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The Dynamic Interrelations between Unequal Neighbors: An Austro-German Case Study

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Abstract

This article investigates the effects and transmission channels of shocks between two asymmetric neighboring countries. In particular, we investigate Austria and Germany which are highly integrated due to their common language and common membership of the European Monetary Union. Generalized impulse response functions reveal that there are large and significant effects of shocks to the German economy on Austria. In contrast, the effects of shocks to the Austrian economy on Germany are barely significant and if they are, their magnitude is comparatively small. Furthermore we can show that multiplier effects are present in Germany but not in Austria and we identify hysteretic properties in Austrian unemployment.

JEL classification: C32, C53, F41

Keywords: Structural Vector Error Correction Model; Open Economies; Economic Integration; Generalized Impulse Response Analysis

1 Introduction

The economic performance of small countries is often determined by large neighbors. In this study we want to identify the transmission channels that ensure these strong interrelations by analyzing the effects of shocks hitting one economy on the neighboring country. In particular we investigate Austria and Germany as they are highly integrated, but nevertheless represent distinct nations. They are both member states of the European Union, they share a common currency and a common language. There are a lot of important sectors where developments in Germany influence the performance of the Austrian economy. Important examples include: The bulk of Austrian exports goes to Germany (see Statistik Austria (2006)), many Germans spend their vacations in Austria, and lots of German workers supply their skills on the Austrian labor market and vice versa. The latter effect has become more and more important during recent years, when the German economy has suffered from a high unemployment rate (see for example Biffl (2006)). As a consequence, the number of German workers in Austria increased substantially. We explicitly address this issue by accounting for cointegration of Austrian and German unemployment. Similar models could prove useful in analyzing interactions between Canada and the United States, between Mexico and the United States, or between Portugal and Spain, to mention only a few examples. Some regional adaptations would be needed because Canada and the United States share a common language but no common currency, the converse holds true for Portugal and Spain, while Mexico shares neither a common language nor a common currency with the United States.

Fenz and Schneider (2007) analyze the reaction of the Austrian economy to German shocks by means of a Structural Vector Autoregressive (SVAR) model. They distinguish between demand, supply and monetary policy innovations and show that the reaction of the Austrian economy to German shocks exhibits on average 44% of the reaction of the German economy itself. In addition, they show that the transmission mechanisms to Austria remained quite stable over time such that the German economy is even nowadays very important for developments in Austria. In contrast, we use a Structural Vector Error Correction model (SVECM) because this model class has some key advantages over SVAR models (see Garratt et al. (2006), Juselius (2007)). First of all, there is no need to impose short-run restrictions, which is an advantage because economists lack consensus on short-run dependence structures. Secondly, we can circumvent the need to impose a causal recursive ordering on endogenous variables. This is done by modeling the cointegration space via long-run economic restrictions instead of arbitrary orthogonalizations and by using generalized impulse response functions that do not rely on a causal recursive ordering of endogenous variables (see Pesaran and Shin (1998)).

In a comparable study Gaggl et al. (2009) investigate the dynamics between the European Monetary Union and the United States, currently representing the dominant economic areas among developed countries. This is done by comparing impulse response functions of a United States model with the eurozone as foreign economy, with those of a eurozone model where the United States represent the foreign economy. However, there are a number of important methodological differences to their approach. First of all, monetary variables do not play such an important role in explaining differences between Austria and Germany, as the Schilling was coupled to the Deutsche Mark over the relevant periods. Furthermore, since 1998, both countries have been members of the European Monetary Union and have shared a common currency. Therefore money aggregates like M0 or M1 do not exist for the individual economies anymore and also the exchange rate has been fixed since 1998. For these reasons, attention has shifted from monetary aggregates to the labor market in our study. The advantage of doing so is that possible interrelations with respect to labor migration can be assessed as well. Another difference concerns the construction of relevant aggregate variables in the foreign economy. Gaggl et al. (2009) construct data for the eurozone from time series of the individual member countries. There are some difficulties associated with such an approach, for instance the choice of accurate weights (see Beyer et al. (2001) for more details), the fact that measurement errors carry over to the constructed aggregates, and that the foreign aggregated series are very smooth as compared to the domestic ones, since shocks are averaged out. In examining the relations between two individual countries there is no need to choose accurate weights, measurement errors are far less important, and shocks are not averaged at all. Therefore one could expect to work with data of higher quality in our case.

The article proceeds as follows: Section 2 describes the general approach we are using, section 3 includes the dynamic economic model that allows to derive long-run restrictions on the cointegration space, in sections 4 and 5 the model is estimated and various specification tests are carried out, finally section 6 summarizes the results, draws various conclusions and highlights scope for further research.

2 The Structural Vector Error Correction Model

In this section we describe the general model class we are using for our analyses in more detail. Furthermore we give a short overview on identification schemes with respect to the cointegration space and with respect to impulse response analysis.

2.1 General Model Formulation

If exogenous variables are allowed for, the most general form of a SVECM with k endogenous variables can be written as (see Garratt et al. (2006)):

$$A\Delta z_t = \tilde{a} + \tilde{b}t + \tilde{\Pi} z_{t-1} + \sum_{i=1}^{\rho_1 - 1} \tilde{\Gamma}_i \Delta z_{t-i} + \sum_{i=0}^{\rho_2 - 1} \tilde{\Psi}_i \Delta x_{t-i} + \epsilon_t, \tag{1}$$

where A represents a $k \times k$ matrix containing the contemporaneous effects of a change in one endogenous variable to the other endogenous variables, Δ denotes the differencing operator, z_t is a $k \times 1$ vector containing the endogenous variables, $\tilde{a}$ is a $k \times 1$ vector of intercepts, the term $\tilde{b}$ describes the coefficients of the time trend, t, $\tilde{\Pi}$ is the matrix of adjustment coefficients to deviations from long-run equilibria defined by stationary linear combinations of endogenous variables in levels, $\tilde{\Gamma}_i$ is a $k \times k$ matrix containing the coefficient estimates of the vector autoregressive part, $\tilde{\Psi}_i$ describes the coefficient matrix of exogenous variables included in the vector x_t and ϵ_t is a $k \times 1$ vector of error terms with variance covariance matrix Ω. Finally ρ_1 and ρ_2 are the unknown lag-lengths of endogenous and exogenous variables respectively. Note that for exogenous variables a lag-length of zero in first differences is possible, i.e. they are allowed to affect endogenous variables contemporaneously. To get to the reduced form of equation (1) it has to be premultiplied by A^{-1}, such that

$$\Delta z_t = a + bt + \Pi z_{t-1} + \sum_{i=1}^{\rho_1 - 1} \Gamma_i \Delta z_{t-i} + \sum_{i=0}^{\rho_2 - 1} \Psi_i \Delta x_{t-i} + u_t, \tag{2}$$

where $a = A^{-1}\tilde{a}$, $b = A^{-1}\tilde{b}$, $\Gamma_i = A^{-1}\tilde{\Gamma}_i$, $\Pi = A^{-1}\tilde{\Pi}$ and $u_t = A^{-1}\epsilon_t$. The variance covariance matrix of u_t can be written as $A^{-1}\Omega(A^{-1})^t$ and is denoted by Σ. Further, Π can be expressed as

$$\Pi = \alpha\beta^t, \tag{3}$$

where α is a $k \times r$ matrix including the adjustment coefficients to deviations from the long-run equilibria and β is a $k \times r$ matrix including the restrictions on the cointegrating relations. In this context r is the number of cointegrating vectors among endogenous variables and via equation (3) also the rank of Π.

2.2 Identifying Short-run and Long-run Restrictions

For exact identification of the long-run relationships, r^2 restrictions must be imposed on β. Johansen (1988) and Johansen (1991) investigate systems defined by equation (3) and provide tests for the rank of Π as well as statistically motivated restrictions for exact identification. These restrictions assume that the column vectors of β are orthogonal to each other, in the sense that in the first column vector the first entry is normalized to one, in the second column vector the second entry is normalized to one and so on, while the other $r^2 - r$ restrictions are zero restrictions on the remaining first $r - 1$ entries of each column vector in β. This procedure is meaningful in a statistical sense, but it renders economic interpretation of equation (3) impossible if $r > 1$ (see also Garratt et al. (2006)). Furthermore the restrictions depend on the ordering of endogenous variables in the SVECM, such that different results can be obtained just by changing the rule of sequencing entries in the z_t-vector. In contrast to this approach, we derive restrictions with the help of economic theory in section 3.

In order to recalculate the structural coefficients of (1) from the reduced form representation (2), additional k^2, so called short-run restrictions, need to be imposed on A and/or Ω (see Garratt et al. (2006)). After this structure is imposed, impulse response analysis can be performed. The most widely used method to implement these short-run restrictions proposed by Sims (1980) requires A to be a lower triangular matrix and Ω to be a diagonal matrix. This procedure assumes a causal recursive ordering of the variables in z_t. Consequently, also impulse response functions vary subject to the choice of ordering the endogenous variables and one is confronted with similar problems as in the case of statistically motivating the identifying restrictions on the cointegrating vectors.

