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A New Approach to Multidimensional Poverty Measurement

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Abstract

We present here a multidimensional poverty index that measures poverty as a function of the extent and the intensity of poverty. Extent is given by the share of the poor in the population. To measure intensity we start by defining individual unidimensional deprivation indices (one for each individual and each dimension) and then aggregating them as a geometric mean. Each individual deprivation index is simply the inverse of the share of individual achievements in the poverty thresholds. Our approach involves an elementary characterization, the determination of a specific formula, and the endogeneous identification of the poor.

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1 Introduction

Multidimensional poverty appears today as an issue of substantial relevance, both theoretically and empirically (see for instance the recent contribution by Alkire and Santos (2010)). The emphasis on multidimensionality arises from the awareness that there may not be a suitable price system that allows aggregating different dimensions into a single number (the so called *income approach*). This might be due to the non-existence of markets for some poverty dimensions (or the imperfection of those markets), the presence of externalities, the nature of public goods of some poverty dimensions, or the inadequacy of market prices to capture the relative importance of the different dimensions (as equilibrium prices depend on the prevailing income distribution in the whole society).

Defining a poverty measure in a truly multidimensional context involves a number of subtle and difficult issues: choosing the appropriate poverty dimensions beyond income or wealth, deciding on whether they all are equally important, fixing sensible thresholds in those dimensions and setting criteria to identify as poor those individuals whose achievements lie partially below them, defining an overall measure of poverty intensity, etc. Those difficulties anticipate that many compromises are required and, indirectly, that the axiomatic approach may be the best way to deal with this type of problem as it makes explicit all those compromises. The reader is referred to the works of Dardadoni (1995), Ravallion (1996), Tsui (2002), Bourguignon and Chakravarty (2003), Lugo and Maasoumi (2008), Alkire and Foster (2008), Wagle (2008), and Chakravarty (2009) for a basic review of the recent literature.

The standard approach to this problem consists of building an indicator in two steps. The first one refers to the extent of poverty. It consists of identifying the poor by some criterion that depends on a vector of reference levels (the poverty thresholds). The second step deals with the intensity of poverty. It aims at providing a measure of the overall deprivation experienced by the poor. Step one answers the question of "Who are the poor?" whereas step two responds to "How poor are the poor?" It is usually understood that a poverty measure should take into account both aspects.

Identifying the poor in a multidimensional context is not as simple as in the single dimensional case. There are two extreme positions on this respect, each one with arguments pro and con. On the one hand, there is the *union approach* that defines poor anyone who is below the reference value in some dimension. On the other hand, we find the *intersection approach* according to which one person is poor if all her achievements are simultaneously below the reference values. There is some consensus in that the first approach strongly overestimates the number of poor and the second one is far too restrictive. That is why there are also some intermediate proposals, as those in Bourguignon and Chakravarty (2003), Alkire and Foster (2007), Lugo and Maasoumi (2008) or Alkire and Santos (2010). Yet there does not seem to be a clear cut principle to decide which intermediate approach to choose.

The literature on the measurement of poverty in a multidimensional context provides a number of alternative formulations, most of them derived axiomatically, that generalize the standard unidimensional decomposable and/or subgroup consistent poverty indices. See for instance Foster, Greer and Thorbecke (1984), Foster and Shorrocks (1991), Chakravarty, Mukherjee and Ranade (1998), Tsui (2002), Bourguignon and Chakravarty (2003), Lugo and Maasoumi (2008) and Alkire and Foster (2008).

We follow here that stream of literature by proposing a closed multidimensional poverty measure that is intuitive, easy to compute, and supported by an elementary characterization. Moreover, our approach implies an endogenous identification of the poor.

We proceed along the following lines. First, we find an index of deprivation for an individual with respect to a single dimension. This index is simply the number of times that the reference value exceeds the agent's achievement in the corresponding dimension. Second, we calculate the overall indicator of poverty intensity by aggregating those deprivation indices in terms of their geometric mean. Concerning the extent of poverty, we adopt the conventional head count ratio over the set of agents who are poor. This set is determined endogenously by the index of poverty intensity and corresponds to all those who are below a poverty hypercurve that goes through the vector of reference values. This entails an intermediate approach on head counting. The final index is simply the poverty intensity index to the power of the head count ratio.

