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Along the New Keynesian Phillips Curve with Nominal and Real Rigidities

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Abstract: The new Keynesian Phillips curve (NKPC) has become central to monetary theory and policy. A seemingly benign NKPC prediction is that trend shocks dominate price level fluctuations at all forecast horizons. Since the NKPC cycle of the U.S. GDP deflator peaks at each of the last seven NBER dated recessions, support for the NKPC is limited. The authors develop monetary business cycle models that contain different combinations of nominal (sticky-price) and real (labor market search) rigidities to understand this puzzle. Simulations indicate that a model combining labor market search and flexible prices is better able to match actual price level movements than sticky-price models do. This model represents a challenge to claims that sticky prices are a key part of the monetary transmission mechanism.

JEL classification: E3, E5

Key words: new Keynesian Phillips curve, sticky prices, labor market search, common trend, common cycle

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1. Introduction

Of all the comebacks of the 1990s, it seems a revival in Phillips curve research was the least anticipated. Unlike earlier Phillips curve research that focused on aggregate demand shocks, recent work aims to identify inflationary expectations. The way inflationary expectations are formed matters for business cycle theory and monetary policy. For example, a Phillips curve dependent more on forward- than backward-looking expectations allows policymakers to disinflate with few costs. This favorable trade-off appears at odds with empirical evidence and the views of policymakers.

Yun (1996) constructs a rational expectations-monetary business cycle model consistent with a revivalist Phillips curve. He assumes monopolistically competitive firms maximize their expected discounted profit stream subject to a sticky price constraint that reflects a nominal rigidity. The solution to the firms' problem can be cast as the new Keynesian Phillips curve (NKPC) in which price expectations are forward-looking and real marginal cost is the fundamental.

The forward-looking NKPC implies a present-value (PV) relation for the price level. The NKPC-PV relation predicts that trend shocks dominate price level movements, in the same way the permanent income hypothesis restricts consumption. If the price level has an economically important cycle, it rejects the NKPC null that only trend shocks matter.

This paper uses NKPC-PV predictions to ask if a sticky price-nominal rigidity is needed by a dynamic stochastic general equilibrium (DSGE) model to generate a NKPC that mimics its empirical counterpart. A Beveridge and Nelson (1981), Stock and Watson (1988), and Vahid and Engle (1993) common trend-common cycle price level decomposition links the empirical and theoretical NKPCs. Thus, we study a key feature of a Phillips curve: its predictions for price level dynamics.

The Beveridge, Nelson, Stock, Watson-Vahid and Engle (BNSW-VE) decomposition of the NKPC provides us with three "moments". The moments are (i) the fraction of price constrained firms, (ii) the NKPC common trend-common cycle decomposition, and (iii) the associated forecast error variance decomposition (FEVD). We use these moments to test the implications of the NKPC-PV restrictions for DSGE models.[1]

Sample NKPC moments are based on U.S. GDP deflator and nominal unit labor cost data that runs from $1960Q1$ to $2001Q4$. Our estimate of NKPC moment (i) has about half of final goods firms being price constrained, which is similar to Sbordone (2002), but smaller than those Gali and Gertler (1999) report. The cycle of NKPC sample moment (ii) is economically important because it peaks at each of the last seven NBER dated recessions. NKPC sample moment (iii) shows trend shocks explain 60 percent of price level variation at a forecast horizon of two years. Thus, NKPC sample moments $(ii) - (iii)$ reject the NKPC-PV predictions.

We solve and simulate a version of the Yun (1996) DSGE model to understand the sources and causes of NKPC sample moments $(i)-(iii)$. Given Yun's results, it is no surprise the synthetic NKPC of his Calvo (1983) staggered price-DSGE model is dominated by trend shocks. Although this matches the NKPC-PV prediction, excess smoothness in the price level of the Yun-sticky price model places this model at odds with NKPC sample moments $(i) - (iii)$.

Chari, Kehoe, and McGratten (2000) also cast doubt on sticky prices being a source of aggregate fluctuations. Ball and Romer (1990), Jeanne (1998), Gali and Gertler (1999), Dotsey and King (2001), Walsh (2002), Trigari (2003b), and Ireland (2003) argue real rigidities solve this

[1] Greenwood and Huffman (1986), Chéron and Langot (1999), Cooley and Quadrini (1999), Ellison and Scott (2000), Walsh (2002), and Krause and Lubik (2003) use DSGE models to study unconditional Phillips curve observations.

problem. For example, Ireland shows that a persistent, exogenous real demand shock is needed for his sticky price model to match U.S. business cycle fluctuations.[2]

Solow (1976) points out a traditional Phillips curve invokes the real rigidity of labor market search, rather than sticky prices, to identify unemployment with the state of aggregate demand. This idea motivate us to combine the Yun-sticky price model with Mortensen and Pissarides (1994) labor market search in the way Andolfatto (1996), Merz (1995), and den Haan, Ramey, and Watson (2000) add it to real business cycle (RBC) models. Although Walsh (2002), Trigari (2003b), and Krause and Lubik (2003) obtain economically interesting results with similar models, our labor market search-sticky price model behaves about as well as the Yun-sticky price model in mimicking NKPC sample moments $(i) - (iii)$. Besides difference in preferences and technologies, Walsh, Trigari, and Krause and Lubik assume a representative household supported by complete insurance, which we construct from the underlying primitives of our labor market search-monetary economy.

We include a labor market search-flexible price model because the sticky price models we study yield poor results. The labor market search-flexible price model produces synthetic NKPC moments $(i) - (iii)$ that match their sample counterparts. Thus, our results suggest that the real rigidity of labor market imperfections is likely responsible for observed price level fluctuations.

The next section presents the NKPC-PV relation, its BNSW-VE decomposition, and reports empirical results. Section 3 reviews the Yun-sticky price model. Model calibration, labor market search, and Monte Carlo results are discussed in section 4. Section 5 concludes.

[2]Other monetary models fit different aspects of U.S. price dynamics. Ireland (1999) and Ruge-Murcia (2003) estimate game-theoretic monetary policy models that capture long-run inflation. Nason and Cogley (1994) show a flexible price-DSGE monetary model replicates short-run price dynamics under a long-run monetary neutrality identification.

2. The New Keynesian Phillips Curve

NKPC estimates are controversially. Sbordone (2002), Gali and Gertler (1999), and Rabanal and Rubio-Ramírez (2003) report empirical success with the NKPC. Fuhrer and Moore (1995), Fuhrer (1997), Roberts (1995, 1997, and 2001), and Rudd and Whelan (2001), among others, test backward-looking Phillips curves against the forward-looking NKPC and reject it. We come at this debate differently because we identify the trend and cyclical components of the NKPC.

2.1 A NKPC Specification

Roberts (1995) shows that several sticky-price models yield the NKPC. Typical is the Calvo (1983) staggered price setting mechanism. Sticky prices arise because only a fraction, $1-\mu$, of monopolistically competitive final goods firms are able to set and commit to a new price, $P_{C,t}$, between dates $t-1$ and t. Aggregate price, P_t, dynamics are restricted by

$$(1) \qquad P_t = [(1-\mu)P_{C,t}^{1-\xi} + \mu\left(\frac{m^*}{\gamma^*}P_{t-1}\right)^{1-\xi}]^{1/(1-\xi)}, \quad 1 < \xi,$$

where ξ, m^*, and γ^* are the demand elasticity, steady state money growth, and non-stochastic growth rate of labor augmenting technology change, respectively. Assume the aggregator of final demand (in physical units) firms face is $Y_{D,t} = \left[\int_0^1 y_{D,j,t}^{(\xi-1)/\xi}dj\right]^{\xi/(\xi-1)}$, where $y_{D,j,t}$ represents the demand firm j faces. This implies the demand schedule of the jth firm is

$$(2) \qquad y_{D,j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\xi} Y_{D,t}$$

where firm j charges $P_{j,t}$ for its output and all firms take $Y_{D,t}$ and P_t as given. Subsequent to cost minimization, profit maximization leads to the forward-looking optimal commitment price

$$(3)\quad P_{C,t} \;=\; \left(\frac{\xi}{\xi - 1}\right)\left[\frac{\mathbf{E}_t\left\{\sum_{i=0}^{\infty}\left(\beta\mu\left[\frac{m^*}{\gamma^*}\right]^{-\xi}\right)^i \Gamma_{t+i}\phi_{t+i}Y_{D,t+i}P_{t+i}^{\xi}\right\}}{\mathbf{E}_t\left\{\sum_{i=0}^{\infty}\left(\beta\mu\left[\frac{m^*}{\gamma^*}\right]^{1-\xi}\right)^i \Gamma_{t+i}Y_{D,t+i}P_{t+i}^{\xi-1}\right\}}\right],\quad 0 \;<\; \beta \;<\; 1,$$

where $\mathbf{E}_t\{\cdot\}$, $\beta^i\Gamma_{t+i}$, and ϕ_t represent the mathematical expectations operator conditional on date t information, the date $t+i$ (stochastic) discount rate all firms face, and real marginal cost.

Sticky price dynamics force monopolistically competitive firms to be forward-looking when price setting. This gives the NKPC its forward-looking character, which is developed by linearizing the price aggregator (1) and the optimal price rule (3), subsequent to detrending, to construct

$$(4)\quad ln[P_t] \;=\; \mu ln[P_{t-1}] \;+\; (1-\mu)\left(1 - \frac{\mu}{\mathcal{B}}\right)\sum_{j=0}^{\infty}\left(\frac{\mu}{\mathcal{B}}\right)^j \mathbf{E}_t ln[\Phi_{t+j}],\quad \mathcal{B} \;\equiv\; \frac{m^*}{\beta\gamma^*},$$

where constants are ignored. The equilibrium law of motion (4) shows the price level is driven by the trend of the "annuity value" of the expected future path of *nominal* marginal cost.

The price dynamics of (4) shows that the price level and nominal marginal cost share a common trend or cointegration relation, $ln[\phi_t] = \varpi_{CI}(ln[P_t]\;\; ln[\Phi_t])'$, where $\varpi_{CI} = [1\;-1]$, if nominal marginal cost is $I(1)$. Subtract $ln[\Phi_t]$ from both sides of equation (4), apply the usual PV algebra, and multiply the result through by minus one to produce

$$ln[\phi_t] \;=\; \left(\frac{\mu}{1-\mu}\right)\Delta ln[P_t] \;-\; \sum_{j=1}^{\infty}\left(\frac{\mu}{\mathcal{B}}\right)^j \mathbf{E}_t\Delta ln[\Phi_{t+j}].$$

Real marginal cost is endogenous and forward-looking because it equals (a multiple of) inflation net of the expected PV of nominal marginal cost growth. Note also that real marginal cost is stationary and acts as the cointegration relation or error correction mechanism in a vector error correction model (VECM) of inflation and nominal marginal cost growth. Given Engle and Issler (1995) refer to cointegration relations as cycle generators, real marginal cost approximates the NKPC cycle.

Another NKPC prediction is that inflation and nominal marginal cost growth share a serially correlated common feature, in the sense of Engle and Kozicki (1993). The NKPC common feature exists if inflation and nominal marginal cost growth form a linear combination whose residual is unpredictable. The NKPC-PV relation (4) produces the linear combination

$$(5) \quad \Delta ln[P_t] - \mu_{\mathcal{B}} \Delta ln[\Phi_t] \quad = \quad \mu^{-1}\mathbf{E}_t \Delta ln[P_{t+1}] - \varsigma_{\Phi,t}, \quad \mu_{\mathcal{B}} \equiv \frac{(1-\mu)(\mathcal{B}-\mu)}{\mu^2},$$

where $\varsigma_{\Phi,t} = \mu_{\mathcal{B}} \{\mathbf{E}_{t-1} \Delta ln[\Phi_t] + (\mathbf{E}_t - \mathbf{E}_{t-1}) \, ln[\Phi_t] + \sum_{j=1}^{\infty} (\mu/\mathcal{B})^j \, (\mathbf{E}_t \, \Delta ln[\Phi_{t+j}] - (\mu/\mathcal{B}) (\mathbf{E}_t - \mathbf{E}_{t-1}) ln[\Phi_{t+j-1}])\}$. The regression (5) yields the NKPC common feature relation if $\varsigma_{\Phi,t}$ annihilates serial correlation in expected inflation to generate (unpredictable) innovations.

2.2 *NKPC Common Trend Prediction: Estimates and Tests*

The proxy for nominal marginal cost is *nominal* unit labor costs, ULC_t, which is measured as the ratio of hourly compensation to output per hour.[3] The price level, P_t, is the GDP deflator. The sample period is $1960Q1$–$2001Q4$, $T = 168$, with lags available beginning with $1955Q1$.

We test for a common trend in P_t and ULC_t using Johansen (1988, 1991) likelihood ratio (LR) tests based on a third-order VECM, case 1* model of Osterwald-Lenum (1992).[4] The LR-max and LR-trace statistics are [8.23, 12.98] and [8.23, 21.20], respectively. The former test rejects a

[3] Sbordone (2002) and Gali and Gertler (1999) show marginal cost equals ULC using the labor demand elasticity of a Cobb-Douglas technology of a monopolistic competitive firm. The Federal Reserve Bank of St.Louis' FRED databank labels the index of hourly compensation (output per hour), non-farm business sector `compnfb` (`ophnfb`).

[4] A LR test for the lag length – beginning with 12 lags – of the VAR of the logs of the price level and ULC cannot reject a four lag model. The AIC gives the same result.

common trend in the price level and ULC and the latter does not based on MacKinnon, Haug, and Michelis (1999) five percent critical values of [9.17, 15.88] for the LR-max test and [9.17, 20.25] for the LR-trace test. The maximum likelihood estimate (MLE) of ϖ_{CT-MLE} is $[1 \ -1.08 \ \ 6.82]$.[5]

We report two additional cointegration tests because Johansen's are inconclusive and can be unreliable. The Engle and Granger (1987) cointegration test yields a $t-$ratio of -3.34, which rejects its null at the five percent level, according to MacKinnon (1991). Boswijk (1994) constructs a Wald test from a simultaneous equations VECM, based on a two-stage least squares (2SLS) estimator. The Wald statistic of 11.82 falls between the ten and five percent critical values Boswijk tabulates (his table B.3). These tests lend support to a model in which $ln[P_t]$ and $ln[ULC_t]$ cointegrate.