A similar argument holds for the reduced form representation in equation (2) without structural restrictions imposed on A. In this case Sims (1980) suggests the Choleski decomposition $\Sigma = PP'$ where P is a $k \times k$ lower triangular matrix, to calculate cumulative orthogonalized scaled impulse response functions according to the formula (see Pesaran and Shin (1998)):

$$\psi^o_{z,j}(h) = B_h P e_j, \tag{4}$$

where $\psi^o_{z,j}(h)$ refers to the orthogonalized scaled cumulative impulse response of endogenous variables in period $t + h$ to an exogenous shock of the error term in equation j in period t, B_h is a matrix containing the cumulative effects of such a shock according to the infinite moving average representation, and e_j is a $k \times 1$ selection vector with zero in all but the jth entry. Note that due to the presence of the matrix P in equation (4) a causal recursive ordering is imposed on the impulse responses.

In this study we use generalized impulse response functions as described in Pesaran and Shin (1998). In contrast to equation (4), they are calculated according to the formula:

$$\psi^g_{z,j}(h) = \frac{B_h \Sigma e_j}{\sqrt{\sigma_{jj}}}, \tag{5}$$

where $\psi^g_{z,j}(h)$ refers to the generalized scaled cumulative impulse response of endogenous variables in period $t + h$ to an exogenous shock of the error term in equation j in period t, and σ_{jj} is the variance of the error term in equation j. Note that the matrix P does not show up in this representation, such that the impulse responses do not depend on the ordering of the endogenous variables. Furthermore, in contrast to orthogonalized impulse responses, all endogenous variables are contemporaneously affected by an exogenous shock in a certain period.

To summarize, we are able to avoid a causal recursive ordering of endogenous variables by deriving restrictions on the cointegration space via long-run economic theory and by making use of generalized impulse response analysis instead of the standard approach proposed by Sims (1980).

3 Theoretically Motivated Long-run Restrictions

To motivate potential restrictions on the parameters of the cointegrating vectors, Garratt et al. (2006) use a loose collection of relationships derived on the basis of arbitrage conditions, accounting identities, solvency requirements and assumptions with respect to the production technology in the investigated economy. This procedure leads to five potential restrictions to be imposed on the cointegration space of their model: The Money Market Equilibrium condition (MME), the Fisher Inflation Parity (FIP), the Interest Rate Parity (IRP), the Purchasing Power Parity (PPP), and an Output Gap (OG) relation. However, Garratt et al. (2006) do not provide any microfoundations for their behavioral equations. In contrast, Gaggl et al. (2009) use a single open economy model to derive the MME, the FIP and the IRP, where the PPP has to hold in order for the other three relationships to be consistent. The OG relation is derived in the same way as in Garratt et al. (2006).

The purpose of this section is to go one step further and set up a model for two open economies that engage in bilateral trade to motivate the IRP and the FIP. In this setting the PPP relationship follows immediately from the underlying preferences of households. The OG relation is a consequence of the production processes in both economies, following neoclassical production functions with labor augmenting technological progress. There is no MME condition because the relevant variables are not included (see Appendix A) since we shift attention from monetary variables to the labor market. Consequently, a Labor Market Condition (LMC) is required, which relates to the decision of individuals to migrate between the two economies. This condition can be motivated by adapting the Gravity Equation concept (see for example Feenstra (2004), Faustino and Leitão (2008)), such that a country with low unemployment attracts workers from a country where unemployment is high.

3.1 Description of the Production Side

The production sides of the two economies are variants of the one described in Garratt et al. (2006) (see also Romer (2001) or Barro and Sala-i-Martin (2004)). Output at home and abroad is produced according to the following constant returns to scale production functions:

$$Y_t = F(K_t, A_t L_t) = A_t L_t F\left(\frac{K_t}{A_t L_t}, 1\right) = A_t L_t f(k_t), \tag{6}$$

$$Y_t^* = F^*(K_t^*, A_t^* L_t^*) = A_t^* L_t^* F^*\left(\frac{K_t^*}{A_t^* L_t^*}, 1\right) = A_t^* L_t^* f^*(k_t^*), \tag{7}$$

where Y_t denotes real output at home, Y_t^* denotes real output in the foreign economy, $F \equiv F^*$ and $f \equiv f^*$ are well behaved production functions, fulfilling the Inada conditions, A_t and A_t^* refer to the technological levels of the two economies, K_t and K_t^* to the aggregate capital stocks of both countries and k_t and k_t^* are the capital stocks per unit of effective labor. With respect to the overall number of employed workers it is assumed that they represent a fraction of the total population:

$$L_t = \delta N_t, \tag{8}$$

$$L_t^* = \delta^* N_t^*, \tag{9}$$

where N_t denotes the number of inhabitants in the home country, N_t^* the number of inhabitants in the foreign country, and δ and δ^* comprise a measure for the fraction of total population employed in the steady state. This formulation implies that the natural unemployment rate is equal to $1-\delta$ in the domestic economy and $1-\delta^*$ abroad. Furthermore it is assumed that technology behaves according to:

$$\eta A_t = \theta A_t^* = \bar{A}_t, \tag{10}$$

where $\bar{A}_t$ is the technological level in the rest of the world and $\eta > 1$ as well as $\theta > 1$ measure incompletenesses of the diffusion process i.e. technology adoption barriers (see Parente and Prescott (1994)). Equation (10) states that the technological levels of the two countries are determined by the world level of technology. Nevertheless there might be difficulties to implement new ideas in both regions, such that gaps between technological levels of the domestic economy, the foreign economy and the rest of the world can remain. Inserting these expressions into the production functions and dividing domestic by foreign output gives:

$$\frac{y_t}{y_t^*} = \frac{\theta\delta}{\eta\delta^*}\frac{f(k_t)}{f(k_t^*)}, \tag{11}$$

where y_t and y_t^* denote per capita output. Equation (11) describes the fact that as long as this ratio is smaller or larger than one, there is an output gap between the two economies, which is determined by the relative size of technology diffusion parameters, the relative size of natural unemployment rates and differences in the capital intensities between the two countries.

3.2 Description of the Consumption Side

In order to get to the FIP, IRP and PPP relations, a dynamic consumer optimization model is set up in discrete time for two open economies with capital mobility restrictions. The underlying structure is that a representative household seeks to maximize its lifetime utility generated by consumption of domestic and foreign goods, subject to a budget constraint, which allows the household to invest its income in domestic capital as well as in domestic and foreign bonds. In addition, a cash-in-advance constraint in the spirit of Clower (1967) is implemented, i.e. individuals are allowed to consume from money holdings but not from capital or bonds in the subsequent period. This means that if households want to consume, they are forced to convert assets that pay a rate of return, into money that does not pay any return but is subject to inflation. This conversion has to take place in period $t-1$, so that in period t individuals own liquid assets, allowing them to buy consumption goods. Therefore the representative household solves the following optimization problem:

$$\max_{C_t, C_t^*} \sum_{t=0}^{\infty} \rho^t U(C_t, C_t^*) \tag{12}$$

subject to

$$C_t + P_t^* C_t^* + K_t + B_t + B_t^* + M_t = (1+r_t)K_{t-1} + w_t L_t + \frac{1+i_t}{1+\pi_t}B_{t-1} + \\ + \frac{1+i_t^*}{1+\pi_t^*}B_{t-1}^* + \frac{M_{t-1}}{1+\pi_t}, \tag{13}$$

$$C_t + P_t^* C_t^* \leq \frac{M_{t-1}}{1+\pi_t}, \tag{14}$$

where ρ is the subjective discount rate, C_t denotes consumption of the domestically produced aggregate which is the numéraire good, C_t^* refers to consumption of the aggregate produced in the foreign country, P_t^* to the price level of foreign goods, K_t is the real capital stock at home, B_t are real bonds issued by the home government, B_t^* stands for real bonds issued by the foreign government, M_t refers to individual's real money holdings, $(1+r_t)$ denotes the capital rental rate which is equal to the real rate of return since we do not allow for depreciation, $(1+i_t)$ and $(1+i_t^*)$ describe the domestic and foreign nominal interest rates on bonds respectively, $(1+\pi_t)$ and $(1+\pi_t^*)$ are the domestic and foreign inflation rates, w_t is the real wage rate and L_t refers to labor supply of households. The left hand side of the budget constraint, equation (13), comprises total household expenditures and savings in period t, whereas the right hand side refers to total household income in the same period[1]. Equation (14) is the cash-in-advance constraint which ensures that expenditures for consumption in period t are not higher than the period t-real value of nominal liquid assets carried over from period $t-1$.

In addition, the following assumptions are implemented: First of all, households inelastically supply all available time on the labor market, i.e. they do not value leisure. As a consequence, there is no decision involved with respect to labor market participation. Instead, no matter how low real wages are, it is optimal for individuals to work, hence there is no voluntary unemployment and L_t is exogenously given by time restrictions and normalized to one. This assumption is implemented since we do not have data with respect to voluntary unemployment. Secondly, as individuals are rational, they do not convert more assets into money than absolutely necessary to finance the optimal amount of consumption in period t. Consequently, the cash-in-advance constraint holds with equality. Lastly, individuals are assumed to have Cobb-Douglas preferences over the two available consumption goods, such that the period utility function can be written as

$$U(C_t, C_t^*) = C_t^{\alpha} C_t^{*1-\alpha}, \tag{15}$$

where α is the budget share of the consumption good produced at home. With these assumptions implemented, one can solve the dynamic optimization problem (see Appendix B). After reformulating the optimality conditions, the following relationships, whose log-

[1]Note that both countries are members of a currency union and so exchange rates are not included as explanatory variables in the SVECM. Consequently, they do not show up in equation (13).

arithmic expressions are estimable versions of the theoretically implied restrictions, can be derived:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_t}, \tag{16}$$

$$\frac{1 + i_t}{1 + \pi_t} = \frac{1 + i_t^*}{1 + \pi_t^*}, \tag{17}$$

$$\frac{CPI_t}{CPI_t^*} = 1, \tag{18}$$

where CPI_t and CPI_t^* denote the consumer price indices at home and abroad respectively. Equation (16) describes the FIP which states that the real rate of return on capital, i.e. the real interest rate, has to be equal to the deflated nominal interest rate. Equation (17) is the IRP in the absence of an exchange rate, stating that the real interest rates in both economies have to be equal to avoid arbitrage rents. Finally, equation (18) is the PPP in the absence of an exchange rate, which relates the price levels between the domestic and foreign economy in terms of consumer price indices, stating that they have to be equal.

