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Abstract

The proportion of informal or unprotected workers in developing countries is large. In developing economies, the fraction of informal workers can be as high as 70% of total employment. For economies with significant informal sectors, business cycle fluctuations and labor market policy interventions can have important effects on the unemployment rate, and also produce large reallocations of workers between "regulated" and "unregulated" jobs. In this paper, we report the main cyclical patterns of one such labor market: Brazil. We then use the empirical regularities found in the data to build, calibrate, and simulate a two-sector search and matching labor market model, in which firms have the choice of hiring workers formally or informally. We find that our model, built in the spirit of traditional search and matching models, can explain well most of the cyclical properties found in the data. We also show that government policies that decrease the cost of formal jobs, or increase the cost of informality, raise the share of formal employment while reducing unemployment.

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1 Introduction

The coexistence of formal and informal jobs in developing economies has attracted the attention of researches and policymakers over the last fifty years. It is estimated that between 30% to 70% of the labor force in developing countries is employed in informal jobs.[1] There is a vast literature which seeks to understand why there are large informal sectors in these countries, and the implications for the rest of the economy.[2] However, there has been little attention given to understanding the flows in and out of unemployment in economies with informal jobs. Due to the existence of large informal sectors, we would expect that the labor markets in these countries display cyclical worker flow patterns different from those in the developed world.[3]

The aim of this paper is to understand, both empirically and theoretically, the way labor markets in economies with large informal sectors respond to cyclical fluctuations and government policies. To that end, we document the basic cyclical labor market facts for a country that has attracted a fair amount of attention in the empirical literature, Brazil, a middle-income country with a large informal sector. We then build, calibrate, and simulate a model to try to account for these empirical facts. Finally, we use this theoretical framework to assess the effects of government policies on the labor market flows and the rates of unemployment and formality.

In Brazil, where labor force surveys allow a clear distinction between formal and informal workers, the share of informality ranges from 35% to 45% of total employment in the 1980s and 1990s. In this period, the unemployment rate remains at an average level of 5.5%, low compared to some OECD countries. The data also shows that, in terms of cyclical fluctuations, the unemployment rate is strongly countercyclical, whereas the share of formal employment is procyclical. Labor market flows also show marked patterns. We find that separations from both formal and informal jobs are countercyclical, especially from informal jobs. The job-finding rate of formal jobs, both from unemployment and directly from informal jobs, is strongly procyclical, while accessions towards informal jobs are much less volatile and do not show such a strong cyclical pattern. We perform an unemployment accounting exercise which shows two important facts: First, almost 80% of the cyclical fluctuations in unemployment can be attributed to changes in the job separations rates. And second, variations in the rate at which workers transition to formality, from unemployment and directly from informal jobs, can account for more than 80% of all the cyclical fluctuations in the share of formal employment.

Guided by the previous evidence, this paper presents a model in the style of the search and matching literature. We incorporate three main features: First, due to the importance of the fluctuations of job separations, we anchor our model in the spirit of the endogenous job destruction literature, such as in Mortensen and Pissarides (1994). Second, we endogenize

[1]See Hart (1973), de Soto (1989), and Schneider and Enste (2000) for extensive reviews of the causes and consequences of this type of employment. See also Djankov et al. (2002) and Schneider (2003) for detailed cross-country estimates of the size of the informal economy for developed and developing countries.

[2]See Fields (2006) for an extensive review of multisectoral models with informal jobs.

[3]See Bosch and Maloney (2008) and Hoek (2002) for comprehensive microeconometric studies of these flows for Brazil and Mexico.

the decision of the firm to hire formal or informal workers in order to explain the different behaviour of the job-finding rates of formal and informal jobs. We assume that firms post "generic" vacancies and, when the worker arrives, they decide whether to establish a formal or an informal relationship. The type of contract chosen depends on the quality of the match and the trade-offs between formal and informal employment. Formal contracts can fully exploit the productivity of the match at the cost of abiding with labor regulations. Informal contracts avoid labor regulations, but firms face a penalty if the government detects them. The third important feature of the model is that, given the quantitative importance of direct flows from informal to formal jobs, we allow for direct transitions from informal into formal employment.

The model highlights the substitutability between formal and informal contracts within similar job types. The assumption that vacancies posted by firms can be filled formally or informally seeks to reflect the idea that jobs are not intrinsically formal or informal. The motivation for modelling this margin is rooted in the fact that most of the changes in the share of formal employment occurs within industries, occupations, and population groups. This suggests that reallocation of labor from formal industries/occupations towards informal industries/occupations is not the main driving force behind changes in the proportion of formal jobs.

We calibrate the model to match basic facts of the Brazilian economy, simulate it, and show that the model does a good job at reproducing the empirical correlations and elasticities of the main variables. The intuition is as follows: As is standard in search and matching models, expansions, or times of positive productivity shocks, foster vacancy creation. This increases the number of meetings between firms and workers, the so-called *meeting effect*. Our model has an extra effect: In good times, firms expand the use of formal contracts, since these types of contracts allow them to take more advantage of the increase in productivity. This is the so-called *offer effect*. These two effects reinforce each other and produce an increase in the job-finding rate for formal jobs. At the same time, we may observe an increase or a decrease in the finding rate for informal jobs, since the two effects act diametrically, which explains the relatively low volatility of this rate in the data. Given the higher profits of firms during booms, destruction of both types of jobs drops. In all, after a positive productivity shock, there is more job creation and less job destruction, which explains the countercyclicality of the unemployment rate. The procyclicality of the share of formal employment in the model depends, however, on the relative strengths of job creation and job destruction in each sector of employment. We show that under a reasonable parametrization, the model can also reproduce the procyclicality of the share of formal employment.

Our theoretical model provides a framework for studying the effects of policy changes on the allocation of workers in developing countries. We examine the impact of changes in five labor market policies: hiring costs, firing costs, payroll taxes, monitoring of the informal sector and fines to informal firms. We find that policies that reduce the cost of formality, or those that increase the cost of informality, produce an increase in the share of formal employment while also reducing unemployment. We show that these interventions affect the labor market not only by modifying the creation of vacancies and the destruction of jobs, but also by changing the firm's hiring standards. This generates a reallocation between formal and informal jobs which has non-neutral effects on the unemployment rate, since informal jobs report much higher separation rates.

We are not the first to analyze the existence of informal employment in a model with search frictions and to use it to understand the impact of policies in the labor market. There is a growing theoretical literature that explores models through which policy can alter the equilibrium in these labor markets. Albrecht et al. (2006) argue that workers' productivity is the major determinant of participation in the informal sector. They use a model with heterogeneous workers, and show that the emergence of informal jobs is rooted in the decision of low-productivity workers to become informally self-employed. Other models assume the exogenous existence of both formal and informal firms posting vacancies. In these models, heterogeneous workers direct their searches towards one of the two types of firms according to their moral costs of operating in the informal sector (Fugazza and Jaques, 2002, and Kolm and Larsen, 2002), to worker's education (Kolm and Larsen, 2004), or to productivity differences (Boeri and Garibaldi, 2006).[4] Other types of models endogenize the firm's choice. Kugler (1999) assumes that firms sort themselves into formal or informal statuses according to their ex-ante productivity levels, and workers are matched randomly into formal and informal firms. Antunes and Cavalcanti (2007) and Bosch (2004) build occupational choice models. In these models, agents are allowed to decide between becoming formal entrepreneurs, informal entrepreneurs, or workers in search of a jobs. Zenou (2008), considers a model where the formal sector is subject to search frictions, whereas the informal market is competitive. He then analyzes the impact of various policies on the size of the formal sector and the equilibrium wages. However, due to the lack of data, this growing body of literature has lacked the empirical scrutiny that its counterpart in developed countries has received (see Shimer, 2005). The contribution of this paper is to build a search and matching model that is consistent with the empirical regularities found in developing countries. This provides a more solid base for the assessment of policy interventions in these labor markets.

The remainder of the paper is organized as follows. The next section summarizes the empirical evidence on worker flows for Brazil. Section 3 presents the steady state model and the equilibrium in the stochastic environment. Section 4 shows the calibration of the model parameters. Section 5 shows the simulation results and compares them with the empirical evidence of Section 2. Section 6 describes the policy experiments. Finally, section 7 summarizes and concludes.

2 Empirical Evidence

We focus our empirical analysis on Brazil, where detailed labor force surveys allow us to compute with precision not only the proportion of informal employment in the economy, but also the gross

[4]Alternatively, Bouev (2002) suggests that workers may search randomly.

movements of workers between employment statuses.[5]

It is widely accepted that formal workers are those working in firms licensed with the government and conforming to tax and labor laws, including minimum wage directives, pension and health insurance benefits for employees, workplace standards of safety, etc. Firms employing informal workers, on the contrary, are largely de-linked from state institutions and obligations, and their employees are not covered by formal labor protections. As is standard in the literature, we define informal employment as the informal micro-entrepreneurs and those salaried workers whose employers do not comply with social security regulations. Using the Brazilian labor survey from 1987 to 2001 (Pesquisa Mensual de Emprego, PME), we identify as informal employment those self-employed workers who are not professionals or technicians, and salaried workers who are not in possession of a work-card or "carteira de trabalho" that entitles them to labor rights and benefits.[6] In what follows, we explain the main facts emerging from the data in Brazil between the first quarter of 1983 to last quarter of 2001.

Figure 1(a) shows the two main indicators of the functioning of the Brazilian labor market, the unemployment rate, u, and the share of formal employment over total employment, π. We can see that the unemployment rate is relatively low compared to OECD countries. Despite major macroeconomic shocks during the 1980's and 1990's, the unemployment rate never increased above 8%. Additionally, informality as a share of total employment comprises a substantial part of total employment, between 35%-45%. We can observe in Table 1 that, consistent with the evidence presented in Figure 1(a), the unemployment rate is strongly countercyclical, with a cross correlation with output of -0.83. Furthermore, it is 5.41 times as volatile as output. The share of formal employment is procyclical and around half as volatile as output.[7] To obtain further insight into the evolution of these two stocks, it is useful to analyze the flows of workers and the transition probabilities between the different states.[8]

Taking advantage of the panel dimension of the PME, we compute the monthly transition probabilities among our three employment states by averaging the number of transitions from one state to another over the stock of the sector of origin. We then take logs, seasonally adjust the result, take quarterly averages of those probabilities, and detrend the series using an HP filter with smoothing parameter 1600. Figures 1(b) - 1(d) show the evolution of the levels of transition probabilities among employment states; the cyclical properties of the series and its correlation with output can be found in Table 1. Several facts merit attention.

First, the probability with which workers find formal jobs from unemployment, the job-

[5]See Appendix A for details on the data. Empirical studies that analyze the Mexican economy, i.e. Bosch and Maloney (2008), find that the cyclical patterns explained here for Brazil are also extensible, for the most part, to Mexico.

[6]See Appendix A for details. Also for a detailed discussion on the issues around the definition of informal workers in Brazil see Bosch and Maloney (2008), Hoek (2002) and Henley et al. (2006)

[7]Since the share of formality and informality add up to one, the correlation of these two variables with output is of the same magnitude, but opposite sign.

[8]The transition probabilities presented in Figure 1 correspond to quarterly averages of monthly transitions rates. Throughout the paper we refer to them as the job-finding and job separation rates, although technically they correspond to discrete transition probabilities. The continuous transition rates exhibit exactly the same cyclical pattern. See Bosch and Maloney (2006) and Bosch et al. (2006) for details.

finding rate for formal jobs JFR_f, is strongly procyclical, whereas that for informal jobs, JFR_i, is only weakly procyclical, with cross-correlations with output of 0.74 and 0.24, respectively. Furthermore, the job-finding rate for formal jobs is more volatile than that for informal jobs, around 3 times more variable. In terms of elasticities, we find that an increase in the Brazilian output of 1% increases the job-finding rate for formal jobs by 3.93%, but that for informal jobs by 10 times less. This evidence suggests a much higher responsiveness in the finding rate for formal jobs than that for informal ones.

Second, the probability of losing a job, the job separation rates JSR_f and JSR_i, are countercyclical for both types of jobs. That is, in bad times there is a higher probability of losing a job. However, the countercyclicality is substantially higher for informal jobs than for formal ones, with cross-correlations with output of -0.75 and -0.28, respectively. Furthermore, the volatility of the job separation rate for informal jobs is also higher. Indeed, the elasticity with respect to output of the job separation rate for informal jobs is more than 4 times higher than that for formal jobs, -4.41 vs -1.01. However, note that despite the large increases in the job separation rate for informal jobs during recessions, the share of formal employment decreases in downturns.