2.3 NKPC Common Feature Prediction: Estimates and Tests

Two tests for a serially correlated common feature employ the canonical correlations, λ, of inflation and ULC growth, conditional on the VECM(3) information set. Inflation and ULC growth share a serially correlated common feature if the smallest $\lambda = 0$. Vahid and Engle (1993) develop a χ^2-common feature test, but a $F-$test exists due to Rao (1973) that has superior small sample properties, according to Engle and Issler (1995). The λ^2s equal 0.0513 and 0.8243, with associated p-values of 0.19 (0.21) and 0.00 (0.00) for the χ^2 ($F-$)test.

Vahid and Engle show a 2SLS regression provides a common feature test and recovers $\varpi_{CF} = [1 \quad -\mu_{\mathcal{B}}]$.[6] The 2SLS estimate of $\mu_{\mathcal{B}}$ equals 0.8203, with a standard error of 0.0800,

[5]The Johansen (1991) test of the theoretical ϖ_{CI} against ϖ_{CT-MLE} is not rejected at standard significance levels.

[6]Gali and Gertler (1999) use generalized method of moments to estimate the NKPC, which imputes inflationary expectations to the instrument vector. Sbordone (2002) minimizes the distance between price level dynamics restricted by a NKPC and the actual price level. This is akin to the instrumental variables estimator of West (1989).

given the VECM(3) regressors are instruments. The LM test of instrument validity cannot reject the null, given a p-value of 0.16 for the statistic 9.32.

We have to calibrate $\mathcal{B}$ to calculate μ_{2SLS}, NKPC sample moment (i), from $\mu_{\mathcal{B},2SLS}$. We set $\beta = 1.03^{-0.25}$, $\gamma = 0.0047$, and $m^* = \exp\{0.0167\}$, where m^* and γ are taken from U.S. data; see section 4.1 for details. The calibration implies the NKPC sample moment (i), μ_{2SLS}, is 0.5292 with a standard error of 0.0081.[7] Thus, firms change prices twice a year, on average.

Another test compares a VECM(3) restricted by the common feature against an unrestricted VECM(3). This LR test has ten degrees of freedom and a p-value of 0.55. Along with tests of the squared canonical correlations and 2SLS instrument validity, the LR test provides evidence that favors the NKPC serially correlated common feature of (5).

2.4 *The BNSW-VE Decomposition of the NKPC*

The BNSW-VE decomposition relies on levels data and ϖ_{CI} and ϖ_{CF}, according to Vahid and Engle (1993). Partition the columns of the inverse of the stack of these vectors into the matrix

$$[\pi_{\cdot,1} \quad \pi_{\cdot,2}] \quad = \quad \begin{bmatrix} \varpi_{CI} \\ \\ \varpi_{CF} \end{bmatrix}^{-1},$$

to recover the NKPC cycle from $\pi_{\cdot,2} \times \varpi_{CI}(ln[P_t] \;\; ln[ULC_t])'$. The trend follows.

Figure 1 contain the NKPC trend and cycle, or NKPC sample moment (ii). The top window of figure 1 shows that the NKPC trend supports traditional views of recent U.S. aggregate price history. The tight labor markets of the mid-1960s coincide with an increase in the NKPC trend.[8]

[7]Vahid and Engle describe a MLE that stacks the common feature regression on top of the ECM(3) of ULC growth. The MLE of $\mu_{\mathcal{B}}$ equals 0.8927, with a standard error of 0.1080. The gives $\mu = 0.5192$ and a standard error of 0.0153.

[8]The price level is less volatile than the NKPC trend because the covariance equals -0.13.

The trend falls with the recession that begins in late 1969. The 1970s sees a rising trend at the time of the first oil price shock. The contractionary monetary policy initiated late in 1979 pushes the NKPC trend below the price level from 1980 until the economic expansion of the mid-1990s. The NKPC trend dips below the price level just before the NBER peak dated 2001 $Q1$.

The NKPC cycle and NBER dated business cycle peaks (vertical dash lines) and troughs (vertical dot-dash lines) appear in the bottom window of figure 1. It shows NKPC cycle peaks at the last seven NBER dated recessions. Since the cycle is a negative (up to a scalar) of real unit labor cost, when it rises it signals recovery from recession. Thus, the NKPC cycle is economically important, consistent with prior views of the Phillips curve, but at odds with the NKPC-PV predictions.

The NKPC sample moment (iii) employs the BNSW-VE decomposition to gauge the contribution of the identified trend shock to movements in the GDP deflator.[9] The FEVDs with respect to the trend are 2.70, 8.80, 26.38, 60.12, 78.37, 86.55, 91.44, and 98.05 percent at 1, 2, 4, 8, 12, 16, 20, and 40 quarter forecast horizons, respectively.[10] Note that trend innovations are responsible for about a quarter of price level fluctuations at a one-year forecast horizon and 60 percent at the end of two years. It takes five years to reach 90 percent.[11] This is evidence against the NKPC because cyclical shocks matter for the price level, at least, through a two-year forecast horizon.

[9]It is not possible to identify the trend (cyclical) shock as a supply (demand) shock.

[10]Engle and Issler (1995) and Issler and Vahid (2001) outline methods to calculate FEVDs, under the BNSW-VE decomposition. Trend innovation are a function of the common trend growth rate, lagged appropriately. Innovations to the cyclical component are the residuals of the cyclical component regressed on the information set of the VECM, lagged j times. Trend and cyclical innovations are orthogonalized by 'regressing' the latter on the former, which asserts trend innovations are prior to cyclical innovations; see footnote 11 and appendix C of Issler and Vahid for details.

[11]The trend shock takes longer to dominate ULC fluctuations, because its FEVDs are 0.28, 2.01, 10.79, 41.34, 65.00, 77.52, 85.41, and 96.65 percent. However, the NKPC places no restrictions on these FEVDs

To summarize, we study three NKPC moments: (i) the 2SLS estimate of the sticky price parameter, μ_{2SLS}, (ii) the NKPC common trend-common cycle decomposition, and (iii) its FEVD. Our evidence lends only weak support to the NKPC because cyclical shocks matter for price level moments. This raises the question of the role of nominal rigidities for price level fluctuations. The next two sections study this question using DSGE monetary models.

3. A Sticky Price DSGE Model

This section reviews the sticky-price DSGE model of Yun (1996). This model combines cash and credit goods, a cash-in-advance (CIA) constraint, and a Calvo-staggered price mechanism into a one-sector growth model.[12] Section 4 integrates a real rigidity into Yun's DSGE model with the labor market-search structure that Merz (1995), Andolfatto (1996), and den Haan, Ramey, and Watson (2000) use in a RBC setting. We also study a flexible price version of this DSGE model.

3.1 The Final Goods Sector

Monopolistically competitive final goods firms take addresses on the unit interval. Producing a differentiated good employs a constant returns to scale (CRS) technology, $F(k-\overline{K}, hZ) \equiv (k-\overline{K})^{\theta}(hZ)^{1-\theta}$, $\theta \in (0, 1)$, where k is capital, $\overline{K}$ is an exogenous minimum capital threshold (e.g., infra-structure) common to all final goods firms, and hZ is productivity augmented hours.[13]

Monopolistic competition in the final goods market forces the associated prices to depend

[12] A slew of sticky price specifications are used in monetary business cycle models. Examples are King and Wolman (1996), Nelson (1998), Ireland (2001a), Kozicki and Tinsley (2001), Sbordone (2001), and Smets and Wouters (2002).

[13] Monopolistically competitive final goods firms must face period-by-period fixed costs. Below, we outline a labor market search structure that precludes fixed labor costs as in Yun (1996) because hours are not priced in a spot market.

on nominal marginal cost, Φ. The jth final good firm sets its price by minimizing its total cost, $\mathcal{TC}_j = R_K\, k_j + W\, h_j$, subject to the CRS technology, where R_K is the nominal rental rate of capital. The first-order necessary conditions (FONCs) are $R_K = \Phi\theta y_j/k_j$ and $W = \Phi(1-\theta)y_j/h_j$. Place these optimality conditions into the cost function and exploit the CRS technology to show, $\mathcal{TC}_j = \Phi y_j$ - $R_K\overline{K}$. Hence, the net profit function of this firm is

$$\frac{D_j}{P} \quad = \quad \left(\frac{P_j}{P} \;-\; \phi\right)\left(\frac{P_j}{P}\right)^{-\xi} Y_D \;-\; \frac{R_K}{P}\overline{K}, \tag{6}$$

given demand schedule (2).

We study economies in which final goods prices are sticky and flexible. When final goods prices are flexible, real marginal cost is constant, $\phi = (\xi - 1)/\xi$, and prices are a constant markup over marginal costs. A final good firm whose behavior is restricted by the Calvo staggered price mechanism (1) solves the intertemporal profit maximization problem

$$\mathbf{E}_t\left\{\sum_{i=0}^{\infty}(\beta\mu)^i\Gamma_{t+i}\left[\left(\left[\frac{m^*}{\gamma^*}\right]^i \frac{P_{C,t}}{P_{t+i}} \;-\; \phi_{t+i}\right)\left(\left[\frac{m^*}{\gamma^*}\right]^i \frac{P_{C,t}}{P_{t+i}}\right)^{-\xi} Y_{D,t+i} \;-\; \frac{R_{K,t+i}}{P_{t+i}}\overline{K}_{t+i}\right]\right\}.$$

The FONC of $P_{C,t}$ leads to the forward-looking price-setting optimality condition (3).

Construction of aggregate dividend and production functions closes the final goods sector. Yun (1996) shows aggregate demand is connected to aggregate supply through the supply price aggregator, $P_{A,t}^{-\xi} \equiv \int_0^1 P_{j,t}^{-\xi}dj$.[14] The definition of aggregate output, $Y_{A,t} \equiv \int_0^1 y_{A,j,t}dj$, and the demand schedule (2) gives $Y_{D,t} = (P_t/P_{A,t})^{-\xi}\, Y_{A,t}$.[15] These facts lead to the aggregate real dividend function, $D_t/P_t = (P_t/P_{A,t})^{-\xi}\,[1-\theta\phi_t]Y_{A,t} - (R_{K,t}/P_t)K_t - (W_t/P_t)h_t$. Since technology

[14]The associated dynamics are $P_{A,t}^{-\xi} = (1-\mu)P_{C,t}^{-\xi} + \mu(m^*\exp\{-\gamma\}P_{A,t-1})^{-\xi}$.

[15]This eliminates $P_{C,t}$ from the state of the economy leaving only current and lagged aggregate prices.

is CRS, market clearing relative prices, $R_{K,t}/P_t$ and W_t/P_t, and the definitions of aggregate capital and hours result in $Y_{A,t} = (K_t - \overline{K}_t)^{\theta}\ (h_t Z_t)^{1-\theta}$, which is the aggregate production function.

3.2 The Household

Households decisions cover consumption, leisure, capital accumulation, and financial portfolios (to hold cash and government bonds). Felicity is summarized by

$$(7) \qquad u(c_{M,t},\ c_{L,t},\ \ell_t) \quad \equiv \quad \psi_1 ln[c_{M,t}] \ + \ (1 - \psi_1) ln[c_{L,t}] \ + \ \psi_3 \frac{\ell_t^{1-\psi_2}}{1 - \psi_2},$$

where $0 < \psi_1 < 1$, $\psi_2 \neq 1$, $0 \leq \psi_3$, $c_{M,t}$, $c_{L,t}$, and ℓ_t $(= 1 - h_t)$ are cash consumption, credit consumption, and leisure, respectively. The household faces the budget constraint

$$(8) \qquad D_t \ + \ R_{K,t}\, k_t \ + \ W_t\, h_t \ + \ (1 + R_{B,t})\, B_{G,t} \ + \ M_t \ - \ A_{t+1}$$

$$= \quad P_t[c_{M,t} \ + \ c_{L,t} \ + \ k_{t+1} \ - \ (1 - \delta_K)\, k_t \ + \ T_t],$$

the CIA constraint

$$(9) \qquad M_t \quad \geq \quad P_t\, c_{M,t},$$

and the wealth constraint

$$(10) \qquad A_t \quad \geq \quad B_{G,t} \ + \ M_t \ - \ X_{t-1},$$

where $0 < \delta_K < 1$, and $D_t,\ B_{G,t},\ M_t,\ A_t,\ T_t$, and X_{t-1} denote dividends the household receives from final good firms, government bonds this household owns at the beginning of date t, cash the household carries over to date t from the end of date $t-1$, nominal wealth it takes from the end of date $t-1$ into the beginning of date t, a lump-sum tax levied on all households, and the total cash injection, respectively. The government pays $R_{B,t}$ on its one-period unit discount bond.

3.3 The Government

The government engages in monetary and fiscal operations. The latter activities involve expenditures, G_t, lump-sum tax collecting, T_t, and issuing one-period unit discount bonds, $B_{G,t+1}$. The monetary operation injects X_t units of cash into the household sector. Hence, the intertemporal budget constraint of the government is

$$(11) \qquad P_t T_t \;+\; (B_{G,t+1} \;-\; B_{G,t}) \;+\; (M_{t+1} \;-\; M_t) \quad = \quad P_t G_t \;+\; R_{B,t} B_{G,t} \;+\; X_t.$$

We let $T_t \;=\; G_t$ at each date t and assume the government spending-output ratio, $g_t \;=\; G_t / Y_{D,t}$, evolves exogenously. Government bonds are restricted to be in *zero* net supply, $B_{G,t+1} = 0$, along the equilibrium path. Cash injections obey $X_t \;=\; M_{t+1} - M_t$ and monetary base growth, m_t (= M_{t+1}/M_t), is assumed to be an exogenous stochastic process to avoid entangling the predictions of our DSGE models with an arbitrary monetary policy rule.

