Raising Revenue With Raffles: Evidence from a Laboratory Experiment

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Abstract

Lottery and raffle mechanisms have a long history as economic institutions for raising funds. In a series of laboratory experiments we find that total spending in raffles is much higher than Nash equilibrium predicts. Moreover, this overspending is persistent as the number of participants in the raffle increases. Subjects as a group do not strategically reduce spending as group sizes increase, in contrast to the comparative statics theory provides. The lack of strategic response cannot be explained by learning direction theory or level-k reasoning models, although quantal response equilibrium can fit the observed distribution of choices. Much of the observed spending levels in the larger groups cannot be explained by financial incentives.

Keywords: Lottery, raffle, contest, laboratory experiments.

JEL classification: C72, C92, D72.

1 Introduction

Lotteries are one of the oldest economic institutions. Chance drawings are reputed to have been used to help finance the construction of the Great Wall of China. The

longevity of lotteries can be attributed directly to their effectiveness at raising money. Proponents of games of chance as a mechanism for raising funds have framed them as a sort of "voluntary tax." U.S. Founding Father Thomas Jefferson wrote in support of lotteries,

> *A lottery is a salutary instrument and a tax...laid on the willing only, that is to say, on those who can risk the price of a ticket without sensible injury, for the possibility of a higher prize.*

The "voluntary tax" argument (see for example Clotfelter and Cook, 1987) has appeared many times throughout history. For example, in 1967-68, the city of Montreal sought to defray some of the expenses from the Expo '67 World's Fair by running a raffle with a prize of up to $100,000 in silver bars.

If lotteries are profitable, then it follows that at least some, and probably many, participants in the lottery are making choices that result in negative expected returns, in the sense that they would be better off in expected monetary terms if they sat out the lottery. In the most prominent national lotteries worldwide, and state lotteries in the U.S., lotteries are structured such that a participant has a very small chance of winning a very large prize. One behavioral explanation for the massive participation in these lotteries is that they offer participants a "chance to buy hope" (see Clotfelter and Cook, 1989, 1990; Cook and Clotfelter, 1993). Another line of argument asserts that people tend to be poor at assessing and processing very small probabilities, so the minuscule probabilities of hitting it big in these lotteries are incorporated in decision-making only after some transformations. The idea of probability weighting has a component of prospect theory (Kahneman and Tversky, 1979). The experimental evidence for probability weighting is mixed; for example, Harbaugh et al. (2002) find that elicited probability weighting functions depend on the frame of the experimental task. This result indicates that, even if a probability weighting function is not per se a well-defined construct in the psychology of an agent's decision-making, framing remains an important component of the decision process.

Lottery mechanisms are frequently used to generate revenue on a much smaller scale. Particularly in the United States, charities, school groups, churches, and the like often hold lotteries, more commonly referred to as "raffles," for fund-raising purposes. In

these cases, grand prizes typically range in value from a few hundred dollars to prizes on the order of a new vehicle. Prizes of this level, while still attractive, do not evoke fantasies of massive riches. The target audience for these raffles is small, with most tickets being purchased by members of a local community. Therefore, the probability of winning the grand prize is orders of magnitude larger than in a state-run lottery. This minimizes the scope at which the explanations cited above are likely to be operable in raffles of this scale. A confounding factor in understanding behavior in these raffles is that, since these are used as fund-raisers for worthy causes, participants may purchase tickets in part or in whole out of charitable motives (see Morgan, 2000). Nonetheless, since raffles are common even when direct appeals for contributions are possible, raffles must offer some additional fund-raising benefits over direct appeals.

We directly investigate the profitability of raffles using a controlled laboratory experiment in which small probabilities, large prizes, and charitable motives are all absent. We systematically vary the number of participants in the raffle across sessions to find the minimum group size needed for a raffle to become profitable. In doing so, we can directly observe the strategic response of agents to the change in group size. Our main result is that a raffle shows a profit when the number of participants is at least four, which is very small compared to most raffles in the field. The behavioral underpinning of this result is that participants do not react strategically to the change in the number of other participants in the raffle. Individual participants spend about the same amount on raffle tickets irrespective of the size of the group. Additionally, as the size of the group increases, participants less frequently adjust their spending based on the previous outcomes. Therefore, our results propose a new explanation for raffle profitability: as the population of participants grows, individuals do not react in a strategically sophisticated way, and as a result, a raffle generates profit.

Our raffle design offers several properties which are favorable for laboratory study. There is a commonly-known number N of participants who have the opportunity to purchase raffle tickets for a fixed prize of \$10. Each participant i simultaneously purchases an amount x_i of tickets; each ticket costs one cent. One ticket is drawn from among those purchased; the purchaser of that ticket wins the prize. Therefore, the chance of participant i winning the prize is $\frac{x_i}{\sum_j x_j}$. Mathematically, this formulation is isomorphic with Tullock's (1980) basic model of contest theory, with the exponent r in his model

set to 1. A main theoretical result in that literature is that when agents are risk-neutral there exists a unique Nash equilibrium where the total amount spent by all players is less than the value of the prize. This result is at odds with the experimental results we report here.

We therefore turn to alternative models of motivation or bounded rationality for further insights into our data. The lottery contest has a clear competitive component, in that at the end of the game there is one and only one winner. Therefore, it is plausible that subjects might care not just about earnings but also about relative performance. In that case, subject behavior would conform to an evolutionarily stable strategy (ESS). In a finite population, Hehenkamp et al. (2004) show that the ESS is greater than the risk-neutral Nash equilibrium, but ESS spending levels decrease as the number of participants increases, which does not match our findings.

Neither Nash equilibrium nor ESS offer predictions to organize the heterogeneity we observe in spending levels. We therefore investigate the predictions of Quantal Response Equilbrium (QRE, McKelvey and Palfrey, 1995), which maintains a mutual best-reply assumption while allowing for noisy observation of expected payoffs. QRE fits the qualitative and quantitative features of our data. Our estimates are consistent with the hypothesis that the financial consequences of actions are less important in determining individual behavior as the group size increases.

We then analyze other behavioral models in order to get some clues as to why individual behavior becomes less financially driven as group sizes increase. Removing the equilibrium assumption, we consider models of level-k reasoning (see, e.g., Stahl and Wilson, 1994, 1995; Nagel, 1995; Ho, Camerer and Weigelt, 1998; Costa-Gomes, Crawford and Broseta, 2001; Costa-Gomes and Crawford, 2006; Crawford and Iriberri, 2007) where players exhibiting higher levels of strategic sophistication choose best responses to the play of players with lower levels of sophistication. Finally, we look at period-by-period adjustment behavior using myopic heuristics and compare them to the predictions of learning direction theory (Selten and Buchta, 1994; and Selten and Chmura, 2008). Although level-k reasoning and learning direction theory result in poor predictions in terms of the observed distribution of choices, they provide a common and consistent message: individuals do not react in a strategically sophisticated way as the population of participants grows.