Finally, in terms of direct transitions between informality and formality, we find that the probability of transitioning from an informal job to a formal job, JJ_{i-f}, is, as expected, strongly procyclical. However, the direct transition rate to formal jobs is less volatile than the transition rate from unemployment. Similarly, the elasticity with respect to output is 1.72, around half as large as the elasticity of the flow from unemployment into formal jobs. The opposite flow, from formal jobs to informal jobs, JJ_{f-i}, is also procyclical, although less volatile than the flow from informal to formal jobs.

The previous evidence suggests that changes in unemployment and the share of formality seem to be explained by changes in the access to formal jobs, both from unemployment and informal jobs, and changes in the separation rate for informal jobs, rather than from changes in the outflows from formal employment. In order to quantify the relative contributions of each flow, we perform an unemployment accounting exercise, in the spirit of other papers in the literature (see Fujita and Ramey, 2008, and Shimer, 2007 for details). The exercise consists of three steps: First, we simulate the steady-state unemployment rate and the share of formal employment for each period using the empirical flows between states from the previous period.[9] Second, we simulate the counterfactual unemployment rates and the share of formal employment when each of the 6 possible flows in our data is allowed to vary individually. Third, we compute the covariance between the detrended counterfactual series and the the detrended unemployment rate and share of formal employment.[10] Table 2 shows this set of covariances as a proportion of the variance of the detrended steady-state unemployment rate and share of formal employment. These covariances can be interpreted as the contribution of the variability of a particular flow to the total

[9]This first step delivers a very good approximation of both series. The cross correlation between the steady state unemployment rate and the actual series is 0.95. The simulation of the steady state share of formal employment is less accurate with a cross correlation of 0.85.

[10]The series are detrended using the Hodrick and Prescott (HP) filter with smoothing parameter 1600.

cyclical variability of either the unemployment rate or the share of formal employment.[11] Two main lessons can be drawn from this exercise. First, the volatility of unemployment is explained in great part by changes in the job separation rate, especially for informal jobs. Approximately, 71% of unemployment variability is attributable to changes in the job separation rate for informal jobs. The contribution of the job-finding rate is 21%, and it is almost fully accounted for by changes in the job-finding rate for formal jobs (16%). And second, fluctuations in the share of formal jobs are mostly due to changes in the probability of transiting towards formality from informality, 82%, with a small fraction due to swings in the job-finding rate for informal jobs, 11%.[12]

In summary, the empirical evidence presented above tells us that (i) unemployment and the share of informality are countercyclical; (ii) job-finding rates are procyclical, but substantially more volatile into formal jobs; (iii) job separation rates are countercyclical, and more variable from informal employment; (iv) direct transitions from informality into formality are procyclical, and (v) the cyclical variability of unemployment is mostly explained by movements in the separation rate for informal workers, while the volatility of the formal employment share is explained by changes in transition rate from informal to formal jobs.

In the following section we build a model to account for these facts, and use it to analyze the extent to which the cyclical patterns in the flows and stock variables are explained by aggregate productivity movements, and to assess policy interventions.

3 The Model

The model is a continuous time search and matching labor market model. In light of the evidence presented above, we introduce three key features in the model. First, in order to account for the existence of formal and informal jobs in the labor market, we introduce a firm-internal decision between two types of contract, formal and informal. Second, given the quantitative importance of the job separation rate in unemployment volatility, we build the model in the spirit of Mortensen and Pissarides (1994), with endogenous job separations. Third, we allow for direct transitions from informal into formal jobs to capture the cyclical variability in the share of formal employment.

As in the standard search and matching model, we assume that firms and workers try to meet in the labor market and form employment relationships, which are composed of one firm and one worker. Matches occur randomly and according to a matching function $m = m(u, v)$, where u is the total number of unemployed workers and v the number of vacancies. The matching

[11] As noted by Fujita and Ramey (2007), the decomposition is not an exact one, and hence the sum of the contributions does not necessarily add up to one. However, we find that it is a very good approximation of both the unemployment and the share of formal employment series.

[12] This is consistent with evidence presented by Bosch and Maloney (2008) for Mexico and Brazil. In that case, the authors consider the contributions of flows between five employment states, including out of the labor force and two sectors of informality. Although the contributions are quantitatively different, qualitatively the results are the same.

technology is homogeneous of degree one, and increasing and concave in both arguments. A firm having a vacancy matches with an unemployed worker according to a Poisson process with arrival rate $q(\theta) = m(u,v)/v$, where $\theta = \frac{v}{u}$ is the market tightness of the economy. Similarly, the arrival rate of vacancies for workers is $q(\theta) = m(u,v)/u$.

Workers are ex-ante identical, but when a match is formed, the productivity of the employment relationship is revealed and heterogeneity arises. This productivity is composed of an aggregate component, p, which is common to all firms, and an idiosyncratic term, ε, which is specific to each match and is initially drawn from a distribution $G : \left[\varepsilon^G_{\min}, \varepsilon^G_{\max}\right] \to [0,1]$. Depending on the productivity level, the firm decides whether to offer a formal or an informal contract. Firms involved in formal productive matches incur the following costs associated with government regulations: payment of a hiring cost, c, upon signing the contract; payroll taxes, where the marginal tax rate is τ; and payment of a firing cost, F, when the formal employment relationship is finished. If the firm chooses to hire the worker as an informal employee, it does not have the burden of the aforesaid costs. However, evading them implies that it can only take advantage of a fraction $\delta \in (0,1)$ of the productivity of the match. This assumption is based on microeconometric analysis of labor markets with large informal sectors, which shows that there is a substantial unexplained wage gap (20% to 30%) between formal and informal workers, even after controlling for observables.[13] We also assume that if an informal firm is discovered by the government, which occurs according to a Poisson process with arrival rate ϕ, it is forced to terminate the match and to pay a fine σ. All of these costs to formal and informal firms are paid to the government and are used in activities outside of the model.

During an employment relationship, ongoing formal and informal matches draw new idiosyncratic productivity levels according to a Poisson process with arrival rates λ_f and λ_i, respectively. These new productivities are i.i.d. across firms and time and drawn from a distribution $H : \left[\varepsilon^H_{\min}, \varepsilon^H_{\max}\right] \to [0,1]$. With the new realization of the idiosyncratic productivity, firms and workers decide whether to continue with the current match, to change the type of contract, or to terminate it. While it is possible that formal firms break a formal contract and start an informal employment relationship with the same worker, we abstract this case from our model. The main reason is that quantitatively, this flow is not crucial to explain the fluctuations of unemployment or the share of formal employment.

Let us now explain the problem of firms and workers in more detail.

3.1 Problem of the Firm

Let V be the present discounted value of posting a vacancy for a firm. Similarly, let $J^l_f(\varepsilon)$ and $J_i(\varepsilon)$ represent the value for a firm of occupied formal and informal jobs respectively, where $l \in \{n, o\}$ identifies the new and ongoing formal matches.

[13]See Gonzaga (2003)and Almeida and Carneiro (2005) for papers referring to the productivity differential between formal and informal jobs in Brazil; see also Marcoullier et al. (1997) for measures of the informal wage gap in Mexico, El Salvador, and Peru.

Given the environment explained previously, the value for a firm of posting a vacancy is:

$$rV = -k + q(\theta) \int_{\varepsilon_{\min}^{G}}^{\varepsilon_{\max}^{G}} \max \left[J_f^n \left(\varepsilon' \right) - c, J_i \left(\varepsilon' \right), V \right] dG \left(\varepsilon' \right) - q \left(\theta \right) V. \tag{1}$$

The interpretation of equation (1) is as follows: Vacancies have a flow cost of k for being opened, and they meet potential employees at a rate $q(\theta)$. If matched with a worker, the idiosyncratic productivity, ε, is revealed and the firm faces three choices. First, it can formalize the relationship, in which case the firm enjoys a value of $J_f^n(\varepsilon)$, but has to pay the hiring cost, c. Second, the firm can avoid regulations by hiring the worker informally, and obtain a value of $J_i(\varepsilon)$. Finally, the firm may prefer to keep searching and retain the vacancy. However, we assume that the distribution of initial productivities is such that the worst initial idiosyncratic productivity realization is high enough to generate match formation.[14]

The introduction of firing costs in formal jobs gives rise to two different value functions for occupied jobs. $J_f^n(\varepsilon)$ is the value for a firm to form a new employment relationship with idiosyncratic productivity ε. These new matches differ from ongoing ones, whose value is $J_f^o(\varepsilon)$, in the wages payed. The reason is that new matches are not subject to the firing cost should a productive relationship not be formed[15]. The value for a filled formal firm with idiosyncratic productivity ε is:

$$rJ_f^l \left(\varepsilon \right) = p + \varepsilon - \left(1 + \tau \right) w_f^l \left(\varepsilon \right) + \lambda_f \int_{\varepsilon_{\min}^{H}}^{\varepsilon_{\max}^{H}} \max \left[J_f^o \left(\varepsilon' \right), V - F \right] dH \left(\varepsilon' \right) - \lambda_f J_f^l \left(\varepsilon \right), \; l \in \{n, o\}, \tag{2}$$

An occupied formal firm produces output as the combination of the general and match specific idiosyncratic productivity, p, and ε, respectively. It pays wages, $w_f^l(\varepsilon)$, and pays payroll taxes, whose marginal rate is τ. It draws a new idiosyncratic productivity ε' according to a Poisson process with arrival rate λ_f, and then decides whether to continue the match, which has a value of $J_f^o(\varepsilon')$, or to dissolve it. If it chooses the latter, it becomes a vacant firm after paying the firing cost F.

Similarly, we can describe the value of an informally filled firm as:

$$rJ_i \left(\varepsilon \right) = \delta \left(p + \varepsilon \right) - w_i \left(\varepsilon \right) + \lambda_i \int_{\varepsilon_{\min}^{H}}^{\varepsilon_{\max}^{H}} \max \left[J_f^n \left(\varepsilon' \right) - c, J_i \left(\varepsilon' \right), V \right] dH \left(\varepsilon' \right) - \lambda_i J_i \left(\varepsilon \right) + \phi \left(V - J_i \left(\varepsilon \right) \right) - \phi\sigma. \tag{3}$$

The value for a filled job for an informal firm is composed of the current flow output net of wages and the continuation value. As explained before, informal jobs are less productive than formal ones, since $\delta < 1$, but on the other hand they avoid paying taxes. The arrival rate of new

[14] An earlier version of this paper relaxed this assumption and allowed matches to be discarded if the initial idiosyncratic productivity was too low. In such a scenario, there is an additional threshold that determines match formation, which coincides with that determining job separation of informal workers. The results of this alternative specification are not qualitatively different from those presented here.

[15] See Chapter 9 of Pissarides (2000) for a discussion on the differences between new and ongoing matches in the presence of firing costs.

idiosyncratic productivities is λ_i, and after drawing the new level, ε', the firm chooses whether to pay the hiring cost and formalize the relationship, to remain informal, or to dissolve the match and create a vacancy. It is also possible that the informal job gets discovered, which happens according to a Poisson process with arrival rate ϕ, in which case the match is forced to dissolve and the firm forced to pay a fine, σ.

3.2 Problem of the Worker

The value functions for the workers are given by

$$rU = b + \theta q(\theta) \int_{\varepsilon^G_{\min}}^{\varepsilon^G_{\max}} \max \left[(W_f^n(\varepsilon'), W_i(\varepsilon'), U \right] dG(\varepsilon') - \theta q(\theta) U, \tag{4}$$

$$rW_f^l(\varepsilon) = w_f^l(\varepsilon) + \lambda_f \int_{\varepsilon^H_{\min}}^{\varepsilon^H_{\max}} \max \left[W_f^o(\varepsilon'), U \right] dH(\varepsilon') - \lambda_f W_f^l(\varepsilon), \quad l \in \{n, o\}, \tag{5}$$

$$rW_i(\varepsilon) r = w_i(\varepsilon) + \lambda_i \int_{\varepsilon^H_{\min}}^{\varepsilon^H_{\max}} \max \left[W_f^n(\varepsilon'), W_i(\varepsilon'), U \right] dH(\varepsilon') - \lambda_i W_i(\varepsilon) + \phi (U - W_i(\varepsilon)). \tag{6}$$

The interpretation of equations (4) to (6) is similar to that of the problem of the firm. Let U represent the present discounted value of an unemployed worker. While searching, the worker gets a flow value of b. He finds jobs at a rate $\theta q(\theta)$, and depending on the type of contract offered by the firm, he enjoys the value of a new formal job $W_f^n(\varepsilon)$, or that of an informal job, $W_i(\varepsilon)$. Once the worker is employed, the value of which is represented in equations (5) and (6), he gets paid a wage depending on the type of contract, formal or informal, and the productivity of the match. The idiosyncratic productivity changes with time, and when it does, the worker decides whether it is beneficial for him to continue working for the firm with the same type of contract as before, to try to change it (if he is currently an informal worker), or to become unemployed and search for another job. Informal workers are also subject to forced separation from the firm if they are discovered by the government.