3.4 Household Optimality

The household maximizes its expected lifetime utility subject to $(8) - (10)$. Lifetime utility is the expectation of the infinite discounted sum of felicity,

$$\mathbf{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j u(c_{M,t+j},\; c_{L,t+j},\; \ell_{t+j}) \right\}, \quad \beta \;\in\; (0,\; 1).$$

This problem yields the consumption-based money demand function

$$\frac{M_t}{P_t} \quad = \quad C_t \left[\frac{\psi_1}{1 \;+\; (1 \;-\; \psi_1) R_{B,t}} \right],$$

where $C_t \;\equiv\; c_{M,t} \;+\; c_{L,t}$. Another implication is the household's stochastic discount factor (SDF)

$$\text{(12)} \qquad \frac{\Gamma_t}{P_t} \quad = \quad \beta \mathbf{E}_t \left\{ \frac{\psi_1}{P_{t+1}\, c_{M,t+1}} \right\}.$$

Firms and the government take the sequence of SDFs, $\{\Gamma_{t+j}\}_{j=0}^{\infty}$, as given, when they discount using (12). The SDF, the CIA constraint (9), and the FONC with respect to $c_{L,t}$ produces

$$\text{(13)} \qquad C_t \quad = \quad \frac{M_t}{P_t} \;+\; \frac{(1 - \psi_1)}{\Gamma_t P_t},$$

which is the household "consumption function".

Optimal choice of employment hours by the typical household involves the usual trade-off between leisure and the rewards of labor market activity. The optimality condition of h_t is

$$\text{(14)} \qquad \frac{\psi_3}{(1 - h_t)^{\psi_2}} \quad = \quad \Gamma_t \frac{W_t}{P_t}.$$

The household supplies labor up to the point at which the dis-felicity of work equals the discounted real wage according to (14). This wage is determined in a perfectly competitive spot market.

The dynamic program the household solves produces two intertemporal optimality conditions. The Euler equation, $\Gamma_t / P_t = \beta \mathbf{E}_t \{(\Gamma_{t+1}/P_{t+1})[1 + R_{B,t+1}]\}$, describes optimal intertemporal choice in the money market. It shows the interaction of the CIA constraint and next period's liquidity preference trade-off between consumption and the government's unit discount bond. The intertemporal trade-off between consumption and capital accumulation is given by

$$\text{(15)} \qquad \Gamma_t \quad = \quad \beta \mathbf{E}_t \left\{ \Gamma_{t+1} \left[\frac{R_{t+1}}{P_{t+1}} \;+\; (1 - \delta_K) \right] \right\},$$

which is determined by the FONC of K_{t+1} and the envelope condition for K_t. Euler equation (15) shows the household is willing to postpone a unit of date t consumption for the return additional capital is expected to yield during date $t + 1$ for date $t + 2$ consumption.

3.5 Aggregate Equilibrium and Optimality

Equilibrium requires the goods, capital, money, government bond, and labor markets to clear. Goods market equilibrium relies on the aggregate resource constraint

$$(16) \qquad \left(\frac{P_t}{P_{A,t}}\right)^{-\xi} Y_t \quad = \quad C_t \; + \; K_{t+1} \; + \; (1 - \delta_K)K_t \; + \; G_t .$$

The aggregate resource constraint (16) adds together the budget and wealth constraints, (8) and (10), of the household, the government's budget constraint (11), and the firm's dividend flow (6). Since the rental market for capital, the money market, the government bond market, and the labor market are perfectly competitive, agents treat the joint stochastic process that generates returns and the nominal wage, $\{R_{K,t+j},\ R_{B,t+j},\ W_{t+j}\}_{j=0}^{\infty}$, as given. The same holds for the exogenous shock process $\{Z_{t+j},\ \overline{K}_{t+j},\ G_{t+j},\ X_{t+j}\}_{j=0}^{\infty}$.

Any candidate equilibrium of the Yun-sticky price must satisfy the optimality conditions and the aggregate resource constraint for $C_t,\ h_t,\ P_t,\ \phi_t$, and K_{t+1}. The optimality condition for consumption is the aggregate resource constraint (16). Optimal labor market activity ties the labor supply schedule embedded in (14) with a firm's FONC with respect to hours

$$(17) \qquad \frac{\psi_3}{(1 - h_t)^{\psi_2}} \quad = \quad \Gamma_t \left(\frac{P_t}{P_{A,t}}\right)^{-\xi} \phi_t (1 - \theta)(K_t - \overline{K}_t)^{\theta}\, h_t^{-\theta}\, Z_t^{1-\theta} .$$

Optimal price behavior requires the consumption function (13), the SDF (12), and $c_{M,t} = M_t / P_t$, which is the CIA constraint (9) in equilibrium and defines money market equilibrium. A flexible price regime equates aggregate consumption to real balances plus (the present-value of) the purchasing power of a dollar. The law of motion of the price level (1) and the optimal commitment price condition (3) restricts the optimal path of ϕ_t in a sticky-price economy. This forces money market adjustment onto C_t and Γ_t. Optimal capital accumulation arises from the Euler equation

$$(18) \qquad \Gamma_t \quad = \quad \beta \mathbf{E}_t \left\{ \Gamma_{t+1} \left[\theta \phi_{t+1} (K_{t+1} - \overline{K}_{t+1})^{\theta-1} \, (h_{t+1} \, Z_{t+1})^{1-\theta} \; + \; (1 - \delta_K) \right] \right\},$$

which rests on the Euler equation (15) and the nominal rental rate of capital. The transversality conditions of the endogenous state variables are sufficient conditions of any candidate equilibrium, where for capital $lim_{j \to \infty} \beta^j \mathbf{E}_t \left\{ \Gamma_{t+j} K_{t+1+j} \right\} = 0$.

4. Comparing Sample and Theoretical NKPCs

This section reports on the calibration, solution strategies, and Monte Carlo experiments. Next, labor market search is placed in the sticky-price model to compare and contrast the implications for the NKPC of this real rigidity with the nominal rigidity of sticky-prices. We complete this study of the NKPC with a flexible price version of our monetary DSGE with labor market search.

4.1 The Calibration and Numerical Solution

We generate an approximate numerical solution of the Yun-sticky price model from the linearized stochastically detrended variants of its optimality conditions, laws of motion, and equilibrium conditions. Stochastic detrending is necessary because labor augmenting technology evolves as a random walk with drift, $Z_{t+1} = Z_t \exp\{\gamma + \varepsilon_{t+1}\}$, $0 < \gamma$, $\varepsilon_{t+1} \sim \mathbf{N}\left(0, \sigma^2_{\varepsilon}\right)$, and money growth is a AR(1), $m_{t+1} = m^{*\,(1-\rho_m)} \, m_t^{\rho_m} \, exp\left\{\eta_{m,t+1}\right\}$, $|\rho_m| < 1$, $\eta_{m,t+1} \sim \mathbf{N}\left(0, \sigma^2_{\eta,m}\right)$, where $ln[m_t] = ln[M_{t+1}/M_t]$ and $\mathbf{E}\{\varepsilon_{t+i}\eta_{m,t+j}\} = 0, \forall\, i, j$.[16] Real aggregates and prices are detrended as $\widehat{U}_{Y,t} = U_{Y,t}/Z_t$ and $\widehat{U}_{P,t} = U_{P,t} Z_t / M_t$, respectively, where $U_{Y,t} = [Y_{D,t}\; Y_{A,t}\; C_t\; K_{t+1}\; G_t]$ and $U_{P,t} = [P_t\; P_{A,t}\; P_{C,t}]$. Detrending $\widehat{W}_t = W_t/M_t$, $\widehat{\Gamma}_t = \Gamma_t Z_t$, and $\widehat{R}_{K,t} = R_{K,t}/P_t$, follows.

[16]We assume the transitory components of $\overline{K}_{t+1}$ and g_{t+1} are non-stochastic.

The numerical solution is obtained by linearizing the detrended aggregate resource constraint (16), the hours schedule (17), the SDF (12), the consumption function (13), the law of motion of the price level that underlies the NKPC-PV relation (4), and the Euler equation of K_{t+1}, (18). The solution we conjecture is

$$\mathcal{K}_{t+1} \quad = \quad \mu_{\mathcal{K}}\mathcal{K}_t \quad + \quad \mu_{\mathcal{E}}\mathcal{E}_t, \tag{19}$$

where $\mathcal{K}_{t+1} = [\widetilde{K}_{t+1} \ \aleph_{t+1} \ \Im_{t+1}]'$, $\widetilde{K}_{t+1} = ln[\widehat{K}_{t+1}/K^*]$, $\aleph_{t+1} = \mathbf{E}_t\widetilde{P}_{t+1}$, $\Im_{t+1} = P_t$, and the exogenous state vector, $\mathcal{E}_t = [\varepsilon_t \ \eta_{m,t}]'$. We seek the unknown elements of the three-by-three matrix $\mu_{\mathcal{K}}$ and the three-by-two matrix $\mu_{\mathcal{E}}$ using methods Zadrozny (1998) and Sims (2000) develop to compute approximate numerical solutions.[17] Given solutions for $\mu_{\mathcal{K}}$ and $\mu_{\mathcal{E}}$, the control system is

$$\mathcal{C}_t \quad = \quad \pi_{\mathcal{K}}\mathcal{K}_t \quad + \quad \pi_{\mathcal{E}}\mathcal{E}_t, \tag{20}$$

where $\mathcal{C}_t = [\widetilde{C}_t \ \widetilde{h}_t \ \widetilde{P}_t \ \widetilde{\phi}_t]'$, $\pi_{\mathcal{K}}$ is a four-by-three matrix, and $\pi_{\mathcal{E}}$ is a four-by-two matrix. The approximate linearized solution drives $\widetilde{K}_{t+1}$ and $\mathcal{C}_t$, which includes $\widetilde{P}_t$, with two lags of price expectations. This imposes the NKPC-PV restrictions on the Yun-sticky price model solution.

We employ sample data and choices made by other studies to calibrate model parameters. Preference parameters β and ψ_1 are 0.9950 and 0.8428, respectively. The latter implies an interest elasticity of money demand of one percent, given the federal funds rate sample mean. We take the other preferences parameters from Andolfatto (1996), $\psi_2 = 2.0$ and $\psi_3 = 2.08$. The technology parameter $\theta = 0.35$ and $\delta_K = 0.0195$. The steady state markup is 1.10, which yields $\xi = 11.0$. The sticky price parameter is calibrated to the NKPC sample moment (i), $\mu_{2SLS} = 0.5292$.

[17] Many sticky price models have a singular leading coefficient matrix in the stochastic difference equation system that arises from linearizing optimality and equilibrium conditions. Sims (2000) describes a solution and provides software.

The calibration of the impulse structure relies on sample data from $1960Q1 - 2001Q4$. The deterministic growth rate $\gamma = 0.0047$ is the sample mean of measured total factor productivity growth and its sample standard deviation is $\sigma_\varepsilon = 0.0117$.

The parameters of the AR(1) process of money growth are based on the Federal Reserve Bank of St. Louis' monetary base series. Its mean growth rate is 0.0166. Estimation of the AR(1) regression of money growth yield $\rho_m = 0.4456$ and $\sigma_{\eta,m} = 0.0068$.[18]

4.2 Monte Carlo Design

We generate 5000 replications of the monetary DSGE models. A replication is 168 observations of the price level and ULC.[19] At each replication, the MLE-cointegrating vector, ϖ_{CI}, of the case 1* VECM(3) is estimated. Conditional on a lag of the cointegration relation and three lags of artificial inflation and ULC growth, the 2SLS regression of inflation on a constant and ULC growth is computed to produce the common feature vector, ϖ_{CF}, and a synthetic estimate of μ_{2SLS}. Synthetic estimates of the ϖ_{CI} and ϖ_{CF} vectors are employed to construct the BNSW-VE decomposition and its FEVD. We report theoretical FEVDs in table 1. Figure 2 contains nonparametric densities of the ensemble of synthetic estimates of μ_{2SLS} and the asymptotic 95 percent confidence interval of the sample μ_{2SLS}. Theoretical NKPC trends and cycles appear in figures 3, 4, and 5.

[18] As previously mentioned, the transitory components of the fixed capital component and government spending are assumed to be nonstochastic. We set g^* at its sample mean, 0.1878. Calibration of $\overline{K}^*$ is problematic. It cannot be constructed without observations on fixed capital. The closest notion is structures, but U.S. capital stock data reveals the ratio of structures to total capital is about 0.23 for the $1960 - 2000$ sample. We assume $\overline{K}^* = 0.025$. Experiments with values between 0.25 and 25 percent have little impact on the Monte Carlo experiments.

[19] We compute 372 artificial observations, but drop the first 204 to remove dependence on initial conditions.

4.3 Yun-Sticky Price Model Experiments

Simulations of the Yun (1996) model reveal it to be at odds with NKPC sample moments $(i) - (iii)$. The mean of theoretical μ_{2SLS} estimates is 0.8264, which places its density (the dashed curve) to the right of the asymptotic 95 percent confidence interval (vertical dotted lines) of the sample $\mu_{2SLS}(= 0.5293)$ in figure 2. Although the Yun-sticky price model is calibrated to NKPC sample moment (i), this model predicts more stickiness in the price level.

Figure 3 has the evidence the Yun (1996) sticky price DSGE model fails to replicate NKPC sample moment (ii). The top window of figure 3 shows that the theoretical NKPC trend falls on top of the sample GDP deflator. Thus, the theoretical NKPC cycle exhibits excess smoothness, which explains the theoretical one-standard deviation confidence bands of figure 3.

Table 1 contains the sample and theoretical FEVDs of the price level with respect to trend shocks and theoretical one-standard deviation confidence intervals of the latter FEVDs (in brackets). The FEVDs of the Yun-sticky-price model and its one-standard deviation confidence intervals are all greater than 97.8 percent, which indicate little uncertainty about these FEVDs. The price level FEVDs of this sticky price model matches the NKPC-PV prediction, but is far away from NKPC sample moment (iii) because actual U.S. price movements are not only driven by trend shocks.

