the proposer takes a legislator at random to give no allocation and splits the surplus evenly among himself and the remaining legislators. So $\sigma^*_i(\overline{\mathbf{p}}^j; s^{t-1}, x^t) = \frac{1}{n}$ for all $j \in I/\{x^t\}$. Notice again that once a proposal in the compromise class, $\overline{\Delta}_1$, has been implemented the equilibrium strategies dictate that all subsequent proposals lie in this set. Hence the set of proposals $\overline{\Delta}_1$ represents a closed class of proposals. The dynamic payoff to the compromise proposal, $\overline{\mathbf{p}}^j$, is therefore

$$U_i(\overline{\mathbf{p}}^j; \alpha^*, \sigma^*) = \begin{cases} (1-\delta)u(\frac{1}{n}) + \delta E_{x^{t+1}}[V_i(\alpha^*, \sigma^*; \overline{\mathbf{p}}^j, x^{t+1})] & \text{if } i \neq j, \\ (1-\delta)u(0) + \delta E_{x^{t+1}}[V_i(\alpha^*, \sigma^*; \overline{\mathbf{p}}^j, x^{t+1})] & \text{if } i = j. \end{cases}$$

If the status quo is an element of the compromise class, the equilibrium strategy is the compromise proposal again, so we can define the recursive payoff when the status quo is in the compromise class as γ for the legislators receiving $\frac{1}{n}$ and ζ for the legislator that is frozen out. With probability $\frac{n}{n+1}$ each legislator receives the same payoff as he did in the previous period, and with probability $\frac{1}{n+1}$ the current loser becomes the proposer, and a new legislator is randomly selected to be frozen out. These payoffs are given by

$$\begin{aligned} \gamma = &\ (1-\delta)u(\tfrac{1}{n}) + \tfrac{\delta}{n+1}\left(n\gamma + [\tfrac{1}{n}\zeta + \tfrac{n-1}{n}\gamma]\right), \\ \zeta = &\ (1-\delta)u(0) + \tfrac{\delta}{n+1}\left(\gamma + n\zeta\right). \end{aligned}$$

Solving for ζ and γ gives

$$\gamma = \tfrac{n(n+1-\delta n)}{(n+1)(n+\delta-\delta n)} u(\tfrac{1}{n}), \tag{3}$$

$$\zeta = \tfrac{n\delta}{(n+1)(n+\delta-\delta n)} u(\tfrac{1}{n}) \ . \tag{4}$$

Although equilibrium strategies are not fully specified, it is possible at this point to explain why a legislator does not deviate from the compromise and cherry-pick $\frac{n}{2}$ legislators to form a minimum winning coalition. This is what defines the lower bound on the discount factor as is explained in the next section.

Definition of $\underline{\delta}$. Consider a deviation from the compromise class in which the proposing legislator employs a cherry-picking strategy. He will attempt to buy-off a minimum winning coalition and extract the remainder of the surplus for himself. The coalition in this case will consist of the legislator who was receiving zero under the status quo and $\frac{n}{2} - 1$ randomly selected legislators who were each receiving $\frac{1}{n}$. The legislator who is receiving zero, will accept zero because $\underline{V}_z \geq \zeta$.

The remaining $\frac{n}{2} - 1$ legislators are receiving a dynamic payoff of γ under the status quo, and the proposer is also receiving γ under the status quo. Hence the best the proposer will be able to do under a deviation is offer an equal amount to himself and the coalition members which results in a stage payoff of $u\left(\frac{2}{n}\right)$. Since the deviation proposal is in the set $\Delta_{\frac{n}{2}<\theta\leq n}$ the continuation strategies dictate proceeding to the no-compromise class and obtaining a continuation payoff of $\frac{\delta}{n+1}$. Hence this deviation is profitable if and only if $\gamma < (1-\delta)u\left(\frac{2}{n}\right) + \frac{\delta}{n+1}$. Conversely such a deviation is prevented if and only if

$$\gamma \geq (1-\delta)u\left(\frac{2}{n}\right) + \frac{\delta}{n+1}. \tag{5}$$

As $\delta \to 0$ this inequality becomes $u\left(\frac{1}{n}\right) \geq u\left(\frac{2}{n}\right)$ which is a contradiction, so the proposer would want to propose this, but as $\delta \to 1$ this becomes $nu\left(\frac{1}{n}\right) \geq 1$, which holds by concavity of $u(\cdot)$. Hence the proposer does not want to propose such a deviation as long as legislators are *patient* enough. So there is a lower bound on the discount factor, $\underline{\delta}$, such that for all $\delta > \underline{\delta}$ the inequality in (5) is satisfied. Lemma 5 in the appendix shows that $\underline{\delta}$ does not contradict the restriction on concavity.

4.1.2 Interior Status-Quo Allocations

Now consider $s^{t-1} \in \Delta_{\frac{n}{2}}$ with $s^{t-1}_{x^t} = 0$. This is considered an interior allocation. As Kalandrakis (2007) notes, it is not necessarily an equilibrium strategy to offer a positive allocation to the legislator with the lowest positive status quo allocation, while freezing out the others. This is because if a single legislator receives a positive allocation in the next period with probability one, his continuation payoff may be so large that his dynamic payoff exceeds the dynamic payoff of other legislators with higher status quo allocations. The proposer in this case could do better by offering the legislator with the higher status quo allocation an amount that would make him indifferent between his status quo and the low continuation payoff. In equilibrium a cherry-picking strategy is used where, with some probability, a share $A(b^*)$ is offered to one of b^* legislators. Following Kalandrakis (2007) and without loss of generality suppose $s^{t-1} = (s_1^{t-1}, \ldots, s_{\frac{n}{2}+1}^{t-1}, 0, \ldots, 0)$ with $0 < s_i^{t-1} \leq s_{i+1}^{t-1}$ and $s_{x^t}^{t-1} = 0$. Given a value $b \in \{1, \ldots, \frac{n}{2} + 1\}$, define the allocation

$$A(b) = u^{-1} \left[\frac{\sum_{i=1}^{b} u(s_i^{t-1})}{b - \frac{\delta n}{2(n+1)}} \right]. \tag{6}$$

This is the value that is demanded by one of b legislators in equilibrium. Define the value b^* as[17]

$$b^* = \begin{cases} \min b \in \{1, \ldots, \frac{n}{2}\} & \text{s.t. } A(b) \leq A(b+1), \\ \frac{n}{2} + 1 & \text{otherwise.} \end{cases}$$

Now define the cherry-picking proposal, $\mathbf{p}^j(A(b^*))$, where legislator j receives $A(b^*)$, the proposer receives $1 - A(b^*)$, and all other legislators receive zero. So

$$p_i^j(A(b^*)) = \begin{cases} A(b^*) & \text{for } i = j, \\ 1 - A(b^*) & \text{for } i = x^t, \\ 0 & \text{otherwise.} \end{cases}$$

The equilibrium proposal strategy is to assign probability $\mu_j(b^*)$ to the cherry-picking proposal $\mathbf{p}^j(A(b^*))$ for all $j \leq b^*$, where $\mu_j(b^*)$ is given by

$$\mu_j(b^*) = \frac{u(A(b^*)) - u(s_j^{t-1})}{u(A(b^*))\frac{\delta n}{2(n+1)}}. \tag{7}$$

The intuition is that for any allocation in $\Delta_{\frac{n}{2}}$, it is a best response for a proposer with a zero status quo allocation to randomize over legislators in such a way that their dynamic payoffs are lower than the next highest status quo allocation legislator. This ensures there is no incentive to deviate to offering any other legislator a positive share.

Now consider the dynamic payoff to the cherry-picking proposal $\mathbf{p}^j(A(b^*))$. This proposal is an element of the set Δ_{n-1} so the equilibrium continuation strategies

[17] Kalandrakis (2007) shows that b^* is unique.

dictate that the no-compromise proposal, $\underline{\mathbf{p}}$, is implemented. The continuation payoffs are therefore $\underline{V}_x$ if legislator i is the proposer and $\underline{V}_z$ otherwise. Hence we can simplify these dynamic payoffs as

$$U_i(\mathbf{p}^j(A(b^*)); \alpha^*, \sigma^*)) = \begin{cases} (1-\delta)u(1-A(b^*)) + \frac{\delta}{n+1} & \text{if } i = x^t, \\ (1-\delta)u(A(b^*)) + \frac{\delta}{n+1} & \text{if } i = j, \\ \frac{\delta}{n+1} & \text{otherwise.} \end{cases}$$

The probability assigned to legislator $i \leq b^*$ being allocated a positive share is μ_i, so define the expected dynamic payoffs to the cherry-picking strategy $\sigma_i^*(\mathbf{p}^j(A(b^*)); s^{t-1}, x^t)$ as $V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0)$. These are given by

$$V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0) = \begin{cases} (1-\delta)u\left(1-A(b^*)\right) + \frac{\delta}{n+1} & \text{if } i = x^t \\ (1-\delta)u\left(A(b^*)\right)\mu_i + \frac{\delta}{n+1} & \text{if } i \leq b^*, \\ \frac{\delta}{n+1} & \text{otherwise.} \end{cases} \tag{8}$$

Given that legislators have concave stage utilities, it seems possible that there would be an incentive for the proposer to deviate to the compromise proposal rather than offering the cherry-picking strategy that leads to a very uncertain payoff. Hence legislators need to be *impatient* enough to avoid such a deviation. The next section defines the first upper bound on the discount factor that prevents such a deviation.

Definition of $\overline{\delta}_1$. The first upper bound ensures that once a cherry-picking allocation has been implemented, legislators no longer have an incentive to propose the compromise. Consider a status quo in $\Delta_{\frac{n}{2}}$ when the proposer has a zero status quo allocation. Equilibrium strategies dictate proposing a cherry-picking allocation such that one legislator receives $A(b^*)$ and the proposer receives $1 - A(b^*)$. The dynamic payoff for the proposer under the equilibrium strategy is therefore $(1-\delta)u(1-A(b^*)) + \frac{\delta}{n+1}$ since continuation strategies dictate remaining in the no-compromise class. First, for the proposer to choose this allocation over the allocation $A(b^*)$ for himself and $1-A(b^*)$ to the coalition member, it must be that $\frac{1}{2} \leq 1-A(b^*)$. This holds by the restriction on concavity. [18] Now consider a deviation where the proposer proposes a compromise allocation, $\overline{\mathbf{p}}^j$. From equation (3) the payoff from such a deviation is γ. To show that this is not a profitable deviation it suffices to show that

[18] See Appendix section 7.7.2.

$$\gamma \leq (1-\delta)u\left(\frac{1}{2}\right) + \frac{\delta}{n+1}(1-\delta) \leq u(1-A(b^*)) + \frac{\delta}{n+1}$$

As $\delta \to 0$ this becomes $u\left(\frac{1}{n}\right) \leq u\left(\frac{1}{2}\right)$, which clearly holds, but as $\delta \to 1$ this becomes $nu\left(\frac{1}{n}\right) \leq \frac{1}{n+1}$, which is a contradiction. Hence this inequality represents an upper bound on the discount factor, which we will call $\overline{\delta}_1$. Lemma 1 ensures that there is a non-degenerate range between $\underline{\delta}$ and $\overline{\delta}_1$.

Lemma 1. *There exists a non-degenerate interval* $[\underline{\delta}, \overline{\delta}_1]$.

