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EFFECTS IN HSIF AND HSI VOLATILITY: A NOTE

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Abstract

In an earlier paper we adopted a Bi-variate BEKK-GARCH framework and employed a systematic approach to examine structural breaks in the Hang Seng Index and Index Futures market volatility. Switching dummy variables were included and tested in the variance equations to check for any structural changes in the autoregressive volatility structure due to the events that have taken place in the Hong Kong market surrounding the Asian markets crisis. In this paper we include measures of daily trading volume from both markets in the estimation. Likelihood ratio tests indicate the switching dummy variables become insignificant and the GARCH effects diminish but remain significant. There is some evidence that the Sequential arrival of Information Model provides a platform to explain these market induced effects when volume of trade is accounted for.

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KEY WORDS: Regulatory change, Multivariate Volatility, Volume of Trade.
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1. INTRODUCTION

There are two prevailing models for explaining the theoretical underpinnings of volume and price variability relationship: the Mixture of Distribution Hypothesis (MDH) and the Sequential arrival of Information Model (SIM). While both models predict a positive correlation between the volume and price variability, the MDH postulates the daily returns are generated by a mixture of distributions, in which the rate of daily information arrival is the stochastic mixing variable. On the other hand, the SIM assumes information arrives in the market in a sequential way and so a series of temporary equilibria are formed prior to the final equilibrium. When there are substantial alterations to market trading activity then the question that arises is whether these theoretical constructs still provide an explanation of market volatility and volume of trade effects. The alternative is to directly model these effects directly employing structural models that measure the impact of these events.

Market frictions such as changes in short sale constraints and various trading rules, could also cause markets to respond to the new information in different styles. Moreover, market events, like the alteration of trading systems and substantial adjustment in initial margins of futures contracts may also have an impact on the price discovery process. The choice of the Hong Kong market is motivated by the occurrence of policy changes and market innovations before, during and after the eruption of the Asian Financial Crisis in 1997. The linking of the Hong Kong dollar to the U.S. dollar since 1983 is a significant motivation for U.S. investors to focus on the Hong Kong market. As well, the much publicized activity following the hedge fund assault on the Hong Kong stock market during the latter part of the 1990’s means this market maintains a high level of international interest.

Between January 1994 and March 2000 a number of market interventions took place in the Hong Kong Stock and Futures Exchange (HKSE and HKFE) markets that could have an impact on the volatility and volume relationship. Reactions to some of the above mentioned factors did help generate regulatory changes in these markets. These events in these markets included introduction of restricted short selling, abolishment and subsequent reintroduction of the uptick rule, the HKFE raised the initial margins of Hang Seng Index Futures (HSIF) and subsequently decreased margins and trading of HSIF migrated to the Hong Kong Automatic Trading System (HKATS). Finally, the HKSE and HKFE merged. These effects were examined by Au-Yeung and Gannon (2005) systematically. The optimal combination of three significant switch points for a bi-variate BEKK-GARCH model was chosen via a likelihood ratio test. The theoretical framework, four prevailing hypotheses and literature regarding the impact on the lead-lag relationship between cash and futures market volatility is discussed in that paper. Features of the data and diagnostic checks for misspecification are also reported in this earlier paper.

Apart from using GARCH representations to model the conditional volatility of spot index and futures, prior research has tempted to explain the conditional volatility with the change in trading volume which works as a proxy for the rate of information arrival. Lamoureux and Lastrapes (1990) included the contemporaneous daily trading volume as an independent variable in the variance equation. They found that contemporaneous volume has significant explanatory power on the conditional volatility of stock returns and the ARCH effects tend to disappear with volume included. On the other hand, Najand and Yung (1991) found the persistence of volatility continues even volume is included. Furthermore, Bessembinder and Seguin (1992) decompose the trading volume and open interest into expected and unexpected components by fitting an ARMA model. They found that conditional volatility could be explained by the unexpected components of trading volume. This research fails to recognize that the endogeneity of the contemporaneous trading volume may result in simultaneity bias. Gannon (1994) had earlier dealt with this issue by specifying a simultaneous set of volatility equations for the cash market volatility, the index futures volatility and volume of trade. That study employed intra-day data sampled at 15 minute intervals. Board, Sandmann & Sutcliffe (2001) further argue that apart from the issue of endogeneity, the coefficient of contemporaneous volume in a GARCH model, employing daily data, is not an estimate of the effect of volume at any single time but in fact of an exponentially weighted average of past values of the volume measure. If the issue is checking the contemporaneous relationship between the asset price volatility and the volume of trade, not the interpretation of the size of the volume of trade coefficient, then likelihood ratio tests designed to measure this contemporaneous effect in the complete system are appropriate. This is the approach employed in this paper.

