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Abstract

The paper develops a banking framework where a welfare comparison is made between non-tradable demand deposit and equity contracts. Contrary to the existing literature that relies heavily on smooth preferences assumption to justify the liquidity insurance superiority of the 'run-prone' debt contracts over the 'run-free' equity contracts, the paper shows that when aggregate consumption uncertainty is introduced, the welfare dominance of deposit contracts emerges for a simpler preference structure as deposit contracts offer more risk-sharing opportunities. The model illustrates that such uncertainty creates a high dispersion between the allocations that can be attained by trading in the secondary market, and therefore the equity contract provides ex ante less risk-sharing to risk-averse consumers than a tailored-made debt contract.

JEL Classification: G21; D81; D82.

Keywords: financial intermediation, aggregate uncertainty, deposit contracts, equity contracts.

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1 Introduction

In the theoretical literature on financial intermediation pioneered by Bryant (1980) and Diamond and Dybvig (1983), demand deposit contracts' ability to provide optimal risk-sharing against consumers' private consumption contingencies at the expense of an illiquid portfolio of assets has been extensively examined, and alternative contractual arrangements have been discussed. Among these alternative contractual arrangements offered by intermediaries is Jacklin (1987)'s proposal of *equity contracts* that attain the optimal level of risk-sharing as a unique equilibrium. That is, financial intermediaries that are entirely financed by equity and permit interim trade of equity claims can also provide consumers with the optimal level of liquidity without the possibility of failure, unlike non-tradable deposit contracts where intermediaries can default on their debt when runs are triggered. However, the existence of these free-of-default alternative contractual forms does not justify the use of deposit contracts in liquidity provision. Therefore, in the literature on equity contracts initiated by Jacklin (1987), it is generally argued that consumption preferences have an important role to play in the ex-ante welfare dominance of demand deposit contracts in the Diamond-Dybvig type banking frameworks.

This paper examines the equity contracts approach on financial intermediation and shows that the welfare superiority of the demand deposit over equity contracts does not necessarily rely on consumers' preference structure, but it can also be substantiated in a simple intertemporal banking environment characterised by uncertainty about consumers' aggregate demand for liquidity. In this framework, aggregate liquidity uncertainty is introduced by considering that consumers' preferences are random at the time period when financial contracts are designed. As such, intermediaries can only make conjectures about the actual realisation of the future aggregate demand for liquidity from an assumed distribution. In this setup, the paper examines the different characteristics of the two contractual arrangements and discusses the contracts' optimality by evaluating their welfare performance.

The seminal paper by Diamond and Dybvig (1983) has provided the building blocks of recent theoretical models in explaining how demand deposit contracts offered by financial intermediaries provide liquidity insurance to consumers and consequently improve on the competitive outcome. Specifically, in an environment where aggregate demand for liquidity is certain, by pooling consumers' endowments, intermediaries can fully diversify away privately observed consumption shocks, uncorrelated across consumers. Hence, through the asset transformation function, intermediaries can attain the optimal risk-sharing allocation and provide risk-averse consumers with the desired liquidity. This liquidity provision, however, leaves intermediaries prone to runs when an extrinsic factor affects depositors' beliefs about the banks' solvency. Therefore, optimal risk-sharing comes at the cost of an illiquid portfolio of assets which leaves banks prone to default.

In the absence of uncertainty about the economic fundamentals, banks' vulnerability to runs has been extensively discussed in the literature, and the welfare implications of policies that prevent or mitigate the effects of bank runs have been widely examined. The most common policies discussed in the literature that enhance public confidence about the soundness of the fractional reserve banking system and avert liquidity problems include the role of a central bank acting as a lender of last resort (e.g. Smith (1984), Allen and Gale (2000)), suspension of demand deposits convertibility schemes (e.g. Chari and Jagannathan (1988), Chari (1989), Wallace (1990)), and deposit insurance schemes (e.g. Freeman (1988), Hazlett (1997)). Another strand of literature has been focused on the design of 'run-proof' deposit contracts. In particular, Green and Lin (2000, 2003) show that a deposit contract can be designed to implement the ex-ante efficient allocation as a unique equilibrium when the payoffs specified by the contract are contingent on depositors' reported preferences which are verifiable but cannot be falsified. When depositors are served on a first-come first-served basis, such distribution mechanism of banks' resources is shown to guarantee truthful revelation of depositors' individual consumption preferences and the inefficient bank-run equilibrium never arises. Peck and Shell (2003) also consider a distribution mechanism such that early withdrawals depend on the withdrawal history when a sequential service constraint is in place but, unlike Green and Lin (2000, 2003), depositors queue outside the bank only if they want to withdraw and cannot observe their position in the line. In this setting, it is shown that there is a unique equilibrium where bank runs can occur with a positive probability.

An alternative contractual arrangement to demand deposit contracts introduced by Jacklin (1987) considers the allocations that can be achieved by a 'banking firm' which issues equity shares rather than debt in return for consumers' endowment (or equity contracts) that can be traded in an ex-post secondary market. In a Diamond-Dybvig setup where consumers have corner preferences, Jacklin (1987) shows that tradable equity contracts are an attractive alternative to deposit contracts as they provide optimal risk-sharing opportunities to depositors against private consumption shocks and eliminate the bank run equilibrium. However, when consumers are considered to have smooth consumption preferences over time, deposit contracts are the dominant contracts in terms of welfare when incentive constraints are not violated and the optimum risk-sharing equilibrium can be attained.

The assumption of smooth preferences coupled with no aggregate uncertainty has also been widely adopted in the literature in evaluating the liquidity risk-sharing performance of these two contractual arrangements. In particular, introducing uncertainty about the fundamentals in their attempt to explain bank runs as triggered by interim information that depositors receive about the impending state of the economy, Jacklin and Bhattacharya (1988) find that deposit contract can be the welfare dominant type of contract. In a similar model, Alonso (1996) reaches the same conclusion even when banks offer 'run-proof' contracts by taking into account the worst possible realisation of the interim information about the state of the fundamentals in the contract design. Haubrich (1988) and Haubrich and King

(1990) make a distinction between liquidity and income risk where they demonstrate that the deposit contract's comparative advantage over an equity contract is in providing liquidity risk-sharing, while Sussman (1992) argues that government intervention in the secondary market can achieve the optimal risk-sharing allocation. Departing from the welfare comparison between the two contractual forms, Gorton and Pennacchi (1990) argue that deposit contracts provide a mechanism to protect uninformed agents being exploited by coalitions of informed agents about the impending state of the economy in the secondary market. In equilibrium, they demonstrate that intermediaries issue both debt and equity contracts where informed agents select to hold equity and uninformed agents select to hold debt.

Thus, it is evident that the results in the literature on the welfare dominance of deposit contracts rely heavily on the assumed preference structure. Following Jacklin (1987)'s argument, the ex-ante welfare inferiority of the equity contracts under smooth preferences is attributed to the restriction in the design of the contract that consumers have the same wealth prior to trade in the secondary market (i.e. the dividend payment). Under smooth preferences, this results in a loss of expected utility in comparison to tailored-made incentive compatible allocations under demand deposit contracts, as different types of consumers have different valuation of consumption in different time periods. However, this distinction between deposit and equity contracts becomes irrelevant in the case of corner preferences where consumers are assumed to consume only once in their lifetime.

The model developed in this paper is based on Diamond and Dybvig (1983) formulation of financial intermediation, and it shows that non-tradable deposit contracts can be ex ante welfare dominant in the presence of aggregate consumption uncertainty and without relying on the commonly used smooth preferences assumption. In contrast to the existing literature, the model demonstrates that when the aggregate demand for liquidity is not ex ante known, deposit contracts can offer more liquidity insurance to risk-averse consumers and dominate in terms of welfare even under a more restrictive preference structure such as corner preferences. In particular, it is assumed that the fraction of early withdrawers is random so that banks can only make inferences about the aggregate early withdrawal demand from an assumed distribution by pooling depositors' endowment, provided that incentives are not distorted due to extrinsic factors. In the presence of uncertainty at the time period when contracts are designed, only second-best allocations can be attained by either contract. Examining the distinguishing features of the two contracts, the attributes that impose tighter constraints to the intermediaries' planning problem are identified, and conclusions are made with respect to the welfare performance of the two contractual arrangements. Indeed, it is shown that a common dividend payment under aggregate consumption uncertainty imposes ex-ante tighter constraints on intermediaries' planning problem. These constraints can generate large variations in the market-clearing price for ex-dividend shares, and consequently large variations in the final equilibrium consumption allocations. It is demonstrated that the equity contract is dominated by a deposit contract because it offers less liquidity insurance in the state of the world that is mostly wanted by risk-averse depositors, and more liquidity insurance when is least desirable. Following Wallace (1988) and similarly to the existing literature, depository claims contrary to equity claims, are not tradable so that ex-post arbitrage opportunities do not arise in the model. Lastly, the importance of the assumed underlying technology structure in concluding about the welfare dominance of the two contracts is highlighted.

Although the paper focuses on the welfare analysis of financial intermediaries with different capital structure, the results can also be extended to welfare comparison between alternative channels of liquidity provision. The contractual arrangements considered in the paper can be interpreted as depository intermediaries that raise capital by issuing non-tradable deposit contracts such as commercial banks and thrifts, and non-depository intermediaries that issue tradable equity claims such as mutual funds. Alternatively, it can be viewed that liquidity is supplied in the economy indirectly through depository intermediaries, or directly by trade of shares of firms with a predetermined dividend policy and access to the productive investments in the economy. As such, the results of the paper provide a welfare evaluation of consumption allocations attainable under different configurations of the financial system of an economy.

The rest of the paper is structured as follows. Section 2 describes the model and the benchmark case of full information. Section 3 analyses the optimal form of the demand deposit and equity contracts. The welfare comparison of these two contracts is described in section 4, and the conclusion is presented in section 5.

2 The Model

The banking environment in this model is similar to Diamond and Dybvig (1983) framework, where uncertainty about depositors' early withdrawal demand is introduced as in Allen and Gale (2005).

Consumers and Preferences: There is a single homogeneous commodity in the economy that can be used for consumption and investment, and three dates indexed by $t = 0, 1, 2$. There is a continuum of measure one of ex-ante identical consumers born at date 0 with an endowment of one unit of the commodity, and nothing thereafter. Consumers receive a 'privately observed' liquidity shock at date 1 and may become either impatient consumers with probability $\pi \in (0, 1)$, or patient consumers with probability $(1 - \pi)$. The liquidity shock affects consumers' preference structure. Consumers are assumed to have corner preferences such that impatient consumers derive utility only from the consumption of the commodity at date 1, whereas patient consumers only from consumption at date 2. Expected utility is given by

$$V(C_1, C_2; \pi) = \pi U(C_1) + (1 - \pi)U(C_2) \tag{1}$$

where C_t denotes consumption at date $t = 1, 2$. The utility function $U(C_t)$

is twice continuously differentiable with $U''(C_t) < 0 < U'(C_t)$ and satisfies the Inada conditions.

