Revealed Political Power

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Abstract

This paper adopts a "revealed preference" approach to the question of what can be inferred about bias in a political system. We model an economy and its political system from the point of view of an "outside observer." The observer sees a finite sequence of policy data, but does not observe either the citizens' preference profile or underlying distribution of political power that produced the policies. The observer makes inferences about distribution of political power as if political power were derived from a wealth-weighted voting system with weights that can vary with the state of the economy. The weights determine the nature and magnitude of the wealth bias. Positive weights on relative income in any period indicate an "elitist" bias in the political system whereas negative weights indicate a "populist" one.

As a benchmark, *any* policy data are shown to be rationalized by *any* system of wealth-weighted voting. However, by augmenting the observer's observations with polling data, nontrivial inference is possible. We show that joint restrictions resulting from the policy and polling data together imply upper and lower bounds on the set of rationalizing biases. These bounds can be explicitly calculated and can be used to discern instances of elitist bias; in other times they show populist bias. Additional restrictions on the preference domain can rule out the unbiased benchmark case of equal representation.

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Key Words and Phrases: wealth-bias, elitist bias, populist bias, weighted majority winner, rationalizing weights, Universal Bias Principle.

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1 Introduction

The principle of *political equality* is widely accepted as a governing philosophy in most democracies. According to this principle, all individuals regardless of income or background should be endowed with the same political power or influence. On paper, electoral processes in most democracies satisfy some rough form of it, often taking the form of "one-man-one-vote" electoral systems. Examples include Winner-take-all Presidential elections (in the U.S. and Latin America) and Proportional Representation in Parliamentary elections (e.g., Western Europe).[1]

It is unlikely, however, that the *de facto* distribution of power in these countries is equal. There is anecdotal evidence, and some systematic evidence, that wealth matters in the political process. For instance, Rosenstone and Hansen (1993) show that the propensity to participate in every reported form of political activity rises with income. Campante (2011) uses campaign contribution data in the 2000 US presidential election to show that increases in income inequality raises the share of contributions coming from relatively wealthy individuals. Bartels (2008) offers a sweeping look at the relation between economic and political inequality. He examines whether economic inequality *creates* political inequality in the policy process. Using data from the Senate Election Study, he finds that Senators' voting records are unresponsive to the preferences of those in the lower third of the income distribution.[2] By contrast, Senators' responsiveness to the middle and upper thirds is virtually linear to income.

These studies all suggest some form of wealth-bias in the political system. They find that the *de facto* allocation of power is such that richer individuals have a disproportionate influence in the policy process. The result is that policies enacted appear to favor wealthier rather than poorer individuals.

The present paper takes a step back by asking whether and how bias can be inferred directly from policies. When, for instance, can the egalitarian distribution of power based on "one-man-one-vote" be ruled out?

To address these issues we model an economy and its political system from the point of view of an "outside observer." The observer observes both policies and income distribution at finitely many dates. He does not observe, however, the underlying preference profile of the citizens whose political choices determined the policies. Instead, the observer knows only that policy preferences are differentiated by income. Richer individuals view policies differently than poorer ones.

The observer's task is to infer something about the underlying distribution of political

[1] There are well known exceptions. In the U.S. representation in the Senate is equal across states, so that voters in small states have disproportionate political power in that governing body.

[2] See Chapter 9 of Bartels (2008). The Senate Election Study consists of survey data conducted after the November elections of 1988, 1990, 1992.

power that generated the observed policies. To make sense of this inference problem, we consider a "detail-free" formulation of political power as follows. Consider a majority voting process that allocates vote shares to each individual depending on his income. This *vote share rule* depends on an unobserved parameter — the "bias weight." The bias weight determines how the individual's income affects his vote share in each state of the economy. If the bias weight were fully known, then one could relate income inequality to political inequality by calculating a "Political Lorenz Curve" — the implied vote share (hence, "political power") of the poorest jth portion of the population, for each possible j.

Hence, to infer the distribution of political power, the observer must infer something about the bias weight. The observer asks: what parameters are consistent with a vote share rule that rationalizes the observed policy as a *Weighted majority winner (WMW)* under an admissible preference profile?

The idea that political bias can be associated with weights in an implicit voting system has been used elsewhere, albeit in different contexts. For instance, the weights given to valence characteristics in probabilistic voting models (Lindbeck and Weibull (1993)) are commonly associated with bias. Bénabou (1996, 2000) explicitly associates bias with wealth-weighted voting in his influential study of the effect of income inequality on incentives for redistribution. In Bénabou's terminology a bias is *elitist* if it is pro-wealth in the sense that a wealthy individual's vote is worth more than a poorer one's. Similarly, a *populist bias* works in reverse: a poorer individual's vote is worth more than a richer one's. An *unbiased* system refers to the standard system of "one-man-one vote" or equal representation.

The results of the present paper provide (i) necessary and sufficient conditions under which there exists a system of wealth-weights that rationalize the data, and (ii) necessary and sufficient conditions under which a *particular* weighting system rationalizes the data. A preliminary result establishes that, without further structure on preferences and/or data, *any* policy data can be rationalized by *any* wealth bias. In other words, in the benchmark case, policy data alone are not very discerning; it is consistent with any type of bias whether it be elitist, populist, or unbiased.

To develop a more meaningful inference, we therefore follow two routes. First, we allow the observer to access additional data in the form of polls. Polls provide data on specific aggregate binary orderings between benchmark policies — typically those that are being considered in the political process. We consider poll data that pit the observed policy against an arbitrary number of alternatives. Second, we restrict the admissible preference domain. Additional restrictions such as single crossing restrictions can, in many cases, rule out certain bias weights.

To understand why it helps to add polling data, consider a poll at some date t that pits the observed policy against a "right-wing" alternative (i.e., an alternative located to the right of the observed policy). Suppose that the poll reveals that portion p_t of the population prefer the observed policy to the alternative. Under single crossing, these individuals belong to the poorest p_t portion of the population. Yet, the fact that the observed policy must have resulted from a weighted voting process tells us that the wealthiest $1-p_t$ portion of the population must have had a weighted vote share smaller than 50%. If this were not the case, then the richest group would have had the clout to veto the observed policy in favor of the alternative. Consequently, the income weights can be no greater than that necessary to lift the $1-p_t$ wealthiest individuals up to the 50% weighted voting threshold. This in turn defines an upper bound for the bias weight — the largest possible bias in favor of wealthy individuals. Similarly, a poll that compares the observed policy to a "left-wing" alternative can be used to infer a lower bound. Both bounds can be explicitly calculated in each state of the economy.

More generally, our main results characterize both sufficient and necessary conditions for a system of wealth weights to rationalize both the policy and poll data. By holding fixed the level of bias, the observer's inference can be refined across time as the data accumulate. The bounds on the bias weights are shown to shrink, allowing the observer to make a more precise prediction as the data accumulate over time.

Finally, two sets of restrictions on the preference domain are considered. One is a single crossing restriction on policy as the state varies. This preference restriction is natural in a growing economy with no distributional change over time. The other is a separability condition on policy and income. Separability is shown to arises naturally in economies with distributional changes but no growth. Each can be shown to rule out the unbiased polity when the data vary non-monotonically.

The paper is organized as follows. Section 2 lays out the economic side of the model. Section 3 then describes the political side: an implied voting process with latent, wealth-weights. Section 4 describes the benchmark result, while Section 5 examines the addition of poll data, using the polling to derive both static bounds and dynamic restrictions on the bias. Later in the section, more exacting assumptions are needed to examine the link between economic and political inequality. Section 6 examines inference under a more restricted preference domain. Section 7 finally concludes with a discussion of extensions. The Appendix follows.

It is worth noting that the more common approach in the literature is one that attributes bias to a specific cause. For example, one prominent theory links bias to differential participation rates among the rich and poor (e.g., Bourguignon and Verdier (2000)). The poor vote less frequently, and so one can argue that wealthier voters have a disproportionate influence on policy. A second type of theory concerns the effect of campaign contributions, for instance, Austen-Smith (1987), Grossman and Helpman (1996), Prat (2002), Coate (2004), Campante (2011). In these models, the money either "buys" influence directly or it affects policy indirectly by changing the electoral odds toward candidates ideologically predisposed toward the rich. Because contributions skew toward the wealthy, policies are biased in their favor. Finally, a third type of theory centers on disenfranchising investments, e.g., Acemoglu and Robinson (2008), made by a wealthy elite in order to disinherit the poor from the political process.

Because the present paper is interested in "working back" from policy, we sidestep specific causal theories. This allows us to avoid using specific parametric assumptions in the model. In this sense, our work parallels Revealed Preference Theory (RPT) which typically examines consistency of consumption data with budget-constrained utility maximization.[3] Our approach follows in the tradition of Afriat (1967) who examines how an individual utility function can be constructed from finite consumption and price data.[4] However, our model involves aggregation of choice and uses a political system to do so. More typically RPT approaches to aggregation follow the general equilibrium tradition, the classic example being the Sonnenschein-Mantel-Debreu result that checks whether an aggregate excess demand function is consistent with the economy-wide aggregation of optimizing choices.[5]

The RPT paradigm has been applied elsewhere to political choices as well. Kalandrakis (2010) for instance finds necessary and sufficient conditions for the results of a series of roll call votes to be rationalized by a voter with quasi-concave utility. Degan and Merlo (2009) use micro-level voting data to examine whether the outcomes of simultaneous, multi-candidate elections can be rationalized by ideological voting behavior.

Our approach is consistent with the standard RPT model since one can interpret the observations as having no intertemporal connection, just as in RPT. However, one can also view the observations as coming from a *fully dynamic* economy populated by infinitely lived citizens. Under the latter interpretation, the data consist of a time series produced by the same underlying polity. Viewed in this way, the present paper extends Boldrin and Montrucchio's (1986) dynamic model of rationalizability of policy rules by a single agent to the case of political aggregation.

2 The Economic Side

This section models the "economic side" of the model from the point of view of an outside observer. The tangible attributes of the economy such as income distribution and policies are observable. The observer does not see either the parametric preferences, or the underlying power distribution that produced the observed policies. Both the observed and unobserved attributes are laid out in the following subsections. The political side is taken up in Section 3.

[3]See Richter (1966) and more recently Varian (2006) for summaries and surveys of RPT developed by Paul Samuelson and others.

[4]See also Varian (1982), Chiappori and Rochet (1987).

[5]References for this result are Sonnenschein (1973), Mantel (1974), Debreu (1974). See also references and recent results in Brown and Kubler (2008) for applications of RPT to general equilibrium theory.

2.1 What the Observer Sees

There are $T < \infty$ observation dates that give the outsider observer a "window" into an ongoing economy. At each observation date $t = 1, \ldots, T$, the observer observes a policy a_t and an aggregate state variable ω_t. The policy could be a tax rate, a public good, or some defined level of redistribution, and is determined by a political process (to be described shortly). The state may be an economy-wide public capital stock, such as public infrastructure. However, it could also represent a summary statistic of ideological characteristics of voters. Formally, $a_t \in A$ with A a compact interval in $\mathbb{R}$, and $\omega_t \in \Omega$ where Ω is a connected subset of $\mathbb{R}$. Each state $\omega_t, t = 1, \ldots, T$ viewed by the observer is assumed to be distinct. Subsequent references to the "data" will be taken to mean the observed sequence $\{a_t, \omega_t\}_{t=1}^{T}$.

The economy is populated by a continuum $I = [0, 1]$ of *citizen-types*. A citizen-type is an index that orders individuals by income, with higher types accorded higher income. A citizen of type $i \in I$ holds income $y(i, \omega_t)$ in period t that depends, potentially, on the value of the state ω_t. The function y is assumed to be continuous and increasing in i, with $y(0, \omega_t) > 0$.[6] The monotonicity of y in i means that higher citizen types are wealthier, state-by-state. The assumption also implies a well defined conditional distribution function $i = F(\tilde{y}; \omega_t)$ corresponding to the proportion of types holding income no greater than $\tilde{y}$ given the state ω_t. The function y is assumed to be known or viewed by the observer.

In the subsequent notation, ω_t and a_t will refer to the on-path observations at date t, while ω and a will connote a generic state and policy, resp., either on the observed path or off it.

2.2 What the Observer Does not See

Consider a citizen of type i. This voter has preferences over policy choices in A expressed by a payoff function $U(i, \omega, a)$. Notice that the type's preferences can vary over the states. The critical assumption is that the outside observer does *not* observe the precise form of function U. However, he knows that U belongs to a set of *admissible* payoff functions satisfying two properties:

(A1) (Single Peakedness) U is continuous in the index i, and single peaked in a.

(A2) (Single Crossing) U satisfies the strict single crossing property in $(a\ ; i)$.[7]

[6] From here on, the term "increasing" will be taken to mean "strictly increasing", and the term "weakly increasing" will be taken to mean "nondecreasing".

