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Trading Volume and the Number of Trades: A Comparative Study Using High Frequency Data

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Abstract

Trading volume and the number of trades are both used as proxies for market activity, with disagreement as to which is the better proxy for market activity. This paper investigates this issue using high frequency data for Cisco and Intel in 1997. A number of econometric methods are used, including GARCH augmented with lagged trading volume and number of trades, tests based on moment restrictions, regression analysis of volatility on volume and trades, normality of returns when standardized by volume and number of trades, and Correlation analysis using volatility generated from GARCH and realized volatility. Our results show that the number of trades is the better proxy for market activity.

Keywords: Trading volume; number of trades; realized volatility, GARCH volatility, Mixture of distribution hypothesis.

I. Introduction

The volatility-volume relation is central to many models in finance and economics. Since the early 1970s, the relation between trading volume and stock prices volatility has been widely investigated in an impressive body of empirical and theoretical literature. The first treatment of the relation goes back to Osborne (1959) in his attempt to model the stock price change as a diffusion process with volatility related to the number of transactions. This was followed by the work of Ying (1966) and Crouch (1970), who find a statistically significance positive correlation between absolute returns and daily volumes for both market indices and individual stocks. Clark (1973) finds a positive relation between squared returns and aggregated volume using daily data from the cotton futures market. Westerfield (1977) finds a similar relation in a sample of returns and volumes for a number of common stocks, as do Tauchen and Pitts (1983) using daily data from the Treasury Bill futures market. Epps and Epps (1976) find a positive relation between the sample variances of returns at given volume levels using transactions from 20 stocks. Harris (1986, 1987) finds a positive correlation between volume and the square of the price change using daily data. Moreover Karpoff (1987, 1988), Lamaourex and Lastrapes (1990, 1994), Liesenfeld (1998, 2001), Richardson and Smith (1994) and Tauchen and Pitts (1983) have all emphasized the role of volume as an activity variable.

All the work cited above on the volatility-volume relation comes under what is known as the Mixture of Distribution Hypothesis (MDH) model. The MDH model assumes volume and volatility are positively correlated, and such correlation arises due to positive association of both volume and volatility to the unforeseen information flow process. The MDH also tells us that volume is the best proxy for market activity; hence we expect the correlation between volume and any volatility proxy to be an increasing function of the accuracy of the volatility measure in use. However, this is not the case with the data we use here where the number of trades is found to show higher correlation with realized volatility than volume, which in turn suggests a volatility-number of trades relation as opposed to the volatility-volume relation implied by the MDH.

The support for the number of trades that we find in this paper is in line with a growing literature which tends to emphasize the role of the number of trades over volume. For example, March and Rock (1986) finds that the net number of trades has similar explanatory power as net volume. Jones *et al.*(1994) argue that trading volume has no informational content beyond that contained in the number of trades. As a result they suggest the use of the number of trades as a substitute for volume. More recent evidence can be found in the work of Easley and O'Hara (1992), Easley *et al.* (1997), Hasbrouk (1999).and Ané and Geman (2000).

This paper adds to the existing literature by comparing volume and the number of trades using high frequency data for Cisco and Intel in (1997). We consider a number of econometric procedures previously used to address the roles of these two activity variables which include: a) The direct test of the MDH model adopted by Richardson and Smith (1996). b) The augmenting of GARCH with volume as in Lamaourex and Lastrapes (1990) but extended to allow for the number of trades. c) The standardization of returns by volume and trades as set out in Harris (1986) and Ané and Geman (2000). Moreover we consider correlation analysis by which we look at the correlation between volume, the number of trades and volatility generated from a variety of commonly used GARCH models - *exponential* GARCH, *threshold* GARCH, GARCH in-the-mean, *fractional* GARCH, *fractional* EGARCH, *two components* GARCH - and realized volatility.

The outline of this paper is as follows. In Section II, we discuss the data and the econometric procedures to be used. In section III, we discuss our findings. We present our conclusions in section IV.

II. Data and Methodology

We use the Cisco and Intel high frequency data for 1997 as used in Ané and Geman (2000). We did not have access to Reuters - the source used by Ané and Geman (2000), so we use data from the Wharton Research Data Services website. We calculate the intra day (9.30 am to 4 pm) returns (r_t), volume (v_t) and the number of trades (n_t) at the 10, 30 and 60 minute and daily time intervals.