Similar to Allen and Gale (2005), aggregate uncertainty is modelled by assuming that the preference shock π is a random variable that takes two possible values $0 < \pi^L < \pi^H < 1$ with respective probabilities q and $1 - q$. The distribution of the liquidity shock is common knowledge at date 0 and uncertainty is resolved after consumption and investment decisions have been made at date 1. The liquidity shock is independently and identically distributed across consumers so that, from the law of large numbers, π also represents the proportion of impatient consumers in the economy. Therefore, there is ex-ante uncertainty about the aggregate demand for liquidity as the fraction of consumers who turn out to be either type is random.

Technologies: There are two risk-free technologies available to all consumers in the economy; a short-term and a long-term technology. The short-term technology is a one-period storage technology with a return of 1 unit at date $t + 1$ for every unit of the commodity invested at date $t = 0, 1$. The long-term technology is a two-period technology with a certain return of $R > 1$ units for every unit of the commodity invested at date 0. If the long-term productive technology is interrupted at date 1, it yields a return equal to the return of storage.

Intermediation and Contracts: As an alternative to the investment in the above technologies, consumers can deposit their endowment in banks which are assumed to have access to all the technologies described above. The banking system consists of a large number of identical banks, perfectly competing on the terms of the contracts offered to consumers at date 0. Hence, without loss of generality, the analysis focuses on the contractual relationship between consumers and a representative financial intermediary that maximises consumers' expected utility subject to constraints. The representative intermediary is assumed to be subject to a sequential service constraint such that depositors are served on a first-come, first-served basis. As such, the intermediary cannot extract any information about consumers' individual consumption preferences which remains private information, and the introduction of incentive compatibility constraints is required to ensure truthful revelation of consumers' preferences. In addition, due to the uncertainty about π when contracts are designed, the aggregate demand for liquidity (i.e. early withdrawals) is not verifiable, and therefore, contracts that offer payments at date 1 contingent on the realisation of π are not enforceable. The representative bank can offer either a menu of demand deposit contracts or an equity contract to the consumers at date 0 in return for their endowment. For each contract, the bank is obliged to pay the amounts specified in the contract.

A menu of demand deposit contracts gives the right to consumers to withdraw either at date 1 or 2. Being unable to distinguish consumers' individual preferences, the bank designs an incentive-compatible menu of deposit contracts such that consumers self-select the payment designed for their consumption profile once the liquidity shock is realised. In addition, because the uncertainty about π is only resolved after early withdrawals have been made, the payment designed for impatient consumers is independent of the state of the world, whereas the payment designed for patient consumers exhausts bank's resources at date 2. However, intermediaries that offer liquidity insurance to risk-averse consumers by issuing debt contracts are always subject to default when bank runs are triggered due to sunspots as in Diamond and Dybvig (1983). As the analysis focuses on the ex ante welfare performance of deposit contracts in the absence of insolvency, the possibility of bank runs is ignored.

Alternatively, the representative intermediary can offer an equity contract which gives consumers the right to receive two payments; a dividend payment $\delta_1 \in (0,1)$ at date 1, and a liquidating dividend payment of $\delta_2 < R$ at date 2. A secondary market opens at date 1 that allows trade of the equity holders' claims to take place. Having realised their individual consumption preferences at date 1, consumers have incentives to participate in the market as they are entitled to receive an additional payment at the date that they do not value consumption. Market forces determine the equilibrium market price which is, therefore, dependent on the prevailing state of the world. The utility from consumption that consumers obtain from an equity contract does not only depend on the terms of the contract, but also on the equilibrium market price in the secondary market, which in turn depends on the realisation of π.

2.1 Full Information

To facilitate the welfare comparison between the two contractual arrangements under incomplete information, the benchmark full-information case is examined first, where the only friction in the economy is the unobservability of consumers' individual consumption preferences. Consider a social planner that invests consumers' endowment in the underlying technologies on their behalf at date 0, and provides consumption allocations that maximise consumers' expected utility. The social planner (or a representative bank with full information) is assumed to realise the state of the world at date 1 and before any consumer is served. The social planner's maximisation problem has the following form:

Problem 2.1

$$\max_{\{C_1^S, C_2^S\}} qV\left(C_1^H, C_2^H; \pi^H\right) + (1-q)\, V\left(C_1^L, C_2^L; \pi^L\right) \tag{2}$$

subject to the budget constraints:

$$\begin{array}{rcll} \pi^S C_1^S & \leq & X^S & \text{at } t=1 \\ (1-\pi^S)C_2^S & \leq & R(1-X^S) + (X^S - \pi^S C_1^S) & \text{at } t=2, \end{array} \tag{3}$$

where $S = H, L$ is the state of the world.

The sequential budget constraints indicate that a proportion $X^S \in (0,1)$ of the investment in the productive technology is liquidated in order to meet the total withdrawal demand at date 1, while the remainder comes to maturity in the next period and finances the withdrawal demand at date 2, given the realisation of the state of the world. Provided that the return from early liquidation of the long-term technology is equal to the return from storage, the feasibility constraint at date 1 holds with equality as it is optimal to invest consumers' endowment in the long-term technology and liquidate a part of this investment in order to meet the demand for early consumption, while keeping the rest invested to finance the demand for late consumption at date 2.

Solving the maximisation problem, the first-order conditions indicate that, independently of the state of the world, the ratio of the marginal utility of consumption between the two periods is equal to the return of the long-term productive technology

$$U'(C_1^S) = RU'(C_2^S). \tag{4}$$

Note that under full-information, the impatient consumers' payoff is contingent on the state of the world. Let $C_1^{S^*}$ and $C_2^{S^*}$ be the positive social optimum (i.e. first-best) payoffs that satisfy the budget constraints and the first-order conditions, and X^{S^*} is the social optimum level of liquidation of the investment in the productive technology, for any $S = H, L$.

Similar to relevant literature, the coefficient of relative risk aversion is assumed to be greater than one as risk-averse consumers seek insurance against the unfortunate event of becoming impatient after the realisation of the liquidity shock at date 1. This condition guarantees that any feasible allocation which transfers consumption from date 2 to date 1 in relation to the autarkic outcome leads to a Pareto-improvement in welfare. [1]

The following relationship characterises the social optimum payoffs:[2]

$$\begin{array}{rcll} 1 < & C_1^{H^*} < C_1^{L^*} & & \text{for } t = 1 \\ C_2^{H^*} < & C_2^{L^*} < R & & \text{for } t = 2. \end{array} \tag{5}$$

Hence, for a coefficient of relative risk aversion is greater than one, the relationship between the social optimum payoffs and the autarkic payoffs

[1] In the absence of intermediaries and prohibition of trade between consumers, given the assumed investment technologies, impatient consumers consume their initial endowment, whereas patient consumers enjoy the full proceeds of the investment of their endowment in the productive technology. Although the autarky allocation is a feasible allocation for the representative bank as it satisfies the budget constraints with equality, it does not necessarily satisfy the first-order condition. The direction of movement of the equilibrium allocations that can Pareto-improve autarky's outcome depends on consumers' risk aversion. Indeed, for a coefficient of relative risk aversion greater than one, $U'(1) > RU'(R)$ as $CU'(C)$ is decreasing in C, and therefore any feasible allocation such that $1 < C_1$ and $C_2 < R$ can attain a higher level of depositors' expected utility.

[2] Note that by substituting the relationship between $C_1^{S^*}$ and $C_2^{S^*}$ obtained from the binding sequential feasibility constraints in equation (3), the first-order condition in equation (4) can be expressed in terms of $C_1^{S^*}$. Differentiation of $C_1^{S^*}$ with respect to π^S yields $\frac{dC_1^{S^*}}{d\pi^S} = \frac{R^2(1-C_1^{S^*})U''(C_2^{S^*})}{(1-\pi^S)\left((1-\pi^S)U''(C_1^{S^*})+\pi^S R^2 U''(C_2^{S^*})\right)} < 0$ since $C_1^{S^*} > 1$ and from the concavity of the utility function. Therefore, for $\pi^H > \pi^L$ it follows that $C_1^{H^*} < C_1^{L^*}$ and correspondingly $C_2^{H^*} < C_2^{L^*}$ from the first-order conditions.

signify an improvement in welfare as risk-averse consumers seek to obtain liquidity insurance. In addition, from the first-order conditions and the concavity of the utility function it can be shown that $1 < C_1^{S^*} < C_2^{S^*} < R$ and consumers self-select the payoff that is designed for their consumption profile. Finally, note that the relationship between the optimal liquidation level for each state is $X^{L^*} < X^{H^*}$ as more resources need to be liquidated at date 1 for a greater number of impatient depositors in order for the optimal risk-sharing allocation to be attained.[3]

3 Intermediation under Incomplete Information

When the banking system is characterised by incomplete information, the representative bank can not distinguish depositors' individual consumption preferences and, contrary to Allen and Gale (2004, 2005), it does not realise the state of the world prior to any withdrawals. The objective function of the representative welfare-maximising bank is identical to the one in the complete information case, but depending on the contractual arrangement in question (i.e. deposit or equity contract), different feasibility and incentive constraints need to be introduced.

3.1 Deposit Contract

Suppose that in return for consumers' endowment at date 0, the bank is offering a menu of demand deposit contracts which provide consumers with the right to withdraw a specified amount of the homogenous commodity at date 1 or date 2. The menu of demand deposit contracts has the form $\{D_1; D_2^S\}$, where D_1 and D_2^S represent the amount of the commodity available to be withdrawn (and consumed) at date 1 and date 2, respectively. Bank's inability to determine the state of the world prior to any withdrawals preclude the contract's payments at date 1 to be contingent on the realisation of π. Therefore, the contracts' payments designed for impatient depositors should be the same across states. After serving the impatient depositors, the bank can determine the state of the world, and therefore, the allocation that is designed for patient depositors is contingent on π and exhausts the resources of the welfare-maximising intermediary. The bank's feasibility constraints have the form:

$$\begin{aligned} \pi^S D_1 &= x^S && \text{for } t = 1 \\ (1-\pi^S) D_2^S &= R\left(1 - x^S\right) && \text{for } t = 2 \end{aligned} \tag{6}$$

for any $S = H, L$.