[7] A function $f(x, y)$ is said to satisfy the *single crossing property in* $(x; y)$ if for all $x > \hat{x}$ and $y > \hat{y}$, $f(x, \hat{y}) - f(\hat{x}, \hat{y})\ (>) \geq\ 0$ implies $f(x, y) - f(\hat{x}, y)\ (>)\ \geq\ 0$, and satisfies *strict single crossing in* $(x; y)$ if $f(x, \hat{y}) - f(\hat{x}, \hat{y})\ \geq\ 0$ implies $f(x, y) - f(\hat{x}, y)\ >\ 0$. The "single crossing" as described here may be more accurately described as "single crossing from below." But because policies have no specific interpretation, notions of "larger" and "smaller" are arbitrary. Hence, without loss of generality, we could also have assumed

The strict single crossing property (A2) implies that in every state, wealthier citizens always prefer larger policies than poorer citizens. The "strictness" is assumed to avoid the trivial case in which all individuals' tastes are identical. Any payoff function U satisfying (A1)-(A2) is referred to as an *admissible preference profile.*

It's worth noting that, restrictiveness in the class of admissible profiles strengthens rather than weakens certain of our results. The reason is that the larger the set of admissible preference orderings, the easier it is to find one that "works" in the sense that a political system can produce the policy data under such preferences. The narrower the class of preferences the more difficult it is for a particular system to have generated the data. Hence, possibility results (i.e., assertions that data *can* be rationalized by a particular system) are stronger under narrower classes of preferences, while impossibility results (i.e., assertions that the data cannot be rationalized) are weaker, all else equal.

2.3 Two Interpretations

At this stage, our goal is to establish a benchmark model under fairly austere assumptions and without parametric structure. This model can be viewed in two ways.

1. The "classic" Revealed Preference Theory (RPT) interpretation. Here, the observer sees $\{a_t, \omega_t\}_{t=1}^T$ and presumes no intertemporal connection between observations. This is either because the data represent different replica economies, or because the data constitute a time series generated by myopic citizens.

2. The Dynamic Economy interpretation. As before the observer sees $\{a_t, \omega_t\}_{t=1}^T$. This time, however, he infers an intertemporal connection and can, in fact, back out a transition rule $\omega_{t+1} = Q(\omega_t, a_t)$ (on path) from the data. Here, the underlying time horizon may be infinite, and policy choices are determined by the aggregated decisions of a forward looking citizenry. Each citizen has long run payoffs determined by U.

The subsequent results apply to either interpretation. However, the first (static) interpretation is easier to describe in concrete terms, and so the subsequent analysis and discussion is cast in terms of Interpretation #1. (The Appendix contains a fuller elaboration of Interpretation #2 and connects it to our main results.) To illustrate, consider the following stylized model of a public goods economy. Voter preferences are given by,

$$u(c_t, G_t) \;=\; c_t \;+\; \frac{G_t^{1-\rho}}{1-\rho} \quad \rho \in [0, 1)$$

where $G_t = \tau_t \int_0^1 y\,(i, \omega_t)\,di = \tau_t \overline{y}\,(\omega_t)$ is a collective good funded by a flat tax τ_t, and $c_t = (1 - \tau_t)\,y\,(i, \omega_t)$ is after-tax private consumption. Though this is framed as a problem of public

single crossing from above.

good provision, notice that if $\rho = 0$, the problem reduces to one of pure redistribution. In terms of the present model, we define $a_t = 1 - \tau_t$ so the problem becomes

$$U(i, \omega_t, a_t) = a_t y(i, \omega_t) + \frac{\left[(1 - a_t)\, \overline{y}(\omega_t)\right]^{1-\rho}}{1 - \rho}.$$

The payoff function U depends on ω_t, not only through income $y(i, \omega)$, but also through $\overline{y}(\omega_t)$. It is easy to verify that this economy is consistent with the model (under Interpretation #1) and Assumptions (A1) and (A2).

3 The Political Side

This Section introduces a measure of political power based on weighted vote shares. The idea roughly is that political power is distributed *as if* each policy is determined by pairwise voting, and each individual's vote is weighted by his income. Political power is therefore determined by how much weight is given to income or wealth such that the observed policy would be a majority winning outcome under a voting scheme that uses such weights. A system's bias in this case is directly associated with these weights.

Formally, we associate the political bias in a given state with a functional parameter $\alpha(\omega)$. Larger values of $\alpha(\omega)$ correspond to greater political weight accorded to the rich. The value $\alpha(\omega)$ is an argument in a continuous integrable function $\lambda : \mathbb{R}^3 \to \mathbb{R}_+$ to be interpreted as follows. A type i with income $y(i, \omega)$ in state ω has weighted vote share $\lambda(y(i, \omega), \alpha(\omega), \omega)$ in a polity with bias $\alpha(\omega)$. In other words, the function λ captures the "share of political power" allocated to each citizen in each state in a potentially biased political system.

The outside observer is assumed to know the function λ, but does not observe the bias function α. This isolates α as the object of interest. Intuitively, λ represents the explicit features (e.g., constitutionally specified voting rules) of the political system, while α captures the nebulous features of the system that are intrinsically hard to observe directly (e.g., effect of lobbying on a senator's vote or of campaign contributions on an election cycle).[8]

The following three properties define the class of weighted votes share rules λ considered here. In each of these, $\tilde{y}$ and $\tilde{\alpha}$ denote real values of the functions y and α, respectively, given a state ω.

(B1) (Normalization) $\int_{y(0,\omega)}^{y(1,\omega)} \lambda(\tilde{y}, \tilde{\alpha}, \omega)\, dF(\tilde{y}; \omega) \;=\; 1.$[9]

[8] We point out, however, that our first result, the benchmark Proposition 0, does not depend on the observer's having any knowledge of λ other than that it satisfies the axioms that follow.

[9] Recall that $F(\tilde{y}; \omega_t)$ is the distribution of types over incomes $\tilde{y}$ as implied by the income process $y(\cdot)$ given ω.

(B2) (Income Monotonicity). The function λ is assumed to be increasing in income level $\tilde{y}$ if $\tilde{\alpha} > 0$, decreasing in income if $\tilde{\alpha} < 0$; constant across income levels if $\tilde{\alpha} = 0$.

(B3) (Strict Single Crossing with Vanishing Tails) The function $\lambda(\tilde{y}, \tilde{\alpha}, \omega)$ satisfies strict single crossing in $(\tilde{\alpha}; \tilde{y})$ with $\lim_{\tilde{\alpha}\to+\infty} \lambda(\tilde{y}, \tilde{\alpha}, \omega) = 0 \quad \forall \tilde{y} < y(1, \omega)$, and $\lim_{\tilde{\alpha}\to-\infty} \lambda(\tilde{y}, \tilde{\alpha}, \omega) = 0 \quad \forall \tilde{y} > y(0, \omega)$.

The theory applies to any weighted voting share rule consistent with these axioms. Axiom (B1) implies that the composite function $\lambda(y(i, \omega), \alpha(\omega), \omega)$ is a density in i. Axiom (B2) asserts that the political power of a citizen varies with his income $\tilde{y}$, and the direction taken by λ depends on the sign of $\tilde{\alpha}$. Political power is increasing in income if $\tilde{\alpha} > 0$, decreasing if $\tilde{\alpha} < 0$, and invariant to income if $\tilde{\alpha} = 0$. Hence, the value $\tilde{\alpha}$ can be thought of as a measure of the extent of wealth bias in state ω. When $\tilde{\alpha} = 0$, the political system may be said to be *unbiased* in the sense that each person's political weight in the distribution is invariant to income, hence all individuals are political equals. Following, for instance, Bénabou (1996), we will refer to $\tilde{\alpha} > 0$ as the case of an *elitist bias* since wealth is rewarded in the political system; the case of $\tilde{\alpha} < 0$ is referred to as a *populist bias* since political power is redistributed away from wealth. We allow that the function α can take values in the entire real line.

A canonical special case is the exponential weighting rule used by Bénabou (2000) in his study of inequality and its effects on redistribution. It's given by

$$\lambda(\tilde{y}, \tilde{\alpha}, \omega) \;=\; \frac{\tilde{y}^{\tilde{\alpha}}}{\int x^{\tilde{\alpha}} \; dF(x; \omega)} \tag{1}$$

In this case $\tilde{\alpha}$ exponentially weights wealth. One can then interpret $1 - \tilde{\alpha}$ as the weight attached to equal vote share or *equal representation* in voting.[10] When, for instance, $\tilde{\alpha} = 1$ then an individual who possesses twice as much income as another has twice as many votes, hence twice as much political power. The cases where $|\tilde{\alpha}| > 1$ are particularly stark in this example since this indicates a distribution of power that disproportionately rewards the fringes of the distribution. Extreme inequality occurs in the limit as $|\tilde{\alpha}| \to \infty$.

To understand the role of Axiom (B3), use the normalization in (B1) to define the distribution function

$$L^p(j; \alpha, \omega) \;=\; \int_0^j \lambda(y(i, \omega), \alpha(\omega), \omega)\, di. \tag{2}$$

We refer to the distribution L^P as the *Political Lorenz curve* since it gives a simple measure of political inequality. It describes the proportion of *political power* held by the poorest j%

[10] To see this more transparently, observe that (1) can be expressed as

$$\lambda(\tilde{y}, \tilde{\alpha}, \omega) \;=\; \frac{\tilde{y}^{\tilde{\alpha}} 1^{1-\tilde{\alpha}}}{\int \tilde{y}^{\tilde{\alpha}} 1^{1-\tilde{\alpha}} \; dF(\tilde{y}; \omega)}$$

of types in state ω. Political inequality, as measured by L^P, can then change over time for two reasons. First, it can change due to changes in the income distribution. Second, it can change due to "structural" changes as captured by changes in $\alpha(\omega)$. Axiom (B3) is the key assumption in guaranteeing monotonicity in these structural changes as shown in the Lemma:

Lemma 1 *For every* $j \in (0,1)$ *and each* ω,

$$L^P(j, \alpha_2, \omega) \; < \; L^P(j, \alpha_1, \omega) \quad \forall \; \alpha_1(\omega) < \alpha_2(\omega).$$

The proof is in the Appendix. Under the Lemma, the absolute value $|\alpha(\omega)|$ can be used to measure the intensity of the bias. Larger positive values correspond to greater elitism in the bias - greater political inequality with weight accorded to wealth. A more negative $\alpha(\omega)$ corresponds to greater populism — again greater political inequality but in reverse. The extra asymptotic conditions in (B3) guarantee that political inequality hits the extremes (power allocated entirely to the richest or the poorest) in the limit as $\alpha(\omega) \to \infty$ or $\to -\infty$.

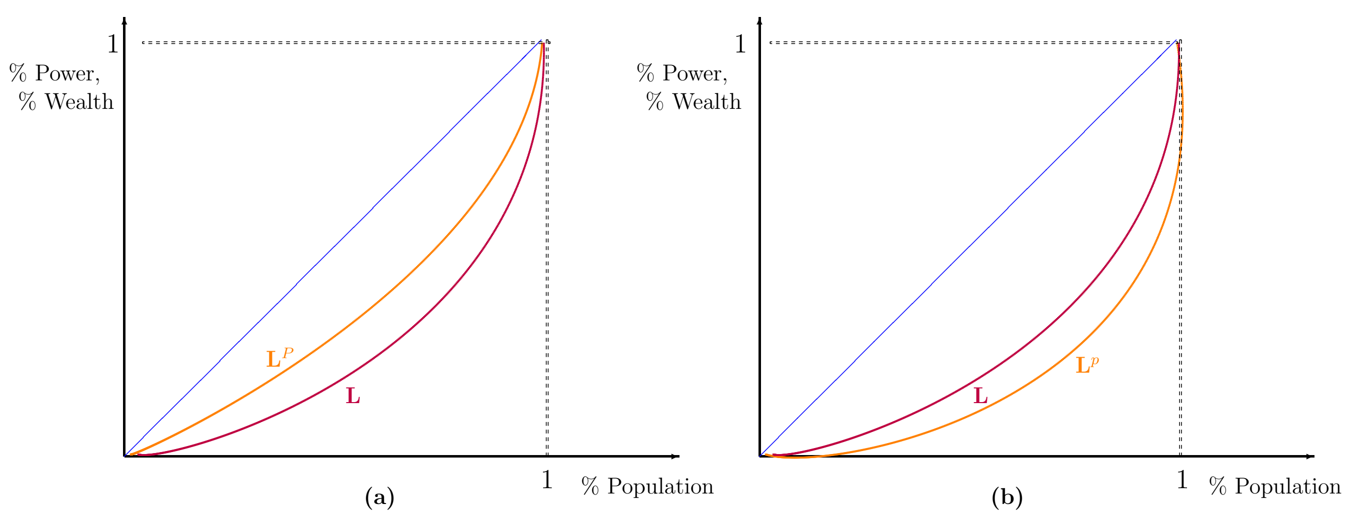

Figure 1: Political Lorenz Curves with Elitist Bias. (a) exhibits dampened bias. (b) exhibits pronounced bias.