We consider a number of econometric procedures / methods as previously used to investigate the volatility-volume and volatility-number of trades relationships. These methods are described below.

The first method draws from the work of Richardson and Smith (1994), Andersen (1996), and Liesenfeld (1998, 2001).and is based on testing the moment restrictions implied by the MDH model using the Generalized Method of Moments (GMM) J- test of overidentifying restrictions.

The MDH assumes that, conditional on the information flow i_t, returns r_t and the observed "market activity" a_t (volume, log volume, the number of trades *etc*.) are independently and normally distributed as:

$$\begin{pmatrix} r_t \\ a_t \end{pmatrix} | \, i_t \sim N\left(\begin{pmatrix} \mu_{r_t} i_t \\ \mu_{a_t} i_t \end{pmatrix}, \begin{pmatrix} \sigma_{r_t}^2 i_t & 0 \\ 0 & \sigma_{a_t}^2 i_t \end{pmatrix} \right), \tag{1}$$

The model implies a set of moment restrictions that can be imposed on the data and evaluated using the GMM J-test of over-identifying restrictions. We consider the moment restrictions set out in equation 4 of Richardson and Smith (1994, p. 106) which can be written as follows:

$$\begin{aligned}
m_1^r &= \mu_r m_1^i \\
m_2^r &= \sigma_r^2 m_1^i + \mu_r^2 m_2^i \\
m_3^r &= 3\mu_r \sigma_r^2 m_2^i + \mu_r^3 m_3^i \\
m_1^a &= \mu_a m_1^i \\
m_2^a &= \sigma_a^2 m_1^i + \mu_a^2 m_2^i \\
m_3^r &= 3\mu_a \sigma_a^2 m_2^i + \mu_a^3 m_3^i \\
m_{11}^{ra} &= \mu_r \mu_a m_2^i \\
m_{12}^{ra^2} &= \sigma_a^2 \mu_r m_2^i + \sigma_a^2 \mu_r m_3^i \\
m_{21}^{ar^2} &= \sigma_r^2 \mu_a m_2^i + \sigma_r^2 \mu_a m_3^i
\end{aligned} \tag{2}$$

where, $m_1^i, ..., m_3^i$ denote the first three (central) moments of information, $m_1^r, ..., m_3^r$ denote the first three unconditional (central) moments of returns, $m_1^a, ..., m_3^a$ denote the first three unconditional (central) moments of activity and $m_{11}^{ra}, m_{12}^{ra^2}, m_{21}^{ar^2}$ denote the co-variances between $(r, a), (r, a^2)$ and (a^2, r).

Murphy and Izzeldin (2007) point out, the parameters μ_r and σ_r^2 are only identified up to scale since i_t is not observed. Suppose i_t is replaced by κi_t with $\kappa > 0$ so that $m_1^i,...,m_3^i$ become $\kappa m_1^i,...,\kappa m_3^i$ in the moment conditions. Then μ_r / κ and σ_r^2 / κ satisfy the new moment conditions. Thus μ_r, σ_r^2 and m_1^i cannot be identified separately. To overcome this problem, we normalize the mean of unobserved information flow process m_1^i to one. Following normalization, the remaining system consists of nine moment conditions and six parameters to be estimated ($\mu_r, \mu_a, \sigma_r^2, \sigma_a^2, m_2^i, m_3^i$). This leaves three over-identifying restrictions which are evaluated using the J-test of over-identifying restrictions. For example, if J > 7.815, then we can reject the moment restrictions at the 5 % level. The activity variable whose moment restrictions best fit the data, can be taken as a good proxy of market activity. In other words, we seek to establish whether a volatility-volume relation or a volatility-number of trades is appropriate for the volatility-activity relation implied by the MDH.

The second method builds on the work of Lamaourex and Lastrapes (1990). Lamaourex and Lastrapes (1990) extends the GARCH model by augmenting the GARCH variance equation with trading volume. This augmented model better fits the data and accommodates for persistence in the GARCH volatility, a result which support the role of volume as a proxy for market activity. We replicate their exercise by adding lagged volume or lagged trades to the GARCH variance equation, and check whether lagged volume or the lagged trades better explains the GARCH effects.