[3] Substituting for $C_1^{S^*}$ and $C_2^{S^*}$ from the binding sequential feasibility constraints in equation (3) into the first-order condition in equation (4), differentiation of X^{S^*} with respect to π^S yields $\frac{dX^{S^*}}{d\pi^S} = \frac{\pi^S R C_2^{S^*} U''(C_2^{S^*}) + X^{S^*} U''(C_1^{S^*})(1-\pi^S)/\pi^S}{(1-\pi^S)U''(C_1^{S^*}) + \pi^S R^2 U''(C_2^{S^*})} > 0$ from the concavity of the utility function. Therefore, $X^{L^*} < X^{H^*}$ from $\pi^L < \pi^H$.

By committing at date 0 to a fixed payoff at date 1, depending on the realisation of the state, let $x^S \in [0,1]$ be the proportion of the investment in the long-term technology that is liquidated in order to meet the total demand for early withdrawals, while the rest remains invested until date 2.

Maximisation of depositors' expected utility given in equation (2) subject to the budget constraints given in equation (6) yields the first-order condition[4]

$$\frac{\left(q\pi^H + (1-q)\pi^L\right) U'\left(D_1\right)}{q\pi^H U'\left(D_2^H\right) + (1-q)\pi^L U'\left(D_2^L\right)} = R. \tag{7}$$

Let D_1^* and $D_2^{S^*}$ be the optimal positive payoffs of a deposit contract which are determined by the feasibility constraints and the above first-order condition.

The following property describes the effect of q on the equilibrium payoffs and on the optimal value function under a deposit contract.

Property 3.1 *D_1^* is strictly decreasing in q, whereas $D_2^{H^*}$ and $D_2^{L^*}$ are strictly increasing in q. The depositors expected utility is strictly decreasing and convex in q.*

(Proof: see Appendix)

Note that when the state of the world is known with certainty (i.e. $q = 0$ or $q = 1$), equation (7) becomes identical to the first-order condition in the social planner's case. In relation to the social optimum payoffs, the above property implies that $D_1^* > 1$ and $D_2^{H^*} < D_2^{L^*} < R$. In particular, the higher the probability of a large number of impatient depositors, the lower the optimal payoff at date 1 will be as a greater proportion of the commodity needs to be liquidated to meet a high demand for liquidity. From the feasibility constraints, this results into higher payoffs for patient consumers as the returns from investment in the productive technology are distributed amongst a smaller number of patient consumers.

In addition, bank's inability to determine depositors' individual consumption preference requires the introduction of an incentive compatibility constraint to ensure that consumers will always truthfully reveal their consumption preferences and has the form

$$U(D_1) \leq qU(D_2^H) + (1-q)U(D_2^L). \tag{8}$$

Consumers who realise at date 1 that they are impatient, will always reveal their true type by withdrawing D_1 to finance early consumption. However, patient consumers have two options; they can either withdraw D_1 at date 1 and store the proceeds for one period, or wait until date 2

[4]Given that the budget constraints as described in equation (6) hold with equality for a welfare maximising intermediary, it can be shown that $x^L/\pi^L = x^H/\pi^H$. Substituting for D_1 and D_2^S into the objective function described in equation (2) and utilising the above relationship, the objective function can expressed in terms of a single choice variable, say x^H. Differentiation with respect to x^H yields the first-order condition.

to withdraw and consume D_2^S. Thus, the above incentive compatibility constraint ensures that patient consumers will not misrepresent their type since the utility that they derive from withdrawing D_1 does not exceed the expected utility they derive from withdrawing the payoff that is designed for their type.

Obviously, the above incentive constraint is satisfied when $D_1^* < D_2^{H^*}$ but this relationship depends on the functional form of the utility and the parameters of the model. Without imposing any additional restrictions in the model, it is shown in the Appendix that the optimal menu of deposit contracts is incentive compatible as the optimal payoffs do not violate the incentive compatibility constraint. However, in order to ensure that the optimal payoffs are positive, the model's specifications are assumed to satisfy $C_1^{L^*} < 1/\pi^H$.

3.2 Equity Contract

The banking firm that offers an equity contract to consumers at date 0 in return for their endowment, issues and sells the contract at a price of 1 unit of the homogeneous commodity, raising capital of 1 unit which is invested in the underlying technologies. The equity contract has the form $\{\delta_1, \delta_2\}$ where payments specified in the contract are: $\delta_1 = \delta$ denotes the dividend payment that consumers receive at date 1, where $\delta \in (0, 1)$; $\delta_2 = R(1 - \delta)$ denotes the liquidating dividend that consumers receive at date 2. The representative intermediary selects δ to maximise the depositors' expected utility from consumption. However, the consumption allocations of the two types of consumers also depend on the market-clearing price which in turn depends on the realised state of the world. The intermediary, anticipating the equilibrium market price for each state of the world, selects the dividend payment to maximise social welfare.

In an attempt to provide a full description of the market forces that determine the attainable allocations under an equity contract, consumers' incentives to trade in the secondary market are examined first. After receiving the dividend payment at date 1, impatient consumers sell their ex-dividend share in the secondary market at a positive price p^S, whereas the patient depositors will use their dividend payment to buy δ/p^S additional shares. Hence, the consumption allocation of impatient and patient consumers, denoted as C_{1E}^S and C_{2E}^S respectively, will be:

$$\begin{aligned} C_{1E}^S &= \delta + p^S \\ C_{2E}^S &= \left(1 + \delta/p^S\right) R(1 - \delta). \end{aligned} \tag{9}$$

In determining the market forces that operate in the secondary market, it is apparent that impatient consumers are always willing to trade their ex-dividend share since they can obtain additional utility of consumption at date 1 by selling it at a positive price. Therefore the supply of ex-dividend shares in the secondary market is perfectly inelastic and equal to the number of impatient consumers; or $Q_S = \pi^S$. On the other hand, the demand for ex-dividend shares derives from patient consumers who use their dividend payment to buy additional shares when this provides them with consumption at date 2 at least equal to the consumption that they could otherwise achieve if they do not participate in the secondary market. Therefore, the demand for ex-dividend equity is given by

$$Q_D = \begin{cases} (1-\pi^S)\delta/p^S & \text{for } p^S \leq R(1-\delta) \\ 0 & \text{for } p^S > R(1-\delta). \end{cases}$$

That is, patient consumers are willing to buy additional ex-dividend shares only if the price they have to pay for each share does not exceed the return that an ex-dividend share yields at date 2.

Consequently, trade in the secondary market determines the equilibrium market price and the resulting consumption allocations. The equilibrium price in the secondary market is

$$p^{S^*} = \begin{cases} (1-\pi^S)\delta/\pi^S & \text{for } \delta \leq \widetilde{\delta_S} \\ R(1-\delta) & \text{for } \delta > \widetilde{\delta_S} \end{cases} \tag{10}$$

where $\widetilde{\delta_S} = \frac{\pi^S R}{\pi^S R + (1-\pi^S)}$ (so that $\widetilde{\delta_L} < \widetilde{\delta_H}$ as $\pi^L < \pi^H$) denotes the threshold value of the dividend payment for which the liquidating dividend payment is equal to the market-clearing price. Thus, the market-clearing price depends on the dividend payment and on the parameters of the model. When $\delta \leq \widetilde{\delta_S}$, the market-clearing price is equal to the ratio of the supply of the commodity by patient consumers to the supply of ex-dividend shares by impatient consumers, and is less than the liquidating dividend. When $\widetilde{\delta_S} < \delta$, the market-clearing price reaches its ceiling value and is equal to the liquidating dividend.

The two possible equilibria that can arise in the secondary market are represented in Figure 1 where the quantity and price of the ex-dividend shares traded are measured on the horizontal and vertical axis, respectively. The supply of shares is perfectly inelastic at π^S, while the demand is initially horizontal at the price for which patient consumers are indifferent to trade, up to the point where, given the dividend payment chosen by the intermediary, there are gains from trade and the demand becomes strictly decreasing and convex thereafter.[5] One possible equilibrium in the market is represented by point A and is referred to *Surplus Equilibrium* as the cost of buying additional ex-dividend shares for patient consumers is less than the returns of this investment. Another possible equilibrium is given by point B and is referred to *Non-Surplus Equilibrium* as the cost of this investment opportunity is equal to its reward.

Substituting for equilibrium price in the secondary market given from equation (10) into the consumption allocations of the two types of depositors given by equation (9), the latter will become:

[5] From the inverse demand function we can observe that $dp/dQ_D = -\delta(1-\pi^S)/Q_D^2 < 0$ and $d^2p/dQ_D^2 = 2\delta(1-\pi^S)/Q_D^3 > 0$.

$$
\begin{aligned}
C_{1E}^{S} &= \begin{cases} \delta/\pi^{S} & \text{for } \delta < \widetilde{\delta_S} \\ \delta + R(1-\delta) & \text{for } \delta \geq \widetilde{\delta_S} \end{cases} \\
C_{2E}^{S} &= \begin{cases} R(1-\delta)/(1-\pi^{S}) & \text{for } \delta < \widetilde{\delta_S} \\ \delta + R(1-\delta) & \text{for } \delta \geq \widetilde{\delta_S}. \end{cases}
\end{aligned} \tag{11}
$$

Anticipating the market equilibrium price p^{S^*}, the bank chooses a dividend payment to maximise consumers' expected utility. Substituting for the consumption allocations of the two types of consumers into the objective function, the bank can determine the optimal dividend payment δ^* and consequently, the resulting equilibrium allocations $C_{1E}^{S^*}$ and $C_{2E}^{S^*}$. Note also from equation (11) that the relationship of the equilibrium payoffs for the two types of consumers in a given state is such that $C_{1E}^{S^*} < C_{2E}^{S^*}$ when the surplus equilibrium is attained, and $C_{1E}^{S^*} = C_{2E}^{S^*}$ when the non-surplus equilibrium is attained.[6]

From these two plausible scenarios that may occur in the secondary market and the two states of the world, the following lemma indicates that there are only two different regions where the optimal dividend payment can lie, and therefore there are two possible configurations of the secondary market that can arise in equilibrium.

Lemma 3.2 *Depending on the parameters of the model, the optimal dividend payment chosen by a welfare maximising intermediary that issues equity shares will be either such that* $\delta^* < \widetilde{\delta_L}$*, or* $\widetilde{\delta_L} \leq \delta^* < \widetilde{\delta_H}$*.*

(Proof: see Appendix)

The first configuration is when $\delta^* < \widetilde{\delta_L}$ which means that the secondary market is in the surplus equilibrium for both states. The second configuration is when $\widetilde{\delta_L} \leq \delta^* < \widetilde{\delta_H}$ which implies that the secondary market is in the surplus equilibrium for the high state, and in the non-surplus equilibrium for the low state.[7] In the proof of the above lemma it is shown that consumers' expected utility is maximised for a dividend payment less than $\widetilde{\delta_H}$, and therefore, the configuration where the secondary market is in the non-surplus equilibrium for both states is never optimal.