4.4 Labor Market Search-Sticky Price Models Experiments

The failure of the Yun-sticky price model indicates the nominal rigidity of sticky prices *alone* cannot explain the NKPC sample moments $(i) - (iii)$. Gali and Gertler (1999), among others, suggest a real rigidity may resolve this problem. We add the real rigidity of Mortensen and Pissarides (1994) job-search that Andolfatto (1996), Merz (1995), and den Haan, Ramey, and Watson (2000) successfully place in RBC models. An appeal of labor market search is the restrictions it places on the Phillips curve, as discussed, for example, by Solow (1976).

Labor market search ties real and nominal activity together with the matching and search technologies available to firms and households. Firms and households engage in job search because hours are bought and sold in the presence of labor market externalities related to the costs of posting vacancies and looking for work. Firms post $v_{j,t}$ plant-job vacancies at a cost of υ per vacancy.[20] The not-employed devote S_t hours to job search which generates felicity and pecuniary costs.

We assume a final good firm operates multiple plants and identify an active plant with a job.[21] Firms with empty-plant jobs and the not-employed are brought together randomly, according to the den Haan, Ramey, and Watson(2000) CRS matching technology

$$\text{(21)} \qquad \mathcal{M}(V_t,\, (1-N_t)S_t) \;=\; \frac{V_t[(1\;-\;N_t)S_t]}{(V_t^{\vartheta}\;+\;[(1\;-\;N_t)S_t]^{\vartheta})^{1/\vartheta}}, \quad 0\;<\;\vartheta,$$

where $V_t(\equiv \int_0^1 v_{j,t}dj)$ is total plant-job vacancies and N_t denotes aggregate employment (or the measure of active plant-jobs). However, the probability of a successful match is influenced indirectly by variation either in posted vacancies or in not-employed search effort. This reflects the labor market externalities associated with search; see the appendix for details.

Job search alters aggregate household felicity (7). When a not-employed household gives up leisure to search a fraction S_t of its one unit of date t time endowment, this household suffers a felicity loss equal to $\psi_5\,(1-S_t)^{1-\psi_4}\,/(1-\psi_4)$, where $\psi_4\;\neq\;1$ and $0\;\leq\;\psi_5$. Since complete income

[20]Total recruitment costs represent a drain on aggregate output. This forces us to assume that υ shares the technology trend, but has a non-stochastic transitory component.

[21]Andolfatto (1996) points out that a CRS production technology in the presence of job search equates a plant-job with an operating plant. Hence, the aggregate measure of plant-jobs and the measure of the employed are equivalent.

and wealth insurance creates an aggregate household that is a weighted average of employed and not-employed households, the leisure component of aggregate household felicity becomes $N_t \psi_3 (1-h_t)^{1-\psi_2}/(1-\psi_2) + (1-N_t)\, \psi_5\, (1-S_t)^{1-\psi_4}(1-\psi_4)$; see the appendix for details.

The wealth constraint of the employed and not-employed differ because the not-employed face transactions costs to job search. We assume these costs rise with search effort at rate φ (> 0) and that the only resource available to pay these costs is the cash injection the not-employed receive from the government. For the not-employed, this adds $-\varphi S_{t-} X_{t-1}$ to the wealth constraint (10). Thus, the employed and not-employed respond differently to the cash injection. Combine the employed and not-employed wealth constraints to obtain the aggregate wealth constraint

$$(22) \qquad A_t \quad \geq \quad B_{G,t} \;+\; M_t \;-\; [1-\varphi(1-N_{t-1})S_{t-1}]X_{t-1},$$

where the weights are N_{t-1} and $1-N_{t-1}$. Within the aggregate household, complete wealth insurance requires the employed to transfer cash to the not-employed to hold the latter harmless for their search costs. The appendix discusses these issues.

Firm and not-employed search frictions place demands on aggregate output. Given complete income and wealth insurance, these costs enter the aggregate resource constraint additively

$$(23) \quad \left(\frac{P_t}{P_{A,t}}\right)^{-\xi} Y_t \quad = \quad C_t \;+\; K_{t+1} \;+\; (1-\delta_K)K_t \;+\; G_t \;+\; (1-N_t)\varphi\frac{X_t}{P_t}S_t \;+\; \upsilon_t V_t,$$

from the wealth constraint of the aggregate household and the aggregate dividend process.[22] The last two terms on the right of the aggregate resource constraint (23) reflect the real resource loss

[22]The appendix outlines the insurance schemes that give rise to the felicity function of the aggregate household and the rest of the economy-wide optimality and equilibrium conditions. Aggregation rests on the capital stocks, dividends received, cash held, and bonds owned by these households to be equal date-by-date. This assumes that employed and not-employed households hold equal endowments of capital and financial wealth at date zero. Further, we assume away

that arises from job search by households and firms, respectively.

Job search precludes a spot market for labor. Rather than a Walrasian auctioneer, a firm and the aggregate household negotiate a labor contract over hours and the real wage to split match surplus at each date the employment relationship exists. Match surplus is the sum of the capitalized value of an active plant-job and the net benefits the aggregate household receives from the ongoing job match. Merz (1995), Andolfatto (1996), and Cooley and Quadrini (1999) assume the aggregate household receives a fixed fraction, ζ, of the surplus at each date t. Thus, the aggregate household's contribution to match surplus equals ζ times the capitalized value of an active plant-job.

The surplus splitting rule together with optimal firm and aggregate household behavior produces the (Nash) equilibrium real wage function

$$(24) \qquad \Gamma_t \frac{W_t}{P_t} h_t \;=\; \zeta \Gamma_t \left[\left(\frac{P_{A,t}}{P_t} \right)^{\xi} \left(1 - \left[\frac{\theta \phi_t}{1 - \overline{K}^*} \right] \right) \frac{Y_{A,t}}{N_t} + \frac{\upsilon_t V_t}{1 - N_t} \right] + (1 - \zeta) \mathcal{H}_{X,t},$$

where $\mathcal{H}_{X,t} = \psi_3\,(1-\psi_2)^{-1}\,(1-h_t)^{1-\psi_2} - \psi_5\,(1-\psi_4)^{-1}\,(1-S_t)^{1-\psi_4} - \varphi \Gamma_t X_t S_t / P_t$ and $\overline{K}^*$ is the steady state of $\overline{K}_t$.[23] Along the equilibrium path, discounted real labor income is a weighted average of that match's value-added to aggregate output and the alternative activities (e.g., non-employment and search) available to the aggregate household. The marginal product of labor plus the foregone costs of fixed capital and firm job search represent the former. The latter is the net impact on felicity of an ongoing plant-job match and foregone transactions job-search costs. Note that the real wage is a function of real marginal cost, ϕ_t, in a sticky price regime. Unlike a spot market in which the

any wealth disparities caused by ownership claims on final goods firms. If employed and not-employed households are initially given equal equity in final goods firms, the dividend flows will be equalized. Also, these results depend on the additive separability of felicity. Sims (1998) discusses related issues.

[23]The appendix constructs the equilibrium real wage process (24).

real wage equals the intersection of labor supply and labor demand (productivity) schedules at each date t, labor market search creates persistence and volatility in the real wage that differs from labor productivity. This creates a monetary transmission mechanism in a labor market search model.

Calibration of the labor market search models follows the process described in section 4.1. The not-employed preference parameters ψ_4 and ψ_5 equal two and 1.37, respectively. The exogenous fixed separation rate is set at 0.0848, which places δ_N within the range Merz (1995), den Haan, Ramey, and Watson (2000), and Andolfatto (1996) use. The calibration of ψ_4, ψ_5, and δ_N help guarantees aggregate hours and employment match their sample counterparts. Cooley and Quadrini (1999) fix $1/\vartheta = 1 - \zeta$ at 0.6. This matters for the NKPC because the first task of a Phillips curve is to describe price behavior. We do the same.[24] The vacancy cost parameter $\upsilon = 0.1050$ is taken from Andolfatto.[25] We assume job-search transactions costs impose a 0.1 percent loss on velocity at the steady state (in terms of sample GDP and the monetary base). This yields $\varphi = 0.3060$.

We solve models with labor market search using methods described in section 4.1. The linearized aggregate household and firm job-search optimality conditions, the aggregate resource constraint (23), and the law of motion of aggregate employment, $N_{t+1} = (1 - \delta_N)N_t + \mathcal{M}_t$, add $\widetilde{N}_{t+1}$ to the state vector $\mathcal{K}_{t+1}$ and $\widetilde{S}_t$ to the control vector $\mathcal{C}_t$. The transversality condition for employment is $lim_{j\to\infty}\beta^j \mathbf{E}_t\left\{\Lambda_{t+j}N_{t+1+j}\right\} = 0$, where Λ_t is the shadow price of a job match.

The theoretical density of μ_{2SLS} generated by the labor market search-sticky price model (dot-dash curve) appears in figure 2. This density is to the right of NKPC sample moment (i)

[24]Since $1-\zeta = 1/\vartheta$, the power the aggregate household exerts on contract negotiations equals the household's share of the match surplus. Thus, the equilibrium real wage is the same as the socially optimal wage; see Hosios (1990).

[25]The steady state is also constructed to make the probabilities that a vacant plant-job is filled and that someone not-employed finds work consistent with the den Haan, Ramey, and Watson calibration.

because only 58 of the 5000 estimates reside within the 95 percent asymptotic confidence interval of the sample estimate of μ_{2SLS}. The mean of synthetic estimates of μ_{2SLS} equals 0.6607, which also signals the labor market search-sticky price model cannot explain NKPC sample moment (i).

Figure 4 presents NKPC moment (ii), the common trend and common cycle, of the labor market search-sticky price model. The theoretical NKPC trend (the top window) closely follows the actual GDP deflator. This explains the smoothness of the one-standard deviation confidence bands of the theoretical NKPC cycle (the bottom window).

The FEVDs of the price level with respect to the trend shock and their one-standard deviation coverage intervals generated by the labor market search-sticky price model appear in the fourth column of table 1. Since the one-standard deviation coverage interval of the FEVD runs from 35 to 96.5 percent at the one-quarter forecast horizon, it suggests a short-run role for cyclical shocks. However, the theoretical one-quarter horizon FEVD of 69 percent and the one year-horizon of nearly 90 percent are closer to the NKPC prediction than to the relevant sample FEVDs. Thus, the labor market search-sticky price model finds it difficult to reproduce NKPC sample moment (iii).

In summary, NKPC sample moments $(i) - (iii)$ fail to be replicated by the Yun- and labor market search-sticky price models. A common element across the two models is that the state vector, $\mathcal{K}_{t+1}$, of their linearized solutions contain price expectations, as in the state system (19) and (20) of the Yun-sticky price model. Since this is the way the NKPC-PV restrictions are imposed in the sticky price models, it explains the upward bias in synthetic estimates of the sticky price parameter, the excess smoothness in the NKPC trend and cycle, and the dominate trend response of the theoretical FEVDs. This suggests the theoretical link between the price level and price expectations needs to be broken for monetary DSGE models to fit NKPC sample moments $(i) - (iii)$.

4.5 Flexible Price-Labor Search Model Experiments

This section reports on a DSGE model that replaces the sticky price mechanism (1) with a flexible price regime. This eliminates the price expectation term, $\mathbf{E}_t\widetilde{P}_{t+1}$, and the stochastically detrended-demeaned price level, $\widetilde{P}_t$, from the equilibrium state system

$$(25) \qquad \begin{bmatrix} \widetilde{K}_{t+1} \\ \widetilde{N}_{t+1} \end{bmatrix} = \mu_{\mathcal{K}} \begin{bmatrix} \widetilde{K}_t \\ \widetilde{N}_t \end{bmatrix} + \mu_{\mathcal{E}} \begin{bmatrix} \varepsilon_t \\ \eta_{m,t} \end{bmatrix},$$

and their lags from the control system that contains the equilibrium price level process

$$(26) \qquad \widetilde{P}_t = \pi_{\mathcal{K},P} \begin{bmatrix} \widetilde{K}_t \\ \widetilde{N}_t \end{bmatrix} + \pi_{\mathcal{E},P} \begin{bmatrix} \varepsilon_t \\ \eta_{m,t} \end{bmatrix},$$

of the labor market search-flexible price model.

Figure 2 shows that the 95 percent asymptotic confidence interval of μ_{2SLS} falls within the density of theoretical μ_{2SLS} estimates produced by the labor market search-flexible price model (the solid line). More than 46 percent of these estimates are contained in the 95 percent asymptotic confidence interval, $[0.5133,\ 0.5450]$. The theoretical mean of μ_{2SLS} is 0.5300, compared to a sample mean of 0.5293. Thus, an econometrician who studies the labor market search-flexible price model would recover the NKPC sample moment (i).

NKPC FEVDs of the labor market search-flexible price model appear in the last column of table 1. Theoretical FEVDs are larger than sample FEVDs at 1, 2, 4, and 8 quarter forecast horizons, but smaller beyond a two-year horizon. However, one-standard deviation confidence intervals of the theoretical FEVDs cover the sample FEVDs, except at one- and two-quarter horizons. Thus, the labor market search-flexible price model matches much of NKPC sample moment (iii) driven only by a random walk technology shock and a money growth process whose AR1 coefficient is 0.45.

The theoretical NKPC trend and cycle of the labor market search-flexible price model appear in figure 5. A weakness of the labor market search-flexible price model is that the theoretical NKPC trend (the top window) and cycle (the bottom window) are more volatile than their sample counterparts. The relative volatility of the NKPC cycle (trend) is 1.5098 (1.3622). This model is able to replicate the persistence of the sample NKPC cycle. The AR1 coefficients from the sample and the theoretical ensemble of the NKPC cycles are 0.9335 and 0.9476, respectively.

Figure 5 also shows that differences between the empirical and theoretical NKPC trends are greatest around peaks and troughs. The one-standard deviation confidence bands cover the sample NKPC cycle, beginning with the mid-1970s. We conclude that the labor market search-flexible price model has more success matching NKPC sample moment (ii), than do the sticky price models.

