Sad-Loser Lottery

Alexander Matros*
University of Pittsburgh

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Abstract. We consider lotteries with reimbursements. It turns out that without loss of generality it is enough analyze lotteries where the winner gets her expenses reimbursed. We find that such a lottery (Sad-Loser) has multiple pure-strategy equilibria. We describe all equilibria and discuss their properties. In particular, we find (1) a sufficient condition for the net total spending to be higher in the Sad-Loser lottery than in the standard lottery, (2) that the Exclusion Principle holds.

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1. Introduction

Lotteries are an important part of our life. They are a big business. Charities raise money, government gets a big share of its revenue through lotteries. For example, California total lottery sales in the fiscal year 2006/2007 reached astronomical $3,318 millions.[1] There is a big literature on lotteries and its applications. Discussions about State Lotteries in America can be found in Clotfelter and Cook (1989, 1990), Cook and Clotfelter (1993). Morgan (2000) and Duncan (2002) provide theoretical contributions. Morgan and Sefton (2000) and Lange, List, and Price (2007) report experimental results. Craig, Lange, List, Price, and Rupp (2006) present field evidence. However, the lottery design (the winner gets the main prize and each player has to pay) is always fixed in the current literature. In this paper, we consider lotteries with reimbursements where different players have different values for the main prize. Once we understand how lotteries with reimbursements work, we can think about applications.

First, we demonstrate (almost without loss of generality[2]) that for our analysis it is enough to consider winner-reimbursed lotteries (which we call Sad-Loser lotteries[3]).

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[1] Based on "California Lottery Report to the Public, Fiscal Year 2006/2007". See http://www.calottery.com/NR/rdonlyres/7F476E30-187B-429F-B566-5027C6444C9B/0/LotteryAR2007English.pdf

[2] See footnote 11.

[3] The name comes from Riley and Samuelson's (1981) example of *Sad-Loser Auction*.

In fact, this type of lotteries is easy to implement: the lottery designer has to announce the following rules

1. Each player is eligible to submit as his lottery bid any positive real number. Each bid corresponds to the same number of lottery tickets.

2. The lottery winner receives the prize and *retains his bid*.

3. All other players lose their bids.

Second, we analyze the Sad-Loser lottery where different players have different values for the main prize in detail. It turns out that there are *multiple* equilibria in pure strategies.[4] We found *all* such equilibria. The equilibria can be of two types: i-type and internal type. i-type equilibria are such that only player i spends a positive amount and all other players spend nothing. The net total spending is zero in *any* i-type equilibrium.

Internal equilibria are such that at least two players are active (spend positive amounts).[5] We demonstrate that the players' expected payoffs are zero in any internal equilibrium. Moreover, we discover a sufficient condition for the expected designer profit (the net total spending) in the Sad-Loser lottery to be higher than the total spending in the standard lottery. This condition is simple and natural: if *all* players are active in the standard lottery, then the expected profit in *any internal equilibrium* in the Sad-Loser lottery is higher than the total spending (designer profit) in the standard lottery. This result can be important for different applications of the Sad-Loser lottery.

In order to understand better how the Sad-Loser lottery works it is important to compare it with the standard lottery where players have different prize values. Hillman and Riley (1989) show that the standard lottery has a set of active high-value players in the unique equilibrium.[6] Stein (2002) describes the equilibrium spending of the active players. As it is usual in the contest literature, higher-value player spends more in the equilibrium in the standard lottery. It turns out that the equilibrium behavior is drastically different in the Sad-Loser lottery. We demonstrate a counter intuitive result that a higher-value (stronger) player *always* spends less than a lower-value (weaker) player and therefore *always* has a lower chance to win the Sad-Loser

[4]It is a standard result in the contest literature that a contest has a unique equilibrium in pure strategies, see for example Szidarovszky and Okuguchi (1997).

[5]Some Sad-Loser lotteries can have an internal equilibrium where *all* players are active.

[6]Fang (2002) and Matros (2006) prove the uniqueness of the equilibrium.

lottery in *any* internal equilibrium.[7]

Another striking observation is that the *Exclusion Principle* holds for the (net) total spending in the Sad-Loser lottery. This result is surprising because the opposite claim is true for the standard lottery, see Fang (2002).

1.1. Contest literature. Contest literature has greatly expanded since Tullock (1980) presented his simple yet powerful rent-seeking model. The general form of the Tullock's contest is

$$\max_{x_i} \frac{x_i^r}{\sum_{j=1}^n x_j^r} V_i - x_i, \; i = 1, ..., n, \tag{1}$$

where $n \geq 2$ players exert effort in order to win one prize, $r > 0$. Player i exerts effort x_i and has a chance $\frac{x_i^r}{\sum_{j=1}^n x_j^r}$ to win prize V_i, if $x_i > 0$. Tullock's initial problem was presented for the same prize values: $V_1 = ... = V_n$. Hillman and Riley (1989) analyze the standard lottery, $r = 1$, with different prize values $V_1 \geq ... \geq V_n > 0$. Nti (1999) describes the unique Nash equilibrium in the two-player case for any $0 < r < 2$ and different prize values.

There are two main questions in the contest literature.[8] The first one is *How to reduce rent-seeking activities?* This question was originally raised in Tullock (1967, 1980), Kruger (1974), and Posner (1975). The second question is *How to maximize the total effort in contests?* Recently, this question attracted a lot of attention. See, for example, Gradstein and Konrad (1999); Moldovanu and Sela (2001) among others. It turns out that the Sad-Loser lottery can surprisingly help to answer both questions. It is possible because of the multiplicity of equilibria. As we mentioned above, the rent-seeking activities are zero (minimized) in *any* i-type equilibrium. Note that i-type equilibrium is easy to implement: the designer has to allow player i to move first. At the same time, since we provide the ranking of different internal equilibria in the Sad-Loser lottery, we are are able to find the equilibrium where the net total spending is maximized: only top two players have to be active in this equilibrium.

