

Bundling and Competition for Slots: Sequential Pricing*

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Abstract

In this paper we study, as in Jeon-Menicucci (2009), competition between sellers when each of them sells a portfolio of distinct products to a buyer having limited slots. This paper considers sequential pricing and complements our main paper (Jeon-Menicucci, 2009) that considers simultaneous pricing.

First, Jeon-Menicucci (2009) find that under simultaneous individual pricing, equilibrium often does not exist and hence the outcome is often inefficient. By contrast, equilibrium always exists under sequential individual pricing and we characterize it in this paper. We find that each seller faces a trade-off between the number of slots he occupies and surplus extraction per product, and there is no particular reason that this leads to an efficient allocation of slots.

Second, Jeon-Menicucci (2009) find that when bundling is allowed, there always exists an efficient equilibrium but inefficient equilibria can also exist due to pure bundling (for physical products) or slotting contracts. Under sequential pricing, we find that all equilibria are efficient regardless of whether firms can use slotting contracts, and both for digital goods and for physical goods. Therefore, sequential pricing presents an even stronger case for laissez-faire in the matter of bundling than simultaneous pricing.

Key words: Bundling, Portfolios, Slots (or Shelf Space), Pure Bundling, Slotting Contracts.

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1 Introduction

There are many situations in which sellers with different portfolios of products compete for limited slots (or shelf space) of a buyer who wants to build up her own portfolio of distinct products. In this situation, sellers may employ bundling as a strategy to win the competition for slots. Even though bundling has been a major antitrust issue and a subject of intensive research, to the best of our knowledge, the literature seems to have paid little attention to competition among portfolios of distinct products and, in particular, no paper seems to have studied how bundling affects portfolios' competition for slots.

Examples of situations we described above are abundant both among digital products and among physical products. For instance, in the movie industry, each movie distributor has a portfolio of distinct movies and buyers (either movie theaters or TV stations) have limited slots. More precisely, the number of movies that can be projected in a season (or in a year) by a theater is constrained by time and the number of projection rooms. Likewise, the number of movies that a TV station can show during prime time of a season (or year) is limited. Actually, allocation of slots in movie theaters has been one of the main issues raised in the movie industry during the last presidential election in France¹. Furthermore, bundling in the movie industry (known as block booking²) was declared illegal in two supreme court decisions in U.S.: *Paramount Pictures* (1948), where blocks of films were rented for theatrical exhibition, and *Loew's* (1962), where blocks of films were rented for television exhibition. In addition, recently in *MCA Television Ltd. v. Public Interest Corp.* (11th Circuit, April 1999), the court of appeals reaffirmed the *per se* illegal status of block booking.

A different situation we have in mind is that of manufacturers' competition for retailers' shelf space. Manufacturers having a portfolio of products may practice bundling (often called full-line forcing) to win competition for slots³ and there have been antitrust cases related to this practice⁴. For instance, the French Competition Authority fined Société

¹Cahiers du Cinema (April, 2007) proposes to limit the number copies per movie since certain movies by saturating screens limits other movies' access to screens and asks each presidential candidate's opinion about the policy proposal.

²Block booking refers to "the practice of licensing, or offering for license, one feature or group of features on the condition that the exhibitor will also license another feature or group of features released by distributors during a given period" (Unites States v. Paramount Pictures, Inc., 334 U.S. 131, 156 (1948)).

³For instance, Procter and Gamble uses 'golden-store' arrangement such that to be considered a golden store, a retailer must agree to carry 40 or so P&G items displayed together. See "P&G has big plans for the shelves of tiny stores in emgering nations", Wall Street Journal, July 17, 2007.

⁴Société des Caves de Roquefort, Conseil de la Concurrence, Decision 04-D-13, 8th April 2004. R.J.

des Caves de Roquefort for using selectivity or exclusivity contracts with supermarket chains.⁵ Furthermore, slotting arrangements, the payment by manufacturers for retail shelf space, have become increasingly important and have been the subject of recent antitrust litigations⁶ and the focus of Federal Trade Commission studies.⁷

In our main paper (Jeon-Menicucci, 2009), we study a simultaneous pricing game among n sellers (or firms) who sell their products to a buyer having $k(> 0)$ number of slots. This paper complements Jeon-Menicucci (2009): we consider a duopoly model of Jeon-Menicucci (2009) and study sequential pricing instead of simultaneous pricing. In the model, each seller i ($=A, B$) has a portfolio of n_i distinct products. We assume that the prototype of each product is already made and call a product a digital good (a physical good) if the cost of producing a copy is zero (strictly positive). The buyer has a unit demand for each product. A product needs to occupy a slot to generate a value. Products have heterogeneous values and the values are independent.⁸ Social efficiency requires the slots to be allocated to the best k products among all products. In this setup, we study how the outcome of competition depends on the nature of products (digital goods versus physical goods) and different bilateral contractual arrangements between each seller and the buyer.

Given a portfolio of products belonging to a firm, we define bundling as a menu contract that specifies a price for every subset of the portfolio. A particular class of bundling contracts is what we call "independent pricing plus a fixed fee". A strategy in this class consists of a fixed fee for the right to buy products in the portfolio and one individual price for each product. There are three interesting special cases of this class. Individual pricing corresponds to the case with zero fixed fee; pure bundling corresponds to the case with zero individual prices; a "technology-renting" strategy is the case in which each individual price is equal to the cost of production.

Interestingly, the change from independent pricing to bundling opens a new contractual dimension, i.e. contracting on slots. Note that under independent pricing, the buyer will

Reynolds Tobacco Co. v. Philip Morris, Inc., 199 F. Supp. 2d 363 (M.D.N.C. 2002). In the second case, Reynolds Tobacco accused Philip Morris for using "Retail Leaders" contract that provides discounts to retailers on its popular Marlboro brand in exchange for the most advantageous display and signage space in retail establishments.

⁵Société des Caves de Roquefort's market share in the Roquefort cheese market was 70% but, through the contract, could occupy eight among all nine brands that Carrefour, a supermarket chain, sold.

⁶See, for instances, American Booksellers Ass'n, Inc. v. Barnes & Noble, Inc., 135 F. Supp. 2d 1031 (N.D. Cal. 2001); Intimate Bookshop, Inc. v. Barnes & Noble, Inc., 88 F. Supp. 2d 133 (S.D.N.Y. 2000); FTC v. H.J. Heinz Co., 116 F. Supp. 2d 190 (D.C.C. 2000), rev'd, 246 F.3d 708 (D.C. Cir. 2001).

⁷See FTC Report (2001) and FTC Study (2003).

⁸In other words, the value that a product generates does not depend on the set of the other products that occupy the slots.

purchase only those products that would occupy a slot and therefore slotting contracts are redundant. In contrast, under bundling, for instance, if all firms offer pure bundles, the buyer may end up buying more products than the slots and hence we need to distinguish bundling with slotting contracts and bundling without slotting contracts. A slotting contract is defined such that if a bundle is sold with a slotting contract, the buyer must allocate a slot to each product in the bundle: exclusive dealing corresponds to a special case in which the number of products in the bundle sold with a slotting contract is equal to k . Therefore, the contractual space increases as we move from individual pricing to bundling without slotting contracts and from bundling without slotting contracts to bundling with the permission of slotting contracts.

The main results from Jeon-Menicucci (2009) are the following. First, under independent pricing, equilibrium often does not exist and hence the market outcome is often inefficient in terms of allocation of slots. Second, when bundling is allowed, there always exists an efficient equilibrium where each firm uses a technology-renting strategy, regardless of whether or not firms can use slotting contracts. Furthermore, if slotting contracts are prohibited and firms sell digital goods, all equilibria are efficient. However, if sellers can use slotting contracts, inefficient equilibria can arise. Furthermore, even if slotting contracts are prohibited, pure bundling can generate inefficient equilibria for physical goods.

The results we obtain by studying sequential pricing are complementary to and consistent with the results obtained by Jeon-Menicucci (2009). We first characterize equilibrium with independent pricing, since such equilibrium always exists under sequential moves. In the equilibrium, each seller faces a trade-off between number of slots to occupy and extracting surplus per product, as does a monopolist charging a uniform price under a demand curve with a negative slope. We find that there is no particular reason that this leads to the efficient allocation of slots. In fact, each seller may end up occupying too many or too few slots.

Second, when bundling is allowed, the trade-off between number of slots and surplus extraction disappears since adding another product into a bundle does not reduce the surplus that a seller can extract from the products already included into the bundle. This is because bundling such as technology renting eliminates internal competition among products belonging to the same seller by making it impossible to purchase a subset of products without paying the fixed fee. Interestingly, we find that all equilibria are efficient regardless of whether firms can use slotting contracts and both for digital goods and physical goods. Sequential pricing eliminates coordination failures between sellers that generate inefficient equilibria under simultaneous pricing. Under simultaneous pricing, given that a rival seller offers a pure bundle of all products (respectively, a pure bundle of all products with a

slotting contract), it is a best response to offer a pure bundle of all products (respectively, a pure bundle of all products with a slotting contract) and Bertrand competition between two pure bundles or two slotting contracts leads to an inefficient equilibrium. In contrast, under sequential pricing, the first mover does not take the offer of the second mover as given but correctly anticipates how the second mover would react to his own offer, and inducing the second mover to offer a pure bundle of all products without or with a slotting contract typically is not optimal for the first mover.

Our paper generates clear-cut policy implications. Under independent pricing, there is no guarantee that competition leads to efficient allocation of slots. On the contrary, when bundling is allowed, efficient allocation of slots is achieved in any equilibrium regardless of whether firms can use slotting contracts and both for physical goods and digital goods. Therefore, sequential pricing presents an even stronger case for laissez-faire in the matter of bundling than simultaneous pricing.

According to the leverage theory, on which the Supreme Court's decisions to prohibit block booking were based, block booking allows a distributor to extend its monopoly power in a desirable movie to an undesirable one. This theory was criticized by Chicago School (see e.g. Bowman 1957, Posner 1976, Bork 1978) since the distributor is better off by selling only the desirable movie at a higher price. As an alternative, Stigler (1968) proposed a theory based on price discrimination⁹, which became a dominant strand (Schmalensee, 1984, McAfee et al. 1989, Shaffer, 1991, Salinger 1995 and Armstrong 1996) at least until Whinston (1990) resuscitated the leverage theory with its first formal treatment (see, for the later work in this line, Choi-Stefanadis 2001, Carlton-Waldman 2002, and Nalebuff 2004).¹⁰ Basically, in Whinston, tying allows an incumbent to commit to be aggressive, which discourages entry if there is a fixed cost of entry. On the contrary, in Jeon-Menicucci (2009), bundling softens competition from rival products and hence it is possible that every firm realizes a (weakly) higher profit when bundling is allowed than when it is prohibited. Then, bundling is unlikely to be an instrument of foreclosure.

⁹However, Kenney and Klein (1983) point out that simple price discrimination explanation is inconsistent with the facts of *Paramount* and *Loew's* and argue that block booking mainly prevents exhibitors from oversearching, (i.e. from rejecting films revealed ex post to be of below-average value). Their hypothesis is empirically tested in a recent paper by Hanssen (2000) but the author finds little support for the hypothesis. But Kenny and Klein (2000) do not agree with Hanssen's analysis.

¹⁰Armstrong-Vickers (2008) is a bit related to our paper since they consider bundling in a symmetric situation: they study competition between two symmetric firms producing two horizontally differentiated products (i.e. consumers are located in a two-dimensional hotelling space). They find that compared to linear pricing, non-linear pricing has the benefit of efficient variable prices (i.e. marginal cost pricing) but the cost of excessive brand loyalty.

Since each firm can bundle any number of products in our paper, we also contribute to the recent literature on bundling a large number of products. More precisely, in a framework of second-degree price discrimination, Armstrong (1999) and Bakos and Brynjolfsson (1999) show that bundling allows a monopolist to extract more surplus since it reduces the variance of average valuations by the law of large numbers. In our paper, since we assume complete information, the rent extraction issue does not arise in a monopoly setting and the law of large number plays no role. In Jeon-Menicucci (2006), we take a framework similar to the one in Jeon-Menicucci (2009) to study bundling electronic academic journals; publishers owning portfolios of distinct journals compete to sell them to a library. The key difference is that competition is generated by the budget constraint of the library instead of the slot constraint. In both papers, we find that bundling is a profitable strategy in terms of surplus extraction. However, contrary to Jeon-Menicucci (2009), Jeon-Menicucci (2006) find that bundling reduces social welfare since if large publishers extract more surplus with bundling, there is less (even zero) budget left for small publishers.

Our efficiency result of bundling is very closely related to the finding in literature on common agency (Bernheim and Whinston (1985, 1986, 1998), O'Brien and Shaffer (1997, 2005)) that shows that when multiple principals deal with a common agency, they can achieve the outcome that maximizes the payoffs of all players. In particular, Bernheim and Whinston (1998) and O'Brien and Shaffer (1997) study the situation when two single-product firms simultaneously offer non-linear tariffs together with exclusive dealing contract to a common retailer and find that the vertically-integrated outcome is obtained.¹¹ However, the papers also find other inefficient equilibria and use either the coalition-proof Nash equilibria (Bernheim and Whinston 1986) or Pareto dominance (Bernheim and Whinston 1998 and O'Brien and Shaffer 1997) to select the equilibrium maximizing joint profits. Our contribution is to show that sequential pricing eliminates the inefficient equilibria. Our paper also differs in its focus on the comparison between independent pricing and bundling in a novel setting of competition among portfolios in the presence of slot constraint.

In what follows, section 2 presents the model. Section 3 characterizes the equilibrium under independent pricing. Section 4 analyzes the case in which bundling is allowed by distinguishing when slotting contracts are prohibited (section 4.1) from when slotting contracts are allowed (section 4.2). Section 5 derives policy implications and concludes the paper.

¹¹O'Brien-Shaffer (2005) show that this result also holds under simultaneous Nash bargaining for the case of N single-product firms.