Using (1) as the canonical example, consider a parametric example with an income process given by

$$y\,(i, \omega) = \exp(i\omega). \tag{3}$$

Here, one can interpret i as the production efficiency of type i and ω as the public capital stock. It is easy to see that $y\,(i, \omega)$ is increasing in (i, ω). In addition, income inequality increases in ω. Combining (3) with the vote share function in (1) yields

$$\lambda\left(y(i,\omega),\alpha(\omega),\omega\right) \;=\; \frac{y\left(i,\omega\right)^{\alpha(\omega)}}{\int_j y\left(j,\omega\right)^{\alpha(\omega)}dj} \;=\; \frac{\exp(\alpha(\omega)\omega i)}{\int_j \exp(\alpha(\omega)\omega j)dj}.$$

This implies a Political Lorenz curve given by

$$L^p\left(j,\alpha,\omega\right) \;=\; \int_0^j \lambda\left(y(i,\omega),\alpha(\omega),\omega\right)di \;=\; \frac{exp(\alpha(\omega)\omega j)-1}{exp(\alpha(\omega)\omega)-1}.$$

This Political Lorenz curve can be compared to the standard, income Lorenz curve given by

$$L(j,\omega) \;=\; \frac{\int_0^j y(i,\omega)di}{\int_0^1 y(i,\omega)di} \;=\;=\; \frac{exp(\omega j)-1}{exp(\omega)-1}. \tag{4}$$

As is standard, L describes the proportion of income held by the lowest j citizen-types in state ω. Figure 1a displays the two Lorenz curves in the case where the Political Lorenz curve exhibits a "dampened" elitist bias. Specifically, $0<\alpha(\omega)<1$, meaning that wealthier individuals have greater political weight than do poorer individuals, however, their increased weight is smaller than their weight in the income distribution. Political inequality therefore lies somewhere between income inequality and full equality. Figure 1b displays the Political Lorenz curve when $\alpha(\omega)>1$. In that case the elitist bias is more pronounced, with political inequality that exceeds income inequality in the degree that the wealthy are accorded power. Note that the two curves coincide in the case where $\alpha(\omega)=1$. Figure 2 illustrates the case of a populist bias, i.e., $\alpha(\omega)<0$. Most theories we are aware of predict an elitist bias if any. Nevertheless, it does not seem sensible to rule out the $\alpha(\omega)<0$ case, *a priori*.

4 Rationalizing Policy Data

Political Lorenz curves have a very simple interpretation. Suppose that policies are determined by some unspecified pairwise voting process. Each time a vote is taken, $\lambda(y(i,\omega),\alpha(\omega),\omega)$ is i's endowment of *vote share* in state ω. Policies are then determined by weighted majority voting where each individual's vote is weighted by his vote share. In the unbiased case ($\alpha(\omega)=0$), policies are determined by a simple majority vote.

Definition 1 A policy a is an α-*Weighted Majority Winner (WMW)* in state ω under admissible profile U if, for all policies $\hat{a}$,

$$\int_{i\in\{j:\,U(j,\omega,a)\geq U(j,\omega,\hat{a})\}} \lambda(y(i,\omega),\alpha(\omega),\omega)\;di \;\;\geq\;\; 1/2$$

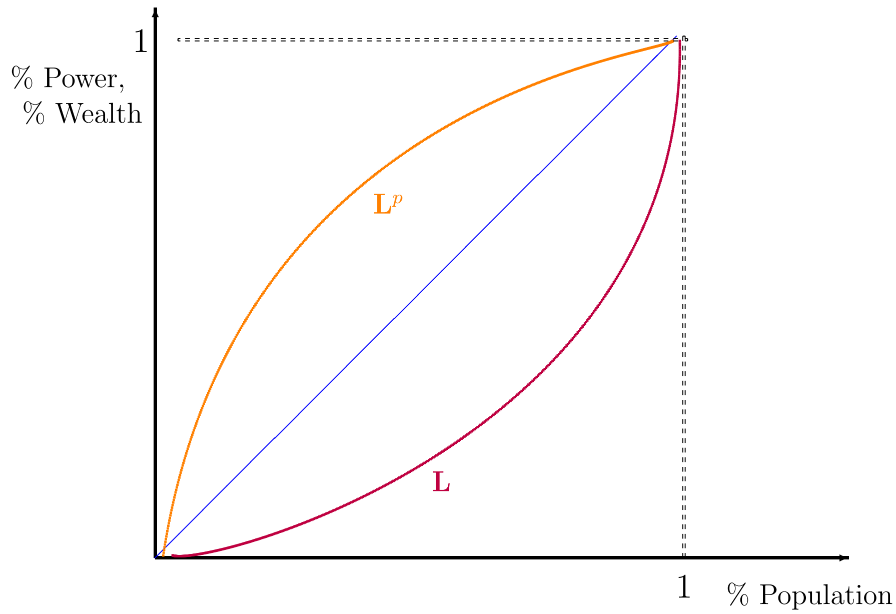

Figure 2: Political Lorenz Curve with Populist Bias

In other words, an α-weighted majority winner, or α-WMW, is a policy that survives against all others in a majority vote when each type i is allocated $\lambda(y(i,\omega),\alpha(\omega),\omega)$ votes and the preference profile is given by U.

Though the vote share function λ is known, the bias function α is not, and this is therefore the object of interest. Notice that if the preference profile U were known precisely to the outside observer, then α could be inferred precisely from observed policies that are generated from α (via the weighting function λ). But because U is not known, it is natural to ask whether observed policies might be "rationalized" by a weighting function α under some admissible preference profile U.

Definition 2 A weighting function α *rationalizes the observer's data* $\{a_t,\omega_t\}_{t=1}^T$ if there exists an admissible preference profile U such that for each $t=1,\ldots,T,$ a_t is an α-weighted majority winner in state ω_t under U.

In words, α rationalizes $\{a_t,\omega_t\}_{t=1}^T$ if the data can be produced by a political system with weighting function α.

One can now ask first whether the observations could have been produced by any weighting function, and second whether the observations could have been produced by a particular type of weighting function. Both questions are addressed by the following preliminary result that serves as a benchmark for rest of the paper.

Proposition 0 (Universal Bias Principle). *Let $\{a_t, \omega_t\}_{t=1}^T$ be any observable data and let α be any weighting function. Then α rationalizes $\{a_t, \omega_t\}_{t=1}^T$.*

Without further information about preference orderings, the Universal Bias Principle (UBP) tells us that all bias weights can be generated by the model. In particular, one can say nothing specific about political inequality, whether it exists or whether its magnitude is large. Since, among all other α, the unbiased polity $\alpha(\omega) = 0 \; \forall \omega$ can also rationalize policy data, it cannot be ruled out. The UBP also reveals that, clearly, single crossing is not necessarily too restrictive since relaxing it will not help.

The proof is straightforward. Standard results (e.g., Gans and Smart (1996)) show that for any admissible profile U, a Median Voter Theorem applies. Namely, any weighting function α admits a weighted majority-winning policy in each state ω. This policy is found to be the most preferred policy of the median type $i = \mu(\omega, \alpha)$ in the Political Lorenz distribution, i.e.,

$$L^P(\mu(\omega, \alpha), \alpha, \omega) \;=\; \frac{1}{2} \tag{5}$$

All that remains is to find an explicit, admissible payoff function U such that the preferred policy of type $i = \mu(\omega, \alpha)$ in state ω is precisely the one that is observed in the data. There are many such Us, for instance,

$$U\,(i, \omega, a) = -\frac{1}{2}\left[a - (i - \mu(\omega, \alpha) + \Psi(\omega))\right]^2,$$

where Ψ is any function such that $\Psi(\omega_t) = a_t$ for each observation date t. It is easy to verify that U is single-peaked in a, continuous and strictly single-crossing in $(a; i)$, with the preferred policy for $\mu(\omega, \alpha)$ as a_t for each t.

In the subsequent sections, it will sometimes prove more convenient to consider inference over $\mu(\omega, \alpha)$ directly rather than over α. The determination of $\mu(\omega, \alpha)$ is shown in Figure 3 for a particular $\alpha(\omega) > 0$.

5 Observing Poll Data

In light of the Universal Bias Principle, there are two possible ways to add predictive content. First, one could add direct information about specific binary rankings. Such information could come, for instance, from polls. Second, one can pare down the class of admissible profiles. Here we consider the first option.

Poll data constitute a natural source of information. This section shows how poll data and policy data together constitute a useful system of cross restrictions that reveal information about political bias. The model is therefore modified to incorporate poll data as follows. Each

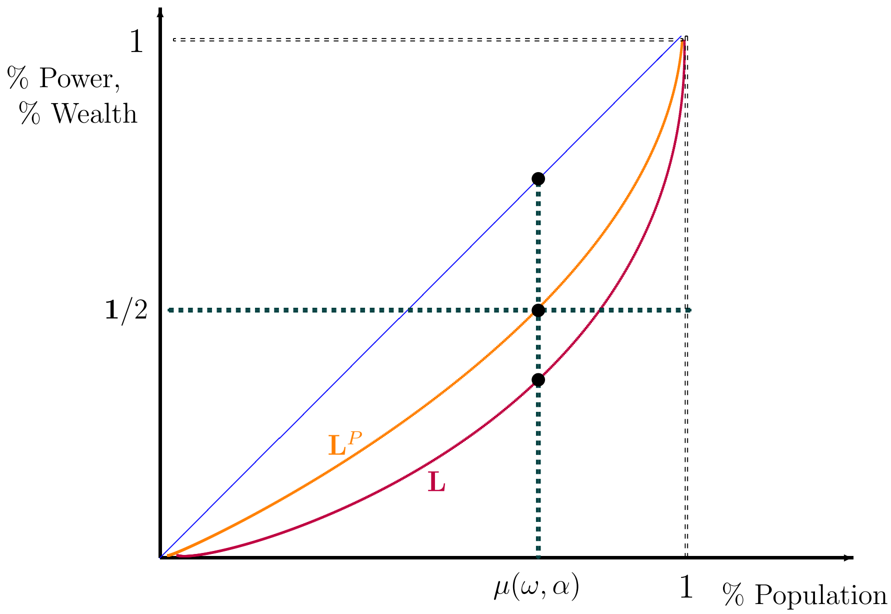

Figure 3: Identifying a Pivotal Voter under an Elitist Bias

period t, an arbitrary number N of binary polls are taken, each of which pit the observed a_t against an alternative policy. Let $a^1 < a^2 < \ldots < a^N$ denote these policy alternatives. Typically, these will be some much discussed policy alternatives, always on the table but not necessarily adopted. The poll data, in this case, consist of a sequence $\{p_t^n\}_{n=1,t=1}^{N,T}$ such that p_t^n is the support rate at date t that (weakly) favors observed policy a_t against alternative a^n. To keep the inference deterministic, we assume that there is no measurement error.

Definition A weighting function α *rationalizes the policy data* $\{a_t, \omega_t\}_{t=1}^T$ *and polling data* $\{p_t^n\}_{t=1,n=1}^{T,N}$ if there exists an admissible U such that

(i) $\forall\ t$, a_t is an α-Weighted Majority Winner in state ω_t under U, and

(ii) $\forall\ t\ \ \forall\ n$, U satisfies

$$p_t^n \;=\; |\{i : \; U(i, \omega_t, a_t) \geq U(i, \omega_t, a^n)\}|$$

This definition extends the earlier notion of rationalizing weights to one that includes polling data. Part (i) is the policy-consistency requirement as before. Part (ii) is a poll-consistency requirement. It requires that the underlying preference profile U be admissible and consistent with both types of data.

5.1 Rationalizing Policy and Poll Data: A Characterization

Since the polling data compare the N alternative policies to the actual policy a_t at each t, we adopt the notational convention of defining a fictitious policy choice $a^{N+1} = \epsilon + \max a$ for some $\epsilon > 0$, and set $p_t^{N+1} = 1$. We proceed to define:

$$n_t^* = \min\{\ n = 1, \ldots, N+1 : \quad a^n > a_t\}.$$

By this definition, $a^{n_t^*}$ is the closest "right-wing" alternative to the observed policy a_t (i.e., closest policy to the right of a_t) , $a^{n_t^*-1}$ is the closest "left-wing" alternative to the observed policy a_t.[11] Recall from (5) that $\mu(\omega, \alpha) = j$ is the pivotal voter in state ω under bias weight $\alpha(\omega)$. We now state our first main result.