Substituting the consumption allocations as given in equation (11) into consumers' expected utility in equation (2) and maximising with respect to δ yields the following first-order condition

$$
\frac{qU'\left(C_{1E}^{H^*}\right) + (1-q)U'\left(C_{1E}^{L^*}\right)}{qU'\left(C_{2E}^{H^*}\right) + (1-q)U'\left(C_{2E}^{L^*}\right)} = R, \tag{12}
$$

[6]From equation (11), $C_{1E}^{S^*}$ is increasing in δ, whereas $C_{2E}^{S^*}$ is decreasing in δ. Since $C_{1E}^{S^*} = C_{2E}^{S^*}$ for $\delta \geq \widetilde{\delta_S}$, then for any lower value of δ for which the surplus equilibrium is attained, it follows that $C_{1E}^{S^*} < C_{2E}^{S^*}$.

[7]Clearly, from the assumption that $\pi^L < \pi^H$, the possibility that the market is in the surplus equilibrium for the low state and in the non-surplus equilibrium for the high state is redundant.

where $C_{1E}^{L*} < C_{2E}^{L*}$ for $\delta^* < \widetilde{\delta_L}$, and $C_{1E}^{L*} = C_{2E}^{L*}$ for $\widetilde{\delta_L} \leq \delta^* < \widetilde{\delta_H}$.

In the proof of lemma (3.2) in the Appendix, it is shown that there is a unique δ^*, and therefore a unique consumption allocation, for each configuration of the secondary market that maximises consumers' expected utility. Let C_{1E}^{S*} and C_{2E}^{S*} denote the positive payoffs for impatient and patient consumers for each state respectively, satisfying the above first-order condition and equation (11). Hence, the relationship between the equilibrium payoffs can be summarised as

$$C_{1E}^{H*} < C_{1E}^{L*} \leq C_{2E}^{L*} < C_{2E}^{H*}. \tag{13}$$

The above relationship indicates that the equity contract offers more liquidity insurance to risk-averse consumers in the low state as the dispersion between the equilibrium payoffs for the two types of consumers is greater in the high state. This is due to the negative effect of π^S on p^{S*} which influences the equilibrium consumption allocations. A high number of impatient consumers implies a high quantity of ex-dividend shares supplied in the secondary market which results in a low equilibrium market price. As a consequence, the consumption of impatient consumers is reduced since they are forced to sell their ex-dividend shares at a low price, and the consumption of patient consumers increases as they can buy a greater number of shares using their dividend payment to finance their consumption at date 2. In terms of Figure 1, an increase in the number of impatient consumers can be represented by a shift of the supply of ex-dividend shares to the right and a leftward shift of the convex segment of the demand for ex-dividend shares, resulting into a lower market-clearing price.

Moreover, the following comparative static property of the equilibrium payoffs and the optimal value function with respect to q provides a greater insight on the performance of the equity contract in terms of welfare.

Property 3.3 *For $\delta^* < \widetilde{\delta_L}$, C_{1E}^{S*} is strictly increasing in q and C_{2E}^{S*} is strictly decreasing in q. For $\widetilde{\delta_L} \leq \delta^* < \widetilde{\delta_H}$, C_{1E}^{L*} becomes strictly decreasing in q (where $C_{1E}^{L*} = C_{2E}^{L*}$). The depositors' expected utility at equilibrium is strictly decreasing and convex in q, for any $\delta^* \in [X_L^*, X_H^*]$.*

(Proof: see Appendix)

The above property suggests that the greater the probability of a large number of impatient consumers, the greater the dividend payment in period 1. A high dividend payment has a positive direct effect on the equilibrium consumption at date 1, and an indirect effect through the resulting high demand for ex-dividend shares which puts an upward pressure on the market-clearing price. Due to the feasibility constraints, this results in lower payoffs at date 2 for both states. In addition, the greater the probability that the state will be high, the lower the liquidity insurance offered by the contract as indicated by equation (13), and therefore the overall expected utility decreases in q.

From the above property, a unique threshold value of q exists, namely $\widetilde{q} \in (0,1]$, for which the optimal configuration of the secondary market changes in the low state. Since $\widetilde{q}$ is the threshold value for which $\delta^*(\widetilde{q}) = \widetilde{\delta_L}$, where $\widetilde{\delta_L}$ is independent of q, from the first-order condition it follows that

$$\widetilde{q} = \frac{U'\left(C_{1E}^{L*}(\widetilde{\delta_L})\right)(R-1)}{U'\left(C_{1E}^{H*}(\widetilde{\delta_L})\right) - U'\left(C_{2E}^{H*}(\widetilde{\delta_L})\right) + U'\left(C_{1E}^{L*}(\widetilde{\delta_L})\right)(R-1)} \tag{14}$$

where $C_{1E}^{L*}(\widetilde{\delta_L}) = C_{2E}^{L*}(\widetilde{\delta_L})$. Note also that $\widetilde{q}$ is positive and less than 1 since $C_{1E}^{H*}(\widetilde{d_L}) \leq C_1^{H*}$ and $C_2^{H*} \leq C_{2E}^{H*}(\widetilde{d_L})$.[8]

Starting from the limit case where the state of the world is known with certainty to be low (i.e. $q = 0$) so that the secondary market is in the surplus equilibrium (i.e. $C_1^{S*} < C_2^{S*}$), as q increases it reaches the threshold value of $\widetilde{q}$ at which the secondary market will be in the non-surplus equilibrium in the low state. As the optimum dividend payment, and consequently the date 1 payoffs, increase with the probability of the number of impatient consumers being high, the equity contract provides more liquidity insurance in the high state. However, as the intermediary has to commit to a fixed dividend payment at date 0, this results in more liquidity insurance than what is socially optimal in the low state. Indeed, when q is sufficiently high (i.e. $\widetilde{q} \leq q$), the contract offers full insurance against the risk of being impatient in the low state.

4 Welfare Evaluation

In each of the maximisation problems that the representative intermediary has to solve in offering either contract as presented before, the intermediary maximises the same objective function but subject to different constraints. Therefore, the welfare comparison of the two contracts focuses on the constraints that characterise each contract's optimal payoffs in relation to the benchmark case of full-information. In addition, optimal payoffs refer to the second-best payoffs as the social optimum allocation cannot be achieved by either contract.

Examining the characteristics of the two contracts, it is evident that, in contrast to the social planner, by offering a menu of demand deposit contracts where depositors are served on a first-come first-served basis following a sequential service constraint, the intermediary realises the state of the world only after π^L depositors have been served in period 1. Therefore, the level of liquidation of the initial investment in the long-term technology is contingent on the state of the world. However, since the uncertainty about π is not resolved prior to early withdrawals, it is not possible for the date 1

[8] The relationship between equity contract's optimal allocation and the social optimum allocation is discussed in the following Section on the comparison of the two contracts.

payoff to be contingent on the state of the world. Hence, the deposit contract is constrained in relation to the social planner case to offer the same payoff to impatient depositors independently of the state. Starting from the social planner's budget constraints and imposing equality on the first period payoffs yields $x^{L^*} < X^{L^*} < X^{H^*} < x^{H^*}$. The relationship between the optimal payoffs is therefore

$$\begin{array}{rl} C_1^{H^*} < D_1^* < C_1^{L^*} & \text{for t=1} \\ D_2^{H^*} < C_2^{H^*} < C_2^{L^*} < D_2^{L^*} & \text{for t=2.} \end{array} \tag{15}$$

The deposit contract eliminates the risk that impatient depositors face due to the uncertainty about the prevailing state, but this risk is borne by patient depositors as the fixed date 1 payoff results in a higher dispersion between date 2 payoffs relative to the social optimum payoffs. In particular, when offering a fixed payment at date 1, if the state turns out to be low, the lower number of withdrawals in period 1 implies that more resources remain invested in the productive technology, and therefore, a higher payoff that patient depositors receive.

On the other hand, when the intermediary offers an equity contract, it commits to a fixed dividend payment where the market-clearing price finally determines the consumption levels for the two states of the world. Comparing the budget constraints for the social planner and the bank that offers an equity contract, the analogy of the dividend payment δ and the level of liquidation X^S become apparent. Clearly, the social planner can adjust the investment portfolio depending on the realisation of the state, whereas the intermediary is constrained to offer a fixed dividend payment and let the market forces determine the equilibrium allocation for each type of consumer. Starting from the social planner's budget constraints at date 1 and imposing the restriction that the amount of the long-term investment liquidated at date 1 is independent of the state yields $X^{L^*} \leq \delta^* \leq X^{H^*}$. The volatility of the market's clearing price, however, results in a higher dispersion between the equilibrium payoffs in relation to the social optimum payoffs. From the budget constraints for the two periods it follows that $C_1^{L^*}$ and $C_2^{H^*}$ increase, whereas at the same time $C_2^{L^*}$ and $C_1^{H^*}$ decrease. Therefore, the relationship between the optimal payoffs between the social planner case and the equity contract case is

$$\begin{array}{rl} C_{1E}^{H^*} < C_1^{H^*} < C_1^{L^*} < C_{1E}^{L^*} & \text{for t=1} \\ \left(C_{1E}^{L^*} \leq\right) C_{2E}^{L^*} < C_2^{L^*} \text{ and } C_2^{H^*} < C_{2E}^{H^*} & \text{for t=2.} \end{array} \tag{16}$$

Evaluating the performance of the two contracts in terms of social welfare, the following result summarises the main findings.

Proposition 4.1 *In a Diamond-Dybvig framework with corner preferences and aggregate consumption uncertainty, when the utility function and the model's parameters are such that the threshold value $q^* \in (0, 1)$ exists, standard demand deposit contracts ex-ante dominate equity contracts in terms of* *welfare for any $q^* < q \leq 1$; otherwise deposit contracts are welfare optimal for any $q \in [0, 1]$.*

(Proof: see Appendix)

The proof of the above proposition is based on the properties with respect to q of consumers' expected utility at equilibrium under each contract, and is depicted in Figure 2 where the horizontal and vertical axis measure q and consumers' expected utility, respectively. As both contracts can achieve the social optimum allocation when the state of the world is known with certainty (i.e. SO_L for $q = 0$ and SO_H for $q = 1$), and are both strictly decreasing and convex in q, the proof of the above statement focuses on the comparison of the slope of the optimal value function for each contractual arrangement at the limit cases where q is known with certainty. It is shown that the expected utility of the deposit contract EV_D^* is steeper than that of the equity contract EV_E^* as q tends to unity. However, the slope of EV_D^* relative to EV_E^* as q tends to zero depends on the parameters of the model. When EV_D^* is flatter than EV_E^* at $q = 0$, the deposit contract dominates for any $q \in [0, 1]$, where EV_D^* is represented by the dashed line and EV_E^* by the bold line. When EV_D^* is steeper relative to EV_E^*, a threshold value $q^* \in (0, 1)$ can be defined, for which EV_D^* and EV_E^* cross, so that the deposit contract dominates for $q^* < q \leq 1$, as it is represented by the solid line. In order to obtain a better understanding of the impact that the model's parameters have in determining the dominance of each contract, the standard budget line-indifference curve analysis is used to examine how changes in q affect consumers' welfare.

