Intercity Trade
and
Convergent versus Divergent Urban Growth

Yannis M. Ioannides*

Department of Economics

Tufts University

Medford, MA 02155

Telephone: 617 627 3294 Fax: 617 627 3917

`Yannis.Ioannides@Tufts.edu`

`http://www.tufts.edu/yioannid/`

June 19, 2008

Abstract

The paper studies intercity trade and growth in an overlapping-generations economy where tradeable goods are produced using a composite of capital, raw labor and intermediates, and are combined in each city to produce a composite. The composite is used for consumption and investment. Tax-financed investment that affects commuting costs endogenizes city size. A combination of weak (strong) diminishing returns and strong (weak) market size effects can lead to increasing (decreasing) returns to scale. Autarkic urban growth may be parallel or divergent. Capital growth in the integrated economy has the same dynamic properties as its counterpart for an economy with autarkic cities but leads to national constant returns to scale.

* This is an outgrowth of material from "Urban Structure, Intercity Trade, and Economic Growth," which was presented at C.R.E.T.E. 2007, Naxos, July 2007 and the University of Cyprus, March 2008. I thank Marcus Berliant, Drusilla Brown, Theodore Palivos and Marios Zachariades for very insightful comments. The usual caveats apply.

`TradeConvDivGrowth 06 19 08.tex`

Contents

INTERCITY TRADE AND
CONVERGENT VERSUS DIVERGENT URBAN GROWTH

1 Introduction

The history of urbanization is closely related to intercity and international trade. Cities grow as they trade with their hinterlands, and with other nearby as well as more distant cities. This process at a national and indeed a global scale is not fully understood. The system-of-cities literature analyses urban development by means of growth in the number of cities [Black and Henderson (1999); Berliant and Wang (2004); Henderson and Ioannides (1981); Ioannides (1994); Ioannides and Overman (2004); Rossi-Hansberg and Wright (2007)]. While changes in the fundamentals may affect relative sizes of cities, it is a key feature of the system-of-cities approach that urban growth occurs through creation of new cities as well as expansion of existing cities. Naturally, the process of economic growth may alter the fundamental determinants of equilibrium city size.

The role of trade in growth is one of the areas that have attracted attention by the new economic geography and the endogenous growth literature. Since in some of those theories, endogenous technological change is driven by specialization, which is in turn driven by the extent of the market, economic openness is naturally underpinning growth in the extent of the market. At the same time, urban economic activity, indeed urban life itself, has been credited for providing one of the key innovation engines of the economy. Such a dynamic role of the urban economy within a national economy has been slow to receive fuller attention, in spite of recognition accorded to the work of Jane Jacobs (1969) by Robert Lucas and others.

Cities are assumed to take the price of the output in which they specialize as given. This is consistent with both the model and reality, if there are many cities of each type, as in Henderson (1974) and Rossi-Hansberg and Wright (2007). In this paper, too, each city is specialized in a single tradeable product, that is produced under conditions of perfect competition. Still, the setting is compatible with terms-of-trade effects. Recalling the terminology of Eaton and Eckstein (1997), we distinguish among the following logical possibilities. If as an economy's population grows, the number of cities grows but their relative sizes do not change, then we say we have *parallel* growth; if the growth rate of smaller cities increases relative to larger cities, then we have *convergent* growth; and if the size of larger cities increases relative to those of smaller cities, then we have *divergent* growth. These different possibilities may be handled only by a model that contains key forces that drive urbanization. Population growth with a constant number of cities increases congestion and reduces welfare and thus disturbs the existing equilibrium. Increase in intracity commuting costs induces emergence of new cities. However, once cities have specialized, the taste for product variety drive the appearance of additional cities, while intercity transportation costs limit the process and acts as a force of agglomeration. There are centrifugal forces, namely congestion and the attendant problem of wasteful commuting and congestion costs in the form of higher rents, and centripetal ones, favoring fewer and larger cities. Generally, the earlier new economic geography works ignored the fact that cities have been growing in size and number, and the reasons why very large cities have appeared. Especially in the third world, cities of a range of sizes may coexist with large diversified cities.

In the remainder of this paper we obtain a precise description of the law of motion in dynamic settings of either autarkic or specialized cities. A combination of weak diminishing returns and strong market size effects can lead to increasing returns to scale in each autarkic city. We extend this model to allow for investment by local governments that reduces urban commuting costs. Under appropriate conditions, unceasing growth sustains a divergent pattern in city sizes. We examine economic growth in an integrated economy, that is in the presence of intercity trade in manufactured goods and free factor mobility within the urban system. The law of motion for capital of the integrated economy has the same dynamic properties as its counterpart for an economy with autarkic cities. Cities specialize and thus an industry with greater economies of scale need not be weighted down and be forced to compete for inputs with another industry, which exhibits lower economies of scale. However, we show that when cities specialize, the advantage of specialization is exactly offset by the effect of its superior performance on the terms of trade. Different specialized cities can grow in parallel, just as autarkic cities can growth in parallel.

2 A Ventura-type Model of Intercity Trade and Economic Growth

Our model of urban growth builds on the model of trade and growth in Ventura (2005), but introduces cities. A number of distinct cities (subject to congestion, unlike Ventura's sites or regions, or different nations), may produce one or both of two tradeable goods X and Y. Each of these goods are produced using physical capital, raw labor and intermediates. The tradeable goods are not consumed directly. Instead, they are combined in each city to produce aggregates, or composite goods, by means of Cobb-Douglas production functions with shares α, and $1-\alpha$, respectively. The aggregates are in turn used for consumption and for investment in physical capital. We start with the case of autarkic cities and refrain from indexing the respective quantities as long as no confusion arises. This specification modernizes Henderson (1987; 1988) for the purpose of examining urban growth in particular. It differs from Rossi-Hansberg and Wright (2007) because of that paper's reliance on the Lucas model to generate sustained growth. It differs from both the Henderson and Rossi-Hansberg and Wright approaches because of its assumption that tradeable goods are not consumed directly. It also allows comparison between autarky and intercity trade which highlights the importance of intercity trade.

A number of individuals $\bar{N}_t$ are born every period and live for two periods. The economy has a demographic structure of the overlapping generations model. We simplify the behavioral model as much as possible by assuming that the individuals born at time t work when young, consume their net labor income net of their savings, and consume again when they are old, C_{1t}, C_{2t+1}, respectively. While numerous refinements of the model are possible, this basic one allows us to bring to the fore the key tradeoffs associated with growth. We assume Cobb-Douglas preferences over first- and second-period consumption for the typical individual,

$$U_t = S^{-S}(1-S)^{-(1-S)} C_{1t}^{1-S} C_{2t+1}^{S}. \tag{1}$$

Net labor supplied by the members of the young generation in a particular city at t is given by $H_t = N_t \left(1 - \kappa N_t^{\frac{1}{2}}\right)$, with N_t the number of the members of the young generation in a particular city at t, $\kappa \equiv \frac{2}{3}\pi^{-\frac{1}{2}}\kappa'$, and κ' the time cost per unit of distance traveled.[1] Each young individual receives net labor income $\left(1 - \kappa N_t^{\frac{1}{2}}\right) W_t$, where W_t denotes the wage rate. Let R_{t+1} be the total return to physical capital, K_{t+1}, in time period $t+1$, that is held by the member of young generation at time t. To the above utility function there corresponds an indirect utility function:

$$R_{t+1}^{S}\left(1 - \kappa N_t^{\frac{1}{2}}\right) W_t. \tag{3}$$

We assume that capital depreciates fully in one period. Under the above utility function, the young save a fraction S of their net labor income. The productive capital stock in period $t+1$, K_{t+1}, is equal to the total savings of the young at time t. Therefore, to preview our growth model, we have: $K_{t+1} = S\left(1 - \kappa N_t^{\frac{1}{2}}\right) W_t$.