There are four aspects that may render our contribution useful. First, we propose a specific formula rather than a family of them, as we believe that for some purposes this approach may help its applicability (at the cost of losing flexibility, obviously). Second, we provide a *friendly* theoretical justification that can be easily followed. Third, we develop a constructive approach based on the measurement of individual unidimensional deprivation. And fourth, the poverty index we propose determines endogenously the number of poor as a function of the vector of poverty thresholds.

2 The model

Let $N = \{1, 2, ..., n\}$ denote a society consisting of n individuals and let $K = \{1, 2, ..., k\}$ be a set of characteristics. Each characteristic corresponds to a variable that approximates one relevant dimension of social development. A *social state* is a positive matrix $Y = \{y_{ij}\}$ with n rows, one for each individual, and k columns, one for each dimension. The entry $y_{ij} \in \mathbb{R}_{++}$ describes the value of variable j for individual i. Therefore, $\mathbb{R}^{nk}_{++}$ is the space of social state matrices and we assume from the outset that all variables are strictly positive and can be described by real numbers.

A vector $\mathbf{z} \in \mathbb{R}^{k}_{++}$ of *reference values* describes the poverty thresholds for all dimensions. Those reference values may be fixed externally (absolute poverty lines) or may depend on the data of the social state matrix itself (relative poverty lines, such as a fraction of the median or the mean value). We shall not discuss here how those thresholds are set, even though the importance of that choice is more than obvious.

We denote by $N_p(Y, \mathbf{z}) \subset N$ the set of poor that results from a social state matrix Y and a vector $\mathbf{z}$ of reference values. The number of poor people is $n_p(Y, \mathbf{z})$, even though we shall also use the simpler notation n_p when no confusion arises.

A poverty index is a mapping $P : \mathbb{R}^{k(n+1)}_{++} \to \mathbb{R}_{+}$ that can be expressed as an increasing function of two arguments, the poverty extent (associated to the number of poor) and the poverty intensity (approximated by some index ψ). That is, $P(Y, \mathbf{z}) = f\left[n_p(Y, \mathbf{z}), \psi(Y, \mathbf{z})\right]$. The following discussion provides a rationale for a specific poverty index of this type.

2.1 Measuring the degree of poverty of the poor

Suppose, for the time being, that the set of poor has already been identified (we take up this question later on). In order to provide a measure of the intensity of poverty we follow two successive steps. First, we define kn_p individual unidimensional deprivation indices, $\pi_{ij}(Y,z)$, $i \in N_p(Y,z)$, $j \in K$. Each of those indices provides an estimate of how far away individual i is from the jth reference value. By imposing some reasonable assumptions we end up with a simple and intuitive formula. Second, we approximate the intensity of poverty by aggregating all those deprivation indices in terms of a function $\psi(Y,z)$ that is characterized by three simple axioms. This function corresponds to the geometric mean of the n_pk values.

2.1.1 An individual unidimensional deprivation index

Let $\pi_{ij} : \mathbb{R}^{k(n_p+1)}_{++} \rightarrow \mathbb{R}_+$ be the function that measures the deprivation suffered by agent i with respect to dimension $j, i \in N_p(Y,z)$, $j \in K$.

To arrive at a precise formula we consider three properties that those indices should satisfy, for all $i \in N_p(Y, \mathbf{z})$, all $j \in K$. The first one, *independence*, establishes that each deprivation index only depends on the value of the concerned agent in the particular dimension and the corresponding reference value.

- **Independence**: Let (Y,z), $(Y',z') \in \mathbb{R}^{k(n_p+1)}_{++}$ be such that $y_{ij} = y'_{ij}$, $z_j = z'_j$. Then, $\pi_{ij}(Y,z) = \pi_{ij}(Y',z')$.

The second one, *linear homogeneity*, requires the index to be homogeneous of degree one in the reference values. That is, if the reference values are doubled without the vector of achievements having changed, then the individual deprivation in each dimension also doubles. Formally:

- **Linear homogeneity in z** : $\pi_{ij}(Y, \lambda\mathbf{z}) = \lambda\pi_{ij}(Y,z)$, for all $\lambda > 0$.