2 The Model

The model is very similar to the model of Jeon-Menicucci (2009). The main difference lies in the timing of the moves: in Jeon-Menicucci (2009) we study simultaneous pricing, while here we analyze sequential moves. Since the game with sequential moves under independent pricing is a bit complex to characterize, for simplicity we consider two sellers.

2.1 Setting

There are two sellers (firms), denoted by $i = A, B$, and a customer, denoted by C ; we use "he" for each seller and "she" for the customer. Seller i has a portfolio of $n_i (\geq 1)$ products, for $i = A, B$, and all products are distinct. We use ij to denote seller i 's j -th best product (for instance, $A2$ represents seller A 's 2nd best product) and $\mathcal{B}_i = \{i1, \dots, in_i\}$ represents i 's portfolio of products; let $\mathcal{B} \equiv \mathcal{B}_A \cup \mathcal{B}_B$. C has a unit demand for each product and has k number of slots. A product needs to occupy a slot to generate a value,¹² and thus the slot constraint generates competition among the products. We use u_i^j to denote the value that C obtains from allocating a slot to product ij ; thus $u_i^1 \geq u_i^2 \geq \dots \geq u_i^{n_i} \geq 0$ for $i = A, B$. Without loss of generality, we suppose that $n_i = k$ for $i = A, B$: in the case in which $n_i \geq k$, it is straightforward that only the k best products of firm i matter; in the case of $n_i < k$, we define $u_i^{n_i+1} = \dots = u_i^k = 0$. We assume that C 's payoff is given by the sum of the values obtained from the purchased products minus her total outlay.¹³

With respect to production costs, we have in mind a situation in which the prototype of each product is already produced, and the cost of (re)production is $c \geq 0$ for every product $ij \in \mathcal{B}$; no cost is incurred by C . Given that we consider heterogenous values, our results can be easily extended to the case of heterogenous production costs $c_i^j \geq 0$ as in Jeon-Menicucci (2009).

Let u^j denote the value that C obtains from the j -th best product among all products in \mathcal{B} ; thus $u^1 \geq u^2 \geq \dots$; we assume that $u^k > \max\{c, u^{k+1}\}$. Hence, the set of the k best products, denoted with B^{FB} , is unique and it is socially optimal that all the slots are occupied by the products in B^{FB} . In what follows, we say that an equilibrium is (socially) efficient if the slots are allocated to the products in B^{FB} . For any $B \subseteq \mathcal{B}$, let

¹²By assuming unit demand, we assume for simplicity that a product can occupy at most one slot in that the value generated from occupying a second slot is zero. This assumption can be relaxed without changing the main results.

¹³Even though we consider one customer, our model can be applied in a straightforward way to a situation with multiple customers as long as each customer operates in a separate market and each seller can price-discriminate the customers.

$U(B)$ represent the total value that the buyer obtains from allocating k slots to the best k products in B ; obviously, if B has less than k number of products, the total value is computed by allocating one slot to each product. In particular, we define

$$U^{FB} \equiv U(B^{FB}) = u^1 + \dots + u^k :$$

and let $B_i^{FB} \equiv B^{FB} \cap \mathcal{B}_i$, with $q_i^{FB} (\geq 0)$ to represent the cardinality of B_i^{FB} (hence, $q_A^{FB} + q_B^{FB} = k$); furthermore, $U_i^{FB} \equiv U(B_i^{FB}) = u_i^1 + \dots + u_i^{q_i^{FB}}$. For the analysis of bundling we also need to define B_i^{SB} as the set of products in $\mathcal{B}_i \setminus B_i^{FB}$ which have value larger than c , and we use q_i^{SB} to represent its cardinality; $U_i^{SB} \equiv U(B_i^{SB}) = u_i^{q_i^{FB}+1} + \dots + u_i^{q_i^{FB}+q_i^{SB}}$ is the value of the products in B_i^{SB} . We let $\bar{q}_i \equiv q_i^{FB} + q_i^{SB}$ denote the number of products in \mathcal{B}_i with value larger than c .

2.2 Contracts and games

In this section, we first describe the bilateral contracts that seller i can propose to C in our model and then introduce the timing of the games that we study.

2.2.1 Bundling without slotting contracts

- Menu of bundles¹⁴

In the absence of slotting contracts (that will be defined later on), the most general bilateral contract between seller i and C is that seller i offers a menu of bundles with prices $\{P_i(B_i)\}_{B_i \subseteq \mathcal{B}_i}$: seller i chooses $P_i(B_i) \geq 0$ for each $B_i \subseteq \mathcal{B}_i$, with $P_i(\emptyset) = 0$. Then, if C buys bundle B_A from seller A and bundle B_B from seller B (some of these sets may be empty),¹⁵ then she pays $P_A(B_A) + P_B(B_B)$. Let $s_i = \{P_i(B_i)\}_{B_i \subseteq \mathcal{B}_i}$ denote a generic pricing schedule of firm i and S_i be the space of pricing schedules for firm i .

- Independent pricing plus a fixed fee

A particular class of menu of bundles is the strategy which is composed of individual prices (p_{i1}, \dots, p_{ik}) and a fixed fee $F_i \geq 0$ such that $P_i(B_i) = F_i + \sum_{ij \in B_i} p_{ij}$ for any (non-empty) $B_i \subseteq \mathcal{B}_i$. In this case, if C wants to buy at least one product from seller i , she must first pay F_i for the right to buy, and then she pays the individual prices of the products

¹⁴Our definition of menu of bundles generalizes the notion of mixed bundling used in the context of two goods. In this case, mixed bundling means that the seller charges a price for each good and another price for the bundle of both goods.

¹⁵In what follows, we simply write that C buys $B_A \cup B_B$.

that she selects to buy. Three particular cases of "independent pricing plus a fixed fee" strategies are of great interest:

* Independent pricing: Independent pricing is an extreme case with $F_i = 0$, thus $P_i(B_i) = \sum_{ij \in B_i} p_{ij}$ for any $B_i \subseteq \mathcal{B}_i$.

* Pure bundling: Pure bundling is another extreme case with $p_{ij} = 0$ for each $ij \in \mathcal{B}_i$ such that $P_i(B_i) = F_i$ for any $B_i \subseteq \mathcal{B}_i$. In other words, pure bundling is a deal of all-or-nothing. However, since we assume free disposal, C can decide not to allocate any slot to any subset of the products in \mathcal{B}_i .

* Technology-renting: A technology renting strategy consists of two elements: firm i rents its production technology to C by charging $p_{ij} = c$ for each $ij \in \mathcal{B}_i$, and then uses a fixed rental fee F_i to extract (part of) C 's surplus. Let TR_i be the set of technology-renting strategies, and $tr_i \in TR_i$ an element of the set.

2.2.2 Bundling with slotting contracts

In what follows, we will distinguish two cases depending on whether slotting contracts are used or not. If seller i does not use any slotting contract, C has full freedom in allocating the slots among all products she purchased. In contrast, if C buys from firm i a bundle B_i with a slotting contract (and $q_i = \#B_i$ is the number of products in B_i), she must allocate q_i number of slots to the products in B_i .

- Exclusive dealing

Exclusive dealing corresponds to the case in which firm i offers a bundle composed of k products with a slotting contract.

Note that under individual pricing, slotting contracts are redundant since C will not buy any product that will not occupy a slot. In section 4.1 we study competition among bundles without slotting contracts, and in section 4.2 we allow for slotting contracts.

2.2.3 Independent pricing

Under independent pricing, firm i chooses p_i^j for his product with value u_i^j . Without loss of generality, we assume $p_i^j > 0$ and define $w_i^j \equiv u_i^j - p_i^j$ as the net value that C obtains from buying this product. Let $\mathbf{p}_i \equiv (p_i^1, p_i^2, \dots, p_i^k)$ and $\mathbf{w}_i \equiv (w_i^1, w_i^2, \dots, w_i^k)$ denote the vectors of prices and of net values for the products of firm i , respectively. It is clear that there is a one-to-one correspondence between \mathbf{p}_i and \mathbf{w}_i , and therefore we can equivalently express firm i 's decision problem in terms of either \mathbf{p}_i or \mathbf{w}_i . However, when we use \mathbf{w}_i we need to

recall that $w_i^j < u_i^j$ for $i = A, B$ and $j = 1, \dots, n_i$. In particular, we will sometimes refer to the condition

$$w_A^1 < u_A^1, \quad w_A^2 < u_A^2, \quad \dots, \quad w_A^k < u_A^k \quad (1)$$

for firm A.

We know that often no equilibrium in pure strategies exists in a simultaneous pricing game under independent pricing (Jeon-Menicucci, 2009). This motivates us to consider a sequential pricing game.

2.3 Timing and tie-breaking rules

We consider the following sequential timing.

- When bundling is prohibited (i.e. under independent pricing),

Stage 1. A chooses \mathbf{p}_A .

Stage 2. After observing \mathbf{p}_A , B chooses \mathbf{p}_B .

Stage 3. After observing \mathbf{p}_A and \mathbf{p}_B , C makes its purchase decision.

- When bundling is allowed

Stage 1. A offers a menu of bundles.

Stage 2. After observing A 's offer, B offers a menu of bundles.

Stage 3. After observing A 's and B 's offers, C decides the bundles (or products) to buy and allocates the slots.

In what follows we use the concept of subgame perfect Nash equilibrium (SPNE) to determine the outcomes of these games. Thus we start with C 's purchases at stage three.

In the game with independent pricing, it is clear that C maximizes her utility by buying the k products with the highest net values, provided that these values are non-negative. However, we also need to specify how C deals with products which have the same net value when the slot constraint is binding. Therefore we introduce the following tie-breaking rules.

T1: If some products have the same net value, C prefers the products offered by firm B .

T1 is motivated by the fact that in our sequential game, given the prices of A 's products, B , as the follower, can always lower slightly the prices of its products to break C 's indifference. Formally, in some cases B has no best reply without this assumption.

T2: If some products offered by the same firm have the same net value, C prefers the products which generate the highest gross values.

T2 is a standard tie-breaking rule.¹⁶

3 Independent pricing

3.1 A preliminary result

Recall that we have set $w_i^j = u_i^j - p_i^j$ for $j = 1, \dots, k$ and $\mathbf{w}_i = (w_i^1, w_i^2, \dots, w_i^k)$, for $i = A, B$. In $\hat{\mathbf{w}}_i \equiv (w_i^{(1)}, w_i^{(2)}, \dots, w_i^{(k)})$ we order instead the net values in a decreasing way, which means that $w_i^{(1)} \geq w_i^{(2)} \geq \dots \geq w_i^{(k)}$. We now prove a simple and intuitive result: for firm i ($i = A, B$) there is no loss of generality in choosing prices such that $w_i^{(j)} = w_i^j$ for all j .

Lemma 1 *Without loss of generality, we can restrict our attention to the case in which $w_i^1 \geq w_i^2 \geq \dots \geq w_i^k$ (i.e., $\mathbf{w}_i = \hat{\mathbf{w}}_i$) for $i=A, B$.*

In particular, lemma 1 implies the following monotonicity condition for firm A, which we will use repeatedly in the remaining of the paper:

$$w_A^1 \geq w_A^2 \geq \dots \geq w_A^k \quad (2)$$

The lemma also implies that when C buys q_i number of products from firm i , she is actually buying i 's products with the highest gross values.

3.2 Stage two

Now we apply backwards induction to firm B , by examining his decision at stage two. Precisely, we take \mathbf{w}_A as given and consider the following questions: given $q \in \{1, \dots, k\}$, is it feasible for B to sell q products? If so, what is the highest profit B can make by selling q products?

Lemma 2 *Given \mathbf{w}_A and $q \in \{1, \dots, k\}$, it is feasible for B to sell q products if and only if $u_B^q > w_A^{k-q+1}$. In this case, the highest profit B can earn by selling q products is $u_B^1 + \dots + u_B^q - q \max\{w_A^{k-q+1}, 0\} - cq$.*

The basic idea of the lemma is that C buys q products from B if and only if these products are among the k products with the highest net values. For instance, consider the

¹⁶This tie-breaking rule is standard in that it is basically equivalent to the following rule applied to two firms producing a homogenous good with different marginal costs. In Bertrand equilibrium, if the cost differential is not large, both firms charge the price equal to the highest marginal cost and the tie is broken by assuming that all consumers buy the good from the firm with the lower marginal cost.

case of $q = 1$. If $w_A^k \geq u_B^1$, then B cannot sell any product because the inequality $w_A^k > w_B^1$ necessarily holds and therefore C will buy k products from A and none from B . If instead $w_A^k < u_B^1$, B succeeds in selling his best product by charging a sufficiently low price p_B^1 such that $w_A^k < u_B^1 - p_B^1$ and p_B^j large for $j \geq 2$. Precisely, from T1, the highest price which induces C to buy B 's best product is $p_B^1 = u_B^1 - \max\{w_A^k, 0\}$. In words, B can sell his best product only if the k -th best product of A gives C a net value that is smaller than the gross value of the best product of B ; only in this case it is possible for B to *block out* the k -th best product of A by pricing aggressively enough his own best product.

For an arbitrary value of q in $\{1, \dots, k\}$, the same argument shows that the inequality $w_A^{k-q+1} < u_B^q$ is necessary, i.e. it must be possible for B to block out the $(k - q + 1)$ -th best product of A by pricing suitably his own q best products. Otherwise, $w_A^{k-q+1} > w_B^q$ holds and therefore C will buy at least $k - q + 1$ products from A , and at most $k - (k - q + 1) = q - 1$ from B . When $w_A^{k-q+1} < u_B^q$, B succeeds in selling q products by charging prices p_B^1, \dots, p_B^q such that $w_B^1 = \dots = w_B^q = \max\{w_A^{k-q+1}, 0\}$ (again, recall T1), or equivalently $p_B^j = u_B^j - \max\{w_A^{k-q+1}, 0\}$ for $j = 1, \dots, q$ and p_B^j large for $j = q + 1, \dots, k$; the resulting profit for B is $u_B^1 + \dots + u_B^q - q \max\{w_A^{k-q+1}, 0\} - cq$.