Consider firstly the case where the surplus equilibrium can be achieved in the secondary market for ex-dividend shares in both states which corresponds to the values of the probability of the high state such that $q \in [0, \widetilde{q})$. The result can be illustrated diagrammatically from the observation that all the feasible allocations can be described by an intertemporal budget constraint. Simplifying for the amount of liquidation (or the dividend payment in case of the equity contract), all optimal allocations satisfy the following intertemporal budget constraint

$$\pi^S C_1^S + (1 - \pi) C_2^S / R \leq 1. \tag{17}$$

From the characteristics of the two contracts, the restrictions of each contractual agreement can be expressed in terms of the consumption allocations. This case is illustrated in Figure 3 where the horizontal axis measures the consumption in period 1 and the vertical axis measures the consumption in period 2. The budget lines for the two states of the world cross at the autarky allocation $C_1 = 1$ and $C_2 = R$. The concavity of the utility function and linearity of budget constraints ensures the existence of a unique optimum allocation for each state that maximises depositors' expected utility. Note that in order to simplify the diagrammatic analysis, a homothetic utility function is considered such that income expansion paths are represented as rays from the origin.[9] The social optimum allocations are located on the crossing points between the two budget lines and the ray from the origin SO which captures the fixed proportionality of the marginal utilities between the two types of depositors, across the two different states. Similarly, the income expansion paths for the equity and deposit contract coincide with the SO for the low state, and for the high state are represented by the rays EC_0 and DC_0, respectively, where $D_2^{H^*} < C_2^{H^*} < C_2^{L^*} < C_{2E}^{H^*}$.[10]

In determining the dominance of each contract at the limit as q tends to 0, from the properties 3.1 and 3.3 it follows that the difference between the slope of the expected utility of the two contracts is equal to the difference between the expected utility that they attain in the high state. Therefore, welfare dominance for values of q in the region around zero, depends on which contract's allocation lies on a higher indifference curve in the high state. Examples provided in the proof of proposition 4.1 in the Appendix show that both possibilities may arise as this depends on the parameters of the model and on the functional form of utility. Hence, if the deposit contract dominates in the region of q around 0, then it remains the dominant contract for any $q \in [0, 1]$. On the other hand, if the equity contract is initially the dominant one, then the threshold value q^* is defined and the equity contract dominates for any $q \in [0, q^*)$.

As q increases, the properties of the optimal payoffs with respect to q suggest that the income expansion paths of the equity contract are rotating downwards whereas the income expansion paths of the deposit contract rotate upwards. In particular, when $q = \widetilde{q}$, an additional condition is introduced in the design of the equity contract as the secondary market is in the non-surplus equilibrium for the low state and the payoffs across the two periods are equal; i.e. $C_{1E}^{L^*} = C_{2E}^{L^*}$. In terms of Figure 3, the income expansion path of the equity contract in the low state is the 45 degrees line and the optimal consumption allocation is now determined by the intersection of the 45 degrees line with the corresponding budget constraint, represented by point F_L.

For higher values of q such that $\widetilde{q} < q \leq 1$, the allocation of the equity contract for the low state does not satisfy the intertemporal budget constraint described in equation (17) with equality. Indeed, it has been shown that when the non-surplus equilibrium is attained in the secondary market for the low state, the common equilibrium payoff for both periods is decreasing in q. In terms of Figure 3, this corresponds to a movement along the 45 degrees line for higher values of q which leads to inferior allocations for the low state inside the budget set. Finally, for $q = 1$, both contracts attain the social optimum allocation for the high state as it is illustrated in Figure 4. The income expansion paths of both contracts coincide with that of the social optimum at SO, whereas for the low state they are represented

[9] This specification of the utility function is only for illustrative purposes as the result of the model hold for any utility function that satisfies the standard neoclassical properties with a coefficient of relative risk aversion greater than one.

[10] Solving for the common dividend payment between the two states yields $C_{2E}^{H^*} = \frac{1-\pi^H}{1-\pi^L} C_2^{L^*} > C_2^{L^*}$ as $\pi^L < \pi^H$.

by the 45 degree line and the ray from the origin DC_1 for the equity and the deposit contract respectively, since $C_{2E}^{L^*} < C_2^{L^*} < C_2^{H^*} < D_2^{L^*}$. The equilibrium allocation of the equity contract lies on the bold segment of the 45 degrees line between points F_L and F_H.[11]

The conclusions about welfare dominance are derived from the characteristics that distinguish these two contracts, as opposed to the contract that can be offered under full information. Clearly, their difference lies in the constraints that characterise each contract and the restrictions that they impose on the payoffs to adjust in each state, once the uncertainty is resolved at the end of date 1. The representative intermediary being unable to observe the state of the world when the contracts are offered at date 0, it loses flexibility in the design of the contracts in relation to the social planner.

Specifically, by offering a demand deposit contract, the bank loses flexibility in terms of the payoff that can be offered at date 1 as it has to commit to a fixed payoff, independent of the state of the world. Hence, although holding the impatient depositors payoff constant eliminates the risk related to the uncertainty about the state of the world in the first time period, this risk is passed to the second time period as it creates greater dispersion between the date 2 payoffs in relation to the social optimum payoffs. In terms of the risk of being impatient (or liquidity risk) which can be captured by the dispersion between the payoffs designed for each type of consumer for a given state, from the relationships described in equation (15) it follows that the deposit contract offers more risk-sharing in the high state than what is socially desirable as $D_2^{H^*} - D_1^* < C_2^{H^*} - C_1^{H^*}$[12] and less risk-sharing in the low state as $C_2^{L^*} - C_1^{L^*} < D_2^{L^*} - D_1^*$. In the graphical representations in Figures 3 and 4 for a homothetic utility function, the risk-sharing provision of the deposit contract is illustrated by the steepness of the income expansion paths relative to the social optimum income expansion path where $D_2^{H^*}/D_1^* < C_2^{H^*}/C_1^{H^*}$ and $C_2^{L^*}/C_1^{L^*} < D_2^{L^*}/D_1^*$.

On the other hand, by offering an equity contract, the bank loses flexibility in terms of the amount of resources that can be liquidated at date 1 (and correspondingly, on the amount of resources that remain invested in the productive technology), as it has to commit to a fixed dividend payment at date 0. Depending on the realisation of the state, trade takes place and the market price for ex-dividend shares adjust to its equilibrium value. As previously shown in the description of the contract, market forces in the secondary market create a high dispersion between the resulting equilibrium consumption allocation in relation to the corresponding social optimum allocation for a given state. In terms of liquidity risk measured as the dispersion between $C_{2E}^{S^*}$ and $C_{1E}^{S^*}$, from the relationships described in equation (16) it

[11] Note that Figure 4 is drawn such that $C_2^{H^*}$ exceeds the full-insurance payoffs in the low state indicated by the point F_L. In case where $C_2^{H^*}$ is lower that the full-insurance payoffs in the low state, the allocation that the equity contract can attain in the low state lies on the segment of the 45 degrees line above point F_H but below $C_2^{H^*}$ since $\pi C_1^{H^*} + (1-\pi)C_2^{H^*} < C_2^{H^*}$.

[12] This relationship holds even when the utility function and the parameters of the model are such that $D_2^{H^*} < D_1^*$.

follows that the equity contract offers less risk-sharing in the high state and more risk-sharing in the low state than the social optimum allocations as $C_2^{H^*} - C_1^{H^*} < C_{2E}^{H^*} - C_{1E}^{H^*}$ and $C_2^{L^*} - C_1^{L^*} > C_{2E}^{L^*} - C_{1E}^{L^*}$. Equivalently, in Figures 3 and 4, the steepness of equity contract's income expansion paths for each state relative to the social optimum expansion path is such that $C_2^{H^*}/C_1^{H^*} < C_{2E}^{H^*}/C_{1E}^{H^*}$ and $C_2^{L^*}/C_1^{L^*} > C_{2E}^{L^*}/C_{1E}^{L^*}$.

In the design of the optimal contract that provides liquidity insurance to risk-averse consumers, it is apparent that committing to a fixed first period payoff is less restrictive than committing to a fixed investment policy when uncertainty is resolved in period 1. The ex-ante dominance of the deposit contract relative to equity contract in terms of consumers' expected utility arises from the fact that the former offers more liquidity insurance in the 'bad state' of the world (i.e. high state) than the equity contract since $D_2^{H^*} - D_1^* < C_2^{H^*} - C_1^{H^*} < C_{2E}^{H^*} - C_{1E}^{H^*}$, which is more valuable ex ante to risk-averse consumers. On the other hand, the equity contract offers more liquidity insurance in the 'good state' of the world (i.e. low state) than the deposit contract since $C_{2E}^{L^*} - C_{1E}^{L^*} < C_2^{L^*} - C_1^{L^*} < D_2^{L^*} - D_1^*$, which is ex-ante less valuable to risk-averse depositors. According to proposition 4.1, this means that as q increases, if the demand deposit is not already the optimal contract, it becomes the dominant one as the state of the world is more likely to be high.

5 Conclusion

This paper shows that, in an economy characterised by a corner preference, when uncertainty about the liquidity shocks is not resolved in the time period when contracts are designed, demand deposit contracts can outperform equity contracts as social welfare is maximised over a less restrictive set of constraints. Indeed, committing to a fixed dividend payment creates large fluctuations on the equilibrium market price, which is reflected by a high dispersion of the resulting equilibrium payoffs relative to social optimum payoffs. As a consequence, equity contracts offer less risk-sharing opportunities against consumption contingencies which are private information to consumers, than deposit contracts that designate incentive compatible allocations to depositors, depending on the realisation of their consumption preferences.