The third one, *scale*, makes the value of the index equal to one when the achievement coincides with the corresponding reference value, $y_{ij} = z_j$. That is,

- **Scale**: $y_{ij} = z_j \Longrightarrow \pi_{ij}(Y,z) = 1$.

The following result shows that those properties lead to an elementary and very intuitive evaluation formula: the inverse of the share of y_{ij} in z_j.

Proposition 1 *An index $\pi_{ij} : \mathbb{R}^{k(n_p+1)}_{++} \rightarrow \mathbb{R}_+$ satisfies independence, linear homogeneity and scale if and only if it takes the form:*

$$\pi_{ij}(Y, \mathbf{z}) = \frac{z_j}{y_{ij}} \qquad [1]$$

Proof. The function defined in equation [1] trivially satisfies those three properties. Let us prove the converse.

Let $F_{ij} : \mathbb{R}^{k(n_p+1)}_{++} \rightarrow \mathbb{R}_+$ be a function that satisfies those three properties. By independence, this function can be fully described in the smaller two-dimensional space. Let then $f : \mathbb{R}^2_{++} \rightarrow \mathbb{R}_+$ be the generic function on the reduced space and let $f(y_{ij}, z_j)$ be a generic value of that function.

That is, $f(y_{ij}, z_j) = F_{ij}(Y,z)$. This function inherits the properties of linear homogeneity and scale. Therefore, by letting $z_j = \lambda y_{ij}$, we have:

$$f\left(y_{ij}, \lambda y_{ij}\right) = \lambda f(y_{ij}, y_{ij}) = \lambda$$

Now observe that, for all $z_j \in \mathbb{R}_{++}$ we can always write $z_j = \lambda y_{ij}$, for some $y_{ij} \in \mathbb{R}_{++}$, some $\lambda > 0$. Therefore,

$$f(y_{ij}, z_j) = \frac{z_j}{y_{ij}}$$

The three properties are trivially independent. ■

This proposition identifies a way of measuring individual deprivation in one dimension that is given by the number of times the reference value exceeds the individual achievement in that dimension. Note that this mapping is continuous and satisfies *scale independence* (the units of measurement do not affect its value). Also that $\pi_{ij}(Y,z) \geq 0$ for all (Y,z), with $\pi_{ij}(Y,z) > 1$ provided $y_{ij} < z_j$.

2.1.2 Measuring the intensity of poverty

Next we want to estimate the overall intensity of poverty by aggregating all individual unidimensional deprivation indices, $\left[\pi_{ij}(Y, \mathbf{z})\right]_{i\in N_p,\ j\in K}$, into a single indicator. Let $\psi : \mathbb{R}_{++}^{k(n_p+1)} \to \mathbb{R}_{+}$ be such a function, that is given by $\psi(Y, \mathbf{z}) = \phi(\mathbf{s})$, where $\mathbf{s} = \left(\mathbf{s}^{(1)}, \mathbf{s}^{(2)}, ..., \mathbf{s}^{(n_p)}\right)$, with $s^{(i)} = (s_{i1}, ..., s_{ik})$, $s_{ij} = \pi_{ij}(Y,z)$, $i \in N_p(Y, \mathbf{z})$, $\phi : \mathbb{R}_{++}^{kn_p} \to \mathbb{R}_{+}$.

We now introduce three requirements on such a function ϕ. The first one, *neutrality*, makes it explicit that all agents in $N_p(Y, \mathbf{z})$ and all dimensions in K enter the evaluation function on an equal foot. Neutrality with respect to the agents is usually known as *anonymity.* Note, however, that we assume that all dimensions are equally important conditional on z.[1]

- **Neutrality**: Let $\mathbf{s} \in \mathbb{R}_{++}^{kn_p}$ and let $p(s)$ denote a permutation of its components. Then, $\phi(\mathbf{s}) = \phi(p\left(\mathbf{s}\right))$.

The second property, *normalization*, fixes the value of the index in the singular case in which all values are identical. It requires function ϕ to take on the very same value. Formally.

[1] Indeed, some differences in the relevance of the poverty dimensions may be introduced while setting the reference values. For instance if we take thresholds as percentages of the median, those can differ across dimensions.

