Location, Information and Coordination

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ABSTRACT. In this paper, we consider K finite populations of boundedly rational agents whose preferences and information differ. Each period agents are randomly paired to play some coordination games.

We show that several "special" (fixed) agents lead the coordination. In a mistake-free environment, all connected fixed agents have to coordinate on the same strategy. In the long run, as the probability of mistakes goes to zero, all agents coordinate on the same strategy. The long-run outcome is unique, if all fixed agents belong to the same population.

1. INTRODUCTION

Numerous studies have shown the importance of social interactions and neighborhood effects in explaining phenomenon such as levels of education, income, production, crime, arising of gangs, ghettos and so on.[1] Therefore, the incorporation of social interactions on behavior is of primary interest. In particular, the influence that some agents exert on others can have a profound impact on the selection of economic outcomes.

In this paper, we consider a coordination problem among heterogeneous agents. Our objective is to analyze the importance of information flows among agents and see whether or not heterogeneous agents are able to find a way to coordinate and on what outcomes. In every day life, social norms, national traditions, and focal points ease agents' coordination problems. For example, when facing the choice of driving on the right or the left side of the road, an agent follows the social norm adopted in her country. Therefore, it is natural to assume that agents can use their neighbors and own *past experience* for guidance in future coordination.

We consider K finite populations of agents, each of which represents a group of agents who share common preferences. Each period, all agents are randomly paired to play some $K \times K$ coordination games. Each population prefers to coordinate on its *preferred* strategy. By *preferred strategy* we mean that when two agents from the

[1] See e.g Becker (1991), Benabou (1993), Glaeser, Sacerdote, and Scheinkman (1996), Jankowski (1991), Venkatesh (1997) among many others.

same population are matched, a unique Pareto efficient outcome can be obtained if both agents coordinate on the so-called preferred strategy. If the matching involves two agents from different populations and the two of them coordinate, several Pareto efficient outcomes can be reached. The information available to each agent differs from one agent to the other. In order to capture this asymmetry, we define a neighbor as an agent from whom one can sample information about past plays, whereas a stranger is an agent one does not have any information about. A convenient way to represent these relationships is as follows. A directed link from agent i to agent j is an information flow from agent i to agent j, as in Bala and Goyal (1998, 2000) and Masson (2005). It means that agent j considers agent i as a neighbor. If the link between agent i and agent j is mutual, as in Jackson and Wolinsky (1996), it means that both agents consider each other as neighbors. Note that contrarily to what is commonly assumed in the literature on social interactions, we do not impose that neighbors share common preferences (are from the same population).

When an agent faces a neighbor, she can access her opponent's past plays and payoffs. We assume that she samples some of her opponent's past plays and plays a myopic best response to this sample that she considers as her opponents' strategy distribution for the current period. This approach is common in the literature, see Kandori, Mailath, and Rob (1993), Young (1993, 1998) among many others. However, the situation differs when an agent is matched with a stranger. In that case, the agent does not have any information about her opponent. The only information at her disposal comes from a sample of her neighbors' past plays. In this case, we assume that the decision rule the agent uses satisfies the following requirements: the decision rule can either select a strategy present in the sample or a best response against *some* subsample.[2] Our motivation for this approach is intuitive: if an agent does not know anything about her current opponent, she may believe that one of her neighbors could have played against her current opponent in the past. Therefore, any past experience from her neighbors could indeed carry some useful information. Notice that *any* imitation rule and *any* belief-based rule satisfy our requirement.

The main question we answer in this paper is *how coordination arises among agents with different preferences when the game they play is what Schelling calls "a mixture of mutual dependence and conflict of partnership and competition."*[3] Our model encompasses many situations. For example, consider a Battle of the Sexes game where men and women from different finite populations are paired to play each period. Note that any two agents can be matched each period. A question is

[2] Note that the "or" is not exclusive. The strategy present in the sample can also be a best response against *some* subsample.

