UNIVERSIDAD CARLOS III DE MADRID
working papers

Working Paper 07-49
Economic Series 24
Mayo 2007

Departamento de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34-91) 6249875

The Forward Premium Anomaly: Can Sticky-Price Models Generate Volatile Foreign Exchange Risk Premia?

Seongman Moon *

Universidad Carlos III de Madrid †

Abstract

Fama's (1984) volatility relations show that the risk premium in foreign exchange markets is more volatile than, and is negatively correlated with the expected rate of depreciation. This paper studies these relations from the perspective of goods markets frictions. Using a sticky-price general equilibrium model, we show that near-random walk behaviors of both exchange rates and consumption, in response to monetary shocks, can be derived endogenously. Based on this approach, the paper provides quantitative results that might explain the forward premium anomaly, which is one of the most important puzzles in international finance.

Keywords: foreign exchange risk premium, forward premium anomaly, random walk behaviors, staggered price setting, interest-sensitive money demand, monetary shocks.

JEL Classification: F31, F41.

* I am grateful to Manuel Santos for his invaluable and encouraging guidance. I also would like to thank to Seung Ahn, Kevin Huang, Kevin Reffet, Richard Rogerson, Makoto Watanabe, and seminar participants at Arizona State University, Cornell University, and Universidad Carlos III for helpful comments and discussions.

† Correspondence: Seongman Moon, Department of Economics, Universidad Carlos III, Calle Madrid 126, Getafe (Madrid), 28903 Spain. Email: smoon@eco.uc3m.es

1 Introduction

This paper studies the forward premium anomaly, which refers to the robust empirical finding that the forward exchange rate is not an optimal predictor of the future spot exchange rate.[1] Simply put, the most puzzling fact is the negative correlation between the forward premium and the expected change in the exchange rate. This apparent departure from uncovered interest parity suggests that a low interest rate currency tends to be depreciated rather than appreciated. To explain this puzzle, Fama (1984) shows that if the market expectation on the future spot exchange rate is rational, then the risk premium should be more volatile than, and negatively correlated with the expected exchange rate change. These results on the volatility relations suggest that, in order to explain the forward premium anomaly, one needs a model that is able to generate: (1) a high volatility of both exchange rate changes and marginal rates of substitution; (2) a low volatility of both expected exchange rate changes and interest rates.

The existing approach, based on pioneering work of Lucas (1982), attempts to explain high volatilities of foreign exchange risk premia with risk aversion parameters and/or habit persistence in preferences.[2] However, previous work using this approach does not provide satisfactory quantitative results. In the present paper, we introduce frictions into the standard macro model. The objective of this paper is to examine if sticky-price general equilibrium models, such as those developed in Chari, Kehoe, and McGrattan(2002) (hereafter CKM), can generate the volatility relations described above.

The paper is motivated by two empirical regularities which have not been given sufficient at-

[1] See Hodrick (1987), Lewis (1995), and Engel (1996) for surveys of empirical evidence on the forward premium anomaly.

[2] Examples include: Backus *et.al.* (1993) for habit persistence; Bekaert *et.al.* (1997) for first-order risk aversion; Bekaert (1996) for habit persistence, consumption duration, and transaction costs. See, also, Lewis (1995) and Engel (1996) for a comprehensive survey.

tention so far in the literature. On the one hand, studies initiated by Meese and Rogoff (1983) find that exchange rates follow a near-random walk.[3] On the other hand, Hall (1978) provides well-known empirical evidence that consumption also closely follows a random walk. The key idea of this paper is to tie the random walk behaviors of both exchange rates *and* consumption, which have been studied separately elsewhere, to the issue of the forward premium anomaly.

Using our sticky-price general equilibrium model, we first show that near-random walk behaviors of both exchange rates and consumption, in response to monetary shocks, can be endogenously derived. Based on this approach, we provide quantitative results that might explain the anomaly. Our model produces Fama's volatility relations described above. In addition, it performs well in terms of generating volatilities and autocorrelations of both exchange rates and consumption observed in the data. However, the volatility of the risk premium is still less than that in the data.

There are two crucial features in the model that are necessary to obtain our results: interest-sensitive money demand and staggered price setting.[4] As shown in Engel and West (2005), the nominal exchange rate closely follows a random walk under certain conditions in a class of asset-pricing models.[5] Since our model with *interest-sensitive money demand* satisfies those conditions, a near-random walk behavior of the nominal exchange rate is endogenously derived in response to monetary shocks: the change in the exchange rate is likely to display large variation but the expected exchange rate change is likely to exhibit small variation. When this channel is combined with the assumption of *staggered price setting*, the marginal utility of consumption is determined similarly to asset prices. Here, a significant degree of sluggish price adjustment is necessary to

[3] See Cheung *et.al.* (2002) for a comprehensive survey.

[4] Here, we consider staggered price setting to generate gradual price adjustments. But other features such as staggered wage setting can also be used as long as they induce sluggish price adjustments.

[5] Engel and West (2005) show that asset prices will exhibit a near-random walk behavior in a class of present-value models if (i) fundamentals have a unit autoregressive root and (ii) the discount factor is close to unity. Our model with persistent money growth rates and interest-sensitive money demand satisfies these two conditions.

obtain a near-random walk behavior of the marginal utility of consumption so that the marginal rate of substitution is likely to display large variation but the interest rate is likely to exhibit small variation.

In addition to these features, we find that the risk premium is determined quite differently between an endowment economy and a production economy. For example, studies that introduce habit persistence in consumption into the Lucas model succeed in increasing variation in the marginal utility of consumption. In these stylized frameworks, the marginal rate of substitution depends mainly on the risk aversion and habit persistent parameters since the equilibrium consumption process is exogenously given. However, raising the degree of risk aversion and/or introducing habit persistence do not help to increase the volatility of the marginal utility of consumption in our sticky-price model with production because a rise in risk aversion is offset by a fall in the elasticity of intertemporal substitution. This result is consistent with those in the equity-premium studies with production economies.[6]

Duarte and Stockman (2005) also use a sticky-price model and study how rational speculation behavior of economic agents affects the risk premium. Their study, motivated by Flood and Rose (1995) and Obstfeld and Rogoff (2000b), pays attention to channels that affect the risk premium and the nominal exchange rate without affecting other macroeconomic variables much. Our study, motivated by Engel and West (2005), focuses on the role of expectations about future fundamentals operating through nominal interest rates, based on the asset market approach to exchange rates. Alvarez *et.al.* (2006) present a monetary model in which asset markets are endogenously segmented and show that the risk premium can be time varying even if the distributions of the fundamentals are time invariant. While they investigate the effects of frictions in asset markets on the risk premium,

[6]For example, see, Rouwenhorst (1995).

we study the effects of frictions in goods markets. Engel (1999) and Obstfeld and Rogoff (2003) analytically show that the foreign exchange risk premium can arise endogenously in sticky price models with a synchronized price setting. We extend their analyses to a more general setting that incorporates both interest-sensitive money demand and staggered price setting, and link persistence of both exchange rates and consumption, which is endogenously derived in response to monetary shocks, to the risk premium as well as to Fama's volatility relations.

2 The Model

We use CKM's two-country monetary general equilibrium model, modified by abstracting from capital accumulation but by introducing an input-output production structure in producing intermediate goods, to study if the sticky-price model can generate a high volatility of the foreign exchange risk premium.[7] The presentation of the model is brief since it is directly drawn from CKM and we follow the same notation as theirs as much as possible.

There are two countries in the world, home (H) and foreign (F). The population of monopolistically competitive intermediate goods producers in each country is normalized to 1. Intermediate goods producers set prices in a staggered way following a variant of the Taylor (1980) staggered nominal price contract. Once prices are set, each intermediate goods producer must meet the forthcoming demand at the same prices. Since gradual price adjustment generates persistent real effects of monetary shocks on consumption, one of our objectives is to analyze the effect of staggered price setting on the risk premium. Markets for intermediate goods are segmented across countries so that consumers cannot engage in arbitrage activities. Intermediate goods producers must set prices in consumer's currency in each market (local currency pricing). Under these two assumptions, inter-

[7] Later, we also consider capital accumulation. However, this does not change the results.

mediate goods producers can discriminate prices across countries and thus the law of one price does not hold. We also study the quantitative effects of segmentation of international goods markets on the risk premium. There is a representative household who lives infinitely in each country. We assume that there exist complete nominal bond markets across countries as well as within each country. The model is driven by exogenous shocks to the growth rates of money supply in each country. In the beginning of each period t, one of many finite states, denoted by s_t, is realized. $s^t = (s^{t-1}, s_t)$ denotes the history of states up to time t and $\pi(s^t)$ is the probability, as of period 0, of a history s^t. In what follows, we mainly describe the economy of the home country. Foreign quantities and prices are attached an asterisk superscript.

The representative home household has preference given by the expected infinite life-time utility function

$$\overline{U}_0 = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(C(s^t), \frac{M(s^t)}{P(s^t)}, L(s^t)) \tag{2-1}$$

where C denotes consumption, $\frac{M}{P}$ denotes real money balances, L is labor, β is the discount factor, and U is a concave instantaneous period utility function.

Both home and foreign households can trade state contingent nominal bonds denominated in the home currency. Let $Q(s^{t+1}|s^t)$ denote the nominal price (in home currency units) of one home state contingent bond paying one unit of home currency at s^{t+1} and 0 otherwise. $B(s^{t+1}|s^t)$ denotes the number of home state contingent bonds held by the home household between s^t and s^{t+1}. The home household's budget constraint (in home currency units) is:

$$P(s^t)C(s^t) + M(s^t) + \sum_{s^{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}|s^t) \leq W(s^t)L(s^t) + M(s^{t-1}) + B(s^t) + \Pi(s^t) + T(s^t) \tag{2-2}$$

where M is nominal money balances; Π represents the profit of the home intermediate firms; and T denotes nominal transfer paid from the home government. $B(s^{t+1}|s^t) \geq -P(s^t)\bar{B}$ is a borrowing

constraint. $\bar{B}$ represents a upper bound of real borrowing of the consumer. The initial conditions are given by $M(s^{-1})$ and $B(s^0)$.