In this paper we employ the same base dataset as Au-Yeung and Gannon (2005) but augment this set to further test volume of trade effects flowing from the stock and futures markets within the optimal switch point model. We employ and extend the BEKK-GARCH bi-variate volatility model of Engle and Kroner (1995) in which contemporaneous conditional volatility is a function of multivariate lagged ARCH, GARCH as well as the covariance terms. This approach allows us to measure the impact of contemporaneous volume via the effect on the Log-Likelihood function and also the impact on structural shift parameters as a complete set. We include measures of the daily trading volume of HSI and HSIF as a contemporaneous independent variable in the respective variance equations. The extension to a class of nested and non-nested models and development of an artificial nested testing framework is an innovation in this paper.

This remainder of this paper is organized as follows. Section II discusses the data and specifies the estimation models. In section III the results of the analysis of the artificial nested testing framework is presented. In this section we also examine and discuss the impact of volume effects on the GARCH model. Section IV concludes the paper.

2. DATA AND METHODOLOGY

2.1 Data Collection

The sample period starts from 1st July 1994 and ends in 31st August 2001. Daily closing price of the HSI and HSIF, number of shares traded for stocks comprising the HSI and daily trading volume for each HSIF contracts within the sample period are collected from Bloomberg and from the HKSE Website.[1] The HSIF nearby (Spot month) contracts are rolled over to the next month contract depending on the trading volume of relevant contracts. In all a total of 1,770 observations are available estimation period.

The first difference of logged HSI and logged HSIF price are employed as the price levels contain a unit root[2]. The daily continuous return is calculated as the formula below,

$$\textit{HSI daily continuous return} \quad R_{1,t} = \ln(P_{1,t} / P_{1,t-1}) \tag{1}$$

[1] The daily closing price and trading volume of HSIF from 4th January 1999 onwards are collected from the Hong Kong Stock Exchange website. There exist a few missing values for the HSIF daily volume, we substitute them with the average of the volumes of the trading date before and after.

[2] Non-synchronous trading in the component stocks of HSI and bid-ask bounce in the HSIF return was found to not affect estimates in the conditional variance equations as the daily closing price of a narrow-based HSI does not exhibit these effects. Dynamic effects in the mean equations were tested and rejected.

$$\textit{HSIF daily continuous return } R_{2,t} = \ln(P_{2,t} / P_{2,t-1}) \qquad (2)$$

where $R_{1,t}$ and $P_{1,t}$ represent the daily continuous return and daily closing price of HSI at time t respectively, and $P_{1,t-1}$ is the daily closing price of HSI at time t-1. Similarly, $R_{2,t}$ and $P_{2,t}$ represent the daily continuous return and daily closing price of HSIF at time t, and $P_{2,t-1}$ is the daily closing price of HSIF at one period prior.

2.2 Model Specification

Lee and Ohk (1992) investigate the variation of return volatility after the trading of futures index in the Korean market by adopting a univariate switching GARCH model. Chang and Gannon (2001) extended this model to a test for multiple switch points in a univariate GARCH framework. Au-Yeung and Gannon (2005) extended the GARCH model to test multiple switch points in a bi-variate BEKK-GARCH framework. We employ the latter model and augment it for contemporaneous volume of trade effects in the HSI and HSIF. We also set up systems of nested and non nested models to test the switch point models against models augmented to account for trading activity.