[3] See Schelling (1960, 1971).

whether men and women coordinate on the same strategy? Another example where agents might find it difficult to coordinate is when they are faced with the choice of a product. In the academic world, for example, where coauthoring makes it critical to coordinate, which of SWP[4] or LaTex should be used for the first draft of a paper. One might prefer LaTex but accommodate SWP's coauthors and the question is: why? What is more important for coordination: the proportion of agents sharing similar preferences, or the information agents can access?

Our main result shows that there exist some "special" agents leading the coordination. Similar agents and their impact on the social behavior of others have been observed by Glaeser, Sacerdote, and Scheinkman (1996) in a study on crime. They called them *Fixed agents*. Bala and Goyal (1998) also studied these peculiar agents, and called them the members of the royal family.

Our first result shows that *connected fixed agents* have to coordinate on the same strategy choice in the short run. This prediction is *independent* from the decision rule agents use when they are matched with a stranger and might lead to the existence of segregated neighborhoods where agents from the same neighborhood play the same strategy.[5] This result implies that within the following common information structures: complete, all links are double-sided, directed star, directed wheel, and directed chain, all agents play the same strategy in the short run.

If there is a small probability that agents can make their choices at random, we obtain a sharp prediction as this probability goes to zero. In particular, if all agents use the following imitation rule: *imitate the strategy which gives the highest payoff in the sample*, *all agents* coordinate on the same strategy in this noisy environment.[6] Moreover, if all fixed agents are from the same population (members of the same royal family), the long-run (noisy) outcome is unique: all agents coordinate on the preferred strategy of the royal family. Our assumption that imitation is a sensible rule to use when access to information is limited is supported by experimental evidence (see Huck, Normann, and Oechssler, 1999) and Hayek's theory of cultural evolution.[7] Theoretical analysis and impact of imitation rules are presented by Robson and Vega-Redondo (1996) and Josephson and Matros (2004).

The paper is organized as follows. A detailed description of the model is given in Section 2. Short run predictions are presented in Section 3. Section 4 describes the long-run outcomes and Section 5 concludes.

[4] Scientific WorkPlace.

[5] See Schelling (1971) for the first discussions about segregated neighborhoods.

[6] We modify the imitation rule to take into account heterogeneity among agents. We assume that an agent imitates a strategy which gives the highest payoff to an agent from the same population. This is always possible since an agent can imitate herself.

[7] See Hayek (1988).

2. The Model

The following subsections characterize the concepts of heterogenous populations, information structure, decision rules and Markov process used in the paper.

2.1. Heterogenous Populations and Payoffs Matrices. Suppose that there exist K finite populations with $n_k \geq 1$ agents in population k, such that $n_1 + ... + n_K = 2n$, for $k = 1, ..., K$. Time is discrete, and in each period, all agents are randomly paired to play some $K \times K$ coordination games. Each agent from population k faces the following payoff matrix

$$\mathbf{A}^k = \begin{pmatrix} a_{11}^k & a_{12}^k & ... & a_{1k}^k & ... & a_{1K}^k \\ a_{21}^k & a_{22}^k & ... & a_{2k}^k & ... & a_{2K}^k \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{k1}^k & a_{k2}^k & ... & a_{kk}^k & ... & a_{kK}^k \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{K1}^k & a_{K2}^k & ... & a_{Kk}^k & ... & a_{KK}^k \end{pmatrix},$$

where $a_{hh}^k > a_{lh}^k$ for all $h, k, l = 1, ..., K$, $h \neq l$; and $a_{kk}^k > a_{ll}^k$ for any $k \neq l$. The first condition $a_{hh}^k > a_{lh}^k$ insures that agents favor coordination. The second condition $a_{kk}^k > a_{ll}^k$ stipulates that an agent from population k prefers to coordinate on strategy k. Therefore, an agent plays different coordination games with agents from different populations. In particular, each population k has a *preferred* strategy k that leads to a unique Pareto Efficient outcome when two agents from this population are matched. This is the case when two men are matched in the Battle of the Sexes game. In this situation, both men have a *preferred* strategy which is to "go to a soccer match". On the other hand, when two agents from different populations are matched, there are at least two Pareto Efficient outcomes. This corresponds to the case where a woman and a man are matched in the Battle of the Sexes game.