4.6 Price Level Fluctuations, Labor Market Search, and the NKPC

Labor market search has difficulties with several important business cycle facts. Cole and Rogerson (1999) point out labor market search models suffer from several weaknesses, among them incorrect predictions about job creation, job destruction, and unemployment flows.[26] Likewise, Walsh (2002) and Trigari (2003a) report that labor market search-monetary DSGE model produce too much nominal volatility, which we confirm with the labor market search-flexible price model.[27] Nevertheless, the labor market search-flexible price model is better able to replicate the NKPC sample moments $(i) - (iii)$, than do the sticky price models we study.

Our results are linked to previous Phillips curve research. For example, the labor market

[26]Trigari (2003b) argues that a combination of nominal rigidities and labor market search solves these problems.

[27]Ireland (2001b) finds a flexible price model in which an interest rate rule defines monetary policy matches inflation volatility in pre-1979 U.S. data. Sticky prices must replace flexible prices to fit inflation volatility in post-1979 data.

search-flexible price experiments explain the importance attached to sticky wage mechanisms, for example, by Jeanne (1998), Erceg, Henderson, and Levin (2000), and Rabanal and Rubio-Ramírez (2003) because the labor market search-flexible price model relies on "stickiness" in the labor market. Our results are also related to King and Watson (1997). They show U.S. data supports a Phillips curve that allows for flexible prices, but in which labor market variables are sticky. According to King and Watson, their evidence fits the sort of Phillips curve Solow (1976) describes. It is also consistent with the state system (25) – (26) of the labor market search-flexible price model because the equilibrium decision rule for N_{t+1} responds to shocks dated t, not $t+1$, and the current price level is flexible with respect to date t shocks.

The real wage equation (24) imposes restrictions on the NKCP trend and cycle of the labor market search-sticky price and -flexible price models. The labor market search models tie the wage contracting process to real unit labor cost, $W_t h_t / P_t Y_{A,t}$, because it equals the right hand side of equation (24), subsequent to dividing by $\Gamma_t Y_{A,t}$. The balanced growth conditions of the models we study impose a theoretical cointegration relation on $W_t h_t / P_t Y_{A,t}$. The theoretical NKPC common feature relation (5) is derived from the real wage generating equation (24) as

$$ln[P_t] - \mu_{\mathcal{B}} ln\left[\frac{W_t h_t}{Y_{A,t}}\right] = (1-\mu_{\mathcal{B}})(ln[Z_t] - ln[M_t] + \widetilde{P}_t)$$

$$+ ln\Bigg[\left(\frac{P_{A,t}}{P_t}\right)^{\xi}\left(1 - \left[\frac{\theta\phi_t}{1-\overline{K}^*}\right]\right)\frac{1}{N_t} + \frac{\upsilon_t V_t}{(1-N_t)Y_{A,t}} + \frac{(1-\zeta)\mathcal{H}_{X,t}}{\zeta\Gamma_t Y_{A,t}}\Bigg],$$

ignoring constants. Engle and Issler (1995) note that the common feature vector ϖ_{CF} applied to levels data generates the NKPC trend because serial correlation in the NKPC cycle is annihilated. Across the DSGE models we study, the theoretical price level is driven in the long-run only by permanent movements in the level of technology, $ln[Z_t]$, and the money stock, $ln[M_t]$.

Theoretical price level fluctuations depend also on changes in the endogenous state vector of the economy. Under a sticky price regime, short- to long-run theoretical price level movements in the Yun and labor market search models are "excessively" smooth because the endogenous state vector, $\mathcal{K}_t$ includes price expectations, whose dynamics are restricted by the NKPC-PV relation (4). Within linearized solutions of the sticky price models, the response of $\widetilde{P}_t$ to $\mathbf{E}_{t-1}\widetilde{P}_t$ and $\widetilde{P}_{t-1}$ reflect these restrictions, as in the control system (20) of the Yun sticky price model.

A flexible price regime does not drive $\widetilde{P}_t$ with price expectations, but instead by capital, employment, and shock impulse as in the state system $(25)-(26)$ of the labor market search-flexible price model. Although the NKPC-PV restrictions no longer govern $\widetilde{P}_t$ in the flexible price model, the price level exhibits "stickiness" because of the impact of labor market search on employment dynamics. This stickiness is enough for the labor market search-flexible price model to mimic NKPC sample moments $(i)-(iii)$. Thus, the real rigidity of labor market search is a prime friction for an economically useful monetary transmission mechanism independent of nominal rigidities.

5. Conclusion

This paper develops a new Keynesian Phillips curve (NKPC) present-value relation, in which *nominal* unit labor cost is the fundamental, rather than real unit labor cost. The NKPC present-value relation restricts the price level to respond only to trend shocks at all forecast horizons. We also show that the NKPC present-value relation has a common cycle-common trend decomposition that is based on Beveridge and Nelson (1981), Stock and Watson (1988), and Vahid and Engle (1993). The NKPC trend-cycle decomposition is used to do the first job of a Phillips curve: to provide a good description of price level dynamics.

The last 40 years of U.S. GDP deflator and nominal unit labor cost data offers weak support of the NKPC. We estimate about half of U.S. final goods firms are price constrained, which is close to estimates reported elsewhere. The NKPC cycle is economically important because it peaks during the last seven NBER dated recessions. Forecast error variance decompositions reveal that trend shocks only begin to account for more than 60 percent of price level movements at forecast horizons of two years or more. Thus, the NKPC prediction that trend shocks dominate price level fluctuations at all forecast horizons is not supported by the data.

We study the implications of the NKPC present-value prediction for the theoretical price level of several dynamic stochastic general equilibrium monetary models. Simulation experiments show that the Yun (1996) model with Calvo (1983) staggered price setting reproduce the NKPC present-value predictions. Hence, a model with only the nominal rigidity of sticky prices generates excess smoothness in the NKPC trend-cycle decomposition.

Earlier Phillips curve models invoke labor market imperfections to explain price level movements. We pursue this idea by adding labor market search to the Yun-sticky price model. Monte Carlo experiments of the labor market search-sticky price model yield NKPC moments that are not qualitatively different from the model with only sticky prices. Unlike the labor market search-sticky price model, its flexible price cousin is better able to match price level fluctuations.

This paper takes up the Chari, Kehoe, and McGratten (2000) challenge to new Keynesian notions that nominal rigidities generate short-run monetary non-neutralities. We broaden the Chari, Kehoe, and McGratten research agenda by studying the monetary transmission mechanism that arises from the real rigidity of labor market search. We identify the real rigidity with labor market search because its externality suggests a role for monetary policy. Since labor market search is only one specification within a large class of real rigidity-DSGE flexible price models, it points to the need to search for an economically meaningful monetary transmission mechanism within this class of models that can be used for policy analysis. We leave this task for future research.

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Table 1. Forecast Error Variance Decomposition

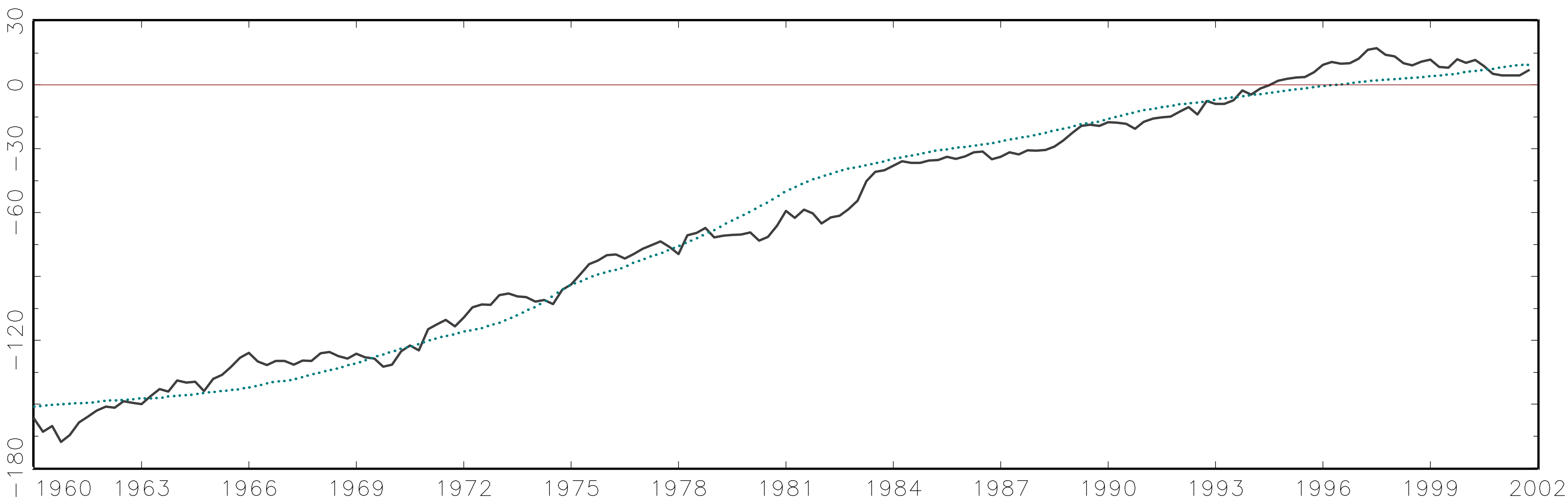
One Standard Deviation Confidence Intervals
FEVDs of Price w/r/t Trend, Generated by DSGE Models

Horizon	Sample PGDP	Yun-Sticky Price Model	Search-Sticky Price Model	Search-Flexible Price Model
1	2.70	98.90	69.22	24.94
		[97.81 99.93]	[34.77 96.53]	[6.92 48.03]
2	8.80	99.33	80.87	37.90
		[98.67 99.96]	[61.21 98.72]	[13.09 69.48]
4	26.38	99.65	89.32	52.90
		[99.31 99.98]	[84.05 99.60]	[23.01 85.63]
8	60.13	99.85	94.32	67.48
		[99.70 99.99]	[94.80 99.88]	[38.70 94.08]
12	78.37	99.91	96.17	74.80
		[99.82 100.00]	[97.40 99.95]	[49.05 96.68]
16	86.55	99.94	97.15	79.18
		[99.88 100.00]	[98.48 99.97]	[56.35 97.77]
20	91.44	99.96	97.75	82.12
		[99.92 100.00]	[99.01 99.98]	[61.93 98.42]
40	98.05	99.99	98.97	89.21
		[99.97 100.00]	[99.76 99.99]	[76.92 99.51]

The values in brackets are the 16th and 84th percentiles of the FEVDs generated from 5000 replications of the DSGE models.

Figure 1: The U.S. Phillips Curve

The Phillips Curve Trend (solid line) and GDP Price Level (dotted line)



The Phillips Curve Cycle and NBER Business Cycle (Peak dash line, Trough dot−dash line)