- **Normalization**: Let $\mathbf{s} \in \mathbb{R}_{++}^{kn_p}$ be such that $s_{ij} = a$ for all i, j. Then, $\boldsymbol{\phi}(s) = a$.

Our last property establishes conditions on the behavior of the function when agent i's deprivation index relative to dimension j changes, from s_{ij} to s'_{ij}, say. The property of *ratio monotonicity* requires that the ratio between the new and the initial values of ϕ be a monotone function of the ratio between s'_{ij} and s_{ij}. Formally:

- **Ratio Monotonicity**: Let s,$s' \in \mathbb{R}_{++}^{kn_p}$ be such that $s'_{ij} \neq s_{ij}$, $s_{hq} = s'_{hq}$, for all $h, q \neq i, j$. Then,

$$\frac{\phi(\mathbf{s}')}{\phi(\mathbf{s})} = g_{ij}\left(\frac{s'_{ij}}{s_{ij}}\right)$$

for some increasing function $g_{ij} : \mathbb{R}_{++} \rightarrow \mathbb{R}_{+}$.

Note that this monotonicity requirement is cardinal in nature and involves a separability feature of the overall index. Moreover, as $g_{ij}(1) = 1$, it follows that $g_{ij}(x) \gtreqless 1$ when $x \gtreqless 1$.

It is also interesting to observe that ratio monotonicity implies a form of the principle of transfers on the space of individual unidimensional deprivation indices, as the following result shows:

Proposition 2 *Let $\phi : \mathbb{R}_{++}^{kn_p} \rightarrow \mathbb{R}_{+}$ be a mapping that satisfies ratio monotonicity and let $\mathbf{s} \in \mathbb{R}_{++}^{kn_p}$ with $s_{ij} > s_{hj}$. Let now $\delta > 0$ be such that $s_{ij} - \delta > s_{ij} + \delta$. Call s' the vector in which we substitute s_{ij}, s_{hj} by $(s_{ij} - \delta)$, $(s_{ij} + \delta)$, respectively. Then, $\phi(s') < \phi(s)$.*

Proof. Without loss of generality, we take $k = 1$, $n_p = 2$ to alleviate notation. Consider the following vectors: $\mathbf{s} = (s_1, s_2)$, $s_1 > s_2$, $s^+ = (s_1,\ s_2 + \delta)$, $s^- = (s_1 - \delta,\ s_2)$, $s' = (s_1 - \delta,\ s_2 + \delta)$. By ratio monotonicity:

$$\begin{aligned}
(1)\quad \frac{\phi(\mathbf{s})}{\phi(\mathbf{s}^+)} &= g_2\left(\frac{s_2}{s_2+\delta}\right) &&\Longrightarrow \phi(\mathbf{s}) < \phi(\mathbf{s}^+) \\
(2)\quad \frac{\phi(\mathbf{s}^-)}{\phi(\mathbf{s})} &= g_1\left(\frac{s_1-\delta}{s_1}\right) &&\Longrightarrow \phi(\mathbf{s}^-) < \phi(\mathbf{s}) \\
(3)\quad \frac{\phi(\mathbf{s}^-)}{\phi(\mathbf{s}')} &= g_2\left(\frac{s_2}{s_2+\delta}\right) &&\Longrightarrow \phi(\mathbf{s}^-) < \phi(\mathbf{s}')
\end{aligned}$$

Now from (1) and (3),

$$\begin{aligned}
\frac{\phi(\mathbf{s}^-)}{\phi(\mathbf{s}')} &= \frac{\phi(\mathbf{s})}{\phi(\mathbf{s}^+)} < \frac{\phi(\mathbf{s}^-)}{\phi(\mathbf{s}^+)} \\
&\Longrightarrow \phi(\mathbf{s}') > \phi(\mathbf{s}^+)
\end{aligned}$$

From this and the implication in (1) we get: $\phi(s') > \phi(s)$. ■

This proposition tells us that a reduction of size δ in the deprivation of a poor agent i with respect to dimension j that is worse off than another poor agent h in the same dimension, more than compensates an identical increase in the deprivation of agent h, provided their relative positions remain unaltered.