[1] If individuals commute to the CBD, consume a unit of land each, and the city extends circularly around it, a city of N residents has a radius $\pi^{-\frac{1}{2}}N^{\frac{1}{2}}$. If individuals are endowed with one unit of leisure and a unit of distance traveled costs κ' units of time, then an individual who travels distance r to the CBD is left with $1 - \kappa' r$ units of time to work. Therefore, the total labor supply of a city with N residents is:

$$H_c(N) = \int_0^{\pi^{-\frac{1}{2}}N^{\frac{1}{2}}} 2\pi r(1-\kappa' r)dr = N\left(1 - \kappa N^{\frac{1}{2}}\right). \tag{2}$$

We develop first the case where all cities are autarkic, that is no intercity trade, and thus produce both final goods, and use them in turn to produce the composite used for consumption and investment.

Each of the final goods, $J = X, Y$, is produced by a Cobb-Douglas production function, with constant returns to scale, using a composite of raw labor and physical capital, with elasticities $1-\phi_J$, and ϕ_J, respectively, and a composite made of intermediates. The shares of the two composites are $u_J, 1-u_J$ respectively. There exists an industry $J-$specific total factor productivity, Ξ_{Jt}. Production conditions for each of two industries J are specified via their respective total cost functions:

$$B_{Jt}(Q_{Jt}) = \left[\frac{1}{\Xi_{Jt}}\left(\frac{W_t}{1-\phi_J}\right)^{1-\phi_J}\left(\frac{R_t}{\phi_J}\right)^{\phi_J}\right]^{u_J}\left[\sum_m P_{Zt}(m)^{1-\sigma}\right]^{\frac{1-u_J}{1-\sigma}} Q_{Jt}, \qquad (4)$$

where Q_{Jt} is the total output of good $J = X, Y$, P_{Zt} is the price of the typical intermediate, elasticity parameters u_J, ϕ_J satisfy $0 < u_J, \phi_J < 1$, and the elasticity of substitution in the intermediates composite σ is greater than 1. The total factor productivity Ξ_{Jt}, summarizes the effect on industry productivity of geography, institutions and other factors that are exogenous to the analysis.[2]

Each of the varieties of intermediates used by industry J are produced according to a linear production function with fixed costs (which imply increasing returns to scale) using the same composite of physical capital and raw labor that is used in the production of manufactured goods X and Y. The shares of the productive factor inputs used are the same as, u_J and $1-u_J$, respectively.[3] The respective total cost function is

$$b_{it}(Z_{Jt}(m)) = \frac{f + cZ_{Jt}(m)}{\Xi_{Jt}}\left[\left(\frac{W_t}{1-\phi_J}\right)^{1-\phi_J}\left(\frac{R_t}{\phi_J}\right)^{\phi_J}\right],$$

and $Z_{Jt}(m)$, the quantity and price of the input variety m used by industry $J = X, Y$. Its price is determined in the usual way from the monopolistic price setting problem,[4] and it is equal to marginal cost, marked up by $\frac{\sigma}{\sigma-1}$:

$$P_{Z,J,t} = \frac{\sigma}{\sigma-1}\frac{c}{\Xi_{Jt}}\left(\frac{W_t}{1-\phi_J}\right)^{1-\phi_J}\left(\frac{R_t}{\phi_J}\right)^{\phi_J}.$$

At the monopolistically competitive equilibrium with free entry, each of the intermediates is supplied at quantity $(\sigma-1)\frac{f}{c}$, and costs $\frac{\sigma f}{\Xi_{Jt}}\left(\frac{W_t}{1-\phi_J}\right)^{1-\phi_J}\left(\frac{R_t}{\phi_J}\right)^{\phi_J}$ per unit to produce. Its producer earns zero profits.

---

[2]This specification combines Anas and Xiong (2003) and Ventura (2005).

[3]This may be generalized to allow for input-output linkages by requiring (see also Fujita, *et al.* (1999), Ch. 14), that each intermediate good industry use its own composite as an input. This is accomplished by introducing as an additional term $\left[\int_0^{M_{it}} p_{it}^{1-\epsilon_i}\right]$ on the r.h.s. of the cost function $b_{it}(Z_{Jt})$.

[4]Dixit and Stiglitz (1976).

3 Growth with Autarkic Cities

We examine first economic growth in an integrated economy consisting of autarkic cities, where each city produces both manufactured goods. When these are available in quantities Q_{Xt}, Q_{Yt}, the quantity of the composite good that may be used for consumption and investment is given by:

$$Q_t = Q_{Xt}^{\alpha} Q_{Yt}^{1-\alpha}.$$

In order not to clutter up notation, we will index cities only when it is necessary. The natural numeraire to use is, just as in Ventura, the composite output itself in every city. Its price is set equal to 1:

$$P_t \equiv 1 \equiv \left(\frac{P_{Xt}}{\alpha}\right)^{\alpha} \left(\frac{P_{Yt}}{1-\alpha}\right)^{1-\alpha}. \tag{5}$$

Therefore, $P_t Q_t = Q_t \equiv P_{Xt} X_t + P_{Xt} Y_t$, and:

$$Q_t = C_t + K_{t+1}.$$

The final good industries X and Y receive share of aggregate spending $\alpha, 1-\alpha$, respectively, of which fractions $1-\phi_X, 1-\phi_Y$, respectively go to labor. Therefore, aggregate labor income as a function of output is:

$$W_t H_t = W_t N_t \left(1 - \kappa N_t^{\frac{1}{2}}\right) = (1-\phi) Q_t, \tag{6}$$

where

$$\phi \equiv \alpha \phi_X + (1-\alpha)\phi_Y.$$

Clearly, $0 < \phi < 1$.

In order to characterize economic growth by means of a law of motion for industrial capital, we express in terms of capital all endogenous quantities that would enter the law of motion. Working with the production function of each final good industry, the output–capital ratio, when the optimal quantity of each intermediate is used, may be written as:

$$\frac{Q_{Jt}}{K_{Jt}} = \left(\frac{\sigma}{\sigma-1}\right)^{-(1-u_J)} m_{Jt}^{\frac{1-u_J}{\sigma-1}} \Xi_{Jt} \left(\frac{K_{Jt}}{H_{Jt}}\right)^{\phi_J - 1}. \tag{7}$$

The range of intermediates used in the production of good J, m_{Jt}, which is referred to as the technology, is endogenously determined in the model and may be expressed as a function of productive factors used (H_{Jt}, K_{Jt}), as:

$$m_{Jt} = \frac{1-u_J}{f\sigma} \Xi_{Jt} H_{Jt}^{1-\phi_J} K_{Jt}^{\phi_J}.$$

This result may be interpreted as follows. Increased factor use raises incentives to specialize, allows the fixed costs of more varieties to be recouped and thus improves productivity. Equ. (7) now becomes:

$$\frac{Q_{Jt}}{K_{Jt}} = \bar{\Xi}_{Jt} H_{Jt}^{\mu_J(1-\phi_J)} K_{Jt}^{\mu_J\phi_J - 1}. \tag{8}$$

where the auxiliary variable μ_J,

$$\mu_J \equiv 1 + \frac{1-u_J}{\sigma - 1},$$

measures the importance of market size effects, and

$$\bar{\Xi}_{Jt} \equiv \left(\frac{\sigma}{\sigma-1}\right)^{-(1-u_J)} \left(\frac{1-u_J}{\sigma}\right)^{\frac{1-u_J}{\sigma}-1} f^{\mu_J} \Xi_{Jt}^{\mu_J}$$

is an augmented measure of industry$-J$ total factor productivity.