Proof. Notice that $\underline{\delta}$ and $\overline{\delta}_1$ together imply $(1-\delta)u\left(\frac{2}{n}\right) + \frac{\delta}{n+1} \leq \gamma \leq (1-\delta)u\left(\frac{1}{2}\right) + \frac{\delta}{n+1}$. Since $4 < n$ there must be a non-degenerate range of δ over which these two conditions hold. ■

Now consider $s^{t-1} \in \Delta_{\theta<\frac{n}{2}} \backslash \overline{\Delta}_1$. This is the remainder of the interior allocations. Let C_j be the set of legislators that are a part of legislator j's coalition. Note that that $|C_j| = \frac{n}{2}$. Define the vector of demands for legislator j's coalition member's as $A^{*j} = (A_1^{*j}, \ldots, A_{\frac{n}{2}}^{*j})$, where $0 \leq A_i^{*j} \leq 1$. This is the set of allocations that makes each of legislator j's coalition members at least indifferent between the current proposal and the status quo, given that legislator j is the proposer. Given a proposer j and demands A^{*j} the implemented proposal is either a cherry picking proposal, $\mathbf{p}(A^{*j})$, or the compromise proposal. In this case the proposal $\mathbf{p}(A^{*j})$ is given by

$$p_i(A^{*j}) = \begin{cases} A_i^{*j} & \text{for } i \in C_j \\ 1 - \sum_{i \in C_j} A_i^{*j} & \text{for } i = x^t \\ 0 & \text{otherwise .} \end{cases}$$

Notice that the cherry-picking proposals previously defined are a special case of the cherry-picking proposal, $\mathbf{p}(A^{*j})$. Generally $\mathbf{p}(A^{*j})$ is in the set $\Delta_{\theta \geq \frac{n}{2}}$. If it is an element of $\Delta_{\frac{n}{2}}$ the equilibrium continuation strategies will be the no-compromise proposal, $\underline{\mathbf{p}}$, if the period $t+1$ proposer has a positive status quo allocation. If the period $t+1$ proposer has a zero status quo allocation the continuation payoffs are given by $V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0)$, so the expected continuation payoffs for the cherry-picking proposal, $\mathbf{p}(A^{*j})$, are given by

$$V_i(A^{*j}) = \begin{cases} \frac{1}{n+1}(\underline{V}_x + \frac{n}{2}\underline{V}_z + \frac{n}{2}V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0)) & \text{for } p_i(A^{*j}) > 0, \\ \frac{1}{n+1}((\frac{n}{2}+1)\underline{V}_z + \frac{n}{2}V_i(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0)) & \text{for } p_i(A^{*j}) = 0. \end{cases} \quad (9)$$

Below I remove the conditioning on (α^*, σ^*) to conserve space. So the dynamic payoffs to the cherry-picking proposal are

$$U_i(\mathbf{p}(A^{*j})) = \begin{cases} (1-\delta)u(A_i^{*j}) + \delta V_i(A^{*j}) & \text{if } i \in C_j, \\ (1-\delta)u(1 - \sum_{i \in C_j} A_i^{*j}) + \delta V_i(A^{*j}) & \text{if } i = j \\ \delta V_i(A^{*j}) & \text{otherwise.} \end{cases}$$

Denote the set of all permutations of cherry-picking proposals where legislator j is the proposer, given a vector of demands, A^{*j}, as $\mathbf{P}(A^{*j})$. The equilibrium strategy is a probability distribution $\mu^{*j}(\cdot)$ over all $\mathbf{p}(A^{*j}) \in \mathbf{P}(A^{*j})$, and over all compromise proposals $\overline{\mathbf{p}}^i \in \overline{\Delta}_1$. The intuition for the probability distribution over cherry-picking proposals here is the same as before. If legislators with the lowest status quo allocations are in the coalition with probability one, in some cases their dynamic payoff exceeds the dynamic payoff to legislators who have higher status quo allocations, hence the proposer would have an incentive to deviate to having these legislators in the coalition. To avoid such a deviation, the equilibrium strategies are a probability distribution over members of the coalition.

Given a status quo allocation, $s^{t-1} = (s_1^{t-1}, \ldots, s_{n+1}^{t-1})$, the vectors A^{*j} and the distributions $\mu^{*j}(\cdot)$ are determined by the fixed point of a map, $\mathbf{B}$, that is discussed in the following section.

4.2 Derivation of $\mu^{*j}(\cdot)$ and A^{*j}

Denote an arbitrary cherry-picking proposal by legislator j as $\mathbf{p}(A^j)$, and, as before, let the set $\mathbf{P}(A^j)$ be all permutations of this cherry-picking proposal, given demands A^j. Now denote for legislator j an arbitrary probability distribution over cherry-picking proposals in $\mathbf{P}(A^j)$ and over all compromise proposals $\overline{\mathbf{p}}^i$ as $\mu^j(\cdot)$, and let $\mu = (\mu^1(\cdot), \ldots, \mu^{n+1}(\cdot))$. Finally, denote the matrix of demands for legislators $j = 1, \ldots, n+1$ as $\mathbf{A} = (A^1, \ldots, A^{n+1})$.

Let $V_i(\mathbf{A}, \mu)$ be the expected continuation payoff for legislator i given demands, $\mathbf{A}$, and probability distributions, μ. This is

$$V_i(\mathbf{A}, \mu) = \frac{1}{n+1} \sum_{j=1}^{n+1} \left[\sum_{\mathbf{p}(A^j) \in \mathbf{P}(A^j)} U_i(\mathbf{p}(A^j)) \mu^j(\mathbf{p}(A^j)) + \gamma \sum_{h \in I/\{i\}} \mu^j(\overline{\mathbf{p}}^h) + \zeta \mu^j(\overline{\mathbf{p}}^i) \right].$$

Hence given a status quo, an arbitrary vector of demands, and probability distributions, the dynamic payoff to the status quo is

$$U_i(s^{t-1}; \mathbf{A}, \mu) = (1-\delta)u(s_i^{t-1}) + \delta V_i(\mathbf{A}, \mu). \tag{10}$$

Assume $p_i(A^j) > 0$, then calculate for all i and all j the value $\hat{A}_i^j(s^{t-1}; \mathbf{A}, \mu)$ as

$$\hat{A}_i^j(s^{t-1}; \mathbf{A}, \mu) = u^{-1}\left[\min\{\max\{0, \frac{1}{1-\delta}\left(U_i(s^{t-1}; \mathbf{A}, \mu) - \delta V_i(A^j)\right)\}, 1\}\right].$$

Define a vector of these values as $\hat{A}^j(s^{t-1}; \mathbf{A}, \mu)$. Last define the set of all distributions over cherry-picking proposals in $\mathbf{P}(A^j)$ and all compromise proposals as $\mathbf{M}(\mathbf{P}(A^j))$. Now let us pick new demands and distributions, $(A^{j\prime}, \mu^{j\prime}) \in \mathbf{B}_j(\mathbf{A}, \mu; s^{t-1})$ where

$$\begin{aligned}
\mathbf{B}_j(\mathbf{A}, \mu; s^{t-1}) = \quad & \{(A^{j\prime}, \mu^{j\prime}) : \\
& (A^{j\prime}, \hat{\mu}^{j\prime}) \in \arg\max\nolimits_{\hat{A}^j, \hat{\mu}^j} U_j(s^{t-1}; \hat{A}^j, A^{-j}, \hat{\mu}^j, \mu^{-j}) \\
& \text{s.t.} \quad \hat{A}^j = \hat{A}^j(s^{t-1}; \mathbf{A}, \hat{\mu}^j, \mu^{-j}) \\
& \qquad\;\; \hat{\mu}^j \in \mathbf{M}(\mathbf{P}(A^j)) \\
& \text{and} \quad \mu^{j\prime}(\mathbf{p}(A^{j\prime})) = \hat{\mu}^{j\prime}(\mathbf{p}(A^j))\}.
\end{aligned}$$

Define

$$\mathbf{B}(\mathbf{A}, \mu; s^{t-1}) = \times_{j=1}^{n+1} \mathbf{B}_j(\mathbf{A}, \mu; s^{t-1}).$$

Lemma 2. *The map $\mathbf{B}(\mathbf{A}, \mu; s^{t-1})$ has a fixed point $(\mathbf{A}^*, \mu^*)$ such that $(\mathbf{A}^*, \mu^*) \in \mathbf{B}(\mathbf{A}^*, \mu^*; s^{t-1})$.*

Proof. See Appendix. ■

We can now formally define the set Γ_j to be the set of status quo allocations where less than $\frac{n}{2}$ legislators have a zero allocation, and from which legislator j will choose to go to compromise. This will be where $\mu^{*j}(\overline{\mathbf{p}}^i) = 1$ for some i. So

$$\Gamma_j = \left\{ s^{t-1} \in \Delta_{\theta<\frac{n}{2}} : \mu^{*j}(\overline{\mathbf{p}}^i) = 1 \text{ for some } i \right\}.$$

Now define the set Γ to be the set of all status quo allocations for which there is some positive probability that some legislator j will choose compromise. That is

$$\Gamma = \left\{ s^{t-1} \in \Delta_{\theta<\frac{n}{2}} : \mu^{*j}(\overline{\mathbf{p}}^i) > 0 \text{ for some } i, \text{ for some } j \right\}.$$

Note that $\Gamma_j \subset \Gamma$. Last define all other allocations in $\Delta_{\theta<\frac{n}{2}}$ excluding the compromise class to be $\Delta^c_{\theta<\frac{n}{2}}$. This is

$$\Delta^c_{\theta<\frac{n}{2}} = \Delta_{\theta<\frac{n}{2}} / (\Gamma \cup \overline{\Delta}_1).$$

We can now provide an illustration of the equilibrium dynamics which will help to give some intuition for the second upper bound on the discount factor. This is given in figure 3 below.

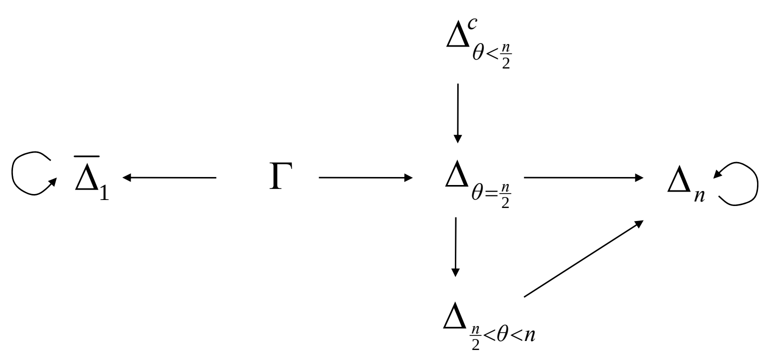

Figure 3: Equilibrium Dynamics

Definition of $\overline{\delta}_2$. The second upper bound on the discount factor $\overline{\delta}_2$ prevents a deviation from the compromise class into the set Γ_j. For the three-legislator case the set Γ_1 is illustrated by the shaded area in Figure 2. The outer boundary of the triangle implies a lower bound on the acceptor's status quo allocation.

Once the compromise class has been reached, at least one legislator must be singled out every period to receive the zero allocation. If the proposer's status quo is non-zero, he gives the zero to the legislator who already had zero, if not, he must select someone at random to receive the zero. This means that within the compromise, there is an element of uncertainty, and legislators may have incentives to "game" the system by proposing a share for themselves that will give a certain continuation payoff of γ. A proposer can do this by making a proposal that is within Γ_j for all j, while making sure he does not have the highest share. This guarantees a continuation payoff of γ.