In view of lemma 2 we define as follows the profit B can make by selling q products, for $q \in \{1, \dots, k\}$:¹⁷

$$\pi_B(q) \equiv \begin{cases} u_B^1 + \dots + u_B^q - q \max\{w_A^{k-q+1}, 0\} - cq & \text{if } u_B^q > w_A^{k-q+1}; \\ 0, & \text{otherwise.} \end{cases}$$

In order to examine how π_B depends on q , we begin by noticing that the higher is q , the more restrictive is the inequality $u_B^q > w_A^{k-q+1}$. Thus, if B is unable to sell q products because $u_B^q \leq w_A^{k-q+1}$, he is *a fortiori* unable to sell $\tilde{q} > q$ products.

Now we consider a case in which $u_B^{q+1} > w_A^{k-q} (\geq w_A^{k-q+1} > 0)$, so that B is able to sell $q + 1$ products (and also fewer than $q + 1$) and we examine how increasing his sales from q to $q + 1$ products affects his profit. When B sells q products, we have seen that he earns a profit of $u_B^1 + \dots + u_B^q - q w_A^{k-q+1} - cq$ by charging prices $p_B^j = u_B^j - w_A^{k-q+1}$ for $j = 1, \dots, q$; these prices are determined by the fact that B needs to block out the $(k - q + 1)$ -th best product of A . If instead he sells $q + 1$ products, B needs to block out the $(k - q)$ -th best product of A , which is weakly more valuable than the $(k - q + 1)$ -th. Prices are then $\hat{p}_B^j = u_B^j - w_A^{k-q}$ for $j = 1, \dots, q + 1$, and $\hat{p}_B^j \leq p_B^j$ for $j = 1, \dots, q$. This generates a loss for B , on his q best products, equal to $q(w_A^{k-q} - w_A^{k-q+1})$. However, now B gains $\hat{p}_B^{q+1} - c = u_B^{q+1} - w_A^{k-q} - c$ (actually, this "gain" is negative if c is large) from the

¹⁷This profit depends also on \mathbf{w}_A , even though we do not emphasize this fact in the notation.

sale of his $(q + 1)$ -th best product. Whether B prefers selling $q + 1$ products rather than q depends on the comparison between the loss $q(w_A^{k-q} - w_A^{k-q+1})$ and the gain $u_B^{q+1} - w_A^{k-q} - c$. In other words, (2) makes B face a *trade-off between quantity (number of slots to occupy) and extracting surplus (per product)*: as B increases the number of products he sells, he must leave more surplus per product to C . This trade-off is similar to the one determining monopoly pricing when a monopolist faces a decreasing demand and charges a single price.

We conclude this subsection with an obvious remark on the optimal q for B . Since $\bar{q}_B = q_B^{FB} + q_B^{SB}$ is the number of products in \mathcal{B}_B with value larger than c , it is clear that B will never choose $q > \bar{q}_B$ since $u_B^q < c$ for any $q > \bar{q}_B$.

3.3 Stage one

We first study the optimal pricing conditional on that A sells $k - q$ products; then we investigate the value of q that maximizes A 's profit.

3.3.1 A's profit when he sells $k - q$ products

Now we consider the first stage of the game in order to determine the profit A can make as a function of the number of products he sells. Hence, suppose that A wants to sell $k - q$ products for $q \in \{0, 1, \dots, k - 1\}$. Then, we inquire whether (i) there exists \mathbf{w}_A that, taking into account the best response by B , induces C to buy $k - q$ products from A ; (ii) within this set of \mathbf{w}_A we identify the vector that maximizes A 's profit.

The conditions that allow A to sell $k - q$ products can be stated by using the following incentive constraints for firm B :

$$(\text{IC}_{q,q'}) \quad \pi_B(q) \geq \pi_B(q') \quad \text{for any } q' \neq q \quad \text{and } q' \in \{1, \dots, k\} \quad (3)$$

In order for C to buy $k - q$ number of products from A , it is necessary that B is not going to block out the $(k - q)$ -th best product of A , nor any better product of A ; accordingly, condition (3) says that B weakly prefers to sell q products rather than $q' > q$, and we assume that if B is indifferent between occupying q slots and occupying $q' > q$ slots, then B occupies q slots.¹⁸ Then A 's profit is given by:

$$\pi_A(k - q) \equiv \sum_{j=1}^{k-q} (w_A^j - w_A^j - c) \mathbf{1}_{[w_A^j \geq 0]}$$

¹⁸Actually, condition (3) covers also the case of $q' < q$, but constraints $(\text{IC}_{q,q'})$ (for $q' < q$) are irrelevant since A is not harmed if B wants to sell $q' < q$ products. In any case, in the proof of Proposition 1 we show that $(\text{IC}_{q,q'})$, for $q' < q$, can be satisfied at no cost.

which, we note, is not affected by $(w_A^{k-q+1}, \dots, w_A^k)$. We investigate below whether there is a set of \mathbf{w}_A which satisfy (3) and, if so, we maximize $\pi_A(k-q)$ in this set. Precisely, we want to maximize $\sum_{j=1}^{k-q} (u_A^j - w_A^j - c) \mathbf{1}_{[w_A^j \geq 0]}$ with respect to q and (w_A^1, \dots, w_A^k) subject to (1)-(3).

We start by observing that when $\bar{q}_B < k$, it is certainly possible for A to sell $k - \bar{q}_B$ products, and that he can do so without leaving any surplus to C on these products. In order to show the details, suppose that A chooses $p_A^j = u_A^j$ for $j = 1, \dots, k - \bar{q}_B$ and p_A^j high enough for $j = k - \bar{q}_B + 1, \dots, k$. In this way, A's \bar{q}_B worst products are not competing with B 's products while A's best $k - \bar{q}_B$ products give C zero surplus. Then, B will reply by charging $p_B^j = u_B^j$ for $j = 1, \dots, \bar{q}_B$, and C will buy $k - \bar{q}_B$ products from A and \bar{q}_B from B , earning no profit.

When A's objective is to induce B to sell only $q (< \bar{q}_B)$ products, as it will become clear later on, B can react in two different ways: *accommodation* or *fighting*. Accommodation means that B contents himself with occupying q or less slots. Fighting means that B tries to occupy more than q slots. Obviously, to achieve his goal, A must choose prices such that B prefers accommodation to fighting, which is equivalent to the property that $(IC_{q,q'})$ is satisfied for all $q' > q$. What makes the case of $q = \bar{q}_B$ straightforward is that B can sell *all* his best \bar{q}_B products by accommodating, and extract the full surplus from them, and thus he will not fight because there is no other product he can sell profitably.

The next proposition characterizes the condition under which A is able to sell $k - q$ products and the profit maximizing vector \mathbf{w}_A (hence, the optimal prices) conditional on selling $k - q$ products. For expositional facility, we introduce the following notation. Given $q \in \{0, 1, \dots, \bar{q}_B - 1\}$, let

$$\mu_q^{k+1-q''} \equiv \frac{1}{q''} [u_B^{q+1} + \dots + u_B^{q''} \mathbf{1}_{[q'' > q+1]} - c(q'' - q)] \quad \text{for } q'' = q + 1, \dots, \bar{q}_B \quad (4)$$

Proposition 1 For a given $q \in \{0, 1, \dots, \bar{q}_B - 1\}$,

(i) a. There exists \mathbf{w}_A that induces C to buy $k - q$ products from A if and only if

$$u_A^{k+1-q''} > \mu_q^{k+1-q''} \quad \text{for } q'' = q + 1, \dots, \bar{q}_B \quad (5)$$

b. Let $\hat{q} \in \{0, 1, \dots, \bar{q}_B - 1\}$ denote the smallest q for which (5) is satisfied [we set $\hat{q} = \bar{q}_B$ if (5) fails to hold for any $q \in \{0, 1, \dots, \bar{q}_B - 1\}$]. Then, (5) is satisfied also for $q = \hat{q} + 1, \dots, \bar{q}_B$.

(ii) For $q \geq \hat{q}$, the profit maximizing \mathbf{w}_A for A is as follows:

a. when $q = 0$, $w_A^1 = \dots = w_A^k = u_B^1$;

b. when $q \in \{1, \dots, \bar{q}_B - 1\}$,

$$w_A^{k-q+1} = w_A^{k-q+2} = \dots = w_A^k = 0 \quad (6)$$

$$w_A^{k-q''+1} = \max\{w_A^{k-q''+2}, \mu_q^{k-q''+1}\} \quad \text{for } q'' = q+1, \dots, \bar{q}_B \quad (7)$$

$$w_A^1 = w_A^2 = \dots = w_A^{k-\bar{q}_B} = w_A^{k-\bar{q}_B+1} \quad \text{if } \bar{q}_B < k, \quad (8)$$

We below give the intuition of the results in Proposition 1; we focus on explaining the profit maximizing \mathbf{w}_A conditional on selling $k - q$ products for $q \geq 1$, described in Proposition 1(ii)b.¹⁹ Given A's objective to sell $k - q$ products, he should structure the prices for his best $k - q$ products (the ones to sell) very differently from the prices for his q worst products (the ones not to sell). On the one hand, for the latter products it is optimal to charge high prices (higher than their values) so that B does not face any competition from them; precisely, (6) reveals that choosing $w_A^{k-q+1} = w_A^{k-q+2} = \dots = w_A^k = 0$ is optimal. The reason is that this pricing maximizes B 's profit from accommodation and hence reduces B 's temptation to fight. In fact, the pricing allows B to extract the full surplus $u_B^1 + \dots + u_B^{q'}$ from C for his best q' products if he wants to sell only $q' \leq q$ products. Then, since $q < \bar{q}_B$, B prefers selling q products rather than less than q , and hence downward incentive constraints (i.e. $(IC_{q,q'})$ for $q' < q$) are trivially satisfied. On the other hand, regarding the best $k - q$ products to sell, the prices should be competitive enough to make it unprofitable for B to fight to sell more than q products. In particular, A cannot extract the full surplus from C on these products since if he attempts to do that, by T1, B can sell all of his best \bar{q}_B products by leaving no surplus per product to C .

To explain the optimal pricing of the best $k - q$ products of A , suppose that B wants to sell $q + 1$ products instead of q products. Lemma 2 shows that B can achieve this goal only if $w_A^{k-q} < u_B^{q+1}$. In this case, B makes a profit equal to $\pi_B(q+1) = u_B^1 + \dots + u_B^{q+1} - (q+1)w_A^{k-q} - c(q+1)$ and we have

$$\pi_B(q+1) - \pi_B(q) = u_B^{q+1} - (q+1)w_A^{k-q} - c$$

As we discussed after Lemma 2, $u_B^{q+1} - (q+1)w_A^{k-q} - c$ is composed of the loss $-qw_A^{k-q}$ on B 's best q products (with respect to selling them at full values) plus the gain $u_B^{q+1} - w_A^{k-q} - c$ from selling the $(q+1)$ -th product. Therefore, $w_A^{k-q} \geq \frac{u_B^{q+1} - c}{q+1} = \mu_q^{k-q}$ is equivalent to $\pi_B(q) \geq \pi_B(q+1)$: note that it is less restrictive than $w_A^{k-q} \geq u_B^{q+1}$. Hence, the smallest value of w_A^{k-q} satisfying $(IC_{q,q+1})$ is $w_A^{k-q} = \mu_q^{k-q}$, as described in (7). In order to deter B from selling $q+2$ products, we can argue as before. A sufficient condition is $w_A^{k-q-1} \geq u_B^{q+2}$,

¹⁹Proposition 1(ii)a is straightforward, as the best way for A to sell k products is to set $w_A^1 = \dots = w_A^k$ equal to the value of B 's best product, u_B^1 , which is possible only if $u_A^k > u_B^1$.

but when $w_A^{k-q-1} < u_B^{q+2}$ we must have:

$$\pi_B(q+2) - \pi_B(q) = u_B^{q+1} + u_B^{q+2} - (q+2)w_A^{k-q-1} - 2c \leq 0,$$

which is equivalent to $w_A^{k-q-1} \geq \mu_q^{k-q-1} = \frac{1}{q+2}(u_B^{q+1} + u_B^{q+2} - 2c)$. Therefore, $(IC_{q,q+2})$ is satisfied if $w_A^{k-q-1} \geq \min\{\mu_q^{k-q-1}, u_B^{q+2}\}$. However, w_A^{k-q-1} should also satisfy the monotonicity condition (2): $w_A^{k-q-1} \geq w_A^{k-q}$. From $w_A^{k-q-1} \geq \min\{\mu_q^{k-q-1}, u_B^{q+2}\}$ and $w_A^{k-q-1} \geq w_A^{k-q}$, we find that the smallest value of w_A^{k-q-1} satisfying $(IC_{q,q+2})$ is $w_A^{k-q-1} = \max\{w_A^{k-q}, \mu_q^{k-q-1}\}$, as described in (7).²⁰ By iterating the argument we obtain the smallest values of w_A^{k-q} , $w_A^{k-q-1}, \dots, w_A^{k-\bar{q}_B+1}$ which satisfy (3), as described in (7). This explains the pricing of the worst $\bar{q}_B - q$ products of A among the $k - q$ products to sell. In the case of $\bar{q}_B = k$, we have found the smallest values of w_A^1, \dots, w_A^k which satisfy (2) and (3). If instead $\bar{q}_B < k$, then the pricing of the best $k - \bar{q}_B$ products of A is determined as follows. Since the variables in $(w_A^1, \dots, w_A^{k-\bar{q}_B})$ do not affect (3), each of them can be set equal to $w_A^{k-\bar{q}_B+1}$ to satisfy the monotonicity condition (2), as described in (8).