3.3 Surplus and Wages

The surplus of a match is defined as the gain by firm and worker of forming a productive match after netting their losses. The existence of hiring and firing costs generates a difference in the wages of new and already established formal relationships, which is reflected in different surpluses. The equations which determine the match surpluses are:

$$S_f^n(\varepsilon) = \left(J_f^n(\varepsilon) - c \right) + W_f^n(\varepsilon) - V - U, \tag{7}$$

$$S_f^o(\varepsilon) = J_f^o(\varepsilon) + W_f^o(\varepsilon) - (V - F) - U, \tag{8}$$

$$S_i(\varepsilon) = J_i(\varepsilon) + W_i(\varepsilon) - V - U. \tag{9}$$

Following the literature, when a firm and a worker first meet, or when an idiosyncratic shock arrives, they choose the wages as the Nash solution to a bargain problem, where β is the bargaining power of the worker. The optimal conditions for wage determination deliver the surplus sharing rules, where firm and worker get a constant fraction of the surplus generated by the match. However, due to the existence of payroll taxes, the formal sector sharing rules differ from the standard ones, in which the worker gets a share of the surplus equal to his bargaining power. The following equations summarize the rules to divide the surplus:

$$\begin{aligned} \left(J_f^n(\varepsilon) - c\right) - V &= \frac{(1-\beta)(1+\tau)}{\beta}\left(W_f^n(\varepsilon) - U\right), &(10)\\ J_f^o(\varepsilon) - (V - F) &= \frac{(1-\beta)(1+\tau)}{\beta}\left(W_f^o(\varepsilon) - U\right), &(11)\\ J_i(\varepsilon) &= \frac{1-\beta}{\beta}\left(W_i(\varepsilon) - U\right). &(12)\end{aligned}$$

Equations (10) to (12) highlight a well-known fact of Nash bargaining. Firms and workers always agree on the type of contract, as well as when to destroy the match. From the hiring point of view, firms are willing to hire a worker formally as long as $J_f^n(\varepsilon) - c > J_i(\varepsilon)$ and $J_f^n(\varepsilon) - c - V > 0$, which necessarily implies that $W_f^n(\varepsilon) > W_i(\varepsilon)$ and $W_f^n(\varepsilon) > U$. Similarly, the firm chooses an informal contract if $J_f^n(\varepsilon) - c < J_i(\varepsilon)$ and $J_i(\varepsilon) - V > 0$.

3.4 Steady State Equilibrium

We are now ready to characterize the steady-state equilibrium of the model. This equilibrium can be reduced to four equations on four variables, θ, ε_R, ε_{d_f} and ε_{d_i}. The four conditions that determine the equilibrium are: The job creation equation; the condition which reflects the formality/informality choice for the firm; and the job destruction equations for formal and informal jobs.[16]

The first equation states that the creation of vacancies is driven by the free entry of firms in the labor market. This implies that in equilibrium, firms post vacancies to the point at which posting an extra vacancy has a present discounted value of zero, that is, $V = 0$. Using this condition, we obtain the following equation:

$$\frac{\beta}{(1-\beta)\, q(\theta)} k = \int_{\varepsilon_{d_i}}^{\varepsilon_R} \frac{\delta(\varepsilon' - \varepsilon_{d_i})}{r + \lambda_i + \phi} dH(\varepsilon') + \int_{\varepsilon_R}^{\varepsilon_{\max}^H} \left[\frac{\varepsilon' - \varepsilon_{d_f}}{r + \lambda_f} - c - F\right] dH(\varepsilon') \tag{13}$$

This condition equates the expected cost of the vacancy to its expected profit, and determines the equilibrium market tightness.

The second condition relates to the hiring decision of the firm. We define ε_R as the level of idiosyncratic productivity that makes the firm indifferent between hiring the worker formally or informally. This can be expressed as

$$J_f^n(\varepsilon_R) - c = J_i(\varepsilon_R), \tag{14}$$

[16]For the derivations of the equilibrium conditions please refer to Appendix B.

which after some algebra delivers the following equation

$$\varepsilon_R = \frac{(r+\lambda_f)(r+\lambda_i+\phi)(c+F)+(r+\lambda_i+\phi)\,\varepsilon_{d_f}-\delta(r+\lambda_f)\,\varepsilon_{d_i}}{r+\lambda_i+\phi-\delta(r+\lambda_f)}. \tag{15}$$

The last two conditions determine the job separation threshold of formal jobs, equation (16), and informal jobs, equation (17). The threshold ε_{d_f} represents the productivity level that makes an ongoing formal job unprofitable. Similarly, the job separation threshold of informal jobs is given by ε_{d_i}. These thresholds satisfy the following two equations:

$$J_f^o\left(\varepsilon_{d_f}\right)+F=0, \tag{16}$$

$$J_i\left(\varepsilon_{d_i}\right)=0. \tag{17}$$

Their explicit formulations are:

$$\varepsilon_{d_f} \;=\; -p+(1+\tau)\,b-rF+\frac{\beta k\theta}{1-\beta}+\Omega-\lambda_f\int_{\varepsilon_{d_f}}^{\varepsilon_{\max}^H}\frac{\varepsilon'-\varepsilon_{d_f}}{r+\lambda_f}dH(\varepsilon'), \tag{18}$$

and

$$\begin{aligned}\varepsilon_{d_i} \;=\;& -p+\frac{b}{\delta}-\frac{\phi\sigma}{\delta}+\frac{\beta k\theta}{\delta(1+\tau)(1-\beta)}+\frac{\Omega}{\delta(1+\tau)}\\ &-\lambda_i\int_{\varepsilon_{d_i}}^{\varepsilon_R}\frac{\varepsilon'-\varepsilon_{d_i}}{r+\lambda_i+\phi}dH(\varepsilon')-\frac{\lambda_i}{\delta}\int_{\varepsilon_R}^{\varepsilon_{\max}^H}\left(\frac{\varepsilon'-\varepsilon_{d_f}}{r+\lambda_f}-c-F\right)dH(\varepsilon'),\end{aligned} \tag{19}$$

where

$$\Omega=\beta\tau\theta q(\theta)\int_{\varepsilon_{\min}^G}^{\varepsilon_R}\frac{\delta\left(\varepsilon'-\varepsilon_{d_i}\right)}{r+\lambda_i+\phi}dG(\varepsilon'). \tag{20}$$

Ω captures the part of the outside option of the worker associated with the fact that informal contracts are not subject to taxation. More specifically, since the informal firm evades taxes, the surplus of an informal match has an extra benefit associated with this tax savings. This is recognized by the unemployed worker and captured in the expected value of future employment. The increase in the outside option of the worker lowers the surplus of every match and raises the minimum productivity that makes the matches profitable.

The job destruction thresholds are both increasing in the market tightness, the flow value of unemployment, and the term Ω, since all of these increase the outside option of the worker and reduce the surplus. They are decreasing in the aggregate productivity and the expected future profits of continuing the match, since, if either of these increase, the surplus of the match increases and the incentive to separate drops.

It is easy to show the existence and uniqueness of the equilibrium in this model. Higher θ increases the left-hand side of equation (13), since more vacancies per unemployed worker increases waiting time for firms. Furthermore, it lowers the right-hand side of equation (13), as both separation thresholds depend positively on θ, and in equilibrium, the formal/informal threshold, ε_R, does not affect the expected profits from posting a vacancy due to the envelope condition. Hence, there is a unique value of θ that satisfies equation (13).

The equilibrium is depicted in Figure 2. This figure shows the equilibrium in the ε_R-θ,[17] and $\varepsilon_{d_f}, \varepsilon_{d_i}$-$\theta$ spaces. Panel (b) shows how equations (13) and (15) determine the equilibrium values of ε_R and θ. Given the equilibrium value of θ, Panel (c) displays the equilibrium for ε_{d_f} and ε_{d_i}.

Once the equilibrium values of the market tightness and the thresholds are obtained, we can retrieve the steady state values of unemployment and formal and informal employment. Normalizing the labor force to unity, the steady state values of these variables are given by

$$u = \frac{\lambda_f H\left(\varepsilon_{d_f}\right) n_f + \left(\lambda_i H\left(\varepsilon_{d_i}\right) + \phi\right) n_i}{\theta q\left(\theta\right)}, \tag{21}$$

$$n_f = \frac{\theta q\left(\theta\right)\left[1 - G\left(\varepsilon_R\right)\right] u + \lambda_i \left[1 - H\left(\varepsilon_R\right)\right] n_i}{\lambda_f H\left(\varepsilon_{df}\right)}, \tag{22}$$

$$n_i = 1 - n_f - u. \tag{23}$$

The steady state level of unemployment is determined by the flows in and out of that state. Flows into unemployment come from occupied formal and informal jobs, which are destroyed at rates $\lambda_f H\left(\varepsilon_{d_f}\right)$ and $\lambda_i H\left(\varepsilon_{d_i}\right) + \phi$ respectively. The rates at which workers flow out of unemployment can be divided in two: The job-finding rate for formal jobs, which is given by

$$JFR_f = \theta q\left(\theta\right)\left[1 - G\left(\varepsilon_R\right)\right], \tag{24}$$

and the job-finding rate for informal jobs, given by

$$JFR_i = \theta q\left(\theta\right) G\left(\varepsilon_R\right). \tag{25}$$

Informal jobs are converted into formal ones if the informal firm draws a high enough idiosyncratic productivity, which occurs at rate $\lambda_i \left[1 - H\left(\varepsilon_R\right)\right]$.

Now that we have explained the steady state of the model, and have established the existence and uniqueness of such equilibrium, we are ready to modify the model to account for aggregate shocks. Such aggregate stochastic movements deliver cyclical fluctuations in the model, which we can use to compare with those observed in the data and hence test the model's performance[18].

[17] In the representation of the equilibrium in Figure 2, we have assumed a positive relationship between ε_R and θ, since that is the relationship we observe in the calibrated model below. Theoretically, this is not necessarily the case, since it depends on the magnitudes of $\frac{\partial \varepsilon_{d_f}}{\partial \theta}$ and $\frac{\partial \varepsilon_{d_i}}{\partial \theta}$. However, the slope of this curve does not qualitatively alter the derivation of the equilibrium.

[18] For the purpose of concision, we have not included here the comparative statics for the model. This exercise can be found in Appendix C for a simpler version of the model with no informal upgrading and payroll taxes. Such a comparative statics exercise provides intuition for the behavior of the steady state of the model following changes in aggregate productivities, and helps to understand the results of the stochastic simulations of the model.