The results derived in this model on the ex ante welfare optimality through the comparison of these two contractual arrangements rely heavily on the assumed structure of the economy's underlying technology. The weak dominance of the long-term technology over the storage technology makes investment decisions at date 0 trivial with regard to the withdrawal uncertainty at date 1. This is because optimality requires full investment of bank's resources in the productive technology, and liquidation at no cost relative to storage of the required amount in order to honour the promised liabilities. This, of course, provides flexibility in the design of the demand deposit contract as the bank can adjust the proportion of the resources it liquidates depending on the prevailing state that is realised at date 1. Therefore, it is clear that when a cost of liquidation is introduced in the model, so that liquidation of the productive technology is costly relative to storage, then bank's investment decisions at date 0 have an important impact on the resulting equilibrium allocations as it loses flexibility in the design of the deposit contract. This imposes additional restrictions on the allocations that are feasible, and consequently erode the dominance of the deposit over the equity contract. However, the performance of the equity contract remains unaffected by the returns from early liquidation of the productive technology. This is partly because the intermediary has always to commit to a fixed dividend payment and also due to the fact that, trade in the secondary market on the claims that are written on the long-term investment creates another 'liquid asset' between dates 1 and 2, as no premature liquidation of the physical investment is required. For example, in the extreme case where investment in the productive technology is irreversible, a depository intermediary is restricted to invest a fixed proportion of its resources in long-term, where at the same time it offers a fixed first period payoff. Obviously, this additional restriction, which is identical to the one of the equity contract offering a single dividend payment, makes maximisation of consumers' expected utility to be under a tighter set of constraints and therefore the equity contract will dominate in terms of welfare.

Figure 1: Surplus and Non-Surplus Equilibria

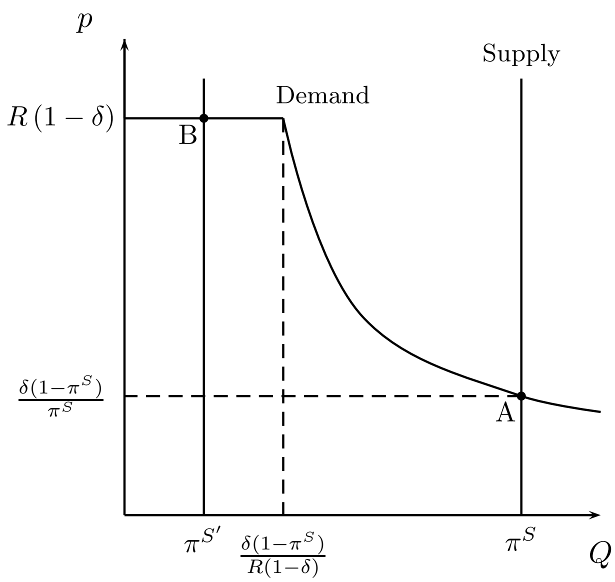


Figure 2: Welfare Comparison of the Contracts

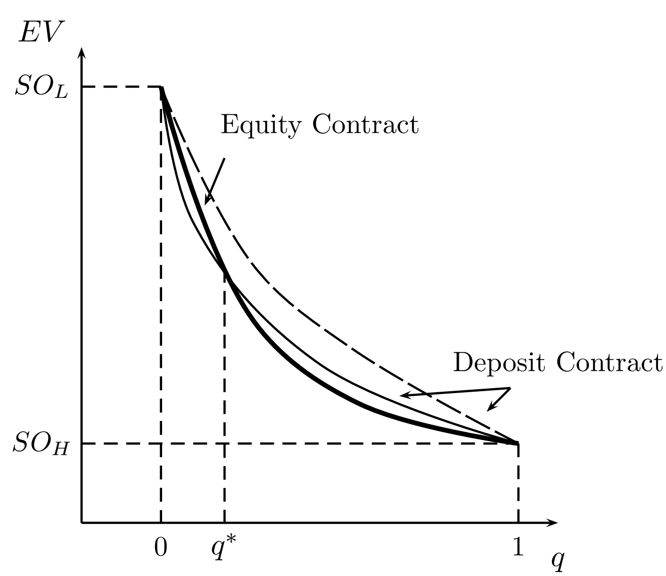


Figure 3: Equilibrium Allocations for $q \leq \widetilde{q}$

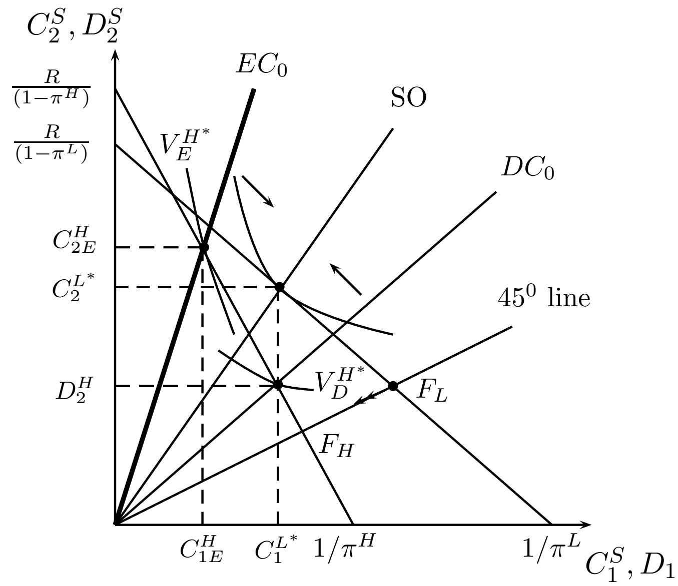
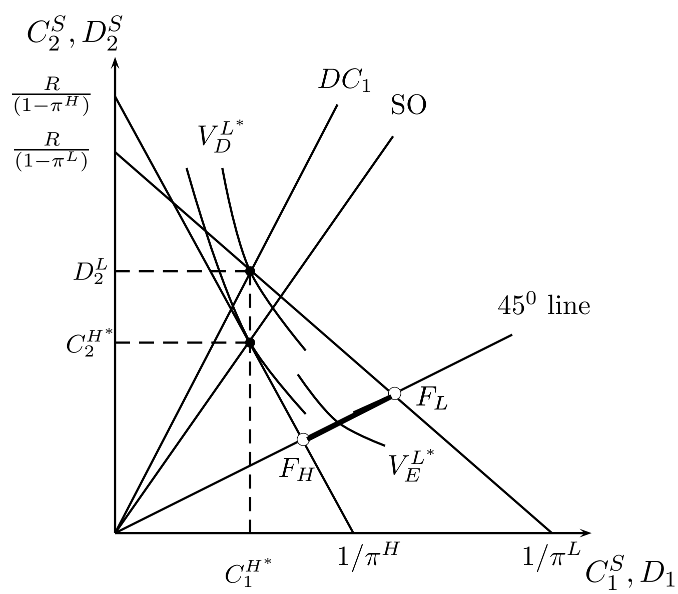


Figure 4: Equilibrium Allocations for $q = 1$



Appendix

Proof of Incentive Compatibility of Deposit Contract's payoffs

Eliminating the return from early liquidation from the sequential budget constraints given in equation (6), the equilibrium allocations for a menu of deposit contracts can be expressed in terms of the date 1 payoff. As such, the objective function given in equation (2), say EV_D, and the incentive constraint in equation (8), say IC, can be expressed in terms of D_1. Differentiation of $EV_D(D_1)$ with respect to D_1 yields

$$\frac{\partial EV_D(D_1)}{\partial D_1} = \overline{\pi}U'\left(D_1\right) - R\left(q\pi^H U'\left(D_2^H\right) + (1-q)\pi^L U'\left(D_2^L\right)\right) \text{ and}$$
$$\frac{\partial^2 EV_D(D_1)}{\partial D_1^2} = \overline{\pi}U''\left(D_1\right) + R^2\left(q\frac{(\pi^H)^2}{1-\pi^H}U''\left(D_2^H\right) + (1-q)\frac{(\pi^L)^2}{1-\pi^L}U''\left(D_2^L\right)\right) < 0,$$

where $\overline{\pi} = q\pi^H + (1-q)\pi^L$ the expected value of π.

To simplify the notation, let $\Phi(D_1) \equiv \partial EV_D(D_1)/\partial D_1$. Ignoring $IC(D_1)$, let D_1^* the unique payoff at date 1 that maximises consumers' expected utility, i.e. $\Phi(D_1^*) = 0$.

Similarly, differentiation of $IC(D_1)$ with respect to D_1 yields

$$\frac{\partial IC(D_1)}{\partial D_1} = U'\left(D_1\right) + R\left(q\frac{\pi^H}{1-\pi^H}U'\left(D_2^H\right) + (1-q)\frac{\pi^L}{1-\pi^L}U'\left(D_2^L\right)\right) > 0.$$

Note that when the state of the world is known with certainty such that $q = 0$ or $q = 1$, the deposit contract attains the social optimum allocation as $\Phi(D_1) = 0$ becomes equivalent to the social planner's first-order condition in equation (4). The social optimum payoffs do not violate the incentive constraint as $C_1^{S^*} < C_2^{S^*}$, and therefore $\Phi(D_1^*) - IC(D_1^*) > 0$ for $q = 0$ or $q = 1$.

Given the standard properties of the assumed utility function, monotonicity of $\Phi(D_1)$ and $IC(D_1)$ in terms of D_1 guarantees the existence of a unique value of D_1, say $\widetilde{D}_1$, such that $\Phi(\widetilde{D}_1) - IC(\widetilde{D}_1) = 0$. In order to prove that the optimal payoffs of the deposit contract do not violate the $IC(D_1)$, it is sufficient to show that $D_1^* < \widetilde{D}_1$, or alternatively $\Phi(\widetilde{D}_1) < 0$, for any q.

Differentiation of $\widetilde{D}_1$ with respect to q provides

$$\frac{d\widetilde{D}_1}{dq} = -\frac{\partial\Phi(\widetilde{D}_1)/\partial q - \partial IC(\widetilde{D}_1)/\partial q}{\partial\Phi(\widetilde{D}_1)/\partial\widetilde{D}_1 - \partial IC(\widetilde{D}_1)/\partial\widetilde{D}_1} < 0,$$

where both the denominator and numerator are negative as

$$\partial\Phi(\widetilde{D}_1)/\partial q = (\pi^H - \pi^L)U'(\widetilde{D}_1) - R\left(\pi^H U'(D_2^H) - \pi^L U'(D_2^L)\right) < 0$$
$$\partial IC(\widetilde{D}_1)/\partial q = U(D_2^L) - U(D_2^H) > 0$$

since $0 < D_2^H < D_2^L$ from $\pi^L < \pi^H$.

Hence, the derivative of $\Phi(\widetilde{D}_1)$ with respect to q will be

$$\frac{d\Phi(\widetilde{D}_1)}{dq} = \partial\Phi(\widetilde{D}_1)/\partial q + \partial\Phi(\widetilde{D}_1)/\partial\widetilde{D}_1 \left(d\widetilde{D}_1/dq\right) < 0.$$

As $\Phi(\widetilde{D}_1)$ is monotonic in q and $\Phi(\widetilde{D}_1) < 0$ when $q = 0$ or $q = 1$, it follows that $\Phi(\widetilde{D}_1) < 0$ for any $q \in [0, 1]$ and therefore the incentive compatibility constraint never binds.