The following result determines the overall evaluation formula to measure the intensity of poverty:

Proposition 3 *An index $\phi : \mathbb{R}^{kn_p}_{++} \to \mathbb{R}_+$ satisfies neutrality, normalization and ratio monotonicity, if and only if it takes the form:*

$$\phi(\mathbf{s}) = \prod_{i \in N_p} \prod_{j \in K} (s_{ij})^{\frac{1}{kn_p}} \qquad [2]$$

Moreover, those properties are independent.

Proof. (i) Let $\mathbf{s} \in \mathbb{R}^{kn_p}_{++}$. By ratio monotonicity and normalization we can write:

$$\begin{aligned} \phi(s_{11}, 1, 1, ..., 1) &= g_{11}(s_{11}) \times \phi(1, 1, ..., 1) = g_{11}(s_{11}) \\ \phi(s_{11}, s_{12}, 1, ..., 1) &= g_{12}(s_{12}) \times \phi(s_{11}, 1, ..., 1) = g_{11}(s_{11}) \times g_{12}(s_{12}) \\ &\dots \\ &\dots \\ \phi(\mathbf{s}) &= \prod_{i \in N_p} \prod_{j \in K} g_{ij}(s_{ij}) \end{aligned}$$

By neutrality, $g_{ij}(.) = g(.)$ for all i, j. Moreover, for the special case in which $s_{ij} = a$ for all i, j, we get:

$$\phi(a, a, ..., a) = [g(a)]^{kn_p} = a$$

which implies $g(a) = a^{1/kn_p}$.

Therefore, we conclude:

$$\phi(\mathbf{s}) = \prod_{i \in N_p} \prod_{j \in K} (s_{ij})^{\frac{1}{kn_p}}$$

(ii) To separate the properties let us consider the following indices (we take the case $k = 1$ for the sake of simplicity in exposition).

(ii,a) $\phi^A(s) = \frac{1}{n_p}\sum_{i\in N_p} s_i$. It satisfies neutrality and normalization but not ratio monotonicity.

(ii,b) $\phi^B(s) = \prod_{i\in N_p} s_i$. It satisfies neutrality and ratio monotonicity but not normalization.

(ii,c) $\phi^C(s) = \prod_{i\in N_p} (s)^{c_i}$, with $\sum_{i\in N_p} c_i = 1$ and $b_i \neq 1/n_p$ for some i. It satisfies normalization and ratio monotonicity but not neutrality. ■

Remark *Note that this function is a particular case of the family of generalized means, that has been repeatedly advocated as a convenient aggregator for multidimensional welfare measurement. See for instance Foster, Greer and Thorbecke (1984), Foster, Lopez-Calva and Szkely (2005), Seth (2009, 2010), Villar (2009). For specific characterizations of the geometric mean in a similar context see Herrero, Martinez and Villar (2010a,b).*

Therefore, under the three properties established for the individual unidimensional deprivation indices and the three properties on the aggregator mapping, we end up with the following formula for the overall measure of intensity of poverty:

$$\psi(Y, \mathbf{z}) = \prod_{i\in N_p}\prod_{j\in K}\left[\frac{z_j}{y_{ij}}\right]^{\frac{1}{kn_p}} \qquad [3]$$

The structure of the formula allows us to estimate the *overall poverty of an individual*, by simply computing:

$$\vartheta_i(Y, \mathbf{z}) = \prod_{j\in K}\left[\frac{z_j}{y_{ij}}\right]^{\frac{1}{k}} \qquad [4]$$

that is, as the geometric mean of all her deprivation indices. Note that this formulation exhibits the following feature: the reduction in the deprivation of dimension q required to compensate an increase in the deprivation of dimension j is smaller the higher the initial level of deprivation in j (decreasing marginal rate of substitution of the individual poverty index across deprivation dimensions).

Similarly, we can have a measure of the overall poverty of society in a given dimension, as:

$$\beta_j(Y, \mathbf{z}) = \prod_{i\in N_p}\left[\frac{z_j}{y_{ij}}\right]^{\frac{1}{n_p}} \qquad [5]$$

Here the decreasing marginal rate of substitution tells us that the the reduction of the deprivation of an agent over the overall dimension is larger the worse off the agent is.