The only way for there to be no such incentive, is if the allocations that are attainable within Γ_j are so small that they offset any gain in continuation payoff. A deviation s^t within Γ_j for all j implies, at best, a continuation payoff equal to γ, so we must have that $(1-\delta)u(s^t_j) + \delta\gamma \leq \gamma$. So all allocations where legislator j does not have the highest share, s^t_j, within Γ_j must satisfy

$$u(s^t_j) \leq \gamma.$$

This boundary cannot be arbitrarily imposed since the boundaries of Γ_j in the next period are determined by the incentives of the next period's proposer (legislator j). So this upper bound on the allocations attainable in Γ_j must be induced by the lower bound on the next period's acceptors' allocations. Figure 4 illustrates this point for the case with three legislators. The horizontal dashed line in figure 4 represents the upper bound on the deviation allocation for legislator 1 that is required to prevent a deviation from the compromise class. The diagonal lines represent the lower bounds on the acceptors' allocations implied by Γ_1.

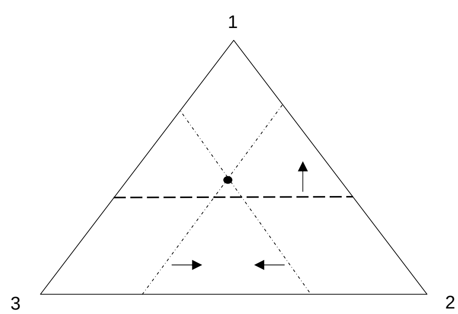

Figure 4: Decreasing δ

The intersection of the diagonal lines, indicated by the black dot, represents the maximum allocation the proposing legislator (legislator 1) can take while still remaining in Γ_1, based on the lower bound on the acceptors' allocations. This maximum must lie below the dashed line in equilibrium. The arrows indicate the direction in which these lines move as δ decreases, so for δ low enough, we achieve the situation depicted in figure 5. This defines the second upper bound on the discount factor, $\overline{\delta}_2$.

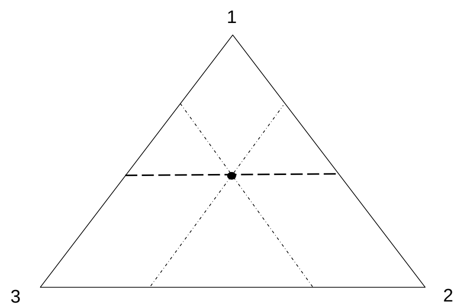

Figure 5: Condition for $\overline{\delta}_2$

The derivation of $\overline{\delta}_2$ is given in Section 7.3 in the Appendix, but below we discuss it intuitively. Defining $\phi = \frac{1}{1-\delta}\left[\gamma - \frac{\delta}{n+1}\right]$, we have $\overline{\delta}_2$ such that for all $\delta \leq \overline{\delta}_2$

$$(1-\delta)u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) + \delta\gamma \leq (1-\delta)\frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right) + \frac{\delta}{n+1}. \tag{11}$$

The right hand side of the expression is the payoff to the coalition member when legislator j is exactly indifferent between compromise and cherry-picking. When the proposer is indifferent he receives an allocation equal to $u^{-1}(\phi)$ under the cherry-picking strategy, so each of the $\frac{n}{2}$ coalition members receive an equal share of $\frac{2}{n}\left[1-u^{-1}(\phi)\right]$. This is equated with the coalition member's status quo payoff, so $(1-\delta)\frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right) + \frac{\delta}{n+1} = (1-\delta)u(s_i^{t-1}) + \delta\gamma$. The left hand side of the inequality is the lower bound on the status quo payoff to coalition members generated by imposing the upper bound on the proposer's status quo allocation of $u^{-1}(\gamma)$. It remains to be verified that $\underline{\delta} < \overline{\delta}_2$ to guarantee a non-degenerate range between $\underline{\delta}$ and $\overline{\delta}_2$. Lemma 3 shows that this is the case.[19]

Lemma 3. *There exists a non-degenerate range* $[\underline{\delta}, \overline{\delta}_2]$.

Proof. In the appendix. ■

The upper bound on the discount factor, $\overline{\delta}$, is given by $\overline{\delta} = \min\{\overline{\delta}_1, \overline{\delta}_2\}$. By lemmas 1 and 3, it is clear that there exists a non-degenerate range $\left[\underline{\delta}, \overline{\delta}\right]$. The equilibrium strategies, together with the incentive analysis in the Appendix section 7.7 and lemmas 2-3 complete the proof of proposition 1.

[19] Calculations of the bounds on the discount factor for specific parameterizations can be found in the appendix in section 7.5.

4.3 Other Compromise Classes

The above characterization focused on the specific compromise class where a single legislator received zero and the remaining legislators split the surplus evenly. However the equilibrium is not restricted to this compromise class. The significance of the compromise class is that it is a distinct set of allocations that signals the compromise, and there must be an incentive compatible algorithm to transition between states in the compromise class, once the compromise class has been reached. Clearly, not every compromise allocation can be sustained in this way. As mentioned before, the perfect compromise is a notable example. The payoffs to the compromise class must allow a non-degenerate set of allocations, Γ_j, and must allow a non-degenerate range of discount factors $[\underline{\delta}, \overline{\delta}]$. The perfect compromise fails the latter condition.

5 Comparative Statics

The intuition that more legislators make it easier for compromise to be sustained is reflected in the comparative statics for the range of discount factors for which this equilibrium holds. We will show below that this range increases as the number of legislators increases.

Proposition 2. *For a large number of legislators, n, the range of discount factors $[\underline{\delta}, \overline{\delta}_1]$ is increasing in n.*

Proof. In the appendix. ■

The proof proceeds by first showing that the lower bound is decreasing for large values of n. Then we show that the difference between the lower bound and the upper bound is increasing as a function of n for large values of n. The result is similar for the range of $[\underline{\delta}, \overline{\delta}_2]$, but because of the complex implicit function we show this for a parametrization of the utility function in appendix 7.5. The intuition for this result is as follows. In this model legislators' incentives to compromise is driven in part by the uncertainty over their future agenda-setting power. As the number of legislators becomes large, this uncertainty increases, thereby increasing legislator's willingness to compromise. This explains the decrease in the lower bound as n becomes large. In addition, as the number of legislators increase, the difference in current period payoff between a no-compromise proposal and a compromise proposal gets larger, making the no-compromise proposal more attractive, which drives up the upper bound. Proposition 2 is illustrated in figure 6.

Figure 6: Bounds on δ as n increases

We would also like to consider what happens to the range on the admissible discount factors as the utility function becomes more concave. Consider two utility functions $u(\cdot)$ and $v(\cdot)$, where $v(\cdot)$ is more concave than $u(\cdot)$ in the sense that $v(s) \geq u(s)$ for all $s \in [0,1]$, but $v(\cdot)$ still satisfies the normalization $v(0) = 0$ and $v(1) = 1$.

Proposition 3. *As the utility function becomes more concave, the lower bound on the discount factor decreases.*

Proof. Denote the lower bound on the discount factor associated with utility function $u(\cdot)$ as $\underline{\delta}_u$ and the lower bound on the discount factor associated with utility function $v(\cdot)$ as $\underline{\delta}_v$. The condition that defines $\underline{\delta}_u$ can be written as

$$\frac{u\left(\frac{2}{n}\right) - \frac{n(n+1-\underline{\delta}_u n)}{(n+1)(n+\underline{\delta}_u - \underline{\delta}_u n)} u\left(\frac{1}{n}\right)}{u\left(\frac{2}{n}\right) - \frac{1}{n+1}} = \underline{\delta}_u.$$

Substituting $v(\cdot)$ for $u(\cdot)$, we have

$$\frac{v\left(\frac{2}{n}\right) - \frac{n(n+1-\underline{\delta}_u n)}{(n+1)(n+\underline{\delta}_u - \underline{\delta}_u n)} v\left(\frac{1}{n}\right)}{v\left(\frac{2}{n}\right) - \frac{1}{n+1}} \leq \underline{\delta}_u.$$

Define a mapping $h : [0,1] \to [0,1]$, such that

$$h(\delta) = \frac{v\left(\frac{2}{n}\right) - \frac{n(n+1-\delta n)}{(n+1)(n+\delta-\delta n)} v\left(\frac{1}{n}\right)}{v\left(\frac{2}{n}\right) - \frac{1}{n+1}}.$$

Lemma 4. *The mapping h is a contraction with modulus $\beta = \frac{nv\left(\frac{1}{n}\right)}{\left[v\left(\frac{2}{n}\right) - \frac{1}{n+1}\right](n+1)}$.*

Proof. In the appendix. ■

Since by lemma 4 h is a contraction, then we know that it has a unique fixed point, and, by definition, this fixed point must be $\underline{\delta}_v$. Also by h increasing in δ, we know that $\underline{\delta}_u \geq h(\underline{\delta}_u) \geq h(h(\underline{\delta}_u)) \geq ....$ Since we know that this process converges to the fixed point, $\underline{\delta}_v$, by induction $\underline{\delta}_u \geq \underline{\delta}_v$.

■

The intuition for this result is that for more concave utility functions, the compromise becomes more attractive. So legislators do not need to be as patient to want to sustain the compromise.

Proposition 4. *As the utility function becomes more concave, the first upper bound on the discount factor, $\overline{\delta}_1$, decreases.*

The proof here is identical to the proof of proposition 3 except that the mapping h is now the mapping $\overline{h}_1 = \frac{v\left(\frac{1}{2}\right) - \frac{n(n+1-\delta n)}{(n+1)(n+\delta-\delta n)} v\left(\frac{1}{n}\right)}{v\left(\frac{1}{2}\right) - \frac{1}{n+1}}$, with modulus $\overline{\beta}_1 = \frac{nv\left(\frac{1}{n}\right)}{\left[v\left(\frac{1}{2}\right) - \frac{1}{n+1}\right](n+1)}$. Again, the intuition is that, as the utility function becomes more concave, the compromise outcome becomes more attractive so legislators need to be even more impatient to want to maintain no compromise. Section 7.6 in the appendix shows that the results for the second upper bound are the same for some common parameterizations. Propositions 3 and 4 are illustrated in figure 7.[20]

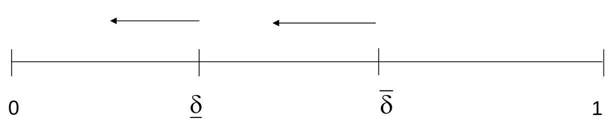

Figure 7: Bounds on δ as $u(\cdot)$ more concave

6 Conclusion

Casual observation indicates that almost all legislative districts in the United States participate in benefits from distributive policies. The theoretical literature on political compromise shows no consensus on whether or not political compromise will be achieved in equilibrium in a general setting [Dixit et al. (2000), Lagunoff (2001), and Kalandrakis (2004)]. We provide a general framework that predicts conditions under which political compromise is achieved, and when compromise is not achieved.

We model legislators with concave utilities that condition strategies only on information that is payoff relevant. We show existence of a set of equilibria that induces

[20]Note that by by lemmas 1 and 3 there always exists a non-degenerate range between $\underline{\delta}$ and $\overline{\delta}$.

a Markov process with two closed classes of proposals: one in which no-compromise is the outcome and the other in which more than a minimum winning majority share in the surplus. We refer to the latter as the compromise outcome.

The question is what determines the outcome in equilibrium. We find that the set of initial allocations that lead to the compromise cannot be too unevenly distributed. If they are unevenly distributed, in equilibrium, proposing legislators have an incentive to propose the no-compromise outcome because they are not too patient, and they can find enough legislators to buy off cheaply. If initial allocations are well distributed among non-proposing legislators the proposer will choose a compromise outcome instead, because a minimum winning coalition is too expensive to buy off.

