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Sharing information in web communities

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Abstract

The paper investigates the formation of information sharing communities. The environment is characterized by the anonymity of the contributors and users, as on the Web. Furthermore information exchange is limited to simple recommendations. When preferences differ, it is argued that a community may be worth forming because it facilitates the interpretation and understanding of the posted information. The admission rule within a community, the quality of information, and the stability of multiple communities are examined.

Keywords value of information, communities, anonymity, preference diversity

JEL classification C71, D83, D85

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1 Introduction

Group structures on the Web such as peer-to-peer (P2P) systems aim at sharing various goods and disseminating information in a fully decentralized way. Quite often, information is not rivalrous and returns to scale are not decreasing. Why then do communities form with a free but restricted access? This paper argues that a basic rationale is related to the value of information. A tremendous quantity of information is posted on the Web, on blogs for instance. In some situations however, all this information is useless. As an illustration, consider a page in which an individual provides her opinion on movies. If she says that it is worth watching movie A, or that she prefers movie A to movie B, do I benefit from this knowledge? If the peer is a critic, and I am pretty aware of her tastes, her judgment may be valuable to me. If instead I have no idea at all about her preferences, I learn nothing. In other words, how useful a person's statement is much depends on whether the preferences of this person are known. As a result, when contributors are anonymous, posted information on topics on which tastes differ may have little value. Furthermore, in these situations, search engines may not be helpful either. Given a query, an engine provides a ranking based on the observed behaviors of those who have experimented the topic. The ranking is valuable to other users only if they share similar tastes and know it. This suggests that a community, by filtering the peers who contribute to a platform, may facilitate the understanding and the usefulness of the conveyed information. This paper investigates in a stylized model the criteria on which this filtering may be determined and whether information is efficiently processed.

There are individuals who regularly look for a piece of advice on a category of 'objects', on movies for instance, and who occasionally post information on a particular object, when they have seen a movie for instance. Objects differ in some characteristics and individuals differ in their tastes. More specifically, objects' characteristics and individuals' preferences are characterized by a single parameter on a circle as in Salop differentiation model (1979). Two crucial features bear on the information process. First, anonymity is preserved as often in P2P. Second, although we assume truthful behavior, language is limited. Individuals are able to describe whether they have enjoyed consuming a particular object but they are unable to describe why, in particular they are unable to describe the object's characteristic.

To analyze the value of the posted recommendations, we follow Blackwell (1953). Recommendations are valuable to a peer if they allow him to make "better" decisions, that is decisions that increase his utility. Anonymity and limited language prevent this to be generally true. Recommendations are useful to users who share similar tastes, and furthermore, posted information without control on the contributors may not only be useless but also detrimental by introducing some noise in the information relevant to other peers. Thus the admission rule in a community is essential in determining the value that each peer derives from the information provided by the community's members. This leads us to analyze preferences over admission rules. Community's members do not fully agree on an admission criterion owing to their differences in tastes, even if all of them benefit from the community. We analyze this divergence and, assuming that a leader/initiator of a community chooses the admission rule, we study how this choice is influenced by the contribution rate,[2] the cost and the probability of finding an answer to a query, and the sharing of ad revenues.

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[2]There is some evidence of "free-riding" in P2P systems. A main question is whether the generated difficulties are severe enough to call for the implementation of incentive schemes (see Feldman et al. 2004 and Ng et al. 2008

Whereas the circle model is the simplest model to explore our ideas, the analysis can be extended to different or more complex settings under heterogeneity in preferences. In particular, the value for a community is explored more informally under vertical differentiation when individuals agree on ranking the objects according to an 'objective' quality but may differ in their willingness to pay. Also, in our basic model, a peer who is contemplating buying a particular object searches for a single recommendation at most. A question is whether the availability of an important number of recommendations, or of an appropriate statistic on those, reveals all the information an individual needs to make a good decision and thereby destroys the value of forming a community. The answer depends on whether preferences really differ. In the circle model, even when a large number of recommendations is available, forming a community that restricts access so as to be homogeneous enough is still valuable, and information is not precise enough for peers to make good decision. This is contrasted with a vertical differentiation model. Hence, due to limited language, when tastes really differ, information exchanges are valuable only between similar individuals.

Finally, we study the coexistence of several communities, called a configuration. To predict which communities might form and be durable, stability concepts borrowed from cooperative game theory are useful. The threat of proposals to form a new community puts constraints on configurations. An interesting question is whether communities form so as to reach some form of efficiency. Indeed there is a trade-off between the amount of information received by peers and its relevance to them: as a community becomes less strict in its admission rule, peers are more numerous but more diverse, which makes recommendations more numerous but less informative. This trade-off is optimally solved at an efficient configuration. It is shown that stability does not imply efficiency nor does efficiency imply stability because of external effects. For instance, it may happen that at an efficient configuration each peer would like his own community to expand and to accept newcomers, which makes the configuration unstable. However expanding communities leads to the suppression of some of them, which has an overall negative effect on welfare because of the decrease in the relevant information.

This paper is related to the growing literature on the search for relevant information on the Internet. The ranking provided by a search engine can be seen as an aggregator of preferences that determines 'objectively' the relative importance of Web pages. As such, it is more valuable under common preferences. This explains why search engine rankings share some similar features as the methods used for measuring 'quality'. Indeed, as acknowledged by Page et al. (1998), the PageRank method used initially by Google has been inspired by the citation analysis techniques (see Slutzki and Volij 2006 who provide axiomatizations of some methods). Also, Dwork et al. (2001) explicitly assume a common underlying ranking and borrow tools from social choice theory in order to study aggregation over several engines. In a circle model as considered here, any anonymous aggregation of preferences over the whole population yields a completely flat ranking, that is the society is indifferent between any two objects, and distinct rankings provided by search engines can only be attributed to chance or to bias. In some sense, a community forms with members whose preferences are homogeneous enough to allow for a useful aggregation.

The formation of communities and preferences diversity are recognized as an important feature of the Web environment. Various algorithms have been proposed for detecting communities through

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for example). Bramoulle and Kranton (2007) and Corbo et al. (2007) provide theoretical analysis of free-riding in a network context.

link structure, as surveyed in Newman (2001). Recent empirical studies analyze the growth of communities as a function of an underlying social network (Backstrom et al. 2006 for instance). This paper investigates, in a specific context, why and how a community forms in the first place (without any prior network). This is also the concern of Asvanund et al. (2004) who investigate how communities may improve the efficiency of search in the Web environment.[3] They consider a hybrid P2P architecture (close to Gnutella 0.6.) in which an 'ultrapeer' forms a club by choosing to link with ordinary peers and these ultrapeers are connected between themselves. Forming a club with peers who are interested in the same topics (recognized by a 'catalog') improves their chances to find answers to their queries. Hence communities are based on the similarity of interests, while here they are based on the similarity of preferences and the adverse selection difficulties due to limited language. The diversity of preferences is also at the root of 'collaborative filtering'. Collaborative filtering is a system that aims at giving tailored recommendations to a user on the basis of his past behaviors and a collection of 'similar' user profiles (Hofmann and Puzicha 1999). Typically, an e-commerce retailer recommends to a user to buy a particular item because another user who has behaved similarly in the past has bought that item. Hence, instead of a recommendation sent by a peer who has experienced an object, the purchase is itself interpreted as a good signal even though the buyer might have disliked it. More importantly, the system is centralized in contrast with P2P. Our model accounts for both the decentralization and anonymity aspects. An individual voluntarily chooses a community and all peers within a community have access to the same recommendations, allowing for anonymity and privacy.

Finally, the paper is related to the literature on belief formation or social learning, in which agents choose actions over time based on information received by others, either by direct communication or by observing actions. Information bears on an unknown state of nature, and its transmission is constrained by a specific setting. For instance information is conveyed by neighbors in a given network structure or the agents whose actions are observed move in a given sequence. A main question analyzed by this literature is whether beliefs converge and whether they converge towards the 'true' state of nature (see Goyal 2005 for a survey). In our model, a 'state of nature' is the objects' characteristics. Because language is limited to simple subjective statements, and by the very fact that communities must be homogeneous enough to be valuable (in a circle model), objects' characteristics are not revealed, even with access to numerous peers' recommendations. Finally, DeMarzo et al. (2003) consider another form of limitation in communication in which information is processed through an exogenous 'listening structure' (or network) with individuals who fail to disentangle what is truly new in the signals they receive over time. The main question investigated here is instead to determine which listening structures might emerge in a setting in which the recommendations given by some individuals are detrimental to others.

The plan of the paper is the following. Section 2 sets up the model and Section 3 studies a single community. Section 4 analyzes configurations of communities. Individuals behavior is investigated when multiple and possibly overlapping proposals to form a community are made, and coordination failure is discussed. How proposals are made is explored and the stability and efficiency properties of configurations are investigated. Proofs are gathered in the final section.

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[3]I thank a referee for pointing out to me this paper.

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2 The model

Consider one category of 'objects', such as movies, or books, or restaurants. Individuals differ in their tastes. Differentiation models in which preferences and objects are characterized by a single characteristic are a stylized and parsimonious way to model these differences. I adopt here the 'circle' model introduced by Salop (1979). Specifically, an object, a movie or a restaurant for instance, is characterized by a point on the circle. Each individual has a most-preferred object specification and his preferences decrease in the distance to this object along the circle. The perfect symmetry is the advantage of this model over Hotelling (1929) model in which characteristics lie in an interval: all characteristics are a priori equally preferred (under the assumptions of uniform distributions that we shall take). We thus avoid technical difficulties due to 'corners'. Also, this means that a characteristic does not specify an intrinsic quality, as in a vertical differentiation model (Shaked and Sutton 1983). Section 3.3 examines how results extend to both Hotelling or vertical differentiation models.

An object's characteristic is specified by a 'point' on the circle t. Individual's preferences are characterized by the most-preferred object specification θ on the circle. An object 'located' at t is called a t-object and similarly an individual 'located' at θ is called a θ-individual. A θ-individual who buys a t-object derives a utility gain $u(d(\theta, t))$ that is non-increasing in the distance to the object $d(\theta, t)$ on the circle between θ and t. If this gain is negative, the individual is better off not buying the object. Function u is non-increasing, identical for all individuals. To fix the idea, at most half of the objects are valuable to an individual: There is a threshold value d^*, $0 < d^* < \pi/2$ for which $u(d) > 0$ for $d < d^*$ and $u(d) < 0$ for $d > d^*$. Furthermore function u is continuous and differentiable except possibly at d^*. As a simple example, consider the situation in which individuals either do or do not enjoy consuming the object, as is represented by the following *binary* function:

$$\text{for some positive } g \text{ and } b, u(d) = g, d < d^*, u(d) = -b, d > d^*. \tag{1}$$

(The utility level at d^* does not matter because the probability of an object being distant of d^* to a person is null.)