We consider the basic GARCH model outlined in Lamaourex and Lastrapes (1990), but with the mean equation given by

$$r_t = c + \sigma_t u_t \tag{3}$$

and the three variance equations as shown below

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 \tag{4a}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 + \alpha_3 v_{t-1} \tag{4b}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 r_{t-1}^2 + \alpha_2 \sigma_{t-1}^2 + \alpha_4 n_{t-1} \tag{4c}$$

We then select the model which best fits the data using the Akaike information criterion (AIC), the significance of the coefficients on v_{t-1} and n_{t-1}, and the level of persistence given by the sum $\alpha_1 + \alpha_2$.

The third method involves comparing the performance of volume and the number of trades in explaining volatility changes. We consider the regressions outlined in Jones *et al.* (1994) and Ané and Geman (2000) which are as follows:

$$\hat{s}_t = a + \beta \Delta v_t + \sum_{j=1}^{12} \rho_j \left| s_{t-j} \right| + e_t \tag{5a}$$

$$\hat{s}_t = b + \gamma \Delta n_t + \sum_{j=1}^{12} \rho_j \left| s_{t-j} \right| + u_t \tag{5b}$$

$$\hat{s}_t = c + \beta \Delta v_t + \gamma \Delta n_t + \sum_{j=1}^{12} \rho_j \left| s_{t-j} \right| + \eta_t \tag{5c}$$

where a, b and c are constants, Δv_t and Δn_t are the first differences of volume and the number of trades, and $\hat{s}_t$ is the Schwert (1990) daily volatility measure.

To generate $\hat{s}_t$, we run a regression of the return r_t over 12 lagged returns as shown in equation 6 below:

$$r_t = \alpha + \sum_{j=1}^{12} \delta_j r_{t-j} + \varepsilon_t \tag{6}$$

We then define $\hat{s}_t$

$$\hat{s}_t = \sqrt{\frac{\pi}{2}} \left| \hat{\varepsilon}_t \right| \tag{7}$$

The standardization $\sqrt{\frac{\pi}{2}}$ follows from an elementary result on the Gaussian distribution which asserts that if $X \sim N(0, \sigma^2)$, then $E(|X|) = \sqrt{2/\pi\sigma}$.

In the fourth method we test one of the assumptions under the MDH which asserts that returns standardized by a good proxy for activity is normally distributed. Clark (1973) shows that returns subordinated/standardized using volume is normally distributed. Ané and Geman (2000) claims that returns standardized by the number of trades are normal. In our exercise we consider returns standardized by volume and returns standardized by the number of trades. The best activity proxy is the one which achieves a higher level of normality for the standardized returns.

Finally, we look at the correlation of volume and trades with volatility generated from GARCH (σ_{garch}), *exponential* GARCH(σ_{egarch}), *threshold* GARCH (σ_{tgarch}), GARCH in-the-mean (σ_{pgarch}), *fractional* GARCH (σ_{fgarch}), *fractional exponential* GARCH $(\sigma_{fegarch})$ and *two components* GARCH (σ_{2garch}). All these models are used extensively in the financial literature and have been found to provide a good fit to financial data. See for example, Bollerslev *et al.* (1994), Engle (2001) and Glosten (1993). We also consider the correlation between volume, trades and realized volatility. Realized volatility is defined as the sum of the intra-day squared returns which, in the absence of micro-structure effects, provides an unbiased and accurate measure of volatility. See, Andersen. *et al.* (2001) and Barndorff-Nielsen and Shephard (2001) for example. The realized volatility in our case is constructed by summing 5 minute intra-day squared returns to the daily interval.

III. Results

[Table 1 around here**]**

Table 1 reports some statistical properties for Cisco and Intel volumes, log volumes, trades and the number of trades. We scale volume by 1/100000 and the number of trades by 1/100 to make the results comparable. The higher mean and standard deviation of v_t and n_t for Intel over Cisco, indicate more activity for Intel. Log v_t and log n_t are more normal relative to v_t and n_t as shown by the Jarque-Bera test statistic. The table also shows the results for the Autoregressive fractionally integrated moving average, Arfima (p, d, q) model applied to $v_t, \log v_t, n_t$, and $\log n_t$. The fractional differencing parameter "d" shows higher values for $\log n_t$. High persistence is a stylized fact of a good volatility model. Hence it follows that a good activity proxy that is highly correlated with volatility should also possess high persistence. Since the number of trades is more persistent than volume indicates that the number of trades has more in common with volatility than volume. The results for v_t and n_t should not be taken as definitive? Since the custom is to apply the Arfima for the log series and not the level series.