Our design and our analysis of the data fill in some gaps in the literature on contests in the laboratory. The results of Millner and Pratt (1989); Potters, de Vries and van Wind (1998); Davis and Reilly (1998); Fonseca (2009) indicate that subjects exceeded the risk-neutral equilibrium predictions for spending.[1] In all these studies, a significant fraction of subjects chooses spending levels which are not rationalizable for risk-neutral participants. The literature has not given much attention to the formal study of the structure of the individual-level spending decisions, although Potters, de Vries and van Wind (1998), in their concluding remarks, hypothesize about the possibility that some players might make their choices in response to previous outcomes. We show that the Quantal Response Equilibrium organizes the distribution of individual spending, and we confirm the conjecture of Potters, de Vries and van Wind (1998) that there are adaptive players in the population, at least when the number of participants is small.

Anderson and Stafford (2003) is the first study to focus specifically on the effects of group size. They employ a one-shot design, so subjects do not have an opportunity to learn or adapt. They find individual spending exceeds the risk-neutral Nash equilibrium prediction, and that individual spending decreases as the number of participants increases. We were not able to replicate their second result in a setting where our participants took part in a series of games. In our case, as the size of the group increases, participants respond less frequently to the outcomes of previous games. In addition, while QRE continues to organize the individual spending levels well, the amount of noise in choices increases. Taken together, these results indicate that the profitability of raffle games even in rather small populations – in the case of our results, with four or more participants – can be explained by a lack of strategic response to changes in the number of other participants.

The paper is organized as follows. We describe the formal model of the game, theoretical predictions, and laboratory procedures in Section 2. Section 3 describes the experimental results. Section 4 considers behavioral theories and provides the main results. Section 5 concludes with a discussion.

[1] The exception to this pattern is the study of Shogren and Baik (1991), who find that individual spending levels match the theoretical prediction; however, they used a design in which subjects interacted in fixed pairs, so their results may be attributable to repeated game effects.

2 Theory and Experimental Design

2.1 Raffle Game

We study a single-prize raffle. There is a commonly-known number of players N, each of whom has an endowment ω. The value of the prize to be raffled is V. This value is the same to all participants, and this fact is commonly known. Participants simultaneously choose how many tickets to buy, with x_i being the choice of participant $i = 1, \ldots, N$. The price of a ticket is normalized to one. Each ticket is equally likely to be chosen; therefore, the chance of participant i winning the prize given a vector of choices $\{x_j\}_{j=1}^N$ is $\frac{x_i}{\sum_{j=1}^N x_j}$.[2] Consider player i and, for notational convenience, define the sum of other players' choices as $Y_i = \sum_{j \neq i} x_j$. Assuming risk-neutrality, the expected payoff to participant i is

$$u_i(x_1, \ldots, x_N) = \omega - x_i + V \cdot \frac{x_i}{x_i + Y_i}. \tag{1}$$

2.2 Rationalizable choices and Nash equilibrium

Given (1) and that other players buy at least one ticket, $Y_i > 0$, player i's best response is

$$x_i^*(Y_i) = \max\{\sqrt{VY_i} - Y_i, 0\}. \tag{2}$$

The best-response function is single-peaked and maximized at $Y_i = \frac{V}{4}$, at which point $x_i^*\left(\frac{V}{4}\right) = \frac{V}{4}$. Therefore, individual choices greater than $\frac{V}{4}$ are never best replies. Tullock (1980) showed that the game has a unique symmetric Nash equilibrium in which

$$x_1^\star = ... = x_N^\star = x^\star(N) = V \cdot \frac{N-1}{N^2}. \tag{3}$$

Note that when $N = 2$, then $Y_i = x_j$, where $i \neq j$ and in the Nash equilibrium $x_1 = x_2 = \frac{V}{4}$, so the Nash equilibrium is equal to the maximum rationalizable spending level.

2.3 Relative performance

Because of the competitive nature of the environment, it is plausible that subjects may care about relative performance in addition to their own earnings level. Under this

[2] We assume that nobody gets the prize if $x_1 = ... = x_N = 0$.

assumption, subjects behavior would be explained by evolutionarily stable strategies (ESS). Hehenkamp, Leininger, and Possajennikov (2004), based on the work of Schaffer (1988, 1989), showed that, in a finite population, the unique ESS for the game we study is

$$x^{ESS} = V/N. \tag{4}$$

This is greater than the Nash equilibrium for all values of N. The intution for this result is that at the Nash equilibrium, an increase in spending decreases expected earnings, by definition; however, the increase in spending decreases other's earnings more than one's own. Therefore, earnings of the agent who spends slightly more than the Nash level, while others spend at the Nash level, are higher.

The Nash and ESS predictions differ most greatly when $N = 2$, and the difference decreases monotonically in N. Also, ESS predicts the raffle will exactly break even for all values of N.

2.4 Protocol

A total of 15 experimental sessions were conducted at PEEL (Pittsburgh Experimental Economics Laboratory) using subjects recruited from the participant pool. There were three sessions with each of the group sizes $N = 2, 3, 4, 5, 9$. Cohort sizes ranged from 12 to 22 subjects. Table 1 summarizes the sessions.

At the beginning of each session, the instructions (see Appendix B) were read aloud. After the instructions were read and clarifying questions answered, subjects completed a questionnaire to check their understanding. Keeping in mind Shogren and Baik's (1991) use of expected-payoff tables, the instructions and questionnaire tested subjects' understanding of the expected payoff consequences of choices. Therefore, maximization of expected payoffs was implicitly suggested to subjects in a light-handed way.

Sessions consisted of 10 periods. Participants were randomly and anonymously assigned into groups of size N each period. In each period, each participant was given an endowment $\omega = 1,200$ tokens. Participants simultaneously selected an integer number of tokens between 0 and 1,200 to spend on a prize worth $V = 1,000$ tokens. Conditional on the amounts contributed, the prize was randomly allocated to one of the participants in the group, with participant i winning the prize with probability $\frac{x_i}{\sum_{j=1}^{N} x_j}$, if $x_i > 0$.

N	session	subjects	Nash	ESS	mean	median	> 250
2	2.1	18	250	500	350.89	300	94 (52%)
2	2.2	20	250	500	338.77	300	105 (53%)
2	2.3	12	250	500	286.68	200	52 (43%)
2	all	50	250	500	330.63	263	251 (50%)
3	3.1	12	222	333.4	240.80	200	48 (40%)
3	3.2	15	222	333.4	246.67	200	49 (33%)
3	3.3	12	222	333.4	361.27	200	54 (45%)
3	all	39	222	333.4	280.12	200	151 (39%)
4	4.1	20	188	250	276.78	200	75 (38%)
4	4.2	16	188	250	377.06	300	86 (54%)
4	4.3	16	188	250	253.51	200	66 (41%)
4	all	52	188	250	300.47	200	227 (44%)
5	5.1	20	160	200	311.81	200	77 (39%)
5	5.2	15	160	200	237.53	100	55 (37%)
5	5.3	15	160	200	417.27	329	83 (55%)
5	all	50	160	200	321.16	200	215 (43%)
9	9.1	18	99	111.11	379.56	200	75 (42%)
9	9.2	18	99	111.11	251.29	100	60 (33%)
9	9.3	18	99	111.11	347.78	200	85 (47%)
9	all	54	99	111.11	326.21	200	220 (41%)

Table 1: Summary of experimental sessions and results

At the end of the 10 periods, one of the 10 periods was selected at random, and subjects' earnings for this portion of the experimental session were determined by the selected period with the exchange rate 100 tokens = \$1. The experimental session then continued with 40 rounds of unrelated games. The overall length of each session was about two hours, with this portion comprising under an hour on average. In addition to their earnings from the rounds, subjects received a \$5 show-up fee.