The society is uniformly distributed on the circle. Individuals who benefit from buying a particular object is given by those located at a distance smaller than d^*. Thus, under perfect information on an object's location, a proportion d^*/π of the people buy it, independently of the location t. But there is imperfect information on new objects. This is the situation we are interested in. New objects are assumed to be *a priori* uniformly distributed on the circle. There is some scope for information sharing: Individuals who have bought an object may post their opinion on it. Our aim is to study the value of a community in gathering and sharing such opinions.

The analysis is conducted in a Von Neumann-Morgenstern setup. Under imperfect information on objects' characteristics, an individual forms some assessment on the location and decides whether to buy a particular object by comparing the expected utility gain from buying it with 0. A weak form of risk aversion is assumed: faced with the lottery of buying two objects with equal probability, a peer prefers not to buy if the sum of the distance is larger than $2d^*$, i.e. $u(d_1) + u(d_2) < 0$ for $d_1 + d_2 > 2d^*$. The assumption is satisfied under risk neutrality, $u(d) = d^* - d$ or under concavity of u since $\frac{1}{2}[u(d_1) + u(d_2)] \leq u(\frac{(d_1 + d_2)}{2}) \leq u(d^*) = 0$. For a binary function (1), risk aversion holds for $b > g$. Risk aversion implies that the expected utility gain derived from buying at 'random' is

negative. We say that u exhibits strict risk aversion if the inequality $u(d_1) + u(d_2) < 0$ holds not only for $d_1 + d_2 > 2d^*$ but also for $d_1 + d_2 = 2d^*$, $d_1 \neq d_2$ (this is satisfied for a strictly concave u).

2.1 Communities

In a community, the role of contributors and users can *a priori* be distinguished. Contributors add to the content by providing information on the objects they have tested while users have access to the posted information. Here the sets of contributors and users are identical. This is induced by the following assumptions. First we assume that there is no intrinsic motive to contribute such as altruism. Thus, for a community to be 'viable' as defined in the next section, contributors are also users so as to draw some benefit. Second, even though there may be no direct cost (nor benefit) in allowing users not to contribute, it may be worth restricting access to contributors simply to encourage them to contribute. In that case users and contributors coincide.

Anonymity and restricted access can be implemented by mechanisms that propagate queries through a P2P network (examples are Gnutella and Freenet, with various architectures, ranging from centralized to fully decentralized systems). A query is sent to neighbors who provide an answer if they have one or otherwise pass the query to their own neighbors and so on until an answer is reached.[4] The system can be anonymous by recognizing members by an address only. Records, which are not public, can keep track of peers' behavior. Sanctions such as exclusion are based on these records and automatic. Records on peers' contributions for instance allow the community to sustain some contribution level by excluding users who contribute too little.

In our model, a proposal to form a community is represented by an arc. It will be assumed that the system can detect whether the recommendations given by the members are indeed compatible with the characteristics in the selected arc. By convention, an arc $[\theta, \theta']$ designates the arc from θ to θ' going clockwise. The *center* of community $[\theta, \theta']$ is $(\theta + \theta')/2$ and its *size* is defined by $(\theta' - \theta)/2$.

We start by considering a single arc and identify a community with the set of all individuals whose characteristics belong to that arc. (Section 4 introduces the distinction between a proposal to form a community -an arc- and the set of individuals who effectively join.) One of our tasks is to determine what is the value of joining a community. This will depend on the whole membership, as made precise in next section.[5]

The technology is characterized by two data: a probability of 'success', which is the probability of finding a recommendation for a particular object in reasonable time and an individual participation cost. In line with decentralized behavior, the size of a community determines the probability of success, which is denoted by $P(\alpha)$ for a community of size α. P is assumed to be increasing and concave. For example, it can be described by a Poisson process $P(\alpha) = 1 - e^{-\lambda \alpha}$ for some positive parameter λ which reflects the contribution rate of the community members and the efficiency of the search mechanism. The participation cost includes the cost for searching and contributing. Normalizing by the average number of requests, it is denoted by c. When it is small, as is likely, it does not play an essential role in the analysis. The probability P and the cost c are first assumed to be given. However contributions have a public good aspect, and a minimal amount of contributions

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[4]See for example Kleinberg and Raghavan (2005), and Arcaute et al. (2007) for a description of decentralized mechanisms and an analysis of the incentive to pass the information.

[5]This is to be contrasted with network formation models such as Jackson and Rogers (2004) in which relationships are formed through bilateral links and under myopic behavior.

can be asked for to increase the success probability P. Section 3.2 investigates this point more closely.

2.2 Signals

Consider a particular object, recognized by a name, say a title for a book or for a movie. Opinions on that object are described by signals. Stating a detailed judgment is difficult. To account for this, signals are assumed to be limited. A signal s on an object takes two values, *yes* or *no*, which are interpreted as a recommendation to buy or not to buy. Since there is no benefit from sending a false signal, signals are assumed to be truthful: a θ-individual having bought a t-object sends *yes* if $u(d(\theta,t)) \geq 0$ and *no* if the inequality is reversed.[6] As will be clear later on, the limitation in communication is not due to the use of a binary signal. What matters is that a signal pertains to feelings (or utility levels) rather than to objective objects' characteristics.

Let us consider a signal on a particular object from a member of community $[\theta, \theta']$. It can be seen as a 'random' recommendation $\tilde{s}$ since the sender is considered as drawn at random from the community. In the sequel $\tilde{s} \in [\theta, \theta']$ refers to a signal sent by a member of community $[\theta, \theta']$.

The value of a signal can be analyzed from the viewpoint of Blackwell (1953). Signal $\tilde{s}$ is valuable to an individual if it enables him to make 'better' decisions in the sense that his expected payoff is increased. More precisely, the signal is used as follows. The joint distribution of $(\tilde{t}, \tilde{s})$ for a signal $\tilde{s}$ sent by a member of community $[\theta, \theta']$ can be computed. After learning the realized value of a signal, peers revise their prior on the characteristic t according to Bayes' formula and decide to buy or not. Clearly a signal that does not change the prior on the object's location is useless. The ignorance of the sender's location in the community has the following consequences.

- (i) A signal $\tilde{s}$ from the whole society is useless.

- (ii) A signal from a community smaller than the whole society may be useful because it changes the prior.

- (iii) Two simultaneous signals may convey less information than each one separately.

Point (i) is straightforward. A signal sent by an individual chosen at random in the whole group does not modify the prior, hence is not informative.

Point (ii) is also clear. Figure 1 illustrates the case of a community $[-\alpha, \alpha]$ and $d^* = \pi/2$. Each peer sends *no* for an object located in $[\alpha + \pi/2, -\alpha - \pi/2]$. Thus the posterior density conditional on the signal being *yes* is null on that arc: the posterior clearly differs from the prior density.[7]

To show Point (iii), consider a signal from a community reduced to a point, say 0, so that the sender's preferences are known. The signal is informative as follows from point (ii) or, more directly,

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[6]Malicious individuals can be incorporated. Under some expectation on their distribution, their presence introduces additional noise in the information inferred by a signal. The same argument applies if individuals make error in their judgment.

[7]More generally, consider the probability of a positive signal from community $[-\alpha, \alpha]$ for a t-object $g^\alpha(t) = P(yes \in [-\alpha, \alpha]|t)$. Each member in the community sends a positive signal *yes* for $t \in [\alpha - d^*, -\alpha + d^*]$, which we call the *acquiescence zone*, $(g^\alpha(t) = 1)$ and each one sends *no* on $[\alpha + d^*, -\alpha - d^*]$, which we call the *refusal zone* $g^\alpha(t) = 0$. Outside these two zones there is *disagreement* and the probability is linear in between. By Bayes'rule, the posterior density conditional on a *yes* $\in [-\alpha, \alpha]$ is equal to $g^\alpha(t)/(2d^*)$.

because a *yes* indicates that the object is in $[-d^*, d^*]$ and a *no* in the complement $[d^*, -d^*]$. Add a signal sent from $[\pi]$. The important point is that, on the receipt of the two signals, it is not known which peer has sent which signal. The two signals, which are always opposite to each other, give no information because the prior is not changed. In this simple example, the new signal not only adds no information but also destroys the information conveyed by the first signal. The reason is that adding a signal introduces an additional source of randomness due to the anonymity of the sender. In contrast, in the standard framework, adding a signal is never harmful because it can simply be ignored.
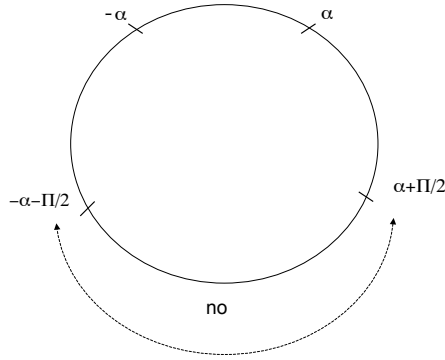

Figure 1: $d^* = \pi/2$

An informative signal has some value to a person if its receipt affects this person's optimal decision. Here, there are three options: to follow the recommendations (that is to buy the object on the receipt of a positive signal and not to buy it on the receipt of a negative signal), or not to listen to them (which gives null utility level), or to take the opposite action of the recommendations. Let us assume for the moment that a person indeed follows the recommendations (this assumption is justified in next proposition). We analyze how the benefit depends on the individual's position in the community and on the size of the community. Specifically, given a community of size α, let $U(\theta, \alpha)$ denote the expected utility of an individual whose distance to the center is θ conditional on having an answer to a query and on following the recommendation. The *a priori* probability for a positive signal is equal to $p = d^*/\pi$ since, given an object at random, each individual says *yes* with probability p. Up to a rotation, the arc may be assumed to be centered at zero. Using that there is no purchase on the receipt of a negative signal gives

$$U(\theta, \alpha) = \frac{d^*}{\pi} E[u(d(\theta, \tilde{t}))|yes \in [-\alpha, \alpha]]. \tag{2}$$

The following proposition states how this value U varies with θ and α.

Proposition 1 *Let $U(\theta, \alpha)$ be the expected value from following a recommendation of a community with size α for an individual whose distance is θ from the center and $V(\alpha) = U(\alpha, \alpha)$ the value for a peer located at an extreme point.*

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- *(i) Given α, $\alpha \leq d^*$, utility $U(\theta, \alpha)$ decreases in the distance θ to the center for $\theta \leq d^*$.*

- *(ii) Given θ, $0 \leq \theta \leq d^*$, utility $U(\theta, \alpha)$ decreases in α, $0 \leq \alpha \leq d^*$.*

- *(iii) There is a value α^{max} smaller than d^* such that $V(\alpha)$ is positive for $\alpha \leq \alpha^{max}$ and negative otherwise.*

- *(iv) Given $\alpha \leq d^* \leq \pi/4$ no individual benefits from taking the opposite recommendation of the community.*

The properties hold for all individuals located at a distance smaller than d^* from the center, hence for some outsiders to the community. Point (i) is natural given the symmetry. It says that the expected benefits derived from following a signal decrease with the distance to the center. According to point (ii), the expected value per signal is greater the smaller the community, that is the less uncertain the sender. This is easy to understand for a peer located at the center ($\theta = 0$). As the size α increases, the objects that he dislikes (distant by more than d^*) are more likely to be recommended and those he likes get less recommended. For an individual who is not at the center, the distribution of signals is 'biased' with respect to his own preferences and as α increases some objects that he likes get more recommended. Increasing α is however still harmful because the distribution of the distance to a peer of the recommended objects becomes riskier in the sense of first order stochastic dominance as shown in the proof. An implication of property (ii) is the superiority of an expert 'everything else being equal'. More precisely, assuming the expert's position perfectly known, her recommendations are more valuable than those of the communities' members. Hence a system in which an expert sends as many signals as the community at the same total cost makes every peer better off. Point (iii) will be important to bound the scope of a community. The point follows from weak risk aversion, which implies that two persons whose distance is larger than $2d^*$ cannot both benefit from following the same recommendations. Finally point (iv) justifies our assumption that either following or ignoring a signal is the optimal strategy provided the tastes are restrictive enough.

