On Fitting Somers' Equation for Seasonally Oscillating Growth, with Emphasis on T-Subzero

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Somers (1988) showed in the previous issue of Fishbyte that the seasonally oscillating version of the von Bertalanffy growth function (VBGF) proposed by Pauly and Gaschütz (1979):

$$L_t = L_\infty\left[1 - \exp\left(-K(t - t_o) - \frac{CK}{2\pi}\sin 2\pi(t - t_s)\right)\right] \qquad \ldots 1)$$

leads to a biased estimation of the "age" at length zero (i.e., "t_o"). He suggested, instead, the more appropriate formula:

$$L_t = L_\infty\left[1 - \exp\left(-K(t - t_o) - \frac{CK}{2\pi}(\sin 2\pi(t - t_s) - \sin 2\pi(t_o - t_s))\right)\right] \qquad \ldots 2)$$

Equations (1) and (2) can be fitted to data either by some non-linear fitting routine or through linearization. In this contribution, we present a method for estimating the parameters of equation (2) through multiple regression analysis. We also present a method for estimating the variance of t_o.

Equation (2) can be linearized as:

$$\ln\left[1 - \frac{L_t}{L_\infty}\right] = -Kt + Kt_o - \frac{CK}{2\pi}\sin 2\pi(t - t_s) + \frac{CK}{2\pi}\sin 2\pi(t_o - t_s) \qquad \ldots 3)$$

where:

b_0	=	$Kt_o + (CK/2\pi)\sin 2\pi(t_o - t_s)$			
b_1	=	$-K$	X_1	=	t
b_2	=	$-(CK/2\pi)\cos 2\pi t_s$	X_2	=	$\sin 2\pi t$
b_3	=	$(CK/2\pi)\sin 2\pi t_s$	X_3	=	$\cos 2\pi t$

and in which only b_0 differs from the corresponding expression for equation (1), for which $b_0 = Kt_o$. Thus, as in the case of the Pauly and Gaschütz model, we have

$$K = -b_1$$

$$C = -2\pi b_2/K\cos(2\pi t_s) = 2\pi b_3/K\sin(2\pi t_s)$$

$$t_s = \tan^{-1}(-b_3/b_2)/2\pi$$

On the other hand, the value of t_o can be obtained only numerically, i.e., by using the Newton-Raphson method of finding the root of the nonlinear equation, viz,

$$t_{o\,(i+1)} = t_{o\,(i)} - f(t_{o\,(i)})/f'(t_{o\,(i)})$$

where:

$$f(t_0) = Kt_0 + (CK/2\pi)\sin 2\pi(t_0 - t_s) - b_0$$

$$f'(t_0) = K + CK\cos 2\pi(t_0 - t_s)$$

with the initial guess of $t_{0(0)} = b_0/K$.

Because b_1, b_2, b_3, are the same as in the model of Pauly and Gaschütz, the variance of L_∞, K, C and t_s can be estimated by the method of Hanumara and Hoenig (1987), not recalled here. This method was adapted here to estimate the variance of t_0. It requires re-expression of b_0 in terms of b_1, b_2 and b_3 and the covariances of b_1, b_2 and b_3 with t_0, i.e.,

$$b_0 = -b_1 t_o - b_2 \sin 2\pi t_o - b_3 \cos 2\pi t_o$$

$$\text{cov}(b_1, t_0) = \frac{\text{cov}(b_1, b_0) + t_o V(b_1) + \sin 2\pi t_o \text{cov}(b_1, b_2) + \cos 2\pi t_o \text{cov}(b_1, b_3)}{-b_1 - 2\pi b_2 \cos 2\pi t_o + 2\pi b_3 \sin 2\pi t_o}$$

$$\text{cov}(b_2, t_0) = \frac{\text{cov}(b_0, b_2) + \sin 2\pi t_o V(b_2) + t_o \text{cov}(b_1, b_2) + \cos 2\pi t_o \text{cov}(b_2, b_3)}{-b_1 - 2\pi b_2 \cos 2\pi t_o + 2\pi b_3 \sin 2\pi t_o}$$

$$\text{cov}(b_3, t_0) = \frac{\text{cov}(b_3, b_0) + \cos 2\pi t_o V(b_3) + t_o \text{cov}(b_1, b_3) + \sin 2\pi t_o \text{cov}(b_2, b_3)}{-b_1 - 2\pi b_2 \cos 2\pi t_o + 2\pi b_3 \sin 2\pi t_o}$$

This leads to:

$$V(t_o) = \Big[V(b_0) + t_o^{\,2} V(b_1) + \sin^2 2\pi t_o V(b_2) + \cos^2 2\pi t_o V(b_3) + 2t_o \sin 2\pi t_o \text{cov}(b_1, b_2)$$
$$+ 2t_o \cos 2\pi t_o \text{cov}(b_1, b_3) + 2\cos 2\pi t_o \sin 2\pi t_o \text{cov}(b_2, b_3)$$
$$+ 2t_o \text{cov}(b_1, b_0) + 2\sin 2\pi t_o \text{cov}(b_2, b_0) + 2\cos 2\pi t_o \text{cov}(b_3, b_0) \Big] * \frac{1}{Q^2}$$

where $Q = -b_1 - 2\pi b_2 \cos 2\pi t_o + \sin 2\pi t_o$

Equation (2) and the method above will replace equation (1) in all software produced at ICLARM, including ETAL I and the various versions of the ELEFAN programs.

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References

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