Proof of Property 3.1

Substituting into the first-order condition (equation (7)) the equilibrium allocations of a menu of deposit contracts which are expressed in terms of D_1^* by eliminating x^S from the sequential budget constraints (equation (6)), differentiation of D_1^* with respect to q yields

$$\frac{d\,D_1^*}{d\,q} = \frac{\left(\pi^L - \pi^H\right) U'(D_1^*) + R\left(\pi^H U'(D_2^{H^*}) - \pi^L U'(D_2^{L^*})\right)}{\left(q\pi^H + (1-q)\pi^L\right) U''(D_1^*) + R\left(q\pi^H U''(D_2^{H*})\frac{\pi^H R}{1-\pi^H} + (1-q)\pi^L U''(D_2^{L*})\frac{\pi^L R}{1-\pi^L}\right)}.$$

The denominator is negative from the concavity of the utility function, while the numerator can be expressed from the first-order condition as

$$\frac{\pi^H\,\pi^L\,R}{q\pi^H + (1-q)\pi^L}\left(U'(D_2^{H^*}) - U'(D_2^{L^*})\right) > 0$$

which has a positive sign since $D_2^{H^*} < D_2^{L^*}$. Therefore, $dD_1^*/dq < 0$, and from the feasibility conditions it follows that $dD_2^{S^*}/dq > 0$.

According to the *Envelope Theorem*, the total effect of a change in q to the optimal value function is equal to the direct effect of q and therefore

$$\frac{dEV_D^*(V)}{dq} = \frac{\partial EV_D^*(V)}{\partial q} = V_D^{H^*} - V_D^{L^*},$$

where $V_E^{S^*}$ denotes consumers' expected utility in a given state as specified in equation (1), and $EV_D^*(V) = qV_D^{H^*} + (1-q)V_D^{L^*}$ the consumers' expected utility at date 0.

Differentiating again with respect to q yields

$$\frac{d^2EV_D^*(V)}{dq^2} = \frac{dD_1^*}{dq}\left((\pi^H - \pi^L)U'(D_1^*) - R\left(\pi^H U'(D_2^{H^*}) - \pi^L U'(D_2^{L^*})\right)\right),$$

which from the first-order condition can be simplified to

$$\frac{d^2EV_D^*(V)}{dq^2} = \frac{\pi^L}{q}\frac{dD_1^*}{dq}\left(RU'(D_2^{L^*}) - U'(D_1^*)\right) > 0.$$

The sign of the above derivative is positive as $dD_1^*/dq < 0$ and $RU'(D_2^{L^*}) < U'(D_1^*)$. The last inequality derives from the relationship between the optimal deposit contract's payoffs and the social optimum payoffs where $D_1^* < C_{1L}^*$ and $D_{2L}^* > C_{2L}^*$, and therefore $U'(D_1^*)/U'(D_2^{L^*}) > R\,(= U'(C_{1L}^*)/U'(C_{2L}^*))$.

As $dEV_D^*(V)/dq$ is monotonically increasing in q, in order to prove that it has a negative sign for any q, it is sufficient to show that $dEV_D^*(V)/dq$ is negative when evaluated at $q = 1$ where

$$\left.\frac{dEV_D^*(V)}{dq}\right|_{q=1} = (\pi^H - \pi^L)U(C_1^{H^*}) + (1-\pi^H)U(C_2^{H^*}) - (1-\pi^L)U\left(\frac{R(1-\pi^L C_1^{H^*})}{1-\pi^L}\right).$$

Note that $EV_D^*(V)/dq|_{q=1}$ is increasing in π^L as

$$\frac{\partial\left(dEV_D^*(V)/dq|_{q=1}\right)}{\partial \pi^L} = U\left(\frac{R(1-\pi^L C_1^{H^*})}{1-\pi^L}\right) - U(C_1^{H^*}) + \frac{R(C_1^{H^*}-1)}{1-\pi^L}U'\left(\frac{R(1-\pi^L C_1^{H^*})}{1-\pi^L}\right) > 0$$

where, following similar reasoning, for $\pi^L = \pi^H$ it becomes $EV_D^*(V)/dq|_{\{q=1,\pi^L=\pi^H\}} = 0$. Therefore, $EV_D^*(V)/dq|_{q=1}$ is negative for any $0 < \pi^L < \pi^H$, and consequently, $EV_D^*(V)/dq$ is negative for any $q \in [0,1]$. Thus, consumers' expected utility is strictly decreasing and convex in q when the equilibrium deposit contract is offered.

Proof of Lemma 3.2

The proof is based on determining the optimal dividend payment for each alternative configuration of the secondary market without initially imposing any restrictions on δ. After imposing the restrictions on δ for which each case is defined, the region where δ^* lies can be established, and the resulting equilibrium configuration in the secondary market can be determined.

Substituting for the consumption allocations given in equation (11) into the objective function for each alternative configuration of the secondary market, consumers' expected utility can be expressed in terms of δ. The concavity of the utility function and linearity of consumption allocations with respect to δ guarantee the existence of a unique dividend payment for each configuration that maximises consumers' expected utility. In particular, for the market configurations where $\delta < \widetilde{\delta_L}$ (say case A) and $\widetilde{\delta_L} \leq \delta < \widetilde{\delta_H}$ (say case B), the optimal dividend payment in each case is an interior solution to the maximisation problem. Note also that $\widetilde{\delta_L}$ is the unique tangency point of consumers' expected utility in these two cases. However, for $\widetilde{\delta_H} \leq \delta$ (say case C), the dividend payment that maximises consumers' expected utility given by $U(\delta + R(1-\delta))$ is a corner solution since $dU/d\delta = -(R-1)U'(\delta + R(1-\delta)) < 0$ for any $\delta \in (0,1)$. In addition, $\widetilde{\delta_H}$ is the unique tangency point of consumers' expected utility in cases B and C. Hence, introducing the constraints on δ for which each case is defined, consumers' expected utility is never maximised for the configuration where the secondary market is in the non-surplus equilibrium for both states. In contrast, cases A and B constitute possible equilibrium configurations for the secondary market depending on the parameters of the model and the utility function.

Proof of Property 3.3

For $\delta^* < \widetilde{\delta_L}$, the equilibrium allocations given in equation (11) can be expressed in terms of C_{1E}^{H*} by simplifying for δ^* as

$$C_{1E}^{L*} = \frac{\pi^H}{\pi^L} C_{1E}^{H*},\ C_{2E}^{H*} = \frac{R(1-\pi^H C_{1E}^{H*})}{1-\pi^H} \text{ and } C_{2E}^{L*} = \frac{R(1-\pi^H C_{1E}^{H*})}{1-\pi^L}.$$

Substituting for the above payoffs in the first-order condition (equation (12)), the latter can be expressed in terms of C_{1E}^{H*}. Differentiating C_{1E}^{H*} with respect to q yields

$$\frac{dC_{1E}^{H*}}{dq} = -\frac{U'(C_{1E}^{H*}) - U'(C_{1E}^{L*}) - R\left(U'(C_{2E}^{H*}) - U'(C_{2E}^{L*})\right)}{qU''(C_{1E}^{H*}) + (1-q)\frac{\pi^H}{\pi^L}U''(C_{1E}^{L*}) + \pi^H R^2\left(q\frac{U''(C_{2E}^{H*})}{1-\pi^H} + (1-q)\frac{U''(C_{2E}^{L*})}{1-\pi^L}\right)} > 0$$

which is always positive since the denominator is negative from the concavity of the utility function, and the numerator is positive from the relationship between the equilibrium payoffs ($C_{1E}^{H*} < C_{1E}^{L*} < C_{2E}^{L*} < C_{2E}^{H*}$) and the concavity of the utility function. Hence, for $\delta^* < \widetilde{\delta_L}$, it follows that $dC_{1E}^{S*}/dq > 0$ and $dC_{2E}^{S*}/dq < 0$ from the feasibility constraints.

In a similar manner, for $\widetilde{\delta_L} \leq \delta^* < \widetilde{\delta_H}$, the consumption allocation for the low state can be written in terms of C_{1E}^{H*} as

$$C_{1E}^{L*} = C_{2E}^{L*} = \pi^H C_{1E}^{H*} + R(1-\pi^H C_{1E}^{H*}),$$

since $\delta^* = \pi^H C_{1E}^{H*}$.

Substituting the above payoffs in the first-order condition and differentiation of C_{1E}^{H*} with respect to q provides

$$\frac{dC_{1E}^{H*}}{dq} = -\frac{U'(C_{1E}^{H*}) - U'(C_{1E}^{L*}) - R\left(U'(C_{2E}^{H*}) - U'(C_{1E}^{L*})\right)}{qU''(C_{1E}^{H*}) + qR^2\frac{\pi^H}{1-\pi^H}U''(C_{2E}^{H*}) + (1-q)\pi^H(1-R)^2 U''(C_{1E}^{L*})} > 0$$

which is positive following similar reasoning as in the case where $\delta^* < \widetilde{\delta_L}$. Hence, $dC_{1E}^{H*}/dq > 0$ and $dC_{2E}^{H*}/dq < 0$, whereas $dC_{1E}^{L*}/dq = -\pi^H(R-1)\,dC_{1E}^{H*}/dq < 0$ when $\widetilde{\delta_L} \leq \delta^* < \widetilde{\delta_H}$.

The above properties of the equilibrium payoffs can be used to determine the properties of the consumers' expected utility with respect to q in equilibrium. From the Envelope Theorem, the comparative static property of the optimal value function ($EV_E^*(V)$) with respect to q for $\delta^* < \widetilde{\delta_L}$ is

$$\frac{d\,EV_E^*(V)}{d\,q} = \frac{\partial EV_E^*(V)}{\partial q} = V_E^{H^*} - V_E^{L^*}$$

Differentiating again with respect to q provides

$$\frac{d^2 EV_E^*(V)}{dq^2} = \pi^H \frac{dC_{1E}^{H^*}}{dq}\left(U'(C_{1E}^{H^*}) - U'(C_{1E}^{L^*}) - R\left(U'(C_{2E}^{H^*}) - U'(C_{2E}^{L^*})\right)\right) > 0$$

which is positive from the relationship between the optimum payoffs and the concavity of the utility function as discussed previously in proving that $dC_{1E}^{H^*}/dq > 0$.

Since $dEV_E^*(V)/dq$ is monotonically increasing in q, evaluation at $\widetilde{q}$ which is the maximum value that q can take for $\delta^* \leq \widetilde{\delta_L}$ yields

$$\left.\frac{dEV_E^*(V)}{dq}\right|_{q=\widetilde{q}} = \pi^H U(C_{1E}^{H^*}(\widetilde{\delta_L})) + (1-\pi^H)U\left(\frac{R(1-\pi^H C_{1E}^{H^*}(\widetilde{\delta_L}))}{1-\pi^H}\right) - U(\pi^H C_{1E}^{H^*}(\widetilde{\delta_L}) + R(1-\pi^H C_{1E}^{H^*}(\widetilde{\delta_L}))) < 0$$

which is negative from Jensen's inequality due to the strict concavity of the utility function, and therefore, $dEV_E^*(V)/dq = V_E^{H^*} - V_E^{L^*} < 0$ for any $q \in [0, \widetilde{q})$. The consumers' expected utility in equilibrium is strictly decreasing and convex when $\delta^* \leq \widetilde{\delta_L}$.