3.5 Stochastic Equilibrium

We now generalize the previous model to incorporate stochastic movements in aggregate productivity. The stochastic equilibrium of the model is as the steady state one, except that the job separations equations (18) and (19) are altered due to the incorporation of the aggregate shock. These two equations differ from their steady state counterparts since when faced with aggregate productivity shocks, firms and workers have to decide hirings and separations, taking into account the probability that the aggregate state of the economy may change. The separation equations in this stochastic environment are

$$\varepsilon_{d_f}(p) = -p + (1+\tau)b - rF + \frac{\beta k\theta(p)}{1-\beta} + \Omega - \lambda_f \int_{\varepsilon_{d_f(p)}}^{\varepsilon^H_{\max}} \frac{\left(\varepsilon' + \varepsilon_{d_f}(p)\right)}{(r+\lambda_f+\mu)} dH(\varepsilon') - \Gamma_f, \quad (26)$$

$$\begin{aligned} \delta\varepsilon_{d_i}(p) &= -\delta p + b - \phi\sigma + \frac{\beta k\theta(p)}{(1+\tau)(1-\beta)} + \frac{\Omega}{1+\tau} - \lambda_i \int_{\varepsilon_{d_i}(p)}^{\varepsilon_R(p)} \frac{\delta\left(\varepsilon' - \varepsilon_{d_i}(p)\right)}{r+\lambda_i+\mu+\phi} dH(\varepsilon') \\ &\quad -\lambda_i \int_{\varepsilon_R(p)}^{\varepsilon^H_{\max}} \left[\frac{\varepsilon' + \varepsilon_{d_f(p)}}{r+\lambda_f+\mu} - c - F\right] dH(\varepsilon') - \Gamma_i, \end{aligned} \quad (27)$$

where Ω is as in equation (20); μ represents the arrival rate of an aggregate shock; $S(p'|p)$ is the conditional distribution of the next state of the economy p', given that we are in p; and Γ_f and Γ_i are the option values generated by the fact that there are fluctuations in the aggregate state p, and are described by the following equations:

$$\Gamma_f = \mu \int_{p_{\min}}^{p_{\max}} \max\left[J^o_f(p',\varepsilon) + F, 0\right] dS(p'|p)$$

$$\Gamma_i = \mu \int_{p_{\min}}^{p_{\max}} \max\left[J^n_f(p',\varepsilon_{d_i}) - c, J_i(p',\varepsilon_{d_i}), 0\right] dS(p'|p).$$

In the presence of aggregate shocks, the law of motion of unemployment, and formal and informal employment are also different. The reason is that upon the arrival of a negative aggregate shock, some jobs, those between the two separation thresholds, are immediately destroyed. Similarly, if a good shock arrives, some informal jobs are immediately upgraded to formal jobs. Let $n_f(\varepsilon)$ be the number of formal workers with idiosyncratic productivity ε at the beginning of the current period. Workers flow out of that pool when they update their idiosyncratic productivity, which occurs at rate λ_f. New workers flow into the pool at different rates depending on their origin. From the formal employment pool, which overall has n_f workers, the inflow rate is $\lambda_f \frac{\partial H(\varepsilon)}{\partial \varepsilon}$; from the pool of informal workers, which contains n_i workers, these flow in at rate $\lambda_i \frac{\partial H(\varepsilon)}{\partial \varepsilon}$ as long as $\varepsilon > \varepsilon_R$; and from the unemployment pool, which counts u workers, these flow in at rate $\theta q(\theta)\frac{\partial G(\varepsilon)}{\partial \varepsilon}$. Therefore, the law of motion for the distribution of formal employment is

$$n_f^{'}(\varepsilon) = \left\{ \begin{array}{ll} n_f(\varepsilon)(1-\lambda_f) + \lambda_f \frac{\partial H(\varepsilon)}{\partial \varepsilon} n_f + \lambda_i \frac{\partial H(\varepsilon)}{\partial \varepsilon} n_i + \theta(p)\, q(\theta(p)) \frac{\partial G(\varepsilon)}{\partial \varepsilon} u & \text{if } \varepsilon > \varepsilon_R(p) \\ n_f(\varepsilon)(1-\lambda_f) + \lambda_f \frac{\partial H(\varepsilon)}{\partial \varepsilon} n_f & \text{if } \varepsilon_R(p) > \varepsilon > \varepsilon_{df}(p) \\ 0 & \text{if } \varepsilon < \varepsilon_{df}(p) \end{array} \right\}.$$

Similarly, the distribution of informal jobs is as follows:

$$n_i^{'}(\varepsilon) = \left\{ \begin{array}{ll} n_i(\varepsilon)(1-\lambda_i) + \lambda_i \frac{\partial H(\varepsilon)}{\partial \varepsilon} N_i + \theta(p)\, q(\theta(p)) \frac{\partial G(\varepsilon)}{\partial \varepsilon} U & \text{if } \varepsilon_R(p) > \varepsilon > \varepsilon_{di}(p) \\ 0 & \text{if } \varepsilon < \varepsilon_{di}(p) \end{array} \right\}.$$

Note that when simulating the model in this stochastic environment, we need to pay particular attention to two groups of workers whose status may change due to an aggregate productivity change, even when they do not vary their idiosyncratic productivity. These two groups are: First, formal and informal workers whose jobs are destroyed due to a negative aggregate technology shock; and second, those informal workers who may be immediately upgraded after a positive aggregate productivity shock.

4 Calibration

To perform the model simulations presented in the following section, we need to choose the forms of the different functions and processes, and assign values for the parameters in the model. The functional forms are chosen following other related studies in the literature. A subset of the parameters are fixed according to what has become standard in the literature, and the remaining parameters are calibrated to match the long-run empirical evidence of Brazil between 1983q1 to 2001q4, which would correspond to the steady state of the model. A summary of the parameter values can be found in Table 4. Let us now explain the calibration procedure in detail.

The time period of the simulation is one quarter. We set the interest rate to $r = 0.019$, which is the quarterly average from 1980 to 2001 of the government primary rate discounted by the consumer price index (see Kanczuk, 2004 for details). The matching function is assumed to be Cobb-Douglas, $m = \eta u^{\xi} v^{1-\xi}$, with unemployment elasticity $\xi = 0.5$, which is in the lower bound of the estimates from Petrongolo and Pissarides (2001). The scaling parameter, η, is calibrated jointly with other model parameters, as will be discussed later. Following the literature, we assume that the bargaining power of workers internalizes the search externalities, that is, $\beta = 0.5$.

The productivity wedge between formal and informal jobs, δ, the monitoring rate, ϕ, and the penalty for detection, σ, summarize the costs of employing informal labor. We estimate δ using the 2001 PME. We regress the log of wages on a number of observables including personal characteristics of the workers, industry and occupational dummies, and a dummy for informal status. This latter dummy gives a value of around 0.81. We set $\delta = 0.81$, which implies that informal contracts are 81% as productive as formal contracts. This is consistent with the findings of Marcouiller et al. (1997) for other Latin American countries, such as El Salvador and Peru. There is very little evidence as to how intensively the government monitors the labor market in search of informal jobs. Almeida and Carneiro (2005) study the impact of the enforcement of regulations in a firm's performance using the Investment Climate Survey collected by the World Bank in a set of Brazilian manufacturing firms. They report that, in their sample of 1641 firms, around 0.5% received some kind of labor regulations related fine. We use this estimate and set the separation of informal jobs due to monitoring to $\phi = 0.005$. The calibration of σ is done jointly with other model parameters, and is explained later.

The Doing Business database at the World Bank provides estimates of the costs of formality in Brazil. The non-wage labor costs in Brazil are 37.3% of formal wages. We use this as a proxy for the payroll taxes, and set $\tau = 0.37$. Furthermore, firms have to spend, on average, around 1 day to register and formalize a job. We take this as a proxy for the output forgone by a formal match due to hiring costs, and since the time period is one quarter, or 90 days, we set $c = 1/90$.

Calibration of the flow value of unemployment, b, has attracted significant attention in the U.S. literature. This parameter captures elements such as the value of leisure, unemployment benefits, home production, and the disutility of work. Shimer (2005) sets it to 0.4, whereas Hagedorn and Manovskii (2008) use a value of 0.955. We choose b to be between these two extreme cases, and set it to $b = 0.6$. Since, as expected, the volatility of the model changes with the value of this parameter, we show how our results change when this parameter is varied.

The arrival rate of idiosyncratic shocks, λ_f and λ_i, are important parameters in the model. As in Mortensen and Pissarides (1994), these variables control the elasticity of the job separation rate. In this case, the parameters are also crucial to determine the overall volatility of the job-finding rate, since ε_R is a function of ε_{d_f} and ε_{d_i}. We set the arrival rate of idiosyncratic productivity shocks to formal jobs, λ_f, to 0.08. This produces a volatility in the job separation rate of formal jobs similar to that in the data. We set the equivalent rate for informal jobs, λ_i, to 0.32, four times higher than its formal counterpart, which is approximately the ratio between the elasticity of the separation rate of formal and informal jobs. Given the importance of these two parameters, we study the response of the model to changes in their values thoroughly in the next section.

We follow Mortensen and Pissarides (1994) and assume that the dynamics of the aggregate productivity are captured by a three state Markov process. Productivity, p, can take three possible values, $\bar{p} - z$, $\bar{p}$, and $\bar{p} + z$, where the parameter z determines the range in the variation of productivity around its mean, $\bar{p}$. The transition probability between any two states is given by $\pi_{ij} = \mu S(p_i, p_j)$, where

$$\pi_{ij} = \begin{bmatrix} \Lambda & \Psi & 1 - \Psi - \Lambda \\ \varphi & 1 - 2\varphi & \varphi \\ 1 - \Psi - \Lambda & \Psi & \Lambda \end{bmatrix}.$$

Note that since the aggregate productivity changes are governed by the transition matrix π_{ij}, we do not need to specify either μ or $S(p_i, p_j)$. Christiano (1990) shows that this specification has a Wold representation for p of the form $p' = \rho p + (1-\rho)\bar{p} + \zeta$. In the previous process, ρ is the coefficient of autocorrelation, and satisfies that $\rho = 2\Lambda + \Psi - 1$; ζ has variance $\sigma_v^2 = \frac{z^2(1-\rho^2)}{\chi}$; and χ is the coefficient of kurtosis, which satisfies that $\chi = 1 + 0.5\frac{\Psi}{\varphi}$. We normalize $\bar{p}$ to unity, and calibrate the remaining parameters of this process to match the cyclical behaviour of labor productivity, measured as GDP per worker, for the Brazilian quarterly data in the period of study. The first order autocorrelation and standard deviation of labor productivity in the data are $\rho = 0.9475$ and $\sigma_v = 0.011$. There are two degrees of freedom in selecting a parametrization for the Markov process since it has four parameters: Ψ, Λ, φ and z. We follow Christiano (1990) and eliminate those degrees of freedom by setting Ψ =0 .0525 and the kurtosis parameter, χ, to be similar to that of a uniform distribution. Therefore, we obtain that Ψ =0 .0525, $\Lambda = 0.9475$, $\varphi = 0.025$ and $z = 0.05$.

We assume that the idiosyncratic productivity, $G(\varepsilon)$ and $H(\varepsilon)$ distributions are uniform in the range $[\varepsilon^G_{\min}, 1]$ and $[\varepsilon^H_{\min}, 1]$, respectively. Therefore, they take the forms

$$G(\varepsilon) = \frac{\varepsilon - \varepsilon^G_{\min}}{1 - \varepsilon^G_{\min}}, \qquad H(\varepsilon) = \frac{\varepsilon - \varepsilon^H_{\min}}{1 - \varepsilon^H_{\min}}. \tag{28}$$

The values of $\varepsilon^G_{\min}$, $\varepsilon^H_{\min}$ are calibrated jointly with the steady state values of the endogenous variables θ, ε_R, ε_{di}, ε_{di}, and the four remaining exogenous parameters in the model: The cost of posting a vacancy, k, the firing cost, F, the scaling parameter in the matching function, η, and the penalty for informal firms to get caught, σ. The 10 values for these parameters and endogenous variables are calibrated such that the steady state of the model matches the long-term properties in the data. In particular, they satisfy the four steady state equilibrium equations (15), (13), (26) and (27), and 6 moments: (i) the unemployment rate, 5.5%; (ii) the fraction of new jobs that are informal, 70% in the data and $G(\varepsilon_R)$ in the model; (iii) the job-finding rate for formal jobs, 30% in the data and $\eta\theta^{\xi}(1 - G(\varepsilon_R))$ in the model; (iv) the net reallocation of workers from informality to formality, 3.5% in the data and $\lambda_i(1 - H(\varepsilon_R))$ in the model[19]; (v) the job separation rate of informal jobs, 10% in the data and $\lambda_i H(\varepsilon_{di}) + \phi$ in the model; (vi) market tightness equal to unity, $\theta = 1$. To the best of our knowledge, there are no estimates of the value of the market tightness in Brazil or any comparable economy, mainly due to the lack of data on vacancies. However, as explained by Shimer (2005), the steady state value of θ is of little importance in the results, since varying it only implies a readjustment of the value of η, leaving everything else unchanged. The calibrated values of $\varepsilon^G_{\min}$ and $\varepsilon^H_{\min}$ are -0.11 and 0.59,

[19] In the data, an average of around 21% of workers in informal jobs transition to formal jobs within a quarter. At the same time, 15% of formal workers transition from formal to informal jobs every quarter. This implies that the net transfer rate from informality to formality is equivalent to 3.5% of existing informal jobs. Since in the model we abstract from the flow of workers from formal to informal jobs, an issue that is the focus of another paper (Bosch and Esteban-Pretel, 2009), to properly match this net reallocation rate in our model we set $\lambda_i(1 - H(\varepsilon_R)) = 0.035$.

respectively[20]. The values for the remaining four parameters are $\eta = 1$, $k = 0.63$, $F = 1.53$, and $\sigma = 8.41$.

Given the previous parametrization, Table 5 reports the equilibrium results for the endogenous variables in the three states of the aggregate shock.