Following on from the above the second moment can be represented by:

$$H_t = C_0' C_0 + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}, \varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}, \varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}' H_{t-1} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \qquad (3)$$

The expanded unrestricted version of the Bi-variate BEKK-GARCH model (Equation (3)) takes the following form:

$$h_{11,t} = c_{11}^2 + a_{11}^2 \varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2 \varepsilon_{2,t-1}^2 + g_{11}^2 h_{11,t-1} + 2g_{11}g_{21}h_{12,t-1} + g_{21}^2 h_{22,t-1}$$

$$\begin{aligned} h_{12,t} = {} & c_{11}c_{21} + a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{21}a_{12} + a_{11}a_{22})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 + g_{11}g_{12}h_{11,t-1} \\ & + (g_{21}g_{12} + g_{11}g_{22})h_{12,t-1} + g_{21}g_{22}h_{22,t-1} \end{aligned}$$

$$h_{22,t} = c_{21}^2 + c_{22}^2 + a_{12}^2\varepsilon_{1,t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2 + g_{12}^2 h_{11,t-1} + 2g_{12}g_{22}h_{12,t-1} + g_{22}^2 h_{22,t-1} \quad (4)$$

To test for any shift in the variance structure event dummy variables for the constant, lagged squared errors and lagged conditional variance enter the MGARCH variance equations (Equation (3)) as follows:

$$H_t = C_0'C_0 + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}, \varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}, \varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}' H_{t-1} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$
$$+ \sum_{i=1}^{k} \left[\begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}_i D_{it} + \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix}_i \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} D_{it} + \begin{bmatrix} p_{11} & 0 \\ 0 & p_{22} \end{bmatrix}_i H_{t-1} D_{it} \right] \quad (5)$$

where $$D_{it} = \begin{cases} 0 & if \quad 1 \le t \le t_i - 1, \\ 1 & if \quad t_i \le t \le T \end{cases}$$

The notation D_{it} stated above represents the dummy variables for the 3 significant different events observed in the Hong Kong markets during the sample period.

D_{1t} ~ dummy variable for the removal of uptick rule

D_{2t} ~ dummy variable for the increase of HSIF initial margins

D_{3t} ~ dummy variable for the trading of HSIF on HKATS

Setting a_{21} and g_{21} equal to zero in equation (3) defines Equation (6) where we can test the causality effect from HSIF to HSI. The log likelihood from these estimations is then compared against that of the model with no off-diagonal restrictions. Previous results to test volatility transmission from HSI to HSIF, with off diagonal terms a_{12} and g_{12} in the conditional volatility equations are set to zero, are clearly rejected.

It follows that there are two competing models: A Bi-Variate BEKK-GARCH model for the HSI and HSIF volatility and a restricted version of this model that imposes HSIF volatility causality onto the HSI volatility. However, if contemporaneous volumes of trade are allowed to enter the volatility equations then there is a set of models that these two are nested within.

We include daily trading volume of HSI and HSIF as an independent variable into the respective variance equations to investigate the volume effect under these bi-variate systems i.e., a 3 switch point model – equation (5):

$$\{\text{with structure (4) or the structure (6)}\} + \begin{bmatrix} w_{11} & 0 \\ 0 & w_{22} \end{bmatrix} \begin{bmatrix} SVol_t \\ FVol_t \end{bmatrix} \qquad (7)$$

where $SVol_t$ and $FVol_t$ denote the daily trading volume of HSI and HSIF at time t respectively.

It follows that we have defined 4 nested and non-nested models:

MODEL 1 Equation (7) with structure (4) imposed

MODEL 2 Equation (7) with structure (6) imposed

MODEL 3 Equation (5) with structure (4) imposed

MODEL 4 Equation (5) with structure (6) imposed

We define Models 1 and 3 “unrestricted” in the sense that the off diagonal terms are not restricted but Model 1 contains Volume of trade so that Model 3 is restricted relative to Model 1.

Model 2 is an off-diagonal restricted version of Model 1 and Model 4 is an off-diagonal restricted version of Model 3.

Models 2 and 3 are nested within Model 1 and Model 4 is nested within both Models 2 and 3. It follows that the non-nested Models 2 and 3 can be compared relative to the above two groupings.

The likelihood ratio test is calculated by comparing the log likelihood of an unrestricted model and a restricted model:

$$D = -2LLR = -2\ln\left(\frac{L_0}{L_1}\right) = -2(\ln L_0 - \ln L_1) \qquad (8)$$

where LLR = Log Likelihood Ratio

L_0 = Value of the likelihood function of the restricted model

L_1 = Value of the likelihood function of the unrestricted model

And the definitions above are for the unrestricted and restricted models.

The statistic D follows a χ-distribution with k degrees of freedom, where k is the number of restrictions in the restricted model.