We note that $\mu_J(1-\phi_J) > 0$, the exponent of labor in (8) above, is positive. Intuitively, increase in labor used raises the output–capital ratio, as the direct positive effect of making physical capital more productive is reinforced by the indirect effect of increasing input variety. Increases in physical capital, on the other hand, have an ambiguous effect on the output–capital ratio, as the sign of $\mu_J\phi_J - 1$ is ambiguous. The direct negative effect of making physical capital abundant and the indirect positive effect of increasing input variety work in opposite directions. Depending upon the magnitudes of market size effects, indicated by μ_J, and diminishing returns, ϕ_J, that is, if $\mu_J\phi_J < (>) 1$, increases in physical capital reduce (increase) the industry's output capital ratio.

Next, in order to derive an expression for the aggregate output capital ratio as function of the aggregate quantities of labor and physical capital, we express industry allocations of productive factors, H_{Xt}, H_{Yt} and K_{Xt}, K_{Yt}, respectively, in terms of their respective aggregate quantities, $H_{Xt} + H_{Yt} = H_t$, $K_{Xt} + K_{Yt} = K_t$. That is:

$$H_{Jt} = \alpha_J \frac{1-\phi_J}{1-\phi} H_t;$$

$$K_{Jt} = \alpha_J \frac{\phi_J}{\phi} K_t,$$

where $\alpha_X = \alpha,\ \alpha_Y = 1-\alpha$. By using these expressions in the right hand sides of (7), the equations for the output–capital ratio in each of the final goods industries, we have

$$\frac{Q_t}{K_t} = \Xi_t^* H_t^{\mu(1-\phi)-v} K_t^{\mu\phi+v-1}, \tag{9}$$

where μ is defined as

$$\mu \equiv \alpha\mu_X + (1-\alpha)\mu_Y = 1 + \frac{1-(\alpha u_X + (1-\alpha)u_Y)}{\sigma-1} > 1,$$

and υ is the "covariance" between μ_J, ϕ_J,

$$\upsilon \equiv \alpha(\mu_X - \mu)(\phi_X - \phi) + (1-\alpha)(\mu_Y - \mu)(\phi_Y - \phi) = -\alpha(1-\alpha)\frac{1}{\sigma - 1}(u_X - u_Y)(\phi_X - \phi_Y),$$

and

$$\Xi_t^* \equiv \left[\alpha^{\mu_X}\left(\frac{1-\phi_X}{1-\phi}\right)^{\mu_X(1-\phi_X)}\left(\frac{\phi_X}{\phi}\right)^{\mu_X\phi_X}\bar{\Xi}_{Xt}\right]^{\alpha}\left[(1-\alpha)^{\mu_Y}\left(\frac{1-\phi_Y}{1-\phi}\right)^{\mu_Y(1-\phi_Y)}\left(\frac{\phi_Y}{\phi}\right)^{\mu_Y\phi_Y}\bar{\Xi}_{Yt}\right]^{1-\alpha}.$$

Equ. (9) is the effective aggregate production function, in that it gives the output–capital ratio for the entire economy after the endogeneity of the range of intermediates (technology) has been accounted for. Just as with the individual final goods industries, increases in labor have unambiguously positive effects on the output–capital ratio, because: $\mu(1-\phi) - \upsilon > 0$. Increases in physical capital, on the other hand, have ambiguous effects on the output-capital ratio. If the "representative" industry has strong diminishing returns and weak market-size effects, $\mu\phi + \upsilon < 1$, then increasing physical capital reduces the output–capital ratio. If the "representative" industry has weak diminishing returns and strong market-size effects, $\mu\phi + \upsilon \geq 1$, increasing physical capital increases the output-capital ratio.

Since young individuals save a fraction S of their labor income, which aggregates to $(1-\phi)Q_t$, and given the expression for the effective aggregate function (9), we have the law of motion for the autarkic urban economy:

$$K_{t+1} = S(1-\phi)\Xi_t^* H_t^{\mu(1-\phi)-\upsilon} K_t^{\mu\phi+\upsilon}. \tag{10}$$

This shows that each autarkic urban economy obeys a Solow growth model with a Cobb-Douglas production function that exhibits increasing returns to scale, as the sum of the factor share coefficients satisfies $\mu \geq 1$.

There is a total number of autarkic cities in the economy at time t, $i \in \mathcal{I}_t$. The only modification needed in the above theory is to account for the net quantity of labor available for production, given city geometry, as a function of city population, $N_{i,t}$. That is, $H_{it} = N_{i,t}\left(1 - \kappa_i N_{i,t}^{\frac{1}{2}}\right)$. The law of motion (10) in city i becomes:

$$K_{i,t+1} = (1-\phi)S\Xi_{i,t}^*\left(N_{i,t}\left(1 - \kappa_i N_{i,t}^{\frac{1}{2}}\right)\right)^{\mu(1-\phi)-\upsilon} K_{i,t}^{\mu\phi+\upsilon}. \tag{11}$$

It is straightforward to show that output of city i satisfies a similar equation,

$$Q_{i,t+1} = \hat{\Xi}_{i,t} Q_{i,t}^{\mu\phi+\upsilon},$$

where $\hat{\Xi}_{i,t}$, a function of parameters, is defined as follows:

At long run equilibrium with free entry, city i would be populated by the maximum possible population,[5] which is given by $\frac{4}{9}\kappa_i^{-2}$, and is therefore proportional to κ_i^{-2}. As we shall see further below, the factor multiplying κ_i^{-2}. may differ, depending upon the intercity trade regime. Suppose that each city is populated with the number of young people that maximizes net labor supply and that the young share housing with the old. Suppose, also, that local geography is uniform, so that commuting costs are the same across all urban sites: $\kappa_i = \kappa$. Then, the number of cities is equal to

$$n_t = \frac{9}{4}\kappa^2 \bar{N}_t.$$

The higher the commuting costs or the larger the national population, the more cities are needed. This is in a nutshell the counterpart here of the case of urban growth as analyzed by Henderson and Ioannides (1981). Unlike their case, however, in this model even if national population is constant, sustained growth is possible, provided that the "representative" industry has weak diminishing returns and strong market-size effects.

3.1 Investment in Urban Transportation

It is reasonable to assume that κ, the unit transportation cost parameter that was defined in Section 2, may be affected through investment in physical capital. This assumption is key to our results. We take no position at the moment on whether or not this would be public capital or private capital. We assume that unit transportation cost κ may be reduced by means of investing physical capital in the transportation system. We posit that

$$\kappa_{i,t+1} \equiv \tilde{\kappa}_i \left(N_{it} k_{g,i,t+1}\right)^{-\eta}, \ \eta, \tilde{\kappa} > 0, \tag{12}$$

where $(\tilde{\kappa}_i, \eta)$ denote parameters, and $k_{g,i,t+1}$ physical capital per young person at time t invested in the transportation system (unit transportation capital, for short). Investment undertaken at time t becomes productive in $t+1$. The unit transportation cost tends to zero asymptotically and in a convex fashion, as total transportation capital tends to infinity.

This assumption has important consequences for our model and our results. In particular, the maximum net labor supply and the young population of city i become functions of transportation investment and given by $\frac{1}{3}n_i^* \left(N_{it} k_{g,i,t+1}\right)^{2\eta}$ and $n_i^* \left(N_{it} k_{g,i,t+1}\right)^{2\eta}$, respectively,

[5] Alternatively, we may choose city population so as to maximize indirect utility of the typical individual at each point in time, given by (3). Using (11), we may express $R_{i,t+1}$, the gross return to capital in terms of $N_{i,t}$, as well. In the absence of free labor mobility, setting city populations in this fashion would fit better a utility maximizing local government. Even in this case, however, the resulting optimum is inversely proportional to the unit commuting costs, with the coefficient of proportionality being different, naturally.

where $n_i^* = 4/9\frac{1}{\kappa_i^2}$. Transportation capital, like production capital, is assumed to depreciate fully after one period.