As we mentioned in section 2, \mathbf{w}_A needs to satisfy (1) since otherwise there exist no prices $p_A^1 > 0, \dots, p_A^k > 0$ such that $w_A^j = u_A^j - p_A^j$ for $j = 1, \dots, k$. Hence, w_A^j must be larger than the profit-maximizing w_A^j characterized in Proposition 1(ii). This is why (5) is necessary and sufficient for A to be able to sell $k - q$ products. Notice that Proposition 1(i)b implies that there is a \hat{q} between 0 and \bar{q}_B such that A is able to sell any number of products between 0 and $k - \hat{q}$, but our arguments above imply that A will always sell at least $k - \bar{q}_B$ products, if $k > \bar{q}_B$.

3.3.2 Maximizing A's profit with respect to q

Since Proposition 1 allows to compute $\pi_A(k - q)$ for any $q \geq \hat{q}$, the profit-maximizing q can be found by comparing $\pi_A(k - \bar{q}_B)$, $\pi_A(k - \bar{q}_B + 1)$, \dots , $\pi_A(k - \hat{q})$. Before seeing a few examples and a useful property of π_A , we can improve our understanding of the problem of A by comparing $\pi_A(k - q)$ with $\pi_A(k - q + 1)$, in order to examine the incentives of A to increase the number of products he sells. Let us use here $w_A^1(q), \dots, w_A^{k-q}(q)$ to denote C's net profits from buying A's products, as determined by (7)-(8), when A sells $k - q$ products.

²⁰Actually, w_A^{k-q-1} must be equal to the highest between w_A^{k-q} and $\min\{\mu_q^{k-q-1}, u_B^{q+2}\}$, but (i) when $\min\{\mu_q^{k-q-1}, u_B^{q+2}\} = \mu_q^{k-q-1}$, we can write $w_A^{k-q-1} = \max\{w_A^{k-q}, \mu_q^{k-q-1}\}$; (ii) when $\min\{\mu_q^{k-q-1}, u_B^{q+2}\} = u_B^{q+2}$, $w_A^{k-q-1} = \max\{w_A^{k-q}, \mu_q^{k-q-1}\}$ still holds because then u_B^{q+2} is much smaller than u_B^{q+1} and it turns out that this implies that $w_A^{k-q} = \mu_q^{k-q}$ is larger than both u_B^{q+2} and μ_q^{k-q-1} .

Then we find

$$\begin{aligned}
w_A^{k-q}(q) &= \mu_q^{k-q}, \\
w_A^{k-q-1}(q) &= \max\{\mu_q^{k-q}, \mu_q^{k-q-1}\}, \\
&\vdots \\
w_A^{k-\bar{q}_B+1}(q) &= \max\{\mu_q^{k-q}, \mu_q^{k-q-1}, \dots, \mu_q^{k-\bar{q}_B+1}\} = w_A^{k-\bar{q}_B}(q) = \dots = w_A^1(q).
\end{aligned}$$

When instead A sells $k - q + 1$ products, we have:

$$\begin{aligned}
w_A^{k-q+1}(q-1) &= \mu_{q-1}^{k-q+1}, \\
w_A^{k-q}(q-1) &= \max\{\mu_{q-1}^{k-q+1}, \mu_{q-1}^{k-q}\}, \\
&\vdots \\
w_A^{k-\bar{q}_B+1}(q-1) &= \max\{\mu_{q-1}^{k-q+1}, \mu_{q-1}^{k-q}, \dots, \mu_{q-1}^{k-\bar{q}_B+1}\} = w_A^{k-\bar{q}_B}(q-1) = \dots = w_A^1(q-1).
\end{aligned}$$

It is straightforward to see from (4) that $\mu_{q-1}^{k+1-q''} > \mu_q^{k+1-q''}$ for any $q'' \in \{q+1, \dots, \bar{q}_B\}$, thus we have $w_A^{k+1-q''}(q-1) > w_A^{k+1-q''}(q)$ for any $q'' \in \{q+1, \dots, k\}$.

The latter inequality is very intuitive: in order to sell one extra product, (i.e. $k - q + 1$ rather than $k - q$ products), A must increase the surplus it abandons to C for all the $k - q$ initial products. Thus, when we compare $\pi_A(k - q + 1) = \sum_{j=1}^{k-q+1} [w_A^j - w_A^j(q-1) - c]$ with $\pi_A(k - q) = \sum_{j=1}^{k-q} [w_A^j - w_A^j(q) - c]$, we see that $\pi_A(k - q + 1)$ contains the additional term $w_A^{k-q+1} - w_A^{k-q+1}(q-1) - c$, which is A's profit on the $(k - q + 1)$ -th product sold, but A's profit on each of his first $k - q$ products is reduced from $w_A^j - w_A^j(q)$ to $w_A^j - w_A^j(q-1)$, as we just proved that $w_A^j(q-1) > w_A^j(q)$ for $j \in \{1, \dots, k - q\}$. In words, as it is the case with B , A also faces a *trade off between quantity and surplus extraction*: as A sells more products, he needs to leave more surplus per product to C . Precisely, as A increases its sales from $k - q$ to $k - q + 1$, inducing B to accommodate becomes more difficult for two reasons. First, B 's ability to fight is now stronger since he can use his q -th best product, with value u_B^q , which was previously sold. Second, B has now less to lose by fighting. Therefore, when A wants to sell one extra product, he should make his products more competitive by leaving C a higher surplus per product in order to induce B not to fight.

We now present a result which simplifies the task of finding the optimal q for A . Precisely, we prove a concavity-like property of π_A which states that the marginal profit for A from selling one extra product is decreasing: the profit increase from selling $k - q + 2$ products instead of $k - q + 1$ is smaller than the profit increase from selling $k - q + 1$ products instead of $k - q$.

Proposition 2 (i) Suppose that it is feasible for A to sell $k - q + 2$ products (i.e. $q - 2 \geq \hat{q}$). Then $\pi_A(k - q + 2) - \pi_A(k - q + 1) \leq \pi_A(k - q + 1) - \pi_A(k - q)$.

(ii) The optimal number of products sold by A , denoted by q_A^* , is characterized as follows:

$$\begin{aligned}\pi_A(q_A^*) &\geq \max\{\pi_A(q_A^* - 1), \pi_A(q_A^* + 1)\} \text{ if } k - \bar{q}_B + 1 \leq q_A^* \leq k - \hat{q} - 1, \\ \pi_A(q_A^*) &\geq \pi_A(q_A^* - 1) \text{ if } q_A^* = k - \hat{q}, \\ \pi_A(q_A^*) &\geq \pi_A(q_A^* + 1) \text{ if } q_A^* = k - \bar{q}_B.\end{aligned}$$

Notice that the concavity-like property of π_A described in Proposition 2(i) implies immediately Proposition 2(ii): in order to test the optimality of q_A^* , it suffices to compare the profit as the number of products sold by A is decreased or increased by one. In what follows, to give further insight, we study some specific settings.

3.3.3 When only the local incentive constraint ($\text{IC}_{q,q+1}$) matters

Let us present first the simple case in which only the local incentive constraint ($\text{IC}_{q,q+1}$) matters. We saw that when A wants to sell $k - q$ products, downward incentive constraints are trivially satisfied but satisfying upward constraints requires A to abandon some surplus to C . We below present a special case in which satisfying only ($\text{IC}_{q,q+1}$) is sufficient to satisfy (3), and this makes it straightforward to derive $\pi_A(k - q)$.

Corollary 1 Suppose that $c = 0$. Given q such that $\hat{q} \leq q \leq \bar{q}_B - 2$, if $u_B^{q+2} \leq \frac{1}{q+1}u_B^{q+1}$ then (5) is equivalent to $u_A^{k-q} > \frac{1}{q+1}u_B^{q+1}$. When this condition is satisfied, (6)-(8) imply $w_A^1 = \dots = w_A^{k-q} = \frac{1}{q+1}u_B^{q+1} > 0 = w_A^{k-q+1} = \dots = w_A^k$; thus $\pi_A(k - q) = u_A^1 + \dots + u_A^{k-q} - \frac{k-q}{q+1}u_B^{q+1}$.

Precisely, if u_B^{q+2} is sufficiently smaller than u_B^{q+1} , it turns out that $\mu_q^{k-q} \geq \mu_q^{k-q-1} \geq \dots \geq \mu_q^{k-\bar{q}_B+1}$ and then (5) is satisfied if and only if $u_A^{k+1-q''} > \mu_q^{k+1-q''}$ holds for $q'' = q + 1$, or equivalently $u_A^{k-q} > \frac{1}{q+1}u_B^{q+1}$. If this condition is satisfied, then the optimal prices for A are such that the products he wants to sell give a constant net value to C equal to $\frac{1}{q+1}u_B^{q+1}$, the value satisfying ($\text{IC}_{q,q+1}$) with equality. If the condition $u_B^{q+2} \leq \frac{1}{q+1}u_B^{q+1}$ holds for every $q \in \{\hat{q}, \dots, \bar{q}_B - 2\}$, then we have

$$\pi_A(k - q + 1) - \pi_A(k - q) = u_A^{k-q+1} - \frac{1}{q}u_B^q - (k - q)\left(\frac{1}{q}u_B^q - \frac{1}{q+1}u_B^{q+1}\right).$$

Note however that the conditions $\frac{1}{\hat{q}+1}u_B^{\hat{q}+1} \geq u_B^{\hat{q}+2}$, $\frac{1}{\hat{q}+2}u_B^{\hat{q}+2} \geq u_B^{\hat{q}+3}$, ..., $\frac{1}{\bar{q}_B-1}u_B^{\bar{q}_B-1} \geq u_B^{\bar{q}_B}$ are somewhat restrictive, since they imply that the values of B 's products decrease quite quickly. This also suggests that in general more than one upward incentive constraints matter as in the examples in the next subsection and in the appendix.

3.3.4 When all B's products have the same value

In this subsection, we characterize the equilibrium when all B's products have the same value and $c = 0$. In appendix, we characterize the equilibrium when B has 3 products of heterogenous values. Suppose that $u_B^1 = u_B^2 = \dots = u_B^{\bar{q}_B} \equiv u_B > 0$. In this case, for $q (= 1, \dots, \bar{q}_B - 1)$ and $q'' (= q + 1, \dots, \bar{q}_B)$, we find that $\mu_q^{k+1-q''} = \frac{q''-q}{q''} u_B$. Thus $\mu_q^{k+1-q''}$ is increasing in q'' . Given q , the profit-maximizing w_A^1, \dots, w_A^{k-q} , determined by (7)-(8), are

$$\begin{aligned} w_A^{k-q} &= \frac{1}{q+1} u_B, w_A^{k-q-1} = \frac{2}{q+2} u_B, \dots, w_q^{k-\bar{q}_B+2} = \frac{\bar{q}_B - q - 1}{\bar{q}_B - 1} u_B, \\ w_q^{k-\bar{q}_B+1} &= \frac{\bar{q}_B - q}{\bar{q}_B} u_B = w_A^1 = \dots = w_A^{\bar{q}_B}. \end{aligned}$$

If $q \geq \hat{q}$, we have that $\pi_A(k-q) = u_A^1 + \dots + u_A^{k-q} - [\frac{1}{q+1} + \frac{2}{q+2} + \dots + \frac{\bar{q}_B - q - 1}{\bar{q}_B - 1} + \frac{\bar{q}_B - q}{\bar{q}_B} (k - \bar{q}_B + 1)] u_B$.

In order to find the optimal q , we exploit lemma 2. Thus, $q = \bar{q}_B$ is optimal if $\pi_A(k - \bar{q}_B) \geq \pi_A(k - \bar{q}_B + 1)$, i.e. if $\frac{u_A^{k-\bar{q}_B+1}}{u_B} \leq \frac{k-\bar{q}_B+1}{\bar{q}_B}$. Finally, for q between 1 and $\bar{q}_B - 1$, q is optimal if $\pi(k - q) - \pi(k - q - 1) \geq 0$ and $\pi(k - q + 1) \leq \pi(k - q)$, i.e.

$$\begin{aligned} \frac{1}{q} + \frac{1}{q+1} + \dots + \frac{1}{\bar{q}_B - 1} + \frac{k - \bar{q}_B + 1}{\bar{q}_B} &\geq \frac{u_A^{k-q+1}}{u_B} \quad \text{and} \\ \frac{u_A^{k-q}}{u_B} &\geq \frac{1}{q+1} + \frac{1}{q+2} + \dots + \frac{1}{\bar{q}_B - 1} + \frac{k - \bar{q}_B + 1}{\bar{q}_B} \end{aligned}$$

3.4 Social optimum vs. market outcome

In this subsection we consider the case in which $k = 2$, $c = 0$ and $u_A^1 \geq u_A^2 > 0$, $u_B^1 \geq u_B^2 > 0$. We use this setting to show that the market outcome can be different from the efficient allocation of the slots.

Let us start by deriving the profit A can make by selling just his best product, that is $q = 1$. In view of Proposition 1, this requires $u_A^1 > \frac{1}{2} u_B^2$ since $\mu_1^1 = \frac{1}{2} u_B^2$, and

$$p_A^1 = u_A^1 - \frac{1}{2} u_B^2, \quad p_A^2 \geq u_A^2.$$

Therefore $\pi_A(1) = u_A^1 - \frac{1}{2} u_B^2$.

Now consider the case in which A wants to sell his two products, which requires $u_A^2 > u_B^1$. Then A sets $w_A^1 = w_A^2 = u_B^1$, and thus $\pi_A(2) = u_A^1 + u_A^2 - 2u_B^1$.

We now compare the social optimum with the market outcome. First, it is easy to find the situation in which B occupies too many slots compared to the efficient allocation. Suppose for instance $u_A^2 > u_B^1 = u_B^2 = u$. Therefore it is socially optimal for A to occupy

all the slots. However, if $\pi_A(2) < \pi_A(1)$, which is equivalent to $u > \frac{2}{3}u_A^2$, then A prefers selling only one product rather than two products.