5 Simulation Results

We simulate the model under the previous parametrization for 1000 quarters, and discard the first 927 periods. This leaves 73 quarters of simulated data, which is the same length as the actual data from Brazil. We repeat this procedure 1000 times, and calculate the average of the main statistics over the 1000 iterations. Table 6 reports the cross-correlations, volatilities, and elasticities, all with respect to output, for the simulated series and the actual data. The main message of this table is that the model does a good job at replicating the correlations of all the variables, and the elasticities of most of them, although it underestimates the volatility of some variables. Let us look at the simulation results in detail.

As we can see in Table 6, the model is capable of capturing the countercyclicality of unemployment and the procyclicality of the share of formal employment well. The model is also able to reproduce the correlations of the other variables in the data. Furthermore, the simulation results show that the model quantitatively reproduces the behavior of accessions towards employment. The job-finding rate from unemployment is strongly procyclical, and its elasticity is very close to that observed in the data (3.65 compared to 3.93 in the data). Similarly, the job-finding rate for informal jobs is much less volatile than its formal counterpart, and does not have a strong cyclical pattern. Finally, the elasticity of the rate at which informal jobs are formalized, 1.22, well matches that in the data, 1.72.

For the baseline parametrization, the model is not able to precisely capture the overall volatility of the economy. The parameter values in the baseline calibration are set to match the relative volatility of the job separation rate for formal jobs. This, however, grossly underestimates the elasticities of the job separation rate of informal jobs almost by a factor of three, -1.95 vs. -4.41. Since the job separation rate for informal jobs is one of the main driving forces of cyclical unemployment, this also implies that the elasticity of the unemployment rate is lower than in the data, -3.26 compared to -4.46. On the other hand, the elasticity of the share of formal employment is very close to actual one, which essentially happens because the model can correctly replicate the accessions towards formality, both from unemployment and from informal jobs. As we show in the sensitivity analysis of the results, it is possible to reconcile the model with the data in terms of volatilities and elasticities using alternative parametrizations.

The intuition for the previous results, and the mechanisms at play in the model, are better understood by analyzing the different thresholds in table 5. Let us look first at the job separation

[20] Although not shown explicitly, the calibrated lower bound of the initial productivity distribution, $\varepsilon^{G}_{\min}$, is higher than the threshold at which informal jobs are destroyed in any of the three aggregate states. This is required to satisfy the assumption that all initial contacts between firms and workers result in match formation.

rate and its negative correlation with output. An increase in productivity, p, reduces the job separation rate for both types of jobs. This is captured in table 5 by the fact that the separation thresholds in both formal and informal jobs fall with productivity. This is a standard result in the literature since, as productivity increases, so does the profitability of jobs, and hence the threshold that makes the value of the job equal to zero also falls.

The second rate to analyze is that of finding a job, which should be divided between the transition rate to formal and informal employment. The job-finding rate for formal jobs is given by $\theta q(\theta)[1 - G(\varepsilon_R)]$. This rate is characterized by two separate effects. The first is the *meeting effect*, which reflects the probability for a worker to meet a firm, and is governed by θ. The second is the *offer effect*, which is the conditional probability of signing a formal contract, $[1 - G(\varepsilon_R)]$. Table 5 suggests that a positive productivity shock increases θ. This is the traditional effect captured in most search and matching models, which implies that an increase in the profitability of both formal and informal jobs fosters vacancy creation. Table 5 also shows that a positive productivity shock lowers ε_R, which increases the conditional probability of offering a formal contract. This occurs because formal jobs can better take advantage of the increase in productivity. Overall, the meeting effect and the offer effect reinforce each other, producing an increase in the chances of an unemployed worker obtaining a formal job. This generates the strong procyclicality of the job-finding rate for formal jobs in the model. Similarly, the variation in the offer effect is the driving force of the conversion of informal jobs into formal ones. This job to job transition rate is procyclical both in the data and in the model, and its elasticity with respect to output is weaker than those accessions from unemployment. The reason is that, in this case, only one effect (the offer effect) is at work.[21]

The job-finding rate for informal jobs is given by $\theta q(\theta) G(\varepsilon_R)$. Once more, it is useful to separate the two effects. In this case, the meeting effect is as before, but the conditional probability of signing an informal contract, the offer effect, is given by $G(\varepsilon_R)$. A positive productivity shock increases the meeting rate, but lowers the informal job offer rate by lowering ε_R. The overall impact on the informal job-finding rate depends on the properties of the distributions $G(\varepsilon)$ and $H(\varepsilon)$. For the baseline parametrization, the model successfully reproduces the low variability of the job-finding rate for informal jobs, but with the opposite sign. This indicates that in our simulations, the offer effect dominates the meeting effect, which is not what seems to occur in the data, since the correlation of output and the job-finding rate for informal jobs is positive. Nevertheless, our results show that changes in the hiring behavior of the firm are a likely candidate to explain the relative stability of the job-finding rate for informal jobs.

[21] In all, note that due to the offer effect mechanism, we circumvent some of the problems experienced by Shimer (2005) for the textbook Pissarides model. Shimer (2005) finds that in the basic model, where only the meeting effect is present, the model response of the job-finding rate to productivity shocks does not generate enough volatility to match the data. In our model, the volatility of the job-finding rate for formal jobs is enhanced by the offer effect, which is also responsible for generating the lower volatility of the job-finding rate for informal jobs.

5.1 Sensitivity Analysis

We now explore the sensitivity of the previous simulation results to changes in the two main parameters of the model: The income when unemployed, b; and the arrival rate of idiosyncratic shocks to informal jobs, λ_i.[22]

As discussed earlier, b is an important determinant of the overall volatility of the model in response to changes in productivity. This can be seen in Table 7, which presents the simulation results under three values of b: 0.4, 0.6 and 0.8. Variations in b, leave the correlations with respect to output virtually unchanged. However, as we increase b, the elasticity of the job separation rate for informal jobs relative to output increases from -1.95 to -2.44. This is due to the fact that as b increases, wages become less responsive to changes in productivity, and the model becomes more volatile. This higher responsiveness of the model brings the elasticity of the unemployment rate closer to the data, -3.99 when $b = 0.8$. Therefore, the results found elsewhere in the literature concerning the increase in the volatility of the model when b increases, i.e. Shimer (2005) and Hagedorn and Manovskii (2008), still apply to our framework. In fact, the maximum value of b for which the model can compute an equilibrium is $b = 0.9$, and although we do not show the results for the sake of brevity, we find that the elasticity of unemployment with respect to output is -4.20, very close to the -4.46 in the data.

We also explore the effect of changes in λ_i in Table 8. As in the previous case, changes in λ_i do not substantially alter the correlations of the model. We have argued before that the arrival rate of shocks determines the elasticity of job separations in the model. This is precisely what happens when we modify λ_i. If we decrease λ_i to 0.16, twice the rate of its formal counterpart, we observe that the elasticity of the job separation rate for informal jobs drops substantially, from -1.95 to -0.32, taking us further away from the empirical data.

Increasing λ_i produces much better results. When $\lambda_i = 0.48$, the elasticity of the separation rate for informal jobs increases substantially, approaching that in the data, -3.78. Furthermore, the model does a better job in replicating the elasticity of unemployment, -3.93, although still falling short of the -4.46 in the data. It is possible to completely reconcile the elasticity of the informal separation rate in the model with the data by allowing $\lambda_i = 0.8$, but in that case the equilibrium implies that the penalty for firms if caught with informal workers necessarily be negative. That is, firms would have to be compensated for hiring informal workers if subject to so much volatility.

6 Policy Changes

The previous section analyzes the performance of the model for different parametrizations. We now use the model, under the preferred parametrization ($b = 0.6$ and $\lambda_i = 0.48$), to assess the

[22]We have also explored changes in the productivity wedge, δ, and the hiring cost, c, but modifying these parameters does not produce significant changes in the behaviour of the simulated variables. These results are available from the authors upon request.

effects of several government policies on the different flows and variables in the labor market. We study the quantitative effects of changes in five policies. Three of these policies directly constrain formal jobs: firing costs, F; hiring costs, c; and payroll taxes, τ. The other two penalize informal jobs: the informality monitoring rate by the government, ϕ, and the penalty imposed to firms if caught with informal workers, σ. The results of these counter-factual policy experiments are shown in Table 9. The main finding indicates that policies that increase the cost of formal employment for the firm, or those that decrease the cost of informality, produce a reduction in the share of formal employment and an increase in unemployment. The main reason is that these policies reduce the accessions to formal employment, both from unemployment and from other informal jobs, while increasing the finding rate for informal jobs. The effect on the separation rates for formal and informal jobs depends on each individual policy. We now analyze the effects of these policies in more detail. We explain first the effects of the formal market policies, and then those for the informal market.

As expected, the three formal job policies directly influence the hiring decisions of the firm. Increasing any of these three policy parameters reduces the job-finding rate for formal jobs, while increasing the finding rate for informal employment. The drop in the formal employment finding rate is due to the reduction in both the meeting and the offer effects, that is, to the decrease in the market tightness, θ, and the decrease in the threshold at which jobs are formalized, ε_R. The main difference between the three policies is the impact on the job separation rate for formal jobs: (i) An increase in the firing costs reduces the job separation rate for formal jobs. This occurs through two channels: A direct one, by making dismissals more costly; and an indirect one, by diminishing the wage demands of workers, since their outside option then becomes less attractive due to the lower finding rates. (ii) Increases in hiring costs, while also reducing the separation rate for formal jobs, only affect this rate through the indirect channel, implying that raising these costs produces a smaller drop in the job separation rate for formal jobs. (iii) Contrary to the previous two policies, payroll taxes increase the job separation rate for formal jobs. This is due to the fact that raising taxes directly reduces the bargaining power of the firms (see Equation (11)), which increases wage demands and raises the job separation rate for formal workers.

Overall, increases in these three policies produce an increase in unemployment and a decrease in the share of formality. Under the baseline calibration, a separate 1% increase in F, τ and c produces increases in unemployment of 0.46%, 0.25%, and 0.01%, and a drop in the share of formal employment of 0.84%, 0.34%, and 0.01%, respectively. The basic intuition is as follows. Higher payroll taxes increase unemployment because they decrease the total job-finding rate and increase the job separation rate for formal jobs, while producing minor reductions in the job-finding rate for informal jobs. Firing and hiring costs generate less job creation, but also less job destruction. Hence, the effect on unemployment is generally ambiguous, although under our parametrization the lower creation effect dominates. Furthermore, in our model there is an additional channel through which labor market policies affect unemployment. Firing costs, hiring costs, and taxes change the composition between formal and informal jobs in favor of the latter. This has non-neutral implications for unemployment, since the higher fraction of informal jobs shifts the labor market from a low job separation rate sector to a high job separation rate sector.

In terms of the informal sector policies, increasing the monitoring rate of informal jobs or the penalty raises the job separation in this sector, while generating a reallocation of informal workers into formal jobs. This policy also reduces the market tightness of the economy, which lowers separation for formal jobs. The reallocation effect seems to dominate, and despite the increase in the separation rate for informal jobs, unemployment decreases. Numerically, a 1% increase in the monitoring rate produces a drop in unemployment and informality of 0.01% and 0.03%, while raising the penalty of noncompliance decreases unemployment and by 0.03% and increases formality 0.07%, respectively.

7 Conclusions

Informal labor markets in developing countries capture a large fraction of employment. In countries such as Brazil, 35% to 45% of the workers are employed in the underground economy, where a lack of protection and regulations is the norm. Understanding the flows of workers between the formal and the informal sectors, as well as from unemployment, is crucial in studying the effects of policies in these countries.

In this paper, we present a set of stylized facts about the cyclical patterns of unemployment, share of formality, and flows of workers in Brazil for the 1980s and 1990s. We then build a two-sector search and matching model of the labor market, which we calibrate and simulate to assess its success in accounting for these empirical facts. We then employ the model to study the effects of policy changes on unemployment and the share of formality in the economy. The model is characterized by the endogenous choice of firms to hire workers either legally or illegally, the possibility for workers to transition from informality to formality after the job has started, and the endogenous destruction of jobs. This paper shows that traditional search and matching models are appropiate to capture the cyclical facts observed in worker flow data. presented here inelsewhereforcrucial that we know little of

We find that the model does a fairly good job of reproducing the correlations and elasticities with respect to output observed in the data, although a different parametrization from the baseline is needed for the model to generate sufficient volatility. In terms of policies, our results highlight that regulations changing the relative incentives to participate in the formal sector substantially affect the allocation of workers among sectors, with non-neutral effects on the unemployment rate. We show that those policies that increase the cost of formality, or decrease the cost of informality, reduce the share of formal employment, as expected. Furthermore, it is these types of policies that tend to increase unemployment.