3.3 Description of the Gravity Equation and Labor Migration

The Gravity Equation (see for example Feenstra (2004), Faustino and Leitão (2008)) is used in International Trade literature to estimate bilateral trade flows. Basically it states that trade between two economies is positively linked to their size and negatively linked to their distance. Compared to trade flows there are several other forces that promote migration, the two most important ones being differences in wage income levels and differences in household labor market perspectives between two economies. Additionally, the interpretation of distance is modified in the literature to account for legal impediments, language barriers, personal reluctance to migrate, or simply bureaucratic obstacles as well (see for example Borjas (1995), Borjas (1996)).

In the case considered here, where it is optimal for individuals to supply all their available time on the labor market, independently of the real wage rate, the income differential between the two economies is not an accurate measure to be included in the specific Gravity Equation. Furthermore, as compared to other areas (for example Mexico and the United States), income levels do not deviate substantially between Austria and Germany, so there is no need to consider them as explanatory variables for bilateral migration. Instead, the difference between the two unemployment rates is a more promising determinant. Therefore the Gravity Equation has the following form:

$$M = \beta_g \frac{(1 - \delta) - (1 - \delta^*)}{D} + u. \tag{19}$$

In this specification, M characterizes migration from Austria to Germany, which is positive if the difference of the unemployment rates $(1 - \delta) - (1 - \delta^*)$ is similarly positive, and negative otherwise. The parameter D is the modified distance parameter, measuring the overall costs of migration as described above, and β_g is the parameter to be estimated[2]. This form of the Gravity Equation ensures that there is migration as long as there is a difference between the unemployment rates. Therefore

$$\frac{1-\delta}{1-\delta^*} = 1 \tag{20}$$

holds in the long-run, stating that home and foreign unemployment rates tend to equalize. Equation (20) can be interpreted as follows: Optimal behavior of individuals ensures that they supply their whole available time on the labor market. In equilibrium there is unemployment in both economies and therefore some agents, who do not find work in the economy with the higher unemployment rate, choose to migrate to the other region, after taking into account the associated costs. This process lasts until the gap between the two unemployment rates, i.e. the fundamental reason for migration, is eventually eliminated[3].

3.4 Implementation of the Restrictions

The theoretically derived equations (16), (17), (18), (11) and (20) have to be matched with the data in Appendix A, and so logarithmic versions are obtained as:

$$\begin{aligned}
\log(1+r_t) &= \log(1+i_t) - \log(1+\pi_t), && (21)\\
\log(1+i_t) - \log(1+\pi_t) &= \log(1+i_t^*) - \log(1+\pi_t^*), && (22)\\
\log(CPI_t) &= \log(CPI_t^*), && (23)\\
\log(y_t) - \log(y_t^*) &= \log(\theta f(k)) - \log(\eta f(k^*)), && (24)\\
\log(1-\delta) &= \log(1-\delta^*), && (25)
\end{aligned}$$

which are deterministic conditions, holding in the theoretical model. The empirically observable time series are subject to various shocks, but economic forces ensure that the described restrictions are fulfilled in the long-run. However, during the adjustment process, they need not be fulfilled with equality. Instead, so called "long-run errors" (Garratt et al. (2006)) describe deviations from these relations in the short-run. As a consequence (21), (22), (23), (24) and (25) have to be augmented by an error term, i.e. reformulated in a stochastic way, to represent cointegrating equations that can be estimated. Recalling that the vector z_t contains the following elements (see Appendix A): $(z_t)^t = (DPAT, PD, RAT, RGER, UAT, UGER, YAT, YGER)$ these cointegrating equations read:

[2]Note that no constant term is included, since the theoretical considerations, especially the inelastic labor supply of households, do not allow other factors to drive migration.

[3]Note that this has an influence on equation (11) in the sense that the ratio $\frac{\delta}{\delta^*}$ disappears and it has to be modified to

$$\frac{y_t}{y_t^*} = \frac{\theta}{\eta}\frac{f(k_t)}{f(k_t^*)},$$

which will be used in the next subsection to describe the logarithmic version of the output gap relation.

$$RAT_t - DPAT_t = \beta_{1,0} + \xi_{1,t+1}, \tag{26}$$

$$RAT_t - RGER_t = \beta_{2,0} + \xi_{2,t+1}, \tag{27}$$

$$PD_t = \beta_{3,0} + \xi_{3,t+1}, \tag{28}$$

$$YAT_t - YGER_t = \beta_{4,0} + \xi_{4,t+1}, \tag{29}$$

$$UAT_t - UGER_t = \beta_{5,0} + \xi_{5,t+1}, \tag{30}$$

where $\beta_{i,0}$ represents the constant and $\xi_{i,t+1}$ the error term, i.e. the long-run error of the respective restriction. The first equation is the stochastic version of the FIP, and consequently $\beta_{1,0}$ represents an estimate for the real interest rate, the second equation refers to the stochastic version of the IRP, such that the estimated value of $\beta_{2,0}$ should be zero, the third relation is the stochastic version of the PPP, so the estimate of $\beta_{3,0}$ should be zero as well, the next equation describes the stochastic OG relation, with $\beta_{4,0}$ being an estimate for $\log(\theta f(k_t)) - \log(\eta f(k_t^*))$, the natural output gap between the two economies, and the last equation is the stochastic counterpart of the LMC, stating that the unemployment rates of the two economies should be equal in the long-run, which implies $\beta_{5,0} = 0$. Note that the long-run errors $\xi_{i,t+1}$ have to be stationary, otherwise the restriction cannot be imposed as an estimable cointegrating equation (see Juselius (2007)). Putting all these things together, yields the following version of equation (2)

$$\Delta z_t = a + bt + \alpha\beta^t z_{t-1} + \sum_{i=1}^{\rho_1 - 1} \Gamma_i \Delta z_{t-i} + \Psi_0 \Delta x_t + u_t, \tag{31}$$

where exogenous variables are only allowed to affect endogenous variables contemporaneously as in Garratt et al. (2006), the vector x_t contains the oil price, so $x_t = (POIL)$, and the matrix β^t has the following form:

$$\beta^t = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{pmatrix}. \tag{32}$$

Again recalling the z_t-vector, the first row of this matrix represents the FIP, the second row the IRP, the third row refers to the PPP, the fourth row to the OG relation, and the last row defines the restrictions implied by the LMC.

4 Estimating the Structural Vector Error Correction Model

In this section we assess the specific form of our model that is estimated afterwards and has to undergo a series of tests for misspecification.

4.1 Lag Selection and Properties of the Cointegrating Relations

First of all we assess the Unit Root properties of the underlying time series. The results of the Augmented Dickey-Fuller test, of the Phillips-Perron test and of the Kwiatkowski-Phillips-Schmidt and Shin test, displayed in Appendix C, suggest to treat price levels in both countries as I(2) and all other variables as I(1). The next step is to determine the optimal number of time lags to be included in the model. Therefore we estimate VAR(ρ) models in levels for $\rho = 1, ..., 4$[4] and display the resulting values of AIC[5] in table 1, where three asterisks indicate the smallest number.

LAGS	AIC	
1	-49.4882	
2	-50.3602	
3	-50.3207	
4	-50.9030	***

Table 1: Model Selection ba means of AIC

AIC favors the specification with four lags, which corresponds to a specification with three lags in first differences. With this result in mind and recalling that the potential restrictions did not allow for time trends, the trace test is carried out to find the number of cointegrating relations between endogenous variables (see Johansen (1995), Juselius (2007)). The corresponding number of cointegrating equations identified by this test is two[6].

Altogether the theoretical discussion revealed the possibility of five cointegrating relations among endogenous variables. However, the trace test indicates that only two of them are present in the data. We can rule out the interest rate parity as a cointegrating equation because there has been convergence in Austrian and German interest rates until 1998 when both countries became part of the currency union. After 1998 the interest rates of Austria and Germany have been the same by construction. Consequently, the resulting long-run errors are not stationary, which would have been required for a valid restriction on the cointegration space. Now the question arises which of the four remaining potential relationships are the "true" ones. This question can be reformulated to ask

[4]Since there are eight endogenous variables in the SVECM and only 146 observations are available for estimation, lag orders higher than 4 would clearly lead to models that could not be estimated in a meaningful way.

[5]Note that this is the optimal criterion if the purpose of the model is forecasting (see Lütkepohl (2005)).

[6]The results of the trace test are available upon request.

which combination of the potential restrictions leads to a model that fits closest to the data generating process. From this point of view the natural way to proceed is to estimate different models with all possible combinations of the four potential long-run restrictions imposed on the two cointegrating vectors and to assess the resulting specifications according to AIC. The corresponding values of this model selection criterion are shown in table 2 for the six possible combinations of the theoretically implied restrictions imposed on the two cointegrating vectors.