Second, the inefficient allocation of slots can take the form in which B occupies too few slots. Suppose now for instance that $u_A^1 < u_B^1 = u_B^2 = u$, so that it is socially optimal for B to occupy two slots. But if $u_A^1 > \frac{1}{2}u$, then A can sell object $A1$ by playing $p_A^1 = u_A^1 - \frac{1}{2}u_B^2$, $p_A^2 \geq u_A^2$, and so he makes a positive profit.

This example illustrates well the point that the market allocation of slots is determined by a double trade-off between quantity and surplus extraction and does not necessarily coincide with the efficient allocation of slots. Summarizing, we have:

Proposition 3 *In the case of independent pricing, the market allocation of slots is determined by a double trade-off between quantity and surplus extraction, and does not necessarily coincide with the efficient allocation of slots: A may end up occupying too many slots or too few slots with respect to the efficient allocation.*

4 Competition with bundling

In this section we consider the case in which bundling is allowed. We study this game using backward induction, starting with stage two. As a tie-breaking rule, here we use a rule similar to T2: if C is indifferent among several combination of bundles, C chooses the bundles in a way to maximize the gross value. In section 4.1, we consider the case in which slotting contracts are prohibited: in section 4.2, we allow firms to use slotting contracts.

4.1 Without slotting contracts

In this subsection, we assume that slotting contracts are prohibited.

4.1.1 Stage two

The first result we present is taken from Jeon-Menicucci (2009), and establishes that firm B does not need to consider a complicated pricing scheme.

Lemma 3 (*Jeon-Menicucci, 2009*) *For any pair of pricing schemes (s_A, s_B) , let $\pi_B \geq 0$ denote the resulting profit of firm B . Then, firm B can make profit π_B also by playing a technology-renting scheme $tr_B \in TR_B$, instead of s_B , such that the fixed fee F_B associated with tr_B is equal to π_B .*

Proof. The lemma follows from Lemma 1 in Jeon-Menicucci (2009). ■

Lemma 3 says that firm B loses nothing by restricting attention to schemes in TR_B , regardless of the strategy used by firm A . In the following example we illustrate a case in which firm B , without bundling, cannot achieve the profit that he can achieve using technology renting.

Example 1 Assume $k = 2$ and $c = 0$. Firm A has two products with values 3 and 1, and firm B has two products with values 4 and 4. Suppose that firm A uses independent pricing with $p_A^1 = p_A^2 = 1$. If firm B also uses independent pricing, the highest profit he can realize is 4. But he can instead use technology-renting and realize a profit of 6.

The reason why independent pricing gives a smaller profit than a technology-renting scheme in the above example is the following. Under independent pricing of B , each product of firm B faces competition from firm A 's best product. This is because, under independent pricing, C has the option of buying (and paying) only one product from firm B . In contrast, under bundling (or technology-renting), such an option does not exist: without paying the fixed fee, no product of firm B is available while after paying the fixed fee, C gets both products of B at the same time. In particular, this implies that bundling gets rid of the trade-off between quantity and surplus extraction that B faces under independent pricing. More precisely, if B wants to sell only one product, he can realize a profit of 4. But if he wants to sell both products with bundling, only the second product needs to match the net value of A 's best product and this is why B obtains a profit of $4 + 2 = 6$. So, when B increases the quantity by selling a second product, it does not affect the profit he makes with the first product.

Lemma 4 *Technology-renting gets rid of the trade-off between quantity and surplus extraction that B faces under independent pricing.*

Lemma 3 and Example 1 together imply

Proposition 4 *(incentive to bundle) Firm B has at least a weak, and sometimes a strict, incentive to practice bundling instead of independent pricing.*

4.1.2 Stage one

Consider now stage one. The next lemma pins down the profit that A realizes.

Lemma 5 *Any SPNE is such that firm A realizes a profit equal to $F_A^* \equiv U_A^{FB} - cq_A^{FB} - (U_B^{SB} - cq_B^{SB})$.*

Proof. We first show that in any SPNE the profit of firm A is not larger than F_A^* . Then we prove that in any SPNE the profit of firm A is not smaller than F_A^* .

The proof for the result that the profit of A is not larger than F_A^* is identical to the proof of Proposition 6(ii) in Jeon-Menicucci (2009), and thus is omitted.

In order to prove that the profit of A is not smaller than F_A^* , suppose that A plays the technology-renting scheme $tr'_A \in TR_A$ such that $F_A = F_A^* - \varepsilon$, with $\varepsilon > 0$ and small. We prove that C will buy at least one product from firm A , and this implies that firm A makes a profit equal to $F_A^* - \varepsilon$. However, since ε can be made as small as wished, we conclude that in no SPNE the profit of A is smaller than F_A^* .

Given that A plays the technology-renting scheme with $F_A = F_A^* - \varepsilon$, we can argue like in the proof of Proposition 3 in Jeon-Menicucci (2009) to show that a best reply for B is the technology-renting scheme $tr_B^* \in TR_B$ such that $F_B = F_B^* \equiv U_B^{FB} - cq_B^{FB} - (U_A^{SB} - cq_A^{SB})$. This technology-renting scheme yields B a profit equal to F_B^* , and thus B cannot make a profit larger than F_B^* . However, B could conceivably find a scheme which yields him a profit F_B^* and induce C to buy no product of A ; we now rule out this case. Precisely, suppose that B plays s_B such that C buys no product of A , but buys bundle B_B from B ; then the profit of C will be $U(B_B) - cq_B - \pi_B$, with $\pi_B \geq F_B^*$ and where q_B is the cardinality of B_B . This payoff needs to be larger than the payoff C can make by purchasing (for instance) only objects offered by A with tr'_A , and in such a case C can make profit $U_A^{FB} + U_A^{SB} - c(q_A^{FB} + q_A^{SB}) - F_A^* + \varepsilon = U_A^{FB} + U_A^{SB} - c(q_A^{FB} + q_A^{SB}) - (U_A^{FB} - cq_A^{FB} - (U_B^{SB} - cq_B^{SB})) + \varepsilon = U_A^{SB} + U_B^{SB} - c(q_A^{SB} + q_B^{SB}) + \varepsilon$. If we maximize $U(B_B) - cq_B - \pi_B$ with respect to B_B , we get $B_B = B_B^{FB} \cup B_B^{SB}$ and C 's utility is $U_B^{FB} + U_B^{SB} - c(q_A^{FB} + q_A^{SB}) - \pi_B$, which is not larger than $U_B^{FB} + U_B^{SB} - c(q_B^{FB} + q_B^{SB}) - (U_B^{FB} - cq_B^{FB} - (U_A^{SB} - cq_A^{SB})) = U_A^{SB} + U_B^{SB} - cq_A^{SB} - cq_B^{SB}$ since $\pi_B \geq F_B^*$. Therefore, no pricing scheme which yields B at least F_B^* can induce C to refuse to buy from A . ■

The value of $F_A^* = U_A^{FB} - cq_A^{FB} - (U_B^{SB} - cq_B^{SB})$ has a simple interpretation. Suppose that C can buy the products in \mathcal{B}_B at the unit price c , and that she is considering how much she is willing to pay for the right to buy the products in \mathcal{B}_A at the unit price c . The answer is F_A^* , which is the marginal benefit that C obtains from having access to products of A at unit cost c taking into account that the products in \mathcal{B}_B are already available at price c . The lemma says that in any SPNE, A realizes a profit exactly equal to this marginal benefit. Before we describe other properties of SPNE, we provide an example to illustrate a case in which firm A , without bundling, cannot achieve the profit that he can achieve with a technology renting strategy.

Example 2 Assume $k = 2$ and $c = 0$. Firm A has two products with values 4 and 4, and

firm B has one product with value 3. Then, under individual pricing, it is optimal for A to sell only one product: then A 's profit is 4. However, under technology-renting, A can realize a profit of 5.

The example illustrates that technology-renting gets rid of the trade-off between quantity and surplus extraction that A faces. If A sells only one product, he realizes a profit of 4. If A sells two products, in addition to 4, he can obtain an extra profit that corresponds to difference between the value of A 's second product and that of B 's product. Hence, selling an additional product does not affect the surplus that A extracts from the first product. This argument generally holds as long as A includes any product in B_A^{FB} .

The proof of Lemma 5 (that shows that a technology-renting strategy allows A to realize the upper bound of his profit) and Example 2 together imply

Proposition 5 (*incentive to bundle*) *Firm A has at least a weak, and sometimes a strict, incentive to practice bundling instead of independent pricing.*

The next lemma shows that any SPNE is efficient.

Lemma 6 *Any SPNE is efficient, that is in any SPNE C buys all the products in B^{FB} .*

Proof. Suppose that, in a certain SPNE, C consumes a bundle of objects $\tilde{B}_A \cup \tilde{B}_B$ (such that $\tilde{B}_A \subseteq \mathcal{B}_A$ and $\tilde{B}_B \subseteq \mathcal{B}_B$) which is different from B^{FB} . Then C 's payoff in this SPNE is $\alpha \equiv \tilde{U}_A + \tilde{U}_B - F_A^* - c\tilde{q}_A - \tilde{P}_B$, in which \tilde{q}_A is the cardinality of \tilde{B}_A and \tilde{P}_B is the total revenue obtained by firm B ; notice that the profit of firm B is equal to $\tilde{P}_B - c\tilde{q}_B$, where \tilde{q}_B is the cardinality of \tilde{U}_B . Now let B deviate by using a technology renting strategy such that $F_B = \tilde{P}_B - c\tilde{q}_B + \varepsilon$, in which $\varepsilon > 0$ is a number close to zero. We prove that C will buy at least one product of firm B , which reveals that B makes profit $\tilde{P}_B - c\tilde{q}_B + \varepsilon$, which is larger than $\tilde{P}_B - c\tilde{q}_B$. Indeed, if C does not buy any product of firm B , then she buys only bundles offered by A and they cannot yield C a payoff larger than α , otherwise we obtain a contradiction with the fact that the initial candidate is a SPNE. However, if C buys $B_B^{FB} \cup B_B^{SB}$ from firm B then she obtains a payoff equal to $\gamma \equiv U_B^{FB} + U_B^{SB} - (\tilde{P}_B - c\tilde{q}_B + \varepsilon) - c(q_B^{FB} + q_B^{SB})$. Given that $F_A^* = U_A^{FB} - cq_A^{FB} - (U_B^{SB} - cq_B^{SB})$, we find that $\alpha \equiv \tilde{U}_A + \tilde{U}_B - (U_A^{FB} - cq_A^{FB} - (U_B^{SB} - cq_B^{SB})) - c\tilde{q}_A - \tilde{P}_B$ and $\gamma > \alpha$ is equivalent to

$$U_A^{FB} + U_B^{FB} - cq_B^{FB} - cq_A^{FB} - \varepsilon > \tilde{U}_A + \tilde{U}_B - c\tilde{q}_A - c\tilde{q}_B$$

When $\varepsilon > 0$ is close to zero, this inequality is true by definition of $U_A^{FB}, U_B^{FB}, q_A^{FB}, q_B^{FB}$. ■

Regarding the profit of firm B , we can only pin down its lower bound and its upper bound. If firm A uses technology-renting strategy, firm B realizes F_B^* as in the proof of

lemma 5. However, if firm A offers only B_A^{FB} , firm A obtains still F_A^* but firm B realizes $U_B^{FB} - cq_B^{FB}$ as in the following example shows.

Example 3 Assume that $k = 2$ and $c = 0$. Firm A has two products with values 8 and 3, and firm B has two products with values 7 and 2. Then $F_A^* = 8 - 2 = 6$ and $F_B^* = 7 - 3 = 4$, but in the following SPNE the profit of firm B is 7:

A plays

$$P_A(\{A1\}) = 6 \quad \text{and} \quad P_A(\{A1, A2\}), P_A(\{A2\}) \quad \text{high} \quad (9)$$

in words, A offers only his best product, at price equal to 6;

B plays a technology renting scheme with F_B determined as follows:

$$\begin{aligned} F_B &= 7 \quad \text{if} \quad A \quad \text{plays} \quad (9) \\ F_B &= 4 \quad \text{if} \quad A \quad \text{does not play} \quad (9) \end{aligned}$$

It is simple to verify that, given the above strategies, C chooses to buy products $B^{FB} = \{A1, B1\}$, and that B's strategy is a best reply to the strategy of firm A. Finally, given the strategy of firm B, it turns out that A cannot make a profit larger than 6.

Since C is going to buy B_B^{FB} , in any SPNE, the profit of firm B is not larger than $U_B^{FB} - cq_B^{FB}$. Summarizing we have

Proposition 6 (i) When bundling is allowed, any SPNE is efficient in terms of allocation of slots.

(ii) In any SPNE, the profit of firm A is F_A^* ; the profit of firm B is at least F_B^* and at most equal to $U_B^{FB} - cq_B^{FB}$.

Proposition 6(i) is especially interesting because in the case of simultaneous moves, Jeon-Menicucci (2009) find that both efficient equilibria (among them, a technology renting equilibrium) and inefficient equilibria exist as in the following example, which is reproduced from example 3 in Jeon-Menicucci (2009):

Example 4 Consider the case in which $k = 2$, $c = 3$, and

$$(u_A^1, u_A^2) = (12, 8), \quad (u_B^1, u_B^2) = (10, 5)$$

With simultaneous moves, the following profile of strategies is a NE:

$$P_A(\{A1\}) = P_A(\{A2\}) = P_A(\{A1, A2\}) = 11; \quad (10)$$

$$P_B(\{B1\}) = P_B(\{B2\}) = P_B(\{B1, B2\}) = 6. \quad (11)$$

In the above example, given the strategies described, C buys \mathcal{B}_A (and gets a payoff of 9). Precisely, C is induced to buy products $A1$ and $A2$, even though the latter product does not belong to B^{FB} , and firm B is unable to sell the superior product $B1$ for the two following reasons. First, given that C buys $A1$, her marginal cost of getting product $A2$ is zero. Second, in order not to make a loss, firm B must charge a price for $B1$ at least equal to $c = 3$ while C 's gain from replacing $A2$ with $B1$ is 2.