Recently, a new area of research has sprung investigating the situation where the contest winner is reimbursed for her expenses. This type of contest has applications in politics, see Matros and Armanios (2007), R&D and industrial organization, see Kaplan, Luski, Sela, and Wettstein (2002).

[7]In the recent paper Cohen and Sela (2005) consider the Sad-Loser lottery. They show that in the two-player case there exists a unique internal equilibrium where the weak contestant wins with higher probability than the stronger one. We demonstrate that this property holds in *all* internal equilibria for any number of players $n \geq 2$.

[8]See Nitzan (1994) and Konrad (2007) for the overview.

Matros and Armanios (2007) analyze contests with reimbursements, $0 < r \leq 1$, but all prize values are the same $V_1 = ... = V_n$ (this approach is similar to Tullock's contest's approach). In this paper, we consider n-player Sad-Loser lotteries, $r = 1$, but analyze different prize values (this approach is similar to Hillman and Riley's contest's approach). Cohen and Sela (2005) consider 2-player Sad-Loser lotteries, describe the unique internal equilibrium, and discuss its properties.

The rest of the paper is organized as follows. Sections 2 and 3 present lotteries with reimbursements and the Sad-Loser lottery. Properties of internal equilibria are studied in Section 4. Section 5 makes a comparison between the Sad-Loser lottery and the standard lottery. Concluding remarks are given in Section 6.

2. The Model

Consider a lottery with reimbursements among $n \geq 2$ risk-neutral players. Players buy simultaneously lottery tickets in order to win one prize. Player i's valuation for the prize is V_i. Suppose that

$$V_1 \geq V_2 \geq ... \geq V_n > 0. \tag{2}$$

The players' valuations are commonly known among the players and we assume that the winner/loser reimbursements are additively separable in the winner and loser spending. Formally, player i buys $z_i \geq 0$ tickets in order to maximize the following function

$$\max_{z_i \geq 0} \frac{z_i}{\sum_{j=1}^{n} z_j} \left(V_i + \pi^W (z_i)\right) + \left(1 - \frac{z_i}{\sum_{j=1}^{n} z_j}\right) \pi^L (z_i) - z_i, \tag{3}$$

where the first term in (3) is the probability to win the lottery, $\frac{z_i}{\sum_{j=1}^{n} z_j} \geq 0$, times the lottery prize for player i, V_i, and the winner's reimbursement, $\pi^W (z_i)$; the second term is the probability to lose the lottery, $\left(1 - \frac{z_i}{\sum_{j=1}^{n} z_j}\right) \geq 0$, times the loser's reimbursement, $\pi^L (z_i)$; and the last term is the cost.[9] In order to find the closed-form solution we look at linear reimbursement functions.[10] We assume that the individual reimbursement depends only on the individual effort

$$\pi^W (z) = \alpha z, \ \pi^L (z) = \beta z, \tag{4}$$

where $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$. The maximization problem (3) is equivalent to the following maximization problem

$$\max_{x_i \geq 0} [V_i + \gamma x_i] \frac{x_i}{\sum_{j=1}^{n} x_j} - x_i, \tag{5}$$

[9] We assume that if $z_1 = ... = z_n = 0$, then nobody wins the prize.

[10] Baye, Kovenock, and De Vries (2005) also examine linear reimbursements.

where

$$x_i = (1-\beta)\, z_i$$

and

$$\gamma = \frac{\alpha-\beta}{1-\beta}.$$

It means that dealing with parameter β introduces an unnecessary degree of freedom to our analysis. What really matters is the extent of reimbursement to a winner given the net spending of a loser; this is entirely captured by the parameter γ and so it is clear that it is (almost) without loss of generality to set $\beta = 0$.[11] We are interested to analyze how winner's reimbursement affects equilibrium behavior in lotteries. In order to emphasize that, we will look at the sad-loser lottery, $\alpha = 1$ or $\gamma = 1$, in the rest of the paper.

3. Sad-Loser Lottery

Consider a Sad-Loser lottery (5) with $\gamma = 1$ among $n \geq 2$ risk-neutral players. The first order condition is

$$[V_i + x_i] \frac{\sum_{j\neq i} x_j}{\left(\sum_{j=1}^n x_j\right)^2} + \frac{x_i}{\sum_{j=1}^n x_j} - 1 = 0. \tag{6}$$

The second order condition is

$$2\frac{\sum_{j\neq i} x_j}{\left(\sum_{j=1}^n x_j\right)^2}\left[1 - \frac{V_i + x_i}{\sum_{j=1}^n x_j}\right] \leq 0,$$

or

$$x_i = 0, \text{ if } \sum_{j\neq i} x_j \geq V_i, \tag{7}$$

$$x_i > 0, \text{ if } 0 \leq \sum_{j\neq i} x_j \leq V_i. \tag{8}$$

We will call player i active if $x_i > 0$. The entry condition (8) and the non-entry condition (7) must hold for active and non-active players respectively in the equilibrium. We will be looking for equilibria in pure strategies. Since each active player has to

[11] It is almost (but not completely) without loss of generality to set $\beta = 0$. The exception is $\beta = 1$ (winner-pay lottery), $\gamma = \frac{\alpha-\beta}{1-\beta}$ is ill-defined. We leave this case outside of the paper.

obtain a non-negative payoff in any equilibrium, the following condition must hold for such players

$$[V_i + x_i] \frac{x_i}{\sum_{j=1}^{n} x_j} - x_i \geq 0,$$

or

$$\sum_{j \neq i} x_j \leq V_i. \tag{9}$$

Note that the second order condition (8) and the non-negative payoff condition (9) coincide for the active players.

Denote the total spending by

$$s(n) = \sum_{j=1}^{n} x_j.$$

Then, the first order condition (6) becomes

$$[V_i + x_i] \frac{s(n) - x_i}{s^2(n)} + \frac{x_i}{s(n)} = 1,$$

or

$$x_i^2 + [V_i - 2s(n)] x_i + s(n)(s(n) - V_i) = 0.$$

Therefore,

$$x_i = s(n) - \frac{V_i \pm V_i}{2}. \tag{10}$$

There are two solutions of the equation (10). They are

$$x_{i1} = s(n) \tag{11}$$

and

$$x_{i2} = s(n) - V_i. \tag{12}$$

First, consider solution (11). This solution and the entry condition (8) describe all i-type equilibria.