Equation [3] makes it clear that our formula to approach the intensity of poverty satisfies *path independence* (that is, one can aggregate individual unidimensional values first across dimensions and then across agents, or viceversa, obtaining the same result). Also observe that the geometric mean is a distribution sensitive measure that penalizes the dispersion of the individual values, relative to the arithmetic mean. In particular, for two distributions with identical mean values it assigns higher value of the intensity of the poverty to that in which the distribution of the y_{ij} values is more disperse.

2.2 On counting the poor

So far we have taken the number of poor, $n_p(Y, \mathbf{z})$, as given. Let us deal with this question now.

In our model the set of poor people is determined endogenously by the very formula that measures the intensity of poverty. To see that observe that, by definition, an agent with $y_{ij} > z_j$, for all j, is non-poor. Consider now equation [4], that provides an index of poverty for an individual. The evaluation of an individual h in the limit case in which $y_{hj} = z_j$, for all $j \in K$, is given by: $\vartheta_h\left(\left(Y_{-h}, \mathbf{z}\right), \mathbf{z}\right) = 1$ (where $\left(Y_{-h}, \mathbf{z}\right)$ describes a social matrix whose hth row is precisely $\mathbf{z}$. Therefore, we can establish that an individual is considered poor if and only if:

$$\vartheta_i(Y, \mathbf{z}) = \prod_{j \in K} \left[\frac{z_j}{y_{ij}}\right]^{\frac{1}{k}} \geq 1$$

From that it follows immediately that the set of the poor, $N_p(Y, \mathbf{z})$, is defined by the following condition:

$$N_p(Y, \mathbf{z}) = \left\{ i \in N \ \ / \ \ \prod_{j \in K} y_{ij} \leq \prod_{j \in K} z_j \right\} \qquad [6]$$

(note that this condition is compatible with the existence of poor individuals with achievements above the poverty thresholds in some dimensions).

Expression [6] permits one to directly identify the poor in the k-dimensional space in which we plot all agents' vectors of characteristics, $\mathbf{y}_i \in \mathbb{R}^k_+$. The poor are all those below the hypercurve defined by $\prod_{j \in K} x_j = \prod_{j \in K} z_j$. This set is larger than the one defined by the intersection approach and smaller than that corresponding to the union approach (see figure 1 below).

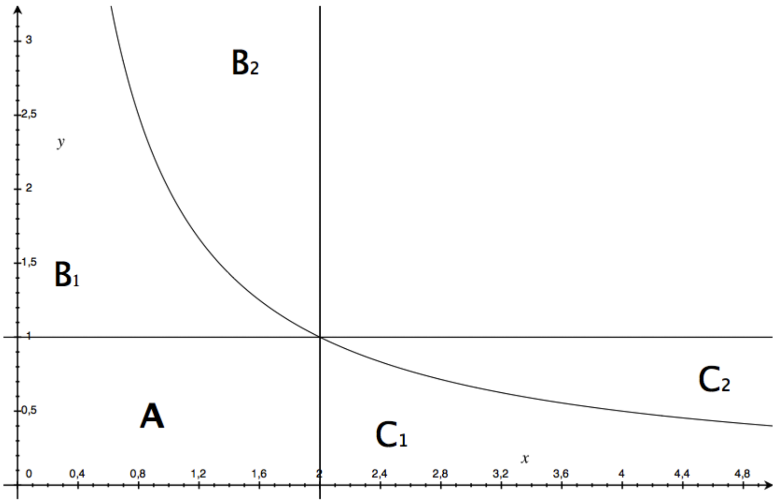

Figure 1: The poverty hypercurve

The figure illustrates a two-dimensional case with $z_1 = 1,\ z_2 = 2$. Each individual agent would be represented by a point in that space. The intersection approach declares poor all those in A, whereas the union approach takes those in the union of $A, B = (B_1 \cup B_2)$, and $C = (C_1 \cup C_2)$. Our approach implies taking those in $A \cup B_1 \cup C_1$.

Therefore our approach to measuring poverty determines endogenously the use of the a precise *intermediate approach* in order to count the poor.