We assume transportation investment takes the form of the same composite that is used for consumption and investment in the production of goods. From among a whole host of possible ways to finance transportation investment, we assume (for simplicity) that the government of city i levies a lump sum tax equal to $k_{g,i,t+1}$ per young individual, which it uses to finance investment in transportation. Therefore, the law of motion for capital (10) becomes:

$$K_{i,t+1} = (1-\phi)SQ_{i,t} - N_{i,t}k_{g,i,t+1}.$$

We assume that the government of city i sets the amount of transportation investment so as to maximize utility of the typical member of generation t, given by (3). At the capital market equilibrium in city i, the gross return to physical capital is equal to marginal product of capital in producing aggregate output, with the external effects on capital being taken as given, that is:

$$R_{i,t+1} = \phi\Xi_t^* H_{i,t+1}^{\mu(1-\phi)-\upsilon} K_{i,t+1}^{\mu\phi+\upsilon-1}.$$

Since net labor supply at time $t+1$ is proportional to $\kappa_{i,t+1}^{-2}$, it may be written in view of (12) as a function of $k_{g,t+1}$. By rewriting the maximand from (3) we have:

$$\left[\phi\Xi_t^* \left[\frac{n_i^*}{3}\left(N_{it}k_{g,i,t+1}\right)^{2\eta}\right]^{\mu(1-\phi)-\upsilon} \left[(1-\phi)SQ_{it} - N_{i,t}k_{g,i,t+1}\right]^{\mu\phi+\upsilon-1}\right]^S \left[(1-\phi)Q_{it} - N_{i,t}k_{g,i,t+1}\right] N_{i,t}^{-1}.$$

Maximization of indirect utility of a young person with respect to $k_{i,g,t+1}$ yields that total investment in transportation, $N_{i,t}k_{g,i,t+1}$, is a constant share of city labor income $(1-\phi)Q_{it}$ and given by [6]:

$$N_{it}k_{g,i,t+1} = \tilde{\eta}(1-\phi)Q_{i,t}, \tag{13}$$

where $\tilde{\eta}$ is a root between 0 and S of the quadratic equation with constant coefficients,

$$(1+S^*+\hat{S})\tilde{\eta}^2 - (S+S^*+\hat{S}+S\hat{S})\tilde{\eta} + S\hat{S} = 0, \tag{14}$$

[6] To see that, taking the log of the maximand above, for simplicity, and then differentiating it with respect to $k_{g,i,t+1}$ and rearranging yields:

$$\hat{S}\frac{1}{k_{g,i,t+1}} - S^*\frac{N_{i,t}}{(1-\phi)SQ_{it} - N_{i,t}k_{g,i,t+1}} - \frac{N_{i,t}}{(1-\phi)Q_{it} - N_{i,t}k_{g,i,t+1}} = 0,$$

where the auxiliary variables $\hat{S}, S^*$ are defined as in the main text. This reduces to a quadratic equation in the unknown quantity $N_{i,t}k_{g,i,t+1}$. It is more convenient to seek a solution, equivalently, for the unknown as $\tilde{\eta} \equiv \frac{N_{i,t}k_{g,i,t+1}}{(1-\phi)Q_{it}}$, transportation investment as a share of labor income. The resulting quadratic equation has time invariant coefficients.

where

$$\hat{S} \equiv 2\eta(\mu(1-\phi)-v)S, \; S^* \equiv S(\mu\phi+v-1),$$

which is defined entirely in terms of parameters. In view of the properties of all the auxiliary parameters we have that the quantity

$$\mu(1-\phi)-v = \frac{1}{\sigma-1}\left[\sigma(1-\alpha\phi_X-(1-\alpha)\phi_Y)+\alpha\phi_X u_X+(1-\alpha)\phi_Y u_Y\right]$$

is positive. Therefore, $\hat{S} > 0$. It can be shown that a root exists of (14) such that $0 < \tilde{\eta} < S$, provided that $S^* > 0$, for which it suffices that the returns to scale are sufficiently strong to ensure that $\mu\phi + v > 1$. Just as before, this can occur if the "representative" industry of the city has weak diminishing returns and strong market size effects.

Therefore, with total investment in transportation being a fraction of total output of city i, the law of motion for productive capital becomes:

$$K_{i,t+1} = (1-\phi)(S-\tilde{\eta})Q_{i,t}. \tag{15}$$

The evolution of city income implied by the law of motion may be characterized as follows. Using the expression for productive capital in period $t+1$ from (15) as a function of city i output at t, and using (13) to express net labor supply also as a function of city i output at t, along with the expression for the output–capital ratio from (9), we obtain the following law of motion for city i output:

$$Q_{i,t+1} = \mathcal{Z}_{i,t+1} Q_{i,t}^{2\eta(\mu(1-\phi)-v)+\mu\phi+v}, \tag{16}$$

where:

$$\mathcal{Z}_{i,t+1} \equiv \Xi^*_{i,t+1}\left(\frac{1}{3}n_i^*\right)^{\mu(1-\phi)-v}\left[\tilde{\eta}(1-\phi)\right]^{2\eta(\mu(1-\phi)-v)}\left[(1-\phi)(S-\tilde{\eta})\right]^{\mu\phi+v}.$$

Clearly, because $\mu(1-\phi)-v$ is positive, a sufficient condition for the economy of city i to exhibit increasing returns to scale is $\mu\phi + v \geq 1$, which is the same condition as for the economy without transportation investment to exhibit increasing returns to scale. A necessary condition may also be obtained. The larger is η, the more effective is transportation capital investment in reducing unit commuting costs, and the more likely it is that the economy of city i exhibit increasing returns to scale.

3.2 Divergent versus Convergent Autarkic Cities

How do city sizes vary over time, when transportation investment is endogenous? A key result of the paper is to characterize exactly the pattern of evolution of city sizes. With total transportation investment evolving according to (13), the optimal number of young at time $t+1$, our definition of *city size*, is

$$\nu_{i,t+1} = n_i^* \left(\tilde{\eta}(1-\phi)Q_{i,t}\right)^{2\eta}, \tag{17}$$

from which we have that:

$$\frac{\nu_{i,t+1}}{\nu_{i,t}} = \left(\frac{Q_{i,t}}{Q_{i,t-1}}\right)^{2\eta}.$$

By using the law of motion for city output derived earlier (16), we may rewrite this in terms of total city output only:

$$\frac{\nu_{i,t+1}}{\nu_{i,t}} = \left(\mathcal{Z}_{i,t}\right)^{2\eta} Q_{i,t-1}^{2\eta(2\eta[\mu(1-\phi)-\upsilon]+\mu\phi+\upsilon-1)}. \tag{18}$$

This renders endogenous the growth rate of city size and provides a direct relationship between the growth rate of city size from time t to period $t+1$ to the growth rate of city output from $t-1$ to t, which in view of (16) may be written in terms of city output in period $t-1$. We note that $\mathcal{Z}_{i,t}$ is larger the smaller is parameter $\tilde{\kappa}_i$ in unit commuting costs. So, a particular advantage of a city's urban transport system, perhaps due to local geography, is translated to a growth effect for the sizes of the respective city type. Relative growth rates depend on respective output ratios. Growth rates of cities are constant, increasing or decreasing, depending upon whether city output is constant, increasing or decreasing.