3. EMPIRICAL RESULTS

In Table I the paired Likelihood ratio tests of Models 1 to 4 with and without volume included and with and without volatility causality from the HSIF to the HSI imposed, in the 3 switch point model, is reported. The results can be summarized as follows:

Models 2 and 3 are rejected by Model 1 (LR statistic, 51.3 and 91.4)

Model 4 is rejected by Model 2 (LR statistic 42.7)

Model 4 is not rejected by Model 3 (LR stastic 2.8)

It follows that the non-nested Model 2 rejects Model 3.

The conclusion from this sequence of tests is that although there was no statistical difference between the two models that did not include volume of trade effects (recall the model that allows volatility causality from the HSI to the HSIF was excluded from analysis because it was clearly rejected) both models with included volume of trade rejects their restricted versions.

<INSERT TABLE I ABOUT HERE>

We consider some specific results and focus on the “unrestricted” models but with volume of trade included/excluded. We only report the results for the model that is clearly preferred (Model 1).

<INSERT TABLE 2 ABOUT HERE>

The switch point model (model 3) is able to capture structural changes in the volatility structure of the HSI and HSIF. Statistically significant events which have taken place in the Hong Kong market: abolishment of uptick rule, increase in initial margins and electronic trading of HSIF, are all significant in that former model and reported in Au-Yeung and Gannon (2005).

However, the results with volumes of trade included reveal that contemporaneous volume is highly significant and positive in both the HSI and HSIF volatility equations. Hence, the contemporaneous trading volume is positively correlated with the conditional volatility, which is consistent with empirical evidence of Lamoureux and Lastrapes (1990). By examining the 3 switching point model with the contemporaneous volume model, it is found that though the size of g_{11} and g_{22} in G_{11} matrix diminishes, they continue to be significant when the contemporaneous volume is included. Similar results are documented by Najand and Yung (1991) who found the volatility persistence continues even the volume effect is accounted for. Moreover, almost all switching dummy variables become insignificant. Under this bi-variate system, it is also noted some off-diagonal elements in the A_{11} and G_{11} matrix which shows the volatility transmission effects between the stock and futures markets switch in terms of significance when contemporaneous volume is included.

To examine whether the daily trading volume of HSI and HSIF follows a regime shift simultaneously with conditional volatilities after the regulatory change, we plot the daily trading volume and the conditional volatility of HSI or HSIF 150 days before and after each event date in the same graph. Figure 1 and Figure 2 present the variation of conditional volatilities and daily trading volume of HSI and HSIF around each market event respectively. The horizontal axis of each graph shows the observations within the event window, whereas the right vertical axis and left vertical axis represent the level of conditional volatilities and daily trading volume of either HSI or HSIF correspondingly. Moreover, the thick dark line shows the variation of conditional volatility, while the thin dark line illustrates the level of daily trading volume around each event date.

<INSERT FIGURES 1 AND 2 ABOUT HERE>

It is noted that there is an abrupt change in the level of conditional volatility of HSI and HSIF return around the abolishment of uptick rule (observation number 431). The level of volatility appears to be lower after the uptick rule is abolished. The level of trading volume of HSIF exhibits a similar pattern with its conditional volatility. The trading volume of HSI does not change correspondingly with its conditional volatility but seems to increase before the event happened. However, the volatility level of trading volume of HSI does appear reduced after the removal of the uptick, which coincides with the result we documented with the level of volatility of HSI.

With regard to the variations around the increase of initial margins, the level of trading volume of HSI and HSIF display very alike movements as their conditional volatilities. However, the mean level of trading volume of both HSI and HSIF seem to be fairly stable around the event date (observation number 832). Again there appears to be spikes in trading volume prior to the event date.

There is a considerable fall in conditional volatilities of HSI and HSIF after the electronic trading of HSIF (observation number 1466). However, the level of trading volume of HSI and HSIF do not demonstrate similar shifts as those of the conditional volatilities. Nevertheless, the volatility of trading volume of HSIF appears to have fallen after the event date. In all, there is no robust evidence that the level of daily trading volume adjusts to a new regime along with this structural event. Overall, the volatility of trading volume seems to be varying upon the occurrence of events.