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A Source and Description of Data

The data for Brazil are drawn from the Monthly Employment Survey (Pesquisa Mensual de Emprego, PME) that conducts monthly household interviews in six of the major metropolitan regions, covering 25% of the national labor market. The PME is structured so as to track each household during four consecutive months and then drop them from the sample for 8 months, after which they are reintroduced for another 4 months. The rotation procedure is such that each month, one-fourth of the sample is substituted by other households to form a new panel. Thus, after 4 months, the entire initial sample has been rotated and after 8 months, a third, different sample is being surveyed. After 12 months, the initial sample is re-encountered. Over a period of two years, three different panels of households are surveyed, and the process starts again with three new panels. We have concatenated the panels from the January 1983 to December 2001. Regrettably, the PME was drastically modified in 2002, and it is not possible to reconcile the new and old definitions of unemployment. The monthly attrition rate in the PME is between 5% and 10% of the sample.

The measure of output is quarterly Gross Domestic Product (GDP), and is obtained from the Brazilian Statistical Institute (IBGE). It is measured at constant prices of 1990.

Defining Informality. Though, generally speaking, there is broad consensus in the literature as to what constitutes an informal worker, the study of transitions raises some particular definitional complexity that we discuss here. Initially, we follow the International Labour Organization (ILO) in dividing employed workers into three sectors: informal salaried, informal self-employed and formal sector workers. Broadly speaking, formal workers are those working in firms licensed with the government and conforming to tax and labor laws, including minimum wage directives, pension and health insurance benefits for employees, workplace standards of safety, etc. Informal workers (both salaried and self-employed), are those owners of firms that are largely de-linked from state institutions and obligations, and those employees who are not covered by formal labor protections. Employers in Brazil are obliged to register their employees by issuing them a working permit or "carteira de trabalho", the signing of which guarantees them access to formal labor protections. Those wage employees without a carteira are therefore considered to be informal salaried.[23] The second group classified as informal is the large number of independent or self-employed workers (between 20% and 30% of the labor force). Brazilian labor surveys enable the description of this micro-entrepreneurial sector by selecting those self-employed workers who are not technicians or professionals.

We choose to pool together these two types of employment and focus on "informal employment" as a whole, based of the lack of protection criteria. Although these two types of employment may have different considerations, they share very similar labor market dynamics. Moreover, both constitute a very flexible "unregulated" source of labor for formal firms.

[23] The selection criterion does not appear critical, as shown in Bosch and Maloney (2008).

B Derivation of the Equilibrium Conditions

Given the free entry of firms, and the existence and uniqueness of ε_R, ε_{d_f} and ε_{d_i} shown through Figure 2, we can rewrite equations (1) to (6) in the following way. The value of the different states for the firm are:

$$k = q(\theta)\left[\int_{\varepsilon_{\min}^G}^{\varepsilon_R} J_i(\varepsilon')\,dG(\varepsilon') + \int_{\varepsilon_R}^{\varepsilon_{\max}^G}\left[J_f^n(\varepsilon') - c\right] dG(\varepsilon')\right], \tag{29}$$

$$(r+\lambda_f)\,J_f^l(\varepsilon) = p + \varepsilon - (1+\tau)\,w_f^l(p,\varepsilon) + \lambda_f\left[\int_{\varepsilon_{d_f}}^{\varepsilon_{\max}^H} J_f^o(\varepsilon')\,dH(\varepsilon') - H\left(\varepsilon_{d_f}\right)F\right],\ l\in\{n,o\}, \tag{30}$$

$$(r+\lambda+\phi)\,J_i(\varepsilon) = \delta(p+\varepsilon) - w_i(\varepsilon) + \lambda_i\left[\int_{\varepsilon_{d_i}}^{\varepsilon_R} J_i(\varepsilon')\,dH(\varepsilon') + \int_{\varepsilon_R}^{\varepsilon_{\max}^H}\left[J_f^n(\varepsilon') - c\right] dH(\varepsilon')\right] - \phi\sigma, \tag{31}$$

and for workers:

$$rU = b + \theta q(\theta)\left[\int_{\varepsilon_{\min}^G}^{\varepsilon_R}\left[W_i(\varepsilon') - U\right] dG(\varepsilon') + \int_{\varepsilon_R}^{\varepsilon_{\max}^G}\left[W_f^n(\varepsilon') - U\right] dG(\varepsilon')\right], \tag{32}$$

$$(r+\lambda)\,W_f^l(\varepsilon) = w_f^l(\varepsilon) + \lambda_f\left[\int_{\varepsilon_{d_f}}^{\varepsilon_{\max}^H} W_f^o(\varepsilon')\,dH(\varepsilon') + H\left(\varepsilon_{d_f}\right)U\right],\quad l\in\{n,o\}, \tag{33}$$

$$(r+\lambda_i+\phi)\,W_i(\varepsilon) = w_i(\varepsilon) + \lambda_i\left[\int_{\varepsilon_{d_i}}^{\varepsilon_R} W_i(\varepsilon')\,dH(\varepsilon') + \int_{\varepsilon_R}^{\varepsilon_{\max}^H} W_f^n(\varepsilon')\,dH(\varepsilon')\right] + \left[\lambda_i H(\varepsilon_{d_i}) + \phi\right]U. \tag{34}$$

Using (29) to (34) together with the sharing rules derived from the bargaining problem, equations (10) to (12), we can obtain the wage functions for each of the three types of matches. These are given by

$$w_f^n(\varepsilon) = \frac{\beta}{1+\tau}\left[p + \varepsilon - (r+\lambda_f)c - \lambda_f F + k\theta\right] + (1-\beta)\left[b + \frac{\tau}{1+\tau}\theta q(\theta)\int_{\varepsilon_{\min}^G}^{\varepsilon_R}\left(W_i(\varepsilon') - U\right) dG(\varepsilon')\right], \tag{35}$$

$$w_f^o(\varepsilon) = \frac{\beta}{1+\tau}\left[p + \varepsilon + rF + k\theta\right] + (1-\beta)\left[b + \frac{\tau}{1+\tau}\theta q(\theta)\int_{\varepsilon_{\min}^G}^{\varepsilon_R}\left(W_i(\varepsilon') - U\right) dG(\varepsilon')\right], \tag{36}$$

$$w_i(\varepsilon) = \beta\left[\delta(p+\varepsilon) - \phi\sigma + \frac{1}{1+\tau}k\theta\right] + (1-\beta)\left[b + \frac{\tau}{1+\tau}\theta q(\theta)\int_{\varepsilon_{\min}^G}^{\varepsilon_R}\left(W_i(\varepsilon') - U\right) dG(\varepsilon')\right]. \tag{37}$$

The employee is compensated for his work with a fraction β, adjusted for taxes, of what the firm gains by hiring him and a fraction $(1-\beta)$ of what it loses for being in the match. By employing the worker, the firm produces output, which makes the wages an increasing function of the productivity of the match. Formal firms, when starting a new match, must pay the hiring cost and eventually the firing cost, both of which reduce the wage of the newly hired worker. However, since once the match is formed the hiring cost is removed, and the firing cost becomes active, the threat point of the worker increases and raises his wages. Informal wages are not affected by these costs, but depend negatively on the fine paid by the firm if caught by the government. All types of workers are also compensated for the firm's saved search cost, $k\theta$. The part of the wage related to the loss due to being employed is the flow value of unemployment, b, and a measure of the value lost to taxes if not hired through an informal contract.

Using the wage equations, (35) to (37), into the value function of the filled firm, (30) and (31), we obtain that the following conditions which are used to characterize the equilibrium:

$$\begin{aligned} J_f^n(\varepsilon) - c &= (1-\beta)\left[\frac{\varepsilon+\varepsilon_{d_f}}{r+\lambda_f} - c - F\right] && (38)\\ J_f^o\left(\varepsilon_{d_f}\right) + F &= (1-\beta)\frac{\varepsilon+\varepsilon_{d_f}}{r+\lambda_f} && (39)\\ J_i(\varepsilon) &= \delta(1-\beta)\frac{\varepsilon-\varepsilon_{d_i}}{r+\lambda_i+\phi} && (40)\end{aligned}$$

We can use the equilibrium condition equations (38) to (40), together with the wage equations (35) to (37), to obtain an explicit function for the value of a filled job for firms. These value functions are:

$$\begin{aligned} J_f^n(\varepsilon) &= \frac{1-\beta}{r+\lambda_f}\left[p+\varepsilon-(1+\tau)b+\frac{r+\lambda_f}{1-\beta}\beta c+\frac{1}{1-\beta}\left(\beta\mu-(1-\beta)\lambda_f\right)F-\frac{\beta}{1-\beta}k\theta\right]\\ &+\frac{1-\beta}{r+\lambda_f}\left[-\Omega+\lambda_f\int_{\varepsilon_{d_f}}^{\varepsilon_{\max}^H}\frac{\varepsilon'-\varepsilon_{d_f}}{r+\lambda_f}dH(\varepsilon')\right], \qquad (41)\end{aligned}$$

$$\begin{aligned} J_f^o(\varepsilon) &= \frac{1-\beta}{r+\lambda_f}\left[(p+\varepsilon)-(1+\tau)b-\frac{\beta r+\lambda_f}{1-\beta}F-\frac{\beta}{1-\beta}k\theta\right]\\ &+\frac{1-\beta}{r+\lambda_f}\left[-\Omega+\lambda_f\int_{\varepsilon_{d_f}}^{\varepsilon_{\max}^H}\frac{\varepsilon'-\varepsilon_{d_f}}{r+\lambda_f}dH(\varepsilon')\right], \qquad (42)\end{aligned}$$

$$\begin{aligned} J_i(\varepsilon) &= \frac{1-\beta}{r+\lambda_i+\phi}\left[\delta(p+\varepsilon)-b-\phi\sigma-\frac{\beta}{(1-\beta)(1+\tau)}k\theta-\frac{\Omega}{1+\tau}\right] \qquad (43)\\ &+\frac{1-\beta}{r+\lambda_i+\phi}\lambda_i\left[\int_{\varepsilon_{d_i}}^{\varepsilon_R}\delta\frac{\varepsilon'-\varepsilon_{d_i}}{r+\lambda_i+\phi}dH(\varepsilon')+\int_{\varepsilon_R}^{\varepsilon_{\max}^H}\left(\frac{\varepsilon'-\varepsilon_{d_f}}{r+\lambda_f}-c-F\right)dH(\varepsilon')\right]\end{aligned}$$

where

$$\Omega = \beta\tau\theta q(\theta)\int_{\varepsilon_{\min}^G}^{\varepsilon_R}\frac{\delta(\varepsilon'-\varepsilon_{d_i})}{r+\lambda_i+\phi}dG(\varepsilon').$$

Given the previous value functions and the equilibrium conditions, we can find the equations which determine the steady state equilibrium.