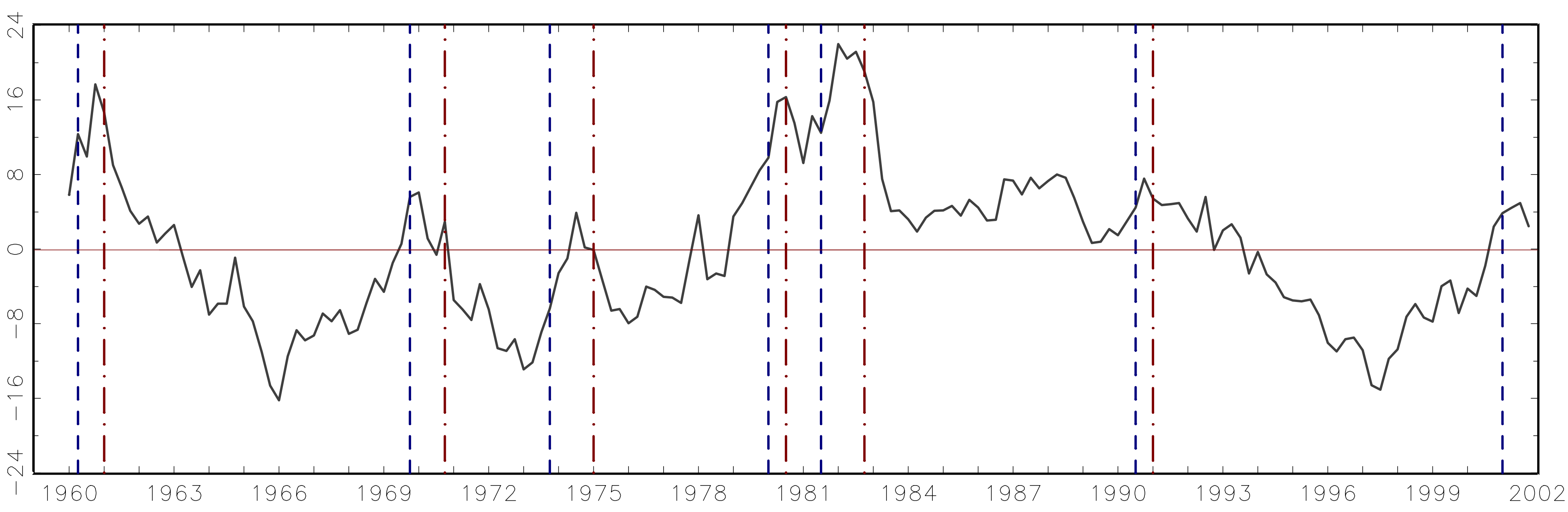


Figure 2: Theoretical Densities of μ

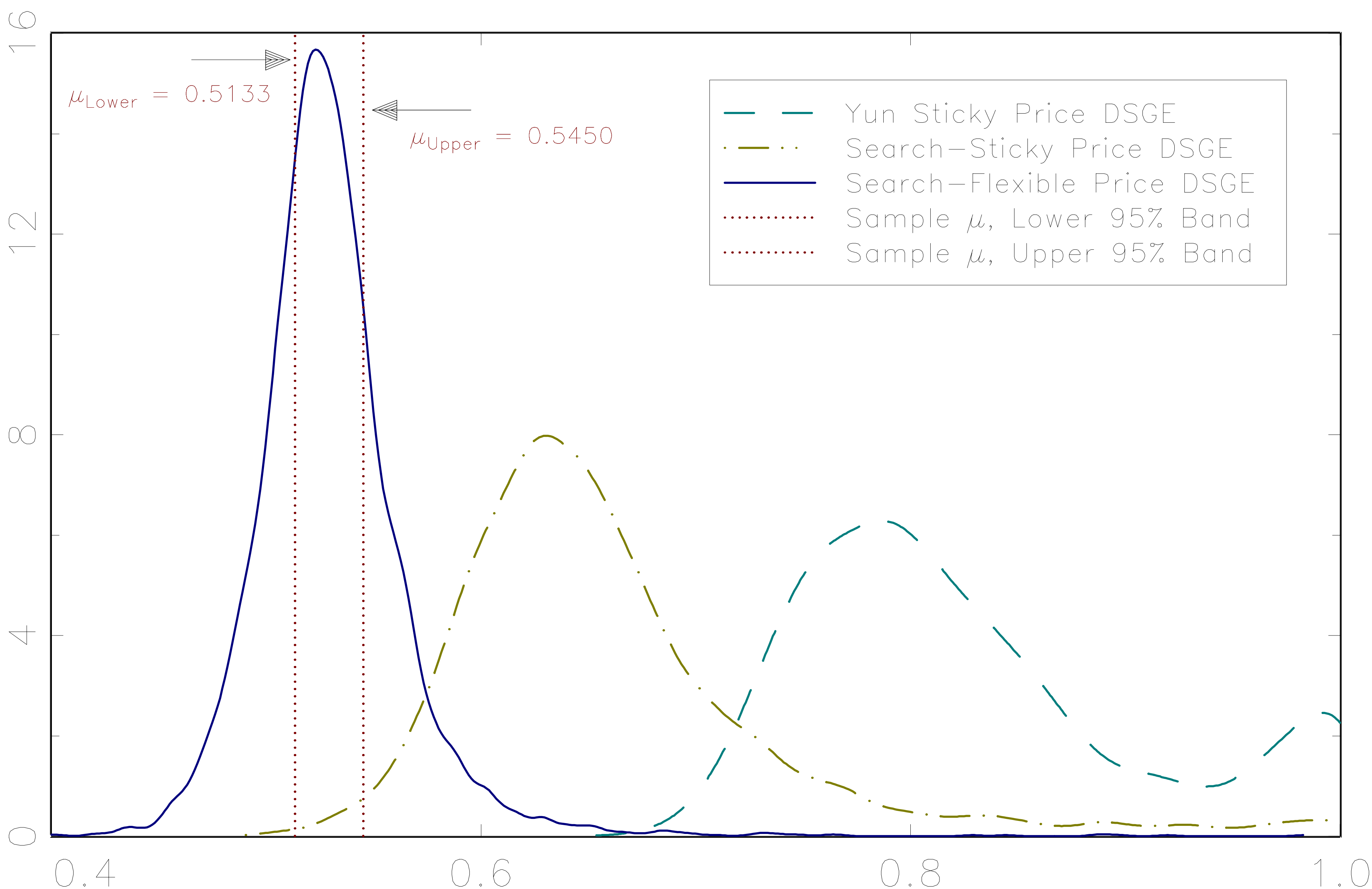
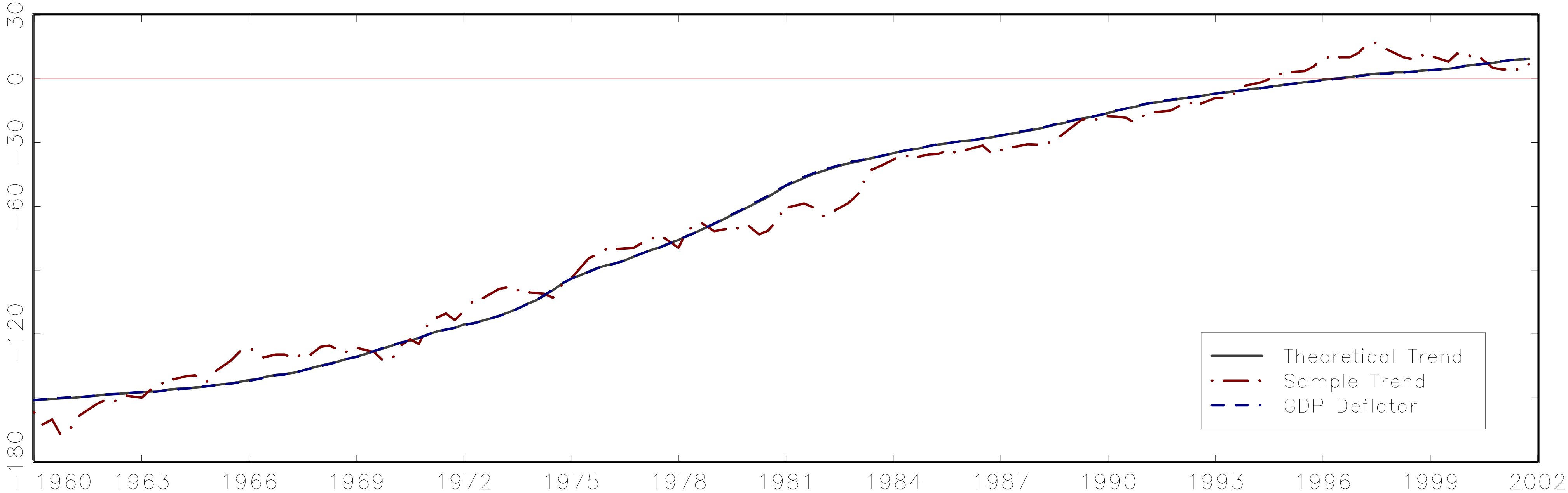


Figure 3: Yun–Sticky Price DSGE Model

Yun–Sticky Price Phillips Curve Trend, Sample Phillips Curve Trend, and Actual Price Level



Sample Phillips Curve Cycle and 1–Standard Deviation Band of Yun–Sticky Price Model

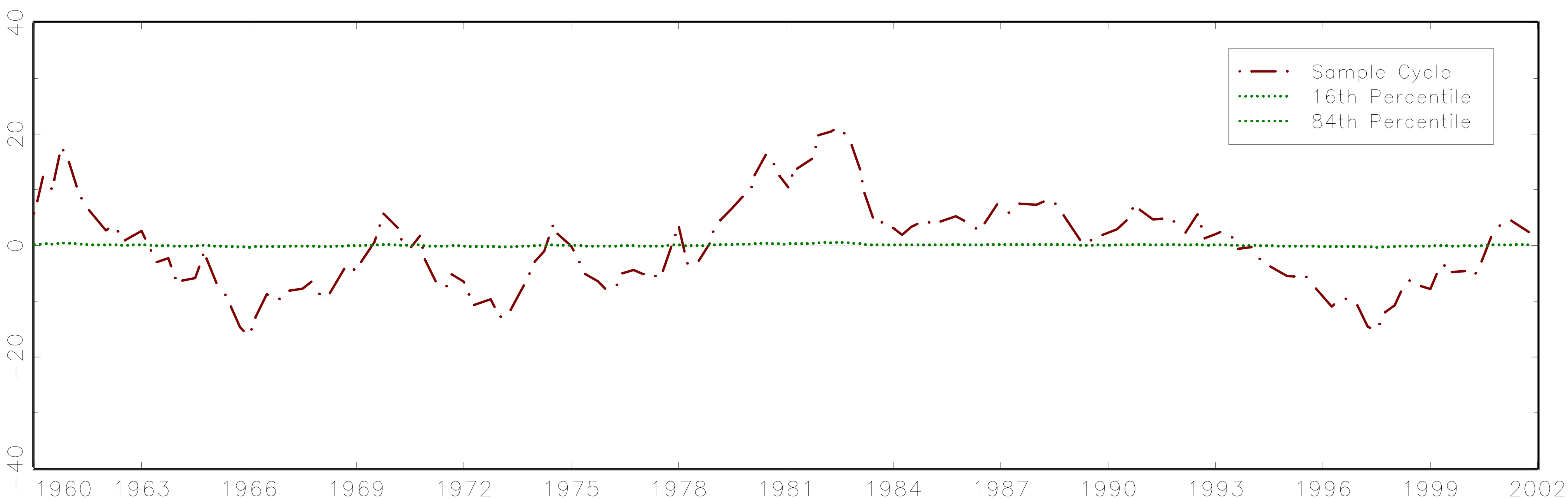
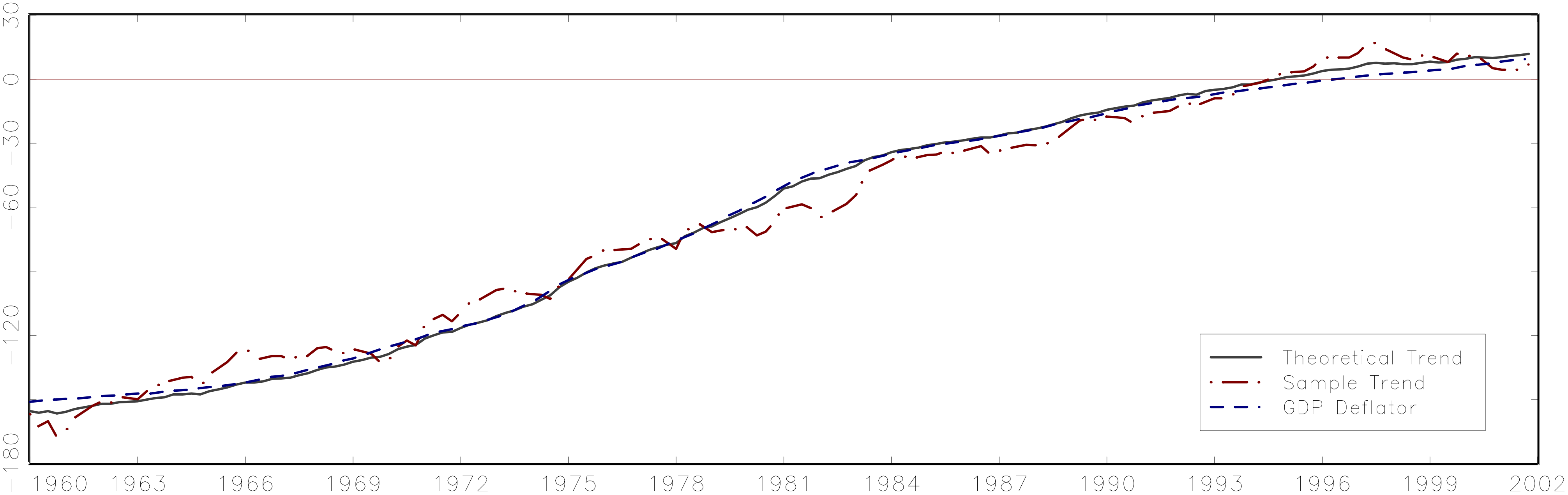


Figure 4: Search–Sticky Price DSGE Model

Search–Sticky Price Phillips Curve Trend, Sample Phillips Curve Trend, and Actual Price Level



Sample Phillips Curve Cycle and 1–Standard Deviation Band of Search–Sticky Price Model