Households are assumed to take prices of goods and labor as given. Then, the home household's first order conditions are derived by maximizing its expected utility subject to the budget constraint and the borrowing constraint (the optimal conditions for the foreign representative household can be derived analogously)

$$\frac{U_m(s^t)}{U_c(s^t)} = 1 - \beta \sum_{s^t} \pi(s^{t+1}|s^t)[\frac{U_c(s^{t+1})}{U_c(s^t)} \frac{P(s^t)}{P(s^{t+1})}] \tag{2-3}$$

$$\frac{W(s^t)}{P(s^t)} = \frac{U_l(s^t)}{U_c(s^t)} \tag{2-4}$$

$$\pi(s^t)\frac{U_c(s^t)}{P(s^t)} Q(s^{t+1}|s^t) = \pi(s^{t+1})\beta \frac{U_c(s^{t+1})}{P(s^{t+1})} \tag{2-5}$$

$$\pi(s^t)\frac{U_c^*(s^t)}{P^*(s^t)\epsilon(s^t)} Q(s^{t+1}|s^t) = \pi(s^{t+1})\beta \frac{U_c^*(s^{t+1})}{P^*(s^{t+1})\epsilon(s^{t+1})} \tag{2-6}$$

where $U_c(\cdot)$ denotes the marginal utility of consumption, $U_m(\cdot)$ denotes the marginal utility of real balances, $U_l(\cdot)$ denotes the marginal disutility from work, $\varepsilon(s^t)$ denotes the nominal exchange rate of home currencies per foreign currency, $\pi(s^{t+1}|s^t) = \frac{\pi(s^{t+1})}{\pi(s^t)}$, and $\beta \sum_{s^t} \pi(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{U_c(s^t)} \frac{P(s^t)}{P(s^{t+1})}$ denotes the inverse of the home gross nominal interest rate. Equation (2-3) shows that money demand for the home household is sensitive to the nominal interest rate. As will be discussed in detail later, this interest-sensitive money demand derived from the utility maximization problem is one of the key mechanisms that generate near-random walk behaviors of both exchange rates and consumption. Equations (2-5) and (2-6) are related to home and foreign nominal intertemporal Euler equations expressed in the home currency for each state. The price, $Q(s^{t+1}|s^t)$, of one state contingent home nominal bond should be equal to the marginal rate of substitution in home consumption between s^t and s^{t+1} weighted by the change in purchasing power of the home currency.

Or, it should be equal to the marginal rate of substitution in foreign consumption weighted by the change in purchasing power of the foreign currency once converted into the home currency.

In each period t, the home competitive representative firm produces a final composite good by using home intermediate goods produced in home country and foreign intermediate goods produced in foreign country according to the following technology:

$$Y(s^t) = [a_1^{1-\rho} Y_H(s^t)^\rho + a_2^{1-\rho} Y_F(s^t)^\rho]^{\frac{1}{\rho}} \tag{2-7}$$

where $Y(s^t)$ denotes the home final composite good, $Y_H(s^t) = [\int_0^1 (Y_H(i, s^t))^\theta di]^{\frac{1}{\theta}}$ denotes a composite good of home intermediate goods, $Y_F(s^t) = [\int_0^1 (Y_F(i, s^t))^\theta di]^{\frac{1}{\theta}}$ denotes a composite good of foreign intermediate goods, $Y_H(i, s^t)$ denotes home intermediate goods i, and $Y_F(i, s^t)$ foreign intermediate goods i. $\frac{1}{1-\rho}$ denotes the elasticity of substitution between home and foreign composite goods, $\frac{1}{1-\theta}$ denotes the elasticity of substitution between differentiated intermediate goods within the country, and a_1 denotes a share for the domestic intermediate goods and determines the ratio of imports to output along with ρ and θ. The home final goods producer takes as given prices $P(s^t)$, $P_H(i, s^t)$ for $i \in [0, 1]$, and $P_F(i, s^t)$ for $i \in [0, 1]$, to maximize its profit given by

$$\max_{\{\{Y_H(i,s^t)\}_{i=0}^1, \{Y_F(i,s^t)\}_{i=0}^1\}} P(s^t)Y(s^t) - \int_0^1 P_H(i, s^{t-1})Y_H(i, s^t)di - \int_0^1 P_F(i, s^{t-1})Y_F(i, s^t)di \tag{2-8}$$

subject to (2-7), where $P(s^t)$ is the price of the final goods, $P_H(i, s^{t-1})$ is the price of home intermediate goods i, and $P_F(i, s^{t-1})$ is the price of foreign intermediate goods i at time t. Intermediate goods prices do not depend on s^t because they are set before period t shocks are realized. These prices are denominated in home currency. From this problem, we can derive input demand functions for home and foreign intermediate goods i:

$$Y_H^d(i, s^t) = a_1 (\frac{P_H(i, s^{t-1})}{P_H(s^t)})^{\frac{1}{\theta-1}} (\frac{P_H(s^t)}{P(s^t)})^{\frac{1}{\rho-1}} Y_H(s^t)$$
$$Y_F^d(i, s^t) = a_2 (\frac{P_F(i, s^{t-1})}{P_F(s^t)})^{\frac{1}{\theta-1}} (\frac{P_F(s^t)}{P(s^t)})^{\frac{1}{\rho-1}} Y_F(s^t)$$

where $P_H(s^t) = [\int_0^1 (P_H(i, s^{t-1}))^{\frac{\theta}{\theta-1}} di]^{\frac{\theta-1}{\theta}}$ and $P_F(s^t) = [\int_0^1 (P_F(i, s^{t-1}))^{\frac{\theta}{\theta-1}} di]^{\frac{\theta-1}{\theta}}$. Using zero profit condition from the above profit maximization problem, the price of the final goods is defined by

$$P(s^t) = [a_1 P_H(s^t)^{\frac{\rho}{\rho-1}} + a_2 P_F(s^t)^{\frac{\rho}{\rho-1}}]^{\frac{\rho-1}{\rho}}. \tag{2-9}$$

The home final goods is distributed to the home representative household and to home intermediate goods producers according to

$$Y(s^t) = C(s^t) + \int_0^1 Z(i, s^t) di \tag{2-10}$$

where $Z(i, s^t)$ denotes a final good purchased by intermediate goods producer i.

The home firm that produces intermediate goods i uses the home labor service as well as the final goods according to the following production function:

$$Y_H(i, s^t) + Y_H^*(i, s^t) = F(L(i, s^t), Z(i, s^t)) = L(i, s^t)^{1-\alpha} Z(i, s^t)^{\alpha} \tag{2-11}$$

where $L(i, s^t)$ denotes labor input, $Z(i, s^t)$ denotes the composite intermediate input, α is the cost share for the intermediate input, and $Y_H(i, s^t)$ and $Y_H^*(i, s^t)$ denote the amounts of intermediate goods i used in the production of home and foreign final goods, respectively. Firms producing intermediate goods are assumed to take as given prices of inputs and of other intermediate goods, while they set prices of their own intermediate goods according to a variant of the Taylor staggered price contract. In each period t, a fraction $\frac{1}{N_p}$ of the intermediate firms choose new prices and fix them for N_p periods. Since firms are assumed to set their prices in the consumer's currency and intermediate goods markets are segmented across countries, the home firm i chooses $P_H(i, s^{t-1})$ in the home currency for sales to the home market and $P_H^*(i, s^{t-1})$ in the foreign currency for sales to

the foreign market to maximize its expected profit given by

$$\max_{\{P_H(i,s^{t-1}),P_H^*(i,s^{t-1})\}} \sum_{\tau=t}^{t+N_p-1} \sum_{s^\tau} Q(s^\tau|s^{t-1})\{P_H(i,s^{t-1})Y_H^d(i,s^\tau) + \varepsilon(s^\tau)P_H^*(i,s^{t-1})Y_H^{d*}(i,s^\tau) - MC(i,s^\tau)(Y_H^d(i,s^\tau)+Y_H^{d*}(i,s^\tau))\} \tag{2-12}$$

subject to $P_H(i,s^{t-1}) = P_H(i,s^t) = \cdots = P_H(i,s^{t+N_p-1})$ and $P_H^*(i,s^{t-1}) = P_H^*(i,s^t) = \cdots = P_H^*(i,s^{t+N_p-1})$. $Y_H^d(i,s^t)$ and $Y_H^{d*}(i,s^t)$ denote home and foreign demand for home good i at time t, respectively, and $MC(i,s^t) = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}W^{1-\alpha}(s^t)P^\alpha(s^t)$ denotes marginal cost for home firm i. We obtain $MC(i,s^t)$ from the following cost minimization problem:

$$\min_{\{L(i,s^t),Z(i,s^t)\}} W(s^t)L(i,s^t) + P(s^t)Z(i,s^t)$$
$$s.\ t.\ Y_H(i,s^t) + Y_H^*(i,s^t) = L(i,s^t)^{1-\alpha}Z(i,s^t)^\alpha.$$

All home firms have the same marginal cost since they have the same production function and face the same input prices.

The home government issues the home currency. Money supplies for the home country are assumed to follow a univariate process of the form

$$\frac{M(s^t)}{M(s^{t-1})} = G(s^t) \tag{2-13}$$

where $G(s^t)$ denotes stochastic home money growth rates. We assume that the process of the money supply for each country evolves independently. The home government runs a balanced budget in each period. So, home nominal transfers are given by

$$T(s^t) = M(s^t) - M(s^{t-1}). \tag{2-14}$$

An equilibrium for this economy is a collection of allocations for the home and foreign final goods producers $Y(s^t)$, $Y^*(s^t)$; allocations and prices for the home and foreign intermediate goods

producers $Y_H(i,s^t)$, $Y_H^*(i,s^t)$, $Z(i,s^t)$, $L(i,s^t)$, $P_H(i,s^{t-1})$, $P_H^*(i,s^{t-1})$, $Y_F(i,s^t)$, $Y_F^*(i,s^t)$ $Z^*(i,s^t)$, $L^*(i,s^t)$, $P_F(i,s^{t-1})$, $P_F^*(i,s^{t-1})$ indexed by $i \in [0,1]$; allocations for home and foreign households $C(s^t)$, $L(s^t)$, $M(s^t)$, $B(s^t)$, $C^*(s^t)$, $L^*(s^t)$, $M^*(s^t)$, $B^*(s^t)$; allocations for the home and foreign governments $T(s^t)$, $T^*(s^t)$; prices $P(s^t)$, $W(s^t)$, $P^*(s^t)$, $W^*(s^t)$, $Q(s^{t+1}|s^t)$; the nominal exchange rate $\varepsilon(s^t)$ that satisfy the following conditions:(i) Optimality of final firms' behavior: taking the prices as given, allocations for both home and foreign final goods producers solve their problems; (ii) Optimality of intermediate firms' behavior: taking all prices except its own as given, prices for both home and foreign intermediate goods producers solve their problems; (iii) Optimality of households' behavior: taking prices as given, allocations for both home and foreign households solve their problems; (iv) Government's budget balance: both home and foreign governments run a balanced budget; (v) All markets clear including both final and intermediate goods markets, labor markets, and bonds markets. To make the economy stationary, all nominal variables are normalized by the level of the relevant money supply.