We show that this equilibrium holds for an intermediate range of discount factors. This range becomes larger as the number of legislators increases, indicating that compromise is *easier* to sustain with a larger number of legislators, contrary to intuition. In addition, as the utility function becomes more concave, both end-points of the range decrease.

Interestingly, we find that the perfect compromise cannot be sustained with the equilibrium strategies we construct (although other compromise classes can be). The reason is that there needs to be some modest punishment for deviating from the compromise class back to allocations that lead directly to compromise. The absence of such a penalty leads to an infeasible range of discount factors that sustains the equilibrium. The question still remains whether the perfect compromise can be sustained as a Markov Perfect equilibrium of this game, but we leave this for future work.

7 Appendix

7.1 Proof of Lemma 2

To prove the map **B** has a fixed point I will employ Kakutani's Fixed Point Theorem. The space of **A** and μ are $[0,1]^{\frac{n(n+1)}{2}}$ and $[0,1]^{(n+1)}$ respectively. These spaces are non-empty, compact and convex. The correspondence **B** is non-empty since $U_j(s^{t-1};\hat{A}^j,A^{-j},\hat{\mu}^j,\mu^{-j})$ is continuous and the space of $\hat{A}^j$ and $\hat{\mu}^j$ are compact so a maximizer exists. The map **B** must also be convex valued since $U_j(s^{t-1};\hat{A}^j,A^{-j},\hat{\mu}^j,\mu^{-j})$ is linear in $\mu^{j\prime}$, and any $A^{j\prime}$ that maximizes $U_j(s^{t-1};\hat{A}^j,A^{-j},\hat{\mu}^j,\mu^{-j})$ must result in the same $\sum_{i\in C_j} A_i^{j\prime}$. By Theorem of the Maximum we can show that **B** is upper hemicontinuous.

For Theorem of the Maximum we already know that $U_j(s^{t-1};\hat{A}^j,A^{-j},\hat{\mu}^j,\mu^{-j})$ is continuous, we need only show $\mathbf{M}(\mathbf{P}(A^j))$ compact, and $\mathbf{M}(\mathbf{P}(A^j))$ continuous. To show $\mathbf{M}(\mathbf{P}(A^j))$ continuous and compact note that there are a finite number of elements of $\mathbf{P}(A^j)$, specifically, $|\mathbf{P}(A^j)| =^n P_{\frac{n}{2}}$. So $\mathbf{M}(\mathbf{P}(A^j))$ is essentially the m-dimensional simplex, where $m =^n P_{\frac{n}{2}}$. This space is compact and continuous.

7.2 Lemma 5

The following lemma shows that there is no contradiction between the lower bound on the discount factor and the restriction on concavity.

Lemma 5. *For all δ such that $\delta \geq \underline{\delta}$, we can also have $\frac{n+1-\delta}{n+1-\delta n} u\left(\frac{1}{n}\right) \leq u\left(\frac{2}{n}\right)$.*

Proof. The restriction on concavity and the lower bound on the discount factor implies

$$\frac{n+1-\delta}{n+1-\delta n} u\left(\frac{1}{n}\right) \leq u\left(\frac{2}{n}\right) \leq \frac{1}{1-\delta}\left[\gamma - \frac{\delta}{n+1}\right].$$

To show that there is no contradiction we must show that it is possible for

$$\frac{n+1-\delta}{n+1-\delta n} u\left(\frac{1}{n}\right) \leq \frac{1}{1-\delta}\left[\gamma - \frac{\delta}{n+1}\right].$$

Simplifying this expression gives

$$1 \leq u\left(\frac{1}{n}\right) \frac{2n^2+n-3\delta n^2-1+2\delta+\delta^2 n^2-\delta^2}{(n+\delta-\delta n)(n+1-\delta n)}$$

As $\delta \to 1$ this becomes $1 \leq nu\left(\frac{1}{n}\right)$, which clearly holds. ∎

7.3 Derivation of $\overline{\delta}$

We would like to ensure that once in the compromise class, the proposer does not have an incentive to deviate to an allocation outside the compromise class, but that would lead to the compromise with certainty. We are interested in the maximum that the proposer, legislator j, can allocate to himself under deviation s^t while remaining in Γ_j. This allocation should satisfy

$$u(s_j^t) \leq \gamma.$$

Since s_j^t cannot be the largest allocation (otherwise the continuation payoff is ξ), it must be smaller than $\frac{1}{2}$ minus the allocations of all other legislators. Because of concavity, for an optimal deviation all other legislators must receive the same allocation, s_i^t, hence $u(s_j^t) \leq u\left(\frac{1}{2}-(n-1)s_i^t\right)$. So if $u\left(\frac{1}{2}-(n-1)s_i^t\right) \leq \gamma$ such a deviation is prevented, or

$$u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) \leq u(s_i^t). \tag{12}$$

For s^t to be in Γ_j the payoff to the proposer under s^t must not exceed the payoff to a cherry-picking proposal. Letting A_i^j denote the demands of the coalition members given an allocation s^t we know that these must satisfy

$$\begin{aligned}(1-\delta)u\left(1-\frac{n}{2}A_i^j\right)+\frac{\delta}{n+1} &\leq \gamma \\ \Rightarrow \frac{2}{n}\left[1-u^{-1}(\phi)\right] &\leq A_i^j.\end{aligned}$$

The maximum deviation for the proposer will put all the acceptors at the lower bound hence for an optimal deviation we will have

$$A_i^j = \frac{2}{n}\left[1-u^{-1}(\phi)\right]. \tag{13}$$

We are interested in the deviation allocations, s_i^t, that results in these A_i^j's for the coalition members. Assuming all continuation strategies lead to the compromise class, the payoff to s^t for a coalition member is given by

$$U_i(s^t) = (1-\delta)u(s_i^t)+\delta\gamma. \tag{14}$$

To calculate their demands under a cherry-picking proposal, this must be set equal to $U_i(\mathbf{p}(A))$. Given that all coalition members are symmetric, they face equal probability of being in the coalition of a proposer with a zero status quo allocation in the next period. Hence $U_i(\mathbf{p}(A))$ is given by

$$U_i(\mathbf{p}(A)) = \frac{(1-\delta)(n+1)}{n+1-\delta}u(A_i^j)+\frac{\delta}{n+1}. \tag{15}$$

Now A_i^j is determined by equality of $U_i(s^t)$ and $U(\mathbf{p}(A))$ so after some algebra we have

$$u(s_i^t) = \frac{n+1}{n+1-\delta}u(A_i^j)-\frac{\delta}{1-\delta}\left(\gamma-\frac{1}{n+1}\right).$$

Substituting for A_i^j from (13) gives.

$$u(s_i^t) = \frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right)-\frac{\delta}{1-\delta}\left(\gamma-\frac{1}{n+1}\right). \tag{16}$$

Finally, substituting the value of $u(s_i^t)$ from (16) into inequality 12 gives the upper bound on the discount factor implicitly defined by

$$\begin{aligned}u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) &\leq \frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right)-\frac{\delta}{1-\delta}\left(\gamma-\frac{1}{n+1}\right) \\ \Leftrightarrow (1-\delta)u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right)+\delta\gamma &\leq (1-\delta)\frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right)+\frac{\delta}{n+1}.\end{aligned}$$

As $\delta \to 1$ this inequality becomes $nu\left(\frac{1}{n}\right) \leq \frac{1}{n+1}$ which by concavity of $u(\cdot)$ is a contradiction, but as $\delta \to 0$ this inequality becomes $u\left(\frac{n-2}{2n(n-1)}\right) \leq u\left(\frac{2(n-1)}{n^2}\right)$ hence it is satisfied. Given that the expressions on the left hand side and right hand side are both continuous in δ, this implies that there exists some δ for which this expression holds with equality and below which the equality is always satisfied. This gives the second upper bound on the discount factor $\overline{\delta}_2$.

7.4 Proof of Lemma 3

Now to show that $\underline{\delta} < \overline{\delta}_2$ we have the following proof.

Proof. The condition for the upper bound on the discount factor is

$$(1-\delta)u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right)+\delta\gamma \leq \frac{n+1}{n+1-\delta}(1-\delta)u\left(\frac{2}{n}[1-u^{-1}(\phi)]\right)+\frac{\delta}{n+1}.$$

The condition for the lower bound on the discount factor is $(1-\delta)u\left(\frac{2}{n}\right)+\frac{\delta}{n+1} \leq \gamma$. To show that there exists a non-degenerate range we must prove that at $\underline{\delta}$ the condition for $\overline{\delta}_2$ is strictly satisfied. Substituting $\frac{\delta}{n+1} = \gamma-(1-\delta)u\left(\frac{2}{n}\right)$, and $\phi = u\left(\frac{2}{n}\right)$ into the condition for $\overline{\delta}_2$ gives

$$\begin{aligned}
&(1-\delta)u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right)+\delta\gamma < \frac{n+1}{n+1-\delta}(1-\delta)u\left(\frac{2}{n}\left[1-u^{-1}\left(u\left(\frac{2}{n}\right)\right)\right]\right)+\gamma-(1-\delta)u\left(\frac{2}{n}\right)\\
&\Leftrightarrow u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right)+u\left(\frac{2}{n}\right) < \frac{n+1}{n+1-\delta}u\left(\frac{2(n-2)}{n^2}\right)+\gamma
\end{aligned}$$

Notice that $u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) < u\left(\frac{1}{2(n-1)}\right)$, so to show the non-degenerate range it suffices to show

$$u\left(\frac{1}{2(n-1)}\right)+u\left(\frac{2}{n}\right) < \frac{n+1}{n+1-\delta}u\left(\frac{2(n-2)}{n^2}\right)+\gamma. \tag{17}$$

Since $n \geq 6$

$$u\left(\frac{1}{2(n-1)}\right) < u\left(\frac{2}{n}\right) \tag{18}$$

$$\Leftrightarrow (1-\alpha-\beta)u\left(\frac{1}{2(n-1)}\right) < (1-\alpha-\beta)u\left(\frac{2}{n}\right) \tag{19}$$

$$\Leftrightarrow u\left(\frac{1}{2(n-1)}\right)+u\left(\frac{2}{n}\right) < (\alpha+\beta)u\left(\frac{1}{2(n-1)}\right)+(2-\alpha-\beta)u\left(\frac{2}{n}\right). \tag{20}$$

This holds for any α and β. Now consider α such that

$$\begin{aligned}
\alpha\left(\frac{1}{2(n-1)}\right)+(1-\alpha)\frac{2}{n} &= \frac{2(n-2)}{n^2}\\
\Leftrightarrow \alpha &= \frac{8(n-1)}{n(3n-4)},
\end{aligned}$$

and β such that

$$\beta\left(\frac{1}{2(n-1)}\right)+(1-\beta)\frac{2}{n}=\frac{1}{n}$$
$$\Leftrightarrow \beta=\frac{2(n-1)}{3n-4}.$$

Given that $0<\alpha<1$, $0<\beta<1$ and by strict concavity of $u(\cdot)$ we know that $\alpha u\left(\frac{1}{2(n-1)}\right)+(1-\alpha)u\left(\frac{2}{n}\right)<u\left(\frac{2(n-2)}{n^2}\right)$ and $\beta u\left(\frac{1}{2(n-1)}\right)+(1-\beta)u\left(\frac{2}{n}\right)<u\left(\frac{1}{n}\right)$. Summing over these two gives

$$\begin{aligned}(\alpha+\beta)u\left(\frac{1}{2(n-1)}\right)+(2-\alpha-\beta)u\left(\frac{2}{n}\right)&<u\left(\frac{2(n-2)}{n^2}\right)+u\left(\frac{1}{n}\right) &(21)\\ &=u\left(\frac{2(n-2)}{n^2}\right)+\frac{\delta}{(n+1)(n+\delta-\delta n)}u\left(\frac{1}{n}\right)+\gamma &(22)\\ &<u\left(\frac{2(n-2)}{n^2}\right)+\frac{\delta}{(n+1)(n+\delta-\delta n)}u\left(\frac{2(n-2)}{n^2}\right)+\gamma &(23)\\ &<u\left(\frac{2(n-2)}{n^2}\right)+\frac{\delta}{n+1-\delta}u\left(\frac{2(n-2)}{n^2}\right)+\gamma &(24)\\ &=\frac{n+1}{n+1-\delta}u\left(\frac{2(n-2)}{n^2}\right)+\gamma. &(25)\end{aligned}$$

Combining this last line with inequality 20 shows that inequality 17 holds. Hence the result is proved.