3 Results

3.1 Data

Our experimental results are summarized in Table 1. Subjects chose spending levels which were integer multiples of 100 in 1913 of the 2450 bids (78.1%), and multiples of 50 a further 236 times (9.6%); overall, 87.7% of choices were multiples of 50. We will proceed with the data analysis using the frame in which subjects appear to process the game. All choices are binned into bins $k = 0, 1, 2, \ldots, 12$, where bin k consists of the choices $[k - 50, k + 50)$. Figure 1 plots the empirical distributions of choices for each of the group sizes. Choice patterns across all group sizes are qualitatively similar and there is no clear ranking of the distributions in terms of first-order stochastic dominance.

Since the distribution of individual spending does not depend in a significant way on the group size, it follows that total spending increases as the group size increases. For group sizes $N = 4, 5$, and 9, we find that total spending exceeds the value of the prize. For example, the average spending for a randomly-constituted nine-person group in the $N = 9$ treatment is 2935.89 tokens, for a prize worth only 1000 tokens. Our data and available data from previous studies are summarized in Table 2 and Figure 2.[3]

3.2 Comparison to theoretical predictions

Figure 3 presents period-by-period average spending levels plotted relative to the Nash equilibrium and ESS predictions. We observe that the average individual spending is above the Nash equilibrium prediction in all periods in all treatments. Mean and median levels of spending in the $N = 2$ and $N = 3$ treatments are roughly comparable to the

[3] In Table 2 and Figure 2 we normalize the prize values to $V = 1,000$ for other experiments.

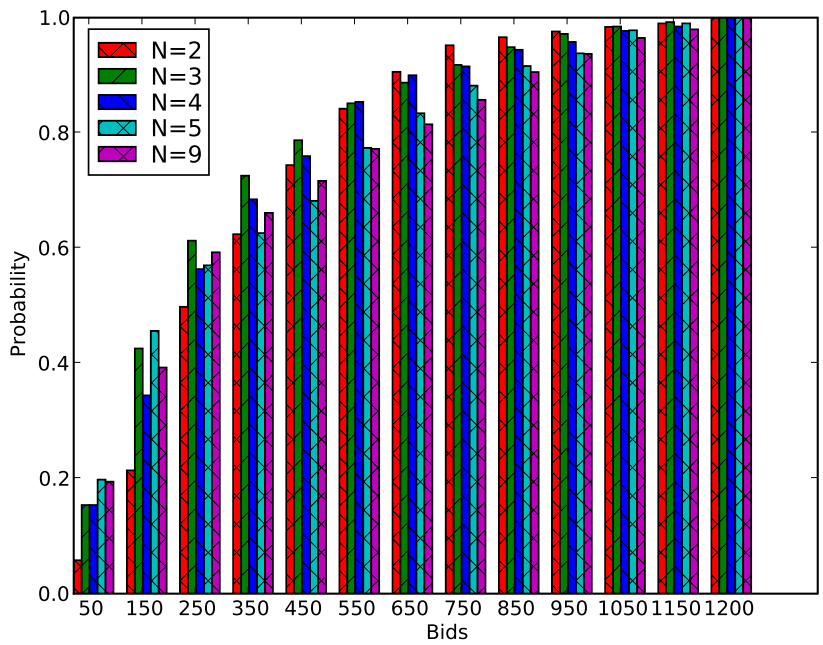

Figure 1: Empirical distribution of choices for $N = 2, 3, 4, 5, 9$, binned in intervals centered around multiples of 100.

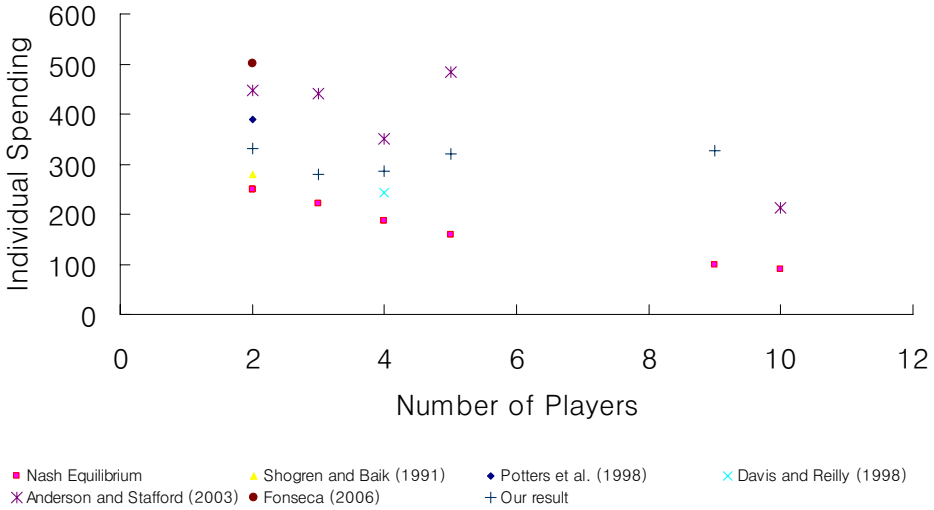

Figure 2: Summary of Lottery Experiments

Authors (Year)	N	Nash (A)	ESS (B)	Experiment (C)	$100 \cdot \frac{(C)-(A)}{(A)}$	$100 \cdot \frac{(C)-(B)}{(B)}$
Millner and Pratt (1989)	2	250	500	280	12.0	-44
Shogren and Baik (1991)	2	250	500	253.4	1.4	-49.3
Potters et al. (1998)	2	250	500	388.5	55.4	-22.3
Davis and Reilly (1998)	4	187.5	250	242.1	29.1	-3.2
Anderson and Stafford (2003)	2	250	500	448	79.2	-10.4
	3	222	333.4	441.7	99	32.5
	4	187.5	250	351.5	87.5	40.6
	5	160	200	483.4	202.1	141.7
	10	90	100	212.7	136.3	112.7
Fonseca	2	250	500	500.4	100.2	0.1
Our result	2	250	500	330.6	32.2	-33.9
	3	222	333.4	280.1	26.2	-16
	4	187.5	250	286.6	52.9	14.6
	5	160	200	321.2	100.8	60.6
	9	98.8	111.11	326.2	230.2	193.6

Table 2: Summary of Lottery Experiments

Nash prediction, but fail to track the Nash prediction for the larger group sizes. ESS is a better predictor than the Nash equilibrium for $N = 3$ and $N = 4$, but it also makes a poor prediction for $N = 5$ and $N = 9$. Note that as N increases, the difference between these two predictions shrinks. Figure 3 indicates that subjects' behaviour in small groups, $N = 2$ and $N = 3$, might be predicted by some weighted average of the Nash equilibrium and ESS. However, it is no longer true for bigger groups, $N = 4$, $N = 5$, and $N = 9$. Table 1 explains why it is the case: for all group sizes, 39% to 50% of individual choices exceed the maximum rationalizable choice of 250. This proportion does not vary systematically with group size.