	PPP + LMC	OG + PPP	OG + LMC	FIP + PPP	FIP + OG	FIP + LMC
AIC	-50.49	-50.42	-50.37	-50.53	-50.51	-50.57

Table 2: AIC for Different Combinations of the Available Restrictions

The best model chosen by AIC includes the FIP and the LMC. Looking at figure 1 indicates that the resulting long-run errors exhibit a stationary behavior. Furthermore, an Augmented Dickey-Fuller test was performed on these long-run errors, which rejected the null hypothesis of unit roots in both cases at the 5% significance level. This leads to the conclusion that a combination of FIP and LMC is the most appropriate choice.

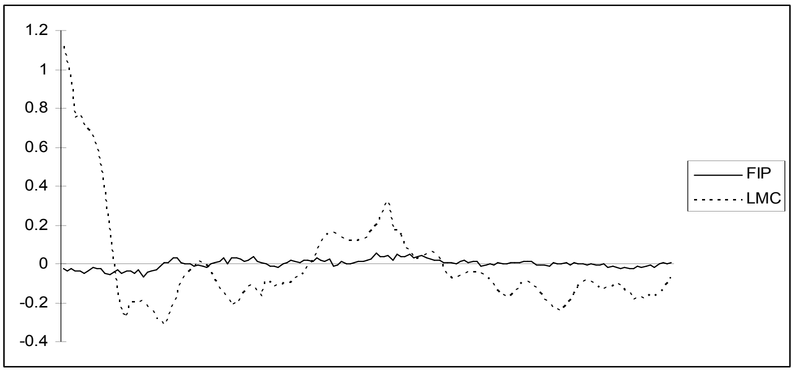

Figure 1: Long-run Equilibrium Errors for FIP and LMC

In addition to the two criteria mentioned above, the validity of the imposed restrictions is tested in section 4.3 by means of a bootstrapped likelihood ratio test suggested by Garratt et al. (2006). This test is not able to reject the restrictions implied by the FIP and the LMC at the 5% significance level. Therefore also from this point of view the model is deemed to be appropriate. With all information gathered so far, the specific model has the following form in our case:

$$\Delta z_t = a + \alpha\beta^t z_{t-1} + \sum_{i=1}^{3} \Gamma_i \Delta z_{t-i} + \Psi_0 \Delta x_t + u_t \tag{33}$$

with

$$\beta^t = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \end{pmatrix} \tag{34}$$

containing the FIP in the first row and the LMC in the second[7] [8].

4.2 Model Fit and Specification Tests

In this section the model is assessed according to the adjusted R^2 and whether specification tests report serious deviations from the underlying assumptions of a VEC-model. In table 3, adjusted R^2 is reported for each of the endogenous variables. Additionally, the p-values of the Jarque-Bera test on normality of the residuals and those for the White test on heteroscedasticity are provided. In the former case, the null hypothesis is that the residuals under consideration are normally distributed, in the latter case the null hypothesis is homoscedasticity.

	D(DPAT)	D(PD)	D(RAT)	D(RGER)
adjusted R^2	0.5384	0.1255	0.4193	0.3047
Jarque-Bera	0.0960	0.2259	0.3679	0.0000
White	0.0798	0.2434	0.0416	0.0008
	D(UAT)	D(UGER)	D(YAT)	D(YGER)
adjusted R^2	0.6090	0.7682	0.4383	0.0740
Jarque-Bera	0.0000	0.0000	0.8313	0.0255
White	0.0022	0.0474	0.0187	0.1722

Table 3: Adjusted R-squared, Jarque-Bera Test and White Test for the Resulting Model

As compared to similar models that allow at most for two lags of endogenous variables and use monetary aggregates instead of unemployment (Gaggl et al. (2009), Garratt et al. (2006)), the fit of the SVECM is quite good. In particular, we obtain large adjusted R^2 values for changes in Austrian inflation, changes in Austrian interest rates, changes in Austrian unemployment and Austrian output growth, which are the main series of interest in the impulse response analysis. In contrast, the model does not perform very well with respect to changes in the price differential and foreign output growth. Since these two variables do not play such an important role in the analyses later on, this is of minor importance.

The White test rejects homoscedasticity with respect to interest rates, unemployment

[7] The model was estimated in EViews, the estimation output is available upon request.

[8] Sensitivity analysis with respect to changes in the underlying model specification have been performed and support our results. They are available upon request.

and Austrian output while the Jarque-Bera test rejects normality of the residuals for the two unemployment series, German interest rates and German output. Consequently, standard errors and t-values are biased and should be interpreted with care. However, parameter estimates are still unbiased and consistent, so there is no need to respecify the model from this point of view.

LAG	D(DPAT)	D(PD)	D(RAT)	D(RGER)
1	0.6040	0.9670	0.9550	0.9330
2	0.8650	0.9420	0.9470	0.9400
3	0.6780	0.9720	0.9880	0.5700
4	0.2460	0.8650	0.8780	0.6430
5	0.3370	0.7300	0.3150	0.7600
6	0.4470	0.8300	0.2840	0.4930
7	0.5420	0.6340	0.3540	0.5650
8	0.6320	0.5570	0.4440	0.5240
9	0.6770	0.6390	0.5230	0.4880
10	0.7330	0.6210	0.4750	0.5810
11	0.7440	0.6510	0.5530	0.6690
12	0.8050	0.1840	0.5570	0.7370
LAG	D(UAT)	D(UGER)	D(YAT)	D(YGER)
1	0.5330	0.9320	0.8880	0.9240
2	0.8230	0.6270	0.9370	0.8820
3	0.8930	0.5140	0.3100	0.9610
4	0.8770	0.2050	0.4440	0.5780
5	0.0550	0.2690	0.2070	0.6880
6	0.0830	0.3790	0.2370	0.7800
7	0.1250	0.3140	0.3200	0.8610
8	0.1760	0.3440	0.2070	0.6210
9	0.2440	0.3210	0.2760	0.6360
10	0.2930	0.3870	0.3560	0.7240
11	0.3270	0.2550	0.3600	0.7440
12	0.1300	0.2390	0.3520	0.5920

Table 4: P-values of the Portmanteau Test on Autocorrelation among Residuals

A serious misspecification would arise if the residuals were correlated. Since we used AIC as the relevant model selection criterion, it is quite unlikely that autocorrelation among residuals is left. However, we additionally compute the Portmanteau test up to a lag-order of twelve. Table 4 contains the corresponding p-values, where the null hypothesis is that the residuals are serially uncorrelated up to the respective lag. The table reveals that, on the 5% significance level, the null hypothesis cannot be rejected for all endogenous variables and all lag-orders.

Regarding the stability of parameters, the CUSUM test is performed and the results are displayed in Appendix D. There are no significant deviations from the null hypothesis that parameter estimates are constant over the whole sample period at the 5% significance level. This means that there is no evidence for the presence of a regime change, i.e. a structural break in the data series.

Since the model passes the most important tests for misspecification, and changes in the model did not help in solving problems with respect to heteroscedasticity and non-normality of the residuals, it was decided to proceed with the current formulation.

4.3 Likelihood Ratio Test on the Validity of Overidentifying Restrictions

Where r cointegrating relations are present, r^2 restrictions are needed to exactly identify the parameters of these relations. In the case considered so far, this means that four restrictions would suffice. Since economic theory provides sixteen restrictions, the system is overidentified. It is possible to assess the validity of overidentifying restrictions by means of a standard likelihood ratio test. However, Garratt et al. (2006) point out that the critical values of this test are biased for small sample sizes so they advocate the use of bootstrapped critical values. We follow a similar approach where the bootstrapping procedure is nonparametric and can be described by the following steps (see for example Johnston and DiNardo (1997), Lütkepohl (2005)):

- Estimate the model and store the fitted values as initial estimates
- Randomly draw residuals with replacement from the residuals of the model obtained in the previous step
- Calculate the mean of the randomly drawn residuals (which should be close to zero) and subtract it from them to get new residuals
- Add the new residuals to the initial estimates
- Estimate the model subject to the overidentifying restrictions and store the value of the log-likelihood ($logl$)
- Estimate the model subject to the exactly identifying restrictions and store the value of the log-likelihood ($logl$)
- Calculate the test statistic as $2(logl(ei) - logl(oi))$, where ei and oi denote the exactly identified and overidentified model respectively, and store this test statistic
- Repeat these steps a number of times and obtain the upper critical value (since it is a one sided test) from the stored test statistics

The resulting distribution of the bootstrapped test statistic is shown in figure 2 for 2000 replications of the algorithm. The upper 5% critical value of this distribution is 52.27, whereas the test statistic obtained by comparing the original overidentified model with the exactly identified one exhibits a value of 37.64. Therefore the validity of the theoretically implied overidentifying restrictions cannot be rejected. Consequently, as the LMC shows up as a restriction on the cointegration space, the labor markets of Austria and Germany seem to be closely related and allowing for labor migration is reasonable.