2.1 The Foreign Exchange Risk Premium

In this section, we derive the foreign exchange risk premium driven by home and foreign monetary volatilities. From equations (2-5) and (2-6), we derive the following risk sharing condition under complete asset markets:

$$\frac{\varepsilon(s^{t+1})}{\varepsilon(s^t)} = \frac{Q^*(s^{t+1}|s^t)}{Q(s^{t+1}|s^t)} \tag{2-15}$$

where the exchange rate is linked to foreign and home nominal marginal rates of substitution. This relation holds regardless of frictions in goods markets such as price rigidities and deviations from PPP. Previous studies based on the Lucas-type exchange economy with complete markets also

derive and use this relation for studying the behavior of the risk premium. For example, Backus *et.al.* (1993) investigate how habit persistence affects the foreign exchange risk premium, while setting the joint stochastic process of exchange rate, inflation, and consumption growth from the data. Instead, we let our sticky-price model generate the behaviors of those variables in response to monetary shocks. Using arbitrage condition (covered interest parity), we now define the forward premium by

$$\frac{F_t}{\varepsilon_t} = \frac{E_t[Q^*_{t+1}]}{E_t[Q_{t+1}]} \tag{2-16}$$

where F_t denotes the forward exchange rate, $E_t[\cdot]$ is a mathematical conditional expectation, and $E_t[Q_{t+1}]$ and $E_t[Q^*_{t+1}]$ denote the inverse of the home and foreign nominal interest rates, respectively. For simplicity, we henceforce suppress notation for state.

In order to derive the foreign exchange risk premium, we take second order approximations around a zero money growth steady state, while ignoring terms higher than second order. Then, the second order approximated version of equation (2-16) is

$$\widehat{f}_t - \widehat{e}_t = E_t[\widehat{q}^*_{t+1}] - E_t[\widehat{q}_{t+1}] + \frac{1}{2}(Var_t[\widehat{q}^*_{t+1}] - Var_t[\widehat{q}_{t+1}]) + V1^*_t - V1_t \tag{2-17}$$

where a hat over a small letter denotes the log deviation of the corresponding capital letter except for the nominal exchange rate: $\widehat{e}_t$ is the log deviation of the nominal exchange rate at time t. $V1_t = \frac{1}{2}(E_t[\widehat{q}_{t+1}])^2 - \frac{1}{2}\widehat{a}_t^2$ denotes other terms from the second order approximation on $A_t = E_t[Q_{t+1}]$ where A_t is the inverse of the gross nominal interest rate.[8] This relation apparently looks similar to

[8]The second order approximation on $A_t = E_t[Q_{t+1}]$ can be taken in the following way:

$$\overline{A}(\frac{A_t - \overline{A}}{\overline{A}}) = E_t[\overline{Q}(\frac{Q_{t+1} - \overline{Q}}{\overline{Q}})]$$
$$\widehat{a}_t + \frac{1}{2}\widehat{a}_t^2 = E_t[\widehat{q}_{t+1} + \frac{\widehat{q}^2_{t+1}}{2}].$$

where $\overline{Q}$ and $\overline{A}$ are the zero money growth steady state values of the corresponding variables, respectively, and $\widehat{q}_{t+1} = \log Q_{t+1} - \log \overline{Q}$. The second equation uses $exp(x) = 1 + x + \frac{x^2}{2} + \cdots$ and ignores terms higher than second

the one obtained by assuming that all relevant variables follow log-normal distributions. But our analysis only concerns with deviations from the steady state values.[9] By taking logs on both sides of equation (2-15) and taking conditional expectations given the information at time t, we derive the log deviation of the expected exchange rate change

$$E_t[\widehat{e}_{t+1}] - \widehat{e}_t = E_t[\widehat{q}^*_{t+1}] - E_t[\widehat{q}_{t+1}]. \tag{2-18}$$

By subtracting equation (2-18) from (2-17) we derive the following foreign exchange rate risk premium

$$rp_t = \widehat{f}_t - E_t[\widehat{e}_{t+1}] = \frac{1}{2}(Var_t[\widehat{q}^*_{t+1}] - Var_t[\widehat{q}_{t+1}]) + V1^*_t - V1_t. \tag{2-19}$$

Equation (2-19) shows that foreign exchange rate risks originate from both home and foreign nominal interest rates: the risk premium increases as relative risks of holding foreign bonds become higher. Using the relation in equation (2-15), we can rewrite the relation for the risk premium in the following way

$$rp_t = \widehat{f}_t - E_t[\widehat{e}_{t+1}] = \frac{1}{2}Var_t[\widehat{e}_{t+1}] - Cov_t[\widehat{e}_{t+1}, \widehat{p}_{t+1}] + Cov_t[\widehat{e}_{t+1}, \widehat{u}_c(t+1)] + V1^*_t - V1_t \tag{2-20}$$

where $\frac{1}{2}Var_t[\widehat{e}_{t+1}] - Cov_t[\widehat{p}_{t+1}, \widehat{e}_{t+1}]$ are related to Jensen's inequalities and $Cov_t[\widehat{e}_{t+1}, \widehat{u}_c(t+1)]$ is interpreted as the true risk premium following Engel (1992).[10] Here, we omit time t variables since

order. Then, we have

$$\widehat{a}_t = E_t[\widehat{q}_{t+1}] + \frac{1}{2}Var_t[\widehat{q}_{t+1}] + \frac{1}{2}(E_t[\widehat{q}_{t+1}])^2 - \frac{1}{2}\widehat{a}_t^2.$$

[9] Backus *et.al.* (2000) also show that if all of conditional moments of $\log Q_{t+1}$ exist, $\log E_t[Q_{t+1}]$ can be expanded in the following way: $\log E_t[Q_{t+1}] = E_t[\log Q_{t+1}] + \frac{1}{2!}Var_t[\log Q_{t+1}] + \cdots$. In this paper, we follow a second order approximation approach because Q_{t+1} is endogenously determined from the general equilibrium model. Further, following Schmitt-Grohé and Uribe (2004) and Kim *et.al.* (2005), we use relations from the Taylor first-order approximation to compute second moments of the variables.

[10] One can derive the same condition by taking the second order approximation on the condition in which a forward position must obtain zero expected utility:

$$E_t[\frac{(F_t - \varepsilon_{t+1})}{\varepsilon_t}Q_{t+1}] = 0.$$

their conditional variances are zero at time t. Engel (1992) shows that the true risk premium is zero in an environment with flexible prices unless monetary shocks are correlated with real shocks; the true risk premium, however, arises endogenously in our model with sticky prices because monetary shocks affect both consumption and exchange rate. In the following sections, we show how the volatility of this true risk premium is linked to the persistent real effects of monetary shocks.

3 Calibration and Estimation of the Foreign Exchange Risk Premium

3.1 Calibration

The parameter values for the benchmark model are reported in Table 1. We begin by choosing parameter values for the utility function specified below:

$$U(C, \frac{M}{P}, L) = \frac{1}{1-\sigma}C^{1-\sigma} + \frac{\kappa_1}{1-\phi}(\frac{M}{P})^{1-\phi} - \frac{\kappa_2}{1+\gamma}L^{1+\gamma} \tag{3-1}$$

where σ is risk aversion and γ is the labor supply elasticity.[11] We set γ to 2 so that the intertemporal elasticity of substitution in labor supply is 0.5, which is within the range of estimates in the empirical labor literature. The discount factor β is set so that an annual interest rate is equal to 4 %. Since preferences are separable between consumption and real money balances, both the consumption and interest elasticities of demand for money are tied with the level of risk aversion. First, we set the consumption elasticity of money demand $\frac{\sigma}{\phi} = 1$, following Mankiw and Summers (1986). Next, we set the level of risk aversion σ at 7 to match the relative standard deviation of the median nominal exchange rate change in the sample to the US consumption growth. This value seems relatively high compared to other studies in the literature. However, our numerical results on the

[11] CKM consider a utility function in which consumption and real money balance are not separable in their benchmark model. Our results do not much change between these two utility functions although variation in the risk premium is slightly reduced when the utility function in CKM is used.

volatilities of the risk premium and exchange rates do not depend much on this parameter value. As can be seen in detail later, the effects of monetary shocks on the marginal utility of consumption are almost irrelevant to the value of σ in our sticky-price model. Rather, the curvature parameter σ mainly affects the relative standard deviations of both the marginal rate of substitution and the real exchange rate to consumption, respectively. We set $\phi = 7$, which governs the interest elasticity of money demand, as a consequence of parameterization on $\frac{\sigma}{\phi}$ and σ. κ_1 is set so that the steady state velocity of money is 1.

We now consider the intermediate goods technology parameters. The cost share α of the composite intermediate goods in the production function (2-11) is calibrated in the following way. We first obtain the steady state ratio of intermediate goods to output

$$\frac{\overline{Z}}{\overline{Y}} = \alpha\theta \tag{3-2}$$

by combining the market clearing conditions for intermediate goods with the optimal condition $\frac{PZ}{WL} = \frac{\alpha}{1-\alpha}$ obtained from the cost minimization problems of intermediate goods producers. We then set $\theta = 0.9$ so that an annual markup is 11 %. This value is the same as the one used in CKM and Huang *et.al.* (2004) and less than those in Christiano *et.al.* (2005) and Bergin and Feenstra (2001) who set the markup to 46 and 50 %, respectively. We finally set $\alpha = 0.7$, following Huang *et.al.* (2004) who find the ratio of intermediate input to the industrial production in the US manufacturing sector is 68 %. We set $N_p = 4$ so that prices are set for one year.

For the final goods technology parameters, we first set ρ so that the elasticity of substitution across countries is 1.5, following Backus *et.al.* (1994). Estimates of this elasticity vary a lot across studies. In general, studies in the RBC literature consider estimates ranging between 1 and 2 as reliable.[12] On the other hand, studies in the international trade literature report higher estimates

[12] e.g., Backus *et.al.* (1994), CKM, and Stockman and Tesar(1995).

ranging between 5 and 10, using micro (sectoral) data.[13] However, our results remain unchanged with respect to changes in this parameter value.[14] We now relate the home bias in the final goods production function to the share of imports and use a US import share of 0.15 to obtain values for a_1 and a_2. Since a_1 and N_p affect the degree of both international and domestic price adjustments and thus the risk premium, we report sensitivity analysis with respect to changes in these parameter values.

The stochastic process for money growth in the home country is given by

$$\log G_t = g_t = (1 - \rho_m)E[g] + \rho_m g_{t-1} + \xi_t \tag{3-3}$$

where ξ_t is a home stochastic disturbance term and $E[g]$ denotes the unconditional mean of home money growth rates. Based on our estimation, we consider that the conditional variances of home money growth rates are time varying and follow a univariate GARCH (1,1) process:

$$h_{t+1} = var(\xi)(1 - \rho_h - \rho_u) + \rho_h h_t + \rho_\xi \xi_t^2 \tag{3-4}$$

where ρ_h denotes the persistent coefficient of conditional variance shocks, ρ_ξ denotes the kurtosis coefficient, $var(\xi)$ denotes the unconditional variance of stochastic disturbances, h_{t+1} denotes the conditional variance of home monetary shocks at time t, and $\rho_h + \rho_\xi < 1$. Then, $\xi_t = \sqrt{h_t}N(0,1)$, where $N(0,1)$ is a random number drawn from the normal distribution with mean zero and variance 1. We assume that the stochastic process for money in the foreign country is the same and the cross correlation between ξ_t and ξ_t^* is zero.