4 Behavioral Predictions

Nash equilibrium and ESS do not do a satisfactory job of capturing the comparative statistics in behavior with respect to the number of participants. Further, they are completely silent in organizing the heterogeneity in choices present in all treatments.

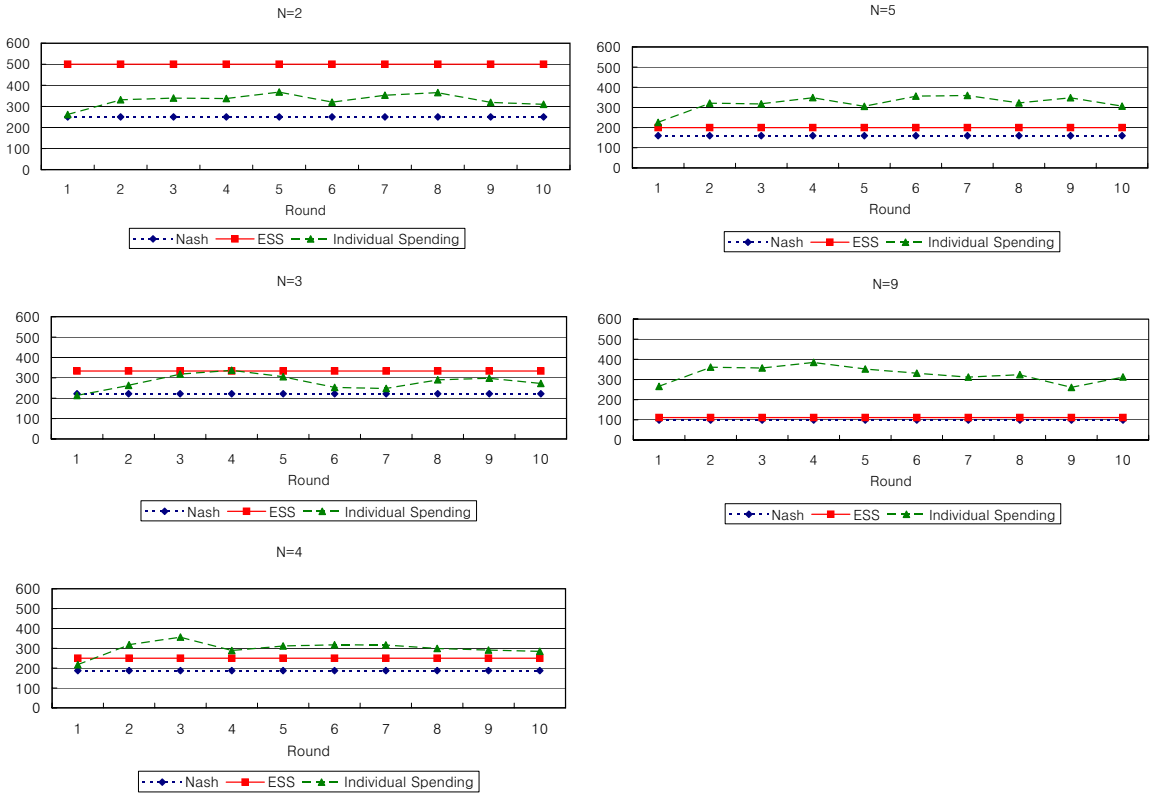

Figure 3: Period-by-period average spending levels.

We therefore turn to behavioral concepts to get a better handle on the data.

We proceed by progressively relaxing assumptions. We start with quantal response equilbrium, which maintains a mutual best-reply assumption while allowing for noisy observation of expected payoffs. Removing the equilibrium assumption, we consider models of level-k reasoning, where players exhibiting higher levels of strategic sophistication choose best responses to the play of players with lower levels of sophistication. Finally, we look at period-by-period adjustment behavior using myopic heuristics, and compare to the predictions of learning direction theory.

4.1 Quantal Response Equilibrium

The Quantal Response Equilibrium (QRE) concept of McKelvey and Palfrey (1995) is a widely-used model of noisy decision-making in games. In a QRE, each player observes the expected payoff to each of his strategy choices with an idiosyncratic shock that is not known to other players or the outside observer. In the most commonly-used version, this noise term is assumed to be i.i.d. across players and strategies, and is drawn from the extreme value distribution with precision parameter λ. This results in the familiar logit specification for choice probabilities (see, for example, Anderson, Goeree and Holt, 2002). To be more precise, if $u_i(x;\pi)$ is the expected utility of player i to playing choice x when all other players choose according to the mixed strategy profile π, then the probability player i will choose x is proportional to $\exp(\lambda u_i(x;\pi))$. For $\lambda = 0$, this reduces to uniform randomization over all strategies; as $\lambda \to \infty$, the set of QRE converges to a subset of the set of Nash equilibria.

The QRE concept has been successful in organizing data in laboratory games. An implication of the QRE is that less costly "mistakes" are made with higher probability. Therefore, in contrast with a pure-strategy Nash equilibrium, a QRE predicts a probability distribution over strategies, with positive probability assigned to each strategy in each player's strategy set. The model does this at the cost of the free parameter λ. Following McKelvey and Palfrey (1995), it is customary to estimate the value of λ against experimental data by likelihood maximization.

Formally, a strategy profile π is a QRE with precision parameter λ if, for every

strategy x, the probability π_x that x is chosen is

$$\pi_x = \frac{\exp(\lambda u_i(x;\pi))}{\sum_y \exp(\lambda u_i(y;\pi))}.$$

In the raffle game, the expected payoff to spending x is

$$u_i(x;\pi) = \omega - x + E_Y \frac{x}{x+Y},$$

for $x > 0$, and $u_i(0;\pi) = \omega$, where the expectation is taken with respect to the mixed strategy profile π. In a QRE,

$$\frac{\pi_x}{\pi_0} = \frac{\exp(\lambda u_i(x;\pi))}{\exp(\lambda u_i(0;\pi))} = \exp(\lambda(u_i(x;\pi) - u_i(0;\pi))).$$

Therefore,

$$\log \pi_x - \log \pi_0 = \lambda(u_i(x;\pi) - u_i(0;\pi)) = \lambda \left(E_Y \frac{x}{x+Y} - x \right).$$

In operationalizing the QRE model, recall that spending levels which are multiples of 100 predominate in the data. Therefore, we compute QRE on the discretized game with spending levels $\{0, 100, \ldots, 1100, 1200\}$ to match the frame subjects appear to use in approaching the game.[4] The precision parameter λ is estimated via maximum likelhiood for each group size N, with all choices in $[k-50, k+50)$ binned together as choice k for each discretized spending level.

Table 3 presents, for each group size N, the count of the number of choices in each bin, and the corresponding empirical frequency. Following these is the QRE fit by maximum likelihood estimation, performed using the Gambit software (McKelvey et al. 2008), with the corresponding log-likelihood. Figure 4 illustrates the close relationship between the distribution of spending levels in the data and the best-fit QRE.

[4]This is similar to the approach taken by Battaglini and Palfrey (2007).