Similarly, in the case where $\widetilde{d_L} \leq d^* < \widetilde{d_H}$, from the Envelope Theorem it follows that

$$\frac{dEV_E^*(V)}{dq} = \frac{\partial EV_E^*(V)}{\partial q} = V_E^{H^*} - U^{L^*} < 0,$$

where $d^* = \pi C_{1E}^{H^*}$ and therefore

$$V_E^{H^*} - U^{L^*} = \pi^H U(C_{1E}^{H^*}) + (1-\pi^H)U\left(\frac{R(1-\pi^H C_{1E}^{H^*})}{1-\pi^H}\right) - U(\pi^H C_{1E}^{H^*} + R(1-\pi^H C_{1E}^{H^*})) < 0$$

from Jensen's inequality due to the concavity of the utility function.

In addition, the second derivative will be

$$\frac{d^2 EV_E^*(V)}{dq^2} = \pi^H \frac{d\,C_{1E}^{H^*}}{d\,q}\left(U'(C_{1E}^{H^*}) - U'(C_{1E}^{L^*}) - R\left(U'(C_{2E}^{H^*}) - U'(C_{1E}^{L^*})\right)\right) > 0$$

which, similar to the case where $q < \widetilde{q}$, is positive from the relationship between the optimum payoffs and the concavity of the utility function.

Thus, the consumers' expected utility is strictly decreasing and convex in $q \in [0, 1]$ when the equilibrium equity contract is offered.

Proof of Proposition 4.1

From properties 3.1 and 3.3, consumers' expected utility under each contractual arrangement is strictly decreasing and convex in q. Hence, in order to conclude about the dominance of each contract, the difference between the slopes of the optimal value function with respect to q of the two contracts is evaluated at $q = 0$ and $q = 1$.

Let $\Delta^*_{ED} = EV^*_E(V) - EV^*_D(V)$ be the difference between the expected utility attained under an equity and deposit contract in equilibrium. Differentiating with respect to q it holds that

$$\frac{d\Delta^*_{ED}}{dq} = \frac{\partial EV^*_E(V)}{\partial q} - \frac{\partial EV^*_D(V)}{\partial q}$$

where for either contract $\partial EV^*/\partial q = V^H - V^L < 0$ from properties 3.1 and 3.3.

Evaluating the above difference at $q = 1$ it follows that

$$\left.\frac{d\Delta^*_{ED}}{dq}\right|_{q=1} = V_D^{L^*} - V_E^{L^*}$$

as both contracts achieve the social optimum in the high state. Note also that since $\widetilde{q} < 1$, the equity contract attains the non-surplus equilibrium in the low state.

Specifically, for $q = 1$ the equilibrium payoffs in the low state for the equity and deposit contracts respectively, will be

$$C^{L^*}_{1E}(q=1) = \pi^H C_1^{H^*} + R(1 - \pi^H C_1^{H^*}) \text{ since } d^* = \pi^H C_1^{H^*},$$
$$D_1^*(q=1) = C_1^{H^*} \text{ and } D_2^{L^*}(q=1) = \frac{R(1-\pi^L C_1^{H^*})}{1-\pi^L}.$$

Therefore, $d\Delta^*_{ED}/dq|_{q=1}$ will be

$$\left.\frac{d\Delta_{ED}}{dq}\right|_{q=1} = \pi^L U(C_1^{H^*}) + (1-\pi^L) U\left(\frac{R(1-\pi^L C_1^{H^*})}{1-\pi^L}\right) - U\left(\pi^H C_1^{H^*} + R(1-\pi^H C_1^{H^*})\right).$$

In order to prove that the deposit contract is the welfare dominant contract at $q = 1$, provided that consumers' expected utility under each contract is strictly decreasing in q, it is sufficient to show that $d\Delta_{ED}/dq|_{q=1} > 0$.

Note that $d\Delta_{ED}/dq|_{q=1}$ is increasing in $C_1^{H^*}$

$$\begin{aligned}\frac{\partial (d\Delta_{ED}/dq|_{q=1})}{\partial C_1^{H^*}} =& \pi^L \left(U'(C_1^{H^*}) - RU'\left(\frac{R(1-\pi^L C_1^{H^*})}{1-\pi^L}\right)\right) \\ &+ \pi^H (R-1) U'(\pi C_1^{H^*} + R(1-\pi^H C_1^{H^*})) > 0.\end{aligned}$$

As the autarky payoff of $C_1^{H^*} = 1$ is the minimum value that $C_1^{H^*}$ can take, evaluation of $d\Delta_{ED}/dq|_{q=1}$ at $C_1^{H^*} = 1$ yields

$$\left.\frac{d\Delta_{ED}}{dq}\right|_{\{q=1,C_1^{H*}=1\}} = \pi^L U(1) + (1-\pi^L)U(R) - U\left(\pi^H + R(1-\pi^H)\right).$$

Given that $\widetilde{q} < 1$, the following condition on the model's parameters should hold for the equity contract to be in the non-surplus equilibrium

$$\pi^H C_1^{H*} + R(1-\pi^H C_1^{H*}) \leq \frac{R}{\pi^L R + 1 - \pi^L}, \text{ or } C_1^{H*} \geq \frac{\pi^L}{\pi^H}\frac{R}{\pi^L R + 1 - \pi^L}.$$

For $C_1^{H*} = 1$, the above condition can be expressed in terms of R as $R \leq \pi^H(1-\pi^L)/\left(\pi^L(1-\pi^H)\right)$. Let $\widetilde{R}$ be the value of R for which the condition holds with equality. As $d\Delta_{ED}/dq|_{\{q=1,C_1^{H*}=1\}}$ is decreasing in R from

$$\frac{\partial\left(\left.\frac{d\Delta_{ED}}{dq}\right|_{\{q=1,C_1^{H*}=1\}}\right)}{\partial R} = (1-\pi^L)U'(R) - (1-\pi^H)U'\left(\pi^H + R(1-\pi^H)\right) < 0,$$

evaluation of $d\Delta_{ED}/dq|_{\{q=1,C_1^{H*}=1\}}$ at $\widetilde{R}$ yields

$$\left.\frac{d\Delta_{ED}}{dq}\right|_{\{q=1,C_1^{H*}=1,\widetilde{R}\}} = \pi^L U(1) + (1-\pi^L)U(\widetilde{R}) - U(\pi^H/\pi^L).$$

In order for the autarky allocation to be the social optimum allocation in the high state, from the first-order condition in the social planner's problem given in equation (4), the coefficient of relative risk aversion should be equal to one. In this case, the utility function takes the logarithmic form and the above expression becomes decreasing in π^L, where $\partial\left(d\Delta_{ED}/dq|_{\{q=1,C_1^{H*}=1,\widetilde{R}\}}\right)/\partial\pi^L = -\log(\widetilde{R}) < 0$. In addition, when evaluated at the maximum value that π^L can take (i.e. $\pi^L = \pi^H$) it is equal to zero. Therefore, as $\pi^L < \pi^H$, $d\Delta_{ED}/dq|_{\{q=1,C_1^{H*}=1,\widetilde{R}\}} > 0$ for any R when the secondary market is in the non-surplus equilibrium, and consequently, $d\Delta_{ED}/dq|_{q=1} > 0$ for any $C_1^{H*} > 1$. Hence, as q approaches 1, the expected utility that consumers derive from the deposit contract is greater than that of the equity contract as illustrated in Figure 2.

Similarly, evaluating the difference between the slopes of the expected utilities at $q = 0$ yields

$$\left.\frac{d\Delta_{ED}}{dq}\right|_{q=0} = V_E^{H*} - V_D^{H*}$$

since both contracts achieve the social optimum for the low state. Substituting for the payoffs provides

$$\left.\frac{d\Delta_{ED}}{dq}\right|_{q=0} =\pi^H U\left(\frac{\pi^L}{\pi^H}C_1^{L^*}\right)+(1-\pi^H)U\left(\frac{R\left(1-\pi^L C_1^{L^*}\right)}{1-\pi^H}\right)-$$
$$\left(\pi^H U(C_1^{L^*})+(1-\pi^H)U\left(\frac{R\left(1-\pi^H C_1^{L^*}\right)}{1-\pi^H}\right)\right)$$

Given that consumers' expected utility under each contract is strictly decreasing in q, for the deposit contract to be the welfare dominant one in the region around $q = 0$ requires $d\Delta_{ED}/dq|_{q=0} < 0$. However, no positive conclusions can be drawn about the sign of the above expression as this depends on the parameters of the model and the functional form of the utility. In particular, for the constant relative risk aversion utility function of the form $U(C) = C^{1-\gamma}/(1-\gamma)$, where $\gamma > 1$ the coefficient of relative risk aversion, algebraic examples can be provided where the above difference can take both positive and negative signs.

From the first-order condition of the social planner and the intertemporal budget constraint, the social optimum payoff at date 1 in the low state is $C_1^{L^*} = \left(\pi^L + (1-\pi^L)R^{\frac{1-\gamma}{\gamma}}\right)^{-1}$. Suppose for example that $\gamma = 2$, $\pi^H = 0.7$ and $\pi^L = 0.4$. For $R = 3.5$, $d\Delta_{ED}/dq|_{q=0} \approx 0.46$, or alternatively $V_E^{H^*} > V_D^{H^*}$. In terms of Figure 2 and given that both $EV_E^*(V)$ and $EV_D^*(V)$ are decreasing in q, this implies that $EV_E^*(V)$ is flatter than $EV_D^*(V)$ at $q = 0$ and therefore the equity contract initially dominates and the threshold value of $q^* \in (0, 1)$ can be defined where $EV_D^*(V)$ is represented by the solid line. In terms of Figure 3, this implies that the allocation of the equity contract in the high state lies on a higher indifference curve than the allocation of the deposit contract as $V_E^{H^*} > V_D^{H^*}$. On the contrary, for $R = 2$, $d\Delta_{ED}/dq|_{q=0} \approx -0.22$, or $V_E^{H^*} < V_D^{H^*}$. Hence, $EV_E^*(V)$ is steeper than $EV_D^*(V)$ at $q = 0$ in Figure 2 and therefore the deposit contract dominates, where $EV_D^*(V)$ is represented by the dashed line. In this case, the equity contract's allocation lies on a lower indifference curve in Figure 3.

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