Let us give two examples, the computation of which are given in the appendix. For a binary function (1), we have $U(\theta, \alpha) = \frac{d^*}{\pi}[g - \frac{(g+b)}{4d^*}(\alpha + \theta^2/\alpha)]$ and $\alpha^{max} = \frac{2gd^*}{(g+b)}$ decreases from d^* to 0 when b/g ranges from 1 to ∞.

For a quadratic function $u(d) = d^{*2} - d^2$, $U(\theta, \alpha) = \frac{d^*}{\pi}[\frac{d^{*2}}{3} - \frac{2}{3}\alpha^2 - \theta^2]$.

2.3 Viable community

Anybody is free not to join a community. A basic requirement is that all community members benefit from it, a property that we call viability. A community is said to be *viable* if each of its members benefits from it, accounting for the failure of search and the participation cost. Since, in the absence of information, an individual does not buy, reservation values are null. Hence, an individual distant of θ from the center of a community of size α is indeed willing to participate if $P(\alpha)U(\theta, \alpha) - c \geq 0$. To cover the cost, the utility from following a signal, U, must be positive. From point (i) of Proposition 1, the peers who achieve the lowest benefit are located at the extreme points of the community. The viability condition can be written simply as

$$P(\alpha)V(\alpha) \geq c \text{ where } V(\alpha) = U(\alpha, \alpha). \tag{3}$$

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From (iii) of Proposition 1, viable communities are of size smaller than d^*, and under a small enough cost, the set of viable sizes is non-empty. Property (ii) points out a trade-off faced by peers: increasing the size increases the probability of getting an answer but decreases the value of an answer. To analyze this trade-off, we make the following assumptions $A0$ and $A1$ throughout the paper, without repeating them. The size α is restricted to be smaller than α^{max} so that utility levels are all positive.

Assumption A0 (concavity assumption) The functions $logU(\theta, \alpha)$ and $logV(\alpha)$ are concave with respect to α in $]0, \alpha^{max}[$ for each $\theta < \alpha^{max}$.

Assumption A1 (elasticity with respect to the extreme)

$$\frac{-U'_\alpha}{U}(0, \alpha) \leq \frac{-V'}{V}(\alpha) \text{ for } \alpha \text{ in }]0, \alpha^{max}[\text{ where } -V'(\alpha) = -[U'_\alpha + U'_\theta](\alpha, \alpha)$$

Under $A0$, PV is log-concave as a product of log-concave functions. Hence the set of viable sizes is an interval $[\underline{\alpha}, \overline{\alpha}]$ which is non-empty for a low enough cost c, (for $c = 0$, $\underline{\alpha}=0$ and $\overline{\alpha} = \alpha^{max}$), which we shall assume. Under $A1$, the relative loss for the center is smaller than the relative decrease in the utility of an individual located at an extreme. The following assumptions B are mostly used for interpretation and mentioned when needed.

Assumptions B (elasticity with respect to position)

$$B_- : \frac{-U'_\alpha}{U}(\theta, \alpha) \text{ decreases with } \theta \qquad \text{or} \qquad B_+ : \frac{-U'_\alpha}{U}(\theta, \alpha) \text{ increases with } \theta$$

Under the elasticity assumptions, either B_- or B_+, the relative loss incurred by a peer due to an increase in the size is monotone in the position, either decreasing or increasing. Both cases are possible as explained below (I have not found general conditions on u under which either assumption holds). Note that $V'(\alpha)$ includes not only the variation due to the size but also the one due to the position, which is negative because U decreases with the distance to the center from (i) of Proposition 1. Hence B_+ implies $A1$ and B_- is compatible with $A1$. For example, assumptions $A0, A1$, and B_- hold for a binary function, and $A0, A1$, and B_+ hold for a quadratic function.

The purpose of next sections is to analyze which communities might form. Before proceeding, it is worthwhile to understand the differences with standard differentiation models. Initiated by Hotelling, they have been used in political science as in Downs model (1957) to analyze political competition of candidates with respect to the ideal positions of the electorate, or in industrial organization, in order to study imperfect competition (pricing, choice of the products). This analysis is closer to the political paradigm because monetary transfers nor firms are allowed.[8] Furthermore, I shall consider 'leaders' who initiate a community and may share some resemblance with political leaders. There are however important differences from political competition. I do not impose a priori constraints on the number of leaders/communities, and the aim of a community is not to win an election but to share information.

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[8] A firm has a well defined objective, to maximize its profit. With transferable utilities, the objective of a community would be to maximize the sum of the utilities. See section 4.3, which analyzes welfare in more detail in a multiple communities setting.

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3   A single community

This section analyzes the formation of a single community, when individuals who may join a community face no other alternative than not joining (multiple opportunities are considered in next section). Of course a requirement is that all community members benefit from it, namely that the community is viable. There are many different viable communities, provided the search cost c is not too large. Furthermore, in a setting without firms nor transferable utilities, there is not a unique criterion to determine which community will form. As a result, community's members may have conflicting views about its scope, i.e., about the membership rule. We analyze this conflict and pay special attention to the choice of a 'leader'.

3.1   Peers' preferences

In practice, a 'leader' initiates a community and possibly defines criteria for accepting peers. The leader in our model does not play an important role -in particular a leader does not influence other peers- but allows us to fix the center of the community. Let α^{θ} denote the value that maximizes the payoff $P(\alpha)U(\theta, \alpha)$, that is the preferred size of a peer who is at a distance θ from the center. The leader's optimal community size is given by the value α^0 provided it is viable.

Proposition 2

1. *Let α^{enl} be the unique solution to the equation $\alpha = inf_{\theta \leq \alpha} \alpha^{\theta}$. If $\alpha^{enl} < \overline{\alpha}$ a viable community is Pareto optimal if and only if its size is larger than α^{enl}: otherwise, slightly enlarging it while keeping the same center makes every peer and every newcomer better off. If $\alpha^{enl} \geq \overline{\alpha}$, only a community of size $\overline{\alpha}$ is Pareto optimal.*

2. *The leader's optimal choice is either (a) the leader's optimum α^0 if it is viable and in this case the community is closed to outsiders, or (b) the maximal viable size $\overline{\alpha}$.*

3. *Under B_+ peers' optimal choices are decreasing the distance to the center. Under B_- peers' optimal choices are increasing so that $\alpha^{enl} = \alpha^0$.*

Point *1* gives bounds on the size of Pareto optimal communities. The first case occurs for c small enough ($\overline{\alpha}$ is decreasing in c whereas α^{enl} does not depend on c). The proof is easy. Note first that the equation $\alpha = inf_{\theta \leq \alpha} \alpha^{\theta}$ defines a unique value α^{enl}: the right hand side is decreasing from α^0 to 0 as α ranges from 0 to α^{max}. This implies that for $\alpha < \alpha^{enl}$ we have $\alpha < inf_{\theta \leq \alpha} \alpha^{\theta}$. Hence, if $\alpha^{enl} < \overline{\alpha}$, for a size smaller than α^{enl}, a slight enlargement with the same center makes every insider better off (because $\alpha < \alpha^{\theta}$ for each $\theta \geq \alpha$) and the new members, who do not belong to any community, are willing to join (because $\alpha < \overline{\alpha}$). When $\alpha^{enl} \geq \overline{\alpha}$, such an enlargement would make outsiders worse off. A reduction in a viable community leaves some peers alone, lowering their utility level to zero, hence cannot lead to a Pareto improvement for the community's members. (Of course, there is no reason why such reduction would not be contemplated by some peers if this is profitable to them. This point is examined in next section together with the formation of multiple communities.)

According to point *2*, when the leader's optimum is not viable, in case (*b*), it is because it is too large: the individual at the extreme just breaks even. In case (*a*), we have $P(\alpha^0)V(\alpha^0) > c$ (except

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in the boundary case where the equality holds and α^0 is equal to $\overline{\alpha}$); thus, close enough outsiders would achieve a positive payoff by joining (by continuity of the payoffs), but they are not allowed to do so. This is why the community is closed.

Point *3* makes precise the direction of possible disagreements within the community when an elasticity assumption B holds. The argument is the following one. At the preferred peer's choice, the relative loss due to an increase in size exactly compensates the relative increase in the probability of success. Under B_+, the relative loss is larger for any other peer further away from the center, who thus prefers a smaller size. This implies that when the leader is unconstrained as in case (a), peers all prefer a smaller size than the leader's choice (and a larger one under B_- by reversing the inequalities). In case (b) instead, under B_-, all peers, including the leader, would benefit from an increase in the community size up to α^0. However, since peers at the extreme of the community just cover their cost, no outsider wants to join.

Illustration To illustrate Proposition 2, we consider a Poisson process $P(\alpha) = 1 - e^{-\lambda\alpha}$ and a binary function, and discuss the impact of the probability of success and ad revenues on the leader's choice.

Let us consider an exogenous increase in the probability of success (other things being equal), here an increase in the parameter λ, as results from an improvement in the technology or from an increase in the number of members due to an increase in Internet users. Figure 2 depicts the maximal viable size $\overline{\alpha}$ (the increasing line) and the leader's optimum α^0 (the decreasing line) as a function of λ. Since the leader's choice is the minimum of these two values, increasing the population has different effects on the leader's choice depending on whether this choice is constrained or not. The following configurations are obtained as λ increases. First, for λ low enough, there is no viable community, second the leader's choice is constrained equal to the maximal viable size, and third the leader can choose his optimum value. This can be explained as follows. Increasing the population within a community makes it more attractive to outsiders. When the community is constrained by viability, for intermediate values of λ, these outsiders are welcome. As a result, the size is increased (note that this argument uses $A1$ only, not B). Instead, when the community is closed, for a large enough λ, increasing the population allows the leader to choose a community restricted to peers whose tastes are more and more similar to his own: the size decreases. The impact of λ on the size directly translates into an impact on the precision of information: as λ increases, information is first made less precise (but the higher chance of getting some information compensates the loss) and then more and more precise.