■

7.5 Proof of Proposition 2

Proof. The difference in the range of the discount factor is simply $\overline{\delta}_1-\underline{\delta}$, so we are interested in the sign of $\frac{\partial}{\partial n}(\overline{\delta}_1-\underline{\delta})=\frac{\partial\overline{\delta}_1}{\partial n}-\frac{\partial\underline{\delta}}{\partial n}$. We will first show that $\frac{\partial\underline{\delta}}{\partial n}$ is negative, and then show that $\frac{\partial\overline{\delta}_1}{\partial n}$ is either negative, but smaller in magnitude, or positive. Define the function

$$f(\delta,n)=(1-\delta)u\left(\frac{2}{n}\right)+\frac{\delta}{n+1}-\gamma,$$

and define the function $g(n)=\underline{\delta}$ such that $f(\underline{\delta},n)=0$. By the implicit function theorem we know that

$$\frac{\partial}{\partial n}g(n)=-\frac{\partial_n f(\delta,n)}{\partial_\delta f(\delta,n)}.$$

We will show that for large values of n this is negative by first showing that the denominator is negative and then showing that the numerator is also negative for large values of n.

Lemma 6. *The partial derivative of $f(\delta,n)$ with respect to δ is negative.*

Proof. Taking a partial derivative of $f(\delta, n)$ with respect to δ gives

$$\partial_\delta f(\delta, n) = -u\left(\frac{2}{n}\right) + \frac{u\left(\frac{1}{n}\right) n}{(n+1)(n+\delta-\delta n)^2} + \frac{1}{n+1}.$$

First observe that

$$\frac{u\left(\frac{1}{n}\right) n}{(n+1)(n+\delta-\delta n)^2} + \frac{1}{n+1} < \frac{u\left(\frac{1}{n}\right) n}{n+1} + \frac{1}{n+1} < \frac{u\left(\frac{1}{n}\right)(n-2)}{n-1} + \frac{1}{n-1}. \tag{26}$$

The last inequality follows because both sides of the inequality are convex combinations of $u\left(\frac{1}{n}\right)$ and 1, but the last expression places a higher weight on 1. Now define a such that

$$(1-a)\frac{1}{n} + a = \frac{2}{n}$$
$$\Leftrightarrow a = \frac{1}{n-1}$$

By strict concavity of $u(\cdot)$ we know that $\frac{n-2}{n-1}u\left(\frac{1}{n}\right) + \frac{1}{n-1} < u\left(\frac{2}{n}\right)$. Combining with (26) gives the desired result. ■

Lemma 7. *The partial derivative of $f(\delta, n)$ with respect to n is negative for large values of n.*

Proof. The partial derivative of $f(\delta, n)$ with respect to n is

$$\partial_n f(\delta, n) = -\frac{2(1-\delta)u'\left(\frac{2}{n}\right)}{n^2} - \frac{\delta}{(n+1)^2} - \frac{u\left(\frac{1}{n}\right)\delta[2n(1-\delta)+1]}{(n+1)^2(n+\delta-\delta n)^2} + \frac{u'\left(\frac{1}{n}\right)(n+1-\delta n)}{n(n+1)(n+\delta-\delta n)}.$$

Taking limits as $\delta \to 0$ this gives

$$\lim_{\delta \to 0} \partial_n f(\delta, n) = -\frac{2u'\left(\frac{2}{n}\right)}{n^2} + \frac{u'\left(\frac{1}{n}\right)}{n^2}.$$

Taking limits of this as $n \to \infty$ we know that $u'\left(\frac{2}{n}\right) \approx u'\left(\frac{1}{n}\right)$ so this term is negative. ■

Lemma 8. *For a large number of legislators, n, the lower bound on the discount factor, $\underline{\delta}$, is decreasing in n.*

This follows from lemmas 6 and 7 we have that the result. Now define the function

$$\overline{f}(\delta, n) = (1-\delta)u\left(\frac{1}{2}\right) + \frac{\delta}{n+1} - \gamma,$$

and define the function $\overline{g}(n) = \overline{\delta}_1$ such that $\overline{f}(\overline{\delta}_1, n) = 0$. Again, by the implicit function theorem we know that

$$\frac{\partial}{\partial n}\overline{g}(n) = -\frac{\partial_n \overline{f}(\delta, n)}{\partial_\delta \overline{f}(\delta, n)}.$$

The partial derivative of $\overline{f}(\delta, n)$ with respect to δ is

$$\partial_\delta \overline{f}(\delta, n) = -u\left(\frac{1}{2}\right) + \frac{u\left(\frac{1}{n}\right)n}{(n+1)(n+\delta-\delta n)^2} + \frac{1}{n+1}.$$

This is larger in absolute value than $\partial_\delta f(\delta, n)$. The partial derivative of $\overline{f}(\delta, n)$ with respect to n is

$$\partial_n \overline{f}(\delta, n) = -\frac{\delta}{(n+1)^2} - \frac{u\left(\frac{1}{n}\right)\delta[2n(1-\delta)+1]}{(n+1)^2(n+\delta-\delta n)^2} + \frac{u'\left(\frac{1}{n}\right)(n+1-\delta n)}{n(n+1)(n+\delta-\delta n)}.$$

Notice that this must be larger than $\partial_n f(\delta, n)$. With a larger denominator and either positive or smaller in absolute value numerator we must have $\frac{\partial \overline{\delta}_1}{\partial n} > \frac{\partial \underline{\delta}}{\partial n}$, so $\frac{\partial \overline{\delta}_1}{\partial n} - \frac{\partial \underline{\delta}}{\partial n}$ must be positive. ■

Since the second upper bound on the discount factor is given by a complex implicit function it is not possible to say much about it without parameterizing the utility function. What follows will show for a common parameterization that the range between the lower bound and the second upper bound is increasing as n increases.

Consider the parameterization $u(s) = s^{\frac{1}{b}}$ with $b > 1$. The lower bound on the discount factor is given by the condition $(1-\delta)u\left(\frac{2}{n}\right) + \frac{\delta}{n+1} \leq \gamma$. Denote the left hand side of this expression as $LHS_l = (1-\delta)u\left(\frac{2}{n}\right) + \frac{\delta}{n+1}$ and the right hand side of this expression as $RHS_l = \gamma$. The upper bound on the discount factor is given by $(1-\delta)u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) + \delta\gamma \leq (1-\delta)\frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right) + \frac{\delta}{n+1}$. Denote the left hand side of this expression as $LHS_u = 1-\delta)u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) + \delta\gamma$ and the right hand side as $RHS_u = (1-\delta)\frac{n+1}{n+1-\delta}u\left(\frac{2}{n}\left[1-u^{-1}(\phi)\right]\right) + \frac{\delta}{n+1}$. We plot these four expressions as a function of δ in figure 9.

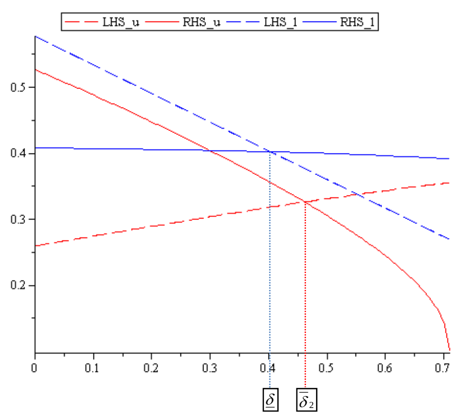

Figure 8: Bounds on δ, $n = 6$, $b = 2$

The blue lines indicate the conditions for the lower bound on the discount factor, with the dashed line being the left hand side of this expression. So for all values where the dashed blue line is below the solid blue line the condition for the lower bound on the discount factor is satisfied. The red lines indicate the conditions for the upper bound on the discount factor, with the dashed red line being the left hand side of the expression. Clearly where the dashed red line is below the solid red line, the conditions for the upper bound on the discount factor is met so the admissible range of the discount factor is indicated between $\underline{\delta}$ and $\overline{\delta}_2$. Notice these values are approximately $\underline{\delta} \approx 0.40$ and $\overline{\delta}_2 \approx 0.46$. Now consider when $n = 10$ as illustrated below.

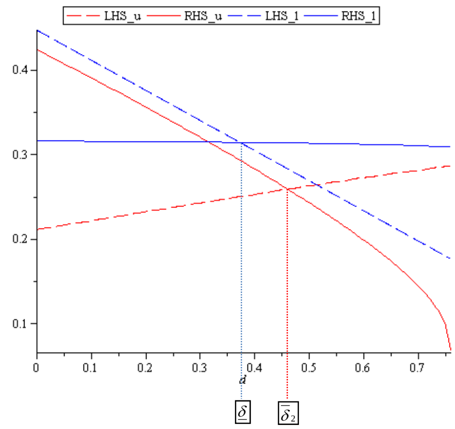

Figure 9: Bounds on δ, $n = 10$, $b = 2$

Notice that the lower bound has decreased significantly, it is now approximately $\underline{\delta} \approx 0.38$, but the upper bound has moved very little and is still approximately $\overline{\delta}_2 \approx 0.46$. These results are consistent for variations of the utility function and variations in n.

7.6 $\overline{\delta}_2$ Decreasing with Concavity

For the parameterization $u(s) = s^{\frac{1}{b}}$ with $b > 1$ the function becomes more concave as b increases. Let us consider when the number of legislators is at it's lower bound, so $n = 6$. By lemma 7.7.2 we know that the range is greater at any higher value of n. Figure 9 illustrated when $u(\cdot)$ is not too concave, so $b = 2$. Now let us consider when $b = 10$, so the utility function becomes more concave. The expressions are plotted in figure 10 below. Notice that now $\underline{\delta} \approx 0.081$ and $\overline{\delta}_2 \approx 0.120$. Hence the entire range has shifted down and the absolute value of the range has decreased.

Consider also an exponential specification of the utility function,

$$u(s) = -\frac{\exp^{-s\lambda}}{1 - \frac{1}{\exp^{\lambda}}} + \frac{1}{1 - \frac{1}{\exp \lambda}}.$$

For this specification increases in the parameter λ represent increases in the concavity of the utility function. The effects of increases in λ display the same result. At $n = 6$ and $\lambda = 2$, $\underline{\delta} \approx 0.46$ and $\overline{\delta}_2 \approx 0.79$. At $n = 6$ and $\lambda = 10$, $\underline{\delta} \approx 0.16$ and $\overline{\delta}_2 \approx 0.23$. So again the range has shifted down and has narrowed in absolute value.