Theorem 1 *Let $\{a_t, \omega_t\}_{t=1}^T$ be any policy data and let $\{p_t^n\}_{t=1,n=1}^{T,N}$ be any polling data such that $p_t^n < 1$ for all n and t. Then:*

1. *There exists an α that rationalizes the data if and only if for each t,*

$$1 - p_t^1 < \ldots < \ 1 - p_t^{n_t^*-2} \ < \ 1 - p_t^{n_t^*-1} \ < p_t^{n_t^*} \ < \ p_t^{n_t^*+1} \ < \ldots < p_t^N \tag{6}$$

2. *Any given α rationalizes the policy and poll data if and only if*

$$1 - p_t^1 < \ldots < 1 - p_t^{n_t^*-2} \ < \ 1 - p_t^{n_t^*-1} \ < \ \mu(\omega_t, \alpha) \ < \ p_t^{n_t^*} \ < \ p_t^{n_t^*+1} \ < \ldots < p_t^N \tag{7}$$

Part 1 can be interpreted as a data restriction, testing directly the validity of the model. Part 2 includes a bias restriction and, as such, speaks more directly to the topic of the paper. The "sufficiency" arguments entail a specific construction of a profile U under which a bias can rationalize the data. The formal arguments appear in the Appendix.

The necessary conditions are more intuitive. The inequalities in (6), for instance, follow directly from (A1) and (A2). If, for instance, we had $p_t^{n_t^*+1} \leq p_t^{n_t^*}$, then by the strict single crossing property we could find an individual $i \in [p_t^{n_t^*+1}, p_t^{n_t^*}]$ who weakly preferred $a_t^{n_t^*+1}$ to a_t and at the same time weakly preferred a_t to $a_t^{n_t^*}$. Since the actions are ordered $a_t < a_t^{n_t^*} < a_t^{n_t^*+1}$, this individual's U would violate single peakedness.

As for (7), to illustrate we take the poll consistency condition in (6) as given and show (7) is implied for any α rationalizes the policy and poll data. Hence, we focus only on those inequalities that bind the weighted pivotal voter $\mu(\omega_t, \alpha)$. Simplify notation by letting

$$r_t \equiv p_t^{n_t^*} \quad \text{and} \quad \ell_t \equiv p_t^{n_t^*-1}, \tag{8}$$

[11] Implicitly, n_t^* depends on the realized policy a_t but we omit the dependence in the notation for simplicity.

Here, r_t is the support rate for observed policy a_t against its closest right-wing alternative, while ℓ_t is the support rate for a_t against its closest left-wing alternative. Then, from Theorem 1, α rationalizes the data only if

$$1 - \ell_t \ < \ \mu(\omega_t, \alpha) \ < \ r_t, \quad t = 1, \ldots, T. \tag{9}$$

These inequalities can be understood from basic equilibrium logic. Suppose, for instance, it were the case that $r_t < \mu(\omega_t, \alpha) < 1$. Then the fraction $(r_t, 1]$ who prefer the closest right-wing alternative $a_t^{n_t^*}$ would exceed half the weighted vote share. This would place the supporters of $a_t^{n_t^*}$ in a position to have vetoed a_t, in which case a_t could not have been a Weighted Majority Winner. If, in fact, $\mu(\omega_t, \alpha) = r_t < 1$, then by continuity of U in i (Assumption (A1)), this pivotal voter would be indifferent between a_t and $a_t^{n_t^*}$, thus violating single peakedness. Consequently, we must have $\mu(\omega_t, \alpha) < r_t$. Similar arguments readily establish that $1 - \ell_t < \mu(\omega_t, \alpha)$.

These inequalities can, in fact, be translated into bounds on the bias itself. Notice that, since the Political Lorenz Curve L^P is decreasing in the weight $\alpha(\omega)$ (holding ω fixed), the pivotal function μ is invertible in the value $\alpha(\omega)$. Hence, let $M(j, \omega)$ denote the *inverse pivotal function*, defined as the map that associates pivotal voter j with the bias weight that would, in fact, yield j as the pivotal voter. Applying M to the inequalities in (9) yields

$$M(1 - \ell_t, \omega_t) \ < \alpha(\omega_t) \ < \ M(r_t, \omega_t), \quad t = 1, \ldots, T. \tag{10}$$

This defines a *bias band*, i.e., an upper and lower bound on the bias as indicated in Figure 4. The bounds of the band in the figure are displayed on the vertical axis. In this particular graph, the range of bias band includes 0, the unbiased weight. It also includes a subinterval of elitist biases, as well as a subinterval of populist ones.

The inverse pivotal function M has a number of uses. It can be used to identify conditions under which an populist or elitist in the bias can be definitively inferred.

Proposition 1 *Suppose that α rationalizes the policy data $\{a_t, \omega_t\}$ and poll data $\{p_t^n\}$, and let r_t and ℓ_t be defined from the poll data as in (8). Then for each $t = 1, \ldots, T$,*

(i) $\alpha(\omega_t) > 0$ *whenever* $\ell_t < 1/2$.

(ii) $\alpha(\omega_t) < 0$ *whenever* $r_t \leq 1/2$.

The bias is therefore elitist if fewer than half the population supports a_t against its closest left-wing alternative. The bias is populist if fewer than half support a_t against its closest right-wing alternative. The result suggests that the basic character of the bias can be identified whenever a policy results with only minority support from the population. This is a suggestive finding given that minority policies are often observed.[12]

[12]The reality, of course, is that in actual political systems, the minority policies pushed through are one component in a higher dimensional policy space. In general the observer would find it more difficult to sort

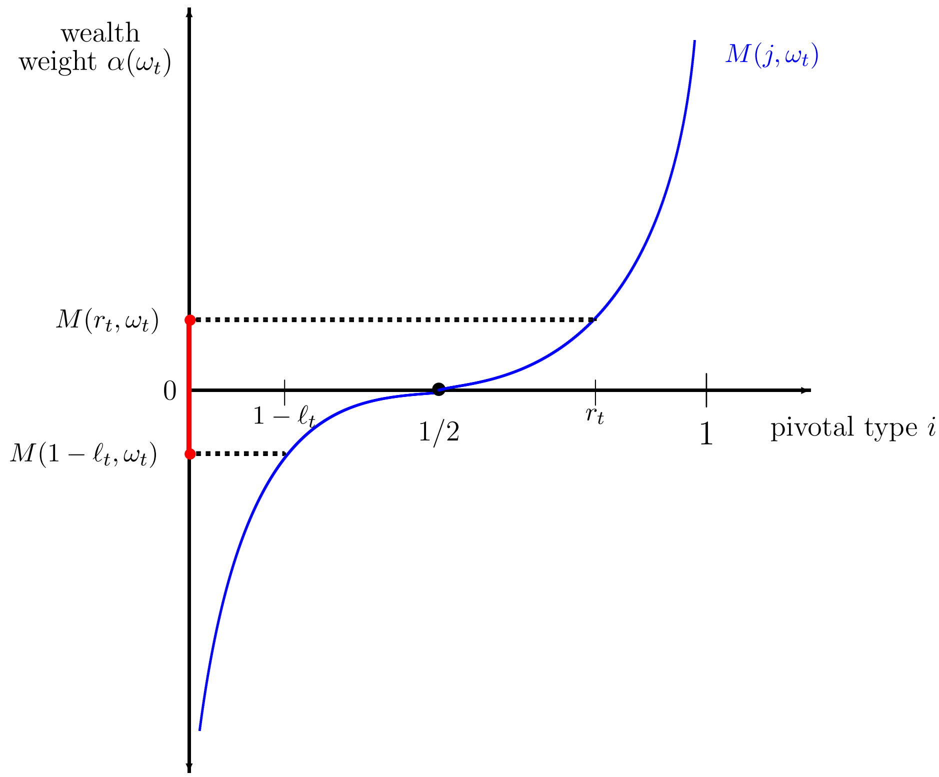

Figure 4: Bias Band and Bounding Function

5.2 The Case of a Stable Bias

By design, the model allows the bias weight α to vary with the state along with income distribution. This affords flexibility to the model but weakens the ability of the observer to make a clear inference. It is quite natural to suppose that institutional features change more slowly than other features of the economy. Consequently we consider the case of a stable bias, i.e., $\alpha(\omega_t) = \bar{\alpha},\ \forall\ t$. In this case the bias is held fixed while the income distribution varies with the state ω_t.

Theorem 1 together with the definition of M implies straightaway:

Proposition 2 *A stable bias $\bar{\alpha}$ rationalizes the data only if*

$$\max_{t=1,\dots,T} M(1-\ell_t, \omega_t) < \bar{\alpha} < \min_{t=1,\dots,T} M(r_t, \omega_t)$$

In other words, observations have a cumulative effect; each observation date serves as a cross

out the effect of bias from these higher dimensional trade offs. We revisit the higher dimensionality problem in the Concluding Section.

check against the observations at other dates. As a result, the *effective* bias band must shrink - at least weakly.

Notice that the cumulative effect of the data as shown in Proposition 2 allows the observer to make real time refinements to the observer's inference of the lower and upper bounds at each observation date t. These are given by $\underline{M}_t = \max_{q=1,\ldots,t} \; M(1-\ell_q, \omega_q)$ and $\overline{M}_t = \min_{q=1,\ldots,t} \; M(r_q, \omega_q)$, respectively. Using Proposition 2, the observer at date t infers that a stable bias $\bar{\alpha}$ rationalizes the data only if $\underline{M}_t < \bar{\alpha} < \overline{M}_t$. By construction, these bounds tighten monotonically so that $\underline{M}_t \leq \underline{M}_{t+1}$... and $\overline{M}_t \geq \overline{M}_{t+1}$...

However, these "real time" refinements also allow the observer to infer something about the sequence of future pivotal voters. Using (9), the observer at date t forecasts a sequence of upper bounds on the weighted pivotal voter given by

$$\mu(\omega_t, \overline{M}_t), \; \mu(\omega_{t+1}, \overline{M}_t), \ldots, \; \mu(\omega_T, \overline{M}_t),$$

and from below,

$$\mu(\omega_t, \underline{M}_t), \; \mu(\omega_{t+1}, \underline{M}_t), \ldots, \; \mu(\omega_T, \underline{M}_t)$$

and at each date $q = t, \ldots, T$,

$$\mu(\omega_q, \underline{M}_t) \; < \; \mu(\omega_q, \bar{\alpha}) \; < \; \mu(\omega_q, \overline{M}_t).$$

At $t+1$, the observer refines his forecast, and draws an entirely new inference using $\overline{M}_{t+1}$ and $\underline{M}_{t+1}$, and so on.

Because the income distribution is arbitrary at this point, these forecasted sequences are not generally ordered. They can jump around from date to date. However, we can order them for the case where income distribution is well ordered. To illustrate this point, consider a vote share rule λ that has the canonical form in (1). Re-indexing dates if necessary, let $\omega_1 < \omega_2 < \cdots < \omega_T$ and suppose that income process y has monotone log differences in the pair (i, ω). That is, for any pair of states, ω and $\hat{\omega}$, the difference $\log y(i, \omega) - \log y(i, \hat{\omega})$ is either increasing in i, or decreasing in i. Many common income processes, including the earlier example in (3) satisfy this condition. Using this restriction income and political inequality can be shown to vary positively or inversely depending on whether $\bar{\alpha}$ is elitist or populist.

Proposition 3 *Suppose y has monotone log differences in the pair (i, ω) and income inequality is increasing in t, i.e., $L(j, \omega_{t+1}) < L(j, \omega_t)$ for all $t = 1, \ldots, T-1$. Then a stable bias $\bar{\alpha} > (<) 0$ rationalizes the data $\{a_t, \omega_t\}$ and $\{p_t^n\}$ only if*

$$\mu(\omega_{t+1}, \bar{\alpha}) > (<) \mu(\omega_t, \bar{\alpha}), \quad \forall t.$$

The interpretation is as follows. When a stable bias is elitist, the weighted pivotal voter is located farther up in the income distribution as inequality increases. When a stable bias is populist, the weighted pivotal voter is located farther down as inequality increases. Note, in particular, that if the income inequality is increasing over time, then Proposition 3 implies that the forecasted bounds on the pivotal voter at any given time t are monotonic: either $\mu(\omega_q, \overline{M}_t) < \mu(\omega_{q+1}, \overline{M}_t)$ or $\mu(\omega_q, \overline{M}_t) > \mu(\omega_{q+1}, \overline{M}_t)$, $\forall q = t, \ldots, T-1$ depending on whether the bound $\overline{M}_t$ is elitist (> 0) or populist (< 0).

As for the argument, notice that both increased income and political inequality are statement about first-order stochastic orderings. In either case, a distribution under, say, ω_2 first-order dominates a distribution under ω_1 if the likelihood ratio is increasing. The log-likelihood ratio of L is

$$\log y\left(i, \omega_2\right) - \log y\left(i, \omega_1\right) - \log\left(\frac{\int_0^1 y\left(s, \omega_2\right) ds}{\int_0^1 y\left(s, \omega_1\right) ds}\right),$$

whereas the log-likelihood ratio of L^P is

$$\bar{\alpha}\left[\log y\left(i, \omega_2\right) - \log y\left(i, \omega_1\right)\right] - \log\left(\frac{\int_0^1 y\left(s, \omega_2\right)^{\bar{\alpha}} ds}{\int_0^1 y\left(s, \omega_1\right)^{\bar{\alpha}} ds}\right).$$

Now suppose that the log difference of y is increasing in i. Then standard argument show that both likelihood ratios are increasing if $\bar{\alpha} > 0$. Similarly, the likelihood ratio for L is increasing, and that for L^P is decreasing if $\bar{\alpha} < 0$.