We discuss now the impact of ad revenues on the leader's choice. To simplify, suppose that peers do not mind ads and that ads do not influence their preferences on the object on which they are searching information. We consider two alternative ways of distributing the revenues generated by ads, which are assumed to be proportional to the number of peers.

First, let the leader capture all ad revenues. In this case the leader's objective is to set up a community that maximizes a combination of his own interests and the revenues. The optimal choice is unchanged if the viability constraint binds. Otherwise, instead of choosing his most preferred size, α^0, the leader chooses a larger size (between α^0 and $\overline{\alpha}$). The more the leader cares about revenues, the closer his choice to the maximal viable size, and, as a result, the less precise the transmitted information is. The effect can be substantial for large λ because the maximal viable size $\overline{\alpha}$ is large and α^0 is small. Whereas a community could be tailored to his specific tastes, the leader chooses
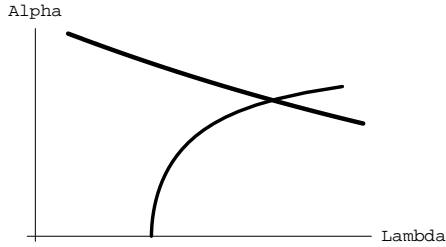
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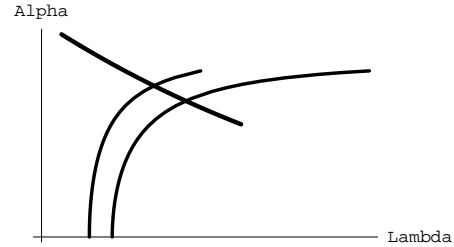Figure 2: $c = 0.1$, $b/g = 1.5$ Figure 3: $c = 0.1$ and $c = 0.07$

looser access criteria so as to capture more ad revenues.

Second, let ad revenues be distributed equally among peers. The effect amounts to diminish cost c, which results in an increase in the maximal viable size and leaves the optimal leader's size unchanged. Hence, the leader's choice is closer and more often identical to his most preferred size. In Figure 2, the maximal viable size is drawn for two distinct values of the cost (the upper increasing line represents α^0 for the lower value of the cost).

3.2 Enticing Contribution

Various factors influence the probability of successful search. Some result from a policy imposed to the community's members such as the enforcement of a minimal contribution rate. We assume here that P is influenced by the peers' contribution rates. A minimum rate is asked for, implemented through records on peers' contributions. The leader now chooses both the size and the minimal contribution rate.

We assume that the peer's participation cost c is an increasing function of his contributions. As a result, no peer will contribute more than the minimum required rate: the supported cost would increase with a null benefit since the impact of a single individual on the success probability is negligible. This is a standard effect in public good provision. Given the minimum required rate λ, let $P(\lambda, \alpha)$ and $c(\lambda)$ denote respectively the probability of success when *each* peer contributes λ and the incurred individual cost. P is non-decreasing and concave in λ and c is non-decreasing and convex.

Without constraint on viability, the leader's optimum is the value of (λ, α) that maximizes $P(\lambda, \alpha)U(0, \alpha) - c(\lambda)$. The maximal viable size now depends on λ; it is denoted by $\overline{\alpha}(\lambda)$.

Proposition 3 *The leader's choice is either (a) the leader's optimum if the size is viable; the community is closed to outsiders and peers who are not at the center would prefer a lower contribution*

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rate, or (b) a smaller contribution rate than the optimum and a size equal to the associated maximal viable size $\overline{\alpha}(\lambda)$; the choice of λ trades off the benefits from increasing contribution and the loss due to a smaller community size (i.e., $\overline{\alpha}(\lambda)$ decreases at the chosen value of λ).

Given the chosen contribution rate, the choice of the size is dictated by the same considerations as in the previous section, which leads to the two cases *(a)* and *(b)*. As for the contribution rate, note that the marginal benefit from increasing the contribution rate, $P_\lambda(\alpha, \lambda)U(\theta, \alpha) - c'(\lambda)$, is decreasing in the distance θ to the center (as U is). When the leader is not constrained as in case *(a)*, the marginal benefit from increasing the contribution rate is null for the center, and all peers with different characteristics would prefer to decrease the contribution rate (and they would all prefer to increase the size if assumption B_- holds or to decrease it B_+ holds). When the leader is constrained as in case *(b)*, the leader's marginal benefit from increasing the contribution rate must be non-negative because otherwise a small reduction in the rate would lead to a Pareto improvement within the community. Hence the leader would benefit from an increase in the contribution rate and from an increase in size. Increasing the rate however incites some peers at the extreme to leave thereby decreasing the size, which results in a trade-off.

3.3 Alternative differentiation models

The previous analysis builds on the trade-off between the amount of information (increasing in the scope of the community) and its relevance (decreasing in the scope). Such a trade-off naturally arises in alternative models, under horizontal or vertical differentiation. In both cases, preferences and objects differ through a single characteristic. Uniform distributions are assumed. My aim here is only to explain why there is still a value for a restricted community, without analyzing precisely its choice.

Hotelling differentiation model. Characteristics (both for objects and individuals) are on an interval instead of a circle. Individuals still value the objects according to their distance to their own location. Most properties extend. In particular, the expected value of a signal to a peer is decreasing in the size of the community, and communities must be of limited size (smaller than d^*) to be viable. Furthermore, when a leader is at a distance larger than $2d^*$ from both boundaries, Proposition 2 exactly applies (because in that case there is no 'corner' effect: the distributions of the recommended objects for any community that might form -the size of which is smaller than d^* by viability- are identical in the circle and line models).

Vertical differentiation model. Objects differ in quality. Individuals all agree on the ranking of quality levels but differ in their willingness to pay. Individuals' preferences are represented by a utility of the form $u(t - \theta)$ in which t is the quality level of the object and θ is the reservation value, that is the minimum quality that makes the object valuable to the individual (normalizing $u(0) = 0$). Individuals' tastes do not widely differ here so that peers may have less incentives to limit the access to a community than in the circle model.

Let us first investigate viability. Viability takes a slightly different form than in the circle model because optimal behavior in the absence of any recommendation may differ across individuals. Specifically, let $B(\theta)$ denote the expected utility per object bought at random for a θ-individual. $B(\theta)$ is decreasing so that individuals with a low enough θ may benefit from buying whereas those with a high enough θ will not. The reservation utility of a θ-peer is therefore the maximum of 0 and

$B(\theta)$.

Now let $U(\theta, [\theta_1, \theta_2])$ denote the expected utility per recommendation for a θ-individual who follows the recommendations given by the peers of community $[\theta_1, \theta_2]$. When no answer to a query is obtained, a peer adopts the strategy that is optimal without information. Hence the overall utility for a θ-peer joining community $[\theta_1, \theta_2]$ is $P(\beta)U(\theta, [\theta_1, \theta_2]) + (1 - P(\beta)) \max(B(\theta), 0) - c$ where $\beta = (\theta_2 - \theta_1)/2$ denotes the community's size. The viability conditions require this value to be larger than the reservation value $\max(B(\theta), 0)$ for each θ in $[\theta_1, \theta_2]$. Using that $B(\theta)$ increases with θ, these conditions simplify into

$$P(\beta)U(\theta, [\theta_1, \theta_2]) - c \qquad \geq 0 \text{ for } \theta = \theta_2$$
$$P(\beta)[U(\theta, [\theta_1, \theta_2]) - B(\theta)] - c \quad \geq 0 \text{ for } \theta = \theta_1.$$

The first condition requires that the more exigent peers, those with θ_2 as reservation value, are at least as well off by searching for a recommendation on an object rather than never buying. If it holds true, it is also satisfied for θ smaller than θ_2. It is violated when the community accepts 'easy' individuals who inflict a loss to a θ_2-peer by inciting too much to buy, which occurs for θ_1 low enough. The second condition says that a peer with the smallest reservation value, θ_1, is at least as well off by searching for a recommendation on an object rather than buying it systematically. If it holds true, it is also satisfied for θ larger than θ_1 (by a simple argument). It is violated when the community accepts individuals who are much more strict than a θ_1-peer, (θ_2 large enough), making him lose many opportunities. To sum up, viability requires preferences not to be too diverse.

Consider now the issue of choosing the scope of a community. Write a community as $[m-\beta, m+\beta]$. The value per signal can be shown to be decreasing in the 'scope', that is $U(\theta, [m-\beta, m+\beta]$ decreases[9] with β whatever θ in $[m - \beta, m + \beta]$. Hence, as in the circle model, increasing the scope and making the senders' preferences more diverse is harmful to any peer everything else being equal. Taking into account the probability of finding the recommendation as a function of the size β, this defines the trade-off between receiving more useful information but less often. Hence, as in horizontal differentiation models, there is a value for a community, possibly closed: peers may want to restrict the access to the community. However, a main difference arises when signals are numerous, as we now investigate.

3.4 Multiple signals

So far, we have assumed that a peer searching for a recommendation stops at the first one she finds. We examine here the situation in which a peer possibly obtains several recommendations, as generated by the following behavior. A peer searches a given lapse of time and considers all the recommendations that she obtains during that lapse. The receipt of $\tilde{n}$ signals is described by a vector of 0 and 1, but the only relevant information is summarized in the number of positive signals, $\tilde{s}_{\tilde{n}}$. An alternative interpretation is that the community provides a statistic on the number of positive and negative recommendations. If all recommendations are recorded, this is the best statistic that

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[9]Up to a normalization, take the objects' characteristics to be distributed on $[0, 1]$. Utility per signal $U(\theta, [m - \beta, m + \beta]$ is equal to $\frac{1}{\beta} \int_{m-\beta}^{m+\beta} u(t - \theta)(t - (m - \beta))dt + \int_{m+\beta}^{1} (u(t - \theta)dt$. A marginal increase in the scope $d\beta$ leads to a marginal variation in utility equal to $\frac{d\beta}{\beta^2} \int_{m-\beta}^{m+\beta} u(t - \theta)[m - t]dt$. Since u is increasing, the inequality $[u(t - \theta) - u(m - \theta)][m - t] \leq 0$ holds for each t. We thus have $\int_{m-\beta}^{m+\beta} u(t - \theta)[m - t]dt \leq u(m - \theta) \int_{m-\beta}^{m+\beta} [m - t]dt = 0$. Thus marginal utility with respect to β is negative for each θ.

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can be provided in our setting. Consider a community C of size α (for the moment we do not need to specify the differentiation model). The expected utility is defined as

$$\hat{U}(\theta, C) = \sum_{n \geq 1} P(\alpha, n) U(\theta, C, n) \qquad (4)$$

where $P(\alpha, n)$ denotes the probability of receiving n signals from a community of size α and $U(\theta, \alpha, n)$ denotes the expected utility derived by a θ-peer upon the receipt of n signals from C under optimal behavior.