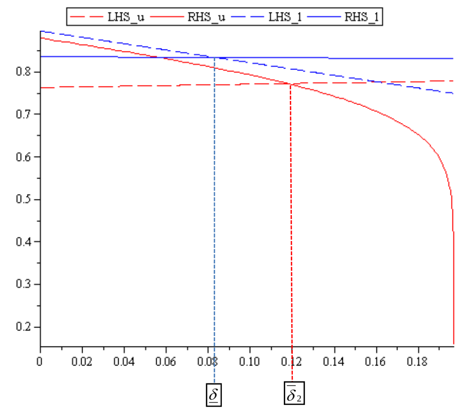

Figure 10: Bounds on δ, $n = 6$, $b = 10$

7.7 Complete Incentives

A complete incentives analysis ensures that for each set indicated in figure 3 there is no incentive to transition to any other set than what the equilibrium strategies dictate. So we proceed by considering status quo allocations in each set.

7.7.1 $s^{t-1} \in \Delta_{\frac{n}{2}<\theta\leq n}$

Consider a status-quo $s^{t-1} \in \Delta_{\frac{n}{2}<\theta\leq n}$. The equilibrium strategies dictate a no-compromise proposal $\underline{\mathbf{p}}$ such that the proposer, x^t, receives payoff $\underline{V}_x$ from equation (1), and all other legislators receive payoffs $\underline{V}_z$ from equation (2). These are

$$\underline{V}_x = \tfrac{n+1-\delta n}{n+1}$$

and

$$\underline{V}_z = \tfrac{\delta}{n+1}.$$

Consider the incentives of legislators to accept the equilibrium proposal. Under the status quo at least $\frac{n}{2}$ non-proposing legislators are receiving payoff $\underline{V}_z$ hence there at least $\frac{n}{2}$ legislators who will accept a payoff of $\underline{V}_z$, hence achieving a majority of votes.

Now consider incentives of the proposer to propose this allocation rather than any other deviation allocation. Consider deviation allocations in the following sets.

Deviation into $\overline{\Delta}_1$

Consider a deviation such that the proposer proposes a compromise allocation $\overline{\mathbf{p}}^i$ for some i. In the compromise class the best available dynamic payoff is γ given by (3). We check that $\gamma \leq \underline{V}_x$. We have

$$\frac{n(n+1-\delta n)}{(n+1)(n+\delta-\delta n)}u\left(\frac{1}{n}\right) \leq \frac{n+1-\delta n}{n+1},$$
$$\Leftrightarrow \delta \leq \frac{1-u(\frac{1}{n})}{1-\frac{1}{n}}.$$

Let $\overline{\delta}_1 = \frac{1-u\left(\frac{1}{n}\right)}{1-\frac{1}{n}}$. To show that this is not profitable deviation it suffices to show that $\overline{\delta} \leq \overline{\delta}_1$ where defined by condition (11). Plugging δ_1 into (11) we obtain:

$$\begin{aligned} LHS &= u\left(\frac{1-2u^{-1}\left(\frac{n^2u\left(\frac{1}{n}\right)-1}{n^2-1}\right)}{2(n-1)}\right) \\ RHS &= -\frac{n^2(1-u\left(\frac{1}{n}\right))}{n^2-1} \end{aligned}$$

Since the RHS of this inequality is negative and the LHS is positive, the condition is violated meaning that $\overline{\delta} < \delta_1$. So $\overline{\delta}$ implies that $\gamma \leq \underline{V}_x$. It is useful to note that at $\overline{\delta}_1$, the value $\phi = 1$. This will be useful below.

Deviation into $\Delta_{\frac{n}{2}<\theta<n}$

Consider a deviation to an arbitrary allocation in $\Delta_{\frac{n}{2}<\theta<n}$, once in this set equilibrium dynamics dictate proceeding to the no compromise set Δ_n. Hence the continuation values of the deviation into $\Delta_{\frac{n}{2}<\theta<n}$ and playing the equilibrium outlined for Δ_n are identical. It thus suffices to compare the stage allocations from the deviation to the equilibrium allocation where the proposer, legislator j, receives $1-\sum_{i\neq j} s_i^t$. Clearly $1-\sum_{i\neq j} s_i^t < 1$ so this is not a profitable deviation.

Deviation into $\Delta_{\theta=\frac{n}{2}}$

Consider a deviation to an allocation in $\Delta_{\theta=\frac{n}{2}}$, once in this set equilibrium dynamics dictate proceeding to the no-compromise set, Δ_n, or if the proposer, x^{t+1}, possesses a zero status quo allocation then proceed to $\Delta_{\frac{n}{2}<\theta<n}$. Under the deviation proposal the proposer receives

$$(1-\delta)u\left(1-\sum_{i\neq j} s_i^t\right) + \delta V_j(s^t). \tag{27}$$

Substituting for the different values of $V_j(\Delta_{\frac{n}{2}}; s_{x^t}^t = 0)$ in equation (8) we obtain the following. If $V_j(\Delta_{\frac{n}{2}}; s_{x^t}^t = 0) = \frac{\delta}{n+1}$, we have that $V_j(s^t) = \frac{1}{n+1}$. As shown above, the continuation value is the same for the proposer but the stage payoff is lower hence this is not a profitable deviation. Now consider if $V_i(\Delta_{\frac{n}{2}}; s_{x^t}^t = 0) = (1-\delta)u\left(A(b^*)\right) + \frac{\delta}{n+1}$, so that he ensures he is in the coalition in the next period. The subsequent proposal strategies are such that the probability assigned to him being in the coalition ensures that his dynamic payoff is exactly equal to $(1-\delta)u\left(A(b^*)\right) + \frac{\delta}{n+1}$, hence this cannot be a profitable deviation. Last, for the case $V_j(\Delta_{\frac{n}{2}}; s_{x^t}^{t-1} = 0) = (1-\delta)u\left(1-A(b^*)\right) + \frac{\delta}{n+1}$. For this to be the continuation from the deviation, it must be that the proposer allocates a zero share to himself. Substituting this into (27), we show that the deviation payoff is equal to

$$\frac{\delta}{n+1}(1-\delta)u(1-A(b^*)) + \frac{\delta^2}{n+1}.$$

This is clearly less than $\underline{V}_x$ since $\frac{\delta}{n+1}u\left(1-A(b^*)\right) < 1$.

Deviation into $\Delta^c_{\theta<\frac{n}{2}}$

Consider a deviation to an allocation in $\Delta^c_{\theta<\frac{n}{2}}$. The payoff from this deviation is given by equation (10). This is

$$U_j(s^t;\mathbf{A}^*,\mu^*) = (1-\delta)u(s^t_j) + \delta V_j(\mathbf{A}^*,\mu^*).$$

Let us consider the continuation payoff $V_j(\mathbf{A}^*,\mu^*)$. Notice that if legislator j is included in any coalition in the next period, equilibrium proposal strategies, μ^{*i}, are such that his payoff from the allocation s^t are equal to or less than his payoff from the next period's cherry-picking allocation, hence

$$U_j(s^t;\mathbf{A}^*,\mu^*) \leq (1-\delta)u(A^{*i}) + \delta V_j(A^{*i}).$$

This payoff we just showed is always lower than $\underline{V}_x$, hence this is not a profitable deviation.

Now consider if he is not included in a coalition in the next period. As shown below in section 7.7.3 this continuation payoff is strictly less than $\frac{1}{n+1}$, and since $1-\sum_{i\neq j} s^t_i < 1$ this deviation is also not profitable.

Deviation into Γ

A deviation into the set Γ may imply a deviation into any arbitrary intersection of Γ_i. Note that if it is any intersection that includes Γ_j, by $\overline{\delta}$ the status quo payoff to being in Γ_j is never greater than γ, and we know again by $\overline{\delta}$ that $\gamma < \underline{V}_x$ hence this is not a profitable deviation. Now consider a deviation into some arbitrary intersection that does not include Γ_j. If the deviation is such that legislator j is included in any other legislator's coalition, by the same arguments above the payoff from the deviation is $U_j(s^t;\mathbf{A}^*,\mu^*) = (1-\delta)u(A^{*i}_j)+\delta V(A^{*i})$, which we showed is always lower than $\underline{V}_x$.

So we must consider when the deviation allocation is not in Γ_j, and the legislator is not included in anyone else's coalition. By the upper bound on the discount factor the minimum stage payoff to coalition members, s^t_i, in Γ_i for any i satisfies, $u\left(\frac{1-2u^{-1}(\gamma)}{2(n-1)}\right) \leq u(s^t_i)$. Hence the stage payoff to any other player, when an allocation is in Γ_i must be less than γ. In continuation, the proposing legislator gets at most $\underline{V}_x$ if he is the proposer, $\frac{\delta}{n+1}$ if he is not the proposer and the legislator does not choose compromise, and γ if he is not the proposer and the legislator chooses compromise. Letting α denote the fraction of legislators that would choose compromise, the payoff for the proposing legislator under such a deviation. s^t is

$$U_j(s^t) \leq (1-\delta)\gamma + \delta\left[\frac{1}{n+1}\underline{V}_x + \left(1-\frac{1}{n+1}-\alpha\right)\frac{\delta}{n+1} + \alpha\gamma\right].$$

Noting that this is a convex combination of values that are no greater than $\underline{V}_x$, this payoff is not greater than $\underline{V}_x$.

7.7.2 $s^{t-1} \in \Delta_{\frac{n}{2}}$

When the status quo is in $\Delta_{\frac{n}{2}}$ equilibrium strategies specify either a no compromise proposal if the proposer has a positive status-quo allocation, or a cherry-picking proposal, $\mathbf{p}^j(A(b^*))$, if the proposer has a status quo allocation of zero. For $s_{x^t}^{t-1} > 0$, where the the equilibrium proposal made is in Δ_n, the incentives analysis is identical to status quo allocations in $\Delta_{\frac{n}{2}<\theta\leq n}$. Below we will show that for $s_{x^t}^{t-1} = 0$, there are no profitable deviations.

Let us first check the incentives of the accepting legislators. The payoff to the legislator receiving zero allocation is $\frac{\delta}{n+1}$, and the payoff under the status quo allocation is $\delta V_i(A^{*j})$, where

$$\delta V_i(A^{*j}) = \frac{\delta}{n+1}\left[\delta + (1-\delta)u(1-A(b^*))\right].$$

Simplifying shows that $\frac{\delta}{n+1} > \delta V_i(A^{*j})$ since $1 > u(1-A(b^*))$. The legislator that is offered a positive status quo allocation is offered $A(b^*)$ with probability μ_i, where $A(b^*)$ and μ_i are calculated to make the accepting legislator exactly indifferent to the status quo.

Now we consider possible deviation proposals by the proposer. Under the equilibrium strategies the proposer receives the payoff $(1-\delta)u(1-A(b^*)) + \frac{\delta}{n+1}$.