It is important to note that in the inefficient equilibrium described in the above example, each firm i realizes a profit lower than its profit F_i^* under the efficient technology-renting equilibrium. Actually, Corollary 2(i) in Jeon-Menicucci (2009) shows that the efficient technology-renting equilibrium Pareto dominates any other equilibrium in terms of sellers' profits. The inefficient equilibrium is thus based on a coordination failure between the two sellers. More precisely, when A takes as given that B plays a pure bundling strategy, offering a pure bundle of all products becomes for A a best response. The main reason for which things are different with sequential moves is that A does not take the strategy of B as given: instead, A knows that B will maximize his profit given A 's offer. Then, A has no reason to play (10), which yields a payoff of 5, strictly lower than the profit $F_A^* - \varepsilon = 7 - \varepsilon$ that he can achieve with a technology-renting strategy (see proof of Lemma 5). Obviously, (11) is not a best reply for firm B against the technology renting strategy of A with $F_A = 7 - \varepsilon$.

4.2 Slotting contracts

In all previous sections, after buying a number of products, C has the freedom to choose the products to occupy the slots. In this subsection, we allow firms to sign slotting contracts such that if C buys q_i number of products from firm i , $i = A, B$, C should allocate exclusively q_i number of slots on i 's products. Introducing slotting contracts does not affect the analysis of independent pricing since under independent pricing, C buys only the products that will occupy slots. However, introducing slotting contracts might affect the analysis of bundling. For instance, we have shown in the proof of Lemma 5 that without slotting contracts, firm A can always occupy q_A^{FB} number of slots with his products in B_A^{FB} . However, if B induces C to sign a slotting contract to occupy more than $k - q_A^{FB}$ slots with B 's products, then A cannot occupy q_A^{FB} number of slots. Similarly, firm A may try to occupy more than q_A^{FB} number of slots with a slotting contract. Then, the question is to know whether it is profitable for i to occupy more than q_i^{FB} number of slots. In what follows, we consider two cases: the case in which each firm is allowed to use slotting contracts in addition to menu of bundles and the case in which each firm is restricted to use a single bundle with

a slotting contract.

4.2.1 General contracts

Consider first the general case in which each firm is allowed to use slotting contracts in addition to menu of bundles. In Jeon-Menicucci (2009), we have considered this case with simultaneous moves, and we have found that some inefficient equilibria exist even for digital goods, due to slotting contracts.²¹

Example 5 (*slotting contracts and inefficiency*) Suppose that $k = 3$, $c = 0$, and

$$(u_A^1, u_A^2, u_A^3) = (10, 7, 6); \quad (u_B^1, u_B^2, u_B^3) = (9, 8, 1)$$

Here $B^{FB} = \{A1, B1, B2\}$, so that $q_1^{FB} = 1$ and $q_2^{FB} = 2$. However, there exists an inefficient NE in which $P_i(B_i)$ is high enough for each $B_i \neq \mathcal{B}_i$, for $i = A, B$, and $P_A(\mathcal{B}_A) = 5$, $P_B(\mathcal{B}_B) = 0$ (this is an example of exclusive dealing described in Section 2). In words, each firm i offers only \mathcal{B}_i through a slotting contract, and Bertrand competition between \mathcal{B}_A and \mathcal{B}_B determines the above prices. In this NE, firm A occupies the three slots even though products $B1$ and $B2$ are both better than $A2$ and $A3$.

With sequential moves, we find that slotting contracts simply do not matter as the following corollary states.

Corollary 2 *Lemma 5 and Proposition 6 hold regardless of whether or not each firm can use slotting contracts.*

Proof. The proofs of Lemma 5 and of Proposition 6 go through even though each firm can use slotting contracts. ■

Therefore, no inefficient outcome like the one in example 5 may occur even if firms can use slotting contracts. The reason, once again, is that firm A can get as close as he wishes to the upper bound of its profit F_A^* by using a suitable technology renting strategy while B cannot gain by using slotting contracts neither in terms of accommodation nor in terms of fighting. To provide further intuition, in the next subsection, we consider the case in which each firm is restricted to offer a single bundle with a slotting contract.

²¹Notice that the inefficiency found in example 5 has a different nature with respect to the inefficiency described in example 4. Indeed, with simultaneous moves and $c = 0$, Jeon-Menicucci (2009) have shown that all NE are efficient, and thus slotting contracts are necessary for the result of example 5.

4.2.2 Single bundle with a slotting contract

Suppose now that each firm i ($=A, B$) offers a single bundle B_i , with a slotting contract, at price P_i . With q_i we denote the number of products in B_i , and it is clear that, given q_i , firm i will include the own q_i best products into the bundle. We use $U_i(q_i)$ to denote the value of the bundle of firm i which includes i 's q_i best products.

We analyze stage two. Then, given q_A and P_A , B has basically two strategies, as we have seen under the independent pricing: B can accommodate or fight. Accommodation means that B is content with $k - q_A$ slots. Fighting means that B tries to occupy more than $k - q_A$ slots. However, if C allocates less than q_A slots to A 's products, this violates the slotting contract. Hence, fighting implies that B induces C to purchase only B_B .

Conditional on accommodation, the maximal number of slots that B can occupy is given by

$$\hat{q}_B(q_A) = \min\{\bar{q}_B, k - q_A\}$$

Then, B can use a slotting contract to occupy $\hat{q}_B(q_A)$ number of slots and can charge $P_B = U_B(\hat{q}_B(q_A))$ and earns a profit $U_B(\hat{q}_B(q_A)) - c\hat{q}_B(q_A)$.

If B wants to fight, he can sell $B_B^{FB} \cup B_B^{SB}$ (a bundle with \bar{q}_B objects) with a slotting contract. Then, P_B is determined by the condition that makes C indifferent between buying A 's bundle and buying B 's bundle.

$$U_B(\bar{q}_B) - P_B = U_A(q_A) - P_A.$$

From this, we obtain the next lemma characterizing B 's optimal strategy at stage 2.

Lemma 7 *Suppose that each firm is restricted to offer a single bundle with a slotting contract and that A offers a bundle B_A of his q_A best products at price P_A . At stage two,*

- (i) *B fights by offering $B_B^{FB} \cup B_B^{SB}$ at price $P_B = U_B(\bar{q}_B) - (U_A(q_A) - P_A)$ if $P_A > U_A(q_A) + U_B(\hat{q}_B(q_A)) - c\hat{q}_B(q_A) - [U_B(\bar{q}_B) - c\bar{q}_B]$;*
- (ii) *Otherwise, B accommodates by selling the bundle with his $\hat{q}_B(q_A)$ best products at price $P_B = U_B(\hat{q}_B(q_A))$.*

Not surprisingly, B prefers fighting (accommodating) when P_A is large (small).

Consider now stage one. As long as P_A satisfies $P_A \leq U_A(q_A) + U_B(\hat{q}_B(q_A)) - c\hat{q}_B(q_A) - [U_B(\bar{q}_B) - c\bar{q}_B]$, B will accommodate and hence A realizes the profit of $P_A - cq_A$. Therefore, A chooses q_A to maximize $U_A(q_A) + U_B(\hat{q}_B(q_A)) - cq_A - c\hat{q}_B(q_A)$. This means that A maximizes the total surplus from k slots and by definition it is optimal to choose $q_A = q_A^{FB}$, which leads to $\hat{q}_B(q_A^{FB}) = q_B^{FB}$. Furthermore, A charges $P_A = U_A^{FB} - (U_B^{SB} - cq_B^{SB})$ and B charges $P_B = U_B^{FB}$.

The next proposition describes the equilibrium.

Proposition 7 *Suppose that each firm is restricted to offer a single bundle with a slotting contract. Then a unique SPNE exists; this SPNE is efficient and the equilibrium strategies are as follows:*

- (i) *A offers B_A^{FB} and charges $P_A = U_A^{FB} - (U_B^{SB} - cq_B^{SB}) = F_A^* + cq_A^{FB}$;*
- (ii) *B plays as described by Lemma 7 and, along the equilibrium path, B offers B_B^{FB} and charges $P_B = U_B^{FB}$.*
- (iii) *C buys both bundles and uses all products in B^{FB} . C's payoff is $U_B^{SB} - cq_B^{SB}$.*

It is interesting to note that when each of A and B is constrained to offer a single bundle with a slotting contract, there is a unique SPNE and it is efficient. This inefficient equilibrium that exists under simultaneous pricing as in the example 5 does not arise under sequential pricing since sequential pricing eliminates such an equilibrium based on coordination failure. The equilibrium arises since A expects B to offer the bundle of all products with a slotting contract. Under sequential pricing, such an expectation makes sense only when A induces B to fight, but inducing B to fight is never optimal for A.

5 Concluding remarks: policy implications

Our results have clear-cut implications for laissez-faire in the matter of bundling. Under independent pricing, each firm faces both internal competition among his own products and external competition from the products of the rival firm, which creates a trade-off between number of slots to occupy and extracting surplus per product. Since each of the two firms faces such a trade-off, the market allocation of slots is determined by this double trade-off between quantity and surplus extraction. There is no particular reason for this to result in an efficient allocation of slots.

In contrast, bundling gets rid the internal competition and thereby eliminates the trade-off between quantity and surplus extraction: as long as a firm increases the size of his bundle by including his first-best products (the products that should occupy the slots in the first-best outcome), he can strictly increase his profit. Therefore, when bundling is allowed, allocation of slots is efficient in any equilibrium regardless of whether firms can use slotting contracts. In particular, when firms can use slotting contracts, firms have no interest to force the customer to buy more than the number of his first-best products since this strictly decreases his profit. The sequential pricing presents a stronger case for laissez-faire than the simultaneous pricing analyzed in Jeon-Menicucci (2009), since under simultaneous pricing some inefficient equilibria can arise due to pure bundling or slotting contracts.

6 Appendix

Proof of Lemma 1

What matters for C 's purchases (hence for A 's and B 's profits) are the vectors $\hat{\mathbf{w}}_A$ and $\hat{\mathbf{w}}_B$. Given $(\hat{\mathbf{w}}_A, \hat{\mathbf{w}}_B)$, suppose that $\mathbf{w}_B \neq \hat{\mathbf{w}}_B$ and let q_B denote the number of products which C purchases from B ; this means that C buys from B the products with net profits $w_B^{(1)}, w_B^{(2)}, \dots, w_B^{(q_B)}$. Let $u_B^{(j)}$ represent C 's gross profit of the product with the net profit $w_B^{(j)}$. Then, B 's profit is given by

$$\pi_B = \sum_{j=1}^{q_B} [u_B^{(j)} - w_B^{(j)}] - cq_B.$$

Now suppose that B chooses prices $\tilde{p}_B^j = u_B^j - w_B^{(j)}$ for $j = 1, \dots, \bar{q}_B$, and denote by \tilde{w}_B^j the resulting net profits for C . Then the same vector $\hat{\mathbf{w}}_B$ as before is obtained and $\tilde{w}_B^1 = w_B^{(1)} \geq \tilde{w}_B^2 = w_B^{(2)} \geq \dots \geq \tilde{w}_B^{\bar{q}_B} = w_B^{(\bar{q}_B)}$. Thus, T1 and T2 imply that C will still purchase q_B number of products from B , and now B 's profit is

$$\tilde{\pi}_B = \sum_{j=1}^{q_B} (u_B^j - \tilde{w}_B^j) - cq_B$$

By definition of u_B^j , $\tilde{\pi}_B$ is at least as large as π_B and, in particular, $\tilde{\pi}_B > \pi_B$ if $\sum_{j=1}^{q_B} u_B^j > \sum_{j=1}^{q_B} u_B^{(j)}$, that is if the products sold initially by B are different from B 's q_B products with the highest net profits.

The above argument applies to firm B since it chooses \mathbf{p}_B after observing \mathbf{p}_A , and thus can take \mathbf{w}_A as given. Conversely, firm A cannot take \mathbf{w}_B as given and the argument must be slightly augmented as follows. If, given \mathbf{w}_A , it is optimal for B to choose prices such that a certain \mathbf{w}_B is obtained, any \mathbf{p}_A which leaves unaltered \mathbf{w}_A leaves unaffected the incentives for firm B , and also his best reply prices. This allows to argue as above for B : in case that $\mathbf{w}_A \neq \hat{\mathbf{w}}_A$, let A choose $\tilde{p}_A^j = u_A^j - w_A^{(j)}$ for $j = 1, \dots, k$ so that $\tilde{w}_B^j = w_B^{(j)}$ for $j = 1, \dots, k$ and the same vector $\hat{\mathbf{w}}_A$ as before is obtained. Then, with respect to the initial situation, (i) B will not change his reply; (ii) C will still buy q_A products of A ; (iii) A 's profit will not decrease.

Proof of Proposition 1

Proof of (i)a There exists \mathbf{w}_A such that C will buy $k - q$ products from A if and only if there exists \mathbf{w}_A which satisfies (1), (2) and (??). Thus, since it is more likely that (1) is satisfied the smaller are w_A^1, \dots, w_A^k , in order to prove (i)a we first find the smallest values

of w_A^1, \dots, w_A^k which satisfy (2) and (??), and then we show that these values satisfy (1) if and only if (5) holds.