An important consequence of endogenizing urban transportation costs, and therefore city size as well, is that the number of cities is also endogenous. This implies, in turn, that if (16), the law of motion of city output, exhibits increasing returns to scale, then city sizes will grow. This follows from the fact that increasing returns to scale in city output are ensured if $\mu\phi+\upsilon$ exceeds 1, while $\mu(1-\phi)-\upsilon$ is always positive. This is due to increasing returns to scale in the urban economy, which originate, of course, in the specification of technology through the use of intermediates. Consequently, whether or not the number of cities grows depends on the rate of growth of population relative to the (endogenous) rate of growth of city size. Furthermore, the economy here consists of a number of autarkic, that is independent cities, which implies a great variety of possible outcomes.

Returning to the typology of Eaton and Eckstein (1997) and referring to (16) and (18), we may note the following about urban growth rates in the long run. Parallel growth would be a knife-edge case, where parameter values allow for a steady state, that is constant city output over time. This requires decreasing returns to scale in city output:

$$2\eta(\mu(1-\phi)-\upsilon)+\mu\phi+\upsilon<1,$$

(and constant $\mathcal{Z}_{i,t}$). In that case, urban growth rates would be equal to zero in the long run. If parameter values give constant returns to scale, that is, if the exponent of city i output in (16) is exactly equal to 1, then the growth rate of city output is $\mathcal{Z}_{i,t+1} - 1$, and the corresponding growth rate in the number of cities is $(\mathcal{Z}_{i,t+1})^{2\eta} - 1$. In that case, we would have parallel growth only if the parameters $\mathcal{Z}_{i,t}$ are constant and equal across all cities. In the case of increasing returns to scale, that is if the exponent of city i output in (16) is greater than 1, we have divergent urban growth, as urban growth rates are larger for large cities. We conclude that convergent growth is not possible in the long run in this model. Convergent growth may well be consistent with transient dynamics, however.

3.3 Economic Integration, Urban Specialization, and Growth

Equ. (11) demonstrates that in addition to differences in terms of city-specific total factor productivities, the $\Xi^*_{i,t}$, cities may also differ in terms of congestion parameters κ_i. If individuals are free to move across cities, then a spatial equilibrium requires that individuals be indifferent as to where they locate. That is, individuals' lifetime utilities are equalized across all cities. This implies in turn conditions on intercity wage patterns. That is, unlike the canonical case in Ventura, *op. cit.,* wages will typically differ across cities at spatial equilibrium. Similarly, if capital is perfectly mobile, it will move so as pursue maximum real returns and in the process equalize them across all cities.

We refer to the case where capital and labor are free to move as *economic integration.* With economic integration, industries will locate where industry productivities, the Ξ_{Jt}'s are the most advantageous ones, and capital will seek to locate so as maximize its return. Unlike the consequences of economic integration as examined by Ventura, *op. cit.,* here urban congestion may prevent industry from locating so as to take greatest advantage of locational factors. Aggregate productivity is not necessarily equal to the most favorable possible in the economy, because free entry of cities into the most advantageous locations may be impeded by competing uses of land as alternative urban sites, at the national level. However, utilities enjoyed by city residents at equilibrium do depend on city populations, and therefore, spatial equilibrium implies restrictions on the location of individuals. We simplify the exposition by assuming that all cities have equal unit commuting costs κ.

We take up first resource allocation under the assumption that cities specialize in the production of tradeable goods. Urban specialization and intercity trade are very important phenomena.[7] We examine the case when each specialized city also produces intermediates

[7] Alexandersson (1956) establishes the facts about specialization of U.S. cities and Henderson (1974) provides the first model of the system of cities as trading entities.

that are used in the production of the traded good. Let Q_{Xit}, Q_{Yjt} denote the total quantities of the traded goods X, Y produced by cities i, j, that specialize in their production, respectively. The situation is symmetrical for the two city types, and therefore, we work with a city of type X. We suppress subscripts that are redundant and write for the nominal wage and the gross rate of return in an $X-$city:

$$W_{Xt} = (1-\phi_X)\frac{P_X Q_X}{H_X},\ R_{Xt} = \phi_X \frac{P_X Q_X}{K_X}, \tag{19}$$

where P_X denotes the local price of traded good X, which is expressed in terms of the local price index, the numeraire. We follow Ventura and again adopt the ideal price index as numeraire, which is equal to one in all cities. We also assume initially that there are no intercity shipping costs for traded goods. With economic integration, the gross rate of return is equalized across all city types, that is:

$$R_t = R_{Xt} = R_{Yt}.$$

Spatial equilibrium requires that indirect utility, (3), be equalized across all cities. In view of free capital mobility, spatial equilibrium requires that:

$$R^S_{t+1}\left(1-\kappa N^{\frac{1}{2}}_{Xt}\right) W_{Xt} = R^S_{t+1}\left(1-\kappa N^{\frac{1}{2}}_{Yt}\right) W_{Yt}. \tag{20}$$

Using the expression for net labor supply in each city yields the following condition for spatial equilibrium:

$$\frac{P_X Q_{Xit}}{P_Y Q_{Yit}} = \frac{1-\phi_Y}{1-\phi_X}\frac{N_{Xit}}{N_{Yit}}. \tag{21}$$

Free capital mobility requires that:

$$\frac{P_X Q_{Xit}}{P_Y Q_{Yit}} = \frac{\phi_Y}{\phi_X}\frac{K_{Xit}}{K_{Yit}}. \tag{22}$$

Therefore, the full set of conditions for economic integration imply a relationship between the ratios of capital per young person in the two types of cities:

$$\frac{K_{Xit}}{N_{Xit}} = \frac{\frac{\phi_X}{1-\phi_X}}{\frac{\phi_Y}{1-\phi_Y}}\frac{K_{Yit}}{N_{Yit}}. \tag{23}$$

Alternatively, since the ratio of city populations, $\frac{N_{Xit}}{N_{Yit}}$, is known at any point in time, economic integration then implies that

$$\frac{K_{Xit}}{K_{Yit}} = \frac{N_{Xit}}{N_{Yit}}\frac{\frac{\phi_X}{1-\phi_X}}{\frac{\phi_Y}{1-\phi_Y}}.$$

The demand for good Y by a city of type X is given by the share of income of city of type X divided by P_Y. It suffices to work with the equilibrium conditions for intercity trade in good X, which may be simply stated as the share of spending on good X by all Y cities is equal to the spending on good Y by all X cities:

$$\frac{Q_Y}{Q_X} = \frac{1-\alpha}{\alpha}\frac{n_X}{n_Y}\frac{P_X}{P_Y}. \tag{24}$$

Substituting into the spatial equilibrium condition (21) yields:

$$\frac{n_X N_X}{n_Y N_Y} = \frac{\alpha(1-\phi_X)}{(1-\alpha)(1-\phi_Y)}. \tag{25}$$

This condition, along with the total labor supply condition, $n_X N_X + n_Y N_Y = \bar{N}$, yields the equilibrium number of cities of each type:

$$n_X = \frac{\bar{N}}{N_X}\frac{\alpha(1-\phi_X)}{\alpha(1-\phi_X)+(1-\alpha)(1-\phi_Y)},\ n_Y = \frac{\bar{N}}{N_Y}\frac{(1-\alpha)(1-\phi_Y)}{\alpha(1-\phi_X)+(1-\alpha)(1-\phi_Y)}. \tag{26}$$

Interestingly, the relative frequencies of the two types of cities differ, *cet. par.,* between the static and dynamic cases, only if $\phi_X \neq \phi_Y$, that it, the capital intensity of production differs among the two manufactured goods. The more labor intensive good requires a greater share of population.