	0	100	200	300	400	500	600	700	800	900	1000	1100	1200	logL
$N = 2$	29	78	142	63	60	49	32	23	7	5	4	3	5	
	0.058	0.156	0.284	0.126	0.120	0.098	0.064	0.046	0.014	0.010	0.008	0.006	0.010	
$\lambda = 0.642$	0.044	0.159	0.198	0.182	0.143	0.102	0.068	0.043	0.027	0.016	0.010	0.006	0.003	-1067.404
$N = 3$	60	106	73	44	24	25	14	12	12	9	5	3	3	
	0.154	0.272	0.187	0.113	0.062	0.064	0.036	0.031	0.031	0.023	0.013	0.008	0.008	
$\lambda = 0.589$	0.107	0.167	0.174	0.154	0.123	0.092	0.065	0.044	0.029	0.019	0.012	0.008	0.005	-849.946
$N = 4$	80	99	114	63	39	49	24	8	15	7	10	4	8	
	0.154	0.190	0.219	0.121	0.075	0.094	0.046	0.015	0.029	0.013	0.019	0.008	0.015	
$\lambda = 0.568$	0.153	0.178	0.167	0.141	0.110	0.082	0.059	0.041	0.027	0.018	0.012	0.008	0.005	-1139.547
$N = 5$	99	129	57	28	28	46	30	24	17	11	20	6	5	
	0.198	0.258	0.114	0.056	0.056	0.092	0.060	0.048	0.034	0.022	0.040	0.012	0.010	
$\lambda = 0.393$	0.172	0.163	0.144	0.122	0.100	0.080	0.062	0.048	0.036	0.027	0.020	0.015	0.011	-1136.628
$N = 9$	105	107	108	37	30	30	23	23	26	17	15	8	11	
	0.194	0.198	0.200	0.069	0.056	0.056	0.043	0.043	0.048	0.031	0.028	0.015	0.020	
$\lambda = 0.318$	0.202	0.168	0.138	0.112	0.090	0.072	0.057	0.045	0.035	0.028	0.022	0.017	0.013	-1227.282

Table 3: Observed data and QRE estimates.

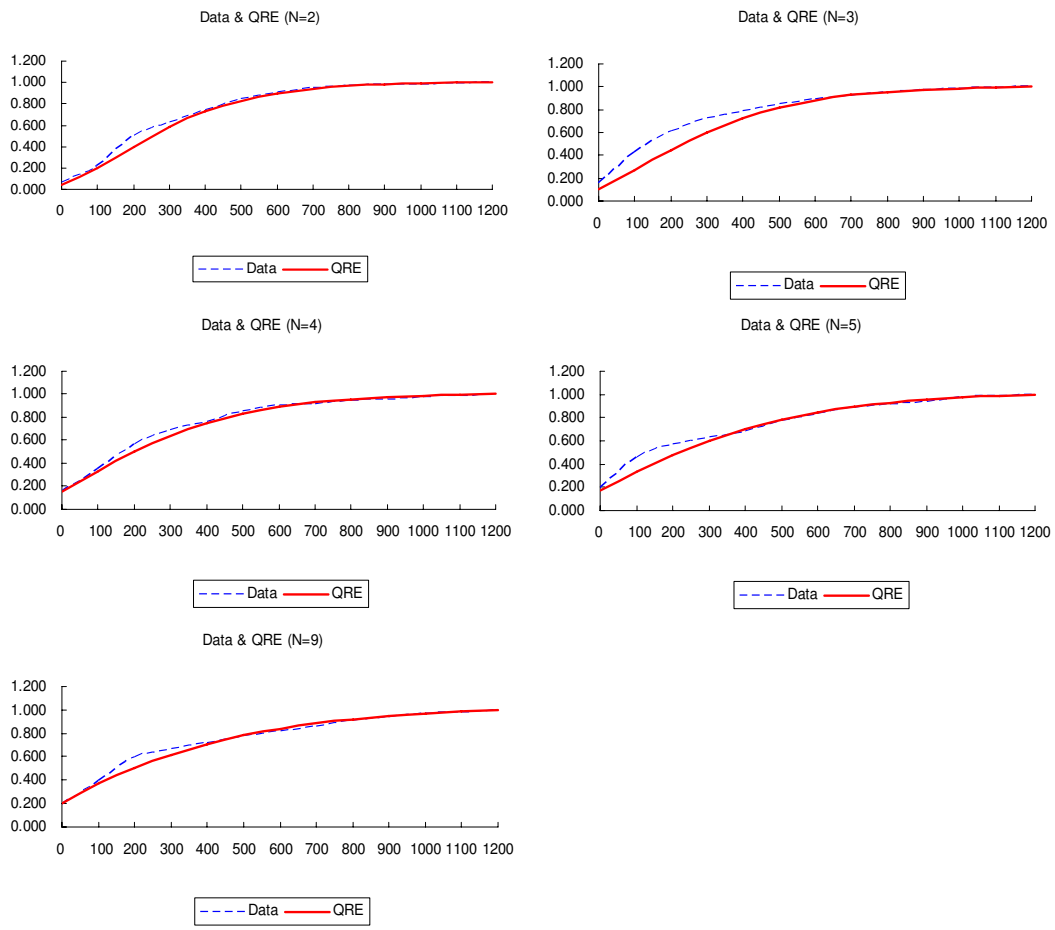

Figure 4: Cumulative Distribution Functions.

The estimated QREs match the qualitative structure of the observed pattern of individual spending levels. Both feature peaks in the distribution around $x = 200$, with long, exponentially-decaying tails for high spending levels. In other words, the presence of the non-rationalizable choices above 250 is well-organized by a logit model of choice.

The maximum likelihood estimates of λ decrease slowly as the group size increases. Intuitively, the λ parameter in QRE controls the relative influence that expected payoffs have on choices, as opposed to the effects of other, unobserved random influences. Lower values of λ correspond to QREs where the unobserved payoff shocks have greater variance, and therefore greater influence on observed choices. Our estimates are consistent with the hypothesis that the financial consequences of actions are less important in

determining behavior as group size increases.

This interpretation of the QRE does not shed light specifically on why financial consequences are less important. It is possible that the larger number of other players in the group increases strategic uncertainty, or makes the strategic problem of formulating a spending choice more difficult.[5] We now turn to other behavioral approaches for clues as to why behavior becomes noisier, and moves further from Nash, as seen by QRE.

4.2 Level-k Reasoning

We now relax the equilibrium assumption which is embedded in QRE and consider the data in light of level-k reasoning (see, e.g., Stahl and Wilson, 1994, 1995; Nagel, 1995; Ho, Camerer and Weigelt, 1998; Costa-Gomes, Crawford and Broseta, 2001; Costa-Gomes and Crawford, 2006; Crawford and Iriberri, 2007). Following the custom in the literature, we suppose that there are three player types. Specifically, we follow the method used by Ho, Camerer and Weigelt (1998) for games with a large strategy space. Start with the assumption that the lowest, Level 0 (L_0) players choose their spending uniformly over $[0, V]$. Level $k > 0$ (L_k) players are assumed to believe that all other players are level L_{k-1} players who make their choices according to the density function $B_{L_{k-1}}(x)$. Believing this, they mentally simulate $N-1$ draws from the density function $B_{L_{k-1}}(x)$ and compute their best response. Such a best response density satisfies

$$B_{L_k} = \sqrt{V \cdot \sum_{i=2}^{N} B^i_{L_{k-1}}} - \sum_{i=2}^{N} B^i_{L_{k-1}}, \tag{5}$$

where $B^i_{L_{k-1}}$ is the i-th draw of a random variable $B_{L_{k-1}}$.