[Table 2 around here]

Table 2 reports the estimated second and third moments of the information flow for Cisco and Intel along with the $\chi^2_{(3)}$ statistic of the J-test of over-identifying restrictions. For all the time intervals considered the bivariate moments with trades achieves a lower value for the J-test than that with volume except for the Cisco (60 minute) case, where the results show more support for volume. These results support the MDH model with both trading volume and the number of trades acting as mixing variables, but with greater emphasis on the role of the number of trades.

[Table 3 around here**]**

Table 3 reports the results of basic and augmented GARCH models, with lagged volume and number of trades. For all cases, GARCH augmented with lagged number of trades shows has lower AIC and lower persistence as given by $\alpha_1 + \alpha_2$. These results show that the number of trades enhances the fit of the GARCH model in a similar or better fashion to volume.

[Table 4 around here**]**

Table 4 reports the results of regression equations (5a, 5b and 5c) as outlined in section II. These provide a method by which to compare the performance of volume and the number of trades in explaining volatility changes. Our results show mixed support for volume and the number of trades. For example, in the Cisco case, the number of trades shows a higher $\bar{R}^2$ relative to volume at all the time intervals considered. At the 60 minute time interval the combined presence of volume and the number of trades renders volume insignificant. Moreover, regressions 5b and 5c are not statistically different from each other. This shows that the number of trades has more explanatory power than volume. Volume contains no extra information to that provided by the number of trades. For Intel, volume shows a higher $\bar{R}^2$ than the number of trades except for the 60 minute time interval, where $\bar{R}^2$ is higher for the number of trades. Moreover, and similar to the Cisco 60 minute case, the presence of the number of trades and volume renders the coefficient of volume insignificant.

If the Cisco results are taken to be more binding (since they tell the same story across all time intervals) we can conclude that the number of trades is more correlated with the Schwert (1990) volatility measure than is volume. Support for the number of trades is consistent with Jones *et al.* (1994) and Ané and Geman (2000) both of whom obtain (from a similar framework) results favoring the number of trades.

[Table 5 around here**]**

Table 5 reports the results for testing the normality of returns standardized by volume or the number of trades, both rescaled to have a mean of unity. Results obtained show returns standardized by the number of trades are more normal than those standardized using volume, as shown by the Jarque-Bera test statistic. The implication is that the numbers of trades possess more filtration power than volume and hence are able to remove some of the factors causing return non-normality. To the best of our knowledge, no study has managed to recover full returns normality using the number of trades or volume as standardizing variables.

[Table 6 around here**]**

Table 6 reports the correlation between the GARCH models, volume and the number of trades at the 60 minute time interval. We consider level correlations and log-correlations. For Cisco, level correlation shows that the number of trades is more closely correlated with GARCH models volatility than is volume. On the contrary, the log-correlation results show that volume is more correlated with these models. In the Intel case, results are mixed. At the level correlation most GARCH models show a higher correlation with the number of trades than with volume, with the exception of $\sigma_{fegarch}$. Using log correlation, σ_{garch}, σ_{tgarch}, and σ_{pgarch} shows a higher correlation with the number of trades than with volume, whereas σ_{egarch}, σ_{fgarch}, $\sigma_{fegarch}$, σ_{2garch} are more correlated with volume than with the number of trades. Therefore the outcome of this exercise is ambiguous and depends on the functional form which best describes the relation between volatility and activity.

[Table 7 around here**]**

Table 7 shows the correlation between realized volatility, volume and the number of trades. Our results show that realized volatility is more correlated with the number of trades than with volume. In contrast with the correlation for GARCH models, this result holds for both level and log-correlation. Given that realized volatility is considered more accurate than GARCH generated volatility, the results in table 7 have greater credibility: the number of trades is a better proxy for market activity than volume.