Observe that optimal behavior is not as simple as in the case of a single signal. In particular different peers may react differently on the receipt of the same signals. Given a number of signals, the optimal strategy for a peer is characterized by a threshold value such that a peer buys the object if the number of positive recommendations exceeds the threshold. The threshold depends on the number of signals and risk aversion. Considering the limit of the situation in which more and more recommendations are available turns out to be instructive and simple. In particular it allows us to answer the natural question of whether a large number of recommendations enables peers to make the right decision. Specifically, do they achieve the maximal payoff, namely the utility level reached under perfect information, when they buy all objects valuable to them and only those ? In the circle model for instance, the maximal payoff is given by $\frac{1}{2d^*} \int_{-d^*}^{d^*} u(t)dt$ for each θ.

To investigate this question, note that, given a t-object, each received signal is distributed according to the probability of a positive signal from the community conditional on t. Let $P(yes \in C|t) = g^C(t)$ denote this probability. By the law of large numbers, the proportion of positive signals, $\tilde{s}_n/n$, converges to $g^C(t)$ when n tends to ∞. Applying this property to the circle and vertical differentiation models gives a clear distinction between the two models, as stated in the following proposition.

Proposition 4 *Let $U(\theta, C, \infty)$ (resp. $U(\theta, C, n)$) be the utility derived by a θ-peer who is told the probability of a positive signal from community C conditional on t, $P(yes \in C|t) = g^C(t)$ (resp. the realization of n signals), and behaves optimally.*

1. *The sequence $U(\theta, C, n)$ is non-decreasing with n and converges (from below) to $U(\theta, C, \infty)$ when n tends to ∞.*

2. *$U(\theta, C, \infty)$ is strictly less than the maximal payoff for θ not in C.*

3. *In a circle model, whatever n, a community C is valuable to all its members only if it is of limited size, less than d^*. $U(0, C, \infty)$ is equal to the maximal payoff. For positive θ, under strict risk aversion, $U(\theta, C, \infty)$ is strictly less than the maximal payoff.*

   In a vertical differentiation model, $U(\theta, C, \infty)$ is equal to the maximal payoff for each θ in C.

The monotony and convergence results stated in point *1* follow from the fact that the statistics $\tilde{s}_n$ are getting more informative in the sense of Blackwell as n increases and converge to $g^C(t)$. These results yield conditions for the convergence of the overall expected utility $\hat{U}(\theta, C)$ as given in (4). Using that $U(\theta, C, n)$ is non-negative (used for n smaller than n_0) and non-decreasing gives $U(\theta, C, \infty) \geq \hat{U}(\theta, C) \geq P(\alpha, n \geq n_0)U(\theta, C, n_0)$ where $P(\alpha, n \geq n_0)$ denotes the probability of

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receiving at least n_0 signals from a community C of size α. Hence we obtain the following inequalities for each n_0:

$$0 \leq U(\theta, C, \infty) - \hat{U}(\theta, C) \leq [U(\theta, C, \infty) - U(\theta, C, n_0)] + P(\alpha, n < n_0)U(\theta, C, n_0)$$

Thus, since on the right hand side the term inside the bracket tends to zero as n_0 increases, the overall utility converges to $U(\theta, C, \infty)$ if the chances of getting few signals, $P(\alpha, n < n_0)$, tend to zero.

To assess the value $U(\theta, C, \infty)$ and to understand the distinction between the differentiation models as stated in point *3*, we need to consider optimal behavior upon the receipt of the probability $g^C(t)$. Observe first that in both differentiation models, the probability of receiving a positive signal from community C is equal to 1 in the acquiescence zone (every peer says *yes*) is null for the objects in the refusal zone (every peer says *no*), and is linear in between. Hence a peer in the community who is told that $g^C(t)$ is 1 (resp. 0) knows for sure that the object is valuable (resp. not valuable) to him. Observe that this is not true for an individual outside the community. Under vertical differentiation for instance, consider $C = [\theta_1, \theta_2]$ and someone who is more exigent than C peers, namely whose θ is larger than θ_2. Knowing that the object is valuable to all members of C ($g^C(t)$ is equal to 1) simply indicates that the quality exceeds the upper bound θ_2, which is not enough for the θ-individual to make the good decision for sure: this explains point *2*.

For a value of $g^C(t)$ strictly between 0 and 1, which indicates that the object is in the disagreement zone, we need to distinguish between the circle and vertical models.

Consider first the circle model. The value $g^C(t)$ reveals the distance to the center $|t|$, hence the location t or $-t$ each with equal probability. As shown in the proof, optimal behavior is characterized by a threshold value (which depends on θ) defined as the positive solution $\hat{t}$ to the equation $u(|\theta + t|) + u(|\theta - t|) = 0$: it is optimal for a θ-peer to buy the object if the value $|t|$ exceeds the threshold $\hat{t}$ and not to buy in the opposite case. For peers who are not located at the center, the threshold value is strictly smaller than d^* (under strict risk aversion). As a result, they lose some valuable purchase opportunities and are strictly worse off than under complete information. Hence, since the value $U(\theta, C, \infty)$ provides an upper bound to $U(\theta, C, n)$, peers -except those at the center- do not reach the maximal payoff even with an access to a large number of recommendations.

Consider now a vertical differentiation model. The probability for an object to be recommended reveals exactly its quality (the value $g^C(t)$ is one to one in the disagreement zone). Hence peers can make the right decision in all circumstances, namely buying the objects of quality larger than their reservation value θ and only those: peers get enough information (at the limit) to reach the maximal payoff. The basic reason for this result is that individuals' rankings are the same. Even though recommendations differ, their aggregation sends a clear message, namely a single cutoff quality, in contrast to a circle model.

This analysis points to important differences between differentiation models in the rationale for forming communities. Under vertical differentiation, in opposite to the circle model, there is no a priori upper bound on the size for a community to be viable. Furthermore, upon the receipt of a large number of signals, all peers within a community obtain almost all the information they need, and not those outside. This suggests that the largest community, without any restriction on membership, should emerge: more recommendations are provided, useful to all. This argument however should be checked more precisely: by arguing directly on the limit probability $g^C(t)$, we are neglecting the

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randomness in the (finite) statistic, the variance of which depends on the size of the community: the larger the size the larger the variance.

4 Configurations of communities

We have so far considered a single community. This section analyzes the coexistence of several communities. Individuals are free to join whatever community is open to them, or not to join any. We shall assume that each individual is willing to join one communiy at most. We first discuss equilibrium behaviors given a set of proposed communities, and next address the question of which proposals are made.

4.1 Equilibrium and coordination issues

Given a set of proposed communities, individuals' choices are based on how much value they expect to derive from a community. We look for an equilibrium under which these expectations are correct. Expectations on others' decisions play an important role and may result in multiple equilibria, some of which entailing some form of coordination failure.

A proposal to form a community is represented by an arc. In the following discussion, it is important to distinguish between the potential members, that is the individuals whose characteristics belong to the arc, and the individuals who decide to join and will form the community. When all potential members join, the community is said to be *full*. The probability $P(\alpha)$ should be interpreted as the success probability only for a full community of size α. If nobody joins the proposal for example, then the probability of success drops to zero. Hence, the viability condition, which is computed under the success probability P, implicitly assumes full communities.

Consider a proposed arc that intersects no other one, for example a single arc as in the previous section. Assume the arc to be viable. Since potential members face no other proposal, joining is 'rational' for them if they expect all others to join. Thus there is an equilibrium in which the full community forms. But, if nobody joins the proposal, then the probability of success drops to zero, making it rational not to join. Thus there is also an equilibrium in which no community forms. Coordination fails since everybody is worse off than at the equilibrium in which they all join. Such coordination failure does not occur under the following expectations. When a proposed arc is the only choice to its potential members, they compute the value of joining the community under maximal participation, that is by assessing the success probability by P.

When proposed arcs overlap, some individuals (in non-negligible number) have the choice between several proposals. The requirement that an individual picks up a preferred community among those open to him does not pin down a well defined behavior. The reason is that the individual's choice is based on the comparison between the utility levels expected from each proposal, and that such levels depend, in turn, on the expected community memberships (in their characteristics and numbers) through the derived distributions of the signals and their impact on the success probability. Hence individuals' decisions and expectations on those interact. As a result, several configurations may be perfectly compatible with rational behavior. The following example illustrates this point.

Let two proposals be overlapping and of identical size, $[-\alpha, \beta]$ and $[-\beta, \alpha]$ where $\alpha > \beta > 0$, as in Figure 4.1 (arcs are 'stretched'). Let the sizes $\alpha + \beta$ and α be viable but not $\alpha - \beta$. There is

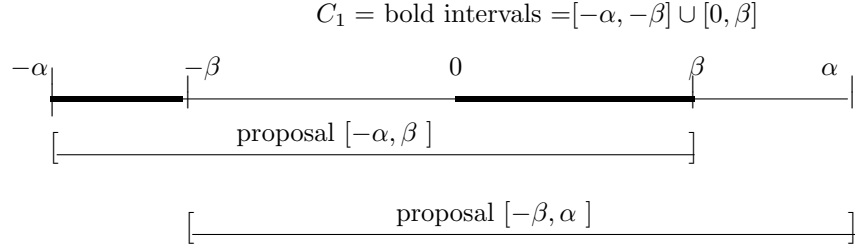$$C_1 = \text{bold intervals} = [-\alpha, -\beta] \cup [0, \beta]$$

Figure 4: Coordination failure

an equilibrium in which the full community $[-\alpha, \beta]$ forms and is the only one to form: under these expectations, all individuals who have the choice between the two proposals are better off by joining proposal $[-\alpha, \beta]$ (this is rational by viability), and the remaining individuals, whose characteristics are in $[\beta, \alpha]$, can only choose proposal $[-\beta, \alpha]$ and do not join because this can only result in a community included in $[\beta, \alpha]$, which is not viable. Similarly, there is an equilibrium in which the full community $[-\beta, \alpha]$ forms. In addition one may consider an equilibrium in which the peers who have the choice between the two proposals split themselves according to a 'cutoff' point. Here, taking 0 as the cutoff point (by symmetry), the configuration with the two communities $[-\alpha, 0]$ and $[0, \alpha]$ form an equilibrium: expecting that each one forms, peers in $[-\beta, \beta]$ prefer the community with the closest center (because communities are of equal size) hence peers in $(-\beta, 0)$ prefer community $[-\alpha, 0]$ to $[0, \alpha]$ and the reverse holds for peers in $(0, \beta)$.