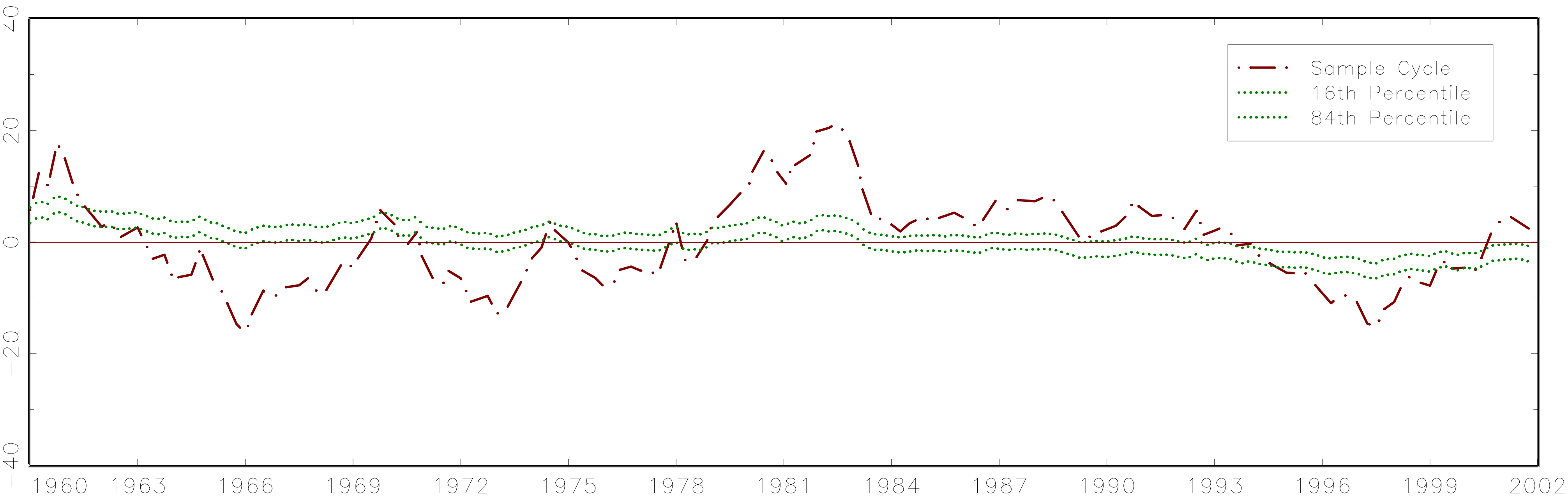
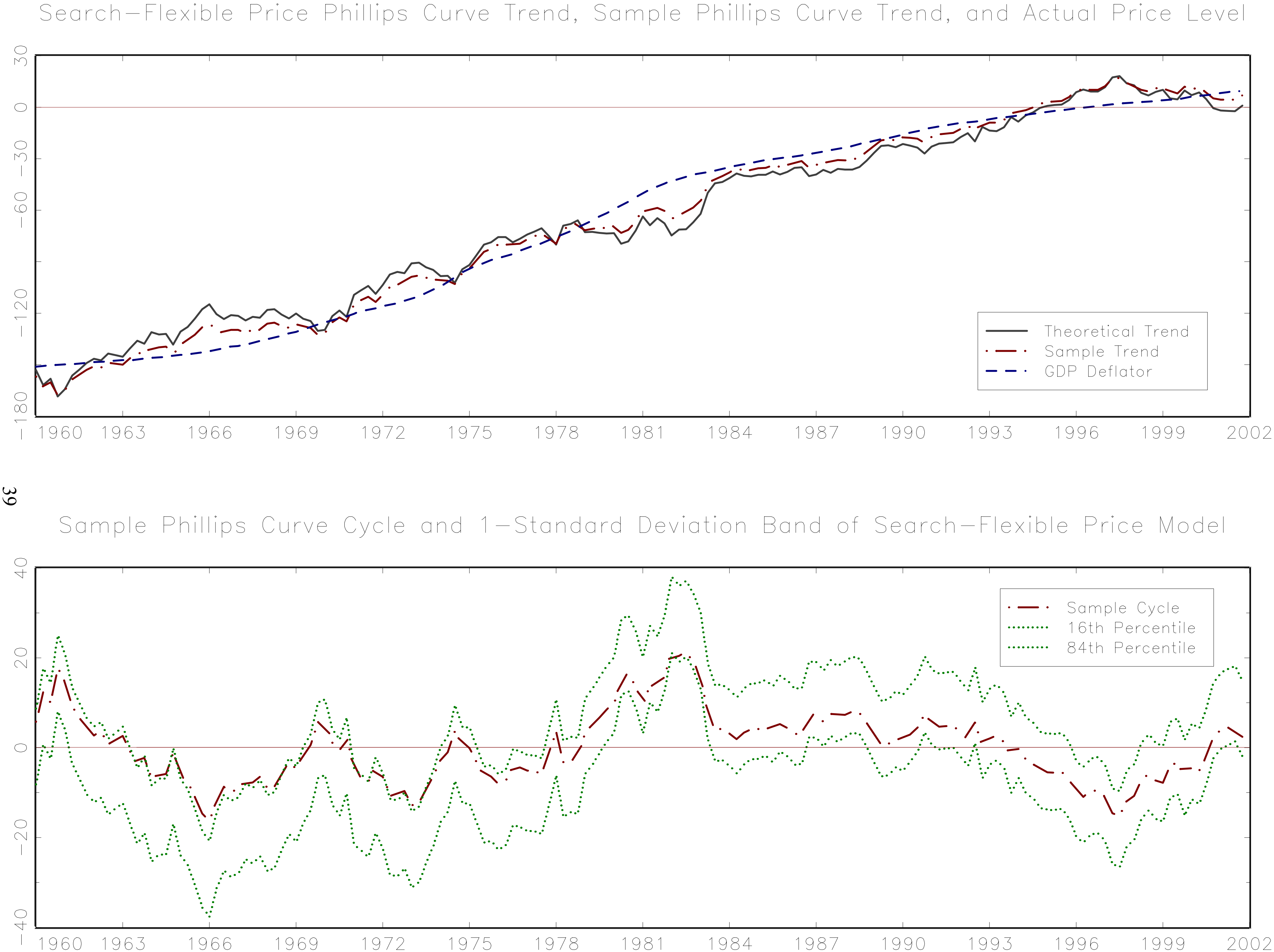


Figure 5: Search–Flexible Price DSGE Model



Appendix

This appendix contains details of the timing of the DSGE models, the labor market search structure, and the income and wealth insurance scheme we develop to construct the aggregate household of the labor market search DSGE models of our paper.

A1. Events and Decisions in Monetary DSGE Models with Labor Market Search

The timing of the economy follows. Households enter date t with the physical and financial assets accumulated at the end of date $t-1$. Date t begins with the exogenous shocks of the economy being realized. Successfully matched firms and households strike labor contracts over real wages and hours. Firms with active plant-job rent capital from households at this moment to combine with contracted hours and the exogenous level of technology to generate output to sell into the final goods market. When the law of motion (1) guides aggregate price dynamics, those firms able to alter their output price do so just prior to production because supply adjusts to meet demand in Keynesian model. Next, the money and bonds market open given households have made decisions about accumulating capital, financial wealth, and cash to carry into date $t+1$. At this moment, firms with latent plant-jobs and not-employed households search for one another. Along with existing job matches (that do not separate), the flow of new job matches are carried forward as date $t+1$ aggregate employment.

A2. Elements of a Monetary DSGE Model with Labor Market Search

Section $A2.1$ develops the optimal search decisions a firm employs to fill its job-plant vacancies. We construct the complete insurance that is the underpinning of the representative household in section $A2.2$. Section $A2.3$ presents the optimality conditions of the aggregate household with respect to labor market search. Aggregate optimality and equilibrium conditions appear in section $A2.4$

A2.1 The Firm and Labor Market Search

Firms either produce with a plant-job, are engaged in search to fill an empty plant-job, or leave it latent. If the final good firm fills a vacancy, the benefit to this firm equals the capitalized value of the active plant-job at date $t+1$, $\mathcal{J}_{t+1}$, with probability $\omega_{V,t}$. An unsuccessful search forces the final good firm to consider the future capitalized value of the latent plant-job, $\mathcal{Q}_{t+1}$, given the probability it is not filled, $1-\omega_{V,t}$. Hence, the capitalized value of a latent job-plant evolves according to $\mathcal{Q}_t = -\Gamma_t Z_{V,t} + \beta\mathbf{E}_t\{\omega_{V,t}\mathcal{J}_{t+1} + (1-\omega_{V,t})\mathcal{Q}_{t+1}\}$, where $Z_{V,t}$ denotes date t job-recruitment costs.[A.1] Free entry into the final good sector requires $\mathcal{Q}_t = 0$ because firms have an incentive to activate a vacancy when $\mathcal{Q}_t > 0$, or to close a plant down when $\mathcal{Q}_t < 0$. Thus, along an equilibrium path the optimality condition

$$\omega_{V,t}\mathbf{E}_t\mathcal{J}_{t+1} = \Gamma_t Z_{V,t}, \tag{A2.1.1}$$

equates the expected capitalized value of a future active plant-job to the discounted cost of filling the vacancy.

Labor market search forces firms to recognize that an active plant-job represents an ongoing employment relationship. Since job matches last more than one date, firms treat active plant-jobs as capitalized assets. This implies the law of motion of aggregate plant-jobs faced by finals goods firms is $N_{t+1} = (1-\delta_N)N_t + \omega_{V,t}V_t$, where the exogenous non-stochastic job separation rate is $\delta_N \in (0,\,1)$ and, on average, $\omega_{V,t}V_t$ vacancies are filled at date t.

An active plant-job's capitalized value can also be measured with its expected discounted profit flow, $\mathcal{J}_t = \mathbf{E}_t\left\{\sum_{i=0}^{\infty}(1-\delta_N)^i\prod_{j=0}^{i}\Gamma_{t+j}\;D_{t+i}\right\}$, where discounting involves $(1-\delta_N)$ because active matches separate at the non-stochastic rate δ_N. This yields the law of motion

$$\mathcal{J}_t = \Gamma_t D_t + (1-\delta_N)\mathbf{E}_t\{\Gamma_t\mathcal{J}_{t+1}\}, \tag{A2.1.2}$$

where the aggregate real dividend process is $D_t/P_t = (P_{A,t}/P_t)^{\xi}\left[1-\theta\phi_t\right]Y_{A,t} - (R_{K,t}/P_t)K_t - (W_t/P_t)N_t h_t - Z_{V,t}V_t$, in the symmetric equilibrium.

A2.2 Risk Sharing in the Household Sector

The employed and not-employed comprise the household sector. These households make decisions about capital accumulation, financial portfolios (to hold cash and government bonds), labor supply (employment, hours, and wages), and job search (effort). The penultimate decisions are associated with the employed while the latter activity pertains to the not-employed.

[A.1] We assume that the transitory component of $Z_{V,t}$ is non-stochastic.

A2.2.1 Risk Sharing in the Household Sector

An employed household enjoys the benefits and suffers the costs of an active job match. The non-pecuniary benefits and costs are summarized by this household's felicity function

$$u(c_{E,M,t},\ c_{E,L,t},\ \ell_t) \quad \equiv \quad \psi_1 ln[c_{E,M,t}] \ + \ (1 - \psi_1) ln[c_{E,L,t}] \ + \ \psi_3 \frac{\ell_{E,t}^{1-\psi_2}}{1 - \psi_2},$$

where $0 < \psi_1 < 1$, $\psi_2 \neq 1$, $0 \leq \psi_3$, $c_{E,M,t}$, $c_{E,L,t}$, and $\ell_{E,t}$ $(= 1 - h_t)$ are cash consumption, credit consumption, and leisure of the employed household, respectively. The employed household faces the budget constraint

$$D_{E,t} \ + \ R_{K,t}\, k_{E,t} \ + \ W_t\, h_t \ + \ (1 + R_{B,t})\, B_{E,G,t} \ + \ M_{E,t} \ - \ A_{E,t+1}$$

$$= \quad P_t[c_{E,M,t} \ + \ c_{E,L,t} \ + \ k_{E,t+1} \ - \ (1 - \delta_K)\, k_{E,t} \ + \ \tau_{N,t} T_{N,t} \ + \ T_t],$$

the cash-in-advance (CIA) constraint $M_{E,t} \ \geq \ P_t\, c_{E,M,t}$, and the wealth constraint

(A2.2.1) $$A_{E,t} \quad \geq \quad B_{E,G,t} \ + \ M_{E,t} \ - \ \tau_{X,t-1} T_{X,t-1} \ - \ X_{t-1},$$

where $\delta_K \in (0,1)$, and $D_{E,t}$, $B_{E,G,t}$, $M_{E,t}$, $A_{E,t+1}$, $\tau_{N,t}$, T_t, $\tau_{X,t-1}$, and X_{t-1} are the dividends the employed household receives from final goods firms, the government bonds this household owns at the beginning of date t, the cash the employed household carries over from the end of date $t-1$, the nominal wealth the employed household takes from the end of date t into the beginning of date $t+1$, the tax levied on employed households to pay for income insurance, $T_{N,t}$, a lump-sum tax, the tax levied on the cash injection received by employed households to pay for the wealth insurance, $T_{X,t}$, that not-employed households receive, and the total cash injection, respectively. Cash earns a zero nominal return. The government pays $R_{B,t}$ on its one-period unit discount bond.

An employed household enjoys an ongoing relationship with a plant-job of a final good firm. The ongoing nature of this relationship occurs because the job match continues from date t into date $t+1$ with probability $1-\delta_N$. Given a not-employed household exerts efforts to move into employed status, the probability a job match occurs is $\omega_{S,t}$. In this case, the law of motion of the measure of employed households becomes

(A2.2.2) $$N_{t+1} \quad = \quad (1 - \delta_N) N_t \ + \ \omega_{S,t}(1 - N_t) S_t,$$

where $\omega_{S,t}(1 - N_t) S_t$ equals the measure of successful job searches by the not-employed.

Besides the obvious difference, an employed household differs from a not-employed household because the latter puts effort into finding a match and employment. This implies the felicity function of the typical not-employed household is

$$u(c_{S,M,t},\ c_{S,L,t},\ \ell_t) \quad \equiv \quad \psi_1 ln[c_{S,M,t}] \ + \ (1 - \psi_1) ln[c_{S,L,t}] \ + \ \psi_5 \frac{\ell_{S,t}^{1-\psi_4}}{1 - \psi_4},$$

where $\psi_4 \neq 1$, $0 \leq \psi_4$, $c_{S,M,t}$, $c_{S,L,t}$, and $\ell_{S,t}$ $(= 1 - S_t)$ are cash consumption, credit consumption, and leisure of the not-employed household, respectively.

In all respects save one, the budget constraint of the not-employed household is the same as the budget constraint of the employed household. The disparity between the budget constraints is the not-employed household receives a government income transfer that replaces wage income. As a result, the budget constraint of the not-employed household is

$$D_{S,t} \ + \ R_{K,t}\, k_{S,t} \ + \ (1 + R_{B,t})\, B_{S,G,t} \ + \ M_{S,t} \ - \ A_{S,t+1}$$

$$= \quad P_t[c_{S,M,t} \ + \ c_{S,L,t} \ + \ k_{S,t+1} \ - \ (1 - \delta_K) k_{S,t} \ + \ (1 - \tau_{N,t}) T_{N,t} \ + \ T_t],$$

where the subscript S denotes the not-employed household. However, the CIA constraint of the not-employed household maintains the form $M_{S,t} \geq P_t\, c_{S,M,t}$.

The wealth constraint of the not-employed household differs from that of a employed household. The not-employed faces transactions costs when it searches. We assume these transactions costs rise with search effort in a linear fashion at rate φ (> 0). Hence, the not-employed household requires cash to engage in job search. Since the not-employed household faces a CIA constraint, the cash injection from the government represents the only available cash to pay the transactions search cost. In this case, the wealth constraint becomes

(A2.2.3) $$A_{S,t} \ \geq \ B_{S,G,t} \ + \ M_{S,t} \ + \ (1 - \tau_{X,t-1} T_{X,t-1}) \ - \ (1 - \varphi S_{t-1}) X_{t-1},$$

where the not-employed household receives a nominal wealth transfer of $1 - \tau_{X,t-1} T_{X,t-1}$. The next section discusses the set of government policies necessary for this transfer and the income transfer to make the distribution of capital and financial wealth independent of household employment histories.