IV. Conclusion

A number of econometric methods including GARCH augmented with lagged volume or number of trades, Tests based on moment restrictions and Correlation analysis using volatility generated from GARCH and realized volatility are considered to decide which is the more appropriate measure of market activity: (i) volume or (ii) the number of trades.

Our general conclusion confirms other findings from recent literature: that the number of trades is a better measure of market activity than volume. Our results show that the volatility-volume relationship implied by the Mixture of Distribution Hypothesis model could also be stated as a volatility-number of trades relationship.

Our study can be extended in various ways. First: to address the question of why the number of trades is a better proxy than trading volume. Work would be necessary at the microstructure level to identify differences and similarities. Second: to extend our results using other measures of volatility, such as implied volatility and realized range. Other measures of correlation might also be examined, for example, Copulas and frequency domain based measures of correlation such as Coherency. Third: to investigate whether forecasts based on GARCH could be enhanced by using GARCH augmented with the number of trades.

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Table 1. Statistical properties of volume and the number of trades

	Cisco				Intel			
	v_t	$\log v_t$	n_t	$\log n_t$	v_t	$\log v_t$	n_t	$\log n_t$
	10 Minute				**10 Minute**			
Mean	2.679	0.667	2.508	0.661	3.784	1.057	4.096	1.180
Std	2.352	0.825	2.099	0.717	3.187	0.762	3.551	0.654
Skewness	2.720	-0.325	3.329	-0.09	3.712	-0.338	4.866	0.254
Kurtosis	16.46	3.782	25.22	4.172	33.68	3.633	53.27	3.429
JB	861111	423	219827	576	405609	349	10677603	180
Arfima (d)	0.278	0.392	0.266	0.420	0.209	0.440	0.243	0.360
S.e	(0.032)	0.025	(0.031)	(0.021)	(0.040)	(0.021)	(0.042)	(0.022)
BIC	35814	15502	34823	10402	40794	12148	44949	7375
ADF	-5.643	-5.317	-5.497	-5.025	-6.766	-6.898	-5.774	-5.650
	30 Minute				**30 Minute**			
Mean	8.099	1.833	7.534	1.795	11.322	2.193	12.256	2.296
Std	6.552	0.727	5.704	0.670	8.867	0.718	9.631	0.647
Skewness	2.6089	-0.240	2.554	-0.154	3.949	-1.065	4.007	-0.548
Kurtosis	14.68	5.253	14.17	5.313	39.28	11.87	37.06	10.450
JB	222297	722	20566	741	187931	11348	166943	7738
Arfima (d)	0.388	0.413	*0.100*	0.436	0.381	0.396	0.415	0.478
S.e	(0.043)	(0.038)	*(0.200)*	(0.000)	(0.039)	(0.039)	(0.038)	(0.036)
BIC	18939	4180	18418	3259	20993	4165	2157	3060
ADF	-5.927	-5.325	-5.542	-5.182	-7.256	-8.389	-6.194	-6.503
	60 Minute				**60 Minute**			
Mean	15.04	2.473	13.999	2.430	21.036	2.834	22.771	2.934
Std	11.857	0.696	10.203	0.650	16.130	0.679	17.297	0.615
Skewness	2.564	-0.544	2.307	-0.485	4.145	-1.262	3.833	-0.704
Kurtosis	13.510	10.500	11.08	10.260	40.310	15.60	31.480	14.070
JB	10032	4211	6343	3930	107160	12116	63832	9132
Arfima(d)	*-0.068*	*0.099*	-0.118	0.477	0.426	0.439	*0.046*	0.445
S.e	*(0.040)*	*(0.048)*	(0.037)	(0.080)	(0.061)	(0.058)	*(0.041)*	(0.058)
BIC	13119	2957	12549	2538	14318	2976	14421	2479
ADF	-5.075	-4.927	-5.208	-4.690	-6.880	-7.212	-6.265	-5.884

Notes: 1.Variables in italics are found not significant at the 5% level.

2. v_t denotes volume, n_t denotes the number of trades., Std denotes standard deviation.,. JB denotes Jarque-Bera test statistic,. BIC denotes Bayesian information criterion, S.e denotes standard error.