The first condition is determined by the free entry of firms, which implies $V=0$ and delivers the following equation

$$\frac{\beta}{(1-\beta)\, q(\theta)} k = \lambda \int_{\varepsilon_{d_i}}^{\varepsilon_R} \frac{\delta(\varepsilon' - \varepsilon_{d_i})}{r+\lambda_i+\phi} dH(\varepsilon') + \lambda \int_{\varepsilon_R}^{\varepsilon_{\max}^H} \left[\frac{\varepsilon' - \varepsilon_{d_f}}{r+\lambda_f} - c - F \right] dH(\varepsilon'). \quad (44)$$

The formal/informal threshold, ε_R, is determined through the condition $J_f^n(\varepsilon_R) - c = J_i(\varepsilon_R)$ and given by

$$\varepsilon_R = \frac{(r+\lambda_f)(r+\lambda_i+\phi)(c+F) + (r+\lambda_i+\phi)\varepsilon_{d_f} - \delta(r+\lambda_f)\varepsilon_{d_i}}{r+\lambda_i+\phi-\delta(r+\lambda_f)}. \quad (45)$$

Using the equilibrium condition for formal destruction, $J_f^o\left(\varepsilon_{d_f}\right) - F = 0$ with equation (42), we obtain the formal destruction threshold

$$\varepsilon_{d_f} = -p + (1+\tau) b - rF + \frac{\beta k \theta}{1-\beta} + \Omega - \lambda_f \int_{\varepsilon_{d_f}}^{\varepsilon_{\max}^H} \frac{\varepsilon' - \varepsilon_{d_f}}{r+\lambda_f} dH(\varepsilon'). \quad (46)$$

Similarly, using the condition for informal destruction, $J_i(\varepsilon) = 0$, we find the informal destruction threshold

$$\begin{aligned} \varepsilon_{d_i} &= -p + \frac{b}{\delta} - \frac{\phi\sigma}{\delta} + \frac{\beta k \theta}{\delta(1+\tau)(1-\beta)} + \frac{\Omega}{\delta(1+\tau)} \\ &\quad -\lambda_i \int_{\varepsilon_{d_i}}^{\varepsilon_R} \frac{\varepsilon' - \varepsilon_{d_i}}{r+\lambda_i+\phi} dH(\varepsilon') - \frac{\lambda_i}{\delta} \int_{\varepsilon_R}^{\varepsilon_{\max}^H} \left(\frac{\varepsilon' - \varepsilon_{d_f}}{r+\lambda_f} - c - F \right) dH(\varepsilon'), \end{aligned} \quad (47)$$

Figure 2 illustrates the firm's hiring and firing decisions. This figure plots the value for the firm of the different contracts against the level of idiosyncratic productivity of the match, ε. It is easy to show that the profitability of occupied jobs is monotonically increasing on the idiosyncratic productivity of the match, and therefore, $J_f^n(\varepsilon) - c$, $J_f^o(\varepsilon) + F$ and $J_i(\varepsilon)$ are upward-sloping. However, the slope of the latter is smaller because of the overall productivity wedge, δ, the existence of monitoring, ϕ, and the larger arrival rate of new idiosyncratic productivity shocks, $\lambda_i > \lambda_f$. As stated by the equilibrium condition (14), the formality/informality threshold, ε_R, is found in the intersection of $J_f^n(\varepsilon) - c$ and $J_i(\varepsilon)$. Similarly, using conditions (16) and (17), ε_{d_f} and ε_{d_i} are obtained in the intersections of $J_i(\varepsilon)$ and $J_f^o(\varepsilon) + F$ with the horizontal axis, respectively. As explained before, we restrict our attention to the range of parameter values where formal and informal jobs coexist, and formal firms do not find it optimal to transform the match into an informal one. Under these conditions, $\varepsilon_R > \varepsilon_{d_i} > \varepsilon_{d_f} > 0$.

C Comparative Statics: Changes in Productivity

Here we present a comparative statics exercise for a simpler version of the steady state model which does not include informal upgrading or payroll taxes. The equilibrium of this simpler version of the model is composed of the same four variables, θ, ε_R, ε_{d_f} and ε_{d_i}, which satisfy the following four equations:

- Free entry condition:

$$\begin{aligned}\frac{k}{q(\theta)} &= (1-\eta)\int_{\varepsilon_R}^{\varepsilon^G_{\max}}\left[\frac{\varepsilon'-\varepsilon_{d_f}}{r+\lambda_f}-(F+c)\right]dG(\varepsilon') \\ &\quad +(1-\eta)\int_{\varepsilon^G_{\min}}^{\varepsilon_R}\delta\frac{\varepsilon'-\varepsilon_{d_i}}{r+\phi+\lambda_i}dG(\varepsilon'). \end{aligned}\tag{48}$$

- Formal-Informal reservation threshold:

$$\varepsilon_R = \frac{(r+\lambda_i+\phi)\left[\varepsilon_{d_f}+(r+\lambda_f)(F+c)\right]-\delta(r+\lambda_f)\varepsilon_{d_f}}{r+\lambda_i+\phi-\delta(r+\lambda_f)}. \tag{49}$$

- Formal destruction:

$$\varepsilon_{d_f} = -p+b-rF+\frac{\beta\theta k}{(1-\beta)}-\frac{\lambda_f}{(r+\lambda_f)}\int_{\varepsilon_{d_f}}^{\varepsilon^H_{\max}}\left(\varepsilon'-\varepsilon_{d_f}\right)dH(\varepsilon'). \tag{50}$$

- Informal destruction:

$$\varepsilon_{di} = \frac{b}{\delta}-p+\frac{\phi\sigma}{\delta}+\frac{\beta\theta k}{(1-\beta)\delta}-\frac{\lambda_i}{r+\phi+\lambda_i}\int_{\varepsilon_{d_i}}^{\varepsilon^H_{\max}}(\varepsilon'-\varepsilon_{d_i})\,dH(\varepsilon'). \tag{51}$$

The remaining important variables of the model, u, n_f, n_i, share of formal employment, and job-finding rates for formal and informal jobs are given by the following six equations:

$$u = \frac{n_f\lambda_f H(\varepsilon_{d_f})+n_i\left(\lambda_i H(W)+\phi\right)}{\theta q(\theta)}, \tag{52}$$

$$n_f = \frac{\theta q(\theta)[1-G(\varepsilon_R)]u}{\lambda_f H(\varepsilon_{d_f})}, \tag{53}$$

$$n_i = 1-n_f-u, \tag{54}$$

$$\pi = \frac{n_f}{n_i+n_f}. \tag{55}$$

$$JFR_f = \theta q(\theta)[1-G(\varepsilon_R)], \tag{56}$$

$$JFR_i = \theta q(\theta)G(\varepsilon_R). \tag{57}$$

Figure 3 illustrates the implication of an increase in productivity[24], p.

An increase in the general productivity parameter shifts both job separation conditions downwards. Similarly, the formal/informal hiring condition also shifts these two conditions downwards, since now firms and workers have more incentives to sign formal contracts. This is because formal jobs are better able to take advantage of the increases in productivity. Finally, the free entry condition is shifted to the right as the expected profit from both types of jobs increases.

While from the graphs we can see that there is an unambiguous increase in market tightness but an ambiguous impact on ε_R, ε_{d_f} and ε_{d_i}, it is easy to see that all three margins decrease. This implies a higher conditional probability of signing formal contracts and lower job separation thresholds in both types of jobs. The following propositions summarize the qualitative predictions of the model.

Proposition 1 *An increase in productivity, p, reduces the job separation rate in both types of jobs.*

This is immediate from figure 3(b). In response to a good productivity shock, both ε_{d_f} and ε_{d_i} decrease. Hence, the job separation rate in both types of jobs, $\lambda_f H(\varepsilon_{d_f})$ and $\lambda_f H(\varepsilon_{d_i}) + \phi$, decrease. However, the model is silent with respect to which of these two probabilities decreases more. This remains a quantitative question, which is addressed in the simulation section of the paper.

Proposition 2 *An increase in productivity, p, unambiguously increases the job-finding rate for formal jobs, but it has an ambiguous effect on the job-finding rate of informal jobs.*

The job-finding rate for formal jobs is given by $\theta q(\theta)[1 - G(\varepsilon_R)]$. It is useful to separate the rate of transition to employment into two effects: The *meeting effect*, which reflects the probability that a worker meets a firm, and is governed by θ; and the *offer effect*, which is the conditional probability of signing a formal contract, $[1 - G(\varepsilon_R)]$. A positive productivity shock increases θ and reduces ε_R. Hence, the meeting effect and the offer effect reinforce each other, which produces an increase in the chances for an unemployed worker to obtain a formal job. This is consistent with the strong procyclicality of the job-finding rate for formal jobs found in Brazil.

Similarly, the job-finding rate for informal jobs is given by $\theta q(\theta)G(R)$. In this case, the conditional probability of signing an informal contract is given by $G(\varepsilon_R)$. A positive productivity shock increases the meeting rate, but lowers ε_R. This last effect lowers the conditional probability of signing an informal contract. The exact effect on the informal job-finding rate depends on the properties of the distributions $G(\varepsilon)$ and $H(\varepsilon)$. Nevertheless, this change in the hiring policy of the firm provides a likely explanation for the relative stability of the job-finding rate for informal jobs. We take up this issue in the numerical simulations of the model.

[24] Note that changes in the unemployment benefits, b, are equivalent, but with opposite sign, to changes in productivity.

Proposition 3 *An increase in productivity, p, unambiguously decreases unemployment, but it has an ambiguous effect on the share of formal employment.*

The unemployment rate is determined by the exit rate from unemployment $\theta q(\theta)$, the job separation rate in each type of job, and the share of formal employment. A positive productivity shock generates lower job separation rates in all jobs, increasing the rate of exit from unemployment. Therefore, in the face of a positive productivity shock, unemployment can only increase if the share of formal employment falls. However, from equation (53) it is easy to see that, if unemployment increases, so does the number of formal workers. Hence, it is not possible to have a drop in the share of formality, since that would imply an increase in the number of informal workers, which is incompatible with the fact that the labor force is constant. Unemployment must, therefore, fall.

The share of formal employment should follow a procyclical pattern dictated by the hiring decisions of the firm. However, the results here are also ambiguous, because quantitative changes in job separation rates will depend on the distributions $G(\varepsilon)$ and $H(\varepsilon)$.

Table 1: Summary Statistics: Brazil 1983-2001.

Correlation Matrix	u	π	JFR_f	JFR_i	JSR_f	JSR_i	JJ_{i-f}	JJ_{f-i}	y	Vol. $\frac{\sigma_x}{\sigma_y}$	Elast. w.r.t y
u	1	-0.67	-0.91	-0.35	0.34	0.95	-0.81	-0.61	-0.83	5.41	-4.46
π		1	0.62	0.01	-0.06	-0.63	0.62	0.27	0.49	0.49	0.24
JFR_f			1	0.39	-0.01	-0.83	0.71	0.52	0.74	5.31	3.93
JFR_i				1	-0.08	-0.26	0.42	0.39	0.24	1.43	0.34
JSR_f					1	0.43	-0.31	-0.26	-0.28	3.54	-1.01
JSR_i						1	-0.77	-0.59	-0.75	5.95	-4.41
JJ_{i-f}							1	0.87	0.73	2.36	1.72
JJ_{f-i}								1	0.55	1.87	1.04
y									1	1	1

Notes: Constructed with quarterly data from the National Urban Labor Survey (ENEU). u is the unemployment rate, π is the share of formal employment, JFR is the job finding rate, JSR is the job separation rate, JJ denotes direct job to job transitions, and the subscripts f and i denote formal and informal respectively. y is quarterly real GDP. The elasticity of x with respect to y corresponds to the coefficient of a regression of $ln(y)$ on the $ln(x)$. All series are smoothed using a moving average filter with a 12 month window and quarterly averaged. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1600.

Table 2: Unemployment Accounting Exercise

	Unemployment Rate	Share of Formal Employment
Job Finding Rates		
JFR_f	0.16	0.11
JFR_i	0.05	-0.01
Job Separation Rates		
JSR_f	0.06	0.09
JSR_f	0.71	-0.04
Job Reallocation Rates		
JJ_{i-f}	0.05	0.82
JJ_{f-i}	-0.03	0.05

Notes: The table shows the contribution to cyclical unemployment and formality of each of the flows among employment states. Constructed with quarterly data from the National Urban Labor Survey (ENEU). JFR is the job-finding rate, JSR is the job separation rate, JJ denotes direct job to job transitions, and the subscripts f and i denote formal and informal respectively. This is computed as the covariance between the steady state unemployment rate (share of formal employment), and the steady state unemployment rate (share of formal employment) resulting of varying each of the six possible flows, as a proportion of the variance of the steady state unemployment rate (share of formal employment).

Table 4: Parameter Configuration

Exogenous Parameters				
$r = 0.019$	$\xi = 0.5$	$\beta = 0.5$	$b = 0.6$	$\bar{p} = 1$
$\tau = 0.37$	$c = 1/90$	$\delta = 0.81$	$\phi = 0.005$	
$\varepsilon^G_{max} = 1$	$\varepsilon^H_{max} = 1$	$\lambda_f = 0.08$	$\lambda_i = 0.32$	
Endogenous Parameters				
$\eta = 1$	$F = 1.53$	$\sigma = 8.41$	$k = 0.63$	
$\varepsilon^G_{min} = -0.11$	$\varepsilon^H_{min} = 0.59$			

Table 5: Results of the Endogenous variables

Parameters	$p = 0.95$	$p = 1$	$p = 1.05$
θ	0.93	1	1.08
ε_R	0.88	0.87	0.86
ε_{di}	0.23	0.22	0.20
ε_{df}	0.44	0.43	0.42

Table 6: Simulation Results: Changes in Productivity

	Cross-corr. with y		Volatility $\frac{\sigma_x}{\sigma_y}$		Elasticity w.r.t y	
	(a) Data	(b) Model	(c) Data	(d) Model	(e) Data	(f) Model
u	-0.83	-0.87	5.41	3.78	-4.46	-3.26
		(0.05)		(1.42)		(1.18)
π	0.49	0.37	0.49	0.41	0.24	0.20
		(0.26)		(0.15)		(0.11)
JFR_f	0.74	0.93	5.31	3.91	3.93	3.65
		(0.04)		(1.12)		(1.10)
JFR_i	0.24	-0.10	1.43	0.68	0.34	-0.03
		(0.80)		(0.07)		(0.54)
JSR_f	-0.28	-0.58	3.54	3.55	-1.01	-2.26
		(0.14)		(2.40)		(1.66)
JSR_i	-0.75	-0.81	5.95	2.38	-4.41	-1.95
		(0.06)		(0.40)		(0.44)
JJ_{i-f}	0.73	0.39	2.36	4.34	1.72	1.22
		(0.17)		(1.52)		(0.76)

Notes: The table shows the correlation, volatility and elasticity with respect to output of the actual series (Data) and the simulated series (Model). The simulated series are computed following the procedure described in section 5. u is the unemployment rate, π is the share of formal employment, JFR is the job finding rate, JSR is the job separation rate, JJ denotes direct job to job transitions, and the subscripts f and i denote formal and informal respectively. Standard errors in parenthesis.