Proposition 1. $(0, ..., 0, x_i, 0, ..., 0)$ *is i-equilibrium, if*

$$x_i \geq \begin{cases} V_1, & \text{if } i > 1, \\ V_2, & \text{if } i = 1. \end{cases} \tag{13}$$

Proof. Consider $(0,...,0,x_i,0,...,0)$, where x_i satisfies (13). It is straightforward to check that condition (8) holds for the active player. All other players are not active because the non-entry condition (7) holds:

$$\sum_{j\neq k} x_j = x_i > V_k \text{ for any } k \neq i.$$

The first order condition (6) holds by our choice of x_i. Therefore, $(0,...,0,x_i,0,...,0)$ is indeed an equilibrium. ■

i-type equilibria in our model are similar to i-type equilibria in the second-price seal-bid auctions where just one bidder - bidder i - places a (very) high bid and all other bidders bid zero. It is highly unlikely that i-type equilibria can actually arise in applications where all players have to make their decisions simultaneously. However, the lottery designer can take advantage of these equilibria if his goal is to reduce the net total spending or rent-seeking activities. The designer should allow one (his favorite) player i to buy lottery tickets first, then allow all other players to buy. If the selected player i buys "enough" tickets (consistent with an i-type equilibrium), then all other players will have a negative expected payoff for any positive amount of expenses. Therefore, all other players buy nothing and player i is reimbursed. Hence, the total net spending or rent dissipation in this equilibrium is zero.

Second, consider solution (12). There must be at least two active players in this type of equilibria. It follows from (12) that

$$\sum_{j\neq i} x_j = V_i, \tag{14}$$

for any active player i. Summing (14) over $k \geq 2$ active players

$$\sum_{i=i_1}^{i_k} \sum_{j\neq i} x_j = \sum_{i=i_1}^{i_k} V_i,$$

or

$$s(k) = \frac{1}{(k-1)} \sum_{j=i_1}^{i_k} V_j. \tag{15}$$

From (14) and (15), we get

$$x_i = s(k) - V_i = \frac{1}{(k-1)} \sum_{i=i_1}^{i_k} V_i - V_i. \tag{16}$$

Expression (16) together with the entry condition (8) describe all internal equilibria.

Proposition 2. *In an internal equilibrium* $(x_1, ..., x_n)$ *with* k *active players, each active player* i *buys lottery tickets according to formula* (16).

Propositions 1 and 2 characterize all equilibria in the Sad-Loser lottery.

3.1. Players' expected payoffs. We calculate players' expected equilibrium payoffs in this subsection. Note that the expected equilibrium payoff of any non-active player is always zero. We start from i-type equilibria.

Proposition 3. *The expected payoff of the active player* i *in the equilibrium of* i*-type is* V_i.

Proof. It follows from (13) and (5). ■

The only active player gets the prize and her expenses are reimbursed. As the result, the net total spending equals to zero. We consider internal equilibria now.

Proposition 4. *The expected payoff of each player in any internal equilibrium is zero.*

Proof. Consider any internal equilibrium with k active players. Suppose that player i is active in this equilibrium. Then condition (16) must hold for player i. Therefore, her expected payoff is

$$[V_i + x_i] \frac{x_i}{s(k)} - x_i = s(k) \frac{x_i}{s(k)} - x_i = 0.$$

■

Proposition 4 gives an intuition for why the Sad-Loser lottery can generate higher expected revenue than the usual lottery: the players expect to receive nothing, hence, the designer should obtain all available expected profit. This result is in contrast with the standard observation in the contest literature: the expected individual payoffs are usually positive. See for example, Tullock (1980); Nitzan (1994); Congleton, Hillman, and Konrad (2007); Konrad (2007).

We consider the internal equilibria in the next section.

4. Internal Equilibria

First, we find the number of internal equilibria. Second, the total equilibrium spending is described. Then, we show that a higher-value (stronger) player *always* exerts less effort than a lower-value (weaker) player and therefore has a lower chance to win the Sad-Loser lottery in *any* internal equilibrium. Finally, the expected total spending is analyzed. In this section, we also show that the *Exclusion Principle* holds for the (net) total spending in the Sad-Loser lottery.

4.1. Number of Internal Equilibria. In turns out that if players' prize values are relatively close, there can be as many as $2^n - (n+1)$ internal equilibria in pure strategies. The following proposition states it formally.

Proposition 5. *Suppose that*

$$V_1 < \min_{2 \le k < n} s(k). \tag{17}$$

Then, there are $2^n - (n+1)$ internal equilibria in pure strategies. In particular, there are $\frac{n!}{(n-k)!k!}$ internal equilibria with $2 \le k \le n$ active players.

Proof. Note that there are exactly $\frac{n!}{(n-k)!k!}$ possibilities to have k active players in the Sad-Loser lottery. Active player equilibrium spending is uniquely determined by expression (16). Therefore, there are at most $\sum_{k=2}^{n} \frac{n!}{(n-k)!k!} = 2^n - (n+1)$ internal equilibria. The entry condition (8) and the non-entry condition (7) must hold for k active and for all other players respectively in an internal equilibrium with k active players. Consider condition (7): player i is non-active ($x_i = 0$), if

$$\sum_{j \neq i} x_j = s(k) \ge V_i.$$

The highest number of internal equilibria is reached if

$$\min_{2 \le k < n} s(k) \ge \max_i V_i = V_1. \tag{18}$$

Condition (17) ensures that there are $2^n - (n+1)$ internal equilibria in pure strategies. ■

If player 1's lottery prize value is much higher than lottery prize values of other players, there are only $(n-1)$ internal equilibria with two active players. Player 1 is active in all of them.