A.2.2.2 The Government

The government engages in monetary, fiscal, (real) income and (nominal) wealth insurance operations. Besides its expenditure, G_t, and tax collecting, T_t, activities, the government injects a total of X_t units of cash into the household sector, and conducts open market operations (OMOs) by issuing one-period unit discount bonds, $B_{G,t+1}$. Governmental social insurance policy provides actuarially fair income and wealth insurance to households. This implies household resource allocations arise without consideration of past employment by any household. The intertemporal budget constraint

$$P_t T_t \ + \ P_t N_t \tau_{N,t} T_{N,t} \ + \ \tau_{X,t} T_{X,t} \ + \ (B_{G,t+1} - B_{G,t}) \ + \ (M_{t+1} - M_t)$$

$$= \quad P_t G_t \ + \ P_t(1 - N_t)(1 - \tau_{N,t}) T_{N,t} \ + \ (1 - \tau_{X,t}) T_{X,t} \ + \ R_{B,t} B_{G,t} \ + \ X_t,$$

records government accounts across this range of activities.

Complete insurance requires the government chooses $\tau_{N,t}$ and $\tau_{X,t}$ to equate the (shadow) prices of the budget constraint and wealth constraint across employed and not-employed households. When complete income insurance prevails, the government sets $\tau_{N,t} = 1 - N_t$ to yield $P_t T_t = W_t h_t$. The income (insurance) tax rate households face equals the probability they will need this insurance. Thus, employed and not-employed households enjoy the same level of consumption.[A.2]

Similarly, the government achieves complete wealth insurance when $\tau_{X,t} = 1 - N_t$ so that $T_{X,t} = -\varphi S_t X_t$.[A.3] For the wealth of the not-employed to be fully insured, government transfers more of the cash injection employed households receive to not-employed households as their search hours rise, taking the cash injection X_{t-1} parametrically.

The government's intertemporal budget constraint becomes

$$\text{(A2.2.4)} \quad P_t T_t \ + \ (B_{G,t+1} - B_{G,t}) \ + \ (M_{t+1} - M_t) \quad = \quad P_t G_t \ + \ R_{B,t} B_{G,t} \ + \ X_t,$$

in the presence of complete income and wealth insurance. Equation (A2.2.4) is the government's intertemporal budget constraint, equation (11), of the paper.

[A.2] This results rests on the felicity of the employed and not-employed being separable in consumption and leisure and log in consumption. Andolfatto (1996) lays out further assumptions and restrictions necessary and sufficient for this type result. In particular, the aggregate household is assumed to be engaged in randomly handing out job matches to its members period-by-period. This separates the worker flows from the flow of ongoing matches at plant-job level. Also, the value function of the aggregate household has to be concave in its arguments.

[A.3] This follows from complete wealth insurance imposing equality on the shadow prices of wealth of employed and not-employed households which requires $\tau_{X,t} T_{X,t} + X_t = (1 - \varphi S_t) X_t - (1 - \tau_{X,t}) T_{X,t}$.

A.2.2.3 The Aggregate Household

The outcome of the government employment insurance program yields the aggregate household. Aggregation of a typical employed and not-employed household produces the value function, $\mathcal{V}_t$, and its Bellman's equation

$$\mathcal{V}_t \quad = \quad \mathbf{Max}_{\{c_{M,t},\, c_{L,t},h_t,\, h_t,\, M_t,\, B_{G,t},\, K_{t+1},\, N_{t+1}\}} \Bigg[\psi_1 ln[c_{M,t}] \; + \; (1 \, - \, \psi_1) ln[c_{L,t}]$$

$$+ \quad N_t \psi_3 \frac{(1 \, - \, h_t)^{1-\psi_2}}{1 \, - \, \psi_2} \; + \; (1 \, - \, N_t) \psi_5 \frac{(1 \, - \, S_t)^{1-\psi_4}}{1 \, - \, \psi_4} \Bigg] \; + \; \beta \mathbf{E}_t \left\{ \mathcal{V}_{t+1} \right\},$$

where $\mathcal{V}_t \; \equiv \; \mathcal{V}(K_t,\, N_t,\, M_t,\, B_{G,t},\, Z_t,\, m_t)$. The constraints the aggregate household faces are

(A2.2.5) $$D_t \; + \; R_{K,t}\, K_t \; + \; W_t\, h_t \; + \; (1 \, + \, R_{B,t})\, B_{G,t} \; + \; M_t \; - \; A_{t+1}$$

$$= \quad P_t[c_{M,t} \; + \; c_{L,t} \; + \; K_{t+1} \; - \; (1 \, - \, \delta_K) K_t \; + \; T_t],$$

and the CIA constraint

(A2.2.6) $$M_t \quad \geq \quad P_t\, c_{M,t}.$$

which follow from the government provision of full income and wealth insurance.[A.4] Given complete wealth insurance, aggregation of the wealth constraints (A2.2.1) and (A2.2.3) produces

(A2.2.7) $$A_t \quad \geq \quad B_{G,t} \; + \; M_t \; - \; [1 - (1 \, - \, N_{t-1}) \varphi S_{t-1}] X_{t-1}.$$

Along with (A2.2.5), (A2.2.6), and (A2.2.7), the aggregate household faces the law of motion of the measure of employed households, (A2.2.2), given the laws of motion of the exogenous shocks.

[A.4]In an equilibrium with complete income and wealth insurance, it becomes apparent that $K_{t+1} = k_{E,t+1} = k_{S,t+1}$, $D_{t+1} = D_{E,t+1} = D_{S,t+1}$, $M_{t+1} = M_{E,t+1} = M_{S,t+1}$, and $B_{G,t+1} = B_{E,G,t+1} = B_{S,G,t+1}$. This assumes that employed and not-employed households hold equal endowments of capital and financial wealth at date zero. Further, we assume away any wealth disparities that are caused by ownership claims on final goods firms. However, if employed and not-employed households are initially given equal equity stakes in final goods firms, the dividend flows will be equalized. Also, these results depend on the additively separable character of the felicity functions, as already noted.

A2.3 The Household and Labor Market Search

The impact of labor market search on aggregate household felicity, the aggregate wealth constraint, and the law of motion of aggregate employment leads to the optimal job-search condition

$$\text{(A2.3.1)} \qquad \frac{\psi_5}{(1 - S_t)^{\psi_4}} + \varphi \Gamma_t \frac{X_t}{P_t} = \omega_{S,t} \Lambda_t,$$

where Λ_t is the shadow price of an active job-match. The optimality condition (A2.3.1) equates the dis-felicity of job-search plus the associated transaction-search costs to the probability, $\omega_{S,t}$, of a successful job match, valued at the shadow price of the marginal match, Λ_t.

The aggregate household's optimal choice of aggregate employment, N_{t+1}, yields the Euler equation

$$\text{(A2.3.2)} \qquad \Lambda_t = \beta \mathbf{E}_t \left\{ \frac{\partial \mathcal{V}_{t+1}}{\partial N_{t+1}} \right\}.$$

The Euler equation (A2.3.2) sets the value of an additional job match equal to the discounted expected value of continuing the match. Labor market optimality also depends on the envelope condition

$$\text{(A2.3.3)} \qquad \frac{\partial \mathcal{V}_t}{\partial N_t} = \mathcal{H}_t + \Gamma_t \frac{W_t}{P_t} h_t + \varphi \Gamma_t \frac{X_t}{P_t} S_t + [(1 - \delta_N) - \omega_{S,t}\, S_t] \Lambda_t,$$

where $\mathcal{H}_t \equiv \psi_3 (1 - \psi_2)^{-1} (1 - h_t)^{1-\psi_2} - \psi_5 (1 - \psi_4)^{-1} (1 - S_t)^{1-\psi_4}$. The envelope condition (A2.3.3) is the sum of the benefits the aggregate household receives from an ongoing job match, the change in felicity from moving a household from not-employed to employed status, the discounted real labor income, the foregone transactions-search costs, and the discounted value of the job match. The latter discounting accounts for the fixed rate of job separation net of the probability that job search is successful given search effort is S_t.

A2.4 Aggregate Optimality and Equilibrium

Remember that firms with plant jobs not in operation and the not-employed meet randomly and that we borrow the den Haan, Ramey, and Watson (2000) CRS matching technology

$$\text{(A2.4.1)} \qquad \mathcal{M}(V_t,\, (1 - N_t) S_t) = \frac{V_t [(1 - N_t) S_t]}{(V_t^{\vartheta} + [(1 - N_t) S_t]^{\vartheta})^{1/\vartheta}}, \quad 0 < \vartheta,$$

to place the not-employed in available job vacancies, where $V_t \equiv \int_0^1 \upsilon_{j,t} dj$ is total plant-job vacancies. Note that the probability of a successful match is influenced indirectly either with variation in posted vacancies or by changes in not-employed search effort. Denote the probability a vacant plant-job is filled $\omega_{V,t}$ $(= \mathcal{M}\ (V_t,\ (1 - N_t)S_t)/V_t)$ and that someone not-employed finds work $\omega_{S,t}$ $(= \mathcal{M}(V_t,\ (1-N_t)S_t)\ /[(1-N_t)S_t])$. The CRS search technology (A2.4.1) bounds $\omega_{V,t}$ and $\omega_{S,t}$ between zero and one. Given these probabilities, the aggregate law of motion of employment is

(A2.4.2) $$N_{t+1} \;=\; (1 - \delta_N)N_t \;+\; \mathcal{M}_t.$$

The stochastic flow of new job matches and non-stochastic rate of job destruction is a weakness of this class of labor market search model.

The optimality condition (A2.1.1) determines if a final good firm operates a plant-job. The surplus rule implies that the left-side of (A2.1.1) equals $[(1 - \zeta)/\zeta]\mathbf{E}_t \partial\mathcal{V}_{t+1}/\partial N_{t+1}$. Equate this expression with the Euler equation (A2.3.2) to find

(A2.4.3) $$\zeta\, \Gamma_t\, Z_{V,t} \;=\; (1 - \zeta)\, \omega_{V,t}\, \Lambda_t.$$

This optimality condition states that the discounted cost of posting job vacancies by firms equals the expected value of the job match to the household.

The optimality condition that completes the labor market substitutes for $\partial\mathcal{V}_{t+1}/\partial N_{t+1}$ in the Euler equation (A2.3.2) with the envelope condition (A2.3.3) and the optimality condition (14) of h_t using the equilibrium real wage generating process. This yields the Euler equation of the value of the marginal match

(A2.4.4) $$\Lambda_t \;=\; \beta\mathbf{E}_t\Bigg\{\zeta\Gamma_{t+1}\left[\left(\frac{P_{A,t+1}}{P_{t+1}}\right)^{\xi}\left(1 - \left[\frac{\theta\phi_{t+1}}{1-\overline{K}^*}\right]\right)\frac{Y_{A,t+1}}{N_{t+1}} \;+\; \frac{\upsilon_{t+1}\, V_{t+1}}{1 \,-\, N_{t+1}}\right] \;-\; \zeta\mathcal{H}_{X,t+1} \;+\; [(1 - \delta_N) \,-\, \omega_{S,t+1}\, S_{t+1}]\Lambda_{t+1}\Bigg\},$$

The current value of a plant-job match is forward-looking. It equals the expected discounted value of the date $t + 1$ equilibrium wage process (24) plus the net probability the match continues into date $t + 1$. The persistence of the endogenous state variable N_{t+1}, its shadow price Λ_{t+1}, and the non-Walrasian equilibrium wage process (24) propagate productivity and money growth shocks.

As noted in the paper Merz (1995), Andolfatto (1996), and Cooley and Quadrini (1999) assume the rule to split match surplus is $\mathcal{S}_t = \mathcal{J}_t + \partial \mathcal{V}_t / \partial N_t$ and that the aggregate household receives a fixed fraction, ζ, of $\mathcal{S}_t$ during each date t.[A.5] Since $\partial \mathcal{V}_t / \partial N_t = \zeta \mathcal{S}_t$ and $(1-\zeta)\ \partial \mathcal{V}_t / \partial N_t = \zeta\ \mathcal{J}_t$, the surplus splitting rule, the envelope condition (A2.3.3), the law of motion of $\mathcal{J}_t$, (A2.1.2), the aggregate dividend function (6), the rental rate of capital, and the optimality condition (A2.1.1) produces the Nash equilibrium real wage generating process

$$\text{(A2.4.5)}\quad \Gamma_t \frac{W_t}{P_t} h_t \;=\; \zeta \Gamma_t \left[\left(\frac{P_{A,t}}{P_t} \right)^{\xi} \left(1 - \left[\frac{\theta \phi_t}{1 - \overline{K}^*} \right] \right) \frac{Y_{A,t}}{N_t} + \frac{\upsilon_t V_t}{1 - N_t} \right] + (1-\zeta) \mathcal{H}_{X,t},$$

where $\mathcal{H}_{X,t} = \psi_3\ (1-\psi_2)^{-1}\ (1-h_t)^{1-\psi_2}$ - $\psi_5\ (1-\psi_4)^{-1}\ (1-S_t)^{1-\psi_4} - \varphi \Gamma_t X_t S_t / P_t$ and $\overline{K}^*$ is the steady state of $\overline{K}_t$. Equation (A2.4.5) is the equilibrium real wage (24) of the paper.

A2.5 The Model Solution

Numerical solution of the labor market search models requires the linearization of the (detrended) optimality and equilibrium conditions: consumption function (13), the cost to firms of posting vacancies (A2.4.3), the search cost of the not-employed (A2.3.1), employed labor supply (14) evaluated using the real wage generating process (A2.4.5), the Euler equation of capital (18), and the law of motion of Λ_{t+1}, (A2.4.4), the law of motion of employment (A2.4.2), and the aggregate resource constraint (23). This adds N_{t+1} to the state vector $\mathcal{K}_{t+1}$ and the shadow price of employment Λ_t to the control vector $\mathcal{C}_t$. The solution algorithm of the labor market search-sticky price model is the same that is described in section 4.2 for the Yun-sticky price model. Note that this solution linearizes the law of motion that generates the NKPC-PV relation (4). Under the flexible price regime, price expectations drop out of the standard solution state vector $\mathcal{K}_{t+1}$ and theoretical real marginal cost, ϕ_t, is eliminated from the control vector $\mathcal{C}_t$. This permits standard solution methods to be engaged, as in Zadrozny (1998).

[A.5] In the negotiations between a final good firm and the aggregate household, $1-\zeta$ reflects the power the aggregate household exerts on equilibrium real wages and hours.