We define *the extent of poverty* by the head count ratio:

$$\eta(Y, \mathbf{z}) = \frac{n_p(Y, \mathbf{z})}{n}$$

2.3 The multidimensional poverty measure

A poverty index is a function $P : \mathbb{R}^{k(n+1)}_{++} \to \mathbb{R}_+$ that associates to each pair $(Y, \mathbf{z})$ a number that synthesizes the overall poverty of the situation given by the social state matrix Y and the vector $\mathbf{z}$ of reference values (to be understood as poverty thresholds). We conceive the multidimensional poverty measure as an increasing function of the extent of poverty, given by the share of poor in the population, and the intensity of poverty, given by the geometric mean of all individual unidimensional deprivation indices.

In order to obtain a formula that satisfies subgroup decomposability (in the multiplicative form determined by our measure of poverty intensity), we propose the following intuitive expression:

$$P(Y,\mathbf{z}) = [\psi(Y,\mathbf{z})]^{\eta(Y,\mathbf{z})} = \prod_{i\in N_p}\prod_{j\in K}\left[\frac{z_j}{y_{ij}}\right]^{\frac{1}{nk}} \qquad [7]$$

This expression tells us that the poverty measure is a monotone transformation of the geometric mean of all individual unidimensional deprivation indices. Note that the head counting element cancels out so that the number of the poor only enters the formula through the selection of those whose deprivation indices are computed.

Equation [7] can also be expressed as:

$$P(Y,\mathbf{z}) = \left[\prod_{i\in N_p}\vartheta_i(Y,\mathbf{z})\right]^{\frac{n_p}{n}} \qquad [8]$$

This can be regarded as a Cobb-Douglas *Social Deprivation Function* (rather than a Social Welfare Function) of the poor, whose arguments are their individual overall deprivation values. Observe that this formula penalizes inequality by the very nature of its multiplicative structure. That is, a change in the social matrix that leaves unchanged the mean values of the achievements of the poor, but involves a more unequal distribution of those achievements $(y_{ij})_{i\in N_p, j\in K}$, increases the value of P.

3 Final comments

We have presented a multidimensional poverty index that is simple, intuitive, easy to get and has been characterized in terms of simple properties. To arrive at that formula we have followed a particular way of approaching the problem that consists of the construction of individual unidimensional deprivation indices as a first step and then aggregating them into an overall measure. This procedure allows for an easy characterization of the poverty index and implies the endogenous identification of the poor. Interestingly enough, the formula turns out to be distribution sensitive, so that the inequality of the achievements among the poor matters.[2]

Our poverty index has also other desirable features. In particular, it trivially the following set of standard properties considered in Bourguignon and Chakravarty (2003):

- *Weak Focus*: the index is independent on the non-poor, provided the reference values are externally given (the formula is defined directly over the set of poor, that is fixed in that case).[3]

- *Symmetry*: the index does not take into account who gets which values, but only the values themselves (this property corresponds to neutrality with respect to the agents and is therefore implied by our neutrality axiom).

- *Monotonicity*: an increase in the individual achievements does not increase poverty (that follows directly from the fact that each π_{ij} is decreasing in y_{ij} and ψ is increasing in π_{ij}).

- *Continuity*: small changes in the achievements induce small changes in the index (our index is the product of continuous real-valued functions).

- *Scale Invariance*: units are immaterial (an immediate consequence of Proposition 1).

- *Principle of Population*: a replica of the population does not change the value of the index (this can be easily checked in equation [7]).

- *One Dimensional Principle of Transfers*: A reduction $\alpha > 0$ in the achievement of poor agent i together with an increase α in the achievement of poor agent h, with $y_{ij} - \alpha > y_{hj} + \alpha$, reduces the overall poverty (this is a direct consequence of the convexity of the π_{ij} function and the structure of ψ).

Note that the index proposed is multiplicatively decomposable by population subgroups, but it does not satisfy *Subgroup Decomposability* in the additive form given in Bourguignon and Chakravarty (2003)[4]

[2] Needless to say other formulae can be obtained following this approach. Yet our focus here is more definiteness than generality: we aimed at obtaining a closed and distribution sensitive poverty index, rather than a family of them.

[3] It does not satisfy *Strong Focus,* as we allow for some trade-offs between characteristics.

[4] This property says the following: If the population is partitioned into subgroups, the overall poverty index corresponds to the weighted average of subgroup poverty values, where the weights correspond to population shares

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