TMD DISCUSSION PAPER NO. 64

## GAMS CODE FOR ESTIMATING <br> A SOCIAL ACCOUNTING MATRIX (SAM) USING CROSS ENTROPY (CE) METHODS

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December 2000

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# GAMS Code for Estimating A Social Accounting Matrix (SAM) Using Cross Entropy (CE) Methods 

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#### Abstract

This paper documents the computer code implementing the CE-SAM estimation technique in GAMS (General Algebraic Modeling System). It defines the estimation problem in a deterministic setting; extends the approach to include a stochastic treatment of errors in control totals; summarizes the equations describing CE technique for estimating a consistent SAM starting from an inconsistent data set estimated with error; and provides the GAMS code.


Key words: Entropy, cross entropy, social accounting matrices, SAM, GAMS

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## 1. Introduction ${ }^{*}$

The paper describes the cross entropy (CE) SAM estimation technique in situations where column sums and macro aggregates represent SAM control totals that may be measured with error. The theory of the estimation technique, comparing it to other methods, is described in S. Robinson, A. Cattaneo, and M. El Said "Updating and Estimating a Social Accounting Matrix Using Cross Entropy Methods" in Economic Systems Research, vol. 13, no. 1, March 2001. ${ }^{1}$ This paper documents the computer code implementing the technique in GAMS (General Algebraic Modeling System) (Brooke, Kendrick, Meeraus, and R. Raman 1998). Section 2 defines the estimation problem in a deterministic setting. Section 3 extends the approach to include a stochastic treatment of errors in the control totals. Section 4 summarizes the equations describing the CE technique for estimating a consistent SAM starting from an inconsistent data set estimated with error. Finally, section 5 provides the GAMS code for the estimation problem both as a nonlinear programming (NLP) problem and a mixed complementarity problem (MCP).

## 2. CE-SAM Estimation: Deterministic Approach

Define $T$ as a matrix of SAM transactions, where $t_{i, j}$ is a payment from column account $j$ to row account $i$, that satisfies the condition:

$$
\begin{equation*}
y_{i}=\sum_{j} t_{i, j}=\sum_{j} t_{j, i} \tag{1}
\end{equation*}
$$

That is, for a SAM, every row sum must equal the corresponding column sum. A SAM coefficient matrix, $A$, is constructed from $T$ by dividing the cells in each column of $T$ by the column sums:

$$
\begin{equation*}
A_{i, j}=\frac{t_{i, j}}{y_{j}} \tag{2}
\end{equation*}
$$

[^0]We assume that we start with information in the form of a prior, $\bar{A}$, which may be based on data from a previous or from scattered, perhaps inconsistent, data from the current year. We also assume that we have exact information on current column sums, $y^{*}$. Applying the Kullback-Leibler (1951) measure of the "cross entropy" (CE) distance between two probability distributions to the CE-SAM estimation, the problem is to find a new set of $A$ coefficients which minimize the cross entropy distance between the prior $\bar{A}$ and the new estimated coefficient matrix.

$$
\begin{align*}
\min _{\{A\}} I & =\left[\sum_{i} \sum_{j} A_{i, j} \ln \frac{A_{i, j}}{\bar{A}_{i, j}}\right]  \tag{3}\\
& =\left[\sum_{i} \sum_{j} A_{i, j} \ln A_{i, j}-\sum_{i} \sum_{j} A_{i, j} \ln \bar{A}\right]
\end{align*}
$$

Subject to:

$$
\begin{gather*}
\sum_{j} A_{i, j} y_{j}^{*}=y_{i}^{*}  \tag{4}\\
\sum_{j} A_{j, i}=1 \text { and } 0 \leq A_{j, i} \leq 1 \tag{5}
\end{gather*}
$$

Note that $x \ln x=0$ if $x=0$. Thus, to solve for equation (3) and allow for zero entries in the SAM in the computer code, we add an epsilon small number to the arguments of the equation. Note also that the system of constraint equations (4) is functionally dependent, since if all but one column and row sum are equal, the last one must also be equal (analogous to Walras' in general equilibrium theory). One equation can be dropped.

## 3. CE-SAM Estimation: Stochastic Approach

Specifying known column sums implies having exact information about control totals in the SAM. Most applications of economic models to real world issues must deal with the problem of extracting results from data or economic relationships with noise. One can generalize to include knowledge about any aggregates or elements of the SAM (e.g., macro aggregates from the national accounts). In this section we generalize our approach to cases where: (i) row and column sums are not fixed parameters but involve errors in measurement; and (ii) macro aggregates are not exact but are measured with error.

The general case starts from assumed prior knowledge of the standard error (perhaps due to measurement error) of the estimate of control totals-a Bayesian prior, not a maintained hypothesis. The estimated error in the $\mathrm{i}^{\text {th }}$ control total can be represented as a weighted sum of elements in a specified error support set:

$$
\begin{equation*}
e_{i}=\sum_{j w t} w_{i, j w t} \bar{v}_{i, j w t} \tag{6}
\end{equation*}
$$

where $e_{i} \quad=$ error value of control total
$w_{i, j w t}=$ error weights estimated in the CE procedure $\left(\sum_{j w t} w_{i, j w t}=1\right)$
$\bar{v}_{i, j w t}=$ error support set
The set $j w t$ defines the dimension of the support set for the error distribution and the number of weights that must be estimated for each error. The prior on the variance of these errors is given by:

$$
\begin{equation*}
\sigma^{2}=\sum_{j w t} \bar{w}_{i, j w t} \cdot \bar{v}_{i, j w t}^{2} \tag{7}
\end{equation*}
$$

where $\bar{w}_{i, j w t}=$ prior weights on the error support set and $\sum_{j w t} \bar{w}_{i, j w t}=1$

Starting with a prior $\sigma$, Golan, Judge, and Miller (1996) suggest picking the $\bar{v}$ s to define a domain for the support set of $\pm 3$ standard errors. In this case, the prior on the weights, $\bar{w}$, are then calculated to yield a consistent prior on the standard error, $\sigma .^{2}$

### 3.1. Case of three-weight error distribution

Assume a prior mean of zero and a given standard error, $\sigma$. With a threeparameter error distribution that is symmetric around zero, the $\bar{v}$ s define the upper and lower bounds for the error distribution, and there are three weights, $\bar{w}$, to be estimated. That is we have:

$$
\begin{align*}
& \bar{v}_{i, 1}=-3 \boldsymbol{\sigma} \\
& \bar{v}_{i, 2}=0  \tag{8}\\
& \bar{v}_{i, 3}=+3 \boldsymbol{\sigma}
\end{align*}
$$

and using (7):

[^1]\[

$$
\begin{equation*}
\sigma^{2}=\bar{w}_{i, 1} \cdot\left(+9 \sigma^{2}\right)+\bar{w}_{i, 2} \cdot(0)+\bar{w}_{i, 3} \cdot\left(9 \sigma^{2}\right) \tag{9}
\end{equation*}
$$

\]

Since the prior weights and support set are symmetric, $\bar{w}_{i, 1}=\bar{w}_{i, 3}$. Solving (9) for the weights, $\bar{w}$, we get:

$$
\begin{align*}
& \bar{w}_{i, 1}=\bar{w}_{i, 3}=\frac{1}{18}  \tag{10}\\
& \bar{w}_{i, 2}=1-\bar{w}_{i, 1}-\bar{w}_{i, 3}=\frac{16}{18}
\end{align*}
$$

### 3.2. Case of five-weight error distribution

For the case of a five-parameter error distribution, there are five weights, $\bar{w}$, to be estimated-the set jwt consists of five elements. We are incorporating more information about the error distribution-more moments, including variance, skewness, and kurtosis. Assuming a prior normal distribution with mean of zero and variance $\sigma^{2}$, the prior on kurtosis is $3 \sigma^{4}$. In this case, the prior weights, $\bar{w}$, are specified so that:

$$
\begin{equation*}
\sum_{j w t} \bar{w}_{i, j w t} \cdot \bar{v}_{i, j w t}^{4}=3 \sigma^{4} \tag{11}
\end{equation*}
$$

in addition to defining the variance as above in (7). The prior weights and support set are also symmetric, so the prior on all odd moments is zero. Choose $\pm 1$ standard error for $\bar{v}_{i, 2}$ and $\bar{v}_{i, 4}$ (which is arbitrary). In this case we get:

$$
\begin{align*}
& \bar{v}_{i, 1}=-3 \boldsymbol{\sigma} \\
& \bar{v}_{i, 2}=-\boldsymbol{\sigma} \\
& \bar{v}_{i, 3}=0  \tag{12}\\
& \bar{v}_{i, 4}=+\boldsymbol{\sigma} \\
& \bar{v}_{i, 5}=+3 \boldsymbol{\sigma}
\end{align*}
$$

and using (7) and (12) we get:

$$
\begin{align*}
\sigma^{2} & =\bar{w}_{i, 1} \cdot\left(9 \sigma^{2}\right)+\bar{w}_{i, 2} \cdot\left(\sigma^{2}\right)+\bar{w}_{i, 3} \cdot(0)+\bar{w}_{i, 4} \cdot\left(\sigma^{2}\right)+\bar{w}_{i, 5} \cdot\left(9 \sigma^{2}\right) \\
3 \sigma^{4} & =\bar{w}_{i, 1} \cdot\left(81 \sigma^{4}\right)+\bar{w}_{i, 2} \cdot\left(\sigma^{4}\right)+\bar{w}_{i, 3} \cdot(0)+\bar{w}_{i, 4} \cdot\left(\sigma^{4}\right)+\bar{w}_{i, 5} \cdot\left(81 \sigma^{4}\right) \tag{13}
\end{align*}
$$

given that $\bar{w}_{i, 1}=\bar{w}_{i, 5}$ and $\bar{w}_{i, 2}=\bar{w}_{i, 4}$ by symmetry, we get

$$
\begin{array}{r}
18 \bar{w}_{i, 1}+2 \bar{w}_{i, 2}=1 \\
162 \bar{w}_{i, 1}+2 \bar{w}_{i, 2}=3
\end{array}
$$

solving (13) for the $\bar{w}$ s

$$
\begin{align*}
& \bar{w}_{i, 1}=\frac{1}{72} \\
& \bar{w}_{i, 2}=\frac{27}{72} \\
& \bar{w}_{i, 3}=\frac{16}{72}  \tag{14}\\
& \bar{w}_{i, 4}=\frac{27}{72} \\
& \bar{w}_{i, 5}=\frac{1}{72}
\end{align*}
$$

Note that all these parameters determine the prior distribution. The estimation procedure yields posterior estimates of all the moments of the error distribution (Golan, Judge, and Miller, 1996). The five parameter specification permits posterior estimation of four moments; mean, variance, skewness, and kurtosis.

## 4. Equations of the CE-SAM Estimation Problem

In this section we provide a mathematical statement of the equations involved in the CE-SAM estimation problem. The statement includes the stochastic formulation to specify errors on column sums, and errors on macro aggregates. In this case we specify two sets of errors with separate weights, $W$ l's and $W 2$ 's, and extend the CE minimand in equation (3) to account for the specification of the error terms as follows:

$$
\begin{gather*}
\min _{\{A, W l, W 2\}} I=\left[\sum_{i} \sum_{j} A_{i, j} \ln A_{i, j}-\sum_{i} \sum_{j} A_{i, j} \ln \bar{A}\right] \\
+\left[\sum_{i} \sum_{j w t} W 1_{i, j w t} \ln W 1_{i, j w t}-\sum_{i} \sum_{j w t} W 1_{i, j w t} \ln \overline{W l}_{i, j w t}\right] \\
+\left[\sum_{k} \sum_{j w t} W 2_{k, j w t} \ln W 2_{k, j w t}-\sum_{k} \sum_{j w t} W 2_{k, j w t} \ln \overline{W 2_{k, j w t}}\right]  \tag{15}\\
{\left[\begin{array}{c}
\text { cross } \\
\text { entropy }
\end{array}\right]=\left[\begin{array}{c}
\text { Ce for } \\
\text { coefficient } \\
\text { matrix }
\end{array}\right]+\left[\begin{array}{c}
\text { CE for weights } \\
\text { on the column } \\
\text { sum errors }
\end{array}\right]+\left[\begin{array}{c}
\text { CE for weights } \\
\text { on macro } \\
\text { aggregate errors }
\end{array}\right]}
\end{gather*}
$$

In this case the minimization problem is to find a set of $A^{\prime} \mathrm{s}, W 1$, s , and $W 2$ 's that minimize cross entropy, where the $W$ 's are treated like the $A$ 's, subject to the following constraints:

## SAM Equation

$$
\begin{gather*}
T_{i, j}=A_{i, j} \cdot\left(\bar{X}_{i}+e_{L_{i}}\right) \\
{\left[\begin{array}{c}
\text { SAM cell } \\
\text { value }
\end{array}\right]=\left[\begin{array}{c}
\text { SAM } \\
\text { coefficient } \\
\text { matrix }
\end{array}\right] \cdot\left(\left[\begin{array}{c}
\text { value for } \\
\text { column sum }
\end{array}\right]+\left[\begin{array}{c}
\text { error term } \\
\text { associated } \\
\text { with column } \\
\text { sum }
\end{array}\right]\right)} \tag{16}
\end{gather*}
$$

where $\bar{X}$ is the prior on the column sums of the SAM matrix.

Row/column sum consistency

$$
\begin{gather*}
Y_{i}=\bar{X}_{i}+e_{i}  \tag{17}\\
{\left[\begin{array}{c}
\text { SAM row } \\
\text { sum }
\end{array}\right]=\left[\begin{array}{c}
\text { value for } \\
\text { column sum }
\end{array}\right]+\left[\begin{array}{c}
\text { error term } \\
\text { associated with } \\
\text { column sum }
\end{array}\right]}
\end{gather*}
$$

Error definition (column sum)

$$
\begin{gather*}
e l_{i}=\sum_{j w t} W l_{i, j w t} \cdot \overline{v l}_{i j w t} \\
{\left[\begin{array}{c}
\text { error term } \\
\text { associated } \\
\text { with column } \\
\text { sum }
\end{array}\right]=\left[\begin{array}{c}
\text { error } \\
\text { weights }
\end{array}\right]+\left[\begin{array}{c}
\text { error } \\
\text { support } \\
\text { values }
\end{array}\right]} \tag{18}
\end{gather*}
$$

Sum of weights on errors (column sum)

$$
\begin{gather*}
\sum_{j w t} W 1_{i, j w t}=1 \\
{\left[\begin{array}{c}
\text { sum of } \\
\text { error } \\
\text { weights }
\end{array}\right]=\left[\begin{array}{c}
\text { add up } \\
\text { to } 1
\end{array}\right]} \tag{19}
\end{gather*}
$$

Row Sum

$$
\begin{gather*}
\sum_{j} T_{i, j}=Y_{i} \\
{\left[\begin{array}{c}
\text { sum of } \\
\text { row } \\
\text { elements }
\end{array}\right]=\left[\begin{array}{c}
\text { row } \\
\text { sum }
\end{array}\right]} \tag{20}
\end{gather*}
$$

## Column sum

$$
\begin{gather*}
\sum_{i} T_{i, j}=\bar{X}_{i}+e_{i} \\
{\left[\begin{array}{c}
\text { summof } \\
\text { column } \\
\text { celenenss }
\end{array}\right]=\left[\begin{array}{c}
\text { fived prior } \\
\text { value or } \\
\text { column sum sum }
\end{array}\right]+\left[\begin{array}{c}
\text { error term } \\
\text { associated } \\
\text { with column } \\
\text { sum }
\end{array}\right]} \tag{21}
\end{gather*}
$$

## Additional macro control totals

The assumption is that one has additional knowledge about the new SAM. For example, aggregate national accounts data may be available for various macro aggregates such as value added, consumption, investment, government, exports, and imports. There also may be information about some of the SAM accounts such as government receipts and expenditures. This information can be summarized as additional linear adding-up constraints on various elements of the SAM while allowing the possibility that additional information might be measured with error. We can write:

$$
\begin{align*}
& \sum_{i} \sum_{j} G_{i, j}^{(k)} T_{i, j}=\boldsymbol{\gamma}^{(k)}+e_{2_{k}} \\
& {\left[\begin{array}{c}
\text { aggregator } \\
\text { matrix }
\end{array}\right]=\left[\begin{array}{c}
\mathrm{k}^{\prime} \text { th control } \\
\text { total }
\end{array}\right]} \tag{22}
\end{align*}
$$

where $G$ is an n-by-n aggregator matrix, which has ones for cells in the aggregate and zeros otherwise. Assume that there are k such aggregation constraints, and $\gamma$ is the value of the aggregate. These conditions are simply added to the constraint set in the cross entropy formulation. The error term $e_{2}$ is associated with macro aggregates. In the example provided in the following section two macro control total equations are included: GDP at factor cost and GDP at market prices.

Error definition (macro totals)

$$
\begin{gather*}
e_{2_{k}}=\sum_{j w t} W 2_{k, j w t} \cdot \bar{\nu}_{k, j w t} \\
{\left[\begin{array}{c}
\text { errorterm } \\
\text { associated } \\
\text { with macro } \\
\text { totals }
\end{array}\right]=\left[\begin{array}{c}
\text { error } \\
\text { weights }
\end{array}\right]+\left[\begin{array}{c}
\text { error } \\
\text { support } \\
\text { values }
\end{array}\right]} \tag{23}
\end{gather*}
$$

Sum of weights on errors (macro totals)

$$
\begin{gather*}
\sum_{j w t} W 2_{k, j w t}=1 \\
{\left[\begin{array}{c}
\text { sum of } \\
\text { error } \\
\text { weights }
\end{array}\right]=\left[\begin{array}{c}
\text { add up } \\
\text { to }
\end{array}\right]} \tag{24}
\end{gather*}
$$

## Additional Cell constraints

Defining the SAM equation (16) over non-zero elements of $A$ guarantees that the zero structure of the original SAM is maintained in the estimated SAM. Fixing all the cells with a prior value of zero to zero greatly reduces the size of the estimation problem. If it is desired to allow a zero entry to become nonzero in the estimated SAM, then the restriction that the SAM equation (16) to be defined over non-zero elements of $A$ must be replaced to include cells which are currently zero but may be nonzero.

## 5. GAMS Code

This section provides the GAMS code implementing the equations described above. It is implemented with macro data from Mozambique. The cross entropy (CE) estimation problem is inherently badly scaled, and solution algorithms often suffer. A number of approaches to improving algorithm performance are available. There are two nonlinear programming solvers that are commonly used in GAMS: MINOS and CONOPT. If the column sums are known without error, then the constraint equations are all linear. In this case, the MINOS solver works well because it is optimized for nonlinear programming problems with linear constraints. If the column sums are assumed to
include errors, then the constraints are nonlinear and the CONOPT solver appears to work better.

Another approach is to convert the nonlinear programming (NLP) problem into a mixed complementarity problem (MCP). The approach is to derive all the first-order conditions and create a set of "shadow price" variables, or Lagrange multipliers, associated with each first-order condition. The resulting problem is square in that it has as many equations as variables, but there is a complementary slackness relationship between the Lagrange multipliers and corresponding first-order conditions. The MCP solver called PATH handles this kind of problem (Dirkse and Ferris, 1995). The PATH solver appears to be very efficient in solving CE problems. The MCP formulation of the problem is given in a separate GAMS "include file" below.

## GAMS code

\$TITLE Cross Entropy SAM Estimation
\$OFFSYMLIST OFFSYMXREF OFFUPPER

*

* CE-SAM illustrates a cross entropy technique for estimating the cells * of a consistent SAM assuming that the initial data are inconsistent
* SAM measured with error. The method is applied to a stylized macro
error, and also all the row and column totals are assumed
* a priow only with error. We assume that the user can specify * and column sums and of the macro control totals.
* Programmed by Sherman Robinson and Moataz El-Said, November 2000.
* Trade and Macroeconomics Division

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* The method is described in S. Robinson, A. Cattaneo and, M. El Said * (2001) "Updating and Estimating a Social Accounting Matrix Using * Cross Entropy Methods." Economic Systems Research, Vol. 13, No, * pp. 47-64.
* Discussion paper \#58 is an earlier version of the Economic

Systems Research paper. A copy can be downloaded from the IFPRI
web page using the following link
http://www.ifpri.cgiar.org/divs/tmd/tmdpubs.htm\#dp

* See also A. Golan, G. Judge, and D. Miller, Maximum Entropy
* Econometrics, John Wiley \& Sons, 1996.
* Data set is based on a SAM developed by C. Arndt, A. S. Cruz, H. T. * Jensen, S. Robinson, and F. Tarp, "Social Accounting Matrices
* for Mozambique - 1994 and 1995." TMD Discussion Paper No. 28, IFPRI * July 1998.
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

SETS

| i sam accounts | / ACT | Activities |
| :--- | :--- | :--- |
|  | COM | Commodities |
|  | FAC | Factors |
|  | ENT | Enterprises |
|  | HOU | Households |
|  | GRE | Govt recurrent expenditures |
|  | GIN | Govt investment |
|  | CAP | Capital account |
|  | ROW | Rest of world |

ii(i) all acounts in i except TOTAL / ACT Activities

| COM | Commodities |
| :--- | :--- |
| FAC | Factors |
| ENT | Enterprises |
| HOU | Households |
| GRE | Govt recurrent expenditures |
| GIN | Govt investment |
| CAP | Capital account |
| ROW | Rest of world / |

macro macro controls /gdpfc2, gdp2 /

* The set jwt defines the dimension of the support set for the error
distribution and the number of weights that must be estimated for each
error. In this case, we specify a five parameter error distribution.
* For a three parameter distribution, jwt is set to /1*3/
jwt set of weights for errors in variables
/ $1 * 5$ /
* ii(i)
$\begin{array}{ll}\text { ii(i) } & =\text { YES; } \\ \text { ii("Total") } & =\text { NO; }\end{array}$
ALIAS (i,j), (ii,jj);

\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

| FAC | ENT |
| ---: | ---: |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 3699.7060 | 0.0 |
| 6031.3080 | 3417.5060 |
| 74.4000 | 165.2000 |
| 0.0 | 0.0 |
| 0.0 | 150.0000 |
| 0.0 | 0.0 |
| 9805.414 | 3732.706 |
|  |  |
| GIN | CAP |
| 0.0 | 0.0 |
| 2118.5000 | 2197.7980 |
| 2518.5000 | 2597.7980 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| -406.2000 | 0.0 |
| 0.0 | 0.0 |
| 1712.3 | 2197.798 |


| CAP | 2163.8570 | 2200.14 |
| :--- | ---: | ---: |
| ROW | 0.0 | 5573.815 |
| Total | 5573.815 |  |

*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Parameters and Scalars \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# PARAMETER

| SAMO (i,j) | Base SAM transact |
| :---: | :---: |
| T0 (i,j) | Matrix of SAM tran |
| T1(i,j) | SAM transactions |
| Abar0 (i,j) | Prior SAM coeffic |
| Abar1 (i, j) | Prior SAM adjusted |
| Target0 (i) | Targets for macro |
| vbar1 (i,jwt) | Error support set |
| vbar2 (macro, jwt) | Error support set |
| wbar1 (i,jwt) | Weights on error |
| wbar2 (macro, jwt) | Weights on error |
| sigmay1(i) | Prior standard er |
| sigmay2 (macro) | Prior standard |
| epsilon | Tolerance to allow |
| ; |  |
| SCALARS |  |
| gdp0 bas | se GDP |
| gdp00 GDP | from final SAM |
| gdpfc0 GDP | at factor cost |
| ; |  |

## \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Initializing Parameters <br> $\operatorname{SAM}(" T O T A L ", j j)=\operatorname{sum}(i i, \operatorname{SAM}(i i, j j))$; <br> SAM(ii,"TOTAL") = sum(jj, SAM(ii,jj))

sam0(i,j)
$=\operatorname{sam}(i, j)$;

## \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

Divide SAM entries by 1000 for better scaling
The SAM is scaled to enhance solver efficiency. Nonlinear solvers are

* more efficient if variables are scaled similarly. In this case,
coefficients to be estimated range between 0 and 1, so SAM values
are also scaled
Scalar scalesam Scaling value /1000/ ;
$\operatorname{sam}(i, j)$
Abar0 $(i i$
= sam(i,j)/scalesam ;
Abar0(ii,jj) \$SAM(ii, jj) = SAM(ii, jj)/SAM("TOTAL", jj) ;
TO ( $\mathrm{ii}, \mathrm{jj}$ )
$=\operatorname{SAM}(\mathrm{i}, \mathrm{j}, \mathrm{j})$;
TO ("TOTAL", jj) = sum(ii, SAM(ii,jj));
TO (ii,"TOTAL") = sum(jj, SAM(ii,jj));
epsilon $=.00001$;


## Display T0, Abar0 ; <br> (Display TH\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# CROSS ENTROPY \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# RED ALERT!!! \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

* The ENTROPY DIFFERENCE
* the SAM are NOT GOOD!!
* The option used here is to detect any negative flows and net them out
of their respective symmetric cells, e.g.
- negative flow column to row is set to zero

The entropy difference method can then be implemented.

* After balancing, the negative SAM values are returned to their
* original cells for printing.

$$
\begin{aligned}
& \text { SET } \\
& \text { red } \\
& ;
\end{aligned}
$$

$$
\begin{aligned}
& \text { SET }(i, j) \quad \text { Set of negative SAM flows } \\
& \text { red }
\end{aligned}
$$

| Parameter |  |
| :--- | :--- |
| redsam (i, $j$ ) | Negative SAM values only |
| rtot $(i)$ | Row total |

redsam(i, j)
rtot (i)
ctot
; ${ }^{\text {ctot }}$
rtot(ii) Column total
ctot (jj)
red (ii, jj) $($ TO (ii, jj) LT 0)
redsam(ii,jj)
redsam(ii,jj) \$red(ii, jj)
redsam(jj,ii) \$red(ii, jj
*Note that redsam includes each entry twice, in corresponding row *and column. So, redsam need only be subtracted from T0.
$\begin{array}{ll}T 1(i i, j j) & =T 0(i i, j j)-\text { redsam (ii, jj); } \\ T 1(\text { "Total", jj) } & =\operatorname{sum}(i i, T 1(i i, j j) ;\end{array}$
$\begin{array}{ll}\mathrm{T1}(\text { ("Total", } \mathrm{jj}) & =\operatorname{sum}(\mathrm{ii}, \mathrm{T1}(\mathrm{ii}, \mathrm{jj})) ; \\ \mathrm{T1}(\mathrm{ii}, \text { "Total") } & =\operatorname{sum}(\mathrm{jj}, \mathrm{T1}(\mathrm{ii}, \mathrm{jj})) ;\end{array}$
redsam("total",jj)
redsam(ii,"total")
sam(ii,"total")
sam("total",jj)
ctot (jj)
= sum(ii, redsam(ii,jj));
$=\operatorname{sum}(j j, r e d s a m(i i, j j))$;
$=\operatorname{sum}(j j, ~ T 1(i i, j j)) ;$
= sum(ii, T1(ii,jj));
$=\operatorname{sum}(j j, \operatorname{T1}(i i, j j)) ;$
= T1 (ii,jj)/sam("total",jj);
display "NON-NEGATIVE SAM" ;
display redsam, T1, Abar0, Abar1, rtot, ctot ;

* Define set of elements of SAM that can be nonzero. In this case, only * elements which are nonzero in initial SAM.

SET NONZERO ( $i, j$ ) SAM elements that can be nonzero ;
NONZERO (ii, jj) $(\operatorname{Abarl}(i i, j j))=$ yes ;
*\#\#\#\# Initializing Parameters after accounting for negative values \#\#\#\#\# * Note that target column sums are being set to average of initial

* Note that target column sums are being set to average of
* row and column sums. Initial column sums or other values
* could have been used instead, depending on knowledge of data quality * and any other prior information.

```
larget0(ii) = (sam(ii,"total") + sam("total",ii))/2;
```

Display gdpfc0, gdp0;
*\#\#\#\#\#\#\#\#\#\#\#\#\#\# Define variable bounds on errors \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# * Start from assumed prior knowledge of the standard error (perhaps due * to measurement error) of the column sums. Below, we assume that al
column sums have a standard error of $5 \%$. This is a Bayesian prior,

* not a maintained hypothesis.
* The estimated error is weighted sum of elements in an error support
set:
$\operatorname{ERR}(i i)=$ SUM(jwt, $w(i i, j w t) * \operatorname{VBAR}(i i, j w t))$
* where the W's are estimated in the CE procedure.

The prior variance of these errors is given by:
(sigmay(ii))**2 $=\operatorname{SUM}(j w t, \operatorname{WBAR}(i i, j w t) *(\operatorname{VBAR}$
(sigmay (in)

* where the WBAR's are the prior on the weights.

The VBARs are chosen er erior on the weights, WBAR support set of $+/-$ standard errors. The prior on the weights, WBAR, are then
to yield the specified prior on the standard error, sigmay

* In Robinson, Cattaneo, and El-Said (2001), we specify prior weights
* (WBAR) that are uniform and set the prior standard error by the
* choice of support set, VBAR. In that paper, we use a three-weight
* specification (jwt /1*3/);
* We define two sets of errors with separate weights, W1 and W2. The
* first is for specifying errors on column sums, the second for errors * on macro aggregates (defined in the set macro)
* First, define standard error for errors on column sums sigmay1 (ii) $=0.05$ * target 0 (ii) ;
* This code assumes a prior mean of zero and a two-parameter
* distribution with specified prior standard error. There are three
* weights, W(ii, jwt), to be estimated. The actual moments are estimated * as part of the estimation procedure
\$ontext
* Set constants for 3-weight error distribution
vbar1(ii,"1") = -3 * sigmay1(ii);

$\begin{aligned}\text { wbar1 (ii, "1" }) & =1 / 18 ; \\ \text { wbar1(ii, } 2 ") & =16 / 18 ; \\ \text { wbar1(i, " } & \\ \text { " }) & =1 / 18 ;\end{aligned}$
\$offtext
* This code assumes a prior mean of zero and a prior value of kurtosis
* consistent with a prior normal distribution with mean zero, variance
* sigmay**2, and kurtosis equal to $3 *$ sigmay**4. The addition of a prior
* on kurtosis requires estimation of 5 weights (jwt $=5$ );

sigmay (ii, jwt) **4
as well as defining the variance as above.
The prior weights and support set are also symmetric, so the prior
or
estimated as parbitrary
* The actual moments are estimated as part of the estimation procedure.
* Set constants for 5-weight error distribution vbar1(ii,"1") = -3 * sigmay1(ii) ;
vbar1(ii,"2") = -1 * sigmay1(ii)
$\begin{array}{ll}\text { vbar1 (ii,"3") } & =0 \\ \text { vbar1 }(i i, " 4 ") & =+1 \text { * sigmay1 (ii); }\end{array}$


| wbar1(ii,"1") | $=1 / 72$ | $;$ |
| ---: | :--- | ---: |
| wbar1(ii,"2") | $=27 / 72$ | $;$ |
| wbar1(ii,"3") | $=16 / 72$ | $;$ |
| wbar1(i, "4") | $=27 / 72$ | $;$ |
| wbar1(ii,"5") | $=1 / 72$ | $;$ |

* Second, define standard errors for errors on macro aggregates

```
sigmay2("gdpfc2") = 0.05*gdpfc0
```

sigmay2("gdp2") $=0.05 *$ gdp0 ;
\$ontext
Set constants for 3-weight error distribution
vbar2(ii,"1") = -3 * sigmay2(ii);
vbar2(ii,"3") $=-3 *$ sigmay2(ii);
wbar2(ii,"1") = 1/18
wbar2(ii,"2") = 16/18
wbar2(ii,"3") = 1/18
\$offtext

* Set constants for 5-weight error distribution
vbar2 (macro, "1") $=-3 * \operatorname{sigmay} 2$ (macro) ;
vbar2 (macro, "2") $=-1$ * sigmay2 (macro) ;
vbar2(macro,"3")
vbar2 (macro, "4") = +1 * sigmay2 (macro) ;
vbar2 (macro, "5") = +3 * sigmay2 (macro) ;

| wbar2 (macro, "1") | $=1 / 72$ | ; |
| ---: | :--- | ---: |
| wbar2 (macro, "2") | $=27 / 72$ | $;$ |
| wbar2 (macro, "3") | $=16 / 72$ | $;$ |
| wbar2 (macro, "4") | $=27 / 72$ | $;$ |
| wbar2 (macro, "5") | $=1 / 72$ | $;$ |

Display vbar1, vbar2, sigmay1, sigmay2 ;
\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# VARIABLES \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

## VARIABLES

A(ii,jj) Post SAM coefficient matrix
TSAM (ii,jj) Post matrix of SAM transactions
Y(ii) row sum of SAM
X(ii)
ERR1 (ii)
ERR2 (macro)
W1 (ii, jwt) DE (macro, jwt GDPF
GDPFC
GDP at factor cost
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# INITIALIZE VARIABLES \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
$\begin{array}{ll}\text { A.L(ii, jj) } & =\text { Abar1 (ii, jj) ; } \\ \text { TSAM.L(ii, jj) } & =T 1(i i, j j) \quad ;\end{array}$
$=T 1(i i, j j)$

| Y.L(ii) | $=$ target 0 (ii) |
| :---: | :---: |
| X.L(ii) | $=$ target 0 (ii) |
| ERR1.L(ii) | $=0.0$ |
| ERR2.L (macro) | $=0.0$ |
| W1.L (ii, jwt) | $=$ wbarl (ii, jwt) |
| W2.L (macro, jwt) | $=$ wbar2 (macro, jwt) |
| DENTROPY.L | = 0 |
| GDPFC.L | $=\mathrm{gdpfc} 0$ |
| GDP.L | = gdp0 |

## \#\#\#\#\#\#\#\#\#\#\#\# CORE EQUATIONS

EQUATIONS

| SAMEQ(i) | row and column sum constraint <br> SAMMAKE(i, j) <br> make SAM flows |
| :--- | :--- |
| ERROREQ(i) | definition of error term 1 |
| ERROR2EQ(macro) | definition of error term 2 |
| SUMW1 (i) | Sum of weights 1 |
| SUMW2 (macro) | Sum of weights 2 |
| ENTROPY | Entropy difference definition |
| ROWSUM(i) | row target |
| COLSUM(j) | column target |
| GDPFCDEF | define GDP at factor cost |
| GDPDEF | define GDP |

;
CORE EQUATIONS
SAMEQ(ii).. Y(ii) =E=X(ii) + ERR1 (ii) ;

SAMMAKE (ii, jj) \$nonzero (ii, jj)

ERROR1EQ(ii).. ERR1(ii) =E= SUM(jwt, W1 (ii, jwt)*vbarl(ii,jwt));
SUMW1 (ii).. SUM(jwt, W1 (ii, jwt)) =E= 1 ;
ENTROPY.. DENTROPY =E= SUM( (ii,jj)\$nonzero (ii, jj), A(ii, jj)*(LOG (A (ii, jj) + epsilon) - LOG(Abar1 (ii,jj) + epsilon)))

SUM( (il, jwt), W1 (il, jwt)
(LOG (W1 (ii, jwt) + epsilon)

- LOG(wbar1(ii, jwt) + epsilon))

SUM ( (macro, jwt), w2 (macro, jwt)

* (LOG (W2 (macro, jwt) + epsilon) - LOG (wbar2 (macro, jwt) + epsilon))) ;
* Note that we exclude one rowsum equation since if all but one colum * and rowsum are equal, the last one must also be equal. Walras' Law * at work.

ROWSUM(ii) $($ NOT SAMEAS (ii, "ROW")). $\quad \operatorname{SUM}(j j, ~ T S A M(i i, j j))=E=Y(i i) ;$ COLSUM(jj).. SUM(ii, TSAM(ii, jj)) =E= (X(jj) + ERR1 (jj)) ;

GDPFCDEF..
GDPFC =E= TSAM("fac","act") + ERR2("gdpfc2") ;

```
GDPDEF..
GDP
=E= TSAM("fac","act") + TSAM("gre","act")
    - TSAM("act","gre") + TSAM("gre","com")
    + ERR2("gdp2") ;
```

ERROR2EQ (macro) . . ERR2 (macro)
$=E=\operatorname{SUM}(j w t, \quad W 2(m a c r o, j w t) * v b a r 2(m a c r o, j w t))$
$\operatorname{SUMW2}$ (macro) . . SUM (jwt, W2 (macro, jwt) ) =E= 1 ;
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Define bounds for cell values \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

* Defining equation SAMMAKE over non-zero elements of A (\$Abarl(ii,jj))
quarantees that the zero structure of the original SAM is maintained
in the estimated SAM. Fixing all the zero entries to zero greatly
reduces the size of the estimation problem. If it is desired to
* the condition SABAR1 (ii, jj) must be replaced with a new set that
does not include cells which are currently zero but may be nonzero.

* 
* Upper and lower bounds on the error weights
W1.LO (ii, jwt)
$=0$;
W1.UP (ii, jwt
W2. LO (macro, jwt)
* Set target column sums, X. If these are not fixed, then the column sum
* constraints will not be binding and the solution values or ERR1 will
* be 0 .
X.FX(ii)
$=$ TARGETO (ii) ;
Fix Macro aggregates.
If these are not fixed, then the macro constraints will not be binding
and the solution values of ERR2 will be zero.
GDP.FX
= GDPO ;
$=$ GDPFCO ;
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# DEFINE MODEL \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
MODEL SAMENTROP / ALL /
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# SOLVE MODEL \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
OPTION ITERLIM
OPTION LIMROW
$=5000$;
OPTION SOLPRINT

$$
\begin{aligned}
& =5000 ; \\
& =0, \quad \text { LIMCOL }=0 ; \\
& =\text { ON; }
\end{aligned}
$$

* SAMENTROP.optfile
$=1 ;$
$=1$
SAMENTROP. HOLDFIXED = 1 ;
option NLP $=$ MINOS5
* OPTION NLP $=$ CONOPT;
* SAMENTROP. WORKSPACE = 25.0;


## -\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Solve statenment \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

SOLVE SAMENTROP using nlp minimizing dentropy ;

## *\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

*\#\#(alternative formulation)\#\#\#\# MCP Formulation \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

* Add code restating the nonlinear-programming (NLP) minimization
* problem as an MCP problem solved using the MCP solver.
*Sinclude CE-MCP.INC


## *\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

| Parameters |  |
| :---: | :---: |
| Macsam1 (i,j) | Assigned new balanced SAM flows from CE |
| Macsam2 (i,j) | Balanced SAM flows from entropy diff x scalesam |
| SEM | Squared Error Measure |
| percent1 $(i, j)$ | percent change of new SAM from original SAM |
| PosUnbal (i, j) | Positive unbalanced SAM |
| PosBalan (i, j) | Positive balanced SAM |
| Diffrnce(i,j) | Differnce btw original SAM and Final SAM in values |
| NormEntrop | Normalized Entropy a measure of total uncertainty |
| ; |  |
| macsam1 (ii,jj) | $=$ TSAM.l(ii, jj) ; |
| macsam1("total",jj) | $)=\operatorname{SUM}(\mathrm{ii}, \operatorname{macsam} 1(i, 1, j))$; |
| macsam1(ii,"total") | ) = SUM (jj, macsaml (ii,jj)) ; |
| macsam2 (i,j) | = macsam1 (i,j) * scalesam ; |
| SEM | $\begin{aligned} &=\operatorname{Sum}((i i, j j), \underset{-}{\operatorname{SQR}(A \cdot L(i i, j j)} \\ &-\operatorname{Abarl}(i i, j j))) / \operatorname{SQR}(\operatorname{card}(i i)) ; \end{aligned}$ |
| percent1 $(i, j) \$(T 1(i, j))=100 *(\operatorname{macsam1}(i, j)-T 1(i, j)) / T 1(i, j)$; |  |
| PosUnbal (i, j) | = T1 (i,j) * scalesam; |
| PosBalan(i,j) = macsam2 $(i, j)$; |  |
| Diffrnce(i,j) = PosBalan(i,j) - PosUnbal (i,j); |  |
|  |  |
|  |  |
|  | Abar1 (ii,jj)* LOG (Abar1 (ii, jj)) |
| display macsam1, macsam2, percent1, sem, dentropy.1, PosUnbal, PosBalan, NormEntrop, Diffrnce ; |  |
| *\#\#\#\#\#\#\#\#\#\#\# Return negative flows to initial cell position \#\#\#\#\#\#\#\#\#\#\#\# |  |
| macsam1 $i \mathrm{i}, \mathrm{j} j) \quad=$ macsam1 $(i i, j j) ~+~ r e d s a m(i i, j j) ~$ |  |
| macsam1 ("total",jj) = SUM(ii, macsam1 (ii,jj)) ; |  |
| macsam1(ii,"total") = SUM (jj, macsam1 (ii,jj) ) ; |  |
| macsam2 $(1, j) \quad=\operatorname{macsam} 1(i, j)$ * scalesam ; |  |
| gdp00 $=$ macsam1("fac","act") + macsam1 ("gre","act") <br>  - macsam1("act","gre") + macsam1("gre","com") |  |
| ; |  |
| display macsam1, macsam2 ; |  |
| display gdp0, gdp00 | 0, gdp.1, gdpfc0, gdpfc.l ; |
| *\#\#\#\#\#\#\# |  |

Parameter ANEW(i,j)

* print some stuff

ANEW("total", jj) = SUM(ii, A.L(ii, jj)) ;
ANEW (ii, "total") $=\operatorname{SUM}(j j, A . L(i i, j j))$;
ABAR1 ("total", jj) = SUM (ii, ABAR1 (ii, jj)) ABAR1 (ii,"total") = SUM (jj, ABAR1 (ii, jj)) ;
Display ANEW, ABAR1 ;
scalar meanerr1, meanerr2 ;
meanerr1 = SUM(ii, abs(err1.l(ii)))/card(ii) ;
meanerr2 $=\operatorname{SUM}$ (macro, abs (err2.1 (macro)))/card (macro) ; display meanerr1, meanerr2 ;

* Use the following code to specify that the column sums are known
* exactly. The errors are thus fixed to zero and two equations are
* dropped from the estimation procedure. The computational gains ar
* dropped from the estimation procedure. The computational gains are
* considerably smaller.


## \#\#\#\# <br> \$ontext <br> *\#\#\#\#\#\#

```
ERR1.FX(ii) = 0.0;
ERR1.FX(ii) = 0.0; ;
MODEL SAMENTROP2 /SAMEQ
    SAMMAKE
    RROR1EQ
    SUMW1
    SUMW2
    SUMW2
    ENTROPY
    COLSUM
    GOLSUM
    GDPFCDEF
```

SAMENTROP2.holdfixed = 1 ;
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Solve statenment \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
SOLVE SAMENTROP2 using nlp minimizing dentropy ;
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
*\#\#\#\#\#\#\#
\$offtext
*\#\#\#\#\#\#
*\#*\#*\#*\#*\#*\#*\#*\#*\#*\#*\#*\#* THE END *\#*\#*\#*\#*\#*\#*\#*\#*\#*\#*\#*\#*\#*\#*

## GAMS code for the MCP Formulation

## File name: CE-MCP.INC

* The code below is a translation of the NLP problem into a
* mixed complementarity problem (MCP), which can be solved
* 
- by Michael Ferris and Jeffrey Horn (1998) at the University
* of Wisconsin. The translation adds "shadow price" or
* complementarity variables for all constraint equations and
* also provides equations for all the first-order conditions
* for minimizing the objective function. The resulting model
* is square with as many variables as equations.
alias (macro,a_macro);
alias(ii,a_ii);
alias(jj,a_jj);
alias(jwt, a_jwt);
*\#\#\#\#\# "SHADOW PRICE" OR COMPLEMENTARITY VARIABLES OR \#\#\#\#\#\#\#\#\# *\#\#\#\#\# LAGRANGE MULTIPLIERS FOR ALL CONSTRAINT EQUATIONS \#\#\#\#\#\#\#\#\#


## variables

m_SAMEQ(i) Multiplier for row and column sum constraint
m_SAMMAKE (i,j)
m_ERROR1EQ(i)
m_ERROR2EQ (macro)
m-SUMW2 (mac
m_SUMW2 (macro)
m_ROWSUM (i)
m_COLSUM
m_COLSUM(j)
m_GDPDEF
; Multiplier for make SAM flows constraint Multiplier for definition of error term 1 Multiplier for Sum of weights 1 constraint Multiplier for Sum of weights 1 constraint Multiplier for row target constraint Multiplier for row target constraint Multiplier for GDP at factor cost constraint Multiplier for GDP at market pricesconstraint
\#\#\#\#\#\#\#\#\#\#\#\#\# EQUATIONS FOR THE FIRST ORDER CONDITIONS \#\#\#\#\#\#\#\#\# -\#\#\#\#\#\#\#\#\#\#\#\#\# FOR MINIMIZING THE OBJECTIVE FUNCTION \#\#\#\#\#\#\#\#\#\#\#

## EQUATIONS <br> d_Y(a_ii) <br> d_X(a_ii) <br> d_ERR1 (a_ii) <br> _- 1 <br> d_W1 (a_ii,a_jwt) <br> d_GDPFC (a_macro, a_jwt) <br> d_GDP <br> ;

d_A(a_ii,a_jj) FOC wrt the choice variable A
d_TSAM (a_ii, a_jj) FOC wrt the variable TSAM
OC wrt the variable Y
FOC wrt the variable $X$
FOC wrt the variable ERR1
OC wrt the variable ERR2
OC wrt the choice variable W1
FOC wrt the macro control variable GDPFC
FOC wrt the macro control variable GDP
*\#\#\#\#\# EQUATION: FOC wrt the choice variable A \#\#\#\#\#\#\#\#\#\#\#\#\#\#\# d_A(a_ii,a_jj) ..
((log(a(a_ii,a_jj)+epsilon)-log (abar1 (a_ii,a_jj)+epsilon))
\$(nonzero (a_ii,a_jj)) + a(a_ii,a_jj)*(1/(a(a_ii,a_jj)+epsilon)) \$(nonzero(a_ii,a_jj)))
 $\left(a \_i i, i i\right)$ and sameas (a_jj,jj))))*(x(jj)+err1(jj)))) =e=0;
*\#\#\#\#\# EQUATION: FOC wrt the variable TSAM \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# d_TSAM(a_ii,a_jj) .. -
m_gdpfcdef*(-(1\$((sameas (a_ii,"fac") and sameas (a_jj,"act"
1)!)!-
m_colsum(a_jj)-
m_gdpdef*(-(1\$((sameas (a_ii,"fac") and sameas (a_ji,"act")) ) +1 ( (sameas (a_ii, "gre") and sameas (a_jj, "act"))) -(1\$( (sameas (a_ii,"act") and sameas (a_jj, "gre")))) +1 ((sameas (a_ii, "gre" ) and sameas (a_jj, "com"))l))-
m_rowsum(a_ii) \$((not sameas (a_ii,"ROW"))) -
m_sammake (a_ii,a_jj) $\$($ nonzero (a_ii,a_jj)) =e= 0 ;
*\#\#\#\#\# EQUATION: FOC wrt the variable Y \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# d_Y(a_ii) .. -
m_sameq(a_ii)-
SUM((ii) \$((not sameas(ii, "ROW"))), m_rowsum(ii)*(-(1\$((sameas (a_ii,ii)))))
$=\mathrm{e}=0$;
\#\#\#\#\# EQUATION FOC wrt the variable x \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# d_X(a_ii) .. -
$\operatorname{SUM}\left(j j, m \_c o l s u m(j j) *\left(-\left(1 \$\left(\left(s a m e a s\left(a \_i i, j j\right)\right)\right)\right)\right)-\right.$
$\operatorname{SUM}\left(i i, m \_\right.$sameq(ii)*(-(1\$((sameas (a_ii,ii))))))-

\$((sameas (a_ii,jj))))) =e=0;
*\#\#\#\#\# EQUATION: FOC wrt the variable ERR1 \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# d_ERR1 (a_ii) .. -
m_error1eq(a_ii)-
SUM(jj,m_colsum(jj)*(-(1\$((sameas (a_ii,jj)))))-
SUM(ii,m_sameq(ii)*(-(1\$((sameas(a_ii,ii))))))-

SUM((ii,jj)\$(nonzero(ii,jj)),m_sammake(ii,jj)*(-a(ii,jj)*(1 \$((sameas (a_ii,jj)))))
$=\mathrm{e}=0$;
*\#\#\#\#\# EQUATION: FOC wrt the variable ERR2 \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# d_ERR2 (a_macro) ..
m_error2eq(a_macro) -
m_gdpfcdef*(-(1\$((sameas (a_macro, "gdpfc2")))))-
m_gdpdef*(-(1\$((sameas (a_macro, "gdp2")))))
$=\mathrm{e}=0$;
\#\#\#\#\# EQUATION: FOC wrt the choice variable W1 \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# d_w1 (a_ii,a_jwt) .. ((log (w1 (a_ii,a_jwt) +epsilon)-log (wbar1 (a_ii , a_jwt)+epsilon))+w1 (a_ii,a_jwt)*(1/(w1 (a_ii,a_jwt)+epsilon)))-

SUM(ii,m_errorleq(ii)*(-(vbar1 (ii,a_jwt) \$(sameas (a_ii,ii)) )) -
m_sumw1 (a_ii)
$=\mathrm{e}=0$
*\#\#\#\#\# EQUATION: FOC wrt the choice variable W2 \#\#\#\#\#\#\#\#\#\#\#\#\#\#\# d_W2 (a_macro, a_jwt) .. ((log (w2 (a_macro, a_jwt) +epsilon) -log (wbar2 (a_macro, a_jwt) +epsilon)) +w2 (a_macro, a_jwt) * (1/ (w2 (a_macro, a_jwt +epsilon)))

SUM (macro, m_error2eq (macro) * (- (vbar2 (macro, a_jwt) \$ (sameas (a_macro ,macro)()))-
m_sumw2 (a_macro)
=e= 0 ;
*\#\#\#\#\# EQUATION: FOC wrt the macro control variable GDPFC \#\#\#\#\#\#\# d_GDPFC .. -
m_gdpfcdef $=e=0 ;$
*\#\#\#\#\# EQUATION: FOC wrt the macro control variable GDP \#\#\#\#\#\#\#\# d_GDP .. -
m_gdpdef $=e=0$
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# DEFINE MODEL \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

* In GAMS the "." is used for pairing the complementarity variables
* In GAMS the "." is used for pairing the complementarity variables
defind d is
* defined over the same sets.

```
MODEL m_SAMENTROP /
```

d_A.A
d_A.A
d_TSAM.TSAM
d_TSAM
d_X.X
d_ERR1.ERR1
d_ERR2.ERR2
d_W1.W1
d_W2.W2
d_GDPFC.GDPFC
d_GDP.GDP
ERROR1EQ.m_ERROR1EQ
ERROR2EQ.m_ERROR2EQ
GDPFCDEF.m_GDPFCDEF
COLSUM.m_COLSUM
GDPDEF.m_GDPDEF
SUMW1 M- SUMW1
SUMW1.m_SUMW1
SUMW2.m_SUMW2
SAMMAKE.m_SAMMAKE
;
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# SOLVE MODEL \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
*Shock the NLP solution
A.L(ii,jj) = 0.9*A.l(ii, jj) ;
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Solve statenment \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# SOLVE m_SAMENTROP using mCP;

## **\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

*Compare NLP and MCP results
Scalar savedent ;
savedent = dentropy. 1 ;
DENTROPY.l = SUM((ii, jj) \$nonzero(ii, jj),
A.l(ii, $j$ ) *(LOG (A.l(ii, $j$ ) + epsilon)

- LOG (Abar1 (ii, jj) + epsilon)))

SUM ((ii, jwt), W1.l(ii, jwt

* (LOG(W1.1(ii, jwt) + epsilon)
- LOG (wbar1 (ii, jwt) + epsilon)))

SUM ( (macro, jwt), W2.l (macro, jwt) (LOG (W2.1(macro, jwt) + epsilon) - LOG (wbar2 (macro, jwt) + epsilon))) ;
option decimals=8
display dentropy.1, savedent
option decimals=3
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# END MCP INCLUDE FILE \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
*\#\#\#\#\#\# NOTE ON THE USE OF "SAMEAS" GAMS COMMAND \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# *\#\#\#\#\# NOTE ON THE USE OF "SAMEAS" GAMS COMMAND \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
$\begin{aligned} & \text { *\#\#\#\#\# Undocumented Feature IN GAMS Manual \#\#\#\#\#\#\#\#\#\#\#\#\#\#\# }\end{aligned}$ \$ontext \$ontex Set Elements

New features in GAMS allow one to introduce conditional statements controlling execution in cases where certain items match up . The
syntax involves using the commands
SAMEAS (setelement1, setelement2)
-
the SAMEAS command returns a true false indicator which is true
if the text string defining the name of set element 1 equals that for setelement 2 and false otherwise. DIAG returns a 1 under equality and a zero otherwise.

For example
$x=\operatorname{sum}((i, j) \$(\operatorname{not} \operatorname{SAMEAS}(i, j)), z(i) * z(j))$;

- or
$x=\operatorname{sum}((i, j) \$(\operatorname{DIAG}(i, j)$ eq 0$), z(i) * z(j))$;
would exclude the cases where $i=j$ from the sum
x=sum((i,j) $($ SAME AS(i,"case1") or SAME AS(j,"case2")), $z(i)+z(j))$
would only include cases where the text for i equaled the string "case1" or the text for $j$ corresponded to "case2."

If interested check the following web address Undocumented Features and Usage Tips
http://agrinet.tamu.edu/mccarl/gamstip.htm
\$offtext
*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# END Note on "SAMEAS" GAMS command \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#

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[^0]:    * The authors would like to thank Channing Arndt, Rebecca Harris, Henning Tarp Jensen, and Finn Tarp for helpful comments on the GAMS code. Also we would like to thank Michael Ferris for providing the MCP version of the estimation problem.
    ${ }^{1}$ An earlier version of the paper can be downloaded in PDF format from the IFPRI web page
    "http://www.ifpri.cgiar.org/divs/tmd/tmdpubs.htm\#dp" division discussion paper No. 58, August 2000.

[^1]:    ${ }^{2}$ In Robinson, Cattaneo, and El-Said (2001), we specify prior weights $\bar{w}$ that are uniform and set the prior standard error by the choice of support set, ( $\bar{v}$ ). In that paper, we use a three-weight specification (jwt $=$ $\{1,2,3\}$ ).