As a simple comparative statics exercise, consider a *ceteris parabis* increase in income inequality from ω_1 to ω_2. Then one can verify that $|M(j, \omega_2)| < |M(j, \omega_1)|$ for all $j \neq 1/2$. In particular, if 0 (the unbiased weight) belongs to the band, then larger income inequality reduces the size of the band around 0. Intuitively, this is not surprising if $\alpha > 0$ since in that case, the pro-wealth bias must be lower to have off-set the greater income inequality. Somewhat more surprising is the fact that when the band is entirely below 0 (populism), greater inequality moves the band closer to 0 as well. In other words the band becomes less populist implying that wealthier individuals receive increased political weight from the bias in addition to increased weight from income alone. Why? Because with a populist system, political inequality is negatively related to income inequality. Hence, holding the bias weight constant, an increase in relative income of the top 10% translates into an weighted decrease in this group's political power. The bias weight must therefore increase to offset this fall in political power due to income change. This dual effect of greater income inequality is displayed in Figure 5.

6 Restricted Preference Domains

In this section, we return to the original premise that the observer sees only policy data (i.e., no polling data) and instead, narrow down class of preference profiles. The idea is that observer knows something more, *a priori*, about the admissible preferences beyond assumptions (A1)

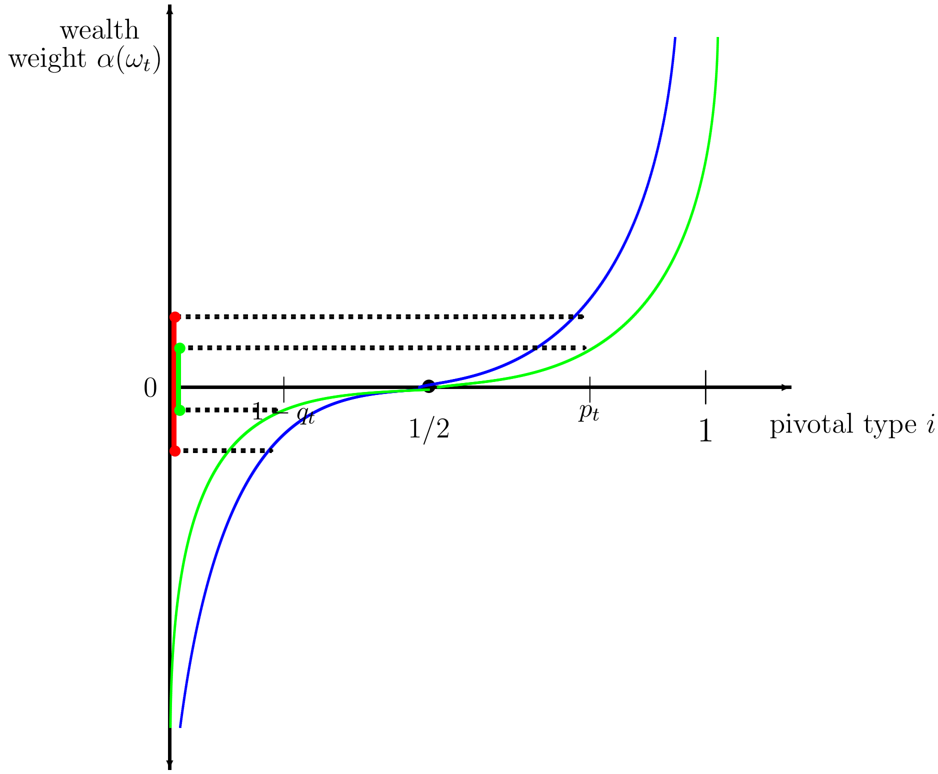

Figure 5: Shrinking Bias Band with Increased Inequality

and (A2). A narrower class of preference profiles potentially narrows the set of rationalizing biases. To illustrate how additional knowledge of preference could be used by the observer, recall the public goods example. Preferences satisfy

$$U\left(i, \omega_t, a_t\right) = a_t y\left(i, \omega_t\right) + \frac{\left[\left(1-a_t\right) \overline{y}\left(\omega_t\right)\right]^{1-\rho}}{1-\rho}. \tag{11}$$

Consider two benchmark income processes. (1) the income process satisfies $y(i, \omega_t) = g(i)\omega_t$ so that the state ω summarizes only aggregate growth effects. There are no distributional changes over time. (2) the income process satisfies $\int y(i, \omega)di = \bar{y}$ so that there are only distributional changes; no aggregate changes in income occur.

Significantly, each of these cases distinctly imply something about U. In the first case where only growth effects exist, U satisfies strict single crossing in $(a; \omega)$. In the second case, where there are only distributional effects, U satisfies separability in income $y(i, \omega)$ and policy a in the sense that $U(i, \omega, a)$ can be expressed as a function $u(y(i, \omega), a)$. These properties have arisen, of course, from a particular payoff function U and a particular income process y. However, because these particular functional forms are standard in, for instance, public provision models, we find it useful to examine the inference problem starting from the general properties that generate each. We therefore ask what the observer can infer about bias, starting directly from:

(A3) U satisfies strict single crossing in the pair $(a\ ;\omega)$ for each i.

Assumption (A3) implies that every type's most preferred policy is (weakly) monotone in the state. This monotonicity restriction is fairly common when the policy is a complementary input in the production process. Returning to the canonical public goods model with preferences given by (11), Assumption (A3) can easily be verified in the "growth only" case where $y(i, \omega_t) = g(i)\omega_t$. To see what this implies, observe that the most preferred policy of a citizen of type i is

$$\tilde{\Psi}(i, \omega_t) \equiv 1 - \frac{1}{\omega_t} \left(\frac{(\int g(j)dj)^{1-\rho}}{g(i)} \right)^{1/\rho}$$

According to the policy definition, the tax rate is $1-\tilde{\Psi}(i, \omega_t)$. The tax rate in the "growth only" case is therefore decreasing in the state. This means that in a growing economy ($\omega_{t+1} > \omega_t$) with no distributional changes, each voter's ideal tax rate is decreasing over time. This does not imply that the observed tax rates are decreasing since the bias itself can move toward voter-types who prefer relatively higher tax rates. This idea is summarized more generally in the result below.

Theorem 2 *Let $\{a_t, \omega_t\}_{t=1}^T$ be any policy data. Then:*

1. *There exists an α that rationalizes the data under preferences in (A1)-(A3).*
2. *Any given α rationalizes policy data $\{a_t, \omega_t\}$ under preferences in (A1)-(A3) if and only if for each pair of observed states ω_t, ω_τ with $\omega_t > \omega_\tau$,*

$$a_t < a_\tau \implies \mu(\omega_t, \alpha) < \mu(\omega_\tau, \alpha) \tag{12}$$

A consequence of Part (2) is that any policy data increasing in the observed state can be rationalized by any α. However, if the data ever decrease in the state, then certain α may not rationalize the data. For instance:

Corollary *Let $\{a_t, \omega_t\}$ be any policy data such that the observed policies decrease whenever the state increases. Then the unbiased weighting function, $\alpha(\omega) = 0$ for all ω, does not rationalize the data.*

The sufficiency proof of Theorem 2 requires a constructive argument just as in Theorem 1. An admissible U must be constructed to satisfy the preference axioms while, at the same time, match the policy data on the observed path whenever type i is the pivotal voter, $\mu(\omega, \alpha)$.

The construction is complicated in this case by the fact that each citizen's optimal policy rule $\tilde{\Psi}(i, \omega_t)$ must be weakly increasing in the state in order to satisfy (A3), even as the actual policy data might be decreasing in the state. To overcome this, we specify a recursive algorithm that exploits the natural bi-monotonicity of the data in a_t and $\mu(\omega, \alpha)$ — as required by the hypothesis in Part (2) of the Theorem. The formal argument is left to the Appendix.

Finally, consider the property:

(A4) U satisfies separability, i.e., $U(i, \omega, a) = u(y(i, \omega), a)$ for all a and $y(\cdot)$.

Whereas (A3) was motivated by the combination of preferences satisfying (11) and a y process with only growth effects, Assumption (A4) is motivated by the combination of (11) and a "distribution only" change in the income process: $\int y(j, \omega_t) dj = \bar{y}$. In the canonical model in (11), this implies that citizen-type i's most preferred policy is

$$\tilde{\Psi}(i, \omega_t) \equiv 1 - \frac{\bar{y}^{(1-\rho)/\rho}}{(y(i, \omega_t))^{1/\rho}}$$

In this case, the citizen's most preferred tax rate depends on i and ω_t exclusively through income $y(i, \omega_t)$. In particular, each citizen's preferred tax $(1 - \tilde{\Psi}(i, \omega_t))$ is decreasing in his income. Assumption (A4) captures the general idea that each citizen's preferred policy rule varies only in his income. The implications are summarized in the result below.

Theorem 3 *Let $\{a_t, \omega_t\}_{t=1}^T$ be any policy data. Then:*

1. *There exists an α that rationalizes the data under preferences in (A1)-(A2) and (A4).*
2. *Any given α rationalizes policy data $\{a_t, \omega_t\}$ under preferences in (A1)-(A2) and (A4) if and only if for any pair of observations,*

$$a_t < a_\tau \implies y(\mu(\omega_t, \alpha), \omega_t) < y(\mu(\omega_\tau, \alpha), \omega_\tau) \tag{13}$$

According to the Theorem, whenever policy is observed to decrease, the weighted median income must decrease as well. To see what this means in the "pure distribution" case, suppose that an increase in inequality over time leads to a drop in median income. In that case a decrease in the observed tax rate would imply a polity with an elitist bias (recall that the tax rate is $1 - a_t$ in each date). This is consistent with results of Bénabou (1996, 2000) that show under an elitist polity, increased inequality is associated with lower levels of redistribution toward the poor.

7 Summary and Extensions

This paper adapts ideas from revealed preference theory to understand political bias. To assess the bias, we formulate a theory of inference based on an outside observer's direct view of policy rather than on indirect measures such as political participation. The theory associates political bias with the weights on a system of wealth-weighted majority voting.

Given fairly standard assumptions ensuring each admissible preference profile admits a weighted majority winner, every weighted system is shown to rationalize every possible policy path. The introduction of polling data rules out "extreme" weighting systems, by imposing upper and lower bounds on the magnitude of the bias. Further restrictions on preferences can rule out certain weighted systems.

As for limitations, the main hurdle in our view is the restriction to one dimensional policies and states. The dimensionality restriction, together with single crossing ensure existence of a majority winner. If policies and states are multi-dimensional, then the single crossing condition on the natural (Euclidian) order is no longer sufficient to ensure majority voting outcomes. At this point one's options are limited. One option is to use a common generalization of (A2), known as "order restrictedness", due to Rothstein (1990). Order restricted preferences are those for which there exists *some* order on the policy space A (other than, presumably, the Euclidian order) under which preferences are single crossing. Under order restricted preferences, wealth-weighted majority winner always exist. Because this is a fairly direct extension, we omit details.

A second and more challenging option is to drop all assumptions that guarantee existence of weighted majority voting. Even in this case, it is possible to articulate a well defined theory, albeit one with few known implications. A weaker equilibrium notion, for instance, the *weighted minmax majority winner (WMMW)* can always be shown to exist. Roughly, WMMW's are policies that garner more support than any other when policies are pitted against their most popular alternatives.[13] It's straightforward to show that the set of WMMW is always nonempty, and coincides with the set of WMW whenever the latter is nonempty. A drawback of this generalization is that since policies are not necessarily well ordered, it is not clear how changes in observed policy map into the wealth distribution. Consequently, it is hard to see how meaningful inference is possible at this level of generality. We have little to say about it at this point, and so we leave it for future consideration.

[13] Formally, a policy a is a *Weighted-Minmax Majority Winner (WMMW)* in state ω if

$$\int_{\{i:\, U(i,\omega,a)\geq U(i,\omega,\hat{a})\}} \lambda(y(i,\omega),\alpha(\omega),\omega)\; di \quad \geq \int_{\{i:\, U(i,\omega,a')\geq U(i,\omega,\hat{a}')\}} \lambda(y(i,\omega),\alpha(\omega),\omega)\; di$$

for all $\hat{a}$, a' and $\hat{a}'$.

8 Appendix

Appendix A: The Dynamic Economy Interpretation

The presentation in the main text does not specify an explicit intertemporal connection between observations. Extending the analysis to a dynamic economy with infinite-lived forward-looking decision makers requires adding three ingredients.

1. The states and policy are connected through a transition function $\omega_{t+1} = Q(\omega_t, a_t)$, which must be known to the participants and may be partly inferred from the data path $\{a_t, \omega_t\}_{t=1}^T$ by the outside observer.

