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On Mortality and Poverty: An Axiomatic Approach With A Modified Index

Haider A. Khan
University of Denver

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Haider A. Khan*
Professor of Economics
GSIS, University of Denver,
Denver, Co. 80208 USA
hkhan@du.edu

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Abstract

The purpose of this note is to analyze a somewhat novel aspect of poverty measurement, namely the impact of premature deaths of poor people on measured poverty. The perverse results one could easily obtain theoretically can be illustrated by using the widely used headcount ratio index. The proposal for a correction offered here involves the introduction of a new axiom of biological stress (ABS) to handle the case of premature mortality among the poor specifically, and to address the issue of increasing biological stress generally. A new headcount ratio index adjusted for the specific biological stress leading to premature deaths among the poor is constructed. It is simple, consistent with ABS, and easy to estimate. It is hoped that given the relevance of the critical demographic variables already acknowledged in the literature, the new index will be of use to the poverty reduction specialists

I. Introduction

What happens to poverty when a poor person dies?[1] A quick and accurate answer is that both headcount ratio and absolute poverty gap will register less poverty. By extension, other things being equal, any poverty index which is an increasing function of either one of these indicators (or both) will also show a decrease in poverty. But is this a really satisfactory answer?

If the rich and poor lived and died alike perhaps the above answer will have seemed satisfactory. But we know that this is not likely to be true. As Chakravarty (2001) and the citations therein show, some poverty indexes- for example the FGT index- seem to be empirically positively correlated with mortality rates. If this is true, then a decrease in poverty when a poor person dies would seem to clash- at least partly- with this finding. More broadly, our intuitions about poverty seem to fit uneasily, if at all, with the proposition that poverty decreases whenever a poor person dies.

A distribution in terms of life expectancies may allow us to pursue the issue further[2]. Demographers typically talk about life expectancies at various ages. For example, e(0) is the life expectancy at age 0, e(5) is the life expectancy at age 5 etc. If we take as our yardstick of how long a life a person can expect to live in a certain society at a certain historical time period, this may be a reasonable way of finding a context for answering the initial question.

What I would like to do in this note is then to ask the following contextual question: if a poor person dies prematurely, what would be a reasonable way of capturing this fact within a poverty index? In what follows, I first introduce a new axiom of poverty measurement to formalize this idea of reasonableness. I then show how to incorporate this idea in one modified version of a commonly used poverty measure which satisfies the new axiom.

II. Axioms of poverty comparisons

Since Sen's (1976) seminal work, axiomatization of poverty measurement has proceeded rapidly. The most important standard axioms in use are as follows.

1. Focus axiom: Only those below the poverty line are relevant. The incomes, consumption or well-being of the rich are not considered.
2. (Strong) Transfer axiom: other things being equal a transfer of income etc. from a poor to a less poor person will increase poverty.
3. A weaker version of the above axiom states that this kind of transfer will not decrease poverty.
4. Monotonicity axiom: Other things being equal, a reduction in the income of a poor person must increase poverty.

[1] I am grateful to Prof. Ravi Kanbur for raising this issue during a conference in honor of Erik Thorbecke at Cornell University. He is, of course, not responsible for the formulation here. See also Kanbur (2000, 2002, 2003, 2004).

[2] Kanbur and Mukherjee (2003) formalized the problem partly in terms of the expected lifetime income of the poor. The formulation here is consistent with their approach but is intended to offer an easily computable measure for policy analysis.

5. P(y,z) is increasing in z where P(.) is the poverty index, y is the income distribution vector and z is the poverty line.

Some commonly used measures- for example, the headcount ratio- violate some of these axioms. For the headcount ratio, in particular the transfer axiom need not hold. In fact, if a very poor person far below the poverty line transfers sufficient money to a person just below the poverty line, and as a result, the less poor person crosses over above the poverty line, the headcount ratio registers less poverty. This is indeed a perverse result. I will show that something similar to this happens in the dying poor case in a later section.

Now, suppose we ask the question: how can we avoid the headcount ratio (and other poverty measures) showing the perverse result that poverty would increase if a poor person died? The answer in axiomatic terms is simple. We can introduce an axiom of biological stress (ABS) in poverty measurement. It could be formulated more broadly, but here we focus on a specific instance of biological stress, i.e., premature mortality of the poor. I give a strong and a weak version of ABS below.