We may now solve for capital allocations in the two types of cities by using in addition the condition for the total supply of capital, $n_X K_{Xt} + n_Y K_{Yt} = K_t$. Thus:

$$K_{Xt} = N_{Xt}\frac{K_t}{\bar{N}}\frac{\phi_X}{1-\phi_X}\frac{1-\alpha\phi_X-(1-\alpha)\phi_Y}{\alpha\phi_X+(1-\alpha)\phi_Y}.$$

Next we compute the real income of the two types of cities. For a city of type X this is equal to $P_X Q_X$. From the definition of the numeraire, we have in every city: $P_X = \alpha^\alpha(1-\alpha)^{1-\alpha}\left(\frac{P_X}{P_Y}\right)^{1-\alpha}$. By using (21) to obtain an expression for the terms of trade, the price ratio, we obtain an expression for the real income of a type X city:

$$Q_X\alpha^\alpha(1-\alpha)^{1-\alpha}\left(\frac{P_X}{P_Y}\right)^{1-\alpha} = \alpha_X^* Q_X^\alpha Q_Y^{1-\alpha}\left(\frac{N_{Xit}}{N_{Yit}}\right)^{1-\alpha},$$

where $\alpha_X^* = \alpha^\alpha(1-\alpha)^{1-\alpha}\left(\frac{1-\phi_Y}{1-\phi_X}\right)^{1-\alpha}$. The real income of a city specializing in good X, $\mathcal{X}_t$, may be expressed, by using (8), in terms of city populations of both types of cities, (N_X, N_Y), and total capital in the economy, K_t, and parameters as follows:

$$\mathcal{X}_t = \mathcal{N}_X\left(\frac{K_t}{\bar{N}}\right)^{\alpha\mu_X\phi_X+(1-\alpha)\mu_Y\phi_Y}, \tag{27}$$

where the auxiliary variable $\mathcal{N}_X$ is defined as a function of city sizes and parameters:

$$\mathcal{N}_X \equiv \alpha_X^* \hat{\Xi}_t N_X^{\alpha\mu_X+1-\alpha}\left(1-\kappa N_X^{\frac{1}{2}}\right)^{\alpha\mu_X(1-\phi_X)} N_Y^{(1-\alpha)\mu_Y-(1-\alpha)}\left(1-\kappa N_Y^{\frac{1}{2}}\right)^{(1-\alpha)\mu_Y(1-\phi_Y)}, \quad (28)$$

the function of parameters $\hat{\Xi}_t$ is defined as:

$$\hat{\Xi}_t \equiv \bar{\Xi}_{Xt}^{\alpha}\bar{\Xi}_{Yt}^{1-\alpha}\left(\frac{\phi_X}{1-\phi_X}\right)^{\alpha\mu_X\phi_X}\left(\frac{\phi_Y}{1-\phi_Y}\right)^{(1-\alpha)\mu_Y\phi_Y}\left(\frac{1-\alpha\phi_X-(1-\alpha)\phi_Y}{\alpha\phi_X+(1-\alpha)\phi_Y}\right)^{\alpha\mu_X\phi_X+(1-\alpha)\mu_Y\phi_Y},$$

and the functions of parameters $\bar{\Xi}_{Xt}, \bar{\Xi}_{Yt}$ were defined earlier. The counterpart of (27) for P_YQ_Y, the real income of a city specializing in good Y, is given by:

$$\mathcal{Y}_t = \mathcal{N}_Y\left(\frac{K_t}{\bar{N}}\right)^{\alpha\mu_X\phi_X+(1-\alpha)\mu_Y\phi_Y}, \quad (29)$$

where $\alpha_Y^* = \alpha^\alpha(1-\alpha)^{1-\alpha}\left(\frac{1-\phi_X}{1-\phi_Y}\right)^{\alpha}$,

$$\mathcal{N}_Y \equiv \alpha_Y^* \hat{\Xi}_t N_X^{\alpha\mu_X-\alpha}\left(1-\kappa N_X^{\frac{1}{2}}\right)^{\alpha\mu_X(1-\phi_X)} N_Y^{(1-\alpha)\mu_Y+\alpha}\left(1-\kappa N_Y^{\frac{1}{2}}\right)^{(1-\alpha)\mu_Y(1-\phi_Y)}. \quad (30)$$

3.3.1 Law of Motion for Integrated Economy

We derive the law of motion for total capital in the integrated economy by recalling that savings per person in an $X-$city is given by $SW_X\left(1-\kappa N_X^{\frac{1}{2}}\right)$. This implies that total savings by both types of cities is given by the share of labor income that is saved: $n_XS(1-\phi_X)P_XQ_X$ and $n_YS(1-\phi_Y)P_YQ_Y$. Therefore, the law of motion for capital becomes:

$$K_{t+1} = S\left[n_X(1-\phi_X)\mathcal{N}_{\mathcal{X}} + n_Y(1-\phi_Y)\mathcal{N}_{\mathcal{Y}}\right]\left(\frac{K_t}{\bar{N}}\right)^{\alpha\mu_X\phi_X+(1-\alpha)\mu_Y\phi_Y}. \quad (31)$$

An important result readily follows from a comparison of (31), the law of motion of the integrated economy, with (10), its counterpart for each autarkic urban economy. That is, the elasticity of total savings with respect to capital for those respective cases coincide:

$$\mu\phi + \upsilon \equiv \alpha\mu_X\phi_X + (1-\alpha)\mu_Y\phi_Y.$$

The intuition of this result may be explained as follows. In the integrated economy, cities specialize and thus an industry with greater economies of scale need not be "set back" and be forced to compete for resources with another industry, that exhibits lower economies of scale. However, when a city specializes, the advantage of specialization is exactly offset by the effect of its superior performance on the terms of trade. In fact, it is a telling sign that this is even "mechanically" so in the above derivations. This, of course, follows from the fact that the terms of trade, P_X/P_Y are evaluated at the general equilibrium of the national economy.

With free movement of labor, individuals seek to maximize their lifetime utility and therefore city populations would tend to their optimum sizes. It is easy to see that this is equivalent to maximizing the value of a city's output. The optimum city sizes follow from the maximization of $\mathcal{N}_X$, with respect to N_X, and $\mathcal{N}_Y$, with respect to N_Y, for $X-$ and $Y-$ types of cities, respectively. Again, the optimum values are inversely proportional to unit commuting costs squared:

$$N_X^* = \left(\frac{2(\alpha\mu_X + 1 - \alpha)}{2(\alpha\mu_X + 1 - \alpha) + \alpha\mu_X(1 - \phi_X)} \right)^2 \frac{1}{\kappa^2}, \tag{32}$$

$$N_Y^* = \left(\frac{2((1-\alpha)\mu_Y + \alpha)}{2((1-\alpha)\mu_Y + \alpha) + (1-\alpha)\mu_Y(1 - \phi_Y)} \right)^2 \frac{1}{\kappa^2}. \tag{33}$$

It is important to note that the factors that multiply $\frac{1}{\kappa^2}$ in the expressions above differ from $\frac{4}{9}$, the one under autarky, the alternative trade regime we examined earlier. Accordingly, the relative frequencies of the two types of cities are determined from (25) as functions of parameters. However, a simple comparison shows that specialized cities are larger than autarkic cities. That is, specialization confers an advantage because an increase in market size allows each industry to support a higher degree of specialization and is accommodated by larger city size.

In the canonical case of an integrated economy of many cities, even if goods and capital move at negligible cost, individuals might not be able to move accordingly, because of "capacity constraints" in cities (in effect). Any geographical distribution of production and factors is possible provided that capacity constraints are satisfied. Capacity constraints prompt of course creation of new cities, a process that is implicit in our approach [Henderson and Ioannides (1981)]. In general, sites may differ in terms of efficiency, reflected in the commuting cost parameter. So, unlike the canonical case examined by Ventura, *op. cit.*, here it is indeed possible to determine production or spending located in each city. This is one of the instances where the system of cities approach differs from the standard setup of international trade theory.