3. Arfima (Autoregressive Fractionally Integrated Moving Average) and d is the fractional differencing parameter.

4. ADF denotes Augmented Dickey Fuller Test. The 5% and 1% critical values are -2.862 and -3.433.

5. Truncation lags for ADF were chosen according to the AIC (Akaike information criterion) and are 37, 20 and 24 for the 10, 30 and 60 minutes frequencies.

Table 2. Estimated moments of information and J-test of over identifying restrictions

	Cisco			Intel		
	m_2^i	m_3^i	$J \sim \chi^2_{(3)}$	m_2^i	m_3^i	$J \sim \chi^2_{(3)}$
	10 Minute			**10 Minute**		
Bivariate moments with volume	0.761 (0.091)	1.769 (0.252)	4.373 (0.224)	1.399 (0.272)	5.145 (1.741)	8.906 (0.031)
Bivariate moments with trades	0.838 (0.044)	2.270 (0.253)	1.916 (0.590)	1.287 (0.268)	5.316 (1.560)	2.212 (0.530)
Moments of re-centered volume	0.707	1.439		0.666	1.969	
Moments of re-centered trades	0.540	0.851		0.578	1.687	
	30 Minute			**30 Minute**		
Bivariate moments with volume	0.781 (0.084)	1.664 (0.311)	0.218 (0.975)	0.751 (0.090)	1.877 (0.466)	6.211 (0.102)
Bivariate moments with trades	0.881 (0.084)	1.926 (0.339)	0.122 (0.989)	0.863 (0.124)	2.220 (0.524)	3.517 (0.318)
Moments of re-centered volume	0.548	0.896		0.506	1.127	
Moments of re-centered trades	0.438	0.558		0.457	0.890	
	60 Minute			**60 Minute**		
Bivariate moments with volume	0.768 (0.102)	1.546 (0.325)	0.612 (0.894)	0.518 (0.069)	0.954 (0.386)	5.203 (0.157)
Bivariate moments with trades	0.765 (0.081)	0.081 (0.263)	1.217 0.749	0.526 (0.070)	0.851 (0.279)	4.688 (0.196)
Moments of re-centered volume	0.468	0.713		0.415	0.782	
Moments of re-centered trades	0.388	0.486		0.395	0.758	

Notes: 1. GMM estimates are based on the following 9 conditions - the first three moments of returns (r), the first three moments of "activity" a and the covariance's of (r, a), (r, a^2), (r^2, a)

2. m_2^i and m_3^i are the second and third moments of the information flow. The values in brackets below m_2^i and m_3^i are standard errors.

3. J denotes the test of over-identifying restrictions and is distributed as a $\chi^2_{(3)}$. At 3 degrees of freedom the critical value at the 5 % significance level is 7.851. The values in brackets below the J-test are p-values

Table 3. GARCH, GARCH + lagged volume and GARCH + lagged number of trades

Models	Cisco GARCH	Cisco GARCH + v_t (-1)	Cisco GARCH + n_t (-1)	Intel GARCH	Intel GARCH + v_t (-1)	Intel GARCH + n_t (-1)
		10 Minute			**10 Minute**	
α_1	0.232 (0.004)	0.202 (0.002)	0.208 (0.000)	0.190 (0.005)	0.164 (0.006)	0.151 (0.006)
α_2	0.724 (0.004)	0.667 (0.003)	0.615 (0.003)	0.755 (0.003)	0.643 (0.007)	0.553 (0.010)
α_3		0.008 (0.000)			0.004 (0.000)	
α_4			0.014 (0.000)			0.0079 (0.000)
$\alpha_1 + \alpha_2$	0.956	0.879	0.823	0.945	0.807	0.704
AIC	9048	8788	8622	4123	3780	3620
		30 Minute			**30 Minute**	
α_1	0.128 (0.008)	0.198 (0.015)	0.185 (0.015)	0.234 (0.017)	0.221 (0.021)	0.164 (0.020)
α_2	0.835 (0.009)	0.558 (0.011)	0.556 (0.011)	0.522 (0.022)	0.222 (0.022)	0.222 (0.021)
α_3		0.145 (0.006)			0.012 (0.000)	
α_4			0.018 (0.005)			0.014 (0.000)
$\alpha_1 + \alpha_2$	0.963	0.756	0.741	0.756	0.443	0.386
AIC	6669	6558	6549	5231	5034	4988
		60 Minute			**60 Minute**	
α_1	0.121 (0.011)	0.093 (0.009)	0.117 (0.010)	0.107 (0.014)	0.098 (0.014)	0.062 (0.012)
α_2	0.839 (0.013)	0.823 (0.018)	0.841 (0.013)	0.791 (0.027)	0.800 (0.026)	0.825 (0.025)
α_3		0.0042 (0.000)			0.0001 (0.000)	
α_4			0.0044 (0.001)			0.001 (0.000)
$\alpha_1 + \alpha_2$	0.960	0.916	0.958	0.898	0.898	0.887
AIC	4668	4666	4671	4028	4029	4021