Proposition 6. *Suppose that*

$$V_1 > \max_{2 \le k < n} s(k). \tag{19}$$

Then, there are $(n-1)$ internal equilibria in pure strategies:

$$\left(\underbrace{V_l, 0, ..., 0,}_{l-1} V_1, 0, ..., 0 \right) \text{ for } l = 2, ..., n. \tag{20}$$

In each internal equilibrium, there are exactly 2 active players.

Proof. Suppose that there exists another internal equilibrium $(x_1, ..., x_n)$ different from (20). There are two cases.

Case 1. Suppose that player 1 is not active in this internal equilibrium. We show that the non-entry condition (7) must be violated for player 1 in this case.

Note that if $x_1 = 0$, then, from the non-entry condition (7), the total spending in this internal equilibrium must be not higher than $\max_{2 \leq k < n} s(k)$. Together with (19), it gives

$$V_1 \leq \max_{2 \leq k < n} s(k) < V_1.$$

Therefore, the non-entry condition (7) is violated for player 1.

Case 2. Suppose that $x_1 > 0$. Then, there are at least $k \geq 3$ active players in this equilibrium. From (16), player 1 should spend

$$0 < x_1 = s(k) - V_1 \leq \max_{2 \leq k < n} s(k) - V_1 < 0.$$

Therefore, there are exactly $(n-1)$ internal equilibria. ■

Propositions 5 and 6 describe the highest and the lowest number of the internal equilibria.

Corollary 1. *There are at least* $(n-1)$ *and at most* $2^n - (n+1)$ *internal equilibria in pure strategies.*

There are two types of equilibria in the Sad-Loser lottery. The first type, i-type equilibria, with (very) high spending by just one player and zero spending from all other players. Since the winner gets reimbursed, player i spends so much that it pushes all other players to stay away from the Sad-Loser lottery, because they have negative expected payoffs for any spending level.

The second type, internal equilibria, where there is a set of active players. Note that all players can be active in an internal equilibrium. This class of equilibria is very intuitive if players have "close" values for the lottery prize.

Example 1. Suppose that $n = 3$ and $V_1 \geq V_2 \geq V_3 > 0$. Then, from Proposition 1, there are the following i-type equilibria in pure strategies:

- 1-type: $(x_1, 0, 0)$, where $x_1 \geq V_2$;
- 2-type: $(0, x_2, 0)$, where $x_2 \geq V_1$;
- 3-type: $(0, 0, x_3)$, where $x_3 \geq V_1$.

From Corollary 1, there are at least 2 internal equilibria in pure strategies:

- $(V_2, V_1, 0)$;
- $(V_3, 0, V_1)$.

If $V_1 \leq V_2 + V_3$, then from Proposition 5 there are 2 other internal equilibria. One with two active players:

- $(0, V_3, V_2)$

and another one with all three active players:

- $\left(\frac{1}{2}[V_2 + V_3 - V_1], \frac{1}{2}[V_1 + V_3 - V_2], \frac{1}{2}[V_1 + V_2 - V_3]\right)$.

4.2. Weaker players win more often. Now, we show how players' lottery prize valuations affect their spending in an internal equilibrium. Denote p_{i_l} to be the probability that player i_l wins the lottery.

Proposition 7. *In each internal equilibrium with k active players,*

$$x_{i_1} \leq ... \leq x_{i_k}$$

and

$$p_{i_1} \leq ... \leq p_{i_k}$$

if and only if

$$V_{i_1} \geq ... \geq V_{i_k}.$$

Proof. Consider any internal equilibrium with k active players. Suppose that players i_1 and i_2 are active in this equilibrium and

$$V_{i_1} \geq V_{i_2}.$$

It follows from condition (16) that

$$x_{i_1} = s(k) - V_{i_1} \leq s(k) - V_{i_2} = x_{i_2}.$$

Note that

$$p_{i_1} = \frac{x_{i_1}}{s(k)} \leq \frac{x_{i_2}}{s(k)} = p_{i_2}.$$

■

Proposition 7 leads to the following surprising conclusion: A stronger player *always* has a lower chance to win the Sad-Loser lottery than a weaker player in *any* internal equilibrium.

Corollary 2. *Suppose that $V_i > V_j$. Consider an internal equilibrium where both players i and j are active. Then $p_i < p_j$.*

This observation together with the (non-)entry conditions establish the following startling trade off: higher lottery prize value promises active participation in more internal equilibria, but *decreases winning chances*. This trade off is unique in the literature.[12] So far, a monotonic relationship was ascertained: higher value would lead to more frequent active equilibrium participation and more aggressive spending which, as the result, would lead to higher winning chances. See, for example, Hillman and Riley (1989); Nti (1999).

4.3. Total spending in internal equilibria. Consider an internal equilibrium $(x_1, ..., x_n)$ with $i_1, ..., i_k$ active players. Then the total spending, from formula (16), is

$$s(i_1, ..., i_k) = \frac{1}{(k-1)} \sum_{i=i_1}^{i_k} V_i.$$

Therefore, the highest equilibrium spending with k active players is achieved if the top k players are active

$$\max_{i_1,...,i_k} s(i_1, ..., i_k) = \frac{1}{(k-1)} \sum_{i=1}^{k} V_i. \tag{21}$$

Our next result shows that the *Exclusion Principal* holds for the total spending in the Sad-Loser lottery.[13] It is important to emphasize that the previous literature indicates, see Fang (2002); Matros (2006), that the Exclusion Principle does not hold in standard lotteries.

Proposition 8. *Consider two internal equilibria with $i_1, ..., i_k$ and $i_1, ..., i_k, i_{k+1}$ active players. Then,*

$$s(i_1, ..., i_k) \geq s(i_1, ..., i_k, i_{k+1}).$$

Proof. Since player i_{k+1} is non-active in the internal equilibrium with $i_1, ..., i_k$ active players, condition (7) gives the following inequality

$$V_{i_{k+1}} \leq s(i_1, ..., i_k) = \frac{1}{(k-1)} \sum_{j=1}^{k} V_{i_j}. \tag{22}$$

[12]Cohen and Sela (2005) notice this effect in the case of two players. They also point out (Proposition 2) that *"in the n player contest ... underdogs* **may** *win with the highest probability."* We prove that this effect holds in *any internal equilibrium.*

[13]For the *Exclusion Principle* see, for example, Baye, Kovenock, and De Vries (1993); Krishna (2002).