Denote a pure strategy $x \in \{1, ..., K\}$ of an agent from population k by a vector $\mathbf{x} = (0, ..., 0, 1, 0, ..., 0)$. Suppose that an agent from population k is matched with an agent from population l. If the agent from population k plays strategy $x \in \{1, ..., K\}$ and the agent from population l plays strategy $y \in \{1, ..., K\}$, then the two agents obtain the following payoffs

$$\pi_k(x, y) = \pi_k(\mathbf{x}, \mathbf{y}) = \mathbf{x}\mathbf{A}^k\mathbf{y}^T$$

and

$$\pi_l(y, x) = \pi_l(\mathbf{y}, \mathbf{x}) = \mathbf{y}\mathbf{A}^l\mathbf{x}^T.$$

The following example illustrates this point.

Example. Battle of the Sexes.

Suppose that there are two populations: **m**en and **w**omen. In each period, all agents are randomly paired to play some 2×2 coordination games. Each agent from the **m**en's population has the following payoff matrix

$$\mathbf{A}^m = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

and each agent from the **w**omen's population has the following payoff matrix

$$\mathbf{A}^w = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}.$$

Let us call the first strategy, $(1,0)$, "go to a **S**occer match," and the second strategy, $(0,1)$, "go to an **O**pera."

Agents can be matched in three ways: man and man, woman and woman, and man and woman. If two men are matched, they play the following coordination game

	S	**O**
S	$2,2$	$0,0$
O	$0,0$	$1,1$

Man vs Man

If two women are matched, they play the following coordination game

	S	**O**
S	$1,1$	$0,0$
O	$0,0$	$2,2$

Woman vs Woman

If a man is matched with a women, they play the following coordination game

	S	**O**
S	$2,1$	$0,0$
O	$0,0$	$1,2$

Man vs Woman

This example illustrates how the populations' heterogeneity is introduced into the payoffs matrices. We now focus on the information structure and adopt some necessary terminology for the description of its asymmetry.

2.2. Information Structure. At time t, each agent from population i chooses a strategy x_i^t from the set $X = \{1, ..., K\}$ according to some behavioral rules (described below) based on past plays' available information. Therefore, the *play* at time t can be defined as $x^t = (x_1^t, x_2^t, ..., x_{2n}^t)$, and the *history of plays* up to time t can be represented by the sequence $h^t = (x^{t-m+1}, ..., x^t)$ of the last m plays.

Define the information structure $\langle \mathcal{V}, \mathcal{T} \rangle$, where $\mathcal{V}$ is the set of agents and $\mathcal{T}$ is the set of information links. Our information structure is similar to the information structure in Bala and Goyal (1998) and Masson (2005). We call agents from whom agent q can access past plays **neighbors** of agent q, and any other agent a **stranger** to agent q. Denote by $Nb(q)$ the set of neighbors of agent q; $St(q)$ the set of strangers of agent q; and $A(q)$ the set of agents who can access agent q's past plays. Note that agents in $Nb(q)$ do not need to share common preferences.

The dichotomy neighbor/stranger can be represented by a directed graph (the information structure $\langle \mathcal{V}, \mathcal{T} \rangle$) where a directed link (flow of information) from agent q to agent g, $\{q \to g\} \in \mathcal{T}$, means that agent g can access information about past plays of agent q, or $q \in Nb(g)$ and $g \in A(q)$. We assume that $q \in Nb(q)$ for any q. The following example illustrates the above definitions.

Example. Suppose that the information structure of the Battle of the Sexes game is as in Figure 1 where agents 1 and 2 are men, and agents 3 and 4 are women.

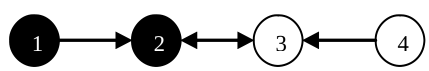

Figure 1. Battle of the Sexes game. Information Structure 1.