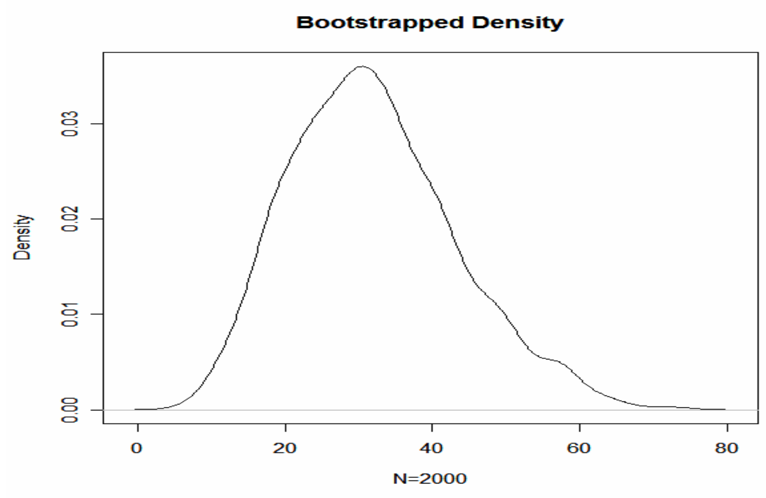

Figure 2: Density-Function of 2000 Bootstrapped Critical Values for the Likelihood Ratio Test on the Validity of Overidentifying Restrictions

5 Generalized Impulse Response Analysis

Since the model performed well with respect to criteria assessing its fit, with respect to model specification tests and with respect to tests on the validity of overidentifying restrictions imposed on the cointegration space[9], it is used to study the impacts of different shocks to domestic and foreign variables in the next two sections.

5.1 Effects of German Shocks on Austrian Variables

In this section the SVECM is used for analyzing the extent to which shocks to German variables influence Austrian inflation, Austrian interest rates, Austrian unemployment and Austrian output growth. For the sake of comparability, shocks to inflation rates and interest rates as well as their respective responses are standardized to represent one percentage point innovations. In contrast, shocks to unemployment and output as well as their responses, are measured in percent deviations from the baseline scenario. The solid lines refer to the point estimates of the generalized impulse response functions and the broken lines represent bootstrapped 95% confidence intervals[10].

At first, a one percentage point shock to the German interest rate is considered and its effects are depicted in figure 3. This shock does not have significant effects on Austrian

[9]We also performed a plausibility check of the generalized impulse response functions by looking at the effects of shocks to Austrian inflation, Austrian interest rates, Austrian unemployment and Austrian per capita output within the Austrian economy itself. This revealed that the generalized impulse response functions have plausible signs and plausible shapes. The results are available upon request.

[10]The bootstrapping procedure required quite similar calculations to those in section 4.3 adapted for impulse response analysis.

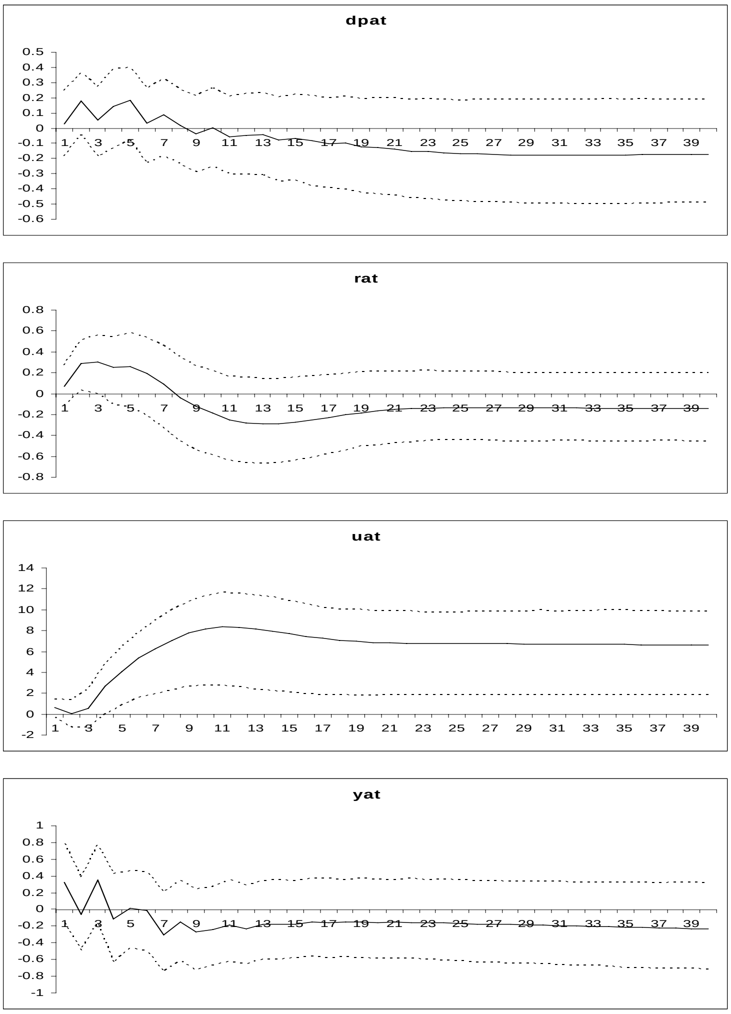

Figure 3: Generalized Impulse Responses to a One Percentage Point Shock to German Interest Rates

output and inflation, but in the short-run the Austrian interest rate increases significantly. Austrian unemployment does not react significantly in the short run, but starts to rise after about four quarters. On the one hand this is caused by lagged effects of the increased domestic interest rate on domestic investments, on the other hand the LMC ensures that rising unemployment in Germany leads to higher unemployment in Austria as well, since some of the newly unemployed seek jobs in Austria. On average Austrian unemployment increases by 6.65% in this scenario. It is interesting to see that these effects are long-lasting, which indicates the presence of hysteresis in Austrian unemployment (see for example Heijdra and van der Ploeg (2002)).

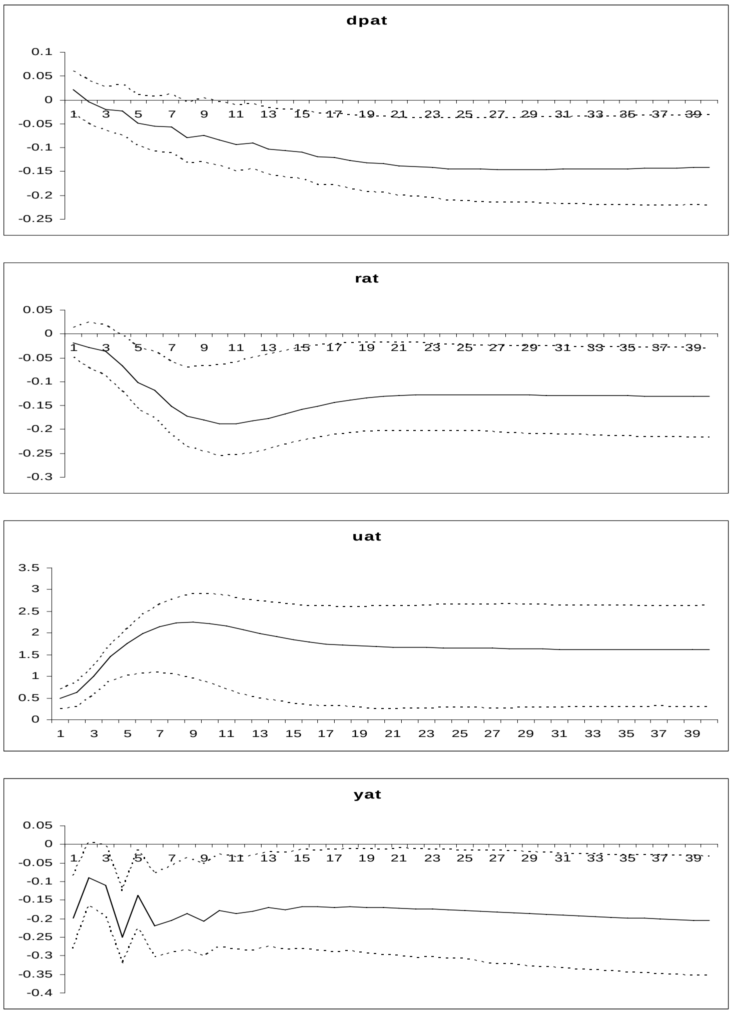

Figure 4: Generalized Impulse Responses to a 1% Shock to German Unemployment

The next scenario to be investigated is a 1% shock to German unemployment with the results shown in figure 4. The effects of this shock again have straightforward interpretations. As unemployment in Germany increases, demand for foreign goods decreases, which lowers Austrian exports and hence Austrian output. This in turn leads to higher unemployment in Austria. Furthermore, Germans who become unemployed will move to Austria, which further increases unemployment there. As another consequence of the aforementioned dynamics, inflationary pressure declines, leaving latitude for the central bank to decrease interest rates. All in all, inflation drops on average by 0.15 percentage points in the long-run. In the short-run the central bank therefore decreases the interest rate, in this case by about 0.18 percentage points, but in the long-run the FIP prevents