As reported in Table 2, the quarterly growth rates in M1 in the US contain strong ARCH components that support our specification for the process of time-varying conditional variances of

[13] e.g., Harrigan (1993), Baier and Bergstrand (2001), and Obstfeld and Rogoff(2000b).

[14] We conduct sensitivity analysis of the results by varying values of the key parameters in the benchmark model and find that our results hold true for a reasonable range of parameter values. We report some of these results in the text. Others are available upon request.

money growth rates. For residual series in the regression of the form (3-3), we apply for ARCH LM tests for conditional homoskedasticity and reject the null hypothesis. This result is consistent with Hodrick (1989) for the monthly growth rates in M1 in the US, Canova and Marrinan (1993) for the monthly and quarterly growth rates in M1 in the US, and Bekeart (1996) for the weekly growth rates in M1 in the US. Parameter values in the AR(1)-GARCH (1,1) model of the forms (3-3) and (3-4) are jointly estimated using quarterly US data for M1 between the second quarter of 1973 and the third quarter of 2003, obtained from the Board of Governors of the Federal Reserve System Database: $\rho_m = 0.66$, $\rho_h = 0.54$, and $\rho_\xi = 0.37$. The unconditional variance of stochastic disturbances are set at 0.015^2 for all experiments.

3.2 Estimation of the Foreign Exchange Risk Premium

To estimate expected returns from currency speculation, we run the OLS regression of the form

$$f_t - e_{t+1} = b_1 + b_2(f_t - e_t) + v_{t+1} \tag{3-5}$$

following Cumby (1988), Backus *et.al.* (1993), and Canova and Marrinan (1993). e_t denotes the log of the price of foreign currency in terms of home currency at quarter t, f_t denotes the log of quarter t home currency price of a one-quarter forward contract specifying delivery of one unit of foreign currency at quarter $t+1$, and v_{t+1} denotes an error term.[15] We assume that the US is the home country. If expectations are rational and uncovered interest parity holds, then both b_1 and b_2 should be zero. Non-zero estimates of b_2 imply that the forward premium $f_t - e_t$ may contain predictable components. We measure fitted values from regression (3-5) by the foreign exchange

[15] We also consider the following form of the regression:

$$\frac{F_t - \varepsilon_{t+1}}{\varepsilon_t} = b_1 + b_2(\frac{F_t - \varepsilon_t}{\varepsilon_t}) + V_{t+1}.$$

But we find that the results are almost the same in both cases.

rate risk premium.

Data consists of quarterly spot and forward rates for the US dollar price of the Japanese yen, the British pound, the French franc, the Italian lira, and the German mark obtained from Data Resources Incorporated (DRI). The series for non-EU currencies run from the second quarter of 1973 to the third quarter of 2003 while the series for the Italian lira and the German mark end in the fourth quarter of 2001. The series for the French franc run from the first quarter of 1980 to the fourth quarter of 2001.

As reported in Panel A in Table 3, we find that the estimated slope coefficients are strictly positive but the French franc and the Italian lila are not statistically significant: estimates of the slope coefficient range from 0.89 for the French franc to 1.84 for the British pound. Previous empirical studies using monthly or weekly series have consistently documented non-zero estimates of the slope coefficient b_2 for a variety of currencies. Although magnitudes of the estimated slope coefficient are less than those in the previous studies, our results using quarterly series are consistent with them. We report properties of expected returns, interpreted as the foreign exchange risk premium in our study, in Panel B in Table 3. Expected returns are volatile and persistent: the standard deviations of these returns range from 0.008 for the French frac to 0.020 for the Japanese Yen and the autocorrelations range from 0.69 for the French franc to 0.91 for the German mark.

To derive Fama's volatility relations, we decompose the estimated slope coefficient into two parts:

$$\widehat{b}_2 = \frac{\widehat{Cov}[f_t - e_t, f_t - e_{t+1}]}{\widehat{Var}[f_t - e_t]} = \widehat{b}_2^{rp} + \widehat{b}_2^{ss} \tag{3-6}$$

where $\widehat{b}_2^{rp} = \frac{\widehat{Cov}[f_t - E_t[e_{t+1}], E_t[e_{t+1} - e_t]] + \widehat{Var}[f_t - E_t[e_{t+1}]]}{\widehat{Var}[f_t - e_t]}$ and $\widehat{b}_2^{ss} = \frac{\widehat{Cov}[f_t - e_t, E_t[e_{t+1}] - e_{t+1}]}{\widehat{Var}[f_t - e_t]}$.[16] $\widehat{b}_2^{rp}$ is

[16]For defining $\widehat{b}_2$, we omit the sample averages of the forward premium and the exchange rate change. This does not change the results because they are very small.

mainly determined by the time-varying risk premium and $\widehat{b}_2^{ss}$ is related to expectation errors. By assuming that $\widehat{b}_2^{ss} = 0$, expectations are rational, and the estimate is consistent, we can derive the following two necessary conditions for obtaining non zero values of the estimated slope coefficient from regression (3-5):

$$Cov(E_t[e_{t+1}] - e_t, rp_t) < 0$$

$$Var(rp_t) > Var(E_t[e_{t+1} - e_t]). \tag{3-7}$$

We call these two conditions Fama's volatility relations and ask whether or not the benchmark model can generate these relations. The implication of the negative correlation between the expected rate of depreciation and the risk premium can be easily seen from excess return on foreign currency, $e_{t+1} - f_t$, which is obtained by selling home currency in the forward market for foreign currency and by using that foreign currency to buy home currency at future spot rate. As the expected depreciation of the dollar becomes higher, that is, the conditional expectation of the future spot exchange rate becomes higher, the higher expected excess return should be required. And thus the risk premium should be negatively related to the expected depreciation.[17]

4 Results

The main question we ask in this paper is whether or not our sticky-price model can produce enough variation in the risk premium to explain the forward premium anomaly. In particular, we are interested if the model can generate Fama's volatility relations. The numerical results in the benchmark economy as well as in other economies are reported in Table 4. The statistics in this table are averages of moments across 1000 simulations with a sample length of 120 periods each.

[17] see, also, Hodrick and Srivastava(1986).

The column labeled with 'Bench' represents the benchmark economy.

The main findings in the benchmark model are: (a) The variance of the risk premium is greater than (but close to) that of the expected rate of depreciation. The variance of the true risk premium is 0.25E-4 while that of the expected depreciation is 0.14E-4. The variances of predictable returns from currency speculation, interpreted as the risk premium, are 0.62E-4 for the French frac, which is the smallest value, 1.12E-4 for the German mark, which is the median value, and 4.08E-4 for the Japanese yen, which is the largest value in the sample. (b) The covariance of the risk premium with the expected rate of depreciation is negative. The cross correlation between these two quantities is -0.74. (c) The correlation between the forward premium and the risk premium is positive but close to zero. The cross correlation between the two quantities is 0.07. (d) The autocorrelation of the risk premium is in the range of our sample, whereas the forward premium is less persistent than those in the data. The autocorrelation of the risk premium is 0.78 in the benchmark model, whereas it is 0.68 for the Italian lira, which is the lowest value, and 0.91 for the German mark, which is the highest value. The autocorrelation of the forward premium is 0.33 in the benchmark model, whereas they are 0.73 for the Italian lira and 0.89 for the German mark, respectively. (e) The benchmark model produces volatilities and autocorrelations of both exchange rates and consumption matched with the data: for example, the standard deviations of both the nominal and real exchange rate changes are 0.061 and 0.067, respectively, while the corresponding sample median values are 0.061 and 0.062. Further, the standard deviation of consumption growth is 0.007, which is the same as that in the US consumption growth. We will discuss the autocorrelations of exchange rates and consumption below.

The main mechanisms for obtaining results (a) and (b) are interest-sensitive money demand and staggered price setting. To begin with studying the role of interest-sensitive money demand in the

determination of the risk premium, we first substitute the home money market clearing condition into equation (2-3) and then take second order approximations:

$$\widehat{p}_t = \frac{\phi\bar{i}}{1+\phi\bar{i}}\widehat{m}_t + \frac{(1+\bar{i})}{1+\phi\bar{i}}\widehat{uc}_t - \frac{1}{1+\phi\bar{i}}E_t[\widehat{uc}_{t+1}] + \frac{1}{1+\phi\bar{i}}E_t[\widehat{p}_{t+1}] + \frac{1}{1+\phi\bar{i}}(\frac{1}{2}Var_t[\widehat{q}_{t+1}] + V1_t + V3_t) \tag{4-1}$$

where $\widehat{p}_t$ denotes the log-linearized home aggregate price index, $\widehat{m}_t$ denotes the log-linearized home money supply, $\widehat{uc}_t$ denotes the log-linearized home marginal utility of consumption, $\bar{i}$ denotes a steady state interest rate, $Var_t[\widehat{q}_{t+1}] + V1_t$ denotes home nominal interest rate risks, and $V3_t = \frac{1}{2}[\bar{i}(-\phi(\widehat{m}_t - \widehat{p}_t) - \widehat{uc}_t)^2]$ is a collection of second order terms derived from second order approximations on the home money market clearing condition.[18] The nominal exchange rate can be derived by using both the home and foreign money market clearing conditions and the risk sharing condition (2-15) from the bonds markets:

$$\widehat{e}_t = \frac{1}{1+\phi\bar{i}}[E_t[\widehat{e}_{t+1}] + \phi\bar{i}\widehat{m}_t^d + (\phi-1)\bar{i}\widehat{re}_t + rp_t - V3_t^d] \tag{4-2}$$

where $\widehat{m}_t^d = \widehat{m}_t - \widehat{m}_t^*$, $V3_t^d = V3_t - V3_t^*$, and $\widehat{re}_t$ denotes the log-linearized real exchange rate. Equation (4-2) shows that the nominal exchange rate is determined in a present value model like as asset prices: the exchange rate is a present discounted sum of expected future fundamentals. Using the solution to equation (4-2), we obtain a relation for the nominal exchange rate change:

$$\begin{aligned} \widehat{e}_{t+1} - \widehat{e}_t &= \frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m}\xi_{t+1}^d + \frac{\rho_m\phi\bar{i}}{1+\phi\bar{i}-\rho_m}g_t^d \\ &+ \frac{\phi\bar{i}}{1+\phi\bar{i}}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s\frac{E_{t+1}[rp_{t+1+s} - V3_{t+1+s}^d] - E_t[rp_{t+s} - V3_{t+s}^d]}{\phi\bar{i}} \end{aligned} \tag{4-3}$$

where $\xi_{t+1}^d = \xi_{t+1} - \xi_{t+1}^*$ and $g_t^d = g_t - g_t^*$. We assume PPP holds for expositional simplicity. One distinct feature of equation (4-3) is that ξ_{t+1}^d dominates the effects of other terms on the

[18] We do not explain the economic interpretations of the second order terms except for the risk premium since our primary concern is the behavior of the risk premium. See, Obstfeld and Rogoff (2000a, 2002) for the economic interpretations of those second order terms.

exchange rate change at $t+1$, which implies that the nominal exchange rate closely follows a random walk.[19] Here, the elastic money demand function derived from a utility maximizing framework in which money enters the utility function plays a key role in amplifying the effects of stochastic disturbances on the nominal exchange rate. As ϕ goes infinity, interest elasticity of money demand becomes zero so that the interest rate effects on the nominal exchange rate would vanish and the nominal exchange rate would no longer follow a near-random walk. This is notable in the sense that only ξ^d_{t+1} terms matter for the determination of the risk premium. And it is one of the reasons why our model with interest-sensitive money demand can generate more volatile risk premia and much less variable expected exchange rate changes than previous studies that have the quantity equation with a unitary income velocity of money. Because of the same reason just mentioned, the model also generates persistence of the exchange rate change closely matched with the data: the autocorrelation of the exchange rate depreciation is -0.00 in the benchmark model, while it ranges from 0.027 for the German mark to 0.159 for the Italian lira in our sample.