Deviation into $\Delta_{\frac{n}{2}\leq\theta\leq n}$

For deviations in $\Delta_{\frac{n}{2}\leq\theta\leq n}$, we rely on the proof in Kalandrakis (2007) but restate his concavity restrictions and show that it is satisfied by the concavity restriction in assumption 1. First to ensure that the amount allocated to the coalition member, $A(b^*)$, is not greater than $1-A(b^*)$, the utility function cannot be too concave, so

$$\frac{(n+1)(n+2)}{(n+1)(n+2)-\delta}u\left(\frac{2}{n+2}\right) \leq u\left(\frac{1}{2}\right).$$

Note that by proposition 1 the utility function satisfies $\frac{n+1-\delta}{n+1-\delta n}u(s_i^t) \leq u(2s_i^t) \Rightarrow \frac{n+1-\delta}{n+1-\delta n}u\left(\frac{2}{n+2}\right) \leq u\left(\frac{4}{n+2}\right)$. Since

$$\frac{(n+1)(n+2)}{(n+1)(n+2)-\delta}u\left(\frac{2}{n+2}\right) < \frac{n+1-\delta}{n+1-\delta n}u\left(\frac{2}{n+2}\right)$$

and because $n \geq 6$

$$u\left(\frac{4}{n+2}\right) \leq u\left(\frac{1}{2}\right).$$

So we have

$$\frac{(n+1)(n+2)}{(n+1)(n+2)-\delta}u\left(\frac{2}{n+2}\right) < u\left(\frac{1}{2}\right).$$

Second, to ensure that a proposer will not prefer another allocation in $\Delta_{\frac{n}{2}}$ to the equilibrium proposal it must also be the case that

$$\frac{n+1}{n+1-\delta}u(s_i^t) \leq u\left(\frac{n}{2}s_i^t\right).$$

Again by assumption 1 we have

$$\frac{n+1}{n+1-\delta}u(s_i^t) \leq \frac{n+1-\delta}{n+1-\delta n}u(s_i^t) \leq u(2s_i^t) \leq u\left(\frac{n}{2}s_i^t\right).$$

Hence these inequalities are satisfied.

Deviation into $\overline{\Delta}_1$

This holds by $\overline{\delta}_1$. See section 4.1.2.

Deviation into $\Delta^c_{\theta<\frac{n}{2}}$

This deviation involves allocating more than $\frac{n}{2}$ players a positive share under the deviation proposal. Equilibrium strategies after the deviation dictates an allocation in $\Delta_{\frac{n}{2}}$, for which the continuation payoffs are no greater than in Δ_n as shown in section 7.7.3. The proposer will have to allocate a positive share to at least one player that possesses a positive share under the status quo, making him at least indifferent to the status quo, as in the equilibrium strategy, and, in addition, allocate some $\varepsilon_j > 0$ to another $\frac{n}{2} - 1$ players also making them at least indifferent to the status quo. This implies a lower stage payoff to proposer than under the equilibrium strategy, while not improving the proposer's continuation payoff. Hence this is not a profitable deviation.

7.7.3 $s^{t-1} \in \Delta^c_{\theta<\frac{n}{2}}$

Consider incentives when the status quo is an element of $\Delta^c_{\theta<\frac{n}{2}}$ and legislator j is the proposer. From these allocations legislator j makes a cherry-picking proposal such that coalition members $i \in C_j$ are offered the demands A_i^{j*}, where A^{j*} and μ^{j*} solve

$$(A^{j*}, \hat{\mu}^{j*}) \quad \in \arg\max_{\hat{A}^j, \hat{\mu}^j} U_j(s^{t-1}; \hat{A}^j, A^{-j*}, \hat{\mu}^j, \mu^{-j*})$$

$$\begin{aligned} \text{s.t.} \quad & \hat{A}^j = \hat{A}^j(s^{t-1}; \mathbf{A}^*, \hat{\mu}^j, \mu^{-j*}) \\ & \hat{\mu}^j \in \mathbf{M}(\mathbf{P}(A^{j*})) \\ \text{and} \quad & \mu^{j*}(\mathbf{p}(A^{j*})) = \hat{\mu}^j(\mathbf{p}(A^{j*}))\}. \end{aligned}$$

So

$$A_i^{j*} = u^{-1}\left[\min\{\max\{0, \frac{1}{1-\delta}\left(U_i(s^{t-1}; \mathbf{A}^*, \mu^*) - \delta V_i(A^{j*})\right)\}, 1\}\right].$$

Let us first check the coalition member's incentives. We must verify that coalition members will prefer this allocation to the status quo. Note that if $A_i^{j*} < 1$, then $U_i(s^{t-1}; \mathbf{A}^*, \mu^*) \leq U_i(\mathbf{p}(A^{*j}))$. We know that if the proposer did not choose the compromise allocation then $\sum_{i\in C_j} A_i^{j*} < 1 -$

$u^{-1}(\phi) < 1$ hence each coalition member's demand must also be less than 1 hence their incentives hold.

Now let us check the proposer's incentives. Given that this solves the maximization problem for the proposer we know the proposer would prefer this to any other cherry-picking allocation that would be accepted, or any compromise allocation. We must also verify that the proposer would prefer this to remaining at the status quo allocation. Under the status quo the proposer has the payoff

$$U_j(s^{t-1}; \mathbf{A}^*, \mu^*) = (1-\delta)u(s_j^{t-1}) + \delta V_j(\mathbf{A}^*, \mu^*).$$

Under the cherry-picking proposal, he has payoff

$$U_j(\mathbf{p}(A^{*j})) = (1-\delta)u\left(1 - \sum_{i \in C_j} A_i^{*j}\right) + \delta V_j(A^{*j}).$$

We will prove that $U_j(s^{t-1}; \mathbf{A}^*, \mu^*) \leq U_j(\mathbf{p}(A^{*j}))$ by first showing that $V_j(\mathbf{A}^*, \mu^*) \leq V_j(A^{*j})$ and then showing that $u(s_j^{t-1}) \leq u\left(1 - \sum_{i \in C_j} A_i^{*j}\right)$. We will first consider the lower bound on $V_j(A^{*j})$. This will be where legislator j is not included in the coalition when the status quo is in $\Delta_{\frac{n}{2}}$ and $s_{x^t}^{t-1} = 0$, so

$$V_j(A^{*j}) = \frac{1}{n+1}\left(\frac{n+1-\delta n}{n+1} + \frac{n}{2}\frac{\delta}{n+1} + \frac{n}{2}\frac{\delta}{n+1}\right) = \frac{1}{n+1}$$

Now let us consider the upper bound on $V_j(\mathbf{A}^*, \mu^*)$. Here $V_j(\mathbf{A}^*, \mu^*)$ is given by

$$V_j(\mathbf{A}^*, \mu^*) = \frac{1}{n+1}\left[U_j(\mathbf{p}(A^{*j})) + \sum_{i \neq j}\left\{\sum_{\mathbf{p}(A^{i*}) \in \mathbf{P}(A^{i*})} U_j(\mathbf{p}(A^{i*}))\mu^{i*}(\mathbf{p}(A^{i*}))\right\}\right]. \tag{28}$$

When legislator j is not the proposer he is in another legislator's coalition if $U_j(s^{t-1}; \mathbf{A}^*, \mu^*)$ is sufficiently low. So at a maximum value of $U_j(s^{t-1}; \mathbf{A}^*, \mu^*)$ legislator j is not included in the other legislator's coalition so $U_j(\mathbf{p}(A^{i*})) = \delta V_j(A^{i*})$ for all $i \neq j$. Substituting this into (28) gives

$$V_j(\mathbf{A}^*, \mu^*) = \frac{1}{n+1}\left[(1-\delta)u\left(1 - \sum_{i \in C_j} A_i^{*j}\right) + \delta V_j(A^{*j}) + \sum_{i \neq j} \delta V_j(A^{i*})\right]. \tag{29}$$

We showed above that $V_j(A^{j*}) = \frac{1}{n+1}$. When any other legislator $i \neq j$ proposes in period t, $V_j(A^{i*})$ is given by

$$V_j(A^{i*}) = \frac{1}{n+1}\left(\left(\frac{n}{2}+1\right)\underline{V}_z + \frac{n}{2}V_j(\Delta_{\frac{n}{2}}; s_{x^{t+1}}^t = 0)\right). \tag{30}$$

Here $V_j(\Delta_{\frac{n}{2}}; s_{x^{t+1}}^t = 0) = (1-\delta)u(1 - A(b^*)) + \frac{\delta}{n+1}$ when legislator j is the proposer in period $t+1$ and is $\frac{\delta}{n+1}$ when all other legislators propose in period $t+1$. Substituting into (30) and simplifying gives

$$V_j(A^{i*}) = \frac{(1-\delta)}{n+1}u(1-A(b^*)) + \frac{\delta}{n+1}. \tag{31}$$

Substituting these values into (28) gives

$$\begin{aligned}V_j(\mathbf{A}^*, \mu^*) &= \frac{1}{n+1}\left[(1-\delta)u\left(1-\sum_{i\in C_j}A_i^{*j}\right) + \delta\left(\frac{1}{n+1} + \sum_{i\neq j}\left\{\frac{(1-\delta)}{n+1}u(1-A(b^*)) + \frac{\delta}{n+1}\right\}\right)\right] \\ &= \frac{1}{n+1}\left[(1-\delta)u\left(1-\sum_{i\in C_j}A_i^{*j}\right) + \delta\left(\frac{1+\delta n}{n+1} + \frac{1-\delta}{n+1}\sum_{i\neq j}u(1-A(b^*))\right)\right].\end{aligned}$$

Notice that the value in square brackets is the convex combination of two values that are less than 1, hence the value in brackets is less than 1 making $V_j(\mathbf{A}^*, \mu^*) \leq \frac{1}{n+1} = V_j(A^{*j})$.

Now we must prove $u(s_j^{t-1}) \leq u\left(1-\sum_{i\in C_j}A_i^{*j}\right)$, or $s_j^{t-1} \leq 1-\sum_{i\in C_j}A_i^{*j}$. First observe that if s_i^{t-1} are equal for all $i \in C_j$, this implies the minimum value of $\sum_{i\in C_j}A^{*j}$ for any given s_j^{t-1}. Now observe that $s_j^{t-1} \leq 1 - ns_i^{t-1}$ since all other legislators must have at least as high a status quo allocation as the coalition members if we assume the legislators with the lowest status quo allocations are always in the coalition. To express $\sum_{i\in C_j}A_i^{*j}$ in terms of s_i^{t-1} we must calculate the value of A_i^{*j} as a function of s_i^{t-1}. Recall that

$$A_i^{j*} = u^{-1}\left[\min\{\max\{0, \frac{1}{1-\delta}\left(U_i(s^{t-1}; \mathbf{A}^*, \mu^*) - \delta V_i(A^{j*})\right)\}, 1\}\right]. \tag{32}$$

Here $U_i(s^{t-1}; \mathbf{A}^*, \mu^*) = (1-\delta)u(s_i^{t-1}) + \delta V_i(\mathbf{A}^*, \mu^*)$. We wish to find the maximum value of $U_i(s^{t-1}; \mathbf{A}^*, \mu^*)$, let's call this value $\overline{U}_i$. This is where legislator i is included in n other legislator's coalition in period t, and recall that if he is included in the coalition then he receives at least $\overline{U}_i$. So

$$\overline{U}_i = (1-\delta)u(s_i^{t-1}) + \frac{\delta}{n+1}\left[U_i(\mathbf{p}(A^{*i})) + n\overline{U}_i\right].$$

The maximum value $U_i(\mathbf{p}(A^{*i}))$ can take is $\underline{V}_x$. Substituting in 33 and simplifying gives

$$\overline{U}_i = \frac{(1-\delta)(n+1)}{n+1-\delta n}u(s_i^{t-1}) + \frac{\delta}{n+1}. \tag{33}$$

Now (32) is equivalent to setting $\overline{U}_i = U_i(\mathbf{p}(A^{*j}))$ and solving for A_i^{*j} when $A_i^{*j} > 0$. So we conjecture that $A_i^{*j} > 0$ and find the minimum value of $U_i(\mathbf{p}(A^{*j}))$. Let's call this minimum value $\underline{U}_i$. Given that we are considering coalition members with equal status quos and hence equal values of A_i^{*j}, they will face equal probability, $\frac{1}{n+1}$, of being in the coalition of the period $t+1$ legislator if his status quo allocation was zero. So

$$\underline{U}_i = (1-\delta)u(A_i^{*j}) + \frac{\delta}{n+1}\left[\underline{V}_x + (n-1)\underline{V}_z + \underline{U}_i\right].$$

Substituting and simplifying gives

$$\underline{U}_i = \frac{(1-\delta)(n+1)}{n+1-\delta} u(A_i^{*j}) + \frac{\delta}{n+1}.$$

Setting $\overline{U}_i = \underline{U}_i$ yields

$$u(A_i^{*j}) = \frac{n+1-\delta}{n+1-\delta n} u(s_i^{t-1}). \tag{34}$$

Notice that $s_i^{t-1} > 0$ by assumption, hence the conjecture that $A_i^{*j} > 0$ is proved true. We wish to have $1 - n s_i^{t-1} \leq 1 - \frac{n}{2} A_i^{*j}$. Substituting from (34) for A_i^{*j} and simplifying gives

$$\frac{n+1-\delta}{n+1-\delta n} u(s_i^{t-1}) \leq u(2s_i^{t-1}).$$

This is satisfied by the restriction on concavity in assumption 1.