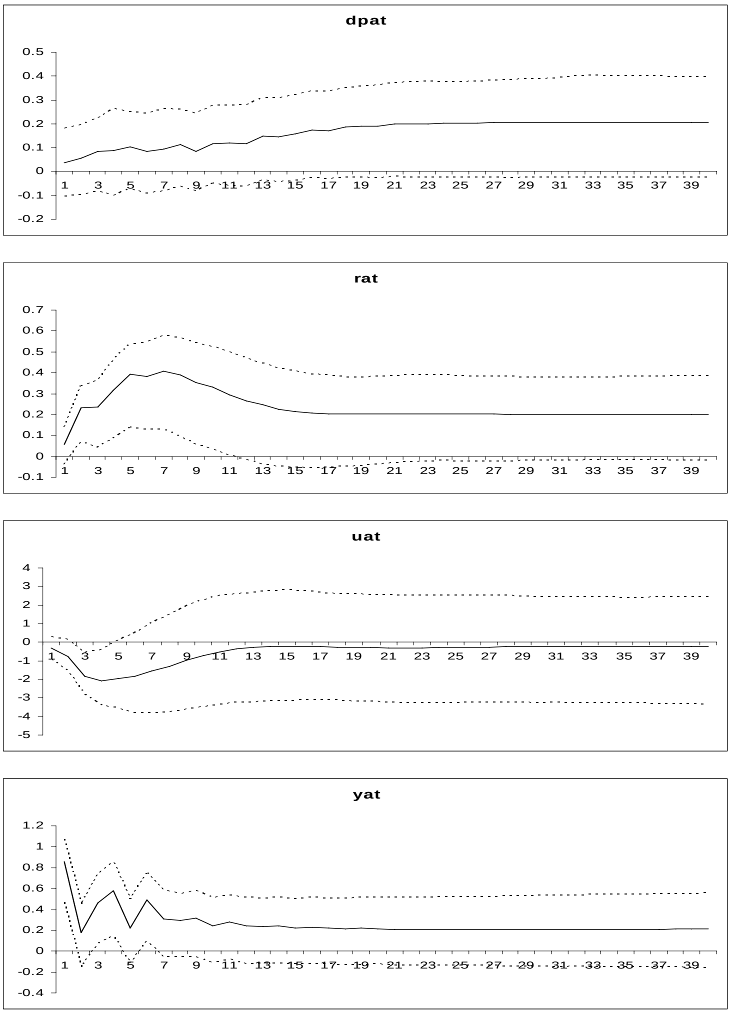

Figure 5: Generalized Impulse Responses to a 1% Shock to German Output

the interest rate from decreasing too much. Conseqently, the interest rate decreases by the same amount as inflation in the long-run.

In the last scenario, a positive 1% shock to German output occurs. Figure 5 reveals that the positive significant spillovers to Austrian output only last for a very short time period. Due to the temporary boom in Germany, demand for Austrian goods rises such that output in Austria stays significantly higher for seven quarters. One can see that there is no multiplier effect at work because the reaction of Austrian output is not hump-shaped. Instead, deviations from the long-run trend of per capita output tend to be short-lived. One explanation for this finding is that Austria represents a very open economy and so additional income is largely spent on goods produced abroad, which reduces the multiplier effect (see for example Blanchard (2003)). Due to the initial boom, unemployment stays significantly lower for five quarters and the interest rate increases significantly for about ten quarters, which in turn prevents the inflation rate from rising significantly.

To summarize the most important insights gained in this section, shocks to German interest rates and unemployment have quite large influences on Austrian unemployment. This is due to labor mobility in the model: The Austrian labor market comes under pressure from two distinct sides in the event of a German economic slowdown. Firstly, Austrian exports to Germany drop and secondly, unemployed Germans migrate to Austria. In addition to these dynamics, the properties of the labor markets mean that exogenous shocks lead to long lasting effects, which is commonly referred to as the phenomenon of hysteresis. Together these facts explain why the Austrian labor market is affected to this extent. In contrast to shocks affecting the German interest rate and unemployment, positive shocks to German output have only transient effects on Austrian variables, which is partly due to the absence of multiplier effects. However, in the short-run, the Austrian interest rate increases significantly and unemployment decreases significantly, which is reasonable according to standard economic arguments.

5.2 Effects of Austrian Shocks on German Variables

The model also allows for investigating the effects on the German economy of shocks felt by Austrian variables. Since the German economy is far larger than the Austrian economy, this case is less interesting. Therefore only the most important issues will be stated here[11]. Again, shocks to inflation rates and interest rates and their responses were standardized to represent one percentage point innovations, whereas shocks to unemployment and output and their responses were measured in percent deviations from the baseline scenario. Since the vector of endogenous variables did not include German inflation, the responses to Austrian shocks can only be traced for interest rates, unemployment and output.

If Austrian inflation is shocked, German interest rates, German unemployment and German output do not react significantly. The same holds true for shocks to Austrian interest rates. Shocks to Austrian unemployment do not have significant effects on German interest rates or German output, but an increase in Austrian unemployment is likely to increase German unemployment as well, although not to the same extent as the corresponding German shock increases Austrian unemployment. The effect of Austrian unemployment on German unemployment is mainly due to the LMC which states that some newly unemployed Austrians will try to find work in Germany.

The most interesting case, depicted in figure 6, is the response of German interest rates, German unemployment and German output to a temporarily boost in Austrian output. It can be seen that German variables do not react as much to a shock of Austrian output as in the reverse case but in the short-run German interest rates rise significantly, German unemployment decreases significantly and German output increases significantly.

[11] The other calculations are available upon request.

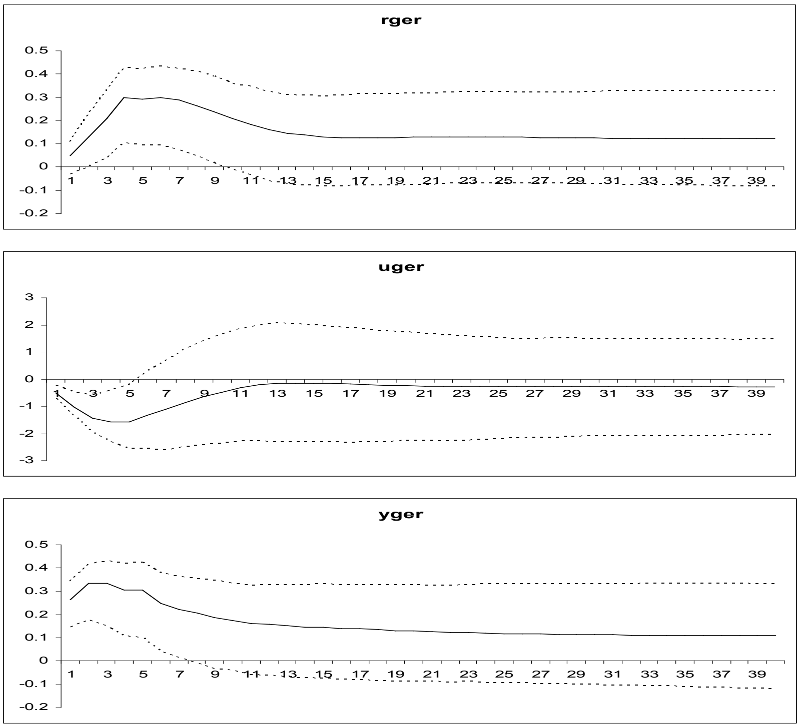

Figure 6: Generalized Impulse Responses of German Variables to a 1% Shock to Austrian Output

Again, these responses can be explained by standard economic arguments but there is one important difference between the response of German output to a positive Austrian output shock and the response of Austrian output to a positive German output shock: In the former case the presence of a multiplier effect is clearly indicated by the third picture in figure 6. German output reacts to an Austrian shock immediately, yet the dynamics are reinforced in the subsequent quarters. This could be explained by the fact that in Germany imports are a lower percentage of GDP than in Austria and thus the degree of openness is lower for the German economy. Consequently, additional income is largely spent on domestic goods and the standard multiplier effect sets in (see for example Blanchard (2003)).

To summarize, shocks that hit Austrian variables barely have any significant effects on the German economy, with some exceptions: Output shocks are able to significantly affect German variables at least in the short-run and due to the LMC, labor market shocks tend to transmit to Germany as well. However, these effects are comparatively small.

6 Conclusions

The Structural Vector Error Correction model presented in this paper is able to reveal the effects and the transmission channels of shocks between unequal neighbors as they are represented by Austria and Germany. The model performs very well with respect to criteria assessing its fit and with respect to tests for misspecification. Associated generalized impulse response functions have plausible signs and shapes that can be explained by standard economic arguments.

The central results are first of all, shocks to German variables have large and significant effects on the Austrian economy whereas shocks to Austria barely have significant effects on Germany. If there are significant effects of Austrian shocks to German variables, the magnitude of these effects is small. Secondly, we are able to identify hysteresis in Austrian unemployment, which is in line with observations of the behavior of unemployment rates in European countries (see for example Heijdra and van der Ploeg (2002)). Thirdly, we have seen that there is a multiplier effect in Germany but not in Austria. This could partly be explained by the fact that the ratio of imports to GDP is smaller in Germany as compared to Austria. Consequently, additional income resulting from a positive output shock is largely spent on domestic goods in Germany and on foreign goods in Austria.

From a policy perspective for Austria, we can conclude that after the economy is hit by an adverse shock, it is more effective to stabilize unemployment than to boost demand. The reason is that the former can prevent unemployment from increasing significantly in the long-run whereas the latter has only small and transient effects on per capita output and unemployment. Furthermore, the standard multiplier effect is not able to work since additional income is largely spent on goods produced abroad.

Acknowledgments

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A Data

The decision which time series to include as endogenous and exogenous variables involves the crucial tradeoff between working with a small model, where parameters can be estimated meaningfully but with the risk of a serious omitted variable bias, and the use of a large model, where the number of observations may be too small to estimate all parameters accurately. Therefore only variables with high explanatory power should be included. Consequently, we use one exogenous and eight endogenous variables, which are described below.