2. Forward-looking individuals correctly forecast future economic policies both on and off the equilibrium path. We restrict attention to *admissible Markov policy rules*, i.e. given any policy data $\{a_t, \omega_t\}_{t=1}^T$ there exists a function $\Psi : \Omega \to A$ satisfying

$$\Psi(\omega_t) \ = \ a_t, \forall \ t = 1, \ldots, T.$$

   The Markov restriction allows for a tractable characterization of the data even as it entails some loss of generality. It seems appropriate in large and anonymous societies where history-dependent enforcement mechanisms would be difficult to implement.

3. The life-time utility is additively separable with a flow payoff $u(\omega, y, a)$ and a known discount factor $\delta \in [0, 1)$. In a Markov equilibrium, this implies that for every (i, ω, a)

$$U(i, \omega, a) = u(\omega, y(i, \omega), a) + \delta U(i, Q(\omega, a), \Psi(Q(\omega, a))). \tag{14}$$

   A payoff function $U(i, \omega, a)$ satisfying (A1), (A2) and Equation (14) is then referred to as a *dynamically admissible* preference profile.

Definition 3 A weighting function α *dynamically rationalizes* the observer's data $\{a_t, \omega_t\}_{t=1}^T$ if there exists a dynamically admissible preference profile U and an admissible Markov policy rule $\Psi(\omega)$ such that $\Psi(\omega)$ is an α-weighted majority winner in every ω under U.

By definition, dynamic rationalization under a given U implies rationalization under the same U. On the other hand, if α rationalizes the observed data under U, then it also dynamically rationalizes the data under the flow payoff

$$u(\omega, y, a) = U(F(\omega, y), \omega, a) - \delta U(F(\omega, y), Q(\omega, a), \Psi(Q(\omega, a))),$$

and

$$\Psi(\omega) = \arg\max U(\mu(\omega, \alpha), \omega, a).$$

As a result, dynamic-rationalization imposes the same testable restrictions as rationalization as defined in Definition 2 in the main text.

Appendix B: Proofs of the Results

Proof of Lemma 1 Let $f(i,\alpha,\omega) = \lambda(y(i,\omega),\alpha,\omega)$. Fix a state ω and let $\alpha_2(\omega) > \alpha_1(\omega)$. Now define

$$D(j) = L^P(j,\alpha_2,\omega) - L^P(j,\alpha_1,\omega) = \int_0^j (f(i,\alpha_2,\omega) - f(i,\alpha_1,\omega))\, di.$$

From strict single crossing property, $f(i_1,\alpha_2,\omega) - f(i_1,\alpha_1,\omega) \geq 0$ implies $f(i_2,\alpha_2,\omega) - f(i_2,\alpha_1,\omega) > 0$ for every $i_2 > i_1$. By definition, $D(0) = 0$ and $D(1) = 0$. As a result, it cannot be the case that $f(i,\alpha_2,\omega) - f(i,\alpha_1,\omega) > 0$ or $f(i,\alpha_2,\omega) - f(i,\alpha_1,\omega) < 0$ for almost all $i \in (0,1)$. Consequently, as a function of i, $f(i,\alpha_2,\omega) - f(i,\alpha_1,\omega)$ crosses zero exactly once and from below. This implies that $L^P(j,\alpha_2,\omega) < L^P(j,\alpha_1,\omega)$ for every $j \in (0,1)$. ∎

Proof of Theorem 1.

Part 1. To prove Part 1, existence of a rationalizing α, we first suppose that Part 2 holds. Suppose that there exists α that rationalizes the data. Then (7) holds by Part 2. This obviously implies (6) which establishes the "necessity" in Part 1. To show the sufficiency part, suppose that (6) holds. It suffices to show that there exists α such that for each ω_t, $\mu(\omega_t,\alpha)$ satisfies (7). Because $\mu(\omega,\tilde{\alpha})$ is continuous in the scalar $\tilde{\alpha}$, we proceed to show this via a full range condition. That is, $\mu(\omega,\tilde{\alpha}) \to 1$ if $\tilde{\alpha} \to +\infty$; $\mu(\omega,\tilde{\alpha}) \to 0$ if $\tilde{\alpha} \to -\infty$. The argument, using Axiom (B3), is as follows. By definition, $L^P(j,\tilde{\alpha},\omega) = \int_0^j \lambda(y(i,\omega),\tilde{\alpha},\omega)\, di$.

(a) If $\lim\limits_{\tilde{\alpha}\to+\infty} \lambda(\tilde{y},\tilde{\alpha},\omega) = 0$, $\forall \tilde{y} < y(1,\omega)$, then $\lim\limits_{\tilde{\alpha}\to+\infty} \lambda(y(i,\omega),\tilde{\alpha},\omega) = 0$, $\forall i < 1$. Hence $\lambda(y(i,\omega),\tilde{\alpha},\omega)$ is a uniformly bounded function for every fixed $i < 1$ when $\tilde{\alpha}$ is large enough. For every $j < 1$, $\int_0^j \lim\limits_{\tilde{\alpha}\to+\infty} \lambda(y(i,\omega),\tilde{\alpha},\omega)\, di = 0$. From Bounded Convergence Theorem on $[0,j]$, one can exchange lim and integral to obtain $\lim\limits_{\tilde{\alpha}\to+\infty} \int_0^j \lambda(y(i,\omega),\tilde{\alpha},\omega)\, di = 0$, or $\lim\limits_{\tilde{\alpha}\to+\infty} L(j,\tilde{\alpha},\omega) = 0$.

(b) If $\lim\limits_{\tilde{\alpha}\to-\infty} \lambda(\tilde{y},\tilde{\alpha},\omega) = 0$, $\forall \tilde{y} > y(0,\omega)$, then $\lim\limits_{\tilde{\alpha}\to-\infty} \lambda(y(i,\omega),\tilde{\alpha},\omega) = 0$, $\forall i > 0$. Hence $\lambda(y(i,\omega),\tilde{\alpha},\omega)$ is a uniformly bounded function for every fixed $i > 0$ when $\tilde{\alpha}$ is small enough. For every $j < 1$, $\int_{1-j}^1 \lim\limits_{\tilde{\alpha}\to-\infty} \lambda(y(i,\omega),\tilde{\alpha},\omega)\, di = 0$. Again using the Bounded Convergence Theorem on $[1-j,1]$, the lim and integral are exchanged to obtain $\lim\limits_{\tilde{\alpha}\to-\infty} \int_{1-j}^1 \lambda(y(i,\omega),\tilde{\alpha},\omega)\, di = 0$, or $\lim\limits_{\tilde{\alpha}\to-\infty} (1 - L(j,\alpha,\omega)) = 0$ and hence $\lim\limits_{\tilde{\alpha}\to-\infty} L(j,\tilde{\alpha},\omega) = 1$.

Part 2. As the necessary part was shown in the main text, it remains to show the sufficiency argument.

Sufficiency. Now we suppose that the inequalities in (7) hold and proceed to show that α rationalizes the data. Consider any payoff U of the form

$$U(i,\omega,a) = -\frac{1}{2}\left(a - \widetilde{\Psi}(i,\omega)\right)^2, \tag{15}$$

where $\widetilde{\Psi}(i,\omega)$ is continuous and increasing in i for every $\omega \in \Omega$. Notice that every U as defined in (15) is admissible. For (A1), observe that U is continuous in i and strictly concave in a (hence single peaked). From the increasing property of $\widetilde{\Psi}$ in i, U is strict single crossing in $(a;i)$, as required in (A2).

We proceed to construct a particular $\widetilde{\Psi}(i,\omega)$ to be consistent with both policy and polling data. For policy data, notice that $\widetilde{\Psi}(i,\omega)$ is the preferred policy choice for type i under ω. By the Median Voter Theorem (for example, Gans and Smart (1996)), α rationalizes the policy data under U in (15) if and only if there exists a $\widetilde{\Psi}$ such that $\widetilde{\Psi}(\mu(\omega_t,\alpha),\omega_t) = a_t$.

To prove consistency of U with polling data comparing a_t to any alternative a^n with $n \geq n_t^*$, it suffices to assume that the type p_t^n is indifferent between a_t and a^n, i.e., $U(p_t^n,\omega_t,a^n) = U(p_t^n,\omega_t,a_t)$. With some algebra, it reduces to

$$\widetilde{\Psi}(p_t^n,\omega_t) = \frac{1}{2}(a^n + a_t),$$

where $\widetilde{\Psi}$ is the function associated with payoff function U as specified in (15). Similarly, when $n \leq n_t^* - 1$, consistency U with polling for a_t against a^n implies that the type $1 - p_t^n$ is indifferent between a_t and a^n, i.e., $U(1-p_t^n,\omega_t,a^n) = U(1-p_t^n,\omega_t,a_t)$, which reduces to

$$\widetilde{\Psi}(1-p_t^n,\omega_t) = \frac{1}{2}(a^n + a_t).$$

To prove that α rationalizes the data under admissible payoff function U of the form in (15), it therefore suffices to construct function $\widetilde{\Psi}$ that is continuous and increasing in i and satisfies the equation systems:

$$\begin{pmatrix} \widetilde{\Psi}\left(1-p_t^1, \omega_t\right) \\ \vdots \\ \widetilde{\Psi}\left(1-p_t^{n_t^*-1}, \omega_t\right) \\ \widetilde{\Psi}\left(\mu(\omega_t, \alpha), \omega_t\right) \\ \widetilde{\Psi}\left(p_t^{n_t^*}, \omega_t\right) \\ \vdots \\ \widetilde{\Psi}\left(p_t^N, \omega_t\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(a^1 + a_t) \\ \vdots \\ \frac{1}{2}(a^{n_t^*-1} + a_t) \\ a_t \\ \frac{1}{2}\left(a^{n_t^*} + a_t\right) \\ \vdots \\ \frac{1}{2}(a^N + a_t) \end{pmatrix} \quad t = 1, \ldots, T. \tag{16}$$

Hence, fix any on-path ω_t. Then $\widetilde{\Psi}(i, \omega_t)$ can be found as a linear spline passing through data points $(0, a^1), (1-p_t^1, \frac{1}{2}(a^1+a_t)), \ldots, (1-p_t^{n_t^*-1}, \frac{1}{2}(a^{n_t^*-1}+a_t)), (\mu(\omega_t, \alpha), a_t), (p_t^{n_t^*}, \frac{1}{2}(a^{n_t^*} + a_t)), \ldots, (p_t^N, \frac{1}{2}(a^N + a_t)), (1, a^N)$. Notice that $\widetilde{\Psi}(i, \omega_t)$ is increasing in i for each ω_t.

Since $\widetilde{\Psi}(i, \omega)$ is not restricted off-path, any $\widetilde{\Psi}(i, \omega)$ increasing in i will serve the purpose. For instance, the construction used in the Universal Bias Principle (Prop. 0) given by $\widetilde{\Psi}(i, \omega) = i - \mu(\omega, \alpha) + \Psi(\omega)$, will work. This concludes the Sufficiency proof. ∎

Proof of Theorem 2.

Part 1. To prove Part 1, existence of a rationalizing α, we first suppose that Part 2 holds. We use the same full range argument for μ as in the Proof of Theorem 1.

Part 2. Since the necessary condition was proven in the paper, it remains "only" to show the sufficiency argument. Hence, fix any policy data and weighting function α that satisfy the implication in (12).