Note that

$$s\left(i_1,...,i_k\right) - s\left(i_1,...,i_k,i_{k+1}\right) =$$

$$\frac{1}{(k-1)}\sum_{j=1}^{k} V_{i_j} - \frac{1}{k}\sum_{j=1}^{k+1} V_{i_j} =$$

$$\frac{1}{k\left(k-1\right)}\left(\sum_{j=1}^{k} V_{i_j} - \left(k-1\right)V_{i_{k+1}}\right) \geq 0, \tag{23}$$

where the last inequality follows from (22). ■

Therefore, the highest total spending in an internal equilibrium is achieved if the top 2 players are active

$$\max_{k\geq 2} s\left(k\right) = \max_{i_1,i_2} s\left(i_1,i_2\right) = V_1 + V_2.$$

Hence, we prove the following result.

Proposition 9. *The total spending in any internal equilibrium is at most $V_1 + V_2$.*

The lowest equilibrium spending with k active players is reached if the bottom k players are active

$$\min_{i_1,...,i_k} s\left(k\right) = \frac{1}{(k-1)}\sum_{i=n-k+1}^{n} V_i. \tag{24}$$

Define the lowest equilibrium spending in any internal equilibrium as

$$\underline{s} = \min_{k\geq 2} s\left(k\right) =$$

$$\min\left\{\left(V_n + V_{n-1}\right), \frac{1}{2}\left(V_n + V_{n-1} + V_{n-2}\right), ..., \frac{1}{(n-1)}\sum_{i=1}^{n} V_i\right\}. \tag{25}$$

The following examples illustrate that the lowest equilibrium spending depends on the lottery prize values.

Example 2. Suppose that

$$V_1 = V_2 = ... = V_n = V > 0.$$

Then

$$\underline{s} = \min\left\{2V, \frac{3}{2}V, ..., \frac{n}{(n-1)}V\right\} = \frac{n}{n-1}V = s(n).$$

Example 3. Suppose that

$$V_n = 1,\ V_{n-1} = 2, ...,\ V_{n-i} = 2^i, ...,\ V_1 = 2^{n-1}.$$

Then

$$\underline{s} = \min\left\{(1+2), \frac{1}{2}(1+2+4), ..., \frac{1}{(n-1)}\sum_{i=0}^{n-1} 2^i\right\} = V_n + V_{n-1} = \min_{i_1,i_2} s(2) = 1+2.$$

4.4. Expected net total spending. Consider the expected net total spending in an internal equilibrium with $i_1, ..., i_k$ active players. Since the net total spending is often the designer's profit, we will call the expected net total spending the expected profit. Then, the expected profit is

$$\pi(i_1, ..., i_k) = \frac{x_{i_1}}{s(k)}[s(k) - x_{i_1}] + ... + \frac{x_{i_k}}{s(k)}[s(k) - x_{i_k}]. \tag{26}$$

From (16),

$$\pi(i_1, ..., i_k) = \frac{s(k) - V_{i_1}}{s(k)}V_{i_1} + ... + \frac{s(k) - V_{i_k}}{s(k)}V_{i_k},$$

or

$$\pi(i_1, ..., i_k) = \sum_{i=i_1}^{i_k} V_i - (k-1)\frac{\sum_{i=i_1}^{i_k} V_i^2}{\sum_{i=i_1}^{i_k} V_i}. \tag{27}$$

The expected profit in an internal equilibrium depends on the number of active players and their lottery prize values. However, if all prize values are the same, the profit is *the same in any* internal equilibrium.

Proposition 10. *Suppose that*

$$V_1 = V_2 = ... = V_n = V.$$

Then, there are $2^n - (n+1)$ *internal equilibria. In any internal equilibrium, the profit is equal to* V*.*

Proof. From Proposition 5, it follows that there are $2^n - (n+1)$ internal equilibria. Consider an internal equilibrium $(x_1, ..., x_n)$ with $i_1, ..., i_k$ active players. From (27), we get

$$\pi(i_1, ..., i_k) = \sum_{i=1}^{k} V - (k-1)\frac{\sum_{i=1}^{k} V^2}{\sum_{i=1}^{k} V} =$$

$$= kV - (k-1)\frac{kV^2}{kV} = V.$$

■

Our next result shows when the *Exclusion Principal* for the expected profit holds in the Sad-Loser lottery.

Proposition 11. *Consider two internal equilibria with $i_1, ..., i_k$ and $i_1, ..., i_k, i_{k+1}$ active players. Then,*

$$\begin{array}{ll} \pi(i_1, ..., i_k) > \pi(i_1, ..., i_k, i_{k+1}), & \text{if } \sum_{j=i_1}^{i_k} V_j \left(V_j - V_{i_{k+1}}\right) > 0, \\ \pi(i_1, ..., i_k) < \pi(i_1, ..., i_k, i_{k+1}), & \text{if } \sum_{j=i_1}^{i_k} V_j \left(V_j - V_{i_{k+1}}\right) < 0. \end{array} \tag{28}$$

Proof. See the Appendix.

The *Exclusion Principal* for the expected profit does not hold for the highest-value player.

Corollary 3. *Consider two internal equilibria with $i_1, ..., i_k$ and $i_1, ..., i_k, i'_{k+1}$ active players. Suppose that*

$$V_{i'_{k+1}} > \max\{V_{i_1}, ..., V_{i_k}\} > 0. \tag{29}$$

Then,

$$\pi(i_1, ..., i_k) < \pi\left(i_1, ..., i_k, i'_{k+1}\right).$$

The *Exclusion Principal* for the expected profit holds for the lowest-value player.