Rational behavior does not prevent coordination failure, as occurs when individuals split themselves in the 'wrong way'. Still in the same example, let individuals who have the choice between the two proposals, those in $[-\beta, \beta]$, split at the cutoff point 0 but make the opposite choice to the one just considered. This results in two communities, each one composed with the individuals whose characteristics belong to disjoint arcs: community C_1 with individuals' characteristics in the two arcs $[-\alpha, -\beta]$ and $[0, \beta]$, and C_2 with the two arcs $[-\beta, 0]$ and $[\beta, \alpha]$. For some parameters α and β, the two communities C_1 and C_2 form an equilibrium.[10] This occurs when individuals in $(-\beta, 0)$ prefer C_2 to C_1 and those in $(0, \beta)$ prefer C_1 to C_2: expecting individuals' choices to lead to the formation of C_1 and C_2 makes it a rational choice. For example, taking a binary function, this occurs for $\beta = 2\alpha/3$ (see computation in the appendix). There is some form of coordination failure, because everybody is better off at the equilibrium with the two communities $[-\alpha, 0]$ and $[0, \alpha]$, where they split at 0 but make the opposite choice. The argument does not depend on symmetry and is more generally valid. Whenever an equilibrium results in two communities each one with two disjoint arcs, everybody would be better off if there was an exchange of peers of equal measure between the two communities: this exchange would keep the probability of success constant for both communities but would improve the quality of transmitted information. From now on, I focus on configurations without coordination failure, in which each community is full, composed with all individuals whose characteristics belong to a single arc. Such a configuration is described by a set of

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[10]The expected utility derived from joining C_1 assuming it forms is

$$P(\alpha/2)[\frac{(\alpha - \beta)}{\alpha} U(\theta, [-\alpha, -\beta]) + \frac{\beta}{\alpha} U(\theta, [0, \beta])] - c$$

because the chances of obtaining the signal from each interval are proportional to their size.

non-overlapping arcs. I focus on symmetric configurations (see however the end of section 4.2). A symmetric configuration is given by n non-overlapping communities of equal size α, with n at most equal to π/α. For n strictly smaller than π/α there are gaps between any two consecutive arcs (we do not necessarily require the gaps to be of equal size). For n equal to π/α, the union of the arcs fill the entire circle and the configuration is said to be without gaps. In what follows, we address two questions. Which configurations may last? Which configurations are optimal (in a sense to be made precise)?

4.2 Stability

A configuration may last if it is stable against proposals to form new communities or to rearrange the existing ones. Specifically, stability requires that no proposal to change is accepted. A proposal is said to be *accepted* if all members in the *new* community are better off than under the standing configuration. This notion is basically the blocking condition of cooperative game theory. This allows us to model a kind of competition between communities in a setting in which there is no unambiguous objective for a community: new proposals must make every peer better off in order to take care of the diversity of preferences.

Given a configuration, consider a proposal to form a new community. As discussed previously, expectations matter for evaluating a proposal. We shall assume that peers evaluate a proposal under the expectation that everybody joins. Denote by $\bar{u}(\theta)$ the utility level achieved by a θ-individual at the standing configuration, where θ is any point on the circle. For a symmetric configuration, $\bar{u}(\theta)$ is given by

$\bar{u}(\theta) = 0$ if the individual does not belong to any community,

$\bar{u}(\theta) = P(\alpha)U(d, \alpha) - c$ if he does and d is the distance to the center.

Write the proposal as $[m - \beta, m + \beta]$ where m is its center and β is its size. Proposal $[m - \beta, m + \beta]$ is said to *block* a standing configuration with utility levels $\bar{u}(\theta)$ if

$$P(\beta)U(|\theta - m|, \beta) - c > \bar{u}(\theta) \text{ for each } \theta \text{ in }]m - \beta, m + \beta[. \tag{5}$$

The left hand side gives the utility evaluated by a θ-individual in the proposal who expects that everybody joins the proposal. Condition (5) requires that such expectations are fulfilling since they make it rational for everybody to join.

I consider here proposals to form a new community that contain a community's leader, who can be assumed to be located at zero. Let us distinguish stability with respect to proposals that are larger in size than the existing community from those that are smaller. A symmetric configuration with communities' size α is *stable against enlargement* (resp. *stable against reduction*) if there is no proposal $[m - \beta, m + \beta]$ which includes zero and with size β larger (resp. smaller) than α that is accepted. Observe that although the proposals contain the leader, they are not necessarily centered at it. Hence an enlargement proposal does not necessarily contain the community $[-\alpha, \alpha]$ and a reduction proposal is not necessarily included in it. The only proposals that are excluded are those that try to attract peers of two consecutive communities without containing a leader, hence that contain less than half of the peers of each one.

Stability against enlargement.   Let α^{ext} be the value that maximizes PV, the payoff to an extreme individual. Observe that α^{ext} is less than α^0, thanks to assumptions *A0* and *A1* (the function

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$\log(PV)$ is concave and is decreasing at α^0 since $\frac{P'}{P}(\alpha^0) + \frac{V'}{V}(\alpha^0) \leq \frac{P'}{P}(\alpha^0) + \frac{U'_\alpha}{U}(0, \alpha^0) = 0$). Recall that α^{enl} is the unique solution to the equation $\alpha = inf_{\theta \leq \alpha} \alpha^\theta$.

Proposition 5 *A symmetric configuration with gaps is stable against enlargement if and only the communities' size is larger than α^{enl}. A symmetric configuration without gaps is stable against enlargement if the communities size is larger than α^{ext}, the maximizer of PV.*

For a configuration with gaps, the same argument as for a single community applies (and the bound follows from point *1* of Proposition 2). For a configuration without gaps, the stability requirement is much weaker. Since communities are adjacent, outsiders, whatever close, achieve a positive payoff at the standing configuration. Thus outsiders are more difficult to attract than in a configuration with gaps, which explains why smaller sizes than the leader's choice are compatible with stability.

Stability against reduction.

Proposition 6 *A symmetric configuration is stable against reduction if the communities' size is smaller than some value α^{red}, where $\alpha^{red} \geq \alpha^0$. Under B_-, one has $\alpha^{red} > \alpha^0$, and under B_+, $\alpha^{red} = \alpha^0$.*

The distinction between configurations with or without gaps is not relevant as far as reduction is concerned. As shown in the proof, stability against proposals that are smaller in size can be checked by considering only proposals that are included in one of the community: such a proposal excludes some of the members of an existing community and outsiders play no role.

Whereas stability to reduction requires communities not to be too large, stability to enlargement requires them to be large enough. From the previous results, both stability conditions are compatible.

Corollary. *A symmetric configuration with communities' size equal to the leader's choice is stable both against enlargement and reduction.*

A direct proof is straightforward since stability is checked against proposals that contain the leader.

Let us consider now the stability against all possible proposals, including those which contain no leader. One cannot rely on general results to prove the existence of a stable configuration (see Demange 2005 for a survey). A first reason is that preferences are characterized by a parameter on a circle (as opposed to an interval). The second reason is more fundamental, linked to adverse selection: the payoffs to the members of a community may decrease with newcomers. Hence even in a Hotelling or a vertical differentiation model, in which the parameter belongs to an interval, known existence results cannot be applied. In our model, although we do not show the existence of a stable configuration, we have no counterexample either. Let us examine the stability conditions against a proposal that contains no leader. Such a proposal attracts peers from two adjacent communities at most. Let it be centered at the adjacent point (it can be shown to be the one that has the more chances to be successful) with size β. We have $\beta \leq \alpha$ since the proposal contains no leader. The proposal is successful if and only if

$$P(\beta)V(\beta) \geq \bar{u}(\alpha - \beta).$$

This condition requires that an individual located at the extreme of the proposal, at distance β from the center of the proposal, hence at a distance $\alpha - \beta$ from the center in the standing configuration,

is willing to join. The condition is thus necessary. It is also sufficient since all individuals in the proposal expect a larger utility than $P(\beta)V(\beta)$ because they are closer to the center of the proposal, and achieves a lower level in the standing configuration, because they are further away to the current center. The inequality is surely not satisfied for β equal to 0 or α. When β runs between 0 and α, the left hand side is hump-shaped and the right hand side increases, which makes it difficult to find conditions on α ensuring that the inequality is not satisfied by any β.

We have so far considered symmetric configurations. Although an analysis of asymmetric configurations is difficult, simple properties can be derived. For a configuration with gaps between each two communities, the same conditions for stability as those stated in Propositions 5 and 6 apply on the size of each community. For a configuration without gaps, the analysis is the same as far as stability against reduction is concerned but is far more complex for the stability against enlargement. A difficulty is that stability does not require a peer at the adjacent point of two communities to be indifferent between both. This is due to the impact of adverse selection and the resulting non-monotony in the payoffs: let a peer at the adjacent point of two communities strictly prefer one community, say C_1 to the other C_2; this is also true for some of his neighbors in C_2 who would like to join C_1. But some peers in C_1 may be hurt by them joining, in which case the enlarged community does not block. As a result, we do not know much about the structure of an asymmetric stable configuration.

4.3   Efficient configurations

Our objective here is to investigate efficiency. In particular we study efficient configurations and their relationships with the stable ones. In a setup as here in which utility is not transferable, there is not a unique welfare criterion to assess efficiency. In line with our framework, we restrict ourselves to *anonymous* criteria. Such a criterion is characterized by a scalar function Φ that assigns a weight $\Phi(v)$ to a utility level v. Φ can be any increasing and continuous function, and is normalized by taking $\Phi(0) = 0$. To simplify the presentation, individual cost c is taken to be nil (which implies that the leader's choice is unconstrained). The welfare reached by a community of size α is given by

$$\int_{-\alpha}^{\alpha} \Phi[P(\alpha)U(\theta, \alpha)]d\theta. \tag{6}$$

and the total welfare at a configuration is the sum of the welfare within each community. Consider a symmetric configuration with viable community size α. Given the size, welfare is maximum at the maximal number communities. We shall neglect integer problems and take that there are $n = \pi/\alpha$ communities. Normalizing by the total size 2π for convenience, this gives the following expression for the welfare reached at a symmetric configuration with community size α

$$W_\Phi(\alpha) = \frac{1}{2\alpha} \int_{-\alpha}^{\alpha} \Phi[P(\alpha)U(\theta, \alpha)]d\theta. \tag{7}$$

In words, the normalized welfare is the average weighted utility per member. A concave function Φ represents an aversion to inequality, and at the opposite a convex function favors dispersion by putting increasingly larger weight on large utility levels.[11] This can be made precise by the following argument. Applying the intermediate value theorem to (7) gives that welfare is equal to

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[11] Observe that a configuration that maximizes a concave welfare criterion is necessarily symmetric.

$\Phi[P(\alpha)U(\hat{\theta}(\alpha), \alpha)]$ for some $\hat{\theta}(\alpha)$, the welfare of a 'representative individual'. The more concave the criterion is, the more distant from the center the representative individual is. Specifically, taking a criterion Φ_1 that is a concave transformation of Φ (i.e., $\Phi_1 = G(\Phi)$ for some G increasing and concave) $\hat{\theta}_1(\alpha)$ is larger than $\hat{\theta}(\alpha)$ (with obvious notation). At the limit, taking increasingly concave criteria, the representative position approaches the extreme position, ($\hat{\theta}(\alpha)$ tends to α) leading to the Rawlsian criterion, which puts weight on the minimum utility level only (a Rawlsian criterion is not associated to a function Φ and is obtained only at the limit). Similarly, an 'elitist' criterion that only values the maximum utility level within a community, here the center's utility level, is obtained as a limit of increasingly convex functions. To sum up, a criterion that is 'very' concave favors individuals with the lowest utility levels in a community, that is individuals close to the extreme, and at the opposite, a 'very' convex criterion favors the individuals with the highest utility levels in a community, those close to the center.