Construction of a Class of Payoff Function

Now consider a payoff function U of the form

$$U(i, \omega, a) = \begin{cases} -\frac{1}{2}\left(1 + \frac{i}{2}\right)\left(1 + \frac{\omega - \min \Omega}{2(\max \Omega - \min \Omega)}\right)\left(a - \widetilde{\Psi}(i, \omega)\right)^2 & \text{if } a \leq \widetilde{\Psi}(i, \omega), \\ -\frac{1}{2}\left(1 - \frac{i}{2}\right)\left(1 - \frac{\omega - \min \Omega}{2(\max \Omega - \min \Omega)}\right)\left(a - \widetilde{\Psi}(i, \omega)\right)^2 & \text{if } a \geq \widetilde{\Psi}(i, \omega), \end{cases} \tag{17}$$

where $\widetilde{\Psi}(i, \omega)$ is continuous and weakly increasing in (i, ω) for every $i \in [0, 1]$ and $\omega \in \Omega$. Notice that $U(i, \omega, a)$ as defined in (17) is continuous in i, and is single-peaked in a, as required in (A1). Graphically, for each fixed (i,ω), $U(i,\omega,a)$ in (17) defines an inverse U-shaped curve in a, which peaks at $a=\widetilde{\Psi}(i,\omega)$ with a maximum $U\left(i,\omega,\widetilde{\Psi}(i,\omega)\right)=0$. For fixed (i,ω), an increase in the value of $\widetilde{\Psi}(i,\omega)$ will lead to a rightward parallel shift of the curve. Alternatively, if we fix the value of the ideal point $\widetilde{\Psi}(i,\omega)$, an increase in i (resp. ω) will rotate the curve counterclockwise along the ideal point $\widetilde{\Psi}(i,\omega)$. These properties give a geometric intuition for the fact that $U(i,\omega,a)$ satisfies strict single-crossing in $(a;i)$ (i.e., (A2)) and in $(a;\omega)$ (i.e., (A3)).[14] Formally, we have

Lemma 2 *Every $U(i,\omega,a)$ defined in (17) satisfies strict single crossing in $(a;i)$ and in $(a;\omega)$.*

Proof of Lemma 2:
We first show that strict single crossing in $(a;i)$ holds, i.e., if $a_2>a_1$, $i_2>i_1$ and $U(i_1,\omega,a_2)-U(i_1,\omega,a_1)\geq 0$, then $U(i_2,\omega,a_2)-U(i_2,\omega,a_1)>0$. Notice that the weak monotonicity of $\widetilde{\Psi}(i,\omega)$ implie that $\widetilde{\Psi}(i_2,\omega)\geq\widetilde{\Psi}(i_1,\omega)$. In addition, $U(i_1,\omega,a_2)-U(i_1,\omega,a_1)\geq 0$ implies that $\widetilde{\Psi}(i_1,\omega)>a_1$, since otherwise the single-peakedness would imply that $U(i_1,\omega,a_2)-U(i_1,\omega,a_1)<0$. Hence, $\widetilde{\Psi}(i_2,\omega)\geq\widetilde{\Psi}(i_1,\omega)>a_1$. We prove the result for two cases. First, $\widetilde{\Psi}(i_2,\omega)\geq a_2>a_1$. Single-peakedness then implies that $U(i_2,\omega,a_2)-U(i_2,\omega,a_1)>0$.
Second, $a_2>\widetilde{\Psi}(i_2,\omega)>a_1$. It then implies that $a_2>\widetilde{\Psi}(i_1,\omega)>a_1$. From the definition of $U(i,\omega,a)$ in (17), we have

$$\begin{aligned}
U(i_2,\omega,a_2) &= -\frac{1}{2}\left(1-\frac{i_2}{2}\right)\left(1+\frac{\omega-\min\Omega}{2(\max\Omega-\min\Omega)}\right)\left(a_2-\widetilde{\Psi}(i_2,\omega)\right)^2,\\
U(i_2,\omega,a_1) &= -\frac{1}{2}\left(1+\frac{i_2}{2}\right)\left(1+\frac{\omega-\min\Omega}{2(\max\Omega-\min\Omega)}\right)\left(a_1-\widetilde{\Psi}(i_2,\omega)\right)^2,\\
U(i_1,\omega,a_2) &= -\frac{1}{2}\left(1-\frac{i_1}{2}\right)\left(1+\frac{\omega-\min\Omega}{2(\max\Omega-\min\Omega)}\right)\left(a_2-\widetilde{\Psi}(i_1,\omega)\right)^2,\\
U(i_1,\omega,a_1) &= -\frac{1}{2}\left(1+\frac{i_1}{2}\right)\left(1+\frac{\omega-\min\Omega}{2(\max\Omega-\min\Omega)}\right)\left(a_1-\widetilde{\Psi}(i_1,\omega)\right)^2.
\end{aligned}$$

Since $i_2>i_1$ and $a_2>\widetilde{\Psi}(i_2,\omega)\geq\widetilde{\Psi}(i_1,\omega)>a_1$, it follows that $\left(1-\frac{i_1}{2}\right)>\left(1-\frac{i_2}{2}\right)>0$ and $\left(a_2-\widetilde{\Psi}(i_1,\omega)\right)^2\geq\left(a_2-\widetilde{\Psi}(i_2,\omega)\right)^2>0$. As a result, we have $U(i_2,\omega,a_2)>U(i_1,\omega,a_2)$. Similarly, $\left(1+\frac{i_2}{2}\right)>\left(1+\frac{i_1}{2}\right)>0$ and $\left(a_1-\widetilde{\Psi}(i_2,\omega)\right)^2\geq\left(a_1-\widetilde{\Psi}(i_1,\omega)\right)^2>0$, which

[14] Notice that U as defined in (17) satisfies the *strict* single crossing property in $(a;i)$ even if the ideal point $\widetilde{\Psi}(i,\omega)$ is only weakly increasing in i. This is in contrast with the construction of the form (15), where a weakly increasing $\widetilde{\Psi}(i,\omega)$ would only imply a (weak) single-crossing property. Because the condition specified in (12) does not rule out a constant path of policy data $\{a_t\}$, the consistency with policy data typically requires a weakly increasing $\widetilde{\Psi}(i,\omega)$. This explains our choice of the form (17) instead of (15).

implies $U(i_2,\omega,a_1) < U(i_1,\omega,a_1)$. Combining both, we have

$$U(i_2,\omega,a_2) - U(i_2,\omega,a_1) > U(i_1,\omega,a_2) - U(i_1,\omega,a_1) \geq 0.$$

This completes the verification of strict single crossing in $(a;i)$. As for strict single crossing in $(a;\omega)$ the proof follows the same steps and so we omit the details. ∎

Construction of a Payoff Function Consistent with Policy Data

To complete the proof, it suffices to construct $\widetilde{\Psi}(i,\omega)$ such that $\widetilde{\Psi}(i,\omega)$ is continuous, weakly increasing in (i,ω), and satisfies $\widetilde{\Psi}(\mu(\omega_t,\alpha),\omega_t) = a_t$.

We first construct it on the finite observed path. The construction is then extended to the remaining states and types. On the finite path, it is convenient to define monotone indices on ω and on i, respectively. Without loss of generality, we suppose that each on-path state ω_t is distinct. Otherwise, we let T denote the number of distinct observations of state variables and ignore repeated observations since they do not add new information. By reordering if necessary, we can define an index t with $t = 1,2,...,T$ such that $\omega_t < \omega_{t+1}, \forall t < T$. The derived sequence of pivotal decision makers is defined as $\{i_t\}_{t=1}^{T}$ such that $i_t = \mu(\omega_t,\alpha)$. For the convenience of extending finite data to the whole range of states and types, we specify two fictional end-point observations as $(\omega_0, i_0, a_0) = (\min\Omega - 1, 0, \min A)$ and $(\omega_{T+1}, i_{T+1}, a_{T+1}) = (\max\Omega + 1, 1, \max A)$.[15]

Similarly, let N be the number of distinct elements in $\{i_t\}_{t=0}^{T+1}$ with $2 \leq N \leq (T+2)$. Define a second index n with $n = 1,2,...,N$ and the corresponding type sequence $\left\{\widetilde{i}_n\right\}_{n=1}^{N}$ with $\widetilde{i}_n \in \{i_t\}_{t=0}^{T+1}$ such that $\widetilde{i}_n < \widetilde{i}_{n+1}, \forall n < N$. In other words, n is a reordering of distinct elements in $\{i_t\}_{t=0}^{T+1}$ such that $\widetilde{i}_n$ is an increasing sequence. Notice that $\widetilde{i}_1 = 0$ and $\widetilde{i}_N = 1$.

We will construct $N \cdot (T+2)$ points of $\widetilde{\Psi}(i,\omega)$, all collectively denoted by $\{\widetilde{a}_{n,t}\}_{n=1,t=0}^{N,T+1}$, such that $\widetilde{a}_{n,t} = \widetilde{\Psi}\left(\widetilde{i}_n, \omega_t\right)$.

Notice first that equilibrium requires that $\widetilde{a}_{n,t} = a_t$ if $\widetilde{i}_n = i_t$. This leaves $(N-1)\cdot(T+2)$ points free for construction. To complete the finite construction, we specify an explicit algorithm to construct a weakly increasing sequence $\{\widetilde{a}_{n,t}\}_{n=1,t=0}^{N,T+1}$.

Algorithm 1 *A recursive algorithm to construct a weakly increasing* $\{\widetilde{a}_{n,t}\}_{n=1,t=0}^{N,T+1}$.

Step 0: Define an initial condition for $t = 0$ *as* $\widetilde{a}_{n,0} = a_0 = \min A$, $\forall 1 \leq n \leq N$.

Step 1: For observation t *with* $1 \leq t \leq T$, *find* $1 \leq n_t^* \leq N$ *such that* $\widetilde{i}_{n_t^*} = i_t$. *Let* $\widetilde{a}_{n_t^*,t} = a_t$.

[15] The specific values of ω_0 and ω_{T+1} are not essential as long as they satisfy $\omega_0 < \min\Omega$ and $\omega_{T+1} > \max\Omega$.

For $1 \leq n \leq N$ and $n \neq n_t^*$, define $\widetilde{a}_{n,t}$ as an average of two points

$$\widetilde{a}_{n,t} = \frac{1}{2}\left(\widetilde{a}_{n,t}^{\min} + \widetilde{a}_{n,t}^{\max}\right),$$

where $\widetilde{a}_{n,t}^{\min}$ and $\widetilde{a}_{n,t}^{\max}$ are defined from $\left\{\widetilde{a}_{n_t^*,t}, \{\widetilde{a}_{n,t-1}\}_{n=1}^{N}\right\}$ in a recursion starting from n_t^* as

$$\widetilde{a}_{n,t}^{\min} = \begin{cases} \max\left\{\widetilde{a}_{n-1,t}, \widetilde{a}_{n,t-1}\right\} & \text{if } n > n_t^*, \\ \widetilde{a}_{n,t-1} & \text{if } n < n_t^* \end{cases}$$

and

$$\widetilde{a}_{n,t}^{\max} = \left\{ \begin{array}{ll} \min\limits_{\left\{t': T+1 \geq t' > t, i_{t'} \geq \widetilde{i}_n\right\}} \{a_{t'}\} & \text{if } n > n_t^*, \\ \min\left\{\widetilde{a}_{n+1,t}, \min\limits_{\left\{t': T+1 \geq t' > t, i_{t'} \geq \widetilde{i}_n\right\}} \{a_{t'}\}\right\} & \text{if } n < n_t^*. \end{array} \right\}.$$

Step 2: If $t < T$, then repeat Step 1 for $t+1$; else go to Step 3.
Step 3: Let $\widetilde{a}_{n,T+1} = a_{T+1} = \max A$, $\forall 1 \leq n \leq N$ and stop.

For each $1 \leq t \leq T$, the Algorithm starts by producing the realized equilibrium policy outcome, $\widetilde{a}_{n_t^*,t} = a_t$. Starting from n_t^*, the Algorithm then proceeds to a two-way recursion towards both the left and right sides of n_t^*. It is easy to see that the Algorithm produces a non-empty sequence of real numbers. In addition, $\widetilde{a}_{n,t} \in A$, $\forall n, t$, since every operation involved, including min, max and mean, is a closed operation. Notice that $\widetilde{a}_{n,0} = \min A$ and $\widetilde{a}_{n,T+1} = \max A$. As a result, we only need to check that $\{\widetilde{a}_{n,t}\}_{n=1,t=1}^{N,T}$ is a weakly increasing sequence in (n, t).

Verification of Algorithm 1. Start from $t = 1$ and we prove the weak monotonicity of $\widetilde{a}_{n,t}$ in n. We do this in two steps.

Step 1: $\widetilde{a}_{n,t}^{\max} \geq \widetilde{a}_{n,t} \geq \widetilde{a}_{n,t}^{\min}$ for every $n \neq n_t^*$. Because $\widetilde{a}_{n,t} = \frac{1}{2}\left(\widetilde{a}_{n,t}^{\min} + \widetilde{a}_{n,t}^{\max}\right)$, it suffices to show that $\widetilde{a}_{n,t}^{\max} \geq \widetilde{a}_{n,t}^{\min}$. We prove the fact for several cases of n. First, $\widetilde{a}_{n,t}^{\max} \geq \widetilde{a}_{n,t}^{\min}$ for $1 \leq n < n_t^*$. From Step 0 of the Algorithm, it follows that $\widetilde{a}_{n,t}^{\max} \geq \min A = \widetilde{a}_{n,t-1} = \widetilde{a}_{n,t}^{\min}$ for every $1 \leq n < n_t^*$. Second, $\widetilde{a}_{n,t}^{\max} \geq \widetilde{a}_{n,t}^{\min}$ for $n = n_t^*+1$. Recall that $a_{t'} \geq a_t$ whenever $t' > t$ and $i_{t'} \geq i_t$. Hence, by taking the minimum we have $\widetilde{a}_{n,t}^{\max} \geq a_t$. In addition, $\widetilde{a}_{n,t}^{\max} \geq \widetilde{a}_{n,t-1} = \min A$. For $n = n_t^* + 1$, it follows that $\widetilde{a}_{n,t}^{\max} \geq \max\{a_t, \widetilde{a}_{n,t-1}\} = \max\{\widetilde{a}_{n-1,t}, \widetilde{a}_{n,t-1}\} = \widetilde{a}_{n,t}^{\min}$. Third, $\widetilde{a}_{n,t}^{\max} \geq \widetilde{a}_{n,t}^{\min}$ for $n_t^* + 1 < n \leq N$. For $n = n_t^* + 2$, notice that $\widetilde{a}_{n,t}^{\max} \geq \widetilde{a}_{n-1,t}^{\max} \geq \widetilde{a}_{n-1,t}$, where the first inequality follows because $\widetilde{i}_{n+1} > \widetilde{i}_n$ so that the set for min operation in the former is a subset of the latter, and the second inequality from the last result $\widetilde{a}_{n-1,t}^{\max} \geq \widetilde{a}_{n-1,t}^{\min}$ so that

$\widetilde{a}^{\max}_{n-1,t} \geq \widetilde{a}_{n-1,t}$ for $n-1 = n^*_t + 1$. Using this and the fact that $\widetilde{a}^{\max}_{n,t} \geq \widetilde{a}_{n,t-1}$, the same argument as in the previous step can establish that $\widetilde{a}^{\max}_{n,t} \geq \widetilde{a}^{\min}_{n,t}$ for $n = n^*_t + 2$. By induction, the same inequality holds for every $n^*_t + 1 < n \leq N$.