Corollary 4. *Consider two internal equilibria with $i_1, ..., i_k$ and $i_1, ..., i_k, i''_{k+1}$ active players. Suppose that*

$$\min\{V_{i_1}, ..., V_{i_k}\} > V_{i''_{k+1}} > 0. \tag{30}$$

Then,

$$\pi(i_1, ..., i_k) > \pi\left(i_1, ..., i_k, i''_{k+1}\right).$$

Using the *Exclusion Principal* we can characterize the lowest and the highest expected profit in the Sad-Loser lottery now.

Proposition 12. *The lowest expected profit is achieved in the internal equilibrium with the two lowest-value active players. This expected profit is* $\frac{2V_{n-1}V_n}{V_{n-1}+V_n}$.

Proof. See the Appendix.

The following result can be proven similar to Proposition 12. We omit the proof.

Proposition 13. *The highest expected profit is achieved in the internal equilibrium with the two highest-value active players. This expected profit is* $\frac{2V_1V_2}{V_1+V_2}$.

Propositions 12 and 13 describe the highest and the lowest boundaries on the expected profit in the Sad-Loser lottery.

Corollary 5. *The expected profit is at least* $\frac{2V_{n-1}V_n}{V_{n-1}+V_n}$ *and at most* $\frac{2V_1V_2}{V_1+V_2}$ *in an internal equilibrium.*

5. Standard vs Sad-Loser lottery

We compare a standard lottery (more common name is an asymmetric contest) with a Sad-Loser lottery in this section. First, we describe the total equilibrium spending in the standard lottery. Then, the total spending in the standard lottery is compared with the (net) total spending in the Sad-Loser lottery. We will only look at internal equilibria in the Sad-Loser lottery in this section.

5.1. Standard Lottery. Hillman and Riley (1989) identify the set of active players in the standard lottery (asymmetric rent-seeking contest) and the total equilibrium spending. Stein (2002) follows Hillman and Riley (1989) and describes the players' equilibrium strategies.

Consider a standard lottery among n risk-neutral players where (2) holds. Players buy simultaneously lottery tickets in order to win one main prize. In particular, player i spends $b_i \geq 0$ in order to win prize V_i. The players' valuations are commonly known among the players. Player i obtains the prize with probability $\frac{b_i}{\sum_{i=1}^n b_i}$, if $b_i > 0$.

Each player i has to solve the following maximization problem

$$\max_{b_i \geq 0} \frac{b_i}{\sum_{j=1}^n b_j} V_i - b_i.$$

Hillman and Riley (1989) demonstrate that the top $1, 2, ..., \overline{\mathbf{n}}$ players are active in the standard lottery and the total spending in the unique equilibrium[14] is

$$T(n) = \frac{(\overline{\mathbf{n}} - 1)}{\overline{\mathbf{n}}} \widehat{V}_{\overline{\mathbf{n}}},$$

where $\widehat{V}_{\overline{\mathbf{n}}}$ is the harmonic mean of the highest $\overline{\mathbf{n}}$ players' prizes

$$\widehat{V}_{\overline{\mathbf{n}}} \equiv \frac{n}{\sum_{j=1}^{n} \frac{1}{V_j}}$$

and non-active player $(\overline{\mathbf{n}} + 1)$ has prize value such that

$$V_{\overline{\mathbf{n}}+1} \leq \frac{(\overline{\mathbf{n}} - 1)}{\overline{\mathbf{n}}} \widehat{V}_{\overline{\mathbf{n}}}. \tag{31}$$

Note that higher-value players spend more than lower-value players and have higher chance to win the standard lottery. Moreover, each active player has a positive expected profit. We compare the standard and the Sad-Loser lotteries in the next subsections.

5.2. Total spending. Since players can be reimbursed in the Sad-Loser lottery, they buy more tickets in this lottery than they buy in the standard lottery. It turns out that this observation hold in *any internal equilibrium.*

Proposition 14. *Total spending in the Sad-Loser lottery is always higher than the total spending in the standard lottery, or*

$$\min_{k \geq 2} s(k) > T(n).$$

Proof. See the Appendix.

The lottery designer usually cares about her profit. So, we compare the net total spending (the expected profit) in the Sad-Loser lottery with the total spending in the standard lottery in the next subsection.

[14]Fang (2002) and Matros (2006) show that there exists a unique equilibrium in the standard lottery.

5.3. The expected profit. As we show in Section 3, there are several internal equilibria in the Sad-Loser lottery. In some of them, the expected profit is higher than the expected profit in the standard lottery.

Proposition 15. *The expected profit in the internal equilibrium* $(V_2, V_1, 0, ..., 0)$ *is higher then the total spending in the standard lottery.*

Proof. The expected profit in the internal equilibrium $(V_2, V_1, 0, ..., 0)$ is $\frac{2V_1V_2}{V_1+V_2} \geq V_2$. Player 2 is always active in the standard lottery and obtains positive payoff. It means that $T(n) < V_2$. ■

The following proposition provides the sufficient condition for the expected profit in *any* internal equilibrium to be higher than the total spending in the standard lottery. It turns out that this condition is very natural: all players have to be active in the standard lottery. This proposition suggests when the designer should run the Sad-Loser lottery instead of the standard lottery.

Proposition 16. *Suppose that all players are active in the standard lottery, or*

$$V_n > \frac{n-2}{n-1}\widehat{V}_{n-1}. \tag{32}$$

Then, the expected profit in any internal equilibrium in the Sad-Loser lottery is higher than the total spending in the standard lottery.

Proof. Proposition 12 shows that the minimal expected profit is achieved in the internal equilibrium with the two lowest-value active players $(n-1)$ and n. This expected profit is equal to $\frac{2V_{n-1}V_n}{V_{n-1}+V_n} \geq V_n$. The total spending in the standard lottery with all n active players must be smaller than a prize value of any player, including the lowest value player n. Therefore, the expected profit in any internal equilibrium is higher than the total spending in the standard lottery. ■

There are several corollaries from Proposition 16. First, if there are just two players.

Corollary 6. *If* $n = 2$*, then the expected profit in the internal equilibrium is higher than the total spending in the standard lottery.*

Proof. Condition (32) always holds for $n = 2$. ■

Seconds, if all lottery values are the same.