A Φ-optimal size is given by the value α^Φ that maximizes W_Φ. To understand the determinants of an optimal size, observe that there are two effects as the size of each community increases: a pure size effect within a community as if each peer could stay at the same distance to the center whereas its community expands, and a position effect, due to the rearrangement of the communities resulting from a reduction in their number. The position effect increases the distance to the center on average, which is detrimental. The size effect may or may not be beneficial to peers depending on their positions and the community's size. An optimal size balances the two effects. These position and size effects are summarized by considering the impact of the size on the 'representative' individual's utility, $P(\alpha)U(\hat{\theta}(\alpha), \alpha)$. An increase in α affects the utility of an individual located at the fixed position $\hat{\theta}(\alpha)$ (size effect) and the representative individual's position (change in $\hat{\theta}(\alpha)$) which is detrimental because the position increases. Consider the binary case for example and take $\alpha = \alpha^0$. Taking Φ linear, $\Phi(v) = v$, welfare is equal to $2\pi P(\alpha)U(\alpha/\sqrt{2}, \alpha)$, that is the 'representative' individual is located at $\alpha/\sqrt{2}$. The size effect is surely beneficial (because B_- is satisfied which implies that all peers within the community benefit from an increase in the size) so that the two effects are in opposite direction. The detrimental position effect can be shown to be stronger than the size effect: α^Φ is smaller than α^0. This is more generally true under an additional assumption $A2$ that evaluates the relative strength of the position and size effects and that we introduce now.

Consider $U(k\alpha, \alpha)$, the utility per signal associated to a fixed relative position k (between 0 and 1) in the community as the size α varies. $U(k\alpha, \alpha)$ is decreasing in α since U is decreasing in both arguments, the position and the size, and the relative loss is given by $\frac{-[kU_\theta + U_\alpha]}{U}(k\alpha, \alpha)$. At $k = 0$ the position effect is null, and the relative loss coincides with $\frac{-U_\alpha}{U}(0, \alpha)$. At $k = 1$, the utility coincides with $V(\alpha)$, and the relative loss is $\frac{-V'}{V}(\alpha)$. Thus, the elasticity assumption $A1$ requires the relative loss to be smaller for $k = 0$ than for $k = 1$. Assumption $A2$ strengthens this condition by requiring the relative loss to be increasing in the relative position k. Writing $k\alpha = \theta$ this is stated as follows.

$$Assumption\, A2 \qquad \frac{-V'}{V}(\alpha) \geq \frac{-[U_\alpha + \frac{\theta}{\alpha}U_\theta]}{U}(\theta, \alpha) \geq \frac{-U_\alpha}{U}(0, \alpha) \text{ for any } \theta \leq \alpha \leq \alpha^{max}.$$

$A2$ holds for the binary and quadratic functions.

Proposition 7 *Under A2, for any criterion Φ strictly positive, increasing and continuous, the optimal Φ-size of communities is larger than the optimal size for the Rawlsian criterion and smaller than the optimal size for the elitist criterion: $\alpha^{ext} < \alpha^\Phi < \alpha^0$.*

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Combining Propositions 6 and 7 yields the following corollary.

Corollary *Under A2 and B_-, a symmetric configuration without gaps and communities' size between α^0 and α^{red} is stable but is suboptimal for any criterion Φ.*

Under B_-, the preferred size of any peer is larger than the leader's one (from Proposition 2). This is why there are communities with a size larger than α^0 that are stable. The configuration is however surely not efficient: reducing each community entails some rearrangement in the communities, which leads to a decrease on average in the distance to communities' centers. These external effects are not taken into account by peers as they evaluate their community. According to Proposition 7 the overall impact on welfare of such a reduction would be positive whatever Φ.

When the size is larger than α^0, negative external effects are large so that the configuration is suboptimal whatever the criteria Φ.

It is difficult to predict which configurations will emerge without making explicit some ad-hoc formation process. At the beginning of a process however communities have few chances to intersect, and peers face a single opportunity at most so that communities should be of an optimal size when considered in isolation. Thus, their size should be optimal, in particular larger than α^0 if B_- holds (since peers unanimously prefer a size at least equal to α^0). Hence an efficient configuration, which requires the community's size to be smaller than α^0, can form only when proposals are made that attract peers from two neighbored communities and result in a split. This suggests that efficiency may be rather difficult to reach under B_-.

5   Concluding remarks.

This paper considers a community as a cluster of individuals with similar preferences. Under anonymity and limited language, the improvement in the value of information determines the scope of a community. This has been analyzed in detail when recommendations are sufficiently rare so that a peer uses the first one obtained, if any. With multiple signals, the value of a restricted community is still positive when individuals truly differ in their tastes, as in a horizontal differentiation model. In contrast, in a vertical differentiation model, in which individuals agree on ranking objects by a quality characteristic and differ only by their minimum valuable quality level, the value of a community may still be positive under a limited number of signals but vanishes as the number of available recommendations per object increases. Hence, limited language and anonymity do not necessarily imply that segregation into communities is valuable. Finally, I investigate the formation of several communities under the assumption that an individual belongs to a single community at most and show that stability and efficiency may enter into conflict.

The analysis could be extended in a variety of directions. A first direction is to analyze the aggregation of recommendations within a community, in particular to see whether some aggregation rules are more likely to be chosen, or some are more apt to ensure some sort of efficiency or stability. Another line of investigation is to allow individuals to join multiple communities. This would require to extend stability notions based on the threat of blocking to situations where coalitions overlap. A third line is to incorporate pricing considerations into the analysis. Interesting questions then bear on the fee structure, namely how (and whether) the recommendations given by peers are remunerated, and how the access to these recommendations is charged. Of course the answer to these questions should depend on the type of organization - say communities akin to co-operatives versus firms that

serve as intermediaries between the providers and the users of information as in two-sided markets. A very interesting issue then is how both types of organizations, communities and firms, interact.

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6   Proofs

PROOF OF PROPOSITION 1. Let $\alpha \leq d^*$. From (2), we have $U(\theta, \alpha) = p \int_{-\pi}^{\pi} f^{\alpha}(t) u(d(\theta, t)) dt$ where f^{α} is the density of objects conditional on the receipt of a *yes* from $[-\alpha, \alpha]$ and $p = P(yes \in [-\alpha, \alpha]) = d^*/\pi$. The prior density of t is $f(t) = 1/(2\pi)$. The density $f^{\alpha}(t)$ and the conditional probability of a positive signal given t, $g^{\alpha}(t) = P(yes \in [-\alpha, \alpha]|t)$, are related by Bayes formula:

$$f^{\alpha}(t) = P(yes \in [-\alpha, \alpha]|t) \frac{f(t)}{P(yes \in [-\alpha, \alpha])} = \frac{1}{2d^*} g^{\alpha}(t).$$

Given t, individuals with a type θ in $[t - d^*, t + d^*]$ say *yes* and others say *no*. Hence

$$g^{\alpha}(t) = P(yes \in [-\alpha, \alpha]|t) = \frac{1}{2\alpha} \int_{-\alpha}^{\alpha} 1_{[t-d^*,t+d^*]}(\theta) d(\theta)$$

In community $[-\alpha, \alpha]$, each member sends a signal *yes* for $t \in [\alpha - d^*, -\alpha + d^*]$, which we call the *acquiescence zone*, and each one sends *no* on $[\alpha + d^*, -\alpha - d^*]$, which we call the *refusal zone*. Outside these two zones there is *disagreement*. Since $\alpha \leq d^* \leq \pi/2$, all zones are non-empty. By symmetry $g^{\alpha}(t) = g^{\alpha}(-t)$. Restricting to $t \geq 0$ one has:

$$g^{\alpha}(t) = \begin{cases} 1 & \text{for } t \in [0, -\alpha + d^*] \text{ acquiescence zone} \\ \dfrac{\alpha + d^* - t}{2\alpha} & \text{for } t \in [-\alpha + d^*, \alpha + d^*] \text{ disagreement zone} \\ 0 & \text{for } t \in [\alpha + d^*, \pi] \text{ refusal zone} \end{cases}$$

which proves footnote 7.

To show the monotonicity of U, it is convenient to rewrite U. Let $F(\theta, \alpha; .)$ denote the distribution of the distance to θ of the objects that are recommended by community $[-\alpha, \alpha]$:

$$F(\theta, \alpha; \delta) = \int_{-\pi}^{\pi} f^{\alpha}(t) 1_{[\theta-\delta,\theta+\delta]}(t) dt. \tag{8}$$

U can be written as

$$U(\theta, \alpha) = p \int_{-\pi}^{\pi} u(\delta) dF(\theta, \alpha; \delta) \tag{9}$$

that is, $U(\theta, \alpha)$ is the expectation of $u(\delta)$ -the utility for an object distant of δ- under the distribution of the distance to θ of the objects that are recommended by community $[-\alpha, \alpha]$ time the probability of a positive recommendation. Function u is decreasing. Hence, U is decreasing with respect to θ or to α (property (i) or (ii)) if the distributions of the distance δ are increasing by first order stochastic

dominance as θ or α increases. Recall that these distributions increase with θ if for any $\delta, 0 \leq \delta \leq \pi$, any θ, θ' with $\theta \leq \theta'$

$$F(\theta', \alpha; \delta) \leq F(\theta, \alpha; \delta)$$

that is the function $F(., \alpha; \delta)$ *decreases* with respect to θ. Similarly the distributions increase with α if function F *decreases* with α.

(i) Let us show that F decreases with positive θ. From expression (8), F is differentiable with respect to θ with a derivative equal to $f^\alpha(\theta + \delta) - f^\alpha(\theta - \delta)$. This term is always non-positive for θ positive because the point $\theta - \delta$ is closer to the center than $\theta + \delta$.