Step 2: $\widetilde{a}_{n,t}$ is weakly increasing in n for $t = 1$. From the construction, $\widetilde{a}^{\min}_{n,t} \geq \widetilde{a}_{n-1,t}$ for $n > n^*_t$ and $\widetilde{a}^{\max}_{n,t} \leq \widetilde{a}_{n+1,t}$ for $n < n^*_t$. Since $\widetilde{a}^{\min}_{n,t} \leq \widetilde{a}_{n,t} \leq \widetilde{a}^{\max}_{n,t}$ as shown in Step 1, we have $\widetilde{a}_{n-1,t} \leq \widetilde{a}_{n,t} \leq \widetilde{a}_{n+1,t}$.

For $1 < t \leq T$, the weak monotonicity of $\widetilde{a}_{n,t}$ in n is shown from an induction argument. Specifically, for each $t > 1$, we assume that $\widetilde{a}^{\max}_{n,t-1} \geq \widetilde{a}_{n,t-1} \geq \widetilde{a}^{\min}_{n,t-1}$ for every $n \neq n^*_{t-1}$, and $\widetilde{a}_{n,t-1}$ is weakly increasing in n, as derived for $t = 1$. Then we revisit the proof of Step 1 and Step 2 as in $t = 1$. It is easy to see that Step 2 is intact, provided that Step 1 holds. For Step 1, a close reading of the proof for $t = 1$ reveals that we only need to reestablish that $\widetilde{a}^{\max}_{n,t} \geq \widetilde{a}_{n,t-1}$, which follows from a series of claims.[16]

Claim 1: $\min\limits_{\{t':t'>t-1, i_{t'} \geq \widetilde{i}_n\}} \{a_{t'}\} \geq \widetilde{a}_{n,t-1}$ for every $1 \leq n \leq N$ and $1 < t \leq T$. For $n \neq n^*_{t-1}$, $\min\limits_{\{t':t'>t-1, i_{t'} \geq \widetilde{i}_n\}} \{a_{t'}\} \geq \widetilde{a}^{\max}_{n,t-1} \geq \widetilde{a}_{n,t-1}$, where the first inequality holds by construction, and the second inequality is true from the assumption of induction. For $n = n^*_{t-1}$, recall that $a_{t'} \geq a_{t-1}$ whenever $t' > t-1$ and $i_{t'} \geq i_{t-1} = \widetilde{i}_n$. Take the minimum to get $\min\limits_{\{t':t'>t-1, i_{t'} \geq \widetilde{i}_n\}} \{a_{t'}\} \geq a_{t-1} = \widetilde{a}_{n^*_{t-1},t-1} = \widetilde{a}_{n,t-1}$.

Claim 2: $\min\limits_{\{t':t'>t, i_{t'} \geq \widetilde{i}_n\}} \{a_{t'}\} \geq \widetilde{a}_{n,t-1}$ for every $1 \leq n \leq N$ and $1 < t \leq T$. Notice that $\min\limits_{\{t':t'>t, i_{t'} \geq \widetilde{i}_n\}} \{a_{t'}\} \geq \min\limits_{\{t':t'>t-1, i_{t'} \geq \widetilde{i}_n\}} \{a_{t'}\}$, since the set for min operation in the former is a subset of the latter. The result then follows from the Claim 1.

Claim 3: $\widetilde{a}_{n^*_t,t} \geq \widetilde{a}_{n^*_t,t-1}$ for every $t > 1$. Notice that $a_t \geq \min\limits_{\{t':t'>t-1, i_{t'} \geq \widetilde{i}_n\}} \{a_{t'}\}$ for $n = n^*_t$, since $a_{t'}$ with $t' = t$ and $i_t = \widetilde{i}_{n^*_t}$ is one member of the constraint set. From the Claim 1, we have $\widetilde{a}_{n^*_t,t} = a_t \geq \widetilde{a}_{n^*_t,t-1}$.

Claim 4: $\widetilde{a}^{\max}_{n,t} \geq \widetilde{a}_{n,t-1}$ for $1 \leq n < n^*_t$. From the definition of $\widetilde{a}^{\max}_{n,t}$ for $1 \leq n < n^*_t$ and Claim 2, we only need to prove that $\widetilde{a}_{n+1,t} \geq \widetilde{a}_{n,t-1}$. Furthermore, it suffices to show that $\widetilde{a}_{n+1,t} \geq \widetilde{a}_{n+1,t-1}$, because $\widetilde{a}_{n+1,t-1} \geq \widetilde{a}_{n,t-1}$ from the weak monotonicity assumption of induction for $t-1$. For $n = n^*_t - 1$, $\widetilde{a}_{n+1,t} = \widetilde{a}_{n^*_t,t} \geq \widetilde{a}_{n+1,t-1}$ from Claim 3. In addition, by repeating the Step 1 as in $t = 1$, we have $\widetilde{a}_{n,t} \geq \widetilde{a}^{\min}_{n,t} = \widetilde{a}_{n,t-1}$ for $n = n^*_t - 1$. But this implies $\widetilde{a}^{\max}_{n,t} \geq \widetilde{a}_{n,t-1}$ for $n = n^*_t - 2$. By induction, the result holds for any $n < n^*_t$.

Claim 5: $\widetilde{a}^{\max}_{n,t} \geq \widetilde{a}_{n,t-1}$ for $n^*_t < n \leq N$. This follows immediately from Claim 2.

[16]Recall that $\widetilde{a}^{\max}_{n,t} \geq \widetilde{a}_{n,t-1}$ holds trivially for $t = 1$ from Step 0 of the Algorithm, which cannot be taken as given any more for $1 < t \leq T$.

To summarize, we just proved that the Algorithm produces a weakly increasing sequence in n for each $0 \leq t \leq T+1$. It remains to show that $\widetilde{a}_{n,t}$ is weakly increasing in t for every $1 \leq n \leq N$. From the construction of $\widetilde{a}_{n,t}^{\min}$, for any t and any $n \neq n_t^*$, $\widetilde{a}_{n,t} \geq \widetilde{a}_{n,t}^{\min} \geq \widetilde{a}_{n,t-1}$. For $n = n_t^*$, from the Claim 3, $\widetilde{a}_{n_t^*,t} \geq \widetilde{a}_{n_t^*,t-1}$. Consequently, $\widetilde{a}_{n,t} \geq \widetilde{a}_{n,t-1}$, $\forall t, n$. This finishes the verification of the Algorithm. ∎

Having constructed the points $\{\widetilde{a}_{n,t}\}_{n=1,t=0}^{N,T+1}$ and corresponding regular grid points $(\{\widetilde{i}_n\}_{n=1}^{N}, \{\omega_t\}_{t=0}^{T+1})$ from the algorithm, all that remains is to extend the construction to the full function $\widetilde{\Psi}(i,\omega)$. For this purpose, a standard bilinear interpolating spline can be used (for an introduction to splines, see Judd (1998)). Specifically, for each $i \in [\widetilde{i}_n, \widetilde{i}_{n+1}]$ and $\omega \in [\omega_t, \omega_{t+1}]$, a unique bilinear piece can be constructed as

$$\widetilde{\Psi}(i,\omega) = b_{n,t}^0 + b_{n,t}^1\, i + b_{n,t}^2\, \omega + b_{n,t}^3\, i\omega, \tag{18}$$

such that $\widetilde{\Psi}(\widetilde{i}_n, \omega_t) = \widetilde{a}_{n,t}$, $\widetilde{\Psi}(\widetilde{i}_n, \omega_{t+1}) = \widetilde{a}_{n,t+1}$, $\widetilde{\Psi}(\widetilde{i}_{n+1}, \omega_t) = \widetilde{a}_{n+1,t}$, and $\widetilde{\Psi}(\widetilde{i}_{n+1}, \omega_{t+1}) = \widetilde{a}_{n+1,t+1}$.

By construction, $\widetilde{\Psi}(i,\omega)$ is continuous in (i,ω). In addition, a bilinear spline preserves the monotonicity property in each dimension: if $\{\widetilde{a}_{n,t}\}_{n=1,t=0}^{N,T+1}$ is a weakly increasing sequence in n (resp. t), then the constructed $\widetilde{\Psi}(i,\omega)$ is weakly increasing in i for each fixed $\omega \in \Omega$ (resp. in ω for each fixed $i \in [0,1]$). Because of the symmetry in (i,ω), it suffices to show the property for i, or $\frac{\partial \widetilde{\Psi}(i,\omega)}{\partial i} = b_{n,t}^1 + b_{n,t}^3 \omega \geq 0$. Notice that $\widetilde{\Psi}(i,\omega)$ is linear in i for each fixed ω, in particular for each ω_t and ω_{t+1}. Hence, $\widetilde{a}_{n+1,t} \geq \widetilde{a}_{n,t}$ and $\widetilde{a}_{n+1,t+1} \geq \widetilde{a}_{n,t+1}$ imply that $\frac{\partial \widetilde{\Psi}(i,\omega_t)}{\partial i} = b_{n,t}^1 + b_{n,t}^3 \omega_t \geq 0$ and $\frac{\partial \widetilde{\Psi}(i,\omega_{t+1})}{\partial i} = b_{n,t}^1 + b_{n,t}^3 \omega_{t+1} \geq 0$. It immediately follows that $\frac{\partial \widetilde{\Psi}(i,\omega)}{\partial i} = b_{n,t}^1 + b_{n,t}^3 \omega \geq 0$ for every $\omega \in [\omega_t, \omega_{t+1}]$.

With the extension to the full function $\widetilde{\Psi}$, the proof of Theorem 2 is complete. ∎

Proof of Theorem 3.

Part 1. It follows from the same full range argument as given in its counterpart in Theorem 2.

Sufficiency in Part 2. Fix any policy data and weighting function α that satisfy the implication in (13). Consider any u of the form

$$u(y,a) = \begin{cases} -\frac{1}{2}\left(1 + \frac{y - y_{\min}}{2(y_{\max} - y_{\min})}\right)(a - g(y))^2 & \text{if } a \leq g(y), \\ -\frac{1}{2}\left(1 - \frac{y - y_{\min}}{2(y_{\max} - y_{\min})}\right)(a - g(y))^2 & \text{if } a \geq g(y), \end{cases}$$

where $y_{\min} = \min\limits_{i \in [0,1], \omega \in \Omega} y(i,\omega)$, $y_{\max} = \max\limits_{i \in [0,1], \omega \in \Omega} y(i,\omega)$, and $g(y)$ is continuous and weakly increasing. It is clear that $u(y,a)$ is continuous in y and single-peaked in a. Following the same argument as in the counterpart of the proof of Theorem 2, $u(y,a)$ satisfies strict single-crossing in $(a;y)$. As a result, any $U(i,\omega,a)$ defined by $U(i,\omega,a) = u(y(i,\omega),a)$ satisfies (A1), (A2) and (A4).

For a given α, define the income of the pivotal decision maker in t as $y_t = y(\mu(\omega_t,\alpha),\omega_t)$. To prove the result, it suffices to show that there exists a $g(y)$ such that $a_t = g(y_t)$. Because $a_t < a_\tau \implies y_t < y_\tau$, we have $y_t = y_\tau \implies a_t = a_\tau$. Hence, by redefining and reordering t if necessary, without loss of generality we can assume that $\{y_t\}_{t=1}^T$ is an increasing sequence in t, and $\{a_t\}_{t=1}^T$ is weakly increasing. For any $1 \le t \le T-1$ and $y \in [y_t, y_{t+1}]$, define

$$g(y) = a_t + \frac{a_{t+1} - a_t}{y_{t+1} - y_t}(y - y_t),$$

and

$$\begin{aligned} g(y) &= \min A + \frac{a_1 - \min A}{y_1} y, \forall y \le y_1, \\ g(y) &= a_T + \frac{\max A - a_T}{\max\limits_{\omega \in \Omega}\{y(1,\omega)\} - y_T}(y - y_T), \forall y \ge y_T. \end{aligned}$$

It is clear that $g(y)$ is weakly increasing and $a_t = g(y_t)$. This finishes the proof. ∎

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