7.7.4 $s^{t-1} \in \Gamma$

Let us first consider s^{t-1} in $\bigcap_{j=1}^{n+1} \Gamma_j$, and the incentives of the proposing legislator. Since $\mu^{*j}(\overline{\mathbf{p}}^i) = 1$ is the solution to the fixed point of the map $\mathbf{B}$ we know that $\overline{\mathbf{p}}^i$ is optimal for the proposer among cherry-picking strategies and among compromise proposals. We also know by the upper bound on the discount factor, $\overline{\delta}$, that the proposer will prefer the compromise to remaining at the the status quo allocation.

Now let us consider the incentives of the accepting legislators. Since the status quo is an element of $\bigcap_{j=1}^{n+1} \Gamma_j$ and all legislators are symmetric, we know by the upper bound on the discount factor, $\overline{\delta}$, that the highest status quo payoff available to any legislator satisfies $(1-\delta)u(s^{t-1}) + \delta\gamma \leq \gamma$. Hence no legislator has an incentive to deviate from accepting the compromise allocation.

Now consider $s^{t-1} \in \Gamma$ where $0 < \mu^{*j}(\overline{\mathbf{p}}^i) < 1$ for some proposing legislator j and some i. Consider the proposing legislator's incentives. Again, we know since $\mu^{*j}(\overline{\mathbf{p}}^i)$ is the solution to the fixed point of the map $\mathbf{B}$ the proposer is indifferent between the compromise proposal $\overline{p}^i$ and some cherry-picking strategy, so the payoff to legislator j when he is the proposer is γ regardless, and this is optimal among cherry-picking proposals and compromise proposals. We must check that it is preferred to the status quo. If legislator j is not the proposer, other legislators will choose either a cherry-picking strategy or a compromise proposal. Under another legislator's cherry-picking strategy, his payoff is highest when he is included in the coalition, and then he is given at most his status quo payoff. Let us call this status quo payoff, U_j, and let's say with probability μ he receives this payoff, and with probability $1-\mu$ he receives γ under a compromise. His status quo payoff can therefore be written down as

$$\begin{aligned} U_j &= (1-\delta)u(s_i^{t-1}) + \delta\left[(1-\mu)\gamma + \mu U_j\right] \\ \Leftrightarrow U_j &= \frac{1-\delta}{1-\delta\mu} u(s_i^{t-1}) + \frac{(1-\delta)\mu}{1-\delta\mu}\gamma. \end{aligned}$$

To show that the proposer has no incentive to deviate from the compromise proposal to the status-quo, we must show that $\gamma \geq U_j$. This is true, by the upper bound on the discount factor. Since we know that they highest status quo payoff to any proposer j when the status quo is in Γ_j satisfies $u(s_i^{t-1}) \leq \gamma$.

Now we must check the incentives of legislators to accept the proposal. Consider a status quo allocation of the form $\left(\varepsilon, \frac{1-\varepsilon}{n}, \ldots, \frac{1-\varepsilon}{n}\right)$. With the proposer receiving ε and all other legislators receiving $\frac{1-\varepsilon}{n}$. This would represent the most restrictive allocation in Γ_j for the proposer. That is, any other allocation in Γ_j must result in a lower payoff for potential coalition members. If at such an allocation acceptors will accept the compromise, then they will accept at any other allocation. Let us denote the payoff to this status quo allocation as U_i. This is given by

$$U_i = (1-\delta)u\left(\frac{1-\varepsilon}{n}\right) + \delta V_i(A^*, \mu^*).$$

We wish to place an upper bound on the continuation payoff $V_i(A^*, \mu^*)$. If an accepting legislator i becomes the proposer, he can obtain a dynamic payoff of no more than $\underline{V}_x$ or $\frac{n+1-\delta n}{n+1}$. If legislator i is not the proposer, and legislator j is the proposer, he can obtain no more than γ. If any of the $n-1$ other legislators is the proposer, since all legislators have equal status quos except legislator j, they have equal probability of being in the coalition hence, they receive payoff U_i again. Otherwise they receive at most $\frac{\delta}{n+1}$. Hence the payoff U_i can be written as

$$\begin{aligned} U_i &= (1-\delta)u\left(\frac{1-\varepsilon}{n}\right) + \frac{\delta}{n+1}\left[\frac{n+1-\delta n}{n} + \gamma + \frac{n-1}{2}U_i + \frac{n-1}{2}\right] \\ \Leftrightarrow U_i &= \frac{2(n+1)(1-\delta)}{2(n+1)-\delta(n-1)}u\left(\frac{1-\varepsilon}{n}\right) + \frac{2\delta}{2(n+1)-\delta(n-1)}\left[\frac{2-\delta}{2} + \gamma\right] \end{aligned}$$

We wish to show that this payoff is no greater than γ. Taking limits as $\varepsilon \to 0$ and simplifying we have

$$\begin{aligned} &\gamma \geq U_i \\ \Leftrightarrow\ &\gamma \geq \frac{2}{2-\delta}(1-\delta)u\left(\frac{1}{n}\right) + \frac{\delta}{n+1}. \end{aligned}$$

By the lower bound on the discount we know that $\gamma \geq (1-\delta)u\left(\frac{2}{n}\right) + \frac{\delta}{n+1}$ and by the restriction on concavity we know that $u\left(\frac{2}{n}\right) \geq \frac{n+1-\delta}{n+1-\delta n}u\left(\frac{1}{n}\right)$. So we have

$$\gamma \geq (1-\delta)u\left(\frac{2}{n}\right) + \frac{\delta}{n+1} \geq \frac{n+1-\delta}{n+1-\delta n}(1-\delta)u\left(\frac{1}{n}\right) + \frac{\delta}{n+1}.$$

It remains to show that $\frac{n+1-\delta}{n+1-\delta n} \geq \frac{2}{2-\delta}$. Simplifying, we show that this is true since $n > 2$.

Clearly the payoff for the proposer if the status quo is in Γ/Γ_j satisfies the above condition by symmetry, that is $U_j \leq \gamma$. And if the status quo is in Γ/Γ_j, since (A^{*j}, μ^{*j}) is the solution to the fixed point of the map $\mathbf{B}$ it must be that $\gamma < (1-\delta)u\left(1 - \sum_{i \in C_j} A^{*j}\right) + \delta V_j(A^{*j})$. Hence $U_j < (1-\delta)u\left(1 - \sum_{i \in C_j} A^{*j}\right) + \delta V_j(A^{*j})$ hence he will have no incentive to deviate to from the equilibrium strategy.

7.7.5 $s^{t-1} \in \overline{\Delta}_1$

Consider a status-quo $s^{t-1} \in \overline{\Delta}_1$. The equilibrium strategies dictate a compromise proposal $\overline{\mathbf{p}}^i$ such that all legislators except legislator i receives payoff γ and legislator i receives payoff ξ. Consider the incentives of legislators to accept. Under the status quo at least $n-1$ non-proposing legislators are receiving payoff γ hence there at least $n-1$ legislators who will accept a payoff of γ, thereby achieving a majority of votes.

Now consider incentives of the proposer to propose this allocation rather than any other deviation allocation. Consider deviation allocations in the following sets.

Deviation into Δ_n

As shown above the payoff from proposing $\underline{\mathbf{p}}$ compared to $\overline{\mathbf{p}}$ would represent a profitable deviation for the proposer $\overline{\Delta}_1$. So we need to check the incentives for coalition members to accept such a proposal. The payoff for coalition members from the deviation would be $\underline{V}_z$ whereas the payoff from the status quo is γ. Clearly

$$\underline{V}_z \leq \gamma$$
$$\Leftrightarrow \delta(n+\delta-\delta n) \leq nu\left(\frac{1}{n}\right)(n+1-\delta n)$$

Hence the proposer will not be able to form a coalition that accepts such a proposal, and the deviation is not possible.

Deviation into Γ

In section 7.7.4 we showed that the dynamic payoff to any status quo in Γ is no greater than γ hence deviating to Γ is never profitable from $\overline{\Delta}_1$.

Deviation into $\Delta_{\frac{n}{2}<\theta<n}$

We have already shown in section 4.1.1 that by $\underline{\delta}$ this is not a profitable deviation.

Deviation into $\Delta_{\frac{n}{2}} \cup \Delta^c_{\theta<\frac{n}{2}}$

In section 4.1.1, $\underline{\delta}$ implied by condition 5 ensures that a deviation from the compromise class to an allocation with cherry picking in $\Delta_{\frac{n}{2}<\theta<n}$ is not possible. Attempting to deviate into $\Delta_{\frac{n}{2}}$, requires compensating an extra player over and above the $\frac{n}{2}-1$ in deviating into $\Delta_{\frac{n}{2}<\theta<n}$. If compensating $\frac{n}{2}-1$ players is not profitable for any proposer, then it follows that compensating $\frac{n}{2}$ players will be even less profitable, hence this deviation is also ruled out.

7.8 Proof of Lemma 4

To show that h is a contraction with modulus $\beta = \frac{nv\left(\frac{1}{n}\right)}{\left[v\left(\frac{2}{n}\right)-\frac{1}{n+1}\right](n+1)}$ we have the following proof.

Proof. The distance between x and y is simply the Euclidean distance $|x-y|$. Hence

$$\begin{aligned}
|h(x)-h(y)| &= \frac{v\left(\frac{2}{n}\right)-\frac{n(n+1-xn)}{(n+1)(n+x-xn)}v\left(\frac{1}{n}\right)}{v\left(\frac{2}{n}\right)-\frac{1}{n+1}} - \frac{v\left(\frac{2}{n}\right)-\frac{n(n+1-yn)}{(n+1)(n+y-yn)}v\left(\frac{1}{n}\right)}{v\left(\frac{2}{n}\right)-\frac{1}{n+1}} \\
&= -\frac{nv\left(\frac{1}{n}\right)}{(n+1)\left[v\left(\frac{2}{n}\right)-\frac{1}{n+1}\right]}\left[\frac{n+1-xn}{n+x-xn}-\frac{n+1-yn}{n+y-yn}\right] \\
&= \frac{nv\left(\frac{1}{n}\right)}{(n+1)\left[v\left(\frac{2}{n}\right)-\frac{1}{n+1}\right]}\left[\frac{x-y}{(n+x-xn)(n+y-yn)}\right] \\
&\leq \frac{nv\left(\frac{1}{n}\right)}{(n+1)\left[v\left(\frac{2}{n}\right)-\frac{1}{n+1}\right]}(x-y).
\end{aligned}$$

■

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