(ii) Let us show that F decreases with respect to α. F is differentiable since function $f^\alpha(t)$ is differentiable almost everywhere with respect to α. Note that f^α has a null derivative except on the disagreement zones. Hence taking the derivative under the integral (8), one has $\partial F / \partial \alpha = I + J$ where

$$I = \int_{-\alpha + d^*}^{\alpha + d^*} f^\alpha_\alpha(t) 1_{[\theta - \delta, \theta + \delta]}(t) dt \text{ where } f^\alpha_\alpha(t) = -\frac{d^* - t}{8 d^* \alpha^2}.$$

and J is defined similarly over the negative disagreement zone. We show $I \leq 0$. Note that f^α_α is antisymmetric around d^*. Making a change of variable, I writes

$$I = \int_{d^*}^{\alpha + d^*} f^\alpha_\alpha(t) \left[1_{[\theta - \delta, \theta + \delta]}(t) - 1_{[\theta - \delta, \theta + \delta]}(2 d^* - t) \right] dt$$

Since f^α_α is positive on $[d^*, \alpha + d^*]$ it suffices to show that the term in brackets is non-positive, or that when $t \in [d^*, \alpha + d^*] \cap [\theta - \delta, \theta + \delta]$ then $2 d^* - t \in [\theta - \delta, \theta + \delta]$. Geometrically, this is true because the middle of $[\theta - \delta, \theta + \delta]$, θ, is by assumption less than d^*, the middle of $[-\alpha + d^*, \alpha + d^*]$. More formally observe that $t \in [d^*, \alpha + d^*] \cap [\theta - \delta, \theta + \delta]$ implies $d^* \leq t \leq \theta + \delta$, which gives $2 d^* - (\theta + \delta) \leq 2 d^* - t \leq d^* \leq \theta + \delta$. Now since θ is by assumption less than d^*, we also have $\theta - \delta \leq 2 d^* - (\theta + \delta)$, and finally $2 d^* - t \in [\theta - \delta, \theta + \delta]$, the desired result. Hence I is surely non-positive. The same argument applies to show $J \leq 0$, which proves $\partial F / \partial \alpha \leq 0$.

(iii) $V(\alpha) = U(\alpha, \alpha)$ is decreasing in α because, from the previous steps, U decreases in both arguments. Since V is positive for α equal to 0 and negative for α larger than d^*, there is a threshold α^{max} below (resp. above) which V is positive (resp. negative).

(iv) The person who could draw the largest benefits from a strategy of buying upon a negative signal is located at the opposite of the leader, at π. With such a strategy, the objects that are bought are in the refusal zone or in the disagreement zone (with some probability). The refusal zone $[\alpha + d^*, -\alpha - d^*]$ is an arc centered at π with size $\delta = \pi - \alpha - d^*$. The expected utility for the peer from buying all the objects is negative when this size is larger than $2 d^*$. Since $\pi - \alpha - d^* \geq \pi - 2 d^*$, this is true for $\pi/4 \geq d^*$. Also all the objects in the disagreement zone are distant from π by more than d^*. Observe that this argument holds more generally when peers search for more than one recommendation or when they receive a statistic of recommendations. ∎

PROOF OF PROPOSITION 2.

Point *1* is proved in the text.

Point *2*. The optimal choice of the leader is the value of α that maximizes $P(\alpha) U(0, \alpha)$ under the viability constraint $P(\alpha) V(\alpha) \geq c$. Viability implies that α is smaller than α^{max}, the value for which $U(\alpha, \alpha) = V(\alpha)$ is null. For such sizes, U and V are positive. The objective and the constraint are

log-concave so that the first order condition is sufficient (alternatively we could work with $logPU$ and $logPV$). Let μ be the multiplier associated with the constraint. The first order condition is

$$P_\alpha U(0,\alpha) + PU_\alpha(0,\alpha) + \mu[P_\alpha V + PV'](\alpha) = 0. \tag{10}$$

If μ is null, the optimal choice is α^0 that maximizes $PU(0,\alpha)$, the leader's optimum. If μ is positive, the constraint binds: α^0 is not viable, i.e. outside the interval $[\underline{\alpha}, \overline{\alpha}]$. Furthermore the solution solves $P(\alpha)V(\alpha) = c$, hence is either $\underline{\alpha}$ or $\overline{\alpha}$. Note that PV increases at $\underline{\alpha}$ and decreases at $\overline{\alpha}$. From (10), the derivatives of PU and PV are of opposite sign. If the latter derivative is positive, PV increases: the solution is $\underline{\alpha}$. Since under $A1$, $P_\alpha V + PV' > 0$ implies $P_\alpha U(0,\alpha) + PU_\alpha(0,\alpha) \geq 0$, it must be that the derivative of PV is non-positive: the solution is $\overline{\alpha}$. Furthermore we surely have $P_\alpha U(0,\alpha) + PU_\alpha(0,\alpha) > 0$ at $\alpha = \overline{\alpha}$.

Let us consider now θ-peers in the community with $0 < \theta$. Under $A0$, the function $logPU$ is concave with respect to α, with a derivative given by $[\frac{P_\alpha}{P}(\alpha) + \frac{U_\alpha}{U}(\theta,\alpha)]$. From $A1$ $[\frac{P_\alpha}{P}(\alpha) + \frac{U_\alpha}{U}(0,\alpha)] \geq 0$ implies $[\frac{P_\alpha}{P}(\alpha) + \frac{U_\alpha}{U}(\theta,\alpha)] > 0$. Thus, if the leader's choice is unconstrained, equal to α^0, the first term is null at α^0, which implies that $logP(\alpha^0)U(\theta,\alpha^0)$ increases for $0 < \theta$: peers not at the center prefer a larger size. If the leader's choice is constrained, we have seen that the first term is strictly positive at the leader's choice $\overline{\alpha}$, hence all peers would prefer a larger size.

Finally observe that when the cost is null, the leader is unconstrained: since the α^0-peer prefers a larger size than the leader, we surely have $U(\alpha^0, \alpha^0) > 0$.

Point *3* is straightforward. ∎

PROOF OF PROPOSITION 3. The optimal choice of the leader is the value that maximizes $P(\lambda,\alpha)U(0,\alpha) - c(\lambda)$ over (λ,α) subject to $P(\lambda,\alpha)V(\alpha) - c(\lambda) \geq 0$. Let μ be the multiplier associated with the constraint. The first order conditions are

$$P_\alpha U(0,\alpha) + PU_\alpha(0,\alpha) + \mu[P_\alpha V + PV'](\alpha) = 0 \tag{11}$$

$$P_\lambda U(0,\alpha) - c'(\lambda) + \mu[P_\lambda V(\alpha) - c'(\lambda)] = 0 \tag{12}$$

When μ is null, the viability constraint does not bind, and the leader can choose its optimal value. The same argument as in proposition 2 yields that for the chosen value of λ other peers would like the size to increase. As for the contribution rate, (12) yields $P_\lambda U(0,\alpha) - c'(\lambda) = 0$, hence $P_\lambda U(\theta,\alpha) - c'(\lambda) \leq 0$ since $U(\theta,\alpha) \leq U(0,\alpha)$: a θ-peer would prefer a smaller contribution rate.

When μ is positive, we know that (11) and $A1$ implies that α is set at the maximal viable size associated to the chosen λ, $\overline{\alpha}(\lambda)$. From the first order condition on λ (12), $P_\lambda U(0,\alpha) - c'(\lambda)$ and $P_\lambda V(\alpha) - c'(\lambda)$ are of opposite sign. Since $U(0,\alpha) > V(\alpha)$ it must be that the former is positive and the latter is negative: the leader would prefer to increase the contribution rate but the individual at the extreme would lose from this increase and quit the community since he just breaks even. Hence the trade-off because $\overline{\alpha}(\lambda)$ decreases. ∎

PROOF OF PROPOSITION 4 Point *1*. As n increases, the statistics $\tilde{s}_n$ (knowing n) are getting more informative in the sense of Blackwell. To see this, given a value s_n create a n-vector of 0 and 1 by assigning at random s_n values 1 and $n - s_n$ values 0. Taking the $n - 1$ first components yields a $n-1$-vector that is distributed according to $n - 1$ signals, which is informationally equivalent to the statistic s_{n-1}. Thus the sequence $U(\theta, C, n)$ increases in n. The sequence $U(\theta, C, n)$ is increasing in n. It converges to $U(\theta, C, \infty)$ since by the law of large numbers, the proportion of positive signals,

$\tilde{s}_n/n$, converges to $g^C(t)$ when n tends to ∞. This gives that $U(\theta, C, \infty)$ is an upper bound to $U(\theta, C, n)$ (which can be also derived by the fact that the statistics $\tilde{s}_n$ is less informative than $g^C(t)$ since it is the result of a sample drawn from that distribution).

Point *2* has been proved in the text, as well as point *3* for vertical differentiation. Let us consider now the circle model. Because of weak risk aversion, two individuals distant of more than $2d^*$ cannot benefit both from the community whatever the number of signals. Let the community be centered at 0 of size larger than d^* and choose θ with $d^* < \theta < \alpha$. We have that $u(|t + \theta|) + u(|t - \theta|) < 0$ whatever t since the sum of the distance $|t + \theta|$ and $|t - \alpha|$) is the maximum of 2θ and $2t$, hence larger than $2d^*$. This implies that $U(\theta, C, n) + U(-\theta, C, n) < 0$ and by symmetry $U(\theta, C, n) < 0$ for all n. Hence individuals at or close to the extreme point behave as if they had no information and never buy. Whatever value for the cost c, they strictly prefer not to join the community.

I prove now that the optimal strategy for a θ-peer on the receipt of $g^C(t)$ is characterized by the threshold $\hat{t}$ defined by the equation $u(|\theta + t|) + u(|\theta - t|) = 0$, The value of $g^C(t)$ gives that the object is either at t or $-t$ with equal probaility. Hence it is optimal to buy if $u(|\theta + t|) + u(|\theta - t|) > 0$ (and not to buy if the expression is < 0). The function is positive at $t = 0$ because the community size is smaller than d^*, is negative at $t = d^*$ for $\theta \neq 0$ because of risk aversion, and is decreasing in t: this proves the claim. For $\theta = 0$, or under risk neutrality the threshold value is d^*.

PROOF OF PROPOSITION 5. Enlarging a community that touches no other community is accepted if all peers and newcomers are better off. This is equivalent to say that the single community is not Pareto-optimal as considered in point *1* of Proposition 2, hence the result.

Consider now a configuration without gaps with community size α larger than α^{ext}. At the standing configuration, the minimum utility level is achieved by an individual located at an extreme point of a community, hence $\bar{u}(\theta) \geq (PV)(\alpha)$ for each θ. Given a proposal to an enlargement of size β larger than α, let us consider an individual located at an extreme point of the proposal. If the proposal is accepted, his expected payoff is $(PV)(\beta)$. This is strictly smaller than $(PV)(\alpha)$ since PV decreases for size larger than α^{ext}: the peer would be strictly worse off by accepting. By continuity, this is also true for individuals close to the extreme of the new community: the proposal is rejected. ∎

PROOF OF PROPOSITION 6 The proof is divided into two steps.

Step 1. This step shows that stability can be checked by considering proposals that are centered at the leader's position. For that, consider $[-m - \beta, -m + \beta]$, a reduction proposal centered at $-m$, $0 < m$. We have $m < \beta < \alpha$ because the leader belongs to the proposal. We show that if this proposal is accepted, then the centered proposal $[-\beta, \beta]$ is also accepted.

Formally, since up to a rotation $U(\theta, [-m - \beta, -m + \beta]) = U(\theta + m, [-\beta, \beta]) = U(\theta + m, \beta)$, the proposal $[-m - \beta, -m + \beta]$ is accepted if