Corollary 7. *Suppose that*

$$V_1 = V_2 = ... = V_n = V. \tag{33}$$

Then, the expected profit in any internal equilibrium is higher than the total spending in the standard lottery.

Proof. Condition (32) always holds if condition (33) is satisfied.

We can see the same result in the following way. Proposition 10 shows that the expected profit in any internal equilibrium is V. Tullock (1980) demonstrates that the total spending is equal to $\frac{n-1}{n}V$ in the standard n-player lottery. ■

The following example illustrates that Proposition 16 provides a sufficient condition. In this example, condition (32) does not hold, but the expected profit is higher in *all internal* equilibria than the total spending in the standard lottery.

Example 4. Suppose that $n = 3$ and $V_1 = V_2 = 10$, $V_3 = 5$. Then, the total spending in the standard lottery (see Hillman and Riley, 1989; Stein, 2002) is $T(3) = 5$. In Example 1, all internal equilibria in the 3-player Sad-Loser lottery are calculated. See Table 2.

values	eq'm $(1,2)$	eq'm $(1,3)$	eq'm $(2,3)$	eq'm $(1,2,3)$
10	10	5	0	2.5
10	10	0	5	2.5
5	0	10	10	7.5
Expected profit	10	6.67	6.67	7

Table 2: Sad-Loser Lottery

Tables 2 shows that the expected profit in the Sad-Loser lottery (in all internal equilibria) is higher than the total spending in the standard lottery.

If condition (32) does not hold, the expected profit can be lower in all but one internal equilibria. The following example illustrates.

Example 5. Suppose that $n = 3$ and $V_1 = V_2 = 10$, $V_3 = 1$. Then, the total spending in the standard lottery is $T(3) = 5$. Based on Example 1, all internal equilibria are calculated for the Sad-Loser lottery in Table 3.

values	eq'm $(1,2)$	eq'm $(1,3)$	eq'm $(2,3)$	eq'm $(1,2,3)$
10	10	1	0	0.5
10	10	0	1	0.5
1	0	10	10	9.5
Expected profit	10	1.818	1.818	1.857

Table 3: Sad-Loser Lottery

The total spending in the standard lottery is higher than the expected profit in all but one internal equilibrium in the Sad-Loser lottery.

5.4. Example. In this subsection, we consider an example which illustrates several propositions above.

Example 6. Suppose that $n = 4$ and $V_1 = 50 > V_2 = 45 > V_3 = 40 > V_4 = 35$. Then, from Proposition 5, there are 11 internal equilibria in pure strategies:

Equilibrium	Total spending	Expected Profit
$(45, 50, 0, 0)$	95	47.368
$(40, 0, 50, 0)$	90	44.444
$(35, 0, 0, 50)$	85	41.176
$(0, 40, 45, 0)$	85	42.353
$(0, 35, 0, 45)$	80	39.375
$(0, 0, 35, 40)$	75	37.333
$(35/2, 45/2, 55/2, 0)$	67.5	44.259
$(30/2, 40/2, 0, 60/2)$	65	41.538
$(25/2, 0, 45/2, 55/2)$	62.5	39.8
$(0, 30/2, 40/2, 50/2)$	60	39.167
$(20/3, 35/3, 50/3, 65/3)$	56.667	40.294

Note that condition (32) holds. Then, the total spending in the standard lottery is

$$T(4) = \frac{4-1}{\sum_{j=1}^{4} \frac{1}{V_j}} = 31.317.$$

We can see that

- the expected profit in any internal equilibrium is higher than the total spending in the standard lottery (Proposition 16);

- the total spending in the Sad-Loser lottery is always higher than the total spending in the standard lottery (Proposition 14);
- the highest expected profit is achieved in the internal equilibrium with the two highest-value active players (Proposition 13);
- the lowest expected profit is achieved in the internal equilibrium with the two lowest-value active players (Proposition 12);
- the total spending in any internal equilibrium is at most $V_1 + V_2 = 95$;
- *Exclusion Principal* holds for the (net) total spending in the Sad-Loser lottery (Propositions 8 and 11).

6. Conclusion

This paper considers the Sad-Loser lottery. All equilibria in pure strategies are found and their properties are discussed. There are several natural extensions of this paper.

It will be interesting to test the results in the experimental laboratory and in the field. We have already started an experimental investigation of the Sad-Loser lottery. In particular, the equilibrium prediction (the counter-intuitive aggressive bidding of weak players) and the Exclusion Principle will be tested.

Another direction is an application of the Sad-Loser lottery to the public good provision. Since, as Morgan (2000) and Duncan (2002) show, lotteries increase provision of public goods, the Sad-Loser lottery might be even a better tool than the standard lottery.

7. Appendix

Proof of Proposition 11. Consider an internal equilibrium $(x_1, ..., x_n)$ with $i_1, ..., i_k$ active players. Suppose that an active player i_{k+1} is added. Then, from (27),

$$\pi(i_1, ..., i_k) - \pi(i_1, ..., i_k, i_{k+1}) =$$

$$\frac{k\left(\sum_{j=i_1}^{i_k} V_j^2 + V_{i_{k+1}}^2\right)\left(\sum_{j=i_1}^{i_k} V_j\right) - (k-1)\left(\sum_{j=i_1}^{i_k} V_j^2\right)\left(\sum_{j=i_1}^{i_k} V_j + V_{i_{k+1}}\right)}{\left(\sum_{j=i_1}^{i_k} V_j + V_{i_{k+1}}\right)\left(\sum_{j=i_1}^{i_k} V_j\right)}$$

$$-\frac{V_{i_{k+1}}\left(\sum_{j=i_1}^{i_k} V_j + V_{i_{k+1}}\right)\left(\sum_{j=i_1}^{i_k} V_j\right)}{\left(\sum_{j=i_1}^{i_k} V_j + V_{i_{k+1}}\right)\left(\sum_{j=i_1}^{i_k} V_j\right)}.$$