The assumptions above allow us to calculate the density of each level type. Note that since any choice greater than 250 is never a best reply to any choice in $[0, 1000]$, the level-k density for any $k > 0$ is truncated at 250 (see Figure 6). Thus, level 0 should be assigned to players who choose any number greater than 250. The crucial problem is how to assign a level type to a player who chooses a number that different types might choose. Take the $N = 2$ case as a clarifying example. Suppose a player chooses

[5] We are not aware of any other studies which specifically address how QRE estimates perform as group sizes are systematically varied.

N	L_0	L_1	L_2
2	36.00	60.00	4.00
3	26.32	65.79	7.89
4	51.92	40.38	7.69
5	64.00	20.00	16.00
9	96.30	1.85	1.85

Table 4: Proportions (1st-Round Data)

N	L_0	L_1	L_2
2	48.00	52.00	0.00
3	36.84	60.53	2.63
4	75.00	17.31	7.69
5	76.00	6.00	18.00
9	90.74	0	9.26

Table 5: Proportions (All-Rounds Data)

200. This choice could come from a L_0 player, from a L_1 player or from a L_2 player. Following Ho, Camerer and Weigelt (1998), we assign this choice to a level-k if and only if this level-k type is more likely to have made that choice than any other level types. Since $B_{L_1}(200) > B_{L_2}(200) > B_{L_0}(200)$, we assign choice 200 to level-1.

Table 4 reports the proportions for each level, using first-round data. The proportion of L_2 type players is small for any $N = 2, 3, 4, 5, 9$. Furthermore, as N increases, the proportion of L_0 type increases and the proportion of L_1 type decreases. When $N = 9$, more than 90% of the subjects are the L_0 type. This observation is very surprising, as we are not aware of any paper in the level-k reasoning literature with similar results. These features are observed not only in the first round, but also when considering spending levels from all rounds.

We follow the methodology of Costa-Gomes and Crawford (2001) to classify subjects whose choices can be consistently identified as fitting into one of the behavior types. We count how many times subject's spending levels are consistent with one of the types. Subjects are classified into a particular type if their behavior is consistent with that type in at least 6 out of the 10 rounds. If there is a tie between two or more types, we classify that subject to the highest type. This classification method is biased in favor of higher-level types. Using this classification method, we find that more than 90% of the subjects are L_0 type when $N = 9$ (see Table 5).

Do subjects iteratively eliminate dominated strategies over time in our experiments, as has been shown, for example, in beauty contest games (Nagel, 1995)? To answer this question, we assume that L_0 learners simply choose the best-response against a weighted average of choices in previous rounds. L_1 learners assume all others are L_0 learners and best respond to anticipated choices by L_0 learners. L_2 learners best respond

to L_1 learners. This approach predicts that no level types choose numbers greater than 250 after the first round. However, choices greater than 250 are observed over all rounds for all $N = 2, 3, 4, 5, 9$ (see Table 6). Subjects do not learn to eliminate dominated strategies in the raffle game. The distributions of spending levels plotted in Figure 5 show that there is no convergence to any one spending level over time.

N	Round	1	2	3	4	5	6	7	8	9	10	Total
	# > 250	18	25	30	29	26	22	24	28	24	25	251
2	Total Subjects	50	50	50	50	50	50	50	50	50	50	500
	% > 250	36	50	60	58	52	44	48	56	48	50	50
	# > 250	10	10	18	20	16	14	12	16	15	17	151
3	Total Subjects	38	38	38	38	38	38	38	38	38	38	380
	% > 250	26	26	47	53	42	37	32	42	39	45	39
	# > 250	14	20	25	24	26	25	28	21	22	22	227
4	Total Subjects	52	52	52	52	52	52	52	52	52	52	520
	% > 250	27	38	48	46	50	48	54	40	42	42	44
	# > 250	15	23	23	24	21	23	23	20	25	18	251
5	Total Subjects	50	50	50	50	50	50	50	50	50	50	500
	% > 250	30	46	46	48	42	46	46	40	50	36	43
	# > 250	19	26	25	24	27	23	18	21	16	21	220
9	Total Subjects	54	54	54	54	54	54	54	54	54	54	540
	% > 250	35	48	46	44	50	43	33	39	30	39	41

Table 6: Choices > 250 over Rounds

4.3 Adaptation and Learning Direction Theory

Selten and Buchta (1994) proposed a simple heuristic model in which agents adjust their choices in response to recent experience. Learning direction theory applies to games where strategy spaces have a clear ordering, such as when strategies are indexed by numbers. The theory asserts that, given an action profile $\{a^t\}$ played in period t, a subject will choose an action a_i^{t+1} in the subsequent period which is closer in the strategy space to the best response against the previous-period choices of others, $\{a_{-i}^t\}$. We apply two formulations of learning direction theory to organize adaptive behavior for subjects over the course of a session.