The data series were made available by the Austrian Institute of Economic Research (WIFO) and were originally obtained from the OECD Economic Outlook and Main Economic Indicators databases. Since there were problems with respect to structural changes in the available series of Austrian Gross Domestic Product, the relevant data of the International Financial Statistics database from the International Monetary Fund was used to construct growth rates of the appropriate variable for each quarter in the sample period. Afterwards the series was reconstructed in levels using the obtained growth rates together with the value of Austrian Gross Domestic Product in the first quarter of 1970 according to the original OECD database. In the estimation procedure and the associated tests, the following variables with the respective transformations were used:

- PD: Price Differential between Austria and Germany calculated as $PAT - PGER$ (see below)
- PAT: log of the Austrian Consumer Price Index (base: first quarter of 2000)
- PGER: log of the German Consumer Price Index (base: first quarter of 2000)
- POIL: log of the import price of crude oil in US-Dollar
- RAT: Austrian interest rates constructed as $log(1+i)$, where i is the nominal interest rate divided by 100
- RGER: German interest rates constructed as $log(1 + i)$, where i is the nominal interest rate divided by 100
- UAT: log of the Austrian unemployment index (base: first quarter of 2000)
- UGER: log of the German unemployment index (base: first quarter of 2000)
- YAT: log of the Austrian per capita gross domestic product index (base: first quarter of 2000)
- YGER: log of the German per capita Gross Domestic Product index (base: first quarter of 2000)

All variables were observed on a quarterly basis starting with the first quarter of 1970 and ending with the second quarter of 2007. Seasonal adjustment was carried out for all price variables and for unemployment using the TRAMO-SEATS procedure implemented in EViews. To calculate per capita variables, Austrian and German population sizes, observed on a yearly basis, were interpolated to obtain quarterly variables. This was done using the method described by Boot et al. (1967), which is implemented in Ecotrim.

In addition, the Austrian Consumer Price Index had to be adjusted for outliers in the first quarter of 1984 and in the first quarter of 1990 due to increases in the rates of consumption taxes. The quarterly inflation rate obtained was replaced by the average of the inflation rate in that quarter during the preceding four years.

B Derivation of the Restrictions

B.1 Fisher Inflation Parity and Interest Rate Parity

The Lagrangian for the representative household's optimization problem can be expressed as:

$$\begin{aligned} L \;=\; & \sum_{t=0}^{\infty} \rho^t \{ C_t^{\alpha} C_t^{*1-\alpha} + \lambda_t [(1+r_t)K_{t-1} + w_t L_t + \frac{(1+i_t)}{(1+\pi_t)} B_{t-1} + \\ & + \frac{(1+i_t^*)}{(1+\pi_t^*)} B_{t-1}^* + \frac{M_{t-1}}{1+\pi_t} - C_t - P_t^* C_t^* - B_t - B_t^* - K_t - M_t] \\ & + \mu_t [\frac{M_{t-1}}{1+\pi_t} - C_t - P_t^* C_t^*] \}, \end{aligned} \tag{35}$$

where λ_t is the Lagrange multiplier for the budget constraint and μ_t represents the Lagrange multiplier for the cash-in-advance constraint. Three necessary first order conditions for an optimum can be obtained by taking the derivative of the Lagrangian with respect to the control variables C_t, C_t^* and M_t and equalizing these derivatives to zero. Other three necessary first order conditions can be obtained by taking the derivative of the Lagrangian with respect to the state variables K_t, B_t and B_t^* and setting them to zero as well. Altogether this leads to six first order conditions reading:

$$\frac{\partial L}{\partial C_t} \overset{!}{=} 0 \;\Rightarrow\; \rho^t [\alpha C_t^{\alpha-1} C_t^{*1-\alpha} - \lambda_t - \mu_t] = 0, \tag{36}$$

$$\frac{\partial L}{\partial C_t^*} \overset{!}{=} 0 \;\Rightarrow\; \rho^t [C_t^{\alpha}(1-\alpha) C_t^{*(-\alpha)} - \lambda_t P_t^* - \mu_t P_t^*] = 0, \tag{37}$$

$$\frac{\partial L}{\partial M_t} \overset{!}{=} 0 \;\Rightarrow\; \rho^{t+1} \left[\frac{\lambda_{t+1}}{1+\pi_{t+1}} + \frac{\mu_{t+1}}{1+\pi_{t+1}} \right] - \rho^t \lambda_t = 0, \tag{38}$$

$$\frac{\partial L}{\partial K_t} \overset{!}{=} 0 \;\Rightarrow\; \rho^{t+1} \lambda_{t+1} (1+r_{t+1}) - \rho^t \lambda_t = 0, \tag{39}$$

$$\frac{\partial L}{\partial B_t} \overset{!}{=} 0 \;\Rightarrow\; \rho^{t+1} \lambda_{t+1} \frac{(1+i_{t+1})}{(1+\pi_{t+1})} - \rho^t \lambda_t = 0, \tag{40}$$

$$\frac{\partial L}{\partial B_t^*} \overset{!}{=} 0 \;\Rightarrow\; \rho^{t+1} \lambda_{t+1} \frac{(1+i_{t+1}^*)}{(1+\pi_{t+1}^*)} - \rho^t \lambda_t = 0. \tag{41}$$

Equations (39) and (40) together lead to:

$$(1 + r_t) = \frac{1 + i_t}{1 + \pi_t} \tag{42}$$

which is the Fisher Inflation Parity (FIP) and equations (40) and (41) together lead to:

$$\frac{1 + i_t}{1 + \pi_t} = \frac{1 + i_t^*}{1 + \pi_t^*} \tag{43}$$

which is the Interest Rate Parity (IRP) in the absence of an exchange rate.

B.2 Purchasing Power Parity

The first order conditions for consumption lead to:

$$C_t = \frac{\alpha}{1-\alpha} P_t^* C_t^*. \tag{44}$$

Plugging the related expressions for C_t and C_t^* into the budget constraint and using the following definitions:

$$\begin{aligned} S_t &= S_t(r_t, i_t, i_t^*, \pi_t, \pi_t^*) \\ &= B_t + B_t^* + M_t + K_t, && (45) \\ I_t &= I_t(r_t, i_t, i_t^*, \pi_t, \pi_t^*) \\ &= w_t L_t + (1 + r_t) K_{t-1} + \frac{M_{t-1}}{1 + \pi_t} + (1 + i_t)\frac{B_{t-1}}{1 + \pi_t} + (1 + i_t^*)\frac{B_{t-1}^*}{1 + \pi_t^*}, && (46) \end{aligned}$$

where S_t denotes a household's savings and I_t denotes the household's income, yields the familiar results for demand:

$$\begin{aligned} C_t &= \alpha(I_t - S_t), && (47) \\ C_t^* &= (1 - \alpha)\frac{I_t - S_t}{P_t^*}, && (48) \end{aligned}$$

which are consequences of the assumed Cobb-Douglas utility functions. These equations imply that a fraction α of the household's income net of savings is spent on the domestic aggregate, whereas a fraction $1 - \alpha$ is spent on the foreign aggregate. Since preferences in both economies are identical, similar expressions hold for demand in the foreign economy. As a consequence, the consumer price index in both countries is a weighted average of the price levels for the goods produced at home and abroad, with α and $1 - \alpha$ representing the weights. Therefore

$$CPI_t = CPI_t^* = \alpha + (1 - \alpha)P_t^*$$

holds, where CPI_t and CPI_t^* denote the consumer price indices in the domestic and foreign economy, and consequently

$$\frac{CPI_t}{CPI_t^*} = 1 \tag{49}$$

has to be fulfilled. This equation is the PPP in the absence of an exchange rate.

C Unit Root Tests

In this section the outputs of the unit root tests are described. Table 5 contains the critical values of the Kwiatkowski-Phillips-Schmidt and Shin test, and table 6 the associated test statistics. Table 7 displays the p-values of the augmented Dickey-Fuller test, and table 8 those of the Phillips-Perron test. The null hypothesis of the ADF-test and the PP-test is equivalent to the assumption that the respective series is nonstationary, whereas the null hypothesis of the KPSS-test is equivalent to the assumption that the series under consideration is stationary.

In the case of price levels, the price differential, oil prices, unemployment and output levels, the correct specification of the test regression is the one including a trend, whereas in the case of interest rates the trend has to be omitted. Three asterisks indicate that the null hypothesis is rejected at the 1% significance level, two asterisks that it is rejected at the 5% significance level and one asterisk indicates that it is rejected at the 10% significance level.

const		const+trend	
α-level	critical value	α-level	critical value
0.01	0.7390	0.01	0.2160
0.05	0.4630	0.05	0.1460
0.10	0.3470	0.10	0.1190

Table 5: Critical Values of the Kwiatkowski-Phillips-Schmidt-Shin Test

	const		const+trend	
UAT	0.6305	**	0.3054	***
UGER	1.2484	***	0.2798	***
PD	0.9298	***	0.3186	***
PAT	1.3810	***	0.3464	***
POIL	0.7568	***	0.2057	**
PGER	1.4151	***	0.3158	***
RAT	0.6305	**	0.1802	**
RGER	0.7022	**	0.0731	
YAT	1.4625	***	0.2235	***
YGER	1.4628	***	0.2289	***
DUAT	0.0442		0.0942	
DUGER	0.4061	*	0.0424	
DPD	0.6012	**	0.1168	
DPAT	1.0504	***	0.1301	*
DPOIL	0.1884		0.1319	*
DPGER	0.8434	***	0.0767	
DRAT	0.0442		0.0333	
DRGER	0.0314		0.0297	
DYAT	0.5107	**	0.1020	
DYGER	0.2080		0.0384	
DDUAT	0.0761		0.0549	
DDUGER	0.1609		0.0708	
DDPD	0.0889		0.0884	
DDPAT	0.1718		0.1529	**
DDPOIL	0.0252		0.0254	
DDPGER	0.1194		0.1146	
DDRAT	0.0761		0.0694	
DDRGER	0.0734		0.0653	
DDYAT	0.1619		0.0863	
DDYGER	0.0636		0.0301	