Note that

$$k\left(\sum_{j=i_1}^{i_k} V_j^2 + V_{i_{k+1}}^2\right)\left(\sum_{j=i_1}^{i_k} V_j\right) -$$

$$-(k-1)\left(\sum_{j=i_1}^{i_k} V_j^2\right)\left(\sum_{j=i_1}^{i_k} V_j + V_{i_{k+1}}\right) - V_{i_{k+1}}\left(\sum_{j=i_1}^{i_k} V_j + V_{i_{k+1}}\right)\left(\sum_{j=i_1}^{i_k} V_j\right) =$$

$$\left(\sum_{j=i_1}^{i_k} V_j\right)\left(\sum_{j=i_1}^{i_k} V_j\left(V_j - V_{i_{k+1}}\right)\right) +$$

$$(k-1)\left[\left(\sum_{j=i_1}^{i_k} V_j^2 + V_{i_{k+1}}^2\right)\left(\sum_{j=i_1}^{i_k} V_j\right) - \left(\sum_{j=i_1}^{i_k} V_j^2\right)\left(\sum_{j=i_1}^{i_k} V_j + V_{i_{k+1}}\right)\right]. \tag{34}$$

Since

$$\left(\sum_{j=i_1}^{i_k} V_j^2 + V_{i_{k+1}}^2\right)\left(\sum_{j=i_1}^{i_k} V_j\right) - \left(\sum_{j=i_1}^{i_k} V_j^2\right)\left(\sum_{j=i_1}^{i_k} V_j + V_{i_{k+1}}\right) =$$

$$V_{i_{k+1}}\left(\sum_{j=i_1}^{i_k} V_j\left(V_{i_{k+1}} - V_j\right)\right),$$

(34) becomes

$$\left(\sum_{j=i_1}^{i_k} V_j\right)\left(\sum_{j=i_1}^{i_k} V_j\left(V_j - V_{i_{k+1}}\right)\right) - (k-1)\,V_{i_{k+1}}\left(\sum_{j=i_1}^{i_k} V_j\left(V_j - V_{i_{k+1}}\right)\right) =$$

$$(k-1)\left(\sum_{j=i_1}^{i_k} V_j\left(V_j - V_{i_{k+1}}\right)\right)\left[\frac{1}{k-1}\left(\sum_{j=i_1}^{i_k} V_j\right) - V_{i_{k+1}}\right].$$

The statement of the proposition follows from the non-entry condition (7), expression (15), and assumption (28). ■

Proof of Proposition 12. Consider an internal equilibrium with $i_1, ..., i_k$ active players where $2 \leq k \leq n$ and

$$V_{i_1} \geq ... \geq V_{i_k}.$$

There are four cases.

Case 1. Suppose that

$$V_{i_1} \geq ... \geq V_{i_k} > V_{n-1} \geq V_n.$$

Then, from Proposition 11,

$$\pi(n-1,n) < \pi(i_k, n-1, n) < ... < \pi(i_1, ..., i_k, n-1, n) <$$
$$\pi(i_1, ..., i_k, n-1) < \pi(i_1, ..., i_k).$$

Case 2. Suppose that

$$V_{i_1} \geq ... \geq V_{i_k} = V_{n-1} \geq V_n.$$

Then, from Proposition 11,

$$\pi(n-1,n) < \pi(i_{k-1}, i_k, n) < ... < \pi(i_1, ..., i_k, n) <$$
$$< \pi(i_1, ..., i_k).$$

Case 3. Suppose that

$$V_{i_1} \geq ... \geq V_{i_{k-1}} = V_{n-1} \geq V_{i_k} = V_n.$$

Then, from Proposition 11,

$$\pi(n-1,n) < \pi(i_{k-2}, i_{k-1}, i_k) < ... < \pi(i_1, ..., i_k).$$

Case 4. Suppose that

$$V_{i_1} \geq ... \geq V_{i_{k-1}} > V_{n-1} \geq V_{i_k} = V_n.$$

First, we show that

$$\pi(n-1,n) \leq \pi(k,n), \text{ for any } 1 \leq k \leq n-1. \tag{35}$$

Note that (35) holds if and only if

$$2\frac{V_{n-1}V_n}{V_{n-1}+V_n} \leq 2\frac{V_k V_n}{V_k+V_n} \iff V_k \geq V_{n-1}.$$

Therefore, from (35) and Proposition 11,

$$\pi(n-1,n) < \pi(i_{k-1}, n) < \pi(i_{k-2}, i_{k-1}, i_k) < ... < \pi(i_1, ..., i_k).$$

Hence, the lowest expected profit is reached in the equilibrium with the two lowest-value players. Players spend V_n and V_{n-1} in this equilibrium. Hence,

$$\min_k \pi(k) = \min_{i_1,i_2} \pi(i_1, i_2) = \frac{2V_{n-1}V_n}{V_{n-1}+V_n}.$$

■

Proof of Proposition 14. It is a well known result, see for example Hillman and Riley (1989), that the total spending in the standard lottery is strictly smaller than the highest prize value,

$$T(n) < V_1. \tag{36}$$

From (25), the lowest equilibrium spending in any internal equilibrium is

$$\min_{k\geq 2} s(k) =$$

$$\min\left\{(V_n + V_{n-1}), \frac{1}{2}(V_n + V_{n-1} + V_{n-2}), ..., \frac{1}{(n-1)}\sum_{i=1}^{n} V_i\right\}.$$

The non-entry condition (7) must hold for player 1 in the internal equilibria with $(n-1, n); (n-2, n-1, n), ..., (2, ..., n-1, n)$ active players. It means that

$$\min\left\{(V_n + V_{n-1}), \frac{1}{2}(V_n + V_{n-1} + V_{n-2}), ..., \frac{1}{(n-2)}\sum_{i=2}^{n} V_i\right\} \geq V_1.$$

Since (36), it must be

$$\min_{2\leq k<n} s(k) > T(n).$$

Consider the remaining case, $\min_{k\geq 2} s(k) = s(n) = \frac{1}{(n-1)}\sum_{i=1}^{n} V_i$. Then, from (16)

$$x_1 = s(n) - V_1 > 0.$$

Therefore,

$$s(n) > V_1 > T(n).$$

■

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