For each agent $q = 1, 2, 3, 4$ we can define

$$Nb(1) = \{1\}, \; A(1) = \{2\}, \; St(1) = \{2, 3, 4\};$$

$$Nb(2) = \{1, 2, 3\}, \; A(2) = \{3\}, \; St(2) = \{4\};$$

$$Nb(3) = \{2, 3, 4\}, \; A(3) = \{2\}, \; St(3) = \{1\};$$

$$Nb(4) = \{4\}, \; A(4) = \{3\}, \; St(4) = \{1, 2, 3\}.$$

In the next subsection, we describe the decision rules agents are assumed to follow.

2.3. Decision Rules. It is important to note that each agent has her own preferences (belongs to a particular population) as well as her own amount of information (from a set of neighbors). Any two agents can be paired in each round and each agent chooses her strategy as follows. Fix integers s and m, where $1 \leq s \leq m$. At time $t+1$, each agent q inspects a sample $(x^{t_1}, ..., x^{t_s})$ of size s taken without replacement from her neighbors' history of size m of plays up to time t, where $t_1, ..., t_s \in \{t-m+1, t-m+2, ..., t\}$. We assume that samples are drawn independently across agents and time.

If an agents is matched with one of her neighbors, she has information about her opponent past plays and plays the best reply against the opponent's strategy distribution in the sample. This approach is intuitive: agents are boundedly rational and expect the play of the game to be "almost" stationary. See Kandori, Mailath, and Rob (1993) and Young (1993, 1998) for discussions.

However, the situation is different if an agent is matched with a stranger. Since no information is available about the opponent, the agent has to select a strategy based on the available information about her neighbors. We assume that the decision rule the agent uses in order to select a strategy satisfies the following requirement: it can either select a strategy present in the sample or a best response against *some* subsample. Our motivation for this approach is as follows: if an agent does not know anything about her current opponent, she believes that one of her neighbors could have played her current opponent in the past. Therefore, it is plausible to believe that she considers her neighbor's information as valuable for her current play. Note that *any* imitation rule and *any* belief-based rule satisfy our requirement.

2.4. Markov Process. Let the sampling process begin in period $t = m+1$ from some arbitrary initial sequence of m plays, h^m. We define a finite Markov chain (call it $B^0 \equiv B^{\mathcal{V},\mathcal{T},m,s,0}$) on the state space $\left[(X)^{2n}\right]^m = H$ (of sequences of length m drawn from the strategy space X), with the information structure $\langle \mathcal{V}, \mathcal{T} \rangle$ and an arbitrary initial state h^m. The process B^0 moves from the current state h to a successor state h' in each period, according to the following transition rule. For each $x_i \in X$, let $p_i(x_i \mid h)$ be the conditional probability that agent i chooses x_i, given that the current state is h. We assume that $p_i(x_i \mid h)$ is independent from t.

The perturbed version of the above process can be described as follows. In each period, there is a small probability $\varepsilon > 0$ that any agent experiments by choosing a random strategy from X instead of applying the described rule. The event that one agent experiments is assumed to be independent from the event that another agent experiments. The resulting perturbed process is denoted by $B^\varepsilon \equiv B^{\mathcal{V},\mathcal{T},m,s,\varepsilon}$. As we will see below, the resulting process B^ε is *ergodic*, making the initial state irrelevant in the long run.

3. Short Run

In order to characterize the short-run outcomes of the model, we first need to adopt some terminology.

Definition 1. *The information structure* $\langle \mathcal{V}, \mathcal{T} \rangle$ *is* **connected** *if for any two agents* $q, g \in \mathcal{V}$ *there exists a sequence of agents* $f_1, ..., f_k \in \mathcal{V}$ *such that* $\{f_l - f_{l+1}\} \in \mathcal{T}$, $l = 1, ..., k-1$, *where link* $-$ *is either link* $\rightarrow$, *or link* $\leftarrow$, *or both; and* $q = f_1$ *and* $g = f_k$.