Table 7: Sensitivity Analysis: Changes in b

	Cross-corr. with y			Volatility $\frac{\sigma_x}{\sigma_y}$			Elasticity w.r.t y		
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
	$b = 0.4$	$b = 0.6$	$b = 0.8$	$b = 0.4$	$b = 0.6$	$b = 0.8$	$b = 0.4$	$b = 0.6$	$b = 0.8$
u	-0.87	-0.87	-0.90	3.06	3.78	4.47	-2.63	-3.26	-3.99
	(0.05)	(0.05)	(0.04)	(1.60)	(1.42)	(1.86)	(1.35)	(1.18)	(1.56)
π	0.18	0.37	0.10	0.38	0.41	0.37	0.13	0.10	0.12
	(0.43)	(0.26)	(0.45)	(0.20)	(0.15)	(0.21)	(0.13)	(0.11)	(0.16)
JFR_f	0.94	0.93	0.96	3.56	3.91	4.04	3.34	3.65	3.86
	(0.04)	(0.04)	(0.04)	(1.09)	(1.12)	(1.26)	(1.02)	(1.10)	(1.16)
JFR_i	-0.12	-0.10	-0.20	0.62	0.68	0.60	-0.13	-0.03	0.19
	(0.77)	(0.80)	(0.95)	(0.11)	(0.07)	(0.37)	(0.49)	(0.54)	(0.66)
JSR_f	-0.30	-0.58	-0.69	2.84	3.55	3.91	-1.77	-2.26	-2.78
	(0.49)	(0.14)	(0.10)	(2.73)	(2.40)	(3.17)	(2.06)	(1.66)	(2.27)
JSR_i	-0.80	-0.81	-0.83	1.91	2.38	2.90	-1.55	-1.95	-2.44
	(0.07)	(0.06)	(0.08)	(0.27)	(0.40)	(0.74)	(0.36)	(0.44)	(0.79)
JJ_{i-f}	0.41	0.39	0.36	3.82	4.34	4.30	1.55	1.22	1.44
	(0.25)	(0.17)	(0.18)	(1.55)	(1.52)	(2.19)	(1.07)	(0.76)	(0.90)

Notes: The table shows the correlation, volatility and elasticity with respect to output of the actual series (Data) and the simulated series (Model) for different values of b. The simulated series are computed following the procedure described in section 5. u is the unemployment rate, π is the share of formal employment, JFR is the job finding rate, JSR is the job separation rate, JJ denotes direct job to job transitions, and the subscripts f and i denote formal and informal respectively. Standard errors in parenthesis.

Table 8: Sensitivity Analysis: Changes in λ_i

	Cross-corr. with y			Volatility $\frac{\sigma_x}{\sigma_y}$			Elasticity w.r.t y		
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
	$\lambda_i = 2\lambda_f$	$\lambda_i = 4\lambda_f$	$\lambda_i = 6\lambda_f$	$\lambda_i = 2\lambda_f$	$\lambda_i = 4\lambda_f$	$\lambda_i = 6\lambda_f$	$\lambda_i = 2\lambda_f$	$\lambda_i = 4\lambda_f$	$\lambda_i = 6\lambda_f$
u	-0.86	-0.87	-0.91	2.89	3.78	3.90	-2.36	-3.26	-3.93
	(0.05)	(0.05)	(0.06)	(1.31)	(1.42)	(1.38)	(1.19)	(1.18)	(1.12)
π	0.19	0.37	0.21	0.40	0.41	0.719	0.17	0.10	0.43
	(0.54)	(0.26)	(0.44)	(0.22)	(0.15)	(0.99)	(0.21)	(0.11)	(1.08)
JFR_f	0.91	0.93	0.91	2.75	3.91	4.68	2.21	3.65	3.83
	(0.06)	(0.04)	(0.15)	(1.92)	(1.12)	(1.58)	(1.81)	(1.10)	(1.51)
JFR_i	0.98	-0.10	-0.04	1.39	0.68	0.49	1.34	-0.03	-0.15
	(0.02)	(0.80)	(0.76)	(0.07)	(0.07)	(0.25)	(0.09)	(0.54)	(0.41)
JSR_f	-0.37	-0.58	-0.45	3.00	3.55	2.39	-1.21	-2.26	-1.35
	(0.09)	(0.14)	(0.28)	(2.55)	(2.40)	(2.63)	(1.28)	(1.66)	(1.76)
JSR_i	-0.52	-0.81	-0.76	0.61	2.38	4.12	-0.32	-1.95	-3.78
	(0.04)	(0.06)	(0.39)	(0.16)	(0.40)	(1.19)	(0.07)	(0.44)	(1.10)
JJ_{i-f}	0.21	0.39	0.67	3.35	4.34	3.09	0.80	1.22	2.05
	(0.31)	(0.17)	(0.15)	(1.46)	(1.52)	(1.42)	(1.00)	(0.76)	(1.07)

Notes: The table shows the correlation, volatility and elasticity with respect to output of the actual series (Data) and the simulated series (Model) for different values of λ_i. The simulated series are computed following the procedure described in section 5. u is the unemployment rate, π is the share of formal employment, JFR is the job finding rate, JSR is the job separation rate, JJ denotes direct job to job transitions, and the subscripts f and i denote formal and informal respectively. Standard errors in parenthesis.

Table 9: Policy Changes: Elasticities

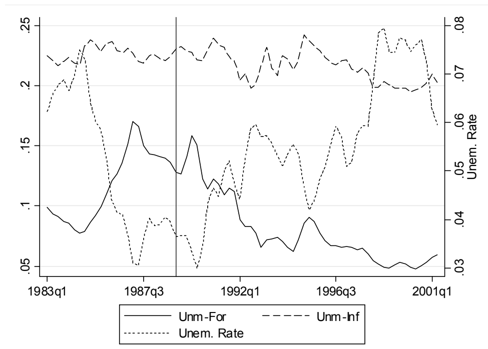
	(a) F	(b) τ	(c) c	(d) σ	(e) ϕ
u	0.46	0.25	0.01	-0.03	-0.02
π	-0.84	-0.33	-0.01	0.07	0.03
y	-0.16	-0.05	-0.01	0.01	0.01
JFR_f	-2.22	-0.70	-0.02	0.14	0.06
JFR_i	0.73	0.23	0.00	-0.06	-0.02
JSR_f	-0.31	0.06	-0.00	-0.01	-0.01
JSR_i	-0.22	-0.08	-0.00	0.06	0.02
JJ_{i-f}	-2.06	-0.6	-0.01	0.14	0.06

Notes: The table shows the elasticity of the steady state value of the series with repect to the policy variables. u is the unemployment rate, π is the share of formal employment, JFR is the job finding rate, JSR is the job separation rate, JJ denotes direct job to job transitions, and the subscripts f and i denote formal and informal respectively. F is the firing cost, τ is the payroll tax, c is the hiring cost, , ϕ is the monitoring rate and σ is the penalty of informality.

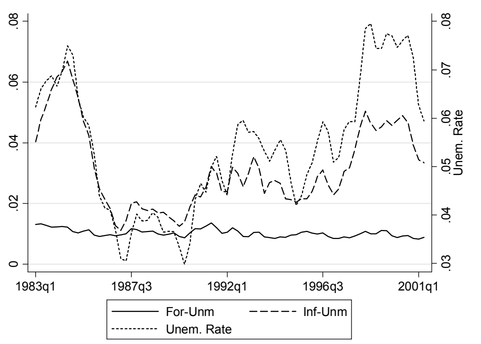
Figure 1: Labor Market in Brazil: 1983-2001

(a) Unemp. Rate and Share of Formal Jobs

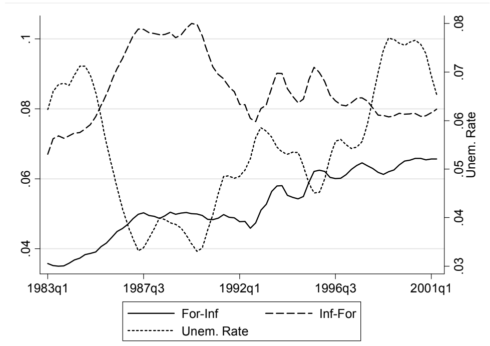
(b) Job Finding Rate and Unemp. Rate



(c) Job Separations and Unemp. Rate



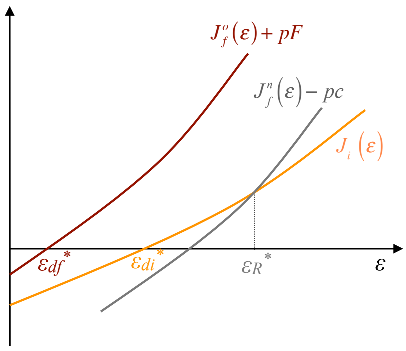
(d) Formal-Informal Flows and Unemp.Rate



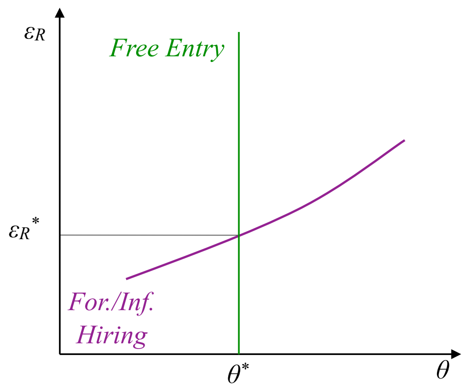
Notes: Series constructed using monthly data, quarterly averaged, from the Monthly Employment Survey (PME). *% For* is the share of formal employment constructed as number of formal workers over total employment. Unemployment rate (Unem. Rate) corresponds to number of unemployed workers over total labor force. Unm-For and Unm-Inf correspond to the average probability of transiting from unemployment into formal and informal employment respectively. For-Unm and Inf-Unm correspond to the average probability of transiting from formal and informal employment into unemployment. For-Inf and Inf-For correspond to the average probability of transiting between formal and informal employment.The series have been smoothed using a moving average filter a with a three-quarter window.

Figure 2: Steady State Equilibrium

(a) Equilibrium Thresholds



(b) Job Creation



(c) Job Destruction

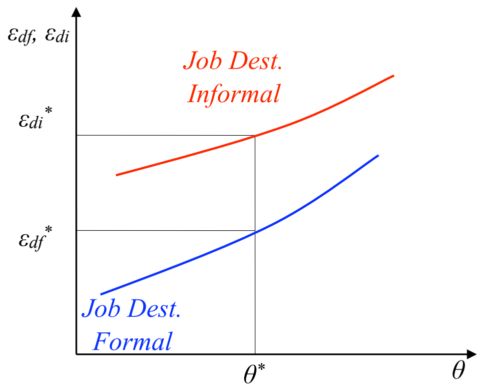
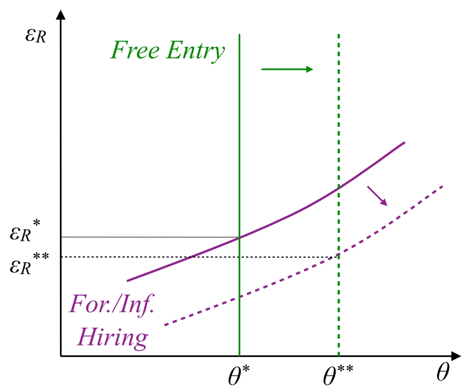


Figure 3: Increase in Productivity

(a) Job Creation



(b) Job Destruction