By Lemma 2, there exists \mathbf{p}_B such that C buys q' products from B if and only if $u_B^{q'} > w_A^{k-q'+1}$. In particular, it is feasible for B to sell $q \in \{1, \dots, \bar{q}_B - 1\}$ products if and only if $u_B^q > w_A^{k-q+1}$. If firm A chooses w_A^{k-q+1} such that $w_A^{k-q+1} \geq u_B^q$, then it would actually sell at least $k - q + 1$ products; thus it must be the case that $u_B^q > w_A^{k-q+1}$. This inequality implies $u_B^{q'} > w_A^{k-q'+1}$ for $q' = 1, \dots, q - 1$. Therefore, for $q' < q$, $(\text{IC}_{q,q'})$ is equivalent to

$$\pi_B(q) - \pi_B(q') = u_B^{q'+1} + \dots + u_B^q - c(q - q') - q \max \left\{ w_A^{k-q+1}, 0 \right\} + q' \max \left\{ w_A^{k-q'+1}, 0 \right\} \geq 0. \quad (12)$$

For $q'' > q$, instead, $u_B^q > w_A^{k-q+1}$ does not imply $u_B^{q''} > w_A^{k-q''+1}$. In case that $u_B^{q''} \leq w_A^{k-q''+1}$, we have $\pi_B(q'') = 0$ and then $(\text{IC}_{q,q''})$ is trivially satisfied. In case that $u_B^{q''} > w_A^{k-q''+1}$, then $(\text{IC}_{q,q''})$ is equivalent to

$$\pi_B(q'') - \pi_B(q) = u_B^{q+1} + \dots + u_B^{q''} - c(q'' - q) - q'' \max \left\{ w_A^{k-q''+1}, 0 \right\} + q \max \left\{ w_A^{k-q+1}, 0 \right\} \leq 0. \quad (13)$$

Therefore, (??) reduces to (12) for $q' = 1, \dots, q - 1$, and to $u_B^{q''} \leq w_A^{k-q''+1}$ and/or (13) for $q'' = q + 1, \dots, \bar{q}_B$.

We first prove that it is convenient to choose $w_A^{k-q+1} = w_A^{k-q+2} = \dots = w_A^k = 0$. For $q'' = q + 1, \dots, \bar{q}_B$, the value of w_A^{k-q+1} which most relaxes (13) is $w_A^{k-q+1} = 0$, and this [together with (2)], implies $w_A^{k-q+2} = \dots = w_A^k = 0$. These values of $(w_A^{k-q+2}, \dots, w_A^k)$ satisfy (12) for any $q' \in \{1, \dots, q - 1\}$ (because $q < \bar{q}_B$) and do not affect $(\text{IC}_{q,q''})$ for $q'' > q$. Thus, with $w_A^{k-q+1} = w_A^{k-q+2} = \dots = w_A^k = 0$ we have taken care of (12). We now turn our attention to (13).

Given $w_A^{k-q+1} = 0$, (13) is equivalent to $w_A^{k-q''+1} \geq \frac{1}{q''} [u_B^{q+1} + \dots + u_B^{q''} - c(q'' - q)]$. In particular, for $q'' = q + 1$ we find

$$w_A^{k-q} \geq \frac{1}{q+1} (u_B^{q+1} - c) \quad (14)$$

This condition is less restrictive than $w_A^{k-q} \geq u_B^{q+1}$, the other way to satisfy $(\text{IC}_{q,q+1})$, and therefore $(\text{IC}_{q,q+1})$ is satisfied if and only if (14) holds – notice that the right hand side of (14) is μ_q^{k-q} . For $q'' = q + 2$, $(\text{IC}_{q,q+2})$ is satisfied if and only if

$$w_A^{k-q-1} \geq \min \left\{ \frac{1}{q+2} (u_B^{q+1} + u_B^{q+2} - 2c), u_B^{q+2} \right\} \quad (15)$$

and since $u_B^{q+1} \geq u_B^{q+2}$, either one can be the minimum in the right hand side of (15).

Likewise, for $q'' = q + 3, \dots, \bar{q}_B$, $(\text{IC}_{q,q''})$ is satisfied if and only if

$$w_A^{k-q''+1} \geq \min \left\{ \frac{1}{q''} (u_B^{q+1} + \dots + u_B^{q''}), u_B^{q''} \right\} = \min \{ \mu_q^{k-q''+1}, u_B^{q''} \}$$

In general, however, we cannot set $w_A^{k-q''+1} = \min \{ \mu_q^{k-q''+1}, u_B^{q''} \}$ for $q'' = q + 1, \dots, \bar{q}_B$ because (2) may be violated. The lowest values for $w_A^{k-q}, w_A^{k-q-1}, \dots, w_A^{k-\bar{q}_B+1}$ which satisfy $(\text{IC}_{q,q''})$ and (2) are given by

$$w_A^{k-q''+1} = \max \left\{ w_A^{k-q''+2}, \min \{ \mu_q^{k-q''+1}, u_B^{q''} \} \right\} \quad \text{for } q'' = q + 1, \dots, \bar{q}_B$$

but we can actually simplify things a bit by proving that this is equivalent to setting

$$w_A^{k-q''+1} = \max \{ w_A^{k-q''+2}, \mu_q^{k-q''+1} \} \quad \text{for } q'' = q + 1, \dots, \bar{q}_B$$

or equivalently $w_A^{k-q''+1} = \max \{ \mu_q^{k-q}, \dots, \mu_q^{k-q''+1} \}$ for $q'' = q + 1, \dots, \bar{q}_B$.

Precisely, we can prove that if $\min \{ \mu_q^{k-q''+1}, u_B^{q''} \} = u_B^{q''}$, then $\max \left\{ w_A^{k-q''+2}, \min \{ \mu_q^{k-q''+1}, u_B^{q''} \} \right\} = w_A^{k-q''+2} = \max \{ w_A^{k-q''+2}, \mu_q^{k-q''+1} \}$. In order to see this fact, suppose that $\min \{ \mu_q^{k-q''+1}, u_B^{q''} \} = u_B^{q''}$ for some $q'' \in \{q + 2, q + 3, \dots, \bar{q}_B\}$, and that this is the smallest q'' with this property. Then $u_B^{q''} \leq \frac{1}{q''} [u_B^{q+1} + \dots + u_B^{q''} - c(q'' - q)]$, or equivalently $u_B^{q''} \leq \frac{1}{q''-1} [u_B^{q+1} + \dots + u_B^{q''-1} - c(q'' - 1 - q)] - \frac{c}{q''-1} \leq \mu_q^{k-q''+2}$. On the other hand, $\min \{ \mu_q^{k-q''+2}, u_B^{q''-1} \} = \mu_q^{k-q''+2}$ by definition of q'' , thus $w_A^{k-q''+2} \geq \mu_q^{k-q''+2}$ and $w_A^{k-q''+1} = \max \{ w_A^{k-q''+2}, u_B^{q''} \} = w_A^{k-q''+2}$. Furthermore $\mu_q^{k-q''+2} \geq \mu_q^{k-q''+1}$ is true because it is equivalent to $u_B^{q''} \leq \frac{1}{q''-1} (u_B^{q+1} + \dots + u_B^{q''-1} + cq)$, which follows from $u_B^{q''} \leq \frac{1}{q''-1} [u_B^{q+1} + \dots + u_B^{q''-1} - c(q'' - 1 - q)] - \frac{c}{q''-1}$. Thus, $w_A^{k-q''+1}$ can be written as $\max \{ w_A^{k-q''+2}, \mu_q^{k-q''+1} \}$, both when $\mu_q^{k+1-q''} < u_B^{q''}$ (this is obvious) and when $\mu_q^{k+1-q''} \geq u_B^{q''}$ (as we just proved).

Finally, we observe that no incentive constraint imposes any restriction on $w_A^1, w_A^2, \dots, w_A^{k-\bar{q}_B}$; thus we can pick $w_A^1 = w_A^2 = \dots = w_A^{k-\bar{q}_B} = w_A^{k-\bar{q}_B+1}$ to satisfy (2).

In this way we have identified the lowest values of w_A^1, \dots, w_A^k which satisfy (2) and (??), and they are described by (6)-(8). However, these values are feasible if and only if they satisfy (1). Clearly, the conditions $w_A^j < u_A^j$ for $j \in \{q + 1, \dots, k\}$ are satisfied given (6). For $j \in \{k - \bar{q}_B + 1, \dots, k - q\}$ we have $w_A^j = \max \{ w_A^{j+1}, \mu_q^j \}$, and thus $w_A^j < u_A^j$ for $j \in \{k - \bar{q}_B + 1, \dots, k - q\}$ if and only if (5) is satisfied. Finally, from $w_A^{k-\bar{q}_B+1} > w_A^{k-\bar{q}_B+1}$ it follows that $w_A^j > w_A^j = w_A^{k-\bar{q}_B+1}$ for $j = 1, \dots, k - \bar{q}_B$. This establishes that A is able to sell $k - q$ products if and only if (5) is satisfied.

Proof of (i)b Now we suppose that (5) is satisfied for a certain $q^* \in \{0, 1, \dots, \bar{q}_B - 2\}$, and show that (5) is satisfied also for $q = q^* + 1$. If A wants to sell $k - q^* - 1$ products, (5) reduces to $w_A^{k+1-q''} > \mu_{q+1}^{k+1-q''}$ for $q'' = q^* + 2, \dots, \bar{q}_B$. This condition holds, as long as

(5) is satisfied, because it involves a subset of the inequalities which appear in (5) and $\mu_{q+1}^{k+1-q''} < \mu_q^{k+1-q''}$ for $q'' = q^* + 2, \dots, \bar{q}_B$.

Proof of (ii) If we assume that (5) is satisfied for a certain q , then it is straightforward to see that the values of w_A^1, \dots, w_A^k determined by (6)-(8) maximize the profit of A. Indeed, (6)-(8) identify the smallest values of w_A^1, \dots, w_A^k which satisfy (2) and (??), and $\pi_A(k - q)$ is decreasing in w_A^1, \dots, w_A^k .

Proof of Proposition 2

Since $\pi_A(k - q) = \sum_{j=1}^{k-q} [w_A^j - w_A^j(q) - c]$, we find

$$\begin{aligned} \pi_A(k - q + 1) - \pi_A(k - q) &= \\ &= u_A^{k-q+1} - w_A^{k-q+1}(q - 1) - \sum_{j=1}^{k-q} [w_{A(q-1)}^j - w_A^j(q)] - c \\ &= u_A^{k-q+1} - w_A^{k-q+1}(q - 1) - [w_A^{k-q}(q - 1) - w_A^{k-q}(q)] - c \\ &\quad - [w_A^{k-q-1}(q - 1) - w_A^{k-q-1}(q)] - \dots - [w_A^1(q - 1) - w_A^1(q)] \end{aligned}$$

and

$$\begin{aligned} \pi_A(k - q + 2) - \pi_A(k - q + 1) &= \\ &= u_A^{k-q+2} - w_A^{k-q+2}(q - 2) - \sum_{j=1}^{k-q+1} [w_A^j(q - 2) - w_A^j(q - 1)] - c \\ &= u_A^{k-q+2} - w_A^{k-q+2}(q - 2) - [w_A^{k-q+1}(q - 2) - w_A^{k-q+1}(q - 1)] \\ &\quad - [w_A^{k-q}(q - 2) - w_A^{k-q}(q - 1)] - [w_A^{k-q-1}(q - 2) - w_A^{k-q-1}(q - 1)] - \\ &\quad \dots - [w_A^1(q - 2) - w_A^1(q - 1)] - c \end{aligned}$$

In order to prove that $\pi_A(k - q + 2) - \pi_A(k - q + 1) \leq \pi_A(k - q + 1) - \pi_A(k - q)$ it suffices to show that

$$w_A^{k-q+1}(q - 1) + [w_A^{k-q}(q - 1) - w_A^{k-q}(q)] + [w_A^{k-q-1}(q - 1) - w_A^{k-q-1}(q)] + \dots + [w_A^1(q - 1) - w_A^1(q)]$$

is smaller (or equal) than

$$\begin{aligned} &w_A^{k-q+2}(q - 2) + [w_A^{k-q+1}(q - 2) - w_A^{k-q+1}(q - 1)] + [w_A^{k-q}(q - 2) - w_A^{k-q}(q - 1)] \\ &+ [w_A^{k-q-1}(q - 2) - w_A^{k-q-1}(q - 1)] + \dots + [w_A^1(q - 2) - w_A^1(q - 1)] \end{aligned}$$

since $u_A^{k-q+2} \leq u_A^{k-q+1}$. In order to accomplish this task, we first prove that

$$w_A^{k-q+1}(q-1) \leq w_A^{k-q+2}(q-2) + w_A^{k-q+1}(q-2) - w_A^{k-q+1}(q-1) \quad (16)$$

and then we show that

$$w_A^{k+1-q''}(q-1) - w_A^{k+1-q''}(q) \leq w_A^{k+1-q''}(q-2) - w_A^{k+1-q''}(q-1) \quad (17)$$

for $q'' = q+1, \dots, k$.

We find from (4) and (7) that $w_A^{k+1-q}(q-1) = \frac{1}{q}u_B^q$, $w_A^{k+2-q}(q-2) = \frac{1}{q-1}u_B^{q-1}$ and $w_A^{k+1-q}(q-2) = \max\{\frac{1}{q-1}u_B^{q-1}, \frac{1}{q}(u_B^{q-1} + u_B^q)\}$. Thus (16) is equivalent to $\frac{2}{q}u_B^q \leq \frac{1}{q-1}u_B^{q-1} + \max\{\frac{1}{q-1}u_B^{q-1}, \frac{1}{q}(u_B^{q-1} + u_B^q)\}$, and it is easy to see that this inequality holds for either value of $\max\{\frac{1}{q-1}u_B^{q-1}, \frac{1}{q}(u_B^{q-1} + u_B^q)\}$.