We assume that the information structure is connected for the remainder of the paper. Next, we define some special agents, called *fixed agents,* who have the ability to influence other agents. Formally,

Definition 2. q^* *is a* **fixed** *agent, if*
(*i*) $\{q^*\} = Nb(q^*)$, *or*
(*ii*) *for any* $g \in Nb(q^*)$, *there exists a sequence of agents* $g_1, ..., g_k$ *such that* $\{g_l \rightarrow g_{l+1}\} \in \mathcal{T}$, $l = 1, ..., k-1$, *and* $g_1 = q^*$ *and* $g_k = g$.

Denote by $F \subseteq \mathcal{V}$ the **set of fixed** agents. Fixed agents propagate information within the information structure. For example, agents 1 and 4 are fixed agents and agents 2 and 3 are not in Figure 1. The following Lemma establishes the existence of fixed agents.

Lemma 1. *Each information structure has a fixed agent.*

Proof. Since each information structure is finite, the claim follows immediately. **End of proof.**

Some information structures can have several fixed agents who might share information.

Definition 3. *Two fixed agents* $q^* \in F$ *and* $g^* \in F$ *are* **connected**, *if there exists a sequence of agents* $q_1, ..., q_k$ *such that* $\{q_l \rightarrow q_{l+1}\} \in \mathcal{T}$, $l = 1, ..., k-1$, *and* $q_1 = q^*$ *and* $q_k = g^*$.

It is obvious that each agent in the sequence of agents $q_1, ..., q_k$ is a fixed agent too. The following example illustrates the definition.

Example. Suppose that the information structure in the Battle of the Sexes game is as in Figure 2. Agents 2 and 3 are two connected fixed agents.

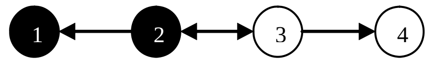

Figure 2. Battle of the Sexes game. Information Structure 2.

It is important to note that there may exist many distinct groups of connected fixed agents within the information structure. There are finitely many, $1 \leq L < \infty$, disjoint groups of connected fixed agents. We denote such groups of connected fixed agents by $F_1, ..., F_L$, where $F_i \cap F_j = \emptyset$, for any $i \neq j$, and $F_1 \cup ... \cup F_L = F$.

Given that the information structures we consider can be very different from one another, we cannot give a detailed description of all absorbing states and sets in general, but we are able to identify larger sets each of which contains an absorbing state or set. In the next section, we will show that this is nonetheless enough to identify the stochastically stable outcomes.

3.1. General Information Structures. Whereas the previous definitions were focused on the information structure, the following definition describes the important states of the Markov process B^0.

Definition 4. *A* **partial convention,** $h_{y_1,...,y_L}$, *is a set of states where all connected fixed agents in group F_j played strategy y_j for the last m periods in each state of the set $h_{y_1,...,y_L}$.*

Note that agents from different groups of connected fixed agents could play different strategies in a partial convention, thus leading the non-fixed agents to play different strategies. The following example illustrates how the strategies of non-fixed agents can vary in partial conventions.

Example. Consider the Battle of the Sexes game with the information structure from Figure 1. We assume that $m = 2$, $s = 1$. Suppose that agents 1 and 2 are from the **m**en's population, and agents 3 and 4 are from the **w**omen's population. Agents are matched at random to play the 2×2 coordination games described in Section 2.1.

Note that agents 1 and 4 are fixed agents, but they are not connected. In a partial convention, each fixed agent played the same strategy two times in the past. If both fixed agents coordinated on strategy S, then the following set of states is a partial convention:

$$h_{S,S} = \{(S, S), (w, x), (y, z), (S, S)\},$$

where $w, x, y, z \in \{S, O\}$ and the first bracket represents the strategy choices of agent 1 in the last two periods, the second bracket shows the strategy choices of agent 2 in the last two periods, and so on. If both fixed agents coordinate on strategy O, then the following set of states is a partial convention:

$$h_{O,O} = \{(O,O), (w,x), (y,z), (O,O)\},$$

where $w, x, y, z \in \{S, O\}$.