Notes: 1. The GARCH mean equation is given by $r_t = c + \sigma_t u_t$. v_t denotes volume and n_t denotes the number of trades.

2. Three specifications for the GARCH variance equation are considered:

a) $\sigma_t^2 = \alpha_0 + \alpha_1 r_t^2 + \alpha_2 \sigma_{t-1}^2$,

b) $\sigma_t^2 = \alpha_0 + \alpha_1 r_t^2 + \alpha_2 \sigma_{t-1}^2 + \alpha_3 v_{t-1}$

,c) $\sigma_t^2 = \alpha_0 + \alpha_1 r_t^2 + \alpha_2 \sigma_{t-1}^2 + \alpha_4 n_{t-1}$

Table 4. Regression estimates for volume and the number of trades

$$r_t = \alpha + \sum_{j=1}^{12} \delta_j r_{t-j} + \varepsilon_t, \quad \hat{s}_t = \sqrt{\frac{\pi}{2}} \left|\hat{\varepsilon}_t\right|$$

$$\hat{s}_t = a + \beta \Delta v_t + \sum_{j=1}^{12} \rho_j \left|s_{t-j}\right| + e_t$$

$$\hat{s}_t = b + \gamma \Delta n_t + \sum_{j=1}^{12} \rho_j \left|s_{t-j}\right| + u_t$$

$$\hat{s}_t = c + \beta \Delta v_t + \gamma \Delta n_t + \sum_{j=1}^{12} \rho_j \left|s_{t-j}\right| + \eta_t$$

	v_t		n_t		v_t and n_t		
	β	$\bar{R}^2$	γ	$\bar{R}^2$	β	γ	$\bar{R}^2$
Cisco							
10 Minute							
Estimates	0.0443	0.200	0.0528	0.208	0.0148	0.0417	0.208
Standard errors	(0.0026)		(0.0026)		(0.0038)	(0.0039)	
30 Minute							
Estimates	0.3822e-02	0.246	0.4627e-03	0.254	0.7663e-03	0.3871e-03	0.254
Standard errors	(0.2323e-03)		(0.2579e-04)		(0.4912e-03)	(0.5943e-04)	
60 Minute							
Estimates	0.0236	0.184	0.0280	0.190	*0.4889e-02*	0.0227	0.190
Standard errors	(0.1743e-02)		(0.1999e-02)		*(0.5546e-02)*	(0.6383e-02)	
Intel							
10 Minute							
Estimates	0.0255	0.194	0.0158	0.186	0.0221	0.00367	0.194
Standard errors	(0.0015)		(0.0011)		(0.00218)	(0.0016)	
30 Minute							
Estimates	0.1821e-02	0.160	0.1425e-03	0.148	0.1859e-02	-0.3809e-05	0.160
Standard errors	(0.1281e-03)		(0.1141e-04)		(0.2810e-03)	(0.2486e-04)	
60 Minute							
Estimates	0.01334	0.142	0.013312	0.153	*-0.4868e-04*	0.013356	0.153
Standard errors	(0.9506e-03)		(0.8929e-03)		*(0.2971e-02)*	(0.2809e-02)	

Notes: 1. Numbers shown in italics are not significant at the 5% level.

2. r_t denotes returns, $\hat{s}_t$ denotes the Schwert (1990) daily volatility measure, v_t denotes volume, and n_t denotes the number of trades.