Consequently, we find that variation in $\frac{1}{2}Var_t[\widehat{e}_{t+1}-\widehat{e}_t]$ is large: the variance of $\frac{1}{2}Var_t[\widehat{e}_{t+1}-\widehat{e}_t]$ is 0.12E-04, which is about half the variance of the true risk premium. Further, the unconditional mean of $Var_t[\widehat{e}_{t+1}-\widehat{e}_t]$ is close to the unconditional variance of $\widehat{e}_{t+1}-\widehat{e}_t$. This is natural because the exchange rate is highly volatile and follows a near-random walk. But variation in $Cov_t[\widehat{p}_{t+1},\widehat{e}_{t+1}]$ is zero because prices do not respond to current monetary shocks in the benchmark model. Our results are consistent with previous studies about Jensen's equalities since those studies mainly focus on the behavior of $Cov_t[\widehat{p}_{t+1},\widehat{e}_{t+1}]$.[20]

To discuss the role of staggered price setting in the determination of the risk premium, we solve

[19] We obtain this result since money growth rates are persistent and the discount factor $\frac{1}{1+\phi\bar{i}}$ is close to one in our present value model. See, Engel and West (2005, pp. 489-451) for the sufficient conditions that generate the results.

[20] For example, see Frankel and Razin (1980), Engel (1984), Cumby (1988), and Backus *et.al.* (1993).

equation (4-1) forward for p_t and rearrange it for the home marginal utility of consumption:

$$\widehat{uc}_t = \frac{1+\phi\bar{i}}{1+\bar{i}}\widehat{p}_t + \frac{(\phi-1)\bar{i}}{1+\bar{i}}\sum_{s=1}^{\infty}(\frac{1}{1+\phi\bar{i}})^s E_t[\widehat{uc}_{t+s}]$$

$$-\frac{\phi\bar{i}}{1+\bar{i}}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s E_t[m_{t+s}] - \frac{1}{1+\bar{i}}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s E_t[\frac{1}{2}Var_{t+s}[\widehat{q}_{t+1+s}] + V1_{t+s} + V3_{t+s}]. \quad (4\text{-}4)$$

Here, consumption must respond to current monetary shocks to clear the money market because the aggregate price index does not change with respect to them as well as changes in the nominal exchange rate.[21] As a result, sticky prices together with interest-sensitive money demand induce the marginal utility of consumption to be apparently determined in a similar way as the nominal exchange rate: the marginal utility of consumption is mainly driven by the discounted sum of current and expected future money supplies. However, in contrast to the determination of the nominal exchange rate, the effects of monetary shocks are also significantly affected by the degree of the price adjustment, which are summarized in the discounted sum of expected future marginal utilities of consumption and current price. For example, when $N_p = 1$, all firms set their prices at the beginning of each period before monetary shocks are realized and fix them only one period. In this case, consumption does not exhibit any persistence even if monetary shocks are persistent because future prices are fully adjusted right after the realization of monetary shocks. On the other hand, if prices are fixed for a certain periods in a staggered way, households would consider the effects of current monetary shocks on their future consumption since some fraction of the firms will not change their future prices in response to them. Therefore, the marginal utility of consumption becomes more volatile as price adjustments become slower.

To see this more precisely, we calculate the marginal rate of substitution in the two extreme cases: the marginal utility of consumption does not exhibit any persistence in the first case and

[21] This transmission mechanism would disappear if prices are flexible because they will immediately adjust in response to monetary shocks. Further, this mechanism will be weakened in the model where PPP holds because the pass-through from nominal exchange rate movements to import prices is significant.

follows a random walk in the second case. In the first case, the marginal rate of substitution is

$$\widehat{uc}_{t+1} - \widehat{uc}_t = \frac{1+\phi\bar{i}}{1+\bar{i}}(\widehat{p}_{t+1} - \widehat{p}_t) - \frac{1+\phi\bar{i}}{1+\bar{i}}(\frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m}\xi_{t+1} + \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m}g_t) + V4_{t+1} - V4_t \tag{4-5}$$

where $V4_t = \frac{1}{1+\bar{i}}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s E_t[\frac{1}{2}Var_{t+s}[\widehat{q}_{t+1+s}] + V1_{t+s} + V3_{t+s}]$ in equation (4-4). In the second case, the marginal rate of substitution is

$$\widehat{uc}_{t+1} - \widehat{uc}_t = \phi(\widehat{p}_{t+1} - \widehat{p}_t) - \phi(\frac{1+\phi\bar{i}}{1+\phi\bar{i}-\rho_m}\xi_{t+1} + \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m}g_t) + V5_{t+1} - V5_t \tag{4-6}$$

where $V5_t = \frac{\phi}{1+\phi\bar{i}}\sum_{s=0}^{\infty}(\frac{1}{1+\phi\bar{i}})^s E_t[\frac{1}{2}Var_{t+s}[\widehat{q}_{t+1+s}] + V1_{t+s} + V3_{t+s}]$.[22] As can be seen in equations (4-5) and (4-6), the marginal rate of substitution can be largely amplified when consumption follows a random walk: the effect of ξ_{t+1} on the marginal rate of substitution is about ϕ times greater than when consumption exhibits no persistence.[23] This example shows that a mechanism that increases persistent real effects of monetary shocks on consumption can also play a significant role in increasing the volatility of the risk premium. In our numerical experiments, when prices are fixed for one year in a staggered way, consumption exhibits a near-random walk: the autocorrelation of consumption growth rates in the benchmark model is -0.01, whereas it is -0.48 in the model with $N_p = 1$.[24] Consequently, variation in the nominal interest rate significantly decreases with respect to the length of contract periods: the variance of the forward premium in the model with $N_p = 1$ is 0.12E-4, whereas it is 0.02E-4 in the benchmark model. The volatility of the marginal rate of substitution, on the other hand, increases with respect to the contract periods: the variance of the true risk premium in the model with $N_p = 1$ is 0.16E-4, while it is 0.25E-4 in the benchmark model. Although staggered price setting increases variation in the risk premium, its quantitative

[22] If consumption follows a random walk, $(\widehat{p}_{t+1} - \widehat{p}_t)$ should be equal to $\frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m}g_t$ once second order terms are ignored.

[23] The analogous number used in CKM is about 2.5. And their estimate of interest elasticity of money demand is similar to that of Mankiw and Summers (1986) and smaller than that of Stock and Watson (1993).

[24] The autocorrelation of the US consumption growth is 0.23.

effects are not so impressive. This may be because staggered price setting, alone, may not be able to generate large persistence in consumption as in CKM (2000) and Christiano, *it.al.*. Hence, in the next section, we consider nominal and real features that prove to lead to longer periods of endogenous price stickiness and thus persistence in real variables in response to monetary shocks.

We now discuss how the benchmark model is likely to produce the negative correlation between the expected depreciation and the risk premium. By taking conditional expectation on equation (4-3), we derive the expected rate of depreciation

$$E_t[\widehat{e}_{t+1}] - \widehat{e}_t = \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m} g_t^d - \frac{1-\rho_c}{1+\phi\bar{i}-\rho_c} rp_t - \frac{1}{1+\phi\bar{i}} \sum_{s=0}^{\infty} (\frac{1}{1+\phi\bar{i}})^s E_t[V3_{t+1+s}^d - V3_{t+s}^d]. \quad (4\text{-}7)$$

where $g_t^d = g_t - g_t^*$ and $\rho_c = \rho_h + \rho_\xi$. For deriving this relation, we use the condition that the risk premium is a function of time-varying conditional variances of home and foreign money growth rates and assume that the real exchange rate is zero for simplicity. Equation (4-7) illustrates the negative relation between the expected depreciation and the risk premium, holding other things constant. Using $\widehat{f}_t - \widehat{e}_t = rp_t + E_t[\widehat{e}_{t+1}] - \widehat{e}_t$, the forward premium can be derived:

$$\widehat{f}_t - \widehat{e}_t = \frac{\phi\bar{i}\rho_m}{1+\phi\bar{i}-\rho_m} g_t^d + \frac{\phi\bar{i}}{1+\phi\bar{i}-\rho_c} rp_t - \frac{1}{1+\phi\bar{i}} \sum_{s=0}^{\infty} (\frac{1}{1+\phi\bar{i}})^s E_t[V3_{t+1+s}^d - V3_{t+s}^d]. \quad (4\text{-}8)$$

Note that equations (4-7) and (4-8) would not be equal due to the presence of the risk premium. Hence, the forward premium anomaly may be reconciled with uncovered interest parity as long as the risk premium is highly volatile as Fama suggests. We obtain this result because exchange rate risks in the nominal exchange rate are transmitted from the home and foreign nominal interest rates via the intertemporal link of interest-sensitive money demand as shown in equation (4-3). This link is absent in a simple cash-in-advance-constraint model in which money demand is independent of interest rates.

To study how much real exchange rate risks affect the risk premium, we compare the benchmark

economy to an economy in which PPP holds. For this, we modify the assumptions of currency pricing and home bias in the final goods production function in the benchmark economy. When prices are preset in the consumer's currency, the law of one price does not hold because there is no pass-through of the exchange rate to import prices. Hence, home monetary shocks mostly affect the home marginal utility of consumption even in the presence of complete asset markets.[25] On the other hand, when intermediate goods prices are set by producers' currency, import prices completely absorb changes in the nominal exchange rate. That is, the relative price between home and foreign goods fluctuates even if prices are unchanged in terms of domestic currencies. As a result, each country's aggregate consumption is internationally diversified. Our experiments show that real exchange risks significantly increase variation in the risk premium. The column labeled with 'PPP' reports statistics from the economy in which prices are set in producer's currency and $a_1 = 0.5$: the variance of the true risk premium is 0.09E-4, which is about twice less than that of the true risk premium in the benchmark economy.