However, fixed agents who are not connected do not need to coordinate in a partial convention. For example, agent 1 could play strategy S in the last two periods, and agent 4 could play strategy O in the last two periods. The following set of states is therefore also a partial convention:

$$h_{S,O} = \{(S,S), (w,x), (y,z), (O,O)\},$$

where $w, x, y, z \in \{S, O\}$. Analogously, if agent 1 played strategy O in the last two periods, and agent 4 played strategy S in the last two periods, the following set of states is also a partial convention:

$$h_{O,S} = \{(O,O), (w,x), (y,z), (S,S)\},$$

where $w, x, y, z \in \{S, O\}$.

Note that non-fixed agents 2 and 3 can switch from strategy S to strategy O (and vice versa) in any partial convention. We will see below that partial conventions contain either exactly one absorbing state or set of the unperturbed process B^0. In our example, the partial convention $h_{S,S}$ ($h_{O,O}$) contains one absorbing state where all agents play strategy S (O). And the partial convention $h_{S,O}$ ($h_{O,S}$) contains one absorbing set where non-fixed agents 2 and 3 can play either S or O. In this latter case, the partial convention and the absorbing set do coincide.

We are now able to complete our study of the absorbing sets and states of the process - short-run outcomes. An **absorbing set** of the unperturbed process B^0 is a minimal set of states such that there is zero probability for the process B^0 of moving from any state in the set to any state outside, and there is a positive probability for the process B^0 of moving from any state in the set to any other state in the set. A singleton absorbing set is called an **absorbing state**.

Definition 5. *A* **uniform convention,** $h_x = h_{x,\dots,x}$, *is a state where* **all agents** *play the same strategy x.*

Note that there exist K uniform conventions.

Proposition 1. *Each uniform convention is an absorbing state.*

Proof. It is evident. **End of proof.**

We now state the main result of this subsection.

Proposition 2. *Each partial convention contains a unique absorbing state or set.*

Proof. Note that each agent in a group of connected fixed agents has to coordinate on the same strategy for the last m periods in any absorbing set. Any partial convention has this property. **End of proof.**

We can narrow down the short-run prediction for the following particular case.

Proposition 3. *If all fixed agents are coordinated on the same strategy x in a partial convention, then this partial convention contains the absorbing state, uniform convention, h_x.*

Proof. It is evident. **End of proof.**

We now take a closer look at several information structures that have been of a particular interest in the network literature.

3.2. Common Information Structures. If the information structure is such that all agents are fixed and connected, or there is just one fixed agent, then the short-run prediction is a state, uniform convention. There are several common information structures where all agents are fixed and connected. For example, (1) if all directed edges are double-sided (the information goes in both directions), (2) if the information structure is a directed wheel, or (3) if the information structure is complete.

Definition 6. *The connected information structure $\langle \mathcal{V}, \mathcal{T} \rangle$ is a* **directed wheel***, if any agent $q \in \mathcal{V}$ has exactly one agent who can access agent q's past plays, $\|A(q)\| = 1$.*

Definition 7. *The information structure $\langle \mathcal{V}, \mathcal{T} \rangle$ is* **complete***, if for any two agents $q, g \in \mathcal{V}$*

$$\{q \to g\}, \{g \to q\} \in \mathcal{T}.$$

Similarly, there are several information structures with just one fixed agent. For example, (4) a directed star, or (5) a directed chain.

Definition 8. *The connected information structure $\langle \mathcal{V}, \mathcal{T} \rangle$ is a* **directed star***, if there exists an agent $q \in \mathcal{V}$ such that every other agent $g \neq q$ has exactly two neighbors, $Nb(g) = \{g, q\}$, and $Nb(q) = \{q\}$.*

Definition 9. *The connected information structure* $\langle \mathcal{V}, \mathcal{T} \rangle$ *is a* **directed chain**, *if there exists an agent* $q \in \mathcal{V}$ *such that every other agent* $g \neq q$ *has exactly two neighbors,* $\|Nb(g)\| = 2$, *and* $Nb(q) = \{q\}$.

The following example illustrates the last definition.