4. CONCLUSION

Results reported in Au-Yeung and Gannons (2005) show a strong reaction to major structural events in the HSIF and HSI volatility equations when trading volume in the respective markets was not fully available and so not accounted for. When trading volumes are included in this bi-variate GARCH framework it is found that the GARCH effects diminish but remain significant for both series. We also find the switching dummy variables and some off-diagonal elements in the matrices become insignificant after the volume effect is included. Therefore, changes in daily trading volume cannot completely explain the rate of information arrival for the daily stock or futures returns in the Hong Kong market. The changes in trading volume may have adjusted prior to and simultaneously to new regimes as a result of the structural events. This analysis provides some evidence that the theoretical structure underlying the SIM may help in explaining events in this market through the volatility/volume relationship.

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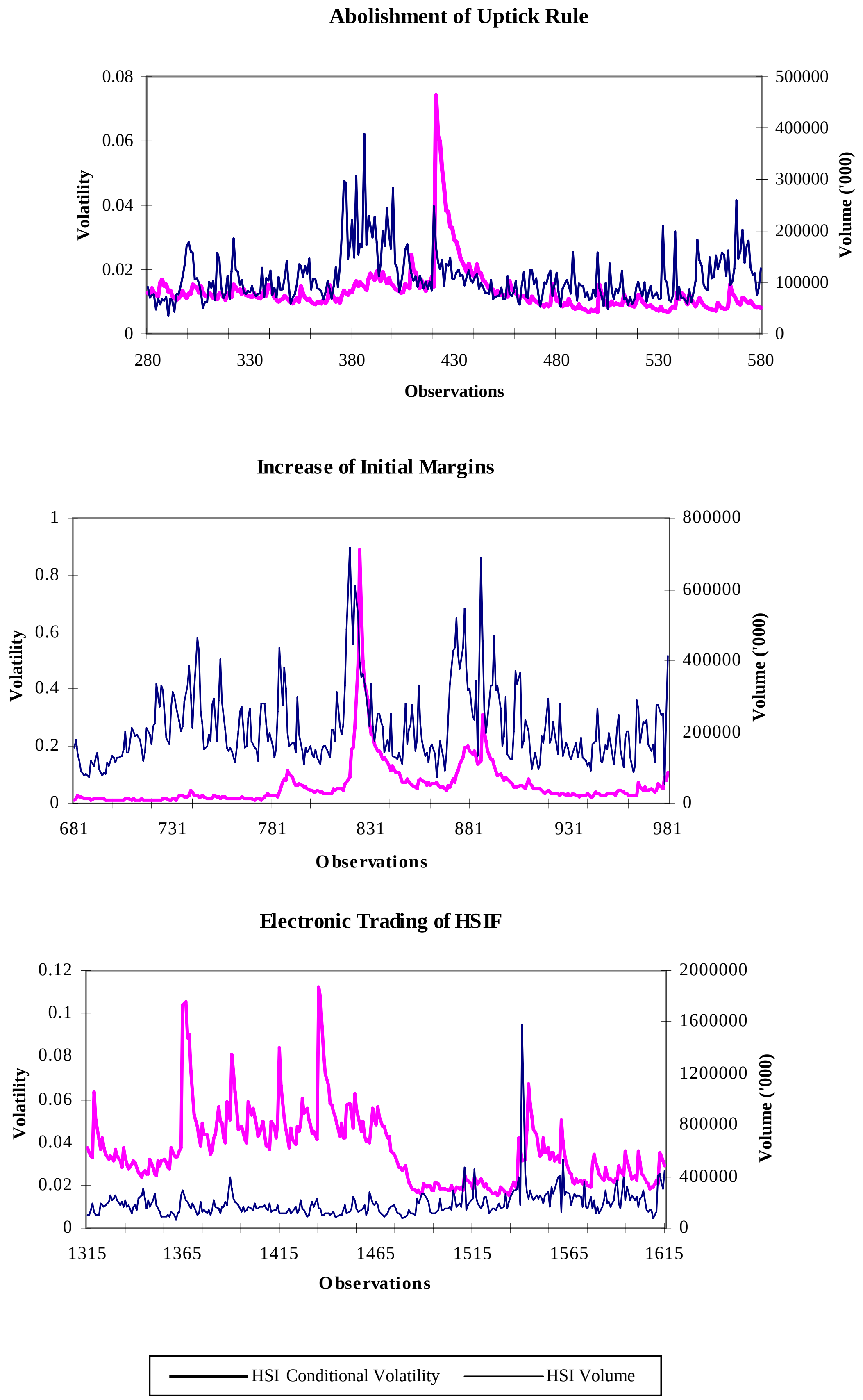