There are two elements in the benchmark model that cause deviations from PPP: one is the segmentation of international goods markets combined with local currency pricing and the other is home bias in the final goods production function. We conduct some experiments to see which of these two elements more significantly affect the volatility of the risk premium. First, we modify the degree of home bias in the final goods production by setting $a_1 = 0.5$, while keeping the assumptions of segmentation of international goods markets and local currency pricing. The results in this economy are very similar to those in the benchmark economy: the variances of the true risk premium and the expected exchange rate change are 0.23E-4 and 0.12E-4, respectively. Second, we modify the assumption of currency pricing in the benchmark model so that the law of one

[25] This may be one of the reasons that both complete and incomplete asset markets have very similar results in CKM.

price holds, while keeping the assumption of home bias in the final goods production function. The results are reported in the column labeled with 'PCPH' in Table 4. The modified model also produces similar results as the benchmark model. However, the results are sensitive to the degree of openness. The variance of the true risk premium is 0.16E-4 when $a_1 = 0.85$, which is a bit lower than that in the benchmark model. But the variance of the true risk premium is 0.09E-4 when $a_1 = 0.5$.

5 Further Analysis

In the benchmark model, we link persistence of the marginal utility of consumption to the volatility of the risk premium. For example, staggered pricing setting increases variation in the marginal rate of substitution because of gradual price adjustments. However, its quantitative effects on the risk premium are not enough to match with the data. Hence, we consider some mechanisms from the monetary business cycle literature that make price adjustments further slower: sticky wages and capital utilization.[26] In addition, we consider habit persistence in consumption that has been widely used for increasing variation in the marginal rate of substitution in both the risk premium and equity premium literatures.[27]

[26] Christiano *et.al.* (2005) provide evidence that their monetary model with these nominal and real features can produce persistence in the aggregate quantities and inflation in the US data.

[27] For example, Jermann (1998) shows that a model with habit persistence and capital adjustment costs can generate the historical equity premium. Further, Boldrin *et.al.* (2001) show that the standard real business cycle model with two modifications of habit persistence and limitations on intersectoral factor mobility succeed in explaining the equity premium and improve the model's business cycle implications.

5.1 Habit Persistence in Consumption

We first investigate quantitative implications of habit persistence for the risk premium. Previous studies find that introducing a non-linear preference specification to an otherwise standard general equilibrium model tends to increase variation in the risk premium because it allows moderate consumption fluctuations to have large impacts on the marginal utility of consumption. For example, Backus *et.al.* (1993) and Moore and Riche (2002) introduce habit persistence in the Lucas two-country general equilibrium model, and find that habit persistence helps to increase volatilities of the risk premium.

To introduce habit persistence in consumption into the benchmark model, we follow Christiano *et.al.* (2005). Preference for the home representative household is given by the following expected utility function:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)[\frac{1}{1-\sigma}(C(s^t)-bC(s^{t-1}))^{1-\sigma}+\frac{1}{1-\phi}(\frac{M(s^t)}{P(s^t)})^{1-\phi}-\frac{1}{1+\gamma}L^{1+\gamma}(s^t)] \tag{5-1}$$

where b indicates habit persistence or consumption durability. If $b = 0$, then the preferences are time additive, if $b > 0$ then consumption exhibits habit persistence, and if $b < 0$ then consumption is durable. In our quantitative study, we set b equal to 0.7, following Christiano *et.al.* (2005).

Our benchmark model with habit persistence generates unrealistically high values of the relative standard deviation of the real exchange rate to consumption, although the absolute volatilities of marginal utility of consumption and exchange rates do not depend much on the risk aversion and habit persistent parameters. Hence, we set $\sigma = 2$ in order to reduce the relative standard deviation of the real exchange rate. Further, we set $\phi = 10.26$ following Christiano *et.al.* (2005). The results from these modifications are reported in the column labeled with 'Habit' in Table 4. Our numerical results show that, in contrast to previous studies that abstract from production, habit persistence

does not play a significant role in the determination of the risk premium. Variations of the risk premium, the forward premium, and the expected depreciation in this modified model do not much increase: The variance of the true risk premium is 0.26E-4, while those of the forward premium and the expected depreciation are 0.05E-4 and 0.14E-4.

The reason why there is not much difference between the two models with and without habit persistence can be easily seen when the money demand function is static

$$\widehat{m}_t - \widehat{p}_t = \widehat{uc}_t. \tag{5-2}$$

Since prices are fixed before monetary shocks are realized, the marginal utility of consumption should change one-for-one with changes in nominal money balances in order to clear money markets regardless of whether or not consumption exhibits habit persistence. This implies that the conditional volatility of the marginal rate of substitution is independent of the risk aversion as well as the habit persistence parameters. That is, in contrast to endowment economies, the effect of habit persistence (or the degree of risk aversion) on the marginal utility of consumption is exactly offset by that of the elasticity of intertemporal substitution.[28] Since only this conditional volatility matters for the determination of the risk premium, the introduction of habit persistence does not much improve the result on the variation of the risk premium in the benchmark model. Similarly, although consumption is more persistent in the habit persistence model than in the benchmark model by construction, it would not help to increase the volatility of marginal rate of substitution in our framework. By inserting equation (5-2) and the foreign counterpart into the risk sharing

[28]See, also, equation (4-5) and (4-6). We also conduct some experiments by varying σ from 1 to 100 to study how the degree of risk aversion affects our results: variances of the true risk premium are 0.24E-4 for $\sigma = 1$ and 0.25E-4 for $\sigma = 100$, while those of the expected depreciation are 0.16E-4 for $\sigma = 1$ and 0.14E-4 for $\sigma = 100$. These results show that rasing the degree of risk aversion does not help to increase the volatilities of the marginal of utility of consumption and exchange rates. Rather, it increases the relative standard deviation of the real exchange rate to consumption because households' incentives for smoothing consumption increase as the elasticity of intertemporal substitution becomes lower: the relative standard deviation of the real exchange rate to consumption is 1.31 for $\sigma = 1$ and 128.56 for $\sigma = 100$.

condition (2-15), one can easily see that the conditional volatilities of changes in both the nominal and real exchange rates do not depend on these two parameters either.

5.2 Sticky Wages

Huang and Liu (2002) and Christiano *et.al.* (2005) find that staggered wage setting can generate more persistent aggregate quantities than staggered price setting. To study this effect on the variation of the marginal rate of substitution, we extend the benchmark model by assuming that labor inputs are differentiated and households set wages according to a variant of the Taylor staggered wage contract.

In the presence of sticky wages, the household's problem is changed, while the problems of the final goods producers and intermediate goods producers remain the same as before. Following Erceg *et.al.* (2002) and Christiano *et.al.* (2005), we introduce the home competitive representative firm that produces aggregate labor $L(s^t)$ by combing a continuum of differentiated labor inputs, indexed by $n = [0, 1]$, using the technology:

$$L(s^t) = (\int_0^1 L(n, s^t)^{\vartheta} dn)^{\frac{1}{\vartheta}} \tag{5-3}$$

where $L(n, s^t)$ denotes home household n's labor service and ϑ denotes substitutability between differentiated labor inputs. The home competitive firm takes as given wages $W(s^t)$, $W(n, s^{t-1})$ for $n = [0, 1]$ to maximize its profits given by

$$\max_{\{L(n,s^t)\}_{n=0}^1} W(s^t)L(s^t) - \int_0^1 W(n, s^t)L(n, s^t)dn \tag{5-4}$$

subject to (5-3). $W(s^t)$ is the aggregate wage rate and $W(n, s^{t-1})$ is the price of home labor input n at time t. Prices of differentiated labor input services do not depend on s^t because they are set

before period t shocks are realized. Wages in home country are denominated in home currency. From this problem, the demand function for labor input n is defined by

$$L^d(n, s^t) = (\frac{W(n, s^{t-1})}{W(s^t)})^{\frac{1}{\vartheta-1}} L(s^t). \tag{5-5}$$

Using the zero profit condition, the price of the composite labor service is defined by $W(s^t) = [\int_0^1 (W(n, s^{t-1}))^{\frac{\vartheta}{\vartheta-1}} dn]^{\frac{\vartheta-1}{\vartheta}}$ and home final labor service is distributed to the home intermediate goods producers according to $L(s^t) = \int_0^1 L(i, s^t) di$. Here, $L(i, s^t)$ denotes the home composite labor service purchased by home intermediate goods producer i.

In the beginning of each period t, a fraction of home households, $n = [0, \frac{1}{N_w}]$, set wage $W(n, s^{t-1})$ and fix it for the subsequent N_w periods. Household n maximizes his expected utility given by

$$\overline{U}_0 = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U(C(n, s^t), \frac{M(n, s^t)}{P(s^t)}, L(n, s^t))$$

subject to the budget constraint and demand for labor input n

$$\begin{aligned} & P(s^t)C(n, s^t) + M(n, s^t) + \sum_{s^{t+1}} Q(s^{t+1}|s^t) B(n, s^{t+1}|s^t) \\ \leq\ & W(n, s^t) L^d(n, s^t) + M(n, s^{t-1}) + B(n, s^t) + \Pi(n, s^t) + T(n, s^t) \quad \forall s^t. \end{aligned} \tag{5-6}$$

Following CKM, we choose initial bond holdings so that each household has the same present discounted value of income. Then, the optimal wage condition for home household n is

$$W(n, s^t) = \frac{\sum_{\tau=t}^{t+N_w-1} \sum_{s^\tau} Q(s^\tau|s^{t-1}) [U_l(n, s^\tau) L^d(n, s^\tau)]}{\vartheta \sum_{\tau=t}^{t+N_w-1} \sum_{s^\tau} Q(s^\tau|s^{t-1}) [\frac{U_c(n, s^\tau)}{P(s^\tau)} L^d(n, s^\tau)]}. \tag{5-7}$$

Equations (2-3) and (2-5) and initial bond conditions guarantee that $U_c(n, s^\tau)$ and $U_m(n, s^\tau)$ are equal across households. For calibration, we set $\vartheta = 0.87$ and $N_w = 4$ from CKM.

The results from this modification are reported in the column labeled with 'Sticky Wages' in Table 4. The modified model improves the benchmark model's performance on the variation of the

true risk premium slightly: The variance of the true risk premium is 0.31E-4, while it is 0.25E-4 in the benchmark model.

5.3 Capital Utilization

In this section, we consider another mechanism that increases persistent real effects of monetary shocks and thus might increase the conditional variation of the marginal rate of substitution: variable capital utilization. For this purpose, we extend the benchmark model by introducing variable capital utilization and investment adjustment costs from Christiano *et.al.* (2005).