Example. Suppose that the information structure is as in Figure 3. Agent 2 is the fixed agent.

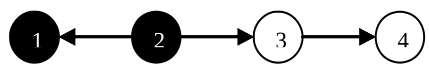

Figure 3. Battle of the Sexes game. Information Structure 3.

The following proposition gives the short-run outcomes for some common information structures.

Proposition 4. *Suppose that at least one of the following conditions holds*

- *all edges are double-sided,*
- *the information structure is a directed wheel,*
- *the information structure is complete,*
- *the information structure is a directed star,*
- *the information structure is a directed chain.*

  Then the Markov process B^0 *converges with probability one to a uniform convention.*

Proof. Note that when the first, second, or third condition holds, all agents are connected fixed agents. If the forth, or fifth condition holds, there is a unique fixed agent. The statement follows from Proposition 3. **End of proof.**

4. Long Run

Many short-run outcomes are candidates for the long-run prediction when agents can make mistakes. Following the literature, we describe properties of the unique stationary distribution of the perturbed Markov process B^{ε}. The main result of this section is that only K states can be the long-run outcomes. These are the uniform conventions.

We use the following definitions.

Definition 10. Stochastic Stability: *A state $h \in H$ is stochastically stable relative to the process B^{ε} if $\lim_{\varepsilon \to 0} \mu_h^{\varepsilon} > 0$ where μ^{ε} is the unique stationary distribution of the process B^{ε}.*

Definition 11. Resistance: *For any two states h, h' the resistance $r(h, h')$ is the total number of mistakes involved in the transition from h to h', if h' is a successor of h; otherwise $r(h, h') = \infty$.*

A **directed tree** is a directed graph (V, E). The vertices, V, represent all possible absorbing sets and the edges, E, represent the transition from one absorbing set to the other. Each edge is assigned a weight which is equal to the corresponding resistance. The resistance of such a directed tree is equal to the sum of the resistances of its edges.

The **stochastic potential** ρ of an absorbing set is the minimum resistance of the tree rooted at this absorbing set. We will use the following well-known result.

Theorem 1. *The only stochastically stable sets of the perturbed Markov process B^{ε} are the absorbing sets with minimum stochastic potential.*

We can now describe the long-run outcomes.

Theorem 2. *For any absorbing set $h \in H$ different from a uniform convention, there exists a sample size s^* and a uniform convention $h_x \in H$, such that for any $s \geq s^* > m/2$ $\rho(h) > \rho(h_x)$.*

Proof. First, note that all connected fixed agents have to coordinate on the same strategy in any absorbing set. Second, if all fixed agents coordinate on the same strategy, the absorbing set is a uniform convention from Proposition 3. Third, it takes at least two mistakes to leave any uniform convention. It is enough to show now that it takes just one mistake to leave an absorbing set different from a uniform convention. It becomes obvious once we see how to leave such a set where all but one (connected) fixed agent(s) coordinate on one strategy.

Consider an absorbing set $h \in H$ different from a uniform convention. It must be the case that at least two strategies are played by the fixed agents. Find a group of connected agents, F_i, who are playing the most popular (among fixed agent) strategy, x. Consider another set of fixed agents, F_j, who are playing another strategy, y. There is a positive probability that all connected fixed agents in this set, F_j, are matched with the fixed agents who play strategy x. Suppose that this matching takes place for $s-1$ periods. As a result, all connected fixed agents in F_j have $s-1$ miscoordinated plays. Suppose that one of the fixed agents in F_j, agent q, makes a mistake and coordinates on the strategy x. There is a positive probability that she and every member from the set $A(q)$ sample her last s plays for the next s periods. Suppose that all connected fixed agents from $A(q)$ and agent q are matched with fixed agents they consider strangers. This will lead to another absorbing set where more fixed agents play strategy x. Continuing in this way, we can see that it takes one mistake to move from one absorbing set to the next until we reach the uniform convention h_x. Hence, $\rho(h) > \rho(h_x)$. **End of proof.**

In a partial convention, connected fixed agents from the same group coordinate on the same strategy. So, if not all fixed agents follow the same strategy and two fixed agents from different groups who play different strategies are matched, only one mistake is needed to have them both coordinating on the same strategy. Therefore, a partial convention is never as stable as a uniform convention, since in order to leave a uniform convention, two mistakes are needed.