It is straightforward, yet algebraically tedious, to show that other things being equal, specialization improves welfare relative to autarky. This follows simply from comparing city output under autarky and under specialization, from (9) and (27), respectively. As with utility comparisons discussed earlier, given the same unit commuting cost, city sizes are assumed to be equal among each type of cities in the specialization case, on one hand, and for all autarkic, on the other. Introduction of iceberg shipping costs, for either final goods or intermediates, does not change these results qualitatively.

A number of remarks are in order. Our treatment of autarky in the growing economy does not assume — in contrast to Anas and Xiong (2003) — intercity trade in intermediates. Here, each city's industries produce the intermediates it uses in its own production. It is still the case with intercity trade and growth, where cities of either type still produce the intermediates they need and do not import any other intermediates from other cities of the same type. This assumption deprives cities of the benefits of a greater variety of intermediates, which over time may grow with the number of cities, but does not affect the returns to scale properties. It is made in order to be able to focus on intercity trade in goods.

3.3.2 Constant Returns to Scale Property at the National Economy

Factor mobility and economic integration alters the economies of scale properties of the model. In particular, we may write real national income as the sum total of real incomes generated in each city, from (27) and (29), respectively, multiplied by the number of cities of each type,

$$\mathcal{Q}_t = n_{Xt}\mathcal{X}_t + n_{Yt}\mathcal{Y}_t. \tag{34}$$

At the optimum city size, the right hand side becomes proportional to

$$(K_t)^{\alpha\mu_X\phi_X+(1-\alpha)\mu_Y\phi_Y}\ \bar{N}^{1-[\alpha\mu_X\phi_X+(1-\alpha)\mu_Y\phi_Y]}.$$

Therefore, the national economy exhibits constant returns to scale with respect to total capital and labor, $(K_t, \bar{N})$. This is, of course, a confirmation in a Dixit-Stiglitz inspired setting, of the result of Rossi-Hansberg and Wright (2007), which is obtained in a Lucas-inspired setting. It is in contrast to Ventura (2005), where integration alters the substitutability among factors, and depends critically on the assumption that there is no congestion for sites that are suitable for urban use, at the national level. The creation of new cities is the margin that eliminates local increasing returns, when they would have been present, and thus confers constant returns to scale at the level of the aggregate economy. This is exactly like the description of industry equilibrium with free entry of firms, each operating with U-shaped average cost curves, may be described as operating with constant returns to scale, with unit cost being equal to the minimum average cost.

Finally, we speculate about possible uses of our results. The availability of the law of motion for the autarkic and for the intercity trade case in closed form, and the fact that they share the same returns to scale properties lend our approach nicely to modeling urban business cycles. With industry-specific stochastic shocks, the behavior of aggregate output is different in the case of autarky versus the case of intercity trade. In addition, without necessarily introducing unemployment, one may study the effects of labor market pooling, when the shocks to the two industries are imperfectly correlated. Such issues have not been examined in the literature. So, in principle, when both types of shocks coexist, one may partially offset the effect on prices thus conferring an advantage to autarky, which then allows for diversification of risks, relative to intercity trade. In this sense, then, the structure of stochastic shocks to industry productivity may give rise to an advantage of autarky versus specialization.

3.4 Investment in Urban Transportation with Specialized Cities

Just as with diversified cities, we retain assumption (12) and consider how investment in urban transportation, financed out of lump-sum taxes in each type of city, may be chosen so as to maximize utility of a typical member of generation at each period in time. Just as in the earlier analysis in (3.1), we assume that city populations are chosen so as to maximize city output, for each type of city. This yields expressions (32 – 33).

By using the expressions for factor prices from (19) and for output-maximizing city sizes from (32) – (33) in the condition for spatial equilibrium across the two types of cities (20), we obtain expressions for utility of young members of generation t who reside in type $-X$ and type $-Y$. At spatial equilibrium, they must be equal. Or:

$$\left[\phi_X \mathcal{N}_{X,t+1}\left(\frac{K_{t+1}}{\bar{N}}\right)^{\tilde{\alpha}-1}\right]^S \left[(1-\phi_X)N_{X,t}^{-1}\mathcal{N}_{X,t}\left(\frac{K_t}{\bar{N}}\right)^{\tilde{\alpha}} - k_{Xg,t+1}\right]$$

$$= \left[\phi_Y \mathcal{N}_{Y,t+1}\left(\frac{K_{t+1}}{\bar{N}}\right)^{\tilde{\alpha}-1}\right]^S \left[(1-\phi_Y)N_{Y,t}^{-1}\mathcal{N}_{Y,t}\left(\frac{K_t}{\bar{N}}\right)^{\tilde{\alpha}} - k_{Yg,t+1}\right], \qquad (35)$$

where $\tilde{\alpha} \equiv \alpha\mu_X\phi_X + (1-\alpha)\mu_Y\phi_Y$, and the expressions for $\mathcal{N}_{X,t}, \mathcal{N}_{X,t+1}; \mathcal{N}_{Y,t}, \mathcal{N}_{Y,t+1}$ are obtained from (28), (30), respectively. Since equalization of the first term on each side above, the gross returns to capital, which ensures capital market equilibrium, has already been employed, it suffices for spatial equilibrium to ensure that wages, net of tax, are equalized:

$$(1-\phi_X)N_{X,t}^{-1}\mathcal{N}_{X,t}\left(\frac{K_t}{\bar{N}}\right)^{\tilde{\alpha}} - k_{Xg,t+1} = (1-\phi_Y)\mathcal{N}_{Y,t}N_{X,t}^{-1}\left(\frac{K_t}{\bar{N}}\right)^{\tilde{\alpha}} - k_{Yg,t+1}.$$

The law of motion, (31), is modified to account for investment in urban transportation:

$$K_{t+1} = S\left[n_X(1-\phi_X)\mathcal{N}_{X,t} + n_Y(1-\phi_Y)\mathcal{N}_{Y,t}\right]\left(\frac{K_t}{\bar{N}}\right)^{\tilde{\alpha}} - n_X N_{X,t}k_{Xg,t+1} - n_Y N_{Y,t}k_{Yg,t+1}. \qquad (31')$$

Maximization of utility of the typical resident in each type of city with respect to $k_{Xg,t+1}, k_{Yg,t+1}$, separately, and subject to the spatial equilibrium constraint yields a solution via one of the roots of a quadratic equation.[8] We may specify parameter restrictions under which the solution is positive and feasible, that is, less than $(1-\phi_X)N_{X,t}^{-1}\mathcal{N}_{X,t}\left(\frac{K_t}{N}\right)^{\tilde{\alpha}}$, $(1-\phi_Y)\mathcal{N}_{Y,t}N_{X,t}^{-1}\left(\frac{K_t}{N}\right)^{\tilde{\alpha}}$, respectively, that is less than the respective wage rates.

Unfortunately, the solution for in the case of specialized cities does not admit a particularly simple expression, unlike in the case of diversified cities. Still, it follows that $k_{Xg,t+1}, k_{Yg,t+1}$, scale with labor income per capita in each type of city. Some qualitative properties readily follow. In view of expressions (32) – (33) both lagged transportation investments $k_{Xg,t}, k_{Yg,t}$, enter via (28), (30) in the solutions for $k_{Xg,t+1}, k_{Yg,t+1}$. This suggests presence of spillovers across the two types of cities in the optimal setting of transportation investment. These spillovers are due, in effect, to the pecuniary externalities associated with use of intermediates in production, which make in turn the terms of trade depend on sizes of both types of cities. This force is not present in the case of autarkic cities and necessitates refinement of the concept of optimum city size in the presence of intercity trade. The solution may also involve complex dynamic dependence, as may be expected from the quadratic solution.