Strong axiom of Biological Stress (SABS): If a poor person dies prematurely, the poverty index should show an increase in poverty.

The weak axiom of Biological Stress does not require the index to necessarily show an increase in the above context. It can be stated as:

WABS: If a poor person dies prematurely, the poverty index should not show a decrease in poverty.

Needless to say, in practice the question of defining and measuring premature death will be a crucial one. The quantified extent of poverty increase ascribable to such 'premature' deaths among the poor as may occur will depend practically on how this issue is handled. For theoretical exposition the simple yardstick of e(0) or life expectancy at birth will be used throughout the rest of this note. Another issue is how to weigh the various types of premature deaths. For example, should infant mortality receive a higher weight than premature poor adult mortality?[3]

III. An Example of Applying ABS: the Headcount Index

As we know, the headcount ratio (H) can violate the transfer axiom. However, it is still the most widely used poverty index. The World Bank's one dollar a day and two dollars a day poverty indexes are both headcount (ratio) indexes in the first place. The simplicity of the measure is probably what explains in large part its popularity.[4] Therefore, the modification of this index can serve an important purpose in policy making by alerting the poverty reduction policy specialists about the problems of using the unmodified index.

In the literature, the headcount ratio is defined as the proportion of population (n) for whom consumption, income or some other suitable measure of living standard (y) is less than the poverty line (z). In other words:

[3] One approach would be to subtract the age at death from e(0) and use a normalized version of this "longevity gap" as the appropriate weight for each age-cohort. Work following this idea is underway.

[4] See for example World Development Report (1990:2001) in particular. UNCTAD's report on LDCs also uses the $1/day and $2/day poverty lines.

$H = \frac{q}{n}$ where q (=1,2, …,q) are (poor)

people with $y_i<z$ (i=1,2, …,q) …….(1)

We now show the following:

Proposition 1: If the number of poor people who die prematurely during the accounting period is $m \leq q$ then standard value of H is equal to $\frac{q-m}{n-m} \leq \frac{q}{n}$ with equality holding if no poor person dies prematurely.

Proof: Define $D = \frac{q}{n} - \frac{q-m}{n-m}$

Clearly, $D = \frac{q(n-m)-n(q-m)}{n(n-m)}$

Rearranging the terms,

$D = \frac{m(n-q)}{n(n-m)} \geq 0$

Therefore, $\frac{q}{n} \geq \frac{q-m}{n-m}$

In other words, $H \geq ADH$

Remarks: It should be noted that ADH=H in two cases:

1. when m=0. This simply means that in the absence of premature deaths among the poor, the two indexes are exactly the same.
2. when q=n. This is the case when everybody is poor and $H \geq ADH$, but equality holds because $\frac{q}{n} = 1 \forall q, h > 0$

However, in the second case the question still remains whether or not a premature death should still be counted as contributing to overall poverty. If the answer to this question is yes then we should still want to find an alternative index.

It should also be noted that when everybody who is poor dies (i.e. q=m) then the index has a zero value. This is perverse if even one of the dead among the poor dies prematurely. Finally, when everybody is poor and everybody dies, the index has a 0/0 form and is truly undefined. However, such cases of total nihilism seem truly morbid and can be excluded from further consideration because practically, in this case the entire community has ceased to exist.

The upshot of the previous discussion is simply that, in general, whenever a poor person dies prematurely, in general Mortality Adjusted Headcount Ratio or ADH<H where ADH

is the 'after-death headcount ratio'. Call this 'the perverse poverty' case for headcount ratio.

The question now is how to use the ABS axioms to modify H so that $ADH \geq H$. The strong inequality is consistent with SABS (& WABS) while the equality is consistent with WABS.

I propose the following modification:

Mortality Adjusted Headcount Ratio, $ADH = \frac{q-m}{n-m} + \frac{m}{n}$ ……..(2)

Proposition 2: It can be shown that $H \leq ADH$ as defined in (2).