Figure 1. Comparison of conditional volatilities and daily trading volume of HSI around each event date

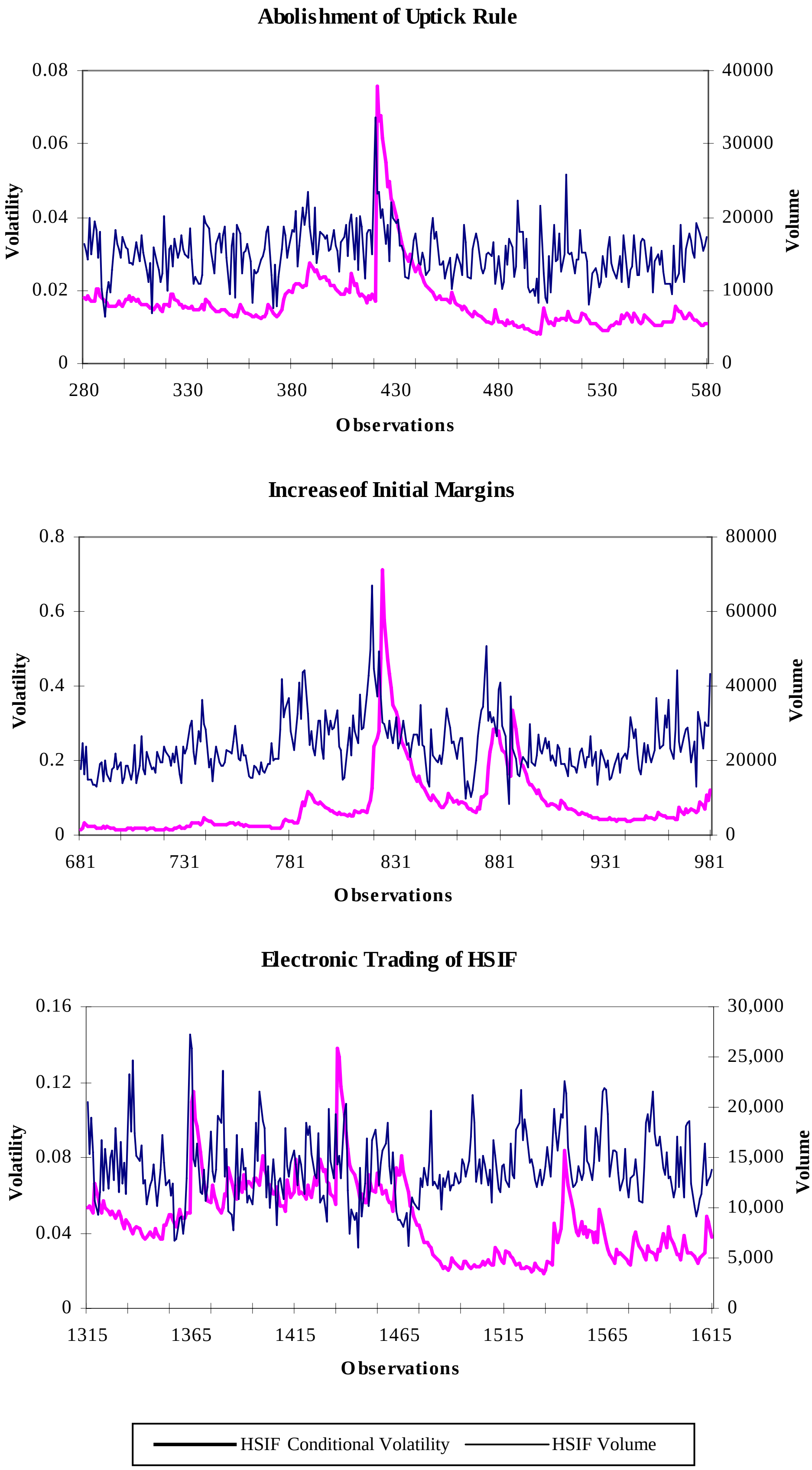

Figure 2. Comparison of conditional volatilities and daily trading volume of HSIF around each event date

Table 1. Likelihood Ratio Tests

Unrestricted Model	L_1	Restricted Model	L_0	D
MODEL1	3368.23	**MODEL2**	3342.58	51.35*
MODEL1	3368.23	**MODEL3**	3322.66	91.14*
MODEL2	3342.58	**MODEL4**	3321.24	42.68*
MODEL3	3322.66	**MODEL4**	3321.24	2.84

* indicates ***D*** is significant at 1% level under Chi-square distribution.