We assume that households own capital and decide how many units of capital services to supply. Accordingly, the home representative household's budget constraint is modified in the following way:

$$\begin{aligned} P(s^t)(C(s^t) + I(s^t) + a(\upsilon(s^t))\overline{K}(s^{t-1})) + M(s^t) + \sum_{s^{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}|s^t) \\ \leq W(s^t)L(s^t) + R^k(s^t)K(s^t) + M(s^{t-1}) + B(s^t) + \Pi(s^t) + T(s^t) \quad \forall s^t. \end{aligned} \tag{5-8}$$

where $I(s^t)$ denotes the purchase of investment goods, $\overline{K}(s^{t-1})$ denotes the physical stock of capital at the end of time $t-1$, $\upsilon(s^t)$ denotes the utilization rate of capital, $a(\upsilon)$ denotes the capital utilization function, $R^k(s^t)$ denotes the price of capital service, $I(s^t) + a(\upsilon(s^t))\overline{K}(s^{t-1})$ denotes the stock of installed capital at time t, and $K(s^t) = \upsilon(s^t)\overline{K}(s^{t-1})$ denotes capital service at time t. The household's capital stock evolves according to:

$$\overline{K}(s^t) = (1-\delta)\overline{K}(s^{t-1}) + F(I(s^t), I(s^{t-1})) \tag{5-9}$$

where δ denotes the depreciation rate of physical capital and $F(I(s^t), I(s^{t-1})) = (1 - S(\frac{I(s^t)}{I(s^{t-1})}))I(s^t)$ denotes investment adjustment costs. We assume the same properties of functions $a(\cdot)$ and $F(\cdot,\cdot)$ as Christiano *et.al.* (2005) for our calibration. In terms of the capital utilization function $a(\cdot)$,

we assume that $\upsilon = 1$ and $a(1) = 0$ in the steady state. And for function $S(\cdot)$, we assume $S(1) = S'(1) = 0$ and $S''(1) > 1$.

The production function for intermediate goods i is modified by:

$$Y_H(i, s^t) + Y_H^*(i, s^t) = F(L(i, s^t), Z(i, s^t), K(i, s^t)) = L(i, s^t)^{(1-\alpha)(1-\alpha_k)} K(i, s^t)^{(1-\alpha)\alpha_k} Z(i, s^t)^{\alpha} \tag{5-10}$$

where α_k is the cost share for capital service and set at $\frac{1}{3}$. Accordingly, the resource constraint is modified in the following way:

$$Y(s^t) = C(s^t) + I(s^t) + a(\upsilon_t)\overline{K}(s^{t-1}) + \int_0^1 Z(i, s^t) di. \tag{5-11}$$

The results from these modifications are reported in the column labeled with "Capital Util" in Table 4. The quantitative performance of the model with capital utilization and investment adjustment costs on the volatilities of the true risk premium, the forward premium, and the expected depreciations is very similar to that of the benchmark model: the variance of the true risk premium is 0.24E-4, while those of the forward premium and the expected depreciation are 0.03E-4 and 0.14E-4, respectively. Finally, we add to the benchmark model all real and nominal frictions that we have been considered: habit persistence in consumption, sticky wages, capital utilization, and investment adjustment costs. The results from these modifications are reported in the column labeled with "All" in Table 4. Again, we find that the model with all these features improves its quantitative performance slightly: the variance of the true risk premium is 0.30E-4, while those of the forward premium and the expected depreciation are 0.04E-4 and 0.23E-4, respectively.

6 Concluding Remarks

Studies such as CKM (2000) and Christiano, *et.al.* (2005) in the monetary business cycle literature focus on developing mechanisms that lead to endogenous price stickiness and thus persistent output movements. Based on their frameworks, in the present paper, we focus on quantitative implications of persistent real effects of monetary shocks for the volatility of the risk premium in foreign exchange markets. In particular, our study links random walk behaviors of both exchange rates and consumption to variation in the risk premium and to Fama's volatility relations in order to account for the forward premium anomaly. In the benchmark model, elastic money demand and persistent money growth produce a near-random walk behavior of the nominal exchange rate. Further, when they interact with the frictions in goods markets that affect the degree of price adjustments, the model can also produce a near-random walk behavior of the marginal utility of consumption. With these features, the benchmark model generates Fama's volatility relations since both the exchange rate and the marginal rate of substitution display large variation, while both the expected depreciation and interest rates exhibit small variation.

However, the risk premium in the benchmark model is less volatile than in the data: the variance of the true risk premium is similar to that of the expected depreciation. We interpret this as staggered price setting, by itself, may not produce enough persistence in the marginal utility of consumption to generate the volatility of the risk premium observed in the data. To improve this, we conduct several experiments using various nominal and real frictions that produce the right persistence in real variables in the monetary business cycle literature. The models with these features improve on the variation of the risk premium although their quantitative effects are not so large. But we do not view this as discouraging. The volatility of price changes in these models is

much larger than in the data. This suggests that there is still room for making price adjustments even slower and thus increasing persistence in the marginal utility of consumption. We leave this for future study.

Reference

Alvarez, F., A. Atkeson, and P. Kehoe (2006), "Time-Varying Risk, Interest Rates and Exchange Rates in General Equilibrium", mimeo, University of Chicago.

Backus, D. K., A. Gregory and C. Telmer (1993), "Accounting for Forward Rates in Markets for foreign Currency", Journal of Finance 48, 1887-1908.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1994), Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?, American Economic Review 84, 84-103.

Baier, S. and J. H. Bergstrand (2001), "The Growth of World Trade:Tariffs, Transport Costs and Income Similarity", Journal of international Economics 53, 1-27.

Basu, S. (1995), "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare", American Economic Review 85, 512-531.

Bekaert, G. (1996), "The Time Variation of Risk and Return in foreign Exchange Markets: A General Equilibrium Perspective", Review of Financial Studies 9, 427-470.

Bekaert, G. and R. J. Hodrick (1992), "Characterizing Predictable Components in Excess Returns on Equity and Foreign Exchange Markets", Journal of Finance 47, 467-509.

Bakaert, G., R. J. Hodrick and D. A. Marshall (1997), "The Implications of First-Order Risk Aversion for Asset Market Risk Premiums", Journal of Monetary Economics 40, 3-39.

Bergin, P. R. and Feenstra, R. C. (2001), "Pricing-to-Market, Staggered Contracts, and Real Exchange Rate Persistence", Journal International Economics 54, 333-359.

Boldrin, M., L. Christiano, and Fisher, J (2001), "Habit Persistence, Asset Returns, and the Business Cycle", American Economic Review, 91, 149-166.

Canova, F. and J. Marrinan (1993), "Profits, Risk, and Uncertainty in Foreign Exchange Markets", Journal of Monetary Economics 32, 259-286.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2000), "Sticky Price Models of Business Cycle: Can Contract Multiplier Solve the Persistence Problem?," Econometrica 68 (5), 1151-1179.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2002), "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?", Review of Economic Studies 69, 533-63.

Christiano L. J., M. Eichenbaum, and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", Journal of Political Economy 113, 1-45.

Chung, W., M. D. Chinn, and A. G. Pascual (2002), "Empirical Exchange Rate Models of the Nineties: Are Any Fit to Survive?" NBER w9393.

Cumby, R. E. (1988),"Explaining Deviations from Uncovered Interest Parity", Journal of Monetary Economics 22, 279-299.

Domowitz, I., R. G. Hubbard, and B. C. Petersen (1988), "Market Structure and Cyclical Fluctuations in US Manufacturing", Review of Economics and Statistics 70, 55-66.

Duarte, M and A. L. Stockman (2005), "Rational Speculation and Exchange Rates", Journal of Monetary Economics 52, 3-29.

Engel, C. M. (1984), "Testing for the Absence of Expected Real Profits from Forward Market Speculation", Journal of International Economics 17, 299-308.

Engel, C. M.(1992), "On the Foreign Exchange Risk Premium in a General Equilibrium Model", Journal of International Economics 32, 305-319.

Engel, C. M.(1996), "The Forward Discount Anomaly and the Risk Premium: A Survey of Recent Evidence", Journal of Empirical Finance 3, 123-192.

Engel, C. M.(1999), "On the Foreign Exchange Risk Premium in Sticky-Price General Equilibrium Models", in International Finance and Financial Crises: Essays in Honor of Robert P. Flood, Peter Isard, Assaf Razin and Andrew Rose, eds., 71-85.

Engel, C. M. and K. D. West (2005), "Exchange Rates and Fundamentals", Journal of Political Economy 113, 485-517.

Erceg, C. J., D. W. Henderson and A. T. Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts", Journal of Monetary Economics 46, 281-313

Fama, E. F. (1984), "Forward and Spot Exchange Rates?" Journal of Monetary Economics 14, 319-338.

Flood, R. P. and A. K. Rose (1995), "Fixing Exchange Rates: A Virtual Quest for Fundamentals", Journal of Monetary Economics 36, 3-37.

Frankel, J. A. and A. Razin (1980), "Stochastic Prices and Tests of Efficiency of Foreign Exchange Markets", Economic Letters 6, 165-170.

Hall, R (1978), "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and evidence", Journal Political Economy 86, 971-987.

Harrigan, J. (1993), "OECD Imports and Trade Barriers in 1983", Journal of International Economics 35, 91-112.

Hodrick (1987), The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets, Harwood Academic Publishers, Chur, Switzerland.

Hodrick (1989), "Risk, Uncertainty, and Exchange Rates", Journal of Monetary Economics 23, 433-459.

Hodrick, R. and S. Srivastatva (1986), "The Covariation of Risk Premiums and Expected Future Spot Exchange Rates", Journal of International Money and Finance 5, 5-21.

Hollifield, B. and P. Sercu (1997), "An Examination of Uncovered Interest Rate Parity in Segmented International Commodity Markets ", Journal of Finance 52, 2145-2170.

Huang, K. X. D. and Z. Liu (2002), "Staggered Price Setting, Staggered Wage Setting, and Business Cycle Persistence", Journal of Monetary Economics 49, 405-433.

Huang, K. X. D., Z. Liu, and L. Phaneuf (2004), "Why does cyclical behavior of real wages change over time", American Economic Review 94, 836-856.

Jorgenson, D. W., F. Gollop, and B. M. Fraumeni (1987), Productivity and U.S. Economic Growth, Cambridge, MA, Harvard University Press.

Jermann, U. (1998), "Asset Pricing in Production Economies", Journal of Monetary Economices, 41, 257-275.

Kim, J., S. Kim, E. Schaumburg, and C. A. Sims (2005), " Calculating and Using Second Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models", mimeo, Princeton University.

Lewis, K. (1995), "Puzzles in International Financial Markets", in Gene Grossman and Kenneth Rogoff (eds.), Handbook of International Economics, vol 3, Amsterdam: North Holland.

Lucas, R. E. Jr. (1982), "Interest Rates and Currency Prices in a Two Country World", Journal of Monetary Economics 22, 3-42.

Mankiw, N. G. and L. H. Summers (1986), "Money Demand and the Effects of Fiscal Policies", Journal of Money, Credit, and Banking 18, 415-429.