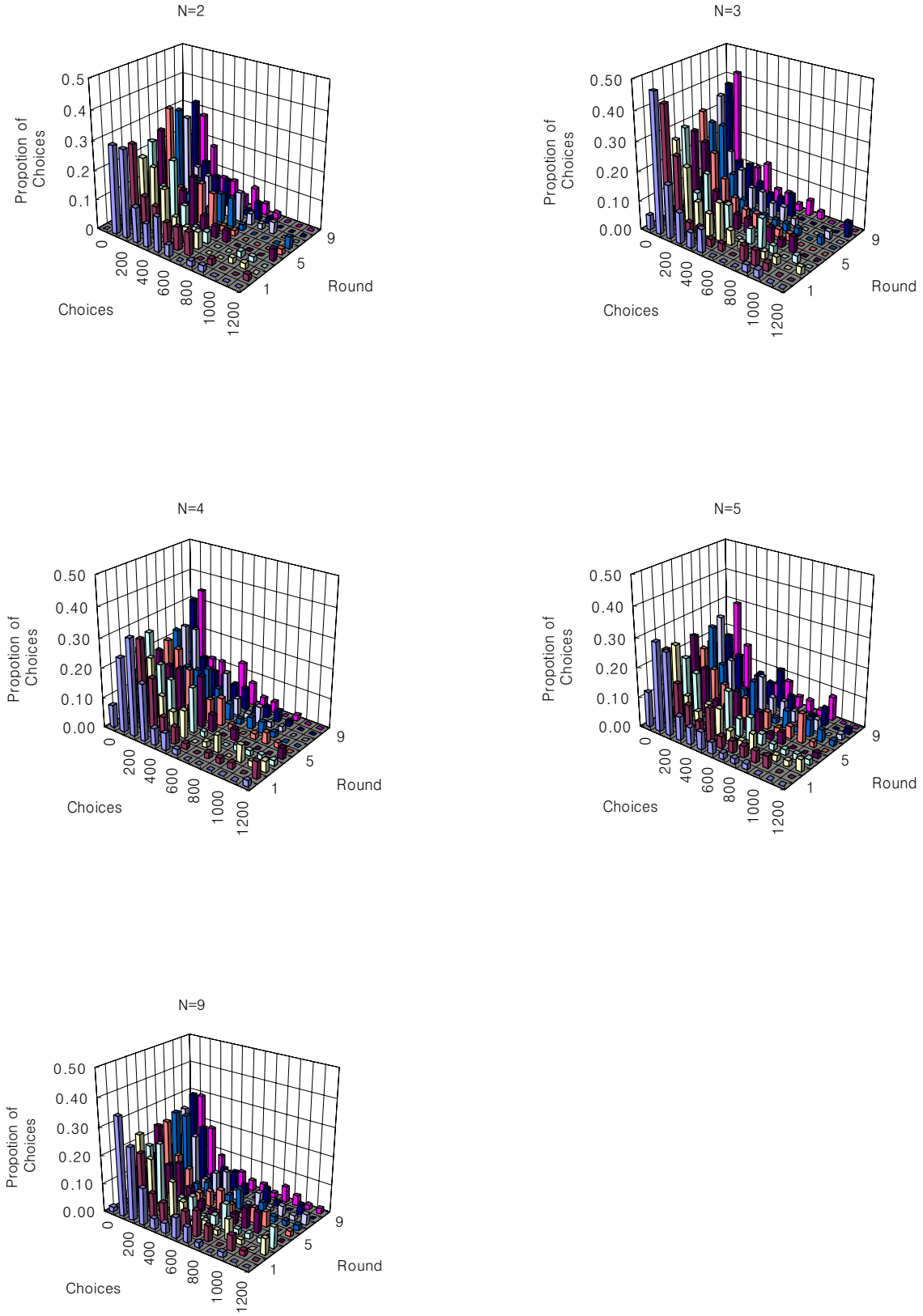

Figure 5: Subjects' Choices over Rounds.

We divide all choices after the first period based on whether the subject making the choice won the prize in the previous period. In each category, we further segregate choices based on whether the subject chose a different spending level than in the previous period. Among those periods where spending levels changed, we tabulate two measures. To get a general sense of whether adaptation over time tends to increase or decrease spending levels, we count the number of times the spending levels increased or decreased relative to the previous period. To specifically test learning direction theory, we tabulate whether subjects' choices moved in the direction predicted by learning direction theory, i.e., in the direction of better responses. Table 7 summarizes these measures.

We observe that adaptation in small lotteries differs from their larger counterparts in two ways. When the number of players is small, players who lost the previous lottery are much more likely to change their choice in the following period compared to players who won the previous lottery. Losing the lottery when the group is small is a stronger stimulus than winning. With larger groups, the asymmetry in response to the lottery disappears, with changes in spending levels occurring equally often after wins and losses. The relative frequency of increases and decreases in spending levels after wins and losses is consistent with a regret explanation. A win is evidence that one has "overpaid," and therefore reduction in the spending level is in order; a loss is evidence that one has not been aggressive enough, and therefore an increase in spending is indicated.

Table 7 illustrates that when the number of players is small, adjustments tend to be in the direction of the best response. As the size of the group increases, the trend is towards random adjustments, with adjustments in the direction of the best response being no more likely than adjustments away from the best response.

Both these trends point to the same conclusion. In small lotteries, the event of losing the lottery is salient, because one expects to win roughly $\frac{1}{N}$ of the lotteries one enters. Losing a lottery in a larger group is less surprising, and so is less of a stimulus. Adaptation in larger group sizes is therefore correspondingly weaker, and changes, both in raw spending levels and in the reliability of moving in the direction of expected-payoff-increasing choices, are more random.

Outcome	Direction	$N = 2$	$N = 3$	$N = 4$	$N = 5$	$N = 9$
Won	Total	75	103	118	95	26
	No change	43 (57%)	53 (52%)	56 (48%)	46 (48%)	10 (39%)
	Higher	9	16	22	21	8
	Lower	23	34	40	28	8
	Better reply	$\frac{21}{32}$ (66%)	$\frac{31}{50}$ (62%)	$\frac{41}{62}$ (66%)	$\frac{29}{49}$ (59%)	$\frac{8}{16}$ (50%)
Lost	Total	87	248	350	355	298
	No change	28 (32%)	102 (41%)	121 (35%)	173 (49%)	131 (44%)
	Higher	37	82	124	102	85
	Lower	22	64	105	80	82
	Better reply	$\frac{36}{59}$ (61%)	$\frac{93}{145}$ (64%)	$\frac{128}{227}$ (56%)	$\frac{72}{157}$ (45%)	$\frac{84}{163}$ (51%)

Table 7: Summary of data for testing learning direction theory.

5 Discussion

We show that in a laboratory setting, individual spending in raffles does not decrease as the number of participants increases. Therefore, the prediction that total expenditure will not exceed the prize fails to hold. We show that this can occur at a surprisingly small group size, with only four participants. The observation in the field that big-money lotteries and smaller raffles can be profitable can be replicated in the laboratory without resorting to very large groups, or to life-changing prize amounts offering a "chance to buy hope."

We organize the data using several behavioral models to investigate the micro-level structure of spending choices underlying the result. These models all point to a similar conclusion: as the number of participants in a raffle increases, choices become less correlated with the underlying financial incentives in the experiment. This is consistent both with increased strategic uncertainty regarding the play of a larger number of coplayers, as well as the weaker ex-post incentive signals received from winning or losing the raffle. This disconnect from earnings results in less adaptation of choices and increased apparent randomness in play, which taken together underlie the observed levels of spending.

A Calculations for Level-k Distributions (N=2)

In this section, we show how to get Level-k distributions. We only provide the detailed calculations for $N = 2$.[6] We suppose that the L_0 type is uniformly distributed on the interval $[0, 1000]$. To get the distribution for L_1 type, let x be a random variable that follows the uniform distribution on the interval $[0, 1000]$. From the best-reply property (5), a random variable of L_1 type, denoted by y, satisfies the following:

$$h(x) = y = \sqrt{1000x} - x.$$

Then the distribution function of the random variable y is

$$\begin{aligned} F_y(y) &= Pr(h(x) \leq y) \\ &= Pr(x + y \geq \sqrt{1000x}) \\ &= Pr(x^2 - 2(500 - y)x + y^2 \geq 0) \\ &= 1 - F_x((500 - y) + \sqrt{500(500 - 2y)}) + F_x((500 - y) - \sqrt{500(500 - 2y)}). \end{aligned}$$

The density function of y is

$$\begin{aligned} f_y(y) &= f_x((500 - y) - \sqrt{500(500 - 2y)})(-1 + \sqrt{\frac{500}{500 - 2y}}) \\ &\quad + f_x((500 - y) + \sqrt{500(500 - 2y)})(1 + \sqrt{\frac{500}{500 - 2y}}) \end{aligned}$$

only if $y \in [0, 250)$. After some rearrangement, we get

$$f_y(y) = \begin{cases} \sqrt{\frac{1}{500(500-2y)}}, & y \in [0, 250) \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

Given the density function (6) and the best-reply property (5), a random variable of L_2 type, denoted by z, satisfies the following:

$$g(y) = z = \sqrt{1000y} - y.$$

Then, the distribution function of the random variable z is

$$\begin{aligned} F_z(z) &= Pr(g(y) \leq z) \\ &= 1 - F_y((500 - z) + \sqrt{500(500 - 2z)}) + F_y((500 - z) - \sqrt{500(500 - 2z)}). \end{aligned}$$

[6] The calculations for $N = 3, 4, 5, 9$ are available upon request.