From Theorem 2 it follows that a uniform coordination must be reached in the long-run. But a sharper prediction can be obtained, if we assume that agents imitate the most successful play in their samples when they are matched with strangers. Call this Markov process BI^{ε}.

Theorem 3. *Suppose that all fixed agents belong to the same population i and there exist at least two distinct groups of fixed agents. Then there exists a sample size s^*, such that for any $s \geq s^*$, the perturbed process BI^{ε} converges with probability one to a uniform convention $h_{i...i}$.*

Proof. We have to select the long-run outcome among different uniform conventions from Theorem 2. Since all fixed agents are from the same population i, they obtain the highest payoff if they coordinate on the strategy i. The imitation rule drives the selection in favor of the uniform convention $h_{i...i}$. Similar results were obtained in Robson and Vega-Redondo (1996) and Josephson and Matros (2004).

Consider a uniform convention $h_{j...j}$, $j \neq i$. It takes just two mistakes in order to leave a uniform convention $h_{j...j}$: two matched fixed agents from different groups switch from strategy j to strategy i. It is obvious that it requires more than two mistakes to leave the uniform convention $h_{i\ldots i}$. **End of proof.**

The intuition for this result is as follows. Theorem 2 insures that only some uniform conventions can be the long-run outcome. Therefore, if all fixed agents share the same preferences (from the same population i), but coordinate on strategy $j \neq i$ in the uniform convention $h_{j\ldots j}$, it is easy to see that only two mistakes are needed to leave the uniform convention $h_{j\ldots j}$. More precisely, two matched fixed agents from distinct groups play strategy i by mistake and obtain the highest possible payoff. Since agents use the imitation rule when matched with strangers, any other fixed agent who can access this information will also play strategy i. Hence, the uniform convention $h_{i\ldots i}$ is the most stochastically stable.

Example. Consider the Battle of the Sexes game from the previous section where the information structure is given by Figure 1. There are only two possible long-run outcomes: all four players coordinate on a unique strategy, i.e. either all of them go to a soccer match or all of them go to an opera. We have a unique long-run prediction only if both fixed agents are from the same population.

Some information structures have a unique fixed agent. This fixed agent's population determines the long-run outcomes: a uniform convention where all agents play the fixed agent's preferred strategy.

Corollary 1. *Suppose that the information structure $\langle \mathcal{V}, \mathcal{T} \rangle$ is either a directed star, or a directed chain and the unique fixed agent is from population i. Then, there exists a sample size s^*, such that for any $s \geq s^*$, the perturbed process BI^{ε} converges with probability one to a uniform convention h_i.*

Proof. Note that any directed star or/and any directed chain have just one fixed agent. The corollary now follows from Theorem 3. **End of proof.**

5. Conclusion

In this paper we consider a population of heterogeneous agents who are randomly paired each period to play some coordination games. We show that the short-run predictions (mistake-free environment) depend on the information structure: connected fixed agents have to coordinate on the same strategy. Whereas in the long run, all agents coordinate on the same strategy. Moreover, the long-run prediction is unique when all fixed agents belong to the same population.

It is interesting to see that despite the divergence of preferences, agents can agree uniformly on one particular strategy in the long run. This coordination is obtained through the "fixed agents" highlighted by Glaeser, Sacerdote, and Scheinkman (1996) in a study on criminal behavior.

The most important result of the paper is the fact that sharing common preferences with a majority of others does not insure a favorable outcome. What matters most is not the number of agents in a population, but rather their "quality." By spreading widely their information, fixed agents insure that their preferences are observed, directly or indirectly, by everyone and therefore influence the plays of others. This is the reason why an agent's location (her population) is less important than her information. This also explains why a minority can sometimes impose its preferences.

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