Table 6: Kwiatkowski-Phillips-Schmidt-Shin Test

	const		const+trend	
UAT	0.1923		0.8290	
UGER	0.2549		0.0178	**
PD	0.0555	*	0.8884	
PAT	0.0002	***	0.0257	**
POIL	0.1215		0.2541	
PGER	0.0310	**	0.2514	
RAT	0.1923		0.2264	
RGER	0.0237	**	0.0271	**
YAT	0.0589	*	0.0178	**
YGER	0.4154		0.2866	
DUAT	0.0000	***	0.0000	***
DUGER	0.0037	***	0.0112	**
DPD	0.0000	***	0.0000	***
DPAT	0.4650		0.1169	
DPOI	0.0000	***	0.0000	***
DPGER	0.1162		0.0455	**
DRAT	0.0000	***	0.0000	***
DRGER	0.0000	***	0.0000	***
DYAT	0.0000	***	0.0000	***
DYGER	0.0000	***	0.0000	***
DDUAT	0.0000	***	0.0000	***
DDUGER	0.0000	***	0.0000	***
DDPD	0.0000	***	0.0000	***
DDPAT	0.0000	***	0.0000	***
DDPOIL	0.0000	***	0.0000	***
DDPGER	0.0000	***	0.0000	***
DDRAT	0.0000	***	0.0000	***
DDRGER	0.0000	***	0.0000	***
DDYAT	0.0000	***	0.0000	***
DDYGER	0.0000	***	0.0000	***

Table 7: Augmented Dickey-Fuller Test

	const		const+trend	
UAT	0.1506		0.8545	
UGER	0.0487	**	0.6716	
PD	0.0571	*	0.8727	
PAT	0.0000	***	0.5547	
POIL	0.1722		0.3884	
PGER	0.0002	***	0.6529	
RAT	0.1506		0.1748	
RGER	0.0780	*	0.1055	
YAT	0.0589	*	0.0178	**
YGER	0.4215		0.2226	
DUAT	0.0000	***	0.0000	***
DUGER	0.0000	***	0.0000	***
DPD	0.0000	***	0.0000	***
DPAT	0.0658	*	0.0011	***
DPOIL	0.0000	***	0.0000	***
DPGER	0.0000	***	0.0000	***
DRAT	0.0000	***	0.0000	***
DRGER	0.0000	***	0.0000	***
DYAT	0.0000	***	0.0000	***
DYGER	0.0000	***	0.0000	***
DDUAT	0.0000	***	0.0000	***
DDUGER	0.0000	***	0.0000	***
DDPD	0.0000	***	0.0001	***
DDPAT	0.0000	***	0.0000	***
DDPOIL	0.0000	***	0.0001	***
DDPGER	0.0000	***	0.0000	***
DDRAT	0.0000	***	0.0000	***
DDRGER	0.0000	***	0.0001	***
DDYAT	0.0000	***	0.0001	***
DDYGER	0.0000	***	0.0000	***

Table 8: Phillips-Perron Test

D CUSUM Test on Parameter Stability

In this section the results of the CUSUM test on stability of the estimated parameters are displayed. In figure 7 the top diagram on the left refers to the equation for D(DPAT), the next one to the right to the equation for D(PD), the left diagram in the second row refers to equation D(DPAT), the right diagram to the equation for D(RGER), the left diagram in the third row refers to the equation for D(UAT), the right diagram to the equation for D(UGER), the left diagram in the last row refers to the equation for D(YAT) and the left diagram to the equation for D(YGER). As can be seen, the test statistic never reaches values in the area of significance, although with respect to German interest rates in the early 1980s it comes very close.

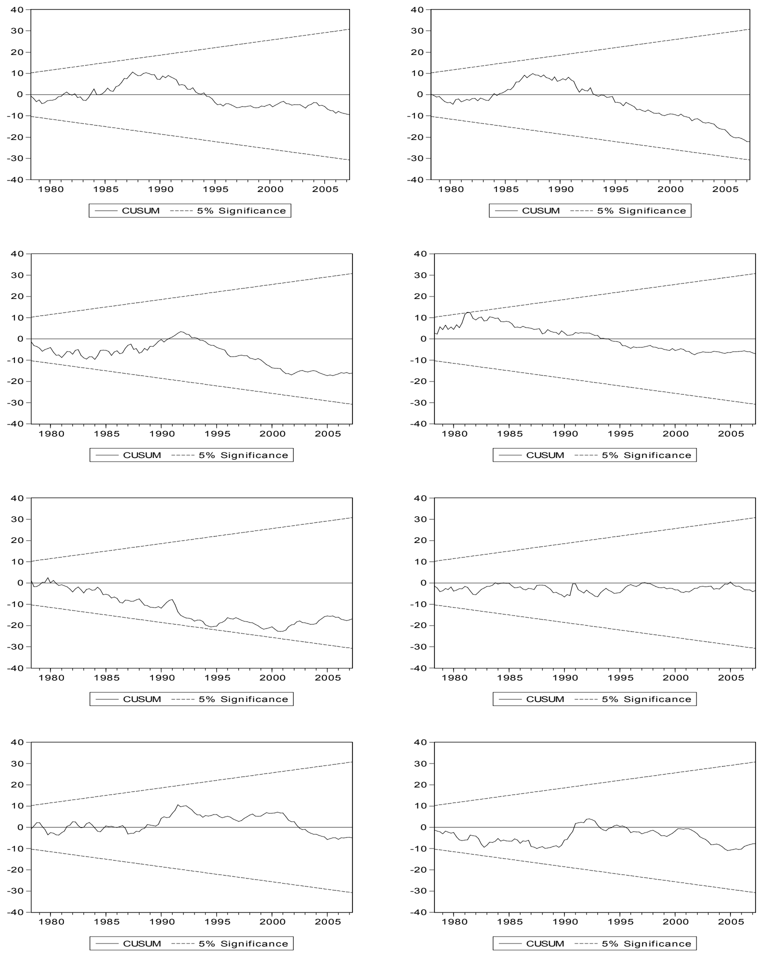

Figure 7: CUSUM Test

References

Barro, R. J. and Sala-i-Martin, X. S. (2004). *Economic Growth*. MIT Press.

Beyer, A., Doornik, J. A., and Hendry, D. F. (2001). Constructing historical euro-zone data. *The Economic Journal*, Vol. 111(No. 469):F102–F121.

Biffl, G. (2006). SOPEMI report on labor migration. Technical report, Austrian Institute of Economic Research.

Blanchard, O. J. (2003). *Macroeconomics*. Prentice Hall.

Boot, J. C. G., Feibes, W., and Lisman, L. H. C. (1967). Further methods on derivation of quarterly figures from annual data. *Applied Statistics*, Vol. 16:65–75.

Borjas, G. (1995). The economic benefits from immigration. *The Journal of Economic Perspectives*, Vol. 9(No. 2):3–22.

Borjas, G. (1996). *Labor Economics*. McGraw-Hill.

Clower, R. (1967). A reconsideration of the microfoundations of monetary theory. *Western Economic Journal*, Vol. 6:1–9.

Faustino, H. and Leitão, N. (2008). Using the gravity equation to explain the Portuguese immigration-trade link. School of Economics and Management. Technical University of Lisbon. Working Papers 12.

Feenstra, R. C. (2004). *Advanced International Trade*. Princton University Press.

Fenz, G. and Schneider, M. (2007). Transmission of Business Cycle Schocks between Unequal Neighbors: Germany and Austria. Working Paper 137, Austrian Central Bank.

Gaggl, P., Kaniovski, S., Prettner, K., and Url, T. (2009). The short and long-run interdependencies between the Eurozone and the USA. *Empirica*, Vol. 36(No. 2):209–227.

Garratt, A., Lee, K., Pesaran, M. H., and Shin, Y. (2006). *Global and National Macroeconometric Modelling: A Long-Run Structural Approach*. Oxford University Press.

Heijdra, B. J. and van der Ploeg, F. (2002). *Foundations of Modern Macroeconomics*. Oxford University Press.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control*, Vol. 12:231–254.

Johansen, S. (1991). Estimation and hypothesis testing of cointegrating vectors in Gaussian vector autoregressive models. *Econometrica*, Vol. 59:1551–1580.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press.

Johnston, J. and DiNardo, J. (1997). *Econometric Methods*. McGraw-Hill.

Juselius, K. (2007). *The Cointegrated VAR Model: Methodology and Applications*. Oxford University Press.

Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*. Springer.

Parente, S. T. and Prescott, E. C. (1994). Barriers to technology adoption and development. *The Journal of Political Economy*, Vol. 102(No. 2):298–321.

Pesaran, M. H. and Shin, Y. (1998). Generalized impulse response analysis in linear multivariate models. *Economic Letters*, Vol. 58(No. 1):17–29.

Romer, D. (2001). *Advanced Macroeconomics*. McGraw-Hill.

Sims, C. (1980). Macroeconomics and reality. *Econometrica*, Vol. 48(No. 1):1–48.

Statistik Austria (2006). Der Außenhandel Österreichs. Technical report, Statistik Austria.