Meese, R. A. and K. Rogoff (1983), "The Out-of-Sample Failure of Empirical Exchange Rates Models: Sampling Error or Misspecification?" in Jacob A. Frankel, (ed.), Exchange Rates and International Macroeconomics, Chicago, University of Chicago Press.

Mehra, R. and E. Prescott (1985), "The Equity Premium: A Puzzle", Journal Monetary Economics 15, 145-168.

Moore, M. J. and M. J. Roche (2002), "Less of a Puzzle: A New Look at the Forward Forex", Journal of International Economics 58, 387-411.

Obstfeld, M. and K. Rogoff (2000a), "New directions for stochastic open economy models", Journal International Economics 50, 117-153.

Obstfeld, M. and K. Rogoff (2000b), "The Six Major Puzzles in International Macroeconomics: Is there a Common Cause?" Ben Bernanke and Kenneth Rogoff (eds.), NBER Macroeconomics Annual, 339-390 Cambridge: MIT Press.

Obstfeld, M. and K. Rogoff (2003), "Risk and Exchange Rates", in Elhanan Helpman and Effraim Sadka (eds.), Contemporary Economic Policy: Essays in Honor of Assaf Razin, Cambridge: Cambridge University Press.

Rouwenhorst, K. G. (1995), "Asset Pricing Implications of Equilibrium Business Cycle Models", in T. Cooley and E. Prescott, (eds.), Modern Business Cycle Theory, pp 1294-1330, Princeton

University Press.

Schmitt-Grohe, S. and M. Uribe (2004), "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function", Journal of Economic Dynamics and Control 28, 755-775

Stock, J. H. and M. W. Watson (1993), "A Simple Estimator of Cointegrating Vectors: in Higher Order Integrated Systems", Econometrica 61, 783-820.

Stockman, A. and L. Tesar (1995), "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements", American Economic Review 85, 168-185.

Taylor, J. B. (1980) "Aggregate dynamics and staggered contracts." Journal of Political Economy 88, 1-23.

Table 1. Parameter Values

The benchmark model	
Preference	$\beta = 0.99$, $\sigma = 7.0$, $\phi = 7.0$, $\gamma = 2.0$
Final goods technology	$\rho = \frac{1}{3}$, $a1 = 0.85$, $a2 = 0.15$
Intermediate goods technology	$\alpha = 0.7$, $\theta = 0.9$, $N_p = 4$
Money Growth Process	$\rho_m = 0.66$, $\rho_h = 0.54$, $\rho_\xi = 0.37$, $var(\xi) = 0.0015^2$
Variations	
Bench1	$N_p = 1$
Habit Persistence	$b = 0.7$, $\sigma = 2.0$, $\phi = 10.26$
Sticky Wage	$\vartheta = 0.87$, $N_w = 4$
PPP	$a1 = 0.5$, $a2 = 0.5$
Capital Utilization	$\delta = 0.025$, $\frac{a''}{a'} = \frac{1}{\beta} - 1 + \delta$

For other economies, we only present parameter values that are different from those in the benchmark economy.

Table 2

Panel A. Diagnostic Tests on the Quarterly Growth Rates in M1 in the US	
$(1-\rho_m)E[g]$	0.01
se	(0.002)
ρ_m	0.43
se	(0.159)
Skewness	0.12
Kurtosis	6.60
$ARCH(12)$	31.80
$Q^2(15)$	25.61
Adjusted R^2	0.18
Panel B. Estimations for the Quarterly Growth Rates in M1 in the US	
$(1-\rho_m)E[g]$	0.01
se	(0.003)
ρ_m	0.66
se	(0.075)
Variance Equation	
$var(\xi) * (1-\rho_h-\rho_\xi)$	0.00
se	(0.000)
ρ_h	0.54
se	(0.148)
ρ_ξ	0.37
se	(0.167)
Skewness	0.71
Kurtosis	3.96
Adjusted R^2	0.11
No. obs	121

The money supply processes in equations (3-3) and (3-4) are used for estimation of quarterly M1 data between Q2 1973 and Q3 2003. $ARCH(12)$ represents ARCH LM test statistic for testing autoregressive conditional heteroskedasticity (ARCH) and $Q^2(15)$ for up to 15th serial correlation in the squared residuals is used for testing ARCH effects in the residuals. 'se' represents standard errors.

Table 3. Estimation of the Risk Premium and Summary Statistics of Exchange Rates

Statistic	Japan	UK	France	Italy	Germany
Panel A: $f_t - e_{t+1} = b_1 + b_2(f_t - e_t) + v_{t+1}$					
b_1	-0.012	0.008	0.005	0.008	-0.003
	(0.008)	(0.006)	(0.008)	(0.007)	(0.007)
b_2	1.696	1.839	0.893	0.701	1.412
	(0.568)	(0.772)	(0.808)	(0.331)	(0.687)
Adjusted R^2	0.088	0.054	0.005	0.016	0.020
No. obs	121	121	87	113	113
Panel B: Fitted values from $f_t - e_{t+1} = b_1 + b_2(f_t - e_t) + v_{t+1}$					
Mean	-0.0002	-0.0021	0.0022	-0.0011	0.0043
Std deviation	0.0202	0.0132	0.0079	0.0092	0.0106
Autocorrelation	0.737	0.844	0.692	0.678	0.909
Panel C: Change in the nominal exchange rate $e_{t+1} - e_t$					
Mean	0.0070	-0.0036	-0.0057	-0.0120	0.0008
Std deviation	0.063	0.052	0.061	0.058	0.061
Autocorrelation	0.121	0.149	0.135	0.159	0.027
Panel D: Excess return $f_t - e_{t+1}$					
Mean	-0.0002	-0.0021	0.0022	-0.0011	0.0043
Std deviation	0.065	0.053	0.062	0.058	0.062
Autocorrelation	0.155	0.189	0.165	0.133	0.063
Panel E: Forward premium $f_t - e_t$					
Mean	0.0068	-0.0058	-0.0035	-0.0131	0.0051
Std deviation	0.011	0.007	0.009	0.013	0.008
Autocorrelation	0.719	0.868	0.703	0.833	0.886
Panel F: Change in the real exchange rate					
Std deviation	0.064	0.056	0.064	0.061	0.062
Autocorrelation	0.150	0.178	0.212	0.229	0.084

Data consists of quarterly spot and one-quarter forward rates for the US dollar price of the Japanese yen, the British pound, the French franc, the Italian lira, and the German mark. The series for the Japanese yen and the British pound run from the second quarter of 1973 to the third quarter of 2003 while the series for the German mark and the Italian lira end in the fourth quarter of 2001. The series for the French franc run from the first quarter of 1980 to the fourth quarter of 2001. For statistics of the real exchange rate, we use CPI data between the second quarter of 1973 to the first quarter of 2000 from CKM. Numbers in parentheses are Newey-West standard errors with 5 lags.

Table 4
Fama's Volatility Relations

Statistics	Data	Bench	Bench1	Habit	Sticy Wages	PCPH	PPP	Capital Util	All
				Panel A: Variance (unit:E-4)					
True rp_t	Na	0.25	0.16	0.26	0.31	0.16	0.09	0.24	0.30
$0.5Var_t(e_{t+1})$	Na	0.12	0.08	0.13	0.14	0.09	0.06	0.12	0.14
$-Cov_t(e_{t+1}, p_{t+1})$	Na	0.00	0.00	0.00	0.00	0.01	0.06	0.00	0.00
$0.5Var_t(e_{t+1}) - Cov_t(e_{t+1}, p_{t+1})$	Na	0.12	0.08	0.13	0.14	0.05	0.00	0.12	0.14
rp_t	0.62	0.13	0.08	0.13	0.17	0.11	0.09	0.12	0.15
$E_t[e_{t+1} - e_t]$	Na	0.14	0.22	0.14	0.18	0.11	0.26	0.14	0.23
fp_t	0.81	0.02	0.12	0.05	0.02	0.05	0.26	0.02	0.04
				Panel B: Cross correlation					
$(E_t[e_{t+1} - e_t], rp_t)$	Na	-0.74	-0.38	-0.92	-0.79	-0.13	-0.43	-0.74	-0.67
$(f_t - e_t, rp_t)$	Na	0.07	-0.08	-0.05	0.02	0.18	0.13	0.02	-0.10
				Panel C: Autocorrelation					
rp_t	0.69	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78
fp_t	0.70	0.33	0.73	0.48	0.32	0.49	0.62	0.33	0.50
$E_t[e_{t+1} - e_t]$	Na	0.63	0.74	0.77	0.65	0.57	0.63	0.63	0.67
				Panel D: Exchange Rates and Consumption					
$Sd(c_{t+1} - c_t)$	0.007	0.007	0.008	0.006	0.007	0.006	0.005	0.007	0.007
$Sd(e_{t+1} - e_t)$	0.061	0.061	0.055	0.062	0.063	0.057	0.050	0.061	0.065
$Sd(rq_{t+1} - rq_t)$	0.062	0.067	0.076	0.068	0.068	0.044	0.000	0.067	0.070
$Auto(c_{t+1} - c_t)$	0.23	-0.01	-0.48	0.46	0.01	-0.01	0.00	0.01	0.56
$Auto(e_{t+1} - e_t)$	0.12	-0.00	0.00	-0.01	0.00	0.02	0.06	0.00	0.00
$Auto(rq_{t+1} - rq_t)$	0.12	-0.01	-0.48	-0.01	-0.00	-0.02	0.00	-0.00	0.01
$Corr(\Delta e_{t+1}, \Delta rq_{t+1})$	0.99	0.91	0.71	0.91	0.94	0.90	0.00	0.91	0.94

Statistics of the risk premium, the forward premium, the nominal exchange rate, and the real exchange rate presented in the column labeled with 'Data' are the values for the French frac in terms of the US dollar. The US consumption data between the second quarter of 1973 and the third quarter of 2003 are obtained from the BEA database and used for producing statistics of consumption growth. Unconditional variances of stochastic disturbances in the processes of both home and foreign money growth rates are set to 0.015^2 for all experiments. 'rp_t' represents the risk premium and 'True rp_t' means the true risk premium. 'Bench' denotes the benchmark model. The followings are the variations of the benchmark model. 'Bench1' denotes the model with $N_p = 1$. 'Habit' denotes the model with habit persistence in consumption and 'Stick Wages' denotes the model in which both prices and wages are fixed for four periods in a staggered way. 'PCPH' denotes the model in which the law of one price holds for each home and foreign good i but $a_1 = 0.85$. 'PPP' denotes the model in which PPP holds and prices are fixed for 4 periods in a staggered way. 'Capital Util' denotes the model with capital utilization and investment adjustment costs and 'All' denotes the model with habit persistence in consumption, sticky wages, capital utilization, and investment adjustment costs. 'Sd' represents standard deviation, 'Auto' represents autocorrelation, and 'Corr' represents cross correlation.