About (17), we start by observing that if the inequalities $\mu_q^{k-q} \leq \mu_q^{k-q-1} \leq \dots \leq \mu_q^{k-\bar{q}_B+1}$ hold, then $w_A^{k+1-q''}(q) = \mu_q^{k+1-q''}$ for $q'' = q+1, \dots, \bar{q}_B$. In the opposite case, $\mu_q^{k+1-q''} > \mu_q^{k+1-(q''+1)}$ for some q'' between $q+1$ and $\bar{q}_B - 1$ and we use $q''(q)$ to denote the smallest q'' for which this inequality holds;²² notice that by using (4) we find that $\mu_q^{k+1-q''} > \mu_q^{k+1-(q''+1)}$ is equivalent to $\mu_q^{k+1-q''} = \frac{1}{q''}(u_B^{q+1} + \dots + u_B^{q''}) > u_B^{q''+1}$. Then it turns out that $\mu_q^{k+1-q''} > \mu_q^{k+1-(q''+1)}$ for $q'' = q''(q) + 1, \dots, \bar{q}_B - 1$,²³ and thus $w_A^{k+1-q''}(q) = \mu_q^{k+1-q''}$ for $q'' = q+1, \dots, q''(q)$, and $w_A^{k+1-q''}(q)$ is constantly equal to $\mu_q^{k+1-q''(q)}$ for $q'' = q''(q)+1, \dots, \bar{q}_B$. Likewise, $\mu_{q-1}^{k+1-q''} > \mu_{q-1}^{k+1-q''-1}$ if and only if $\mu_{q-1}^{k+1-q''} = \frac{1}{q''}(u_B^q + u_B^{q+1} + \dots + u_B^{q''}) > u_B^{q''+1}$, and we let $q''(q-1)$ denote the smallest q'' between q and $\bar{q}_B - 1$ for which this inequality holds. Notice that $q''(q-1) \leq q''(q)$ because $\mu_{q-1}^{k+1-q''} - \mu_q^{k+1-q''} = \frac{1}{q''}u_B^q > 0$. Finally, $\mu_{q-2}^{k+1-q''} > \mu_{q-2}^{k+1-q''-1}$ if and only if $\mu_{q-2}^{k+1-q''} = \frac{1}{q''}(u_B^{q-1} + u_B^q + u_B^{q+1} + \dots + u_B^{q''}) > u_B^{q''+1}$, and we let $q''(q-2)$ denote the smallest q'' between $q-1$ and \bar{q}_B for which this inequality is satisfied; we have $q''(q-2) \leq q''(q-1)$ because $\mu_{q-2}^{k+1-q''} - \mu_{q-1}^{k+1-q''} = \frac{1}{q''}u_B^{q-1} > 0$. Thus, as q'' goes from $q+1$ to \bar{q}_B , $w_A^{k+1-q''}(q-2)$ may become constant at some point, but not later than $w_A^{k+1-q''}(q-1)$, which in turn will not become constant (if it will) later than $w_A^{k+1-q''}(q)$.

Now we prove that (17), or equivalently

$$2w_A^{k+1-q''}(q-1) \leq w_A^{k+1-q''}(q-2) + w_A^{k+1-q''}(q), \quad (18)$$

is satisfied for $q'' = q+1, \dots, k$.

²²If $\mu_m^{k-m} \leq \mu_m^{k-m-1} \leq \dots \leq \mu_m^{k-n_B+1}$, then we set $m''(m) = n_B$. A similar remark applies to $m''(m-1)$ and $m''(m-2)$ defined below.

²³We know that $\mu_m^{k+1-m''} > \mu_m^{k+1-(m''+1)}$ is equivalent to $u_B^{m+1} + \dots + u_B^{m''} > m''u_B^{m''+1}$, and when this inequality is satisfied at $m'' = m''(m)$ we find that it is satisfied also at $m'' = m''(m) + 1$ since $u_B^{m''+1} \geq u_B^{m''+2}$.

Step 1 The case of $q + 1 \leq q'' < q''(q - 2)$. Then $w_A^{k+1-q''}(q - 2) = \frac{1}{q''}(u_B^{q-1} + u_B^q + u_B^{q+1} + \dots + u_B^{q''})$, $w_A^{k+1-q''}(q - 1) = \frac{1}{q''}(u_B^q + u_B^{q+1} + \dots + u_B^{q''})$ and $w_A^{k+1-q''}(q) = \frac{1}{q''}(u_B^{q+1} + \dots + u_B^{q''})$. As a consequence, (18) reduces to $2u_B^q \leq u_B^{q-1} + u_B^q$, which is satisfied.

Step 2 The case of $q''(q - 2) \leq q'' < q''(q - 1)$. Then $w_A^{k+1-q''}(q - 2) = \frac{1}{q''(q-2)}(u_B^{q-1} + u_B^q + u_B^{q+1} + \dots + u_B^{q''(q-2)}) > \frac{1}{q''}(u_B^{q-1} + u_B^q + u_B^{q+1} + \dots + u_B^{q''})$, $w_A^{k+1-q''}(q - 1) = \frac{1}{q''}(u_B^q + u_B^{q+1} + \dots + u_B^{q''})$ and $w_A^{k+1-q''}(q) = \frac{1}{q''}(u_B^{q+1} + \dots + u_B^{q''})$. We know that (18) would hold if $w_A^{k+1-q''}(q - 2)$ were equal to $\frac{1}{q''}(u_B^{q-1} + u_B^q + u_B^{q+1} + \dots + u_B^{q''})$, thus (18) *a fortiori* holds since $w_A^{k+1-q''}(q - 2) > \frac{1}{q''}(u_B^{q-1} + u_B^q + u_B^{q+1} + \dots + u_B^{q''})$.

Step 3 The case of $q''(q - 1) \leq q'' < q''(q)$. Then $w_A^{k+1-q''}(q - 2) = \frac{1}{q''(q-2)}(u_B^{q-1} + u_B^q + u_B^{q+1} + \dots + u_B^{q''(q-2)})$, $w_A^{k+1-q''}(q - 1) = \frac{1}{q''(q-1)}(u_B^q + u_B^{q+1} + \dots + u_B^{q''(q-1)}) > \frac{1}{q''}(u_B^q + u_B^{q+1} + \dots + u_B^{q''})$ and $w_A^{k+1-q''}(q) = \frac{1}{q''}(u_B^{q+1} + \dots + u_B^{q''})$. We know from step 2 that (18) holds at $q'' = q''(q - 1) - 1$. As q'' increases to $q''(q - 1)$, and beyond, $w_A^{k+1-q''}(q - 1)$ and $w_A^{k+1-q''}(q - 2)$ remain constant while $w_A^{k+1-q''}(q)$ increases. Thus (18) is still satisfied.

Step 4 The case of $q''(q) \leq q'' \leq \bar{q}_B$. Then $w_A^{k+1-q''}(q - 2) = \frac{1}{q''(q-2)}(u_B^{q-1} + u_B^q + u_B^{q+1} + \dots + u_B^{q''(q-2)})$, $w_A^{k+1-q''}(q - 1) = \frac{1}{q''(q-1)}(u_B^q + u_B^{q+1} + \dots + u_B^{q''(q-1)})$ and $w_A^{k+1-q''}(q) = \frac{1}{q''(q)}(u_B^{q+1} + \dots + u_B^{q''(q)})$. We know from step 3 that (18) holds at $q'' = q''(q) - 1$. As q'' increases to $q''(q)$, and beyond, we have that $w_A^{k+1-q''}(q - 1)$, $w_A^{k+1-q''}(q - 2)$ and $w_A^{k+1-q''}(q)$ all remain constant; thus (18) still holds.²⁴

Step 5 The case of $q'' = \bar{q}_B + 1, \dots, k$. From (8) we see that in this case (18) is reduced to $2w_A^{k+1-\bar{q}_B}(q - 1) \leq w_A^{k+1-\bar{q}_B}(q - 2) + w_A^{k+1-\bar{q}_B}(q)$, and we have proved in step 4 that this inequality is satisfied.

Proof of Corollary 1

We know from Proposition 1 that $w_A^{k-q} = \frac{1}{q+1}u_B^{q+1}$ and that $w_A^{k+1-q''} = \max\{\frac{1}{q+1}u_B^{q+1}, \frac{1}{q+2}(u_B^{q+1} + u_B^{q+2}), \dots, \frac{1}{q''}(u_B^{q+1} + \dots + u_B^{q''})\}$ (recall that $\mu_q^{k+1-q''} = \frac{1}{q''}(u_B^{q+1} + \dots + u_B^{q''})$) for $q'' = q + 2, \dots, \bar{q}_B$. Given that $\frac{1}{q+1}u_B^{q+1} \geq u_B^{q+2}$, we infer that $\frac{1}{q+1}u_B^{q+1} \geq u_B^{q+3} \geq \dots \geq u_B^{q''}$. This implies that $\frac{1}{q''}(u_B^{q+1} + \dots + u_B^{q''}) \leq \frac{1}{q''}[u_B^{q+1} + \frac{1}{q+1}u_B^{q+1}(q'' - q - 1)] = \frac{1}{q+1}u_B^{q+1}$. Thus, $\max\{\frac{1}{q+1}u_B^{q+1}, \frac{1}{q+2}(u_B^{q+1} + u_B^{q+2}), \dots, \frac{1}{q''}(u_B^{q+1} + \dots + u_B^{q''})\} = \frac{1}{q+1}u_B^{q+1}$ and $w_A^{k+1-q''} = \frac{1}{q+1}u_B^{q+1}$ for $q'' = q + 2, \dots, \bar{q}_B$.

Characterization of independent pricing game when $\bar{q}_B = 3$

²⁴By invoking very similar argument to the ones used in steps 1-4 we can deal with the case in which $m''(m - 2) = m''(m - 1)$, or $m''(m - 1) = m''(m)$, or $m''(m - 2) = m''(m - 1) = m''(m)$. We skip the details for the sake of brevity.

Suppose that $\bar{q}_B = 3$. In order to sell $k - 3$ products, A sets

$$p_A^1 = u_A^1, \quad p_A^2 = u_A^2, \quad \dots, \quad p_A^{k-3} = u_A^{k-3}, \quad p_A^{k-2} \geq u_A^{k-2}, \quad p_A^{k-1} \geq u_A^{k-1}, \quad p_A^k \geq u_A^k.$$

and then B chooses $p_B^1 = u_B^1, p_B^2 = u_B^2, p_B^3 = u_B^3$. Hence, $\pi_A(k - 3) = u_A^1 + u_A^2 + \dots + u_A^{k-3}$. In order to find $\pi_A(k - 2)$ we have to consider (IC_{2,3}), which is given by

$$(IC_{2,3}) \quad w_A^{k-2} \geq \frac{1}{3}u_B^3.$$

Therefore, A chooses

$$p_A^1 = u_A^1 - \frac{1}{3}u_B^3, \quad p_A^2 = u_A^2 - \frac{1}{3}u_B^3, \quad \dots, \quad p_A^{k-2} = u_A^{k-2} - \frac{1}{3}u_B^3, \quad p_A^{k-1} \geq u_A^{k-1}, \quad p_A^k \geq u_A^k.$$

which is feasible only if $u_A^{k-2} > \frac{1}{3}u_B^3$. Then, B plays $p_B^1 = u_B^1, p_B^2 = u_B^2$. Hence, $\pi_A(k - 2) = u_A^1 + u_A^2 + \dots + u_A^{k-2} - \frac{k-2}{3}u_B^3$.

In order to find $\pi_A(k - 1)$ we need to consider both (IC_{1,2}) and (IC_{1,3}), which are given by:

$$(IC_{1,2}) \quad w_A^{k-1} \geq \frac{1}{2}u_B^2.$$

$$(IC_{1,3}) \quad w_A^{k-2} \geq \max\left\{\frac{1}{2}u_B^2, \frac{1}{3}(u_B^2 + u_B^3)\right\}.$$

Hence, satisfying the incentive constraints is feasible if $u_A^{k-1} > \frac{1}{2}u_B^2$ and $u_A^{k-2} > \max\left\{\frac{1}{2}u_B^2, \frac{1}{3}(u_B^2 + u_B^3)\right\}$. Then, A chooses

$$\begin{aligned} p_A^j &= u_A^j - \max\left\{\frac{1}{2}u_B^2, \frac{1}{3}(u_B^2 + u_B^3)\right\} \text{ for } j = 1, \dots, k - 2; \\ p_A^{k-1} &= u_A^{k-1} - \frac{1}{2}u_B^2, \quad p_A^k \geq u_A^k. \end{aligned}$$

Then $\pi_A(k - 1) = u_A^1 + u_A^2 + \dots + u_A^{k-2} + u_A^{k-1} - (k - 2) \max\left\{\frac{1}{2}u_B^2, \frac{1}{3}(u_B^2 + u_B^3)\right\} - \frac{1}{2}u_B^2$.

Finally, A is able to sell k products if and only if $u_A^k > u_B^1$, and then $\pi_A(k) = u_A^1 + u_A^2 + \dots + u_A^{k-2} + u_A^{k-1} + u_A^k - k u_B^1$.

In order to fix the ideas, suppose that $u_B^2 > 2u_B^3$, so that $\max\left\{\frac{1}{2}u_B^2, \frac{1}{3}(u_B^2 + u_B^3)\right\} = \frac{1}{2}u_B^2$. Then, from Proposition 2(ii), we see for instance that it is optimal for A to sell $k - 2$ products if $\pi_A(k - 2) \geq \max\{\pi_A(k - 1), \pi_A(k - 3)\}$, which is equivalent to $u_A^{k-2} \geq \frac{k-2}{3}u_B^3$ and $u_A^{k-1} \leq \frac{k-2}{2}(u_B^2 - u_B^3) + \frac{1}{2}u_B^2$. The first inequality implies that the gain on the $(k - 2)$ -th sold by A, $u_A^{k-2} - \frac{1}{3}u_B^3$, is larger than his loss on the $k - 3$ products, $\frac{k-3}{3}u_B^3$, with respect to selling them at full prices. The second inequality means that selling the $(k - 1)$ -th product yields a profit of $u_A^{k-1} - \frac{1}{2}u_B^2$ but results in a loss of $\frac{k-2}{2}(u_B^2 - u_B^3)$, which is larger than $u_A^{k-1} - \frac{1}{2}u_B^2$.

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