Proof: For this, we write the difference between H and ADH as defined in (2) as:

$$H - ADH = \frac{q}{n} - (\frac{q-m}{n-m} + \frac{m}{n}) = \frac{m(m-q)}{n(n-m)} .....(3)$$

In (3) the R.H.S. is less than or equal to zero, since $m \leq q$

Therefore, $ADH \geq H$

since $0 < q \leq n$ and $0 < m \leq q$

Therefore, in general $ADH \geq H$ with equality holding when $m=q$.

Notice that a more general transformation where $ADH = \frac{q-m}{n-m} + f(\frac{m}{n})$ where f takes on non-negative values will also work. But the interpretation of (2) is straightforward, and intuitively appealing. It is also computationally simpler.

The interpretation of ADH is as follows. The first term on the right hand side in (2) gives us 'the perverse poverty case' (PPC) which I have argued clashes with our intuitions about measuring poverty in the context of prematurely dying poor. In order to correct for this, we have added a second term which is the proportion of the poor who die prematurely. This is simply another special headcount ratio. However, because of the inability of the standard headcount ratio to take this special ratio, accounting for premature deaths among the poor, into account, the standard H gives us the PPC. It is as if the prematurely dying poor people are invisible, and indeed they are so under the standard H-measurement.

What we have done with the second term in (2) is then to simply have made visible what was kept invisible by the standard measurement. It should be emphasized that this exercise in *visibility of the poor* is based on the standard demographic concept of biological stress and its impact. Clearly more complex weighting schemes and other modifications are possible but this initial approximation gives us a quantitative estimate that can be derived from pieces of data readily available from demographic surveys and standard household expenditure surveys used in poverty analysis. Thus, ADH as it is given in equation 2, has the virtues of simplicity, transparency, ready policy evaluation capability, and therefore straightforward practical applications.

One welfare economic issue left unsettled by the form of the second term is whether it gives too much or too little weight to these premature deaths among the poor. It seems to me that the additive term where all poor people who die prematurely receive equal weight is consistent with the standard headcount ratio exercise where differences among the poor are ignored. Indexes that give the poorer people who die prematurely greater weight will in all likelihood show even greater poverty. In fact, a simple weighting scheme that gives a weight of one to the richest poor person who dies prematurely and increases this to m(>1) for the poorest person who dies prematurely will increase the value of the second term in (2) by a factor of m(m+1)/2. Given the additive form of the index, this will clearly lead to an increase in the value of overall ADH. However, for the sake of ease of application, only the simple form is presented here.

IV. Summary and Conclusions

The purpose of this note was to analyze a somewhat novel aspect of poverty measurement, namely the impact of premature deaths of poor people on measured poverty. The perverse results one could easily obtain theoretically were illustrated by using the widely used headcount ratio index. My proposal for a correction involves the introduction of a new axiom to handle the case of premature mortality among the poor specifically, and to address the issue of increasing biological stress generally. Accordingly, the new axiom is called an axiom of biological stress (ABS). A new headcount ratio index adjusted for the specific biological stress leading to premature deaths among the poor is constructed. It is simple, consistent with ABS, and easy to estimate. It is hoped that given the relevance of the critical demographic variables already acknowledged in the literature, the new index will be of use to the poverty reduction specialists.

There are many extensions that are possible. These may go in two related directions in general. One is to further modify the modified headcount ratio presented here-perhaps along the lines suggested in section III[5], or in a different direction. The second is to look at other poverty indexes and see to what extent these, too, may need to be modified.

More generally, demographic factors related to biological stress signals other than just premature mortality among the poor may be considered as well. As Chakravarty (2001) has shown, a neat separation between poverty and destitution studies on the one hand, and the biological stress signals as these relate to the poor may be preventing the realization of intellectual gains from trade and from strategic complementarities in these two areas of research. However, the case of each type of biological stress signal should be treated separately before attempting to combine their effects. Thus we must begin by carefully taking stock of the specificities of these stress signals as in the case of premature mortality among the poor, and then proceed to examine systematically the set of theoretically plausible relations between these stress signals and various indexes of poverty.

[5] As mentioned in footnote 3, a related issue is whether the difference between e(0) and the actual age at death should be given some weight. We could also simply add up the number of years thus lost and either use this as a separate measure or include it in some fashion in ADH consistent with the ABS axioms.

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