Table 5. Recovering normality using re-centered (volume and the number of trades)

	Cisco			Intel		
	r_t	$r_t / \sqrt{v_t}$	$r_t / \sqrt{n_t}$	r_t	$r_t / \sqrt{v_t}$	$r_t / \sqrt{n_t}$
	10 Minute			**10 Minute**		
Skewness	-2.776	-0.181	-6.840	-0.794	1.548	0.287
Kurtosis	121.404	97.338	461.169	49.082	39.314	18.001
JB	4.82e+06	3.05e+06	7.31e+07	766908	479126	81298
p-values	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	30 Minute			**30 Minute**		
Skewness	-1.303	-0.120	-0.312	0.658	3.593	1.053
Kurtosis	36.926	27.623	17.433	16.733	86.345	17.728
JB	142202	74473	25603	24341	895162	28309
p-values	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	60 Minute			**60 Minute**		
Skewness	-1.041	-0.548	-0.469	0.253	2.808	0.834
Kurtosis	15.259	7.545	6.340	5.102	56.090	11.363
JB	10412	1471	810	323	197475	5036
p-values	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Notes: 1. JB denotes Jarque-Bera test statistic.

2. v_t and n_t denotes volume and trades and which have been re-centered prior to the standardization so that to have a mean of unity.

Table 6. Correlation coefficients for volume and number of trades with GARCH models

	σ_{garch}	σ_{egarch}	σ_{tgarch}	σ_{pgarch}	σ_{fgarch}	$\sigma_{fegarch}$	σ_{2garch}	v_t	n_t
Cisco (60 Minute)									
Level - Correlation									
v_t	0.361	0.389	0.364	0.408	0.375	0.391	0.363	1.000	
n_t	0.374	0.406	0.373	0.419	0.388	0.408	0.377	0.954	1.000
Log - Correlation									
	σ_{garch}	σ_{egarch}	σ_{tgarch}	σ_{pgarch}	σ_{fgarch}	$\sigma_{fegarch}$	σ_{2garch}	v_t	n_t
v_t	0.462	0.466	0.459	0.463	0.474	0.469	0.463	1.000	
n_t	0.408	0.413	0.411	0.419	0.417	0.416	0.410	0.871	1.000
Intel (60 Minute)									
Level - Correlation									
	σ_{garch}	σ_{egarch}	σ_{tgarch}	σ_{pgarch}	σ_{fgarch}	$\sigma_{fegarch}$	σ_{2garch}	v_t	n_t
v_t	0.318	0.315	0.298	0.292	0.314	0.283	0.317	1.000	
n_t	0.329	0.326	0.329	0.326	0.316	0.281	0.324	0.945	1.000
Log - Correlation									
	σ_{garch}	σ_{egarch}	σ_{tgarch}	σ_{pgarch}	σ_{fgarch}	$\sigma_{fegarch}$	σ_{2garch}	v_t	n_t
v_t	0.316	0.321	0.300	0.301	0.328	0.313	0.323	1.000	
n_t	0.323	0.319	0.324	0.320	0.310	0.276	0.317	0.755	1.000

Notes: 1. v_t denotes volume and n_t denotes the number of trades.

2. σ_{garch} denotes volatility from GARCH, σ_{egarch} denotes volatility from *exponential* GARCH, σ_{tgarch} denotes volatility from *threshold* GARCH, σ_{pgarch} denotes volatility from GARCH in-the-mean, σ_{fgarch} denotes volatility from *fractional* GARCH, $\sigma_{fegarch}$ denotes volatility from *fractional exponential* GARCH, and σ_{2garch} denotes volatility from *two component* GARCH.

Table 7. Correlation coefficients for volume and number of trades with realized volatility

	Cisco (Daily)					
	Level - Correlation			Log - Correlation		
	rv_t	v_t	n_t	rv_t	v_t	n_t
rv_t	1.000			1.000		
v_t	0.564	1.000		0.652	1.000	
n_t	0.642	0.956	1.000	0.704	0.955	1.000
	Intel (Daily)					
	Level - Correlation			Log - Correlation		
rv_t	1.000			1.000		
v_t	0.649	1.000		0.637	1.000	
n_t	0.668	0.934	1.000	0.688	0.858	1.000

Notes: 1. JB denotes Jarque-Bera test statistic, rv_t denotes realized volatility, v_t denotes volume, and n_t denotes the number of trades.