The unrestricted version of the BEKK-GARCH model takes the following form:

$$h_{11,t} = c_{11}^2 + a_{11}^2\varepsilon_{1,t-1}^2 + 2a_{11}a_{21}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^2\varepsilon_{2,t-1}^2 + g_{11}^2 h_{11,t-1} + 2g_{11}g_{21}h_{12,t-1} + g_{21}^2 h_{22,t-1}$$

$$h_{12,t} = c_{11}c_{21} + a_{11}a_{12}\varepsilon_{1,t-1}^2 + (a_{21}a_{12} + a_{11}a_{22})\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}a_{22}\varepsilon_{2,t-1}^2 + g_{11}g_{12}h_{11,t-1} + (g_{21}g_{12} + g_{11}g_{22})h_{12,t-1} + g_{21}g_{22}h_{22,t-1}$$

$$h_{22,t} = c_{21}^2 + c_{22}^2 + a_{12}^2\varepsilon_{1,t-1}^2 + 2a_{12}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^2\varepsilon_{2,t-1}^2 + g_{12}^2 h_{11,t-1} + 2g_{12}g_{22}h_{12,t-1} + g_{22}^2 h_{22,t-1} \quad (4)$$

Conversely, a_{21} and g_{21} are set equal to zero when we test the causality effect from HSIF to HSI. (6)

We include daily trading volume of HSI and HSIF as an independent variable into the respective variance equations to investigate the volume effect under these bi-variate systems i.e., a 3 switch point model:

{with structure (4) or the structure (6)} + $\begin{bmatrix} w_{11} & 0 \\ 0 & w_{22} \end{bmatrix} \begin{bmatrix} SVol_t \\ FVol_t \end{bmatrix}$ (7)

It follows that we have defined 4 nested and non-nested models:

MODEL 1	Equation (7) with structure (4) imposed
MODEL 2	Equation (7) with structure (6) imposed
MODEL 3	Equation (5) with structure (4) imposed
MODEL 4	Equation (5) with structure (6) imposed

TABLE 2 UNRESTRICTED (no diagonal restrictions) MODEL 1

3 Switching Points Model(k=3) with Volume

		Value	P value	Value	P value
c_{11}	c_{21}	0.0118	0.0818	0.0280	0.0000
	c_{22}			-1.1226	0.3624
a_{11}	a_{21}	0.1735	0.0039	0.1521	0.0057
a_{12}	a_{22}	0.5334	0.0000	-0.2154	0.0008
g_{11}	g_{21}	0.4439	0.0000	0.4390	0.0000
g_{12}	g_{22}	-0.0434	0.5582	0.9722	0.0000
$d_{11(1)}$	$d_{22(1)}$	0.1340	0.6032	-1.0390	0.0012
$q_{11(1)}$	$q_{22(1)}$	0.0090	0.2693	0.0034	0.5752
$p_{11(1)}$	$p_{22(1)}$	-0.0189	0.4007	0.0279	0.1789
$d_{11(2)}$	$d_{22(2)}$	-0.7580	0.2474	1.6480	0.0346
$q_{11(2)}$	$q_{22(2)}$	-0.0083	0.3303	-0.0017	0.8464
$p_{11(2)}$	$p_{22(2)}$	0.0128	0.6180	-0.0054	0.8099
$d_{11(3)}$	$d_{22(3)}$	0.1690	0.8442	-1.1910	0.2216
$q_{11(3)}$	$q_{22(3)}$	-0.0150	0.1805	0.0130	0.3112
$p_{11(3)}$	$p_{22(3)}$	0.0404	0.2603	-0.0309	0.3219
w_{11}	w_{21}	0.4420	0.0000	0.0354	0.0372
Log Likelihood			3368.231		