4 Brief Summary and Conclusions

The paper adapts the Anas–Xiong (2003) static model of urban structure and combines it with key features of the model of trade and growth in Ventura (2005) in order to explore intercity trade and growth. The economy is populated by identical individuals and is assumed to have an overlapping generations demographic structure. Individuals work when they are

[8] Specifically, by substituting for $k_{Yg,t+1}$ from the condition that expresses equalization of wages into (31') and then using the resulting expression for K_{t+1} in each of the expressions for utility in (35), we may obtain necessary conditions for utility maximization in the form of quadratic equations. For $k_{Xg,t+1}$ specifically, this equation is:

$$(2S\eta(\alpha\mu_X+1-\alpha)+S(\tilde{a}-1)+1)\,k_{Xg,t+1}^2-\left[(2S\eta(\alpha\mu_X+1-\alpha)+S(\tilde{a}-1))\bar{W}_{Xt}\right.$$
$$\left.+((2S\eta(\alpha\mu_X+1-\alpha)+1)\bar{K}_{t+1}\right]k_{Xg,t+1}+2S\eta(\alpha\mu_X+1-\alpha)\bar{W}_{Xt}\bar{K}_{t+1}=0,$$

where we have ignored some inessential constants and defined the auxiliary variables $\bar{W}_{Xt}, \bar{K}_{t+1}$ as follows: $\bar{W}_{Xt}\equiv(1-\phi_X)N_{X,t}^{-1}\mathcal{N}_{X,t}\left(\frac{K_t}{N}\right)^{\tilde{\alpha}}$,

$$\bar{K}_{t+1}\equiv\bar{N}^{-1}\left[S\frac{\bar{N}}{N_X}n_X(1-\phi_X)\mathcal{N}_{\mathcal{X}}+(1-S)\left(n_Y\frac{N_Y}{N_X}(1-\phi_X)\mathcal{N}_{X,t}-n_Y(1-\phi_Y)\mathcal{N}_{Y,t}\right)\right]\left(\frac{K_t}{\bar{N}}\right)^{\tilde{\alpha}}.$$

The quadratic equation for $k_{Yg,t+1}$ is symmetrical.

young and consume in both periods of their lives. They work where they are born, but may move in the second period of their lives. Cities produce either one or both of two tradeable goods. These goods are used to produce a composite, which is used in turn for final consumption and investment. The manufactured goods are produced using raw labor, physical capital and intermediates. In the autarkic case, each city is self-sufficient in both the manufactured goods and intermediates used in their production. Capital comes from the savings of the young. Our assumptions allow us to obtain a precise characterization of the law of motion, just like in Ventura (2005). So, a combination of weak diminishing returns and strong market size effects can lead to increasing returns to scale in each autarkic city.

We extend the model of autarkic cities to allow for investment by local governments that reduces intra-urban unit commuting costs and thus makes city size depend on equilibrium in the capital market. We are able to solve for transportation investment in closed form. Under appropriate conditions, unceasing growth sustains a divergent pattern in city sizes. Finally, we examine economic growth in the presence of intercity trade in manufactured goods and free factor mobility. Individuals will locate so as to equalize utility and capital so as to equalize returns. In the integrated economy, cities specialize and thus an industry with greater economies of scale need not be weighted down and be forced to compete for resources with another industry, which exhibits lower economies of scale. However, when cities specialize, their specialization advantage is exactly offset by the effect of its superior performance on the terms of trade, under our assumptions. We also extend the model of specialized cities to allow for investment by local governments to reduce commuting costs. This reveals that there are spillovers across city types that must be recognized in setting optimal city size. Different specialized cities grow in parallel, just as autarkic cities can growth in parallel. One of our key results is that the law of motion for capital of the integrated economy has the same dynamic properties as its counterpart for an economy with autarkic cities. The key results of the paper are exploring conditions for divergent versus convergent urban growth under alternative intercity trade regimes.

There are numerous issues which fall within the broader scope of the paper and deserve further attention. The study of the urbanization process along with intercity economic integration, the relationship between different patterns of shipping costs and patterns of specialization, and issues of policy stand out as particularly important. Patterns of specialization among cities map to different predictions about urban hierarchies and associated city size distributions. Another issue of particular importance is modeling proximity between cities and possible competition among alternative sites.

5 References

Alexandersson, G. 1956. *The Industrial Structure of American Cities.* Lincoln, Nebraska: University of Nebraska Press.

Anas, Alex, and Kai Xiong. 2003. "Intercity Trade and the Industrial Diversification of Cities." *Journal of Urban Economics* 54: 258–276.

Berliant, Marcus, and Ping Wang. 2004. "Dynamic Urban Models: Agglomeration and Growth." in *Urban Dynamics and Growth: Advances in Urban Economics,* Pieter Nijkamp and Roberta Capello, eds., New York, NY: Elsevier.

Black, Duncan, and J. Vernon Henderson. 1999. "A Theory of Urban Growth." *Journal of Political Economy* 107(2): 252–284.

Ciccone, Antonio, and Robert E. Hall. 1996. "Productivity and the Density of Economic Activity." *American Economic Review* 86: 54–70.

Dixit, Avinash K., and Joseph E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review* 67(3): 297–308.

Duranton, Gilles, and Diego Puga. 2004. "Micro-Foundations of Urban Agglomeration Economies." In *Handbook of Regional and Urban Economics, Vol. 4,* ed. Henderson, J. Vernon, and Jacques-Francois Thisse, Chapter 48, 2063–2117. Amsterdam: Elsevier.

Eaton, Jonathan, and Zvi Eckstein. 1997. "Cities and Growth: Theory and Evidence from France and Japan," *Regional Science and Urban Economics* 27: 443–474.

Fujita, Masahisa, Paul M. Krugman, and Anthony J. Venables. 1999. *The Spatial Economy: Cities, Regions, and International Trade.* Cambridge, MA: MIT Press.

Henderson, J. Vernon. 1974. "The Size and Types of Cities," *American Economic Review* 64(4): 640–656.

Henderson, J. Vernon. 1987. "Systems of Cities and Inter-City Trade." In *Systems of Cities and Facility Location,* ed. Hansen, Pierre, Martin Labbé, Dominique Peeters, and Jacques-Francois Thisse, and J. Vernon Henderson, 71–119. Chur: Harwood Academic Publishers.

Henderson, J. Vernon. 1988. *Urban Development: Theory, Fact, Illusion.* Oxford: Oxford University Press.

Henderson, J. Vernon, and Yannis M. Ioannides. 1981, "Aspects of Growth in a System of Cities." *Journal of Urban Economics* 10: 117-139.

Henderson, J. Vernon, and Jacques-Francois Thisse. 2004. *Handbook of Regional and Urban Economics, Vol. 4, Cities and Geography.* Amsterdam: Elsevier.

Ioannides, Yannis M. 1994, "Product Differentiation and Economic Growth in a System of Cities." *Regional Science and Urban Economics* 24: 461–484.

Ioannides, Yannis M., and Henry G. Overman. 2004. "Spatial Evolution of the U.S. Urban System." *Journal of Economic Geography* 4, 2, 1–26.

Jacobs, Jane. 1969, *The Economy of Cities.* New York: Random House.

Rossi-Hansberg, Esteban, and Mark L. J. Wright. 2007. "Urban Structure and Growth," *Review of Economic Studies* 74, 597–624.

Ventura, Jaume. 2005. "A Global View of Economic Growth." In *Handbook of Economic Growth,* ed. Aghion, Philippe, and Steven N. Durlauf, Ch. 22, 1419–1497, Amsterdam: Elsevier.