It is straightforward to calculate the density function of z:

$$f_z(z) = \begin{cases} \sqrt{\frac{1}{500(500-2(500-z)+2\sqrt{500(500-2z)})}}(-1+\sqrt{\frac{500}{500-2z}}), & y \in [0, 250) \\ 0, & \text{otherwise.} \end{cases}$$

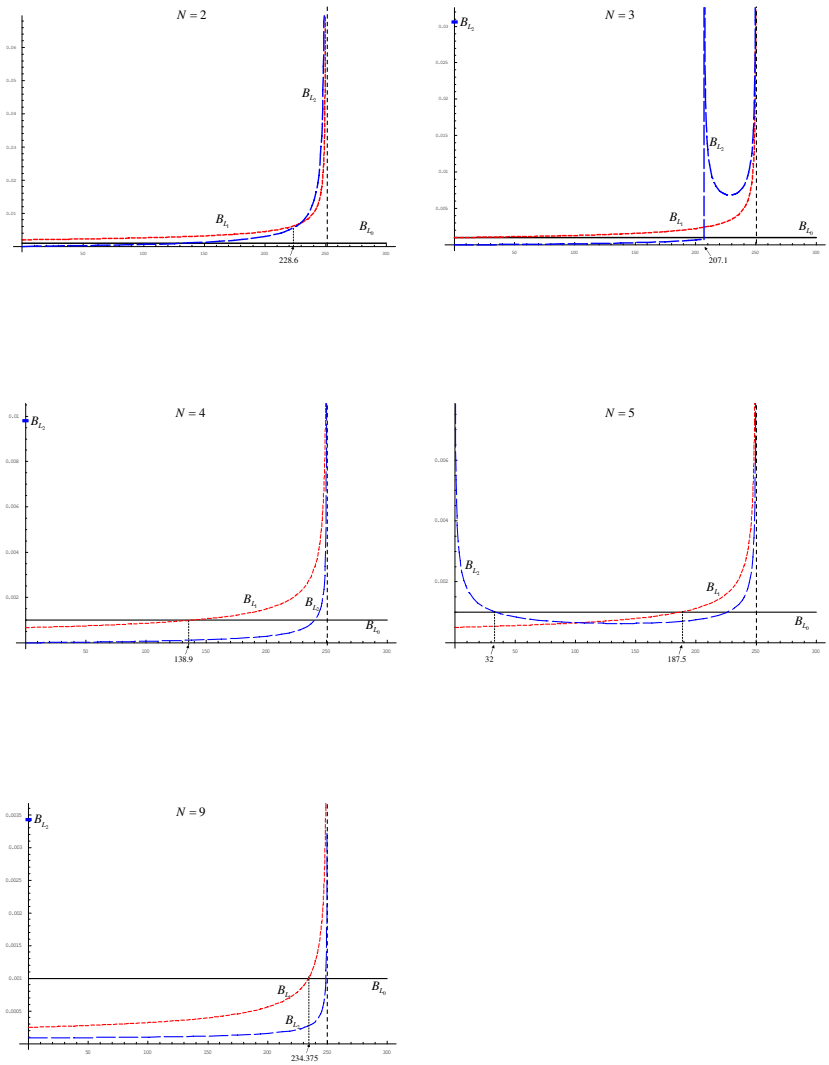

Figure 6: Densities

Figure 6 illustrates all densities.

B Appendix: Instruction (N=3)

This is an experiment related to economic decision making. Over approximately the next hour and a half, you will be asked to participate in several lottery games. For simply showing up to this experiment you have already received a flat sum of $5. In addition, the decisions you make and your luck in the lottery will determine your final monetary payoff for this experiment session, which in fact, may total as much as $27 (including the show up fee). You will be paid in private once the session has been completed.

There are several people participating in this session. However, during any particular round, you will be competing against only two other people, identified by participant numbers. It is extremely important that you do not communicate with any of the other subjects. If at any time you have a question, please raise your hand and if need be, I will come over to where you are sitting in order to answer your question privately. There will be a lottery game and it will be played for 10 rounds.

Lottery Prize

In every game in which you will participate, the lottery prize is equal to 1,000 tokens. At the beginning of each round, you will have 1,200 tokens and the opportunity to choose the number of tokens you would like to contribute to the lottery game. Keep in mind that one token corresponds to one lottery game ball and another chance to win the lottery prize. After all contributions are noted on the contribution slip and on your computer screen, a winner will be selected at random. After all 10 rounds have been completed, one lottery round from among those played will be chosen at random and you will be compensated in dollars according to your token earnings during the selected round (plus the $5 for participating, which you have already received). To be more specific, you will earn $1 for every 100 token remaining in your private fund (calculated as simply your initial wealth of 1,200 tokens minus the number of tokens you chose to contribute to the lottery game). Additionally, you will earn $10 more if you win during that round. For example, if you chose to contribute 100 tokens during the selected round, your private wealth will be 1,100 tokens (1,200 - 100), plus your lottery winnings, if applicable. If you won the lottery, your total token wealth at the end of the round would be equal

to 2,100 (1,100 + 1,000). Your total dollar payoff would be equal to your tokens times the exchange rate plus the additional $5 simply for participating in the experiment. If the describe outcome happens in the selected round your total dollar payoff would be equal to your token times the exchange rate, namely $21, plus the additional $5 for participating in the experiment. If these instructions are unclear, please raise your hand now. If not, we will begin the first lottery.

1-10 rounds: Lottery Game

The lottery prize is equal to 1,000 tokens. You have 1,200 tokens, 2 opponents, and the opportunity to choose the number of tokens you would like to contribute to the lottery game. Keep in mind that one token corresponds to one lottery game ball and another chance to win the lottery prize. If you contribute X tokens and your opponents TOGETHER contribute Y tokens, your chance to win the lottery is $\frac{X}{X+Y}$.

QUIZ 1

1. Assume that your contribution is 100 tokens and your opponents' total contribution is 900 tokens. What is your chance to win the lottery?
(a) 100 / 900
(b) 100 / 1,000
(c) 100 / 800
(d) 800 / 900
(e) 900 / 1,000

2. Assume that your contribution is 900 tokens and your opponents' total contribution is 100 tokens. What is your chance to win the lottery?
(a) 100 / 900
(b) 100 / 1,000
(c) 800 / 900
(d) 900 / 1,000
(e) 900 / 900

Your expected payoff in each round is (Your probability to win the prize) times (The prize value) minus (Your contribution): $\frac{1{,}000 \cdot X}{X+Y} - X$.

QUIZ 2

3. Assume that your contribution is 100 tokens and your opponents' total contribution is 900 tokens. What is your expected payoff?

(a) -100

(b) 0

(c) 100

(d) 900

(e) 1,000

4. Assume that your contribution is 900 tokens and your opponents' total contribution is 100 tokens. What is your expected payoff?

(a) - 900

(b) - 100

(c) 0

(d) 100

(e) 900

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