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ASYMMETRIC MARKET SHARES, ADVERTISING, AND PRICING:
EQUILIBRIUM WITH AN INFORMATION GATEKEEPER

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Asymmetric Market Shares, Advertising, and Pricing: Equilibrium with an Information Gatekeeper

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Abstract

We analyze the impact of market share on advertising and pricing decisions by firms that sell to loyal, non-shopping customers and can advertise to shoppers through an information intermediary or “gatekeeper.” In equilibrium the firm with the smaller loyal market advertises more aggressively but prices less competitively than the firm with the larger loyal market, and there is no equilibrium in which both firms advertise with probability 1. The results differ significantly from earlier literature which assumes all prices are revealed to shoppers and finds that the firm with the smaller loyal market adopts a more competitive pricing strategy. The predictions of the model are consistent with advertising and pricing behavior observed on price comparison websites such as Shopper.com.

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1 Introduction

The increased adoption of the Internet as a commercial platform has led to an array of online information intermediaries or “gatekeepers” that provide consumers with price and product information. Websites such as Shopper.com and Mysimon.com enable consumers to easily compare prices for a homogeneous good offered for sale by several online retailers.¹ In traditional markets newspapers, trade journals, and brokers often serve the role of gatekeepers. Gatekeepers provide price information to shoppers who search for the lowest possible price. Other consumers, who we classify as “loyal,” bypass the gatekeeper and purchase directly from a preferred retailer. Baye and Morgan (2001) explicitly incorporate a gatekeeper in a clearinghouse model.² A critical simplifying assumption in their analysis and in subsequent clearinghouse models with a gatekeeper is that loyal customers are allocated equally across firms; firms have symmetric market shares. A central finding by Baye and Morgan is that in equilibrium firms adopt symmetric mixed pricing and advertising strategies. This important result provides an explanation for price dispersion in online markets, even in the presence of an information intermediary capable of providing complete price information to consumers. However, observed patterns of price dispersion in many online markets reveal persistent differences in pricing and advertising behavior across firms.³ Figure 1 provides an illustrative example based on prices advertised on the price comparison site Shopper.com by Dell and Buy.com for a SanDisk 2GB Secure Digital Card. Dell advertised a price in 84 percent of the observations while Buy.com advertised in only 42 percent. In addition, whenever both firms advertised, Buy.com advertised the lower price. These observed patterns are at odds with the theoretical prediction that equilibrium pricing and advertising strategies are symmetric.

In light of this discrepancy between theory and observed behavior, in their survey of the literature

¹According to E-consultancy, in the UK, price comparison websites generated revenues between 120 and 140 million Euros in 2005.

²Information providers or “gatekeepers” are not unique to the Internet or new to the economics literature. In his seminal work, Stigler (1961) argued that if the size of a market characterized by price dispersion increased sufficiently, then it would become profitable for a third-party to collect and sell information about the distribution of prices. Moreover, Stigler predicted that there would be a “tendency toward monopoly in the provision of information.” In later theoretical work, the information providers envisioned by Stigler appeared in the background of several models of equilibrium price dispersion. For example, Salop and Stiglitz (1977) assume that for a fee consumers obtain full price information. Varian (1980) assumes that some consumers have access to a list of prices across different retailers. These informed consumers buy at the lowest advertised price. Other consumers are uninformed and shop randomly.

³Empirical work has documented widespread price dispersion in online markets as well as asymmetric pricing behavior by firms. The literature offers a number of possible explanations (including brand equity, reputation, product availability, website design, or customer service). Baye, Morgan and Scholten (2006) present a nice survey of this literature.

on price dispersion in online markets Baye, Morgan and Scholten (2006) note that “little is known about the general clearinghouse model with asymmetric consumers. . . Further theoretical work on clearinghouse models with consumer asymmetries and positive listing fees would be a useful addition to the literature.” In this paper we explore how asymmetries in the allocation of consumers across firms impact advertising and pricing decisions in a market with an information gatekeeper. We present a duopoly model in which a larger share of the loyal customers is allocated to one of the two firms. The model is particularly pertinent to online markets which are often characterized by firms with familiar names which capture a large share of the market and smaller websites which are unknown to many consumers.

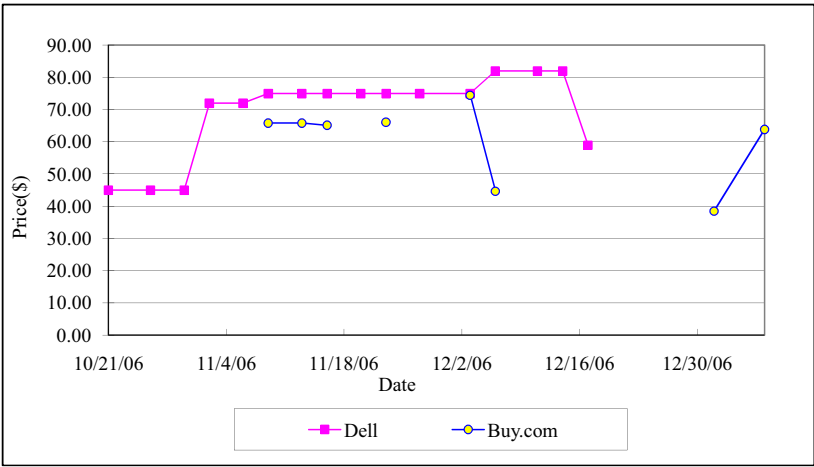


Figure 1: Price for SanDisk 2 GB Secure Digital Card on Shopper.com

Our analysis demonstrates that asymmetric loyal market shares impact firm advertising and pricing behavior in ways that are not revealed by the analysis with symmetric market shares. The probability a firm advertises through the gatekeeper is decreasing in the size of the firm’s loyal market. The firm with the smaller loyal market is more likely to advertise in an attempt to increase its market share by capturing shoppers, but it prices less competitively. The equilibrium mixed strategy pricing distribution of the this firm is less competitive than the pricing strategy adopted by the firm with the larger loyal market. The firm with more loyal customers avoids the opportunity cost of discounting its price by adopting a lower advertising probability. However, when it does advertise, it adopts a more competitive pricing strategy so that the higher opportunity cost of selling to loyal customers at a discounted price is justified by the higher probability of capturing the shoppers. These results are consistent with the example presented in Figure 1. They also help

explain the absence of well known retailers on some price comparison sites.⁴

Our model also extends the literature on pricing strategies adopted by asymmetric firms in the absence of a gatekeeper. Narasimhan (1988) considers equilibrium pricing strategies in a market in which firms have asymmetric loyal customer segments and shoppers automatically observe the price charged by each firm. (Firms make no advertising decision in Narasimhan’s model.) In a duopoly model he finds that the firm with the *larger* loyal segment charges *higher* prices on average. The firm with the smaller loyal segment prices more aggressively because it has less to lose from charging a lower price to its loyal base in order to attract shoppers. These results are extended to the n firm case by Baye, Kovenock and de Vries (1992). They demonstrate that only the firms with the two smallest loyal segments compete for shoppers. These two firms adopt mixed strategy pricing distributions and the smallest firm prices more aggressively than the second smallest firm. All larger firms advertise the monopoly price and sell only to their loyal, non-shopping customers.⁵ These results highlight the central trade-off facing a firm in a market with both loyal customers and shoppers; setting a low price to attract shoppers requires selling at a discount to loyal customers who are willing to pay a higher price, while setting a high price to extract more surplus from loyal customers fails to attract shoppers and results in fewer units being sold.

By explicitly incorporating the firm’s decision about whether or not to advertise, our model allows firms a broader set of strategies with which to balance the trade-off between charging a high price to extract surplus from loyal customers and charging a low price to capture shoppers. In a market with a gatekeeper a firm can increase the probability that it captures shoppers either by advertising with a very high probability, so it is likely to advertise and capture the shoppers when the competition chooses not to advertise, or by setting a low price when it does advertise to ensure that it captures the shoppers even if the competition also chooses to advertise. Importantly, attempting to capture shoppers by advertising with a high probability does not necessitate charging a low price and suffering the coincident reduction in surplus extracted from loyal customers.

In contrast to the previous literature, we find that the firm with the *larger* loyal segment charges a

⁴For example, Baye, Morgan and Scholten’s (2004) data set of over four million prices for consumer electronics products collected from Shopper.com in the period from August 2000 through March 2001 did not contain a single observation from circuitcity.com or bestbuy.com. Circuit City established an online retail presence in 1999, and Best Buy followed in 2000. As Baye, Morgan and Scholten (2004) note, this absence also may be explained by the fact that Circuit City and Best Buy were relatively new participants in the online market at the time their data were collected.

⁵Kocas and Kiyak (2006) show that these results continue to apply even if reservation prices differ across consumers.

lower price, on average, when it chooses to advertise, than the firm with the smaller loyal segment. This does not imply that the firm with the larger loyal segment ignores the opportunity cost of selling to its loyal base at a discounted price. Rather, it balances this trade-off by choosing a lower advertising probability. Through this advertising strategy the firm with the larger loyal segment is less likely to sell to its loyal customers at a discount and is more likely to concede shoppers than is the firm with the smaller loyal segment. However, when it does advertise, the larger firm discounts more heavily than its competition and is more likely to capture the shoppers.

Our model is introduced in section 2. The firms' optimal strategies are derived in section 3. Section 4 presents the optimal strategy of the gatekeeper, and equilibrium analysis is presented in section 5. Section 6 provides concluding remarks.

2 The Model

Our model builds on Baye and Morgan (2001) and Narasimhan (1988) to investigate the role of asymmetric loyal customer segments in a market with an information gatekeeper. We assume a continuum of consumers, each of whom has a reservation price r for one unit of a homogeneous good. The measure of consumers is normalized to unity. The good is provided by two firms that produce the good at a constant marginal cost m . Without loss of generality, we set $m = 0$. Each firm establishes a price p_i , $i = 1, 2$ for the good. A firm may choose to advertise its price through a monopoly information gatekeeper which charges a fee Φ for advertising services. Consumers fall into one of three categories. A fraction L_1 of the consumers are loyal to firm 1. A fraction L_2 are loyal to firm 2. The remaining fraction $S = 1 - L_1 - L_2$ are shoppers. We assume $L_2 < L_1$, so that firm 2 is arbitrarily designated as the firm with the smaller loyal customer base, and we assume $S > 0$. Loyal customers purchase only from their preferred firm (provided the price does not exceed r). Shoppers have no firm preference. Rather, they purchase at the lowest price advertised through the gatekeeper. If neither firm advertises a price, or if both firms advertise the same price, then the shoppers randomly choose one of the two firms and purchase from that firm if its price does not exceed r .⁶ Firms cannot discriminate between shoppers and non-shoppers; they charge the same

⁶The assumption that shoppers are indifferent between firms when no price is advertised is reasonable because in equilibrium any firm that does not advertise a price with the gatekeeper will charge the monopoly price r . The nature of our results do not change if shoppers are allocated to the firms in proportion to each firm's loyal customer base when neither firm advertises a price.

price to all consumers.⁷ This framework is equivalent to the Baye and Morgan (2001) framework with two firms if $L_1 = L_2$, and to the Narasimhan (1988) framework if $\Phi = 0$.

We consider subgame perfect equilibria of the following three stage game. In the first stage the gatekeeper sets the advertising fee Φ . In the second stage the firms observe Φ and then simultaneously determine the prices p_i , $i = 1, 2$ they will charge and the probabilities α_i , $i = 1, 2$, that they will advertise with the gatekeeper. Finally, consumers make purchase decisions. Provided prices do not exceed r , non-shoppers purchase from their preferred firms and shoppers purchase at the lowest price available.

3 The Firms' Problem

Each firm must determine the probability α_i that it will advertise with the gatekeeper and the price p_i that it will charge. The optimal values of α_i and p_i must balance the trade-off between charging the reservation value r to extract the maximum possible surplus from loyal customers versus seeking to capture shoppers by advertising more frequently and setting a lower price while incurring the advertising fee Φ . To characterize the potential gain from advertising, we compare the expected return for a firm that chooses not to advertise with the expected return if the firm does advertise.

A firm i that does not advertise sells to its loyal customers, plus one-half of the shoppers if shoppers do not find a price advertised at the gatekeeper's site. Because the competing firm j advertises with probability α_j , the expected number of customers who purchase from firm i when it does not advertise is $L_i + \frac{1}{2}(1 - \alpha_j)S$. Because the firm's profit from any given transaction is maximized by selling at the reservation price r , a firm that chooses not to advertise will charge a price of r . Thus, the expected profit $E\pi_i^N(r)$ of a firm i if it chooses not to advertise is

$$E\pi_i^N(r) = \left(L_i + \frac{1}{2}(1 - \alpha_j)S \right) r. \quad (1)$$

If firm i advertises, then it sells to its L_i loyal customers as well as to all S shoppers only if firm i 's price is the lowest advertised price. If firm i 's price is not the lowest advertised price, then it sells only to its L_i loyal customers. Let $F_j(p) = \Pr(p_j \leq p)$ denote the advertised price distribution function adopted by firm j . Assuming firm i advertises a price p , the probability that firm j advertises and charges a price $p_j < p$ is $\alpha_j F_j(p)$. Therefore, the probability that shoppers

⁷See Chen, Iyer, and Pudmanabhan (2002), and Baye and Morgan (2002) for models in which sellers discriminate between subscribers and non-subscribers. These papers do not consider asymmetric loyal markets segments.

purchase from firm i is $1 - \alpha_j F_j(p)$,⁸ and the expected profit $E\pi_i^A(p)$ of firm i if it chooses to advertise is

$$E\pi_i^A(p) = (L_i + (1 - \alpha_j F_j(p)) S) p - \Phi. \quad (2)$$

The advertising fee Φ is central to the firm's decision. In particular, if $\Phi \geq rS/2$, then neither firm will advertise because the maximum gain a firm can achieve by advertising is $rS/2$.⁹ In the analysis below we assume $0 < \Phi < rS/2$.

3.1 Preliminaries

In this subsection we present several preliminary results that place restrictions on the firms' equilibrium behavior. These restrictions facilitate the analysis of the equilibrium pricing and advertising strategies presented in subsection 3.2. Because these preliminary results are similar to results in the existing literature and rely on well know arguments, proofs are presented in the appendix. In the analysis below, let \underline{p}_i denote the minimum price charged by firm i , $i = 1, 2$.

Lemma 1 *Assume that both firms advertise with strictly positive probability. Then in any equilibrium neither firm adopts a pure pricing strategy when advertising.*

Lemma 2 *In any equilibrium the lower support of each firm's equilibrium mixed strategy advertised price distribution must be the same, $\underline{p}_i = \underline{p}_j \equiv \underline{p}$.*

Lemma 3 *In equilibrium each firm's mixed strategy advertised price distribution must be atomless over the interval $[\underline{p}, r)$ and must be defined continuously over the interval $[\underline{p}, r]$. Furthermore, only one firm can have a mass point at the upper support r .*

Lemma 4 *In any equilibrium both firms must advertise with strictly positive probability, and there is no equilibrium in which both firms advertise with probability $\alpha_1 = \alpha_2 = 1$.*

⁸This probability assumes that neither firm has a mass points at the price p so a situation in which both firms advertise the same price is not a concern. The analysis in section 3.1 below demonstrates that mass points cannot exist in equilibrium, except at the upper support of the price distribution for one firm.

⁹To see this, note that if neither firm advertises, then firm i will charge the reservation price r and capture 1/2 of the shoppers. The largest possible gain from advertising occurs when firm i advertises the reservation price r and firm j does not advertise. In this case firm i captures the remaining 1/2 of the shoppers at the reservation price r which increases firm i 's profit by $rS/2$.

3.2 The Firms' Optimal Strategies

Each firm must determine both an optimal advertising strategy and an optimal pricing strategy conditional on the decision to advertise. If a firm does not advertise, then it charges the monopoly price r . The preliminary results presented in subsection 3.1 imply that in any equilibrium, both firms advertise with strictly positive probability. In addition, at least one of the firms must adopt a mixed advertising strategy. Conditional upon advertising, the preliminary results imply that both firms adopt mixed strategy advertised price distributions. The equilibrium advertised price distributions must be continuous and have a common support $[\underline{p}, r]$, they can have no mass points, except at the upper support r , and only one firm's distribution, at most, can have a mass point at r .

At this point, it is useful to define \tilde{p}_i as the minimum price that firm i will ever consider advertising through the gatekeeper. In particular, \tilde{p}_i equates the return $\tilde{p}_i(L_i + S) - \Phi$ from advertising and capturing the shoppers with the return from not advertising and charging the reservation price r . Thus, \tilde{p}_i satisfies

$$\tilde{p}_i(L_i + S) - \Phi = E\pi_i^N(r).$$

Note that if $\tilde{p}_i < \tilde{p}_j$, then firm i will advertise with probability $\alpha_i = 1$. This holds because Lemma 2 implies that the lower support \underline{p} of each firm's equilibrium advertised price distribution must be the same. Thus, if $\tilde{p}_i < \tilde{p}_j$, then $\tilde{p}_i < \underline{p}$ because the common lower support \underline{p} of the advertised price distributions cannot be less than the lowest price \tilde{p}_j that firm j would ever choose to advertise. Because $p_i(L_i + S) - \Phi$ is strictly increasing in p_i and firm i can capture all shoppers by advertising $\underline{p} > \tilde{p}_i$, it follows that firm i 's return $\underline{p}(L_i + S) - \Phi$ from advertising the price $p_i = \underline{p}$ with probability one exceeds the expected return $E\pi_i^N(r)$ gained by not advertising and charging the reservation price r . The fact that the two firms may be willing to establish different minimum advertised prices but must adopt the same minimum price \underline{p} in equilibrium enables us to place additional structure on the equilibrium advertising strategies.

Proposition 5 *The firm with the larger loyal customer base (firm 1) will advertise with probability $\alpha_1 < 1$ in any equilibrium.*

Proof. Suppose $\alpha_1 = 1$. Then Lemma 4 implies that $\alpha_2 < 1$. By definition of \tilde{p}_i , $\alpha_1 = 1$ and $\alpha_2 < 1$ imply

$$\tilde{p}_1 = \frac{L_1 r + \Phi}{L_1 + S} + \frac{(1 - \alpha_2) r S / 2}{L_1 + S} > \frac{L_2 r + \Phi}{L_2 + S} = \tilde{p}_2$$

where the inequality follows from the fact that $L_1 > L_2$ by assumption. Lemma 2 and $\tilde{p}_2 < \tilde{p}_1$ imply $\tilde{p}_2 < \underline{p}$, so the expected return $\underline{p}(L_1 + S) - \Phi$ that firm 2 obtains by advertising $p_2 = \underline{p}$ and capturing all shoppers is greater than $E_2^N(r)$. But this implies it is optimal for firm 2 to advertise with probability $\alpha_2 = 1$ which contradicts $\alpha_2 < 1$. ■

Proposition 5 further narrows the set of possible equilibrium advertising strategies; in any equilibrium either both firms adopt mixed advertising strategies, or firm 1 adopts a mixed advertising strategy and firm 2 advertises with probability one. As demonstrated in Proposition 6 below, which of these two cases occurs depends upon the advertising fee Φ established by the gatekeeper.

Proposition 6 *If the gatekeeper sets the advertising fee Φ such that $0 < \Phi \leq \frac{rS(L_1-L_2)}{2L_1+S-L_2}$, then firm 1 adopts a mixed advertising strategy*

$$\alpha_1 = \frac{L_2 + S}{L_1 + S} \left(1 - \frac{\Phi}{rS}\right),$$

and firm 2 adopts a pure advertising strategy $\alpha_2 = 1$, and the mixed advertised price strategies are characterized by the cumulative distribution functions

$$\begin{aligned} F_1(p) &= \frac{r(p(L_1 + S) - (rL_1 + \Phi))}{p(rS - \Phi)}, \text{ and} \\ F_2(p) &= 1 - \frac{L_1(r - p) + \Phi}{pS} \end{aligned}$$

on the interval $[\underline{p}, r]$ where $\underline{p} = \frac{rL_1 + \Phi}{L_1 + S}$.

If the gatekeeper sets Φ such that $\frac{rS(L_1-L_2)}{2L_1+S-L_2} \leq \Phi < rS/2$, then both firms adopt mixed advertising strategies

$$\begin{aligned} \alpha_1 &= 1 - \frac{2\Phi}{rS}, \text{ and} \\ \alpha_2 &= \frac{(rS - 2\Phi)(2L_1 + S - L_2)}{rS(L_2 + S)}, \end{aligned}$$

and the mixed advertised price strategies are characterized by the cumulative distribution functions

$$\begin{aligned} F_1(p) &= \frac{r(pS - 2\Phi - L_2(r - p))}{p(rS - 2\Phi)}, \text{ and} \\ F_2(p) &= \frac{r(L_1 + S)(p(L_2 + S) - rL_2 - 2\Phi)}{p(rS - 2\Phi)(2L_1 + S - L_2)}, \text{ on } [\underline{p}, r], \end{aligned}$$

on $[\underline{p}, r]$ where, $\underline{p} = \frac{rL_2 + 2\Phi}{L_2 + S}$.

Proof. See Appendix.

The results of Proposition 6 enable us to more carefully consider how the size of each firm's loyal customer base impacts the firm's decision to advertise as well as how competitively it prices when the firm does choose to advertise. Propositions 7 and 8 provide two general results. Additional comparative statics results are explored in the subsequent discussion.

Proposition 7 *In any equilibrium the firm 2 (with the smaller loyal market) advertises with a higher probability than the firm 1, i.e., $\alpha_2 > \alpha_1$.*

Proof. If firm 2 adopts $\alpha_2 = 1$, then Proposition 5 implies $\alpha_1 < \alpha_2$. If firm 2 sets $\alpha_2 < 1$, then from Proposition 6

$$\alpha_2 - \alpha_1 = \frac{(rS - 2\Phi)(2L_1 + S - L_2)}{rS(L_2 + S)} - \left(1 - \frac{2\Phi}{rS}\right) = \frac{(rS - 2\Phi)(2L_1 - 2L_2)}{rS(L_2 + S)} > 0$$

where the inequality follows from the assumptions that $\Phi < rS/2$ and $L_1 > L_2$. ■

Proposition 8 *In any equilibrium, conditional upon advertising, the price p_2 advertised by firm 2 is stochastically larger (in the sense of first-order stochastic dominance) than the price p_1 advertised by firm 1. Furthermore, the equilibrium advertised price distribution of firm 2 (and only firm 2) always has a mass point at the monopoly price r .*

Proof. The price p_2 advertised by firm 2 is stochastically larger than p_1 if $1 - F_2(p) \geq 1 - F_1(p)$ for all $p \in [\underline{p}, r]$. Alternatively, p_2 is stochastically larger than p_1 if $F_2(p) \leq F_1(p)$ for $p \in [\underline{p}, r]$.

Suppose $0 < \Phi \leq \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$. Then for $\underline{p} \leq p < r$,

$$F_1(p) - F_2(p) = \frac{\Phi(L_1(p - r) + pS - \Phi)}{pS(rS - \Phi)}.$$

The denominator is positive because $\Phi < rS/2$ by assumption. Evaluated at $\underline{p} = \frac{rL_1 + \Phi}{L_1 + S}$, the numerator equals 0. Because the numerator is strictly increasing in p , the numerator is strictly positive for all $p > \underline{p}$.

Now suppose $\frac{rS(L_1 - L_2)}{2L_1 + S - L_2} \leq \Phi < rS/2$. Then for $\underline{p} \leq p < r$,

$$F_1(p) - F_2(p) = \frac{r(L_1 - L_2)(p(L_2 + S) - rL_2 - 2\Phi)}{p(rS - 2\Phi)(2L_1 + S - L_2)}.$$

The denominator is positive because $\Phi < rS/2$ and $L_2 < L_1$. Evaluated at $\underline{p} = \frac{rL_2 + 2\Phi}{L_2 + S}$ the numerator is 0. Because the numerator is strictly increasing in p , the numerator is strictly positive for all $p > \underline{p}$.

Finally, note that $F_1(r) = 1$ for any value of $\Phi < rS/2$, while $F_2(r) < 1$, so firm 2's advertised price distribution has a mass point at r while firm 1's does not. ■

Propositions 7 and 8 imply that although firm 1 is less likely to advertise, firm 1 adopts a more aggressive pricing strategy when it does advertise. This is the exact opposite of the results found by Narasimhan (1988) and Baye, Kovenock and de Vries (1992). In our model, the ability to refrain from competition by choosing not to advertise serves as a substitute for pricing less aggressively for firm 1. This alternative does not exist in the Narasimhan and Baye, Kovenock and de Vries models. As in their models, firm 1 has more to lose from its loyal base if it prices aggressively to attract shoppers. However, rather than advertising relatively high prices (on average) to minimize this loss, our results demonstrate that when firms choose advertising strategies, firm 1 hedges against this loss by advertising with a lower probability. Because the opportunity cost of advertising a price less than r and failing to capture the shoppers is greater for firm 1 than for firm 2, it only makes sense for firm 1 to advertise if it is intent on capturing the shoppers. This intent is reflected in the fact that under the equilibrium strategies p_2 is stochastically larger than p_1 . Although firm 2 adopts a less competitive advertised price distribution, because the magnitude of the opportunity represented by the shoppers (as a percent of its total market) is greater for firm 2, firm 2 adopts a higher advertising probability. Using the taxonomy of Fudenberg and Tirole (1984), we might think of the small firm as adopting a “puppy-dog” strategy in which it advertises relatively high prices in an attempt to appear non-threatening to firm 1 while simultaneously adopting a higher advertising probability so that on average it only captures shoppers in the event that firm 1 chooses not to advertise.

Comparative statics results with respect to the advertising fee Φ shed further light on the equilibrium strategies. For example, an increase in Φ results in less competitive advertising and pricing behavior. The reduction in competition appears in several ways. First, as Φ increases, firms are less likely to advertise (and so, are more likely to charge the monopoly price r). In particular, α_1 is decreasing in Φ , and α_2 is decreasing in Φ if $\Phi \geq \frac{rS(L_1-L_2)}{2L_1+S-L_2}$, while if $\Phi < \frac{rS(L_1-L_2)}{2L_1+S-L_2}$, then $\alpha_2 = 1$, but an increase in Φ moves Φ closer the range in which $\alpha_2 < 1$ and is decreasing in Φ . Second, as Φ increases, the minimum price \underline{p} that either firm will advertise increases. Finally, as Φ increases, $F_i(p)$ decreases for $i = 1, 2$, so the expected advertised price, conditional on the firm deciding to advertise, increases. The effects all result in less competitive pricing as summarized in the following proposition.

Proposition 9 *The expected price paid by any given buyer is increasing in the advertising fee Φ .*

It is also interesting to consider the impact of an increase in Φ on firm profits. While an increase in Φ results in less competitive pricing behavior, so consumers receive less surplus, the firms also pay a higher advertising fee to the gatekeeper. Thus, the overall impact on firm profitability is unclear. Whether firm profit is increasing or decreasing in Φ depends upon whether both firms adopt mixed advertising strategies. Using the results of Proposition 6, firm profits can be calculated as a function of Φ . If $0 < \Phi < \frac{rS(L_1-L_2)}{2L_1+S-L_2}$, then $E\pi_1^N = E\pi_1^A = rL_1$ and $E\pi_2^A = \frac{rL_1(L_2+S)-\Phi(L_1-L_2)}{L_1+S} > E\pi_2^N$. Thus, if Φ is sufficiently small so that firm 2 advertises with probability $\alpha_2 = 1$, then firm 1's profit is independent of the advertising fee, while firm 2's profit is strictly decreasing in Φ . However, if $\frac{rS(L_1-L_2)}{2L_1+S-L_2} \leq \Phi < rS/2$, then $E\pi_1^N = E\pi_1^A = \frac{rL_2(L_1+S)+\Phi(2L_1+S-L_2)}{L_2+S}$ and $E\pi_2^A = E\pi_2^N = rL_2 + \Phi$, so profits for both firms are strictly increasing in Φ . These results are summarized in the following proposition.

Proposition 10 *If $0 < \Phi < \frac{rS(L_1-L_2)}{2L_1+S-L_2}$, then firm 1's expected profit is independent of the advertising fee Φ , while firm 2's expected profit is strictly decreasing in Φ . If $\frac{rS(L_1-L_2)}{2L_1+S-L_2} \leq \Phi < rS/2$, then expected profits for both firms are strictly increasing in Φ .*

The difference in the impact of an increase in Φ on firm profits follows from the fact that when Φ is small, firm 2 operates at a margin at which the benefit from advertising is strictly greater than the benefit from not advertising. If Φ is small, then firm 2 relies heavily on advertising and sales to shoppers as a source of expected revenue, as evidenced by the fact that $\alpha_2 = 1$ and $E\pi_2^A > E\pi_2^N$. As a result, the gatekeeper can extract additional surplus from firm 2 by increasing Φ , and firm 2's expected profit is decreasing in Φ . Firm 1, on the other hand, only advertises to the extent that advertising generates an expected return equal to the return rL_1 that can be obtained by selling only to its loyal base. Because $\alpha_2 = 1$, the return to firm 1 from not advertising depends only on its loyal base (and is independent of the fraction of shoppers in the market). As a result, the gatekeeper is unable to extract additional surplus from firm 1 by changing Φ .

If Φ is large, then, somewhat surprisingly, an increase in Φ actually generates an increase in the expected profits of both firms. Propositions 6 and 9 imply that an increase in Φ leads to less competitive pricing and advertising strategies, and the firms capture more of the surplus. However, that this does not imply that the expected profit of the gatekeeper is decreasing in Φ . As Φ increases, the gatekeeper's expected revenue is a larger fee multiplied by smaller advertising probabilities, so

the total could be either increasing or decreasing. Proposition 12 below demonstrates that there are conditions under which it is optimal for the gatekeeper to establish an advertising fee $\Phi > \frac{rS(L_1-L_2)}{2L_1+S-L_2}$.

The degree of asymmetry in the firms' loyal market segments also impacts equilibrium strategies. As evidenced by Proposition 6, firm 2 will advertise with probability $\alpha_2 = 1$ if $\Phi \leq \frac{rS(L_1-L_2)}{2L_1+S-L_2}$. While it is intuitive that firms are more likely to advertise if Φ is small, the bound on Φ provides additional insight into this decision. The term $L_1 - L_2$ can be interpreted as the degree of asymmetry in the loyal market segments. For any positive advertising fee $\Phi < rS/2$, if the asymmetry $L_1 - L_2$ is sufficiently large, then $\alpha_2 = 1$, while if $L_1 - L_2$ is sufficiently small, then $\alpha_2 < 1$. When the asymmetry is large, which implies firm 1 has a much larger share of the loyal customers in the market, then firm 2 advertises as aggressively as possible to ensure that it captures the shoppers whenever firm 1 does not advertise.

3.3 Special Cases: $L_1 = L_2$ and $\Phi = 0$

As noted in the introduction, the Baye and Morgan model and the Narasimhan model can be viewed as special cases of our model; in the Baye and Morgan model $L_1 = L_2$, and in the Narasimhan model $\Phi = 0$. The connection between these models and our model can be verified using the results of Proposition 6. If $L_1 = L_2 = L$, then $\frac{rS(L_1-L_2)}{2L_1+S-L_2} = 0$ so $\Phi \geq \frac{rS(L_1-L_2)}{2L_1+S-L_2}$ always applies. Thus, Proposition 6 implies $\alpha_1 = \alpha_2 = 1 - 2\Phi/rS$, $F_1(p) = F_2(p) = \frac{1}{\alpha} \left(1 - \frac{(r-p)\frac{1-S}{2} + \frac{1}{2}rS(1-\alpha) + \Phi}{pS} \right)$, and $\underline{p} = \frac{r(1-S)+4\Phi}{1+S}$. These results are consistent with those of Baye and Morgan with $n = 2$ and adjustment for our unit demand assumption.

Alternatively, if $L_1 > L_2$ and $\Phi = 0$ as in Narasimhan, then $\alpha_1 = \frac{L_2+S}{L_1+S}$, $F_1(p) = \frac{pL_1+pS-rL_1}{pS}$, $F_2(p) = 1 - \frac{L_1(r-p)}{pS}$, and $\underline{p} = \frac{rL_1}{L_1+S}$. These equilibrium expressions appear to conflict with Narasimhan who finds $\alpha_1 = 1$ and $F_1(p) = 1 + \frac{L_2}{S} - \frac{L_1r(L_2+S)}{Sp(L_1+S)}$. However, these two seemingly different equilibria generate the same profits for the two firms. When the advertising fee is zero, we have $F_2(r) = 1$, so firm 2 does not have a mass point at the reservation price. Thus, for firm 1, advertising and charging r generates the same expected profit as not advertising and charging r , because in either case firm 1 sells only to its loyal customers and pays a zero advertising fee to earn a profit of rL_1 . From another perspective, the expected advertised price distribution $\alpha_1 F_1(p)$ implied by our model is

$$\alpha_1 F_1(p) = 1 + \frac{L_2}{S} - \frac{L_1r(L_2+S)}{Sp(L_1+S)},$$

which is equivalent to the pricing distribution found by Narasimhan. More generally, if $\Phi = 0$,

then any convex combination of the equilibrium that is the special case of Proposition 6 with $\Phi = 0$ and the equilibrium that is characterized in Narasimhan is an equilibrium, and all of these equilibria generate the same profits for both firms. The equilibrium found by Narasimhan is just one equilibrium from a continuum of equilibria.

4 The Gatekeeper's Problem

The gatekeeper correctly anticipates the advertising strategies adopted by the firms and chooses its advertising fee Φ to maximize its own expected profit. We assume the gatekeeper has a fixed setup cost k , so its expected profit $E\pi_G$ is

$$E\pi_G = (\alpha_1 + \alpha_2) \Phi - k. \quad (3)$$

The gatekeeper maximizes his expected profit subject to the constraint that firms will choose their advertising strategies optimally as derived in Proposition 6. When choosing the optimal fee, the gatekeeper faces a trade-off between profit per advertisement and the probability the firms choose to advertise. A higher fee will raise the profit per advertisement, but will reduce the probability the firms advertise. Proposition 11 characterizes the gatekeeper's optimal strategy.

Proposition 11 *The optimal advertising fee Φ^* for the gatekeeper is*

$$\Phi^* = \begin{cases} rS/4 & \text{if } L_1 < 1/3 + 2L_2/3 \\ \frac{rS(L_1 - L_2)}{2L_1 + S - L_2} & \text{if } L_1 \geq 1/3 + 2L_2/3. \end{cases}$$

Proof. See Appendix.

Figure 2 demonstrates the gatekeeper's optimal strategy. The shaded areas represent all the possible equilibrium combinations of L_1 and L_2 given the restriction that $L_1 > L_2$ and $L_1 + L_2 < 1$. The closer the pair (L_1, L_2) is to the 45-degree line, the less asymmetric are the firms' loyal market shares. When the asymmetry is relatively small, so the point (L_1, L_2) is in the vertically shaded area, then the optimal fee for the gatekeeper is $\Phi^* = rS/4$ and both firms adopt mixed advertising strategies. With greater asymmetry, so the point (L_1, L_2) is in the horizontally shaded region, the gatekeeper's optimal fee is $\Phi^* = \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$. If $L_1 > (1 + 2L_2)/3$, then $\Phi^* = \frac{rS(L_1 - L_2)}{2L_1 + S - L_2} > rS/4$, and Φ^* increases as $L_1 - L_2$ increases. Thus, if the market is sufficiently asymmetric, then the gatekeeper's advertising fee increases with the asymmetry. The primary reason for this is that if the market is sufficiently asymmetric, then firm 2 advertises with probability $\alpha_2 = 1$. The gatekeeper

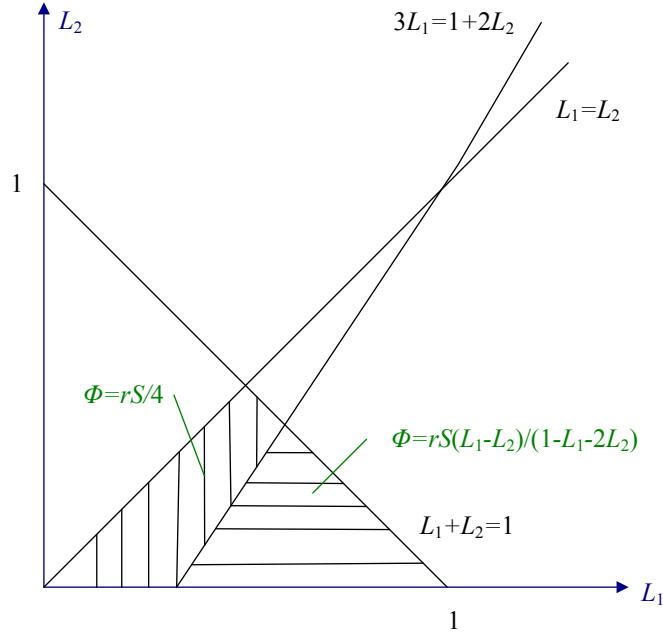


Figure 2: The Gatekeeper's Optimal Advertising Fee

responds to firm 2's strong desire to advertise by increasing Φ to extract surplus from firm 2 to the point at which any further increase in Φ would cause firm 2 to revert to a mixed advertising strategy. However, if the market is more symmetric, so $L_1 \leq (1 + 2L_2)/3$, then the gatekeeper's fee depends only on the monopoly price r and the fraction S of shoppers in the market. It is independent of the allocation of the $1 - S$ loyal customers across the two firms. Reasoning for this result is explored in more detail in Section 5 below.

5 Equilibrium

In this section we utilize the results from Propositions 6 and 11 to characterize the equilibrium strategies as a function of the exogenous parameters r , L_1 , L_2 , and S , and to conduct comparative statics analysis. The equilibrium strategies are found by substituting the optimal advertising fee Φ^* derived in Proposition 11 into the firm strategies derived in Proposition 6.

Proposition 12 *If $L_1 < 1/3 + 2L_2/3$, then there is a unique subgame perfect equilibrium in which*

$$\begin{aligned}\Phi^* &= rS/4, \\ \alpha_1^* &= 1/2, \\ \alpha_2^* &= \frac{(2L_1 + S - L_2)}{2(L_2 + S)} > \frac{1}{2},\end{aligned}$$

$$F_1(p) = 1 - \left(\frac{S + 2L_2}{S} \right) \left(\frac{r - p}{p} \right),$$

$$F_2(p) = \left(\frac{S + L_1}{S + 2L_1 - L_2} \right) \left(1 - \left(\frac{S + 2L_2}{S} \right) \left(\frac{r - p}{p} \right) \right),$$

and

$$\underline{p} = \frac{r(2L_2 + S)}{2(L_2 + S)}.$$

If $L_1 \geq 1/3 + 2L_2/3$, then there is a unique subgame perfect equilibrium in which

$$\Phi^* = \frac{rS(L_1 - L_2)}{2L_1 + S - L_2},$$

$$\alpha_1^* = \frac{S + L_2}{2L_1 + S - L_2},$$

$$\alpha_2^* = 1,$$

$$F_1(p) = 1 - \left(\frac{2L_1 - L_2}{S} \right) \left(\frac{r - p}{p} \right),$$

$$F_2(p) = \left(\frac{S + L_1}{S + 2L_1 - L_2} \right) \left(1 - \left(\frac{2L_1 - L_2}{S} \right) \left(\frac{r - p}{p} \right) \right),$$

and

$$\underline{p} = \frac{r(2L_1 - L_2)}{2L_1 + S - L_2}.$$

As might be expected, when compared with the equilibrium in Narasimhan's model, pricing behavior is less competitive when firms can choose not to advertise. In particular, the expected price charged by either firm in a market with a gatekeeper is higher than the expected price charged by the corresponding firm when advertising is compulsory.¹⁰ As mentioned in subsection 3.2, when the option to advertise is introduced, firm 1 protects the profit opportunity represented by its larger loyal customer base by advertising less frequently than firm 2. As shown in proposition 12, $\alpha_1^* < \alpha_2^*$. Narasimhan finds that firm 1 protects this profit opportunity by advertising a higher price (on average) than firm 2 to avoid selling at a discounted price to its larger loyal base. However, when the advertising decision is endogenous, firm 1 protects the profit opportunity represented by its large loyal base by advertising less frequently and selling to its loyal customers at the reservation

¹⁰This follows from the fact that in Narasimhan's model $\alpha_i = 1$ for $i = 1, 2$ by assumption, and that the equilibrium price distribution $\alpha_i F_i(p)$ over the interval (\underline{p}, r) in our model is strictly less than the equilibrium price distribution over this interval for the corresponding firm in Narasimhan's model. In addition, while the upper support of the equilibrium price distributions is r in both models, the lower support in a market without a gatekeeper is strictly less than \underline{p} .

price r . When firm 1 does advertise, it does so with the intent of capturing shoppers by pricing more competitively than firm 2 as reflected in the fact that $F_1^*(p) > F_2^*(p)$.

We can also consider the role of the gatekeeper on the firms' incentive compete by utilizing the gatekeeper's services. Narasimhan finds that firm 2 prefers the Nash game over collusion when firm 1's loyal segment is relatively large (in particular if $L_1 > 2L_2$).¹¹ In a market with a gatekeeper both firms always prefer the collusive outcome over the Nash game.¹² Firm 2's preference for the Nash game when L_1 is large disappears because although there is a benefit to firm 2 from advertising aggressively to capture shoppers, the gatekeeper is able to extract much of this benefit through the advertising fee. As a result, both firms achieve higher profits if they choose not to advertise. However, the collusive outcome is not an equilibrium of the game in which collusion is an option because if either firm advertises with probability zero, the gatekeeper will establish an advertising fee such that the other firm chooses to advertise with probability 1 and captures all of the shoppers.

The equilibrium results demonstrate that with a gatekeeper, the firms do not engage in Bertrand competition even as the proportion of shoppers becomes arbitrarily large. In particular, as $S \rightarrow 1$, we have $L_1 < 1/3 + 2L_2/3$. Thus, Proposition 12 implies $\alpha_2 \rightarrow 1/2 = \alpha_1$, the equilibrium advertised price distributions both converge to $F_1(p) = F_2(p) = 1 - (r - p)/p$, and the minimum price $\underline{p} \rightarrow r/2$. The advertising fee $\Phi^* \rightarrow r/4$, the expected profits of both firms converge to $r/4$, and the gatekeeper's expected profit is $r/4$ as well. As $S \rightarrow 1$, the equilibrium price is strictly greater than the Bertrand price of 0, and both of the firms and the gatekeeper earn strictly positive expected profits. These results stand in stark contrast to the results of Narasimhan and other equilibrium search models in which the equilibrium converges to Bertrand competition as $S \rightarrow 1$. More generally, it can be shown that the expected price in a market with a gatekeeper is strictly greater than the expected price in the Narasimhan model.¹³ As $S \rightarrow 0$, the gatekeeper becomes irrelevant (the optimal advertising fee $\Phi^* \rightarrow 0$), and the equilibrium converges to the collusive outcome in which

¹¹If L_1 is small ($L_1 < 2L_2$), then both firms prefer the collusive outcome in Narasimhan's model.

¹²If the firms collude by choosing not to advertise and posting the reservation price r , then profit for each firm is $E\pi_i^N(r) = (L_i + S/2)r$. Profit in the equilibrium with the gatekeeper depends upon the size of the loyal segments L_1 and L_2 . If $L_1 < 1/3 + 2L_2/3$, then $E\pi_1 = \frac{(2SL_1 + 3SL_2 + 4L_1L_2 + S^2)}{4(L_2 + S)}r$ and $E\pi_2 = (L_2 + S/4)r$. If $L_1 \geq 1/3 + 2L_2/3$, then $E\pi_1 = rL_1$ and $E\pi_2 = \frac{(-L_2^2 + 2L_1L_2 + SL_1)}{S + 2L_1 - L_2}r$. Because $L_1 + S/2 > \frac{(2SL_1 + 3SL_2 + 4L_1L_2 + S^2)}{4(L_2 + S)}$ and $L_2 + S/2 > L_2 + S/4$, both firms prefer the collusive outcome over the equilibrium with advertising if $L_1 < 1/3 + 2L_2/3$. Similarly, because $L_1 + S/2 > L_1$ and $L_2 + S/2 > \frac{(-L_2^2 + 2L_1L_2 + SL_1)}{S + 2L_1 - L_2}$ both firms prefer the collusive outcome even if $L_1 \geq 1/3 + 2L_2/3$.

¹³In particular, letting F_1^N and F_2^N denote the equilibrium price distributions of the firm the the larger loyal base and smaller loyal base, respectively, in Narasimhan's model, $\alpha_1 F_1^* < F_1^N$, and $\alpha_2 F_2^* < F_2^N$, so in a market with a gatekeeper firms place more weight on higher prices than they do in a market in which shoppers automatically observe prices. Furthermore, the minimum advertised price is strictly greater in a market with a gatekeeper.

neither firm advertises and both charge the monopoly price r .

Proposition 12 also reveals interesting features of the impact of total surplus r and of the distribution of customers across the three segments L_1 , L_2 , and S on equilibrium behavior. Regardless of the size of the loyal segments L_1 and L_2 , an increase in r has no impact on the equilibrium advertising probabilities α_1^* and α_2^* . As r increases, firms do not reduce their advertising probabilities even though the surplus that can be extracted from loyal customers has increased. Rather, as r increases, both firms adopt less competitive advertised price strategies so that the expected revenue from capturing shoppers increases.¹⁴ Reducing α_i^* as r increases is not optimal because doing so would limit the firm's chance of capturing shoppers to whom a higher price is being charged (on average). As expected, an increase in r increases profit for both firms and the gatekeeper.

Comparative statics analysis of the equilibrium strategies with respect to L_1 , L_2 , and S is complicated by the fact that the overall market size is fixed. As a result, an increase in the size of any one of the three market segments requires a reduction in at least one of the other two segments. The final reallocation of customers must be accounted for in the comparative statics analysis. The comparative statics results for each of the equilibrium strategies presented in Proposition 12 with respect to L_1 , L_2 , and S are presented in the Appendix.

Many of these comparative statics results are consistent with expectations. For example, if the difference in loyal customer segments is small, so $L_1 < 1/3 + 2L_2/3$, then an increase in firm 2's loyal base L_2 causes firm 2 to reduce its advertising probability α_2^* and to price less competitively when it does advertise ($dF_2/dL_2 < 0$ and $dp/dL_2 > 0$). Somewhat less obvious is the fact that firm 1 also will price less competitively when L_2 increases ($dF_1/dL_2 < 0$), even if the increase in firm 2's loyal base is the result of a direct transfer of customers from firm 1 to firm 2. An increase in the proportion S of shoppers in the market leads to more competitive advertised prices. These results also suggest that dispersion in advertised prices decreases as L_2 increases but increase as S increases. The impact of an increase in firm 1's loyal segment is more complex. If a sufficiently large fraction of the increase in L_1 is the result of a transfer from firm 2's loyal segment, then both firms will price more competitively, the minimum price will decrease, and price dispersion will increase. However, if a relatively large fraction of the increase in L_1 comes from S , then firm 1 prices less competitively and \underline{p} increases. Firm 2 adjusts its advertised price distribution by placing less weight in the tails of the distribution; $F_2(p)$ decreases for small values of p less than some critical value \hat{p}

¹⁴In particular, the lower support \underline{p} of the advertised price distributions is increasing in r and both $F_1(p)$ and $F_2(p)$ are decreasing in r .

and increases for $p > \hat{p}$.

If the difference in the size of the two firms' loyal segments is large, so $L_1 \geq 1/3 + 2L_2/3$, then only an increase in L_1 has an unambiguous impact on pricing behavior. As L_1 increase, firms price less competitively in equilibrium. This is driven by firm 1's incentive to forego advertising ($d\alpha_1^*/dL_1 < 0$) and charge the monopoly price to its growing loyal customer base. As firm 1 advertises less aggressively, firm 2 adopts a less competitive advertised price distribution in order to collect higher prices from the shoppers it is now more likely to capture.

The equilibrium results also enable us to consider whether an increase in the proportion of shoppers makes firms better or worse off. While one might expect that an increase in S leads to more competitive pricing and results in a transfer of surplus from firms to the gatekeeper and consumers, this is not always the case. If $L_1 < 1/3 + 2L_2/3$, then firm 2's expected profit is increasing in S if less than 1/4 of the increase in S comes from firm 2's loyal base. While firm 2's loyal base is slightly diminished, it is more than able to make up for the loss in loyal customers by capturing the now larger pool of shoppers. Similarly, if a large fraction of the increase in S comes from firm 2's loyal base, then it is possible for firm 1's profit to increase as S increases. If $L_1 \geq 1/3 + 2L_2/3$, then an increase in S always reduces firm 1's profit, but firm 2's profit is still increasing in S if a sufficiently small fraction of the increase in S comes from L_2 . Not surprisingly, in all cases, an increase in S is beneficial to the gatekeeper.

6 Conclusions

We consider a duopoly market in which two firms, 1 and 2, have asymmetric loyal market shares, L_1 and L_2 , with $L_1 > L_2$, and a fraction S of the population shops for the lowest price advertised through an information gatekeeper. Our analysis demonstrates that in equilibrium the firm with the smaller loyal market is more likely to advertise its price to shoppers, but adopts a less competitive advertised price distribution than the firm with the larger loyal market. This contrasts with earlier models by Narasimhan (1988) and Baye, Kovenock and de Vries (1992) which find that in the absence of a gatekeeper, the firm with the smaller loyal market adopts a more competitive pricing strategy.

A market with a gatekeeper, provides firms two strategic options, both pricing and advertising, for balancing the trade-off between competing for shoppers and charging a higher price to loyal, non-shopping customers. In contrast to earlier models, we allow a firm to choose not to advertise

at all, in which case it sells only to its loyal customers. This strategy has the advantage of extracting the highest possible surplus from non-shopping, loyal customers, but it concedes shoppers. We demonstrate that the ability to choose not to advertise leads to equilibrium pricing strategies which differ dramatically from the strategies found in the earlier literature. In particular, firm 1's advertised price is lower (on average) than firm 2's. Because the opportunity cost of competing for shoppers is higher for firm 1, when it does advertise, firm 1 prices more aggressively to ensure it captures the shoppers. However, because firm 1 has a larger loyal customer base, it is also more likely than firm 2 to simply choose not to advertise and to sell to its loyal customers at the monopoly price. Firm 2, on the other hand, advertises more aggressively but prices less competitively. In this sense firm 2's strategy is consistent with a "puppy-dog" approach as characterized by Fudenberg and Tirole (1984). Firm 2 adopts a less competitive advertised price strategy so that in the event that both firms choose to advertise, firm 2 appears less threatening to firm 1. As a result, both firms price less competitively than they would in the absence of a gatekeeper.

The fact that competition is less severe in a market with a gatekeeper does not imply that firms prefer operating in a market with a gatekeeper over a market in which shoppers readily observe all prices. However, our analysis demonstrates that this is the case if the difference in the size of the loyal market segments L_1 and L_2 is sufficiently small. If $L_1 < 1/4 + 3L_2/4$, then both firms are better off in a market with a gatekeeper. Such market conditions would support a collaborative effort by firms to develop a gatekeeper institution which gives each firm control over if and when its price is advertised to shoppers. This may explain why several major airlines partnered to launch the travel price comparison site Orbitz.com and why price comparison websites such as Shopper.com, where prices are only advertised when submitted by a firm, have persisted even with the development of shopbot websites which use search technology to automatically collect prices from a number of sellers at the shopper's request. If the difference in the size of the loyal segments is large, then firm 1 still prefers a market with a gatekeeper, but firm 2 is strictly better off in a market in which shoppers observe all prices.

7 Appendix

Proof of Lemma 1. Assume that firm i adopts a pure pricing strategy p_i when advertising. Suppose that firm j 's advertised pricing strategy includes a minimum price (under either a mixed or a pure strategy) of $r \geq \underline{p}_j > p_i$. Then firm j makes sales to shoppers only if firm i does not advertise, which implies firm j should employ a pure strategy of $p_j = r$ when it advertises. But if firm j adopts a pure strategy of $p_j = r$ when advertising, then firm i can strictly increase its expected profit by raising its advertised price by some $\varepsilon > 0$ sufficiently small so that $p_i + \varepsilon < r = p_j$, which implies firm i 's initial pure strategy is not optimal, so $\underline{p}_j > p_i$ cannot occur in equilibrium.

Next, suppose that firm j 's advertised pricing strategy includes a minimum price of $\underline{p}_j < p_i \leq r$. Then firm j 's expected profit is strictly increasing in \underline{p}_j , so i 's pure strategy cannot satisfy $\underline{p}_j < p_i$.

Finally, suppose $\underline{p}_j = p_i$. Then for $\varepsilon > 0$ and sufficiently small

$$E\pi_j^A(p_j - \varepsilon) - E\pi_j^A(p_j) = \frac{p_j \alpha_i S}{2} - \varepsilon(L_j + S) > 0.$$

where the inequality follows from the facts that $\alpha_i > 0$ by assumption and that ε can be arbitrarily small. Thus, $\underline{p}_j = p_i$ cannot occur in any equilibrium. ■

Proof of Lemma 2. Suppose that $\underline{p}_i < \underline{p}_j$. Because $F_j(p) = 0$ for all $p \in [\underline{p}_i, \underline{p}_j)$, $E\pi_i^A(p_i)$ is strictly increasing over this interval, so $\underline{p}_i < \underline{p}_j$ is suboptimal for firm i . ■

Proof of Lemma 3. To verify that neither firm can have a mass point in the interval $[p, r)$, suppose one firm j has a mass point at a price p^m where $r > p^m \geq \underline{p}$. Then for $\varepsilon > 0$, the expected profit of firm i if it advertises a price of $p^+ \equiv p^m + \varepsilon$ is

$$E\pi_i^A(p^+) = p^+ (L_i + (1 - \alpha_j F_j(p^+))S) - \Phi.$$

Similarly, firm i 's expected profit when advertising a price $p^- \equiv p^m - \varepsilon$ is

$$E\pi_i^A(p^-) = p^- (L_i + (1 - \alpha_j F_j(p^-))S) - \Phi.$$

If $p^m = \underline{p}$, then because $F_j(p^m) - F_j(p^-)$ is at least as large as the strictly positive probability mass assigned to the price p^m , there exists an $\varepsilon > 0$ such that $E\pi_i^A(p^-) > E\pi_i^A(p^m)$. But this implies that $\underline{p}_i < \underline{p}$ which contradicts Lemma 2, so firm j cannot have a mass point at \underline{p} . If $p^m > \underline{p}$, then in order for both p^+ and p^- to be part of firm i 's equilibrium mixed pricing strategy, firm i 's expected profit must be equal for both of these prices. However, because firm j has a mass point at p^m , there exists an $\varepsilon > 0$ such that $E\pi_i^A(p^-) > E\pi_i^A(p^+)$, and firm i will never choose to advertise in the

interval $[p^m, p^+]$. But this implies that $F_i(p)$ must be constant over the interval $[p^m, p^+]$, so firm j can strictly increase its expected profit by shifting its mass point from p^m to p^+ . This contradicts our initial assumption that firm j has a mass point at p^m .

To verify that F must be continuous over $[\underline{p}, r]$, first assume that neither firm advertises a price in the interval (p_l, p_u) where $\underline{p} \leq p_l < p_u \leq r$. Because, as just shown, firm j cannot have a mass point at p_l , for any $p \in [p_l, p_u)$, $F_j(p)$ is a constant equal to $F_j(p_l)$. Thus, for any advertised prices $\tilde{p} \in (p_l, p_u)$, $E\pi_i^A(\tilde{p}) - E\pi_i^A(p_l) = (L_i + (1 - \alpha_j F_j(p_l))S)(\tilde{p} - p_l) > 0$, so firm i would strictly prefer advertising the price \tilde{p} over the price p_l which contradicts the assumption that neither firm advertises in the interval (p_l, p_u) . Next, assume that only one firm, j , chooses not to advertise in some interval (p_l, p_u) , so $F_j(p)$ is a constant equal to $F_j(p_l)$ for all $p \in [p_l, p_u)$. This implies that for any two prices p_1 and p_2 in (p_l, p_u) with $p_1 > p_2$ we have $E\pi_i^A(p_1) > E\pi_i^A(p_2)$, so firm i would never advertise p_2 . But this contradicts the assumption that only firm j chooses not to advertise prices in the interval (p_l, p_u) .

To verify that only one firm can have a mass point at r , suppose that advertised pricing strategies for both firms have a mass point at $\bar{p} = r$. Then, because each firm's expected profit is continuous in its advertised price, there exists an $\varepsilon > 0$ such that $E\pi_i^A(\bar{p} - \varepsilon) - E\pi_i^A(\bar{p}) > 0$ for $i = 1, 2$ ■

Proof of Lemma 4. Suppose that one firm j sets $\alpha_j = 0$. Then the competing firm i will charge the reservation price r if it advertises, and the gain from doing so as opposed to not advertising is

$$E\pi_i^A(r) - E\pi_i^N(r) = (S + L_i)r - \Phi - \left(L_i + \frac{1}{2}S\right)r = rS/2 - \Phi > 0$$

where the inequality follows from the fact that $\Phi < rS/2$ by assumption. Thus, if firm j sets $\alpha_j = 0$, then firm i will set $\alpha_i = 1$ and advertise a price $p_i = r$. However, if firm i follows this strategy, then by advertising a price $r - \varepsilon$ with probability $\alpha_j = 1$, where $\varepsilon > 0$ is small, firm j gains

$$E\pi_j^A(r) - E\pi_j^N(r) = (S + L_j)(r - \varepsilon) - \Phi - L_j r = rS - (S + L_j)\varepsilon - \Phi > rS/2 - (S + L_j)\varepsilon > 0$$

where the first inequality follows from the fact that $\Phi < rS/2$, and the second inequality follows from ε being arbitrarily small. Thus, firm j will prefer to deviate from $\alpha_j = 0$ so it is optimal for both firms to advertise with strictly positive probability.

Next, suppose $\alpha_1 = \alpha_2 = 1$. Then the return for each firm i must satisfy $E\pi_i^A(p) \geq E\pi_i^N(r)$ for all $p \in [\underline{p}, r]$. If neither firm has a mass point at r or if firm i but not firm j has a mass point at

r (recall that Lemma 3 implies that at most one firm can have a mass point at r), then because $F_j(r) = 1$ and $\alpha_j = 1$,

$$E\pi_i^A(r) - E\pi_i^N(r) = rL_i - \Phi - rL_i = -\Phi < 0$$

which implies $E\pi_i^A(r) < E\pi_i^N(r)$, so firm i prefers not to advertise. ■

Proof of Proposition 6. Proposition 5 implies $\alpha_1 < 1$. This implies $\tilde{p}_1 \geq \tilde{p}_2$. First consider possible equilibria with $\alpha_2 = 1$. If $\alpha_2 = 1$, then $\tilde{p}_2 \leq \tilde{p}_1$ simplifies to

$$\frac{rL_2 + \frac{1}{2}(1 - \alpha_1)rS + \Phi}{L_2 + S} \leq \frac{rL_1 + \Phi}{L_1 + S}. \quad (4)$$

Because firm 1 sets $\alpha_1 < 1$, firm 1 must be indifferent between advertising and not advertising. This implies $E\pi_1^N(r) = E\pi_1^A(p)$ or, when $\alpha_2 = 1$,

$$p(L_1 + (1 - F_2(p))S) - \Phi = L_1r. \quad (5)$$

It follows that the equilibrium advertised price distribution $F_2(p)$ of firm 2 must satisfy

$$F_2(p) = 1 - \frac{L_1(r - p) + \Phi}{pS}$$

Lemma 3 implies $F_2(\underline{p}) = 0$, which implies $\underline{p} = \frac{rL_1 + \Phi}{L_1 + S}$. Furthermore, $F_2(r) = 1 - \frac{\Phi}{rS}$, so firm 2's mixed advertised price distribution has a mass point at the reservation price r , whence Lemma 3 implies $F_1(r) = 1$ because only one firm can have a mass point at r . Because firm 2 must be indifferent between advertising any price in the support of its distribution, we have $E\pi_2^A(\underline{p}) = E\pi_2^A(r)$. Substituting $F_1(r) = 1$, this implies

$$\underline{p}(L_2 + S) - \Phi = r(L_2 + (1 - \alpha_1)S) - \Phi.$$

Substituting the expression for \underline{p} and solving for α_1 yields

$$\alpha_1 = \frac{L_2 + S}{L_1 + S} \left(1 - \frac{\Phi}{rS}\right).$$

Similarly, $E\pi_2^A(p) = E\pi_2^A(\underline{p})$ implies

$$p(L_2 + (1 - \alpha_1 F_1(p))s) - \Phi = \underline{p}(L_2 + s) - \Phi.$$

Substituting the expression for \underline{p} and solving for $F_1(p)$ yields

$$F_1(p) = \frac{p(L_1 + S) - (rL_1 + \Phi)}{pS(1 - \frac{\Phi}{rS})}.$$

It is easy to verify that $F_1(\underline{p}) = 0$ and $F_1(r) = 1$. Finally, substituting the expression for α_1 into equation (4) implies $\Phi \leq \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$. Otherwise the condition $\tilde{p}_2 \leq \tilde{p}_1$ is violated.

Now suppose $\alpha_2 < 1$. Because $\alpha_1 < 1$ also must hold in any equilibrium, the arguments in the proof of Proposition 5 imply that in equilibrium we must have $\tilde{p}_1 = \tilde{p}_2$, which is equivalent to

$$\frac{rL_1 + \frac{1}{2}rS(1 - \alpha_2) + \Phi}{L_1 + S} = \frac{rL_2 + \frac{1}{2}rS(1 - \alpha_1) + \Phi}{L_2 + S}. \quad (6)$$

Because both firms must adopt mixed advertised price distributions, in any equilibrium $E\pi_1^A(p) = E\pi_1^A(\underline{p})$ and $E\pi_2^A(p) = E\pi_2^A(\underline{p})$ or

$$pL_1 + pS(1 - \alpha_2 F_2(p)) - \Phi = \underline{p}(L_1 + S) - \Phi \quad (7)$$

$$pL_2 + pS(1 - \alpha_1 F_1(p)) - \Phi = \underline{p}(L_2 + S) - \Phi \quad (8)$$

In addition, because both firms adopt mixed advertising strategies, both firms must be indifferent between advertising and not advertising so $E\pi_i^N(r) = E\pi_i^A(p)$ for any $p \in [\underline{p}, r]$. This implies

$$p(L_i + (1 - \alpha_j F_j(p))S) - \Phi = L_i r + \frac{1}{2}(1 - \alpha_j)rS. \quad (9)$$

At this point, it is not clear whether either firm will have a mass point at r . Lemma 3 implies there are three possibilities; firm 1 has a mass point, firm 2 has a mass point, or neither firm has a mass point. If neither firm has a mass point, then $F_1(r) = F_2(r) = 1$, and when evaluated at the price r equation (9) implies $\alpha_1 = \alpha_2 = 1 - \frac{2\Phi}{rS}$. But, substituting $\alpha_1 = \alpha_2 = 1 - \frac{2\Phi}{rS}$ into equation (6) yields $L_1 = L_2$ which contradicts our assumption that $L_1 > L_2$. Thus, exactly one firm must have a mass point at r . Suppose firm 1 has a mass point at r . Letting $j = 1$ and $i = 2$ in equation (9) and solving for $F_1(p)$ yields

$$F_1(p) = \frac{1}{\alpha_1} \left(1 - \frac{rL_2 - pL_2 + \frac{1}{2}rS(1 - \alpha_1) + \Phi}{pS} \right). \quad (10)$$

Similarly, letting $j = 2$ and $i = 1$ yields $\alpha_2 = 1 - \frac{2\Phi}{rS}$. Substituting this expression for α_2 into equation (6), solving for α_1 , and substituting this expression into equation (10) yields

$$F_1(p) = \frac{r(pSL_1 + pS^2 + pL_1L_2 + pL_2S - rL_1L_2 - 2\Phi S - rSL_1 - 2\Phi L_2)}{p(rS - 2\Phi)(2L_2 + S - L_1)}.$$

But this implies $F_1(r) = \frac{S + L_2}{S + 2L_2 - L_1} > 1$ which contradicts the definition of a cumulative distribution function. It follows that firm 2's equilibrium advertised price distribution must have a mass point at r . This implies $F_1(r) = 1$, so

$$\alpha_1 = 1 - \frac{2\Phi}{rS} \quad (11)$$

Substituting this expression for α_1 into equation (6) yields

$$\alpha_2 = \frac{(rS - 2\Phi)(2L_1 + S - L_2)}{rS(L_2 + S)}.$$

Substituting $\alpha_1 = 1 - \frac{2\Phi}{rS}$ into equation (8) and evaluating at $p = \underline{p}$ (note that lemma 3 implies $F_1(\underline{p}) = 0$) yields $\underline{p} = \frac{rL_2 + 2\Phi}{L_2 + S}$. Finally, substituting the expressions for α_1 , α_2 and \underline{p} into equations (7) and (8) generates the expressions for $F_1(p)$ and $F_2(p)$ stated in the Proposition. Notice that the solution satisfies required properties. Given the assumption that $\Phi < rS/2$, it follows that $0 < \alpha_1 < 1$ and that $\alpha_2 > 0$. Also, it can be verified that $F_1(\underline{p}_1) = F_2(\underline{p}_2) = 0$, $F_1(r) = 1$, and $F_2(r) = \frac{L_1 + S}{2L_1 + S - L_2} < 1$. Therefore, at equilibrium, firm 2 will have mass point at r and firm 1 will not have positive density at r . Finally, to see that the equilibrium with $\alpha_2 < 1$ only applies if $\Phi > \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$, note that $\alpha_2 \geq 1$ if $\Phi \leq \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$. ■

Proof of Proposition 11. Since the equilibrium firm behavior depends upon the gatekeepers fee Φ , the approach of the proof is to find the optimal fee first constraining Φ to the range in which $\alpha_2 = 1$, or $0 < \Phi \leq \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$, and then constraining Φ to the range in which $\alpha_2 < 1$, or $\frac{rS(L_1 - L_2)}{2L_1 + S - L_2} < \Phi < rS/2$, and then to note that the optimal fee corresponds to the local maximum which generates the greatest profit for the gatekeeper.

Recall that the gatekeeper's expected profit is

$$E\pi_G = (\alpha_1 + \alpha_2)\Phi - k.$$

Suppose $0 < \Phi \leq \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$. Then Proposition 6 implies $\alpha_1 = \frac{L_2 + S}{L_1 + S}(1 - \frac{\Phi}{rS})$ and $\alpha_2 = 1$, so

$$E\pi_G = \left(\frac{L_2 + S}{L_1 + S} \left(1 - \frac{\Phi}{rS}\right) + 1 \right) \Phi - k.$$

Maximizing subject to the constraint that $0 < \Phi \leq \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$ yields a corner solution $\Phi^* = \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$. (The unconstrained maximum is $\Phi^* = \frac{rS}{2} \left(1 + \frac{L_1 + S}{L_2 + S}\right) > \frac{rS}{2}$ which is outside the acceptable range for Φ).

Now suppose that $\frac{rS(L_1 - L_2)}{2L_1 + S - L_2} \leq \Phi < rS/2$. Then Proposition 6 implies $\alpha_1 = 1 - \frac{2\Phi}{rS}$ and $\alpha_2 = \frac{(rS - 2\Phi)(2L_1 + S - L_2)}{rS(L_2 + S)}$, so

$$E\pi_G = \frac{rS - 2\Phi}{rS} \left(\frac{S + L_1}{S + L_2} \right) 2\Phi - k.$$

Maximizing with respect to Φ subject to the constraint that $\frac{rS(L_1 - L_2)}{2L_1 + S - L_2} < \Phi < rS/2$ yields $\Phi^* = \frac{rS}{4}$ if $rS/4 > \frac{rS(L_1 - L_2)}{2L_1 + S - L_2}$, i.e., if $L_1 < (1 + 2L_2)/3$. Furthermore, if $L_1 < (1 + 2L_2)/3$, then the

gatekeeper's expected profit from setting $\Phi^* = rS/4$ is $\frac{1}{4}rS\frac{L_1+S}{L_2+S}$ which exceeds the expected profit from setting a fee of $\frac{rS(L_1-L_2)}{2L_1+S-L_2}$ at the corner solution with $\alpha_2 = 1$. Therefore, if $L_1 < (1 + 2L_2)/3$, then $\Phi^* = rS/4$ is the advertising fee that achieves a global maximum of the gatekeeper's expected profit. On the other hand, if $L_1 \geq (1 + 2L_2)/3$, then the constrained maximization yields $\Phi^* = \frac{rS(L_1-L_2)}{(1+L_1-2L_2)}$, which is the same solution when $\alpha_2 = 1$. As a result, it will also be the global maximum. ■

Comparative Statics Results for Section 5

Equilibrium Comparative Statics if $L_1 < 1/3 + 2L_2/3$

Change In	With respect to		
	L_1	L_2	S
Φ^*	0 if $\frac{dS}{dL_1} = 0$, - if $\frac{dS}{dL_1} < 0$	0 if $\frac{dS}{dL_2} = 0$, - if $\frac{dS}{dL_2} < 0$	+
α_1^*	0	0	0
α_2^*	+	-	+ iff. $\frac{dL_1}{dS} > \frac{-(S+L_2)}{1+S}$
F_1	+ iff. $\frac{dS}{dL_1} > \frac{-S}{L_2+S}$	-	+
F_2	+ if $\frac{dS}{dL_1} \geq \frac{-S}{L_2+S}$ If $\frac{dS}{dL_1} < \frac{-S}{L_2+S}$, then there is a $\hat{p} \in (\underline{p}, r)$ such that $dF_2/dL_1 > 0$ iff. $p > \hat{p}$.	-	+
\underline{p}	+ iff. $\frac{dS}{dL_1} < \frac{-S}{L_2+S}$	+	-
Equilibrium Comparative Statics if $L_1 \geq 1/3 + 2L_2/3$			
Change In	With respect to		
	L_1	L_2	S
Φ^*	+ iff $\frac{dS}{dL_1} > \frac{S(L_1+L_2-2)}{2L_1^2-2L_1L_2-L_1+2L_2^2-2L_2+1}$	-	+ iff $\frac{dL_1}{dS} < \frac{2L_1^2-2L_1L_2-L_1+2L_2^2-2L_2+1}{S(L_1+L_2-2)}$
α_1^*	-	+	+ iff $\frac{dL_2}{dS} < -\frac{S+L_1}{S+1}$
α_2^*	0	0	0
F_1	-	+ iff $\frac{dS}{dL_2} > \frac{-3S}{2(L_1+S)-L_2}$	+
F_2	-	+ if $\frac{dS}{dL_2} \geq \frac{-3S}{2S+2L_1-L_2}$ If $\frac{dS}{dL_2} < \frac{-3S}{2S+2L_1-L_2}$, then there is a $\hat{p} \in (\underline{p}, r)$ such that $dF_2/dL_2 > 0$ iff $p > \hat{p}$.	+ if $\frac{dL_2}{dS} > -\frac{S+L_1}{S+1}$ If $\frac{dL_2}{dS} \leq -\frac{S+L_1}{S+1}$, then there is a $\hat{p} \in (\underline{p}, r)$ such that $dF_2/dL_2 > 0$ iff $p < \hat{p}$
\underline{p}	+	- iff $\frac{dS}{dL_2} > \frac{-3S}{2L_1+2S-L_2}$	-

References

- [1] Baye, Michael R., Dan K. Kovenock and Caspar G. de Vries, "It Takes Two to Tango: Equilibria in a Model of Sales," *Games and Economic Behavior*, **4** (1992), 493-510.
- [2] Baye, Michael R., and John Morgan, "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets," *American Economic Review*, **91** (2001), 454-474.
- [3] Baye, Michael R. and John Morgan, "Information Gatekeepers and Price Discrimination on the Internet," *Economic Letters*, **76** (2002) 47-51.
- [4] Baye, Michael R., John Morgan, and Patrick Scholten. "Price Dispersion in the Small and in the Large: Evidence from an Internet Price Comparison Site," *Journal of Industrial Economics*, **52** (2004), 463-496.
- [5] Baye, Michael R., John Morgan, and Patrick Scholten, "Information, Search, and Price Dispersion," in *Handbook on Economics and Information Systems* (T. J. Hendershott, Ed.), Amsterdam: Elsevier, 2006.
- [6] Chen, Yuxin, Ganesh Iyer, and V. Padmanabhan, "Referral Infomediaries," *Marketing Science*, **21** (2002), 412-434.
- [7] Fudenberg, Drew, and Jean Tirole. "The Fat-Cat Effect, The Puppy-Dog Ploy, and the Lean and Hungry Look," *American Economic Review*, **74** (1984), 361-68.
- [8] Kocas, Cenk, and Tunga Kiyak. "Theory and Evidence on Pricing by Asymmetric Oligopolies," *International Journal of Industrial Organization*, **24** (2006), 83-105.
- [9] Narasimhan, C. "Competitive Promotional Strategies." *Journal of Business*, **61** (1988), 427-449.
- [10] Salop, Steven C., and Joseph E. Stiglitz, "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion." *Review of Economic Studies*, **44** (1997), 493-510.
- [11] Stigler, George J. "The Economics of Information," *Journal of Political Economy*, **69** (1961), 213-225.
- [12] Varian, Hal R., "A Model of Sales," *American Economic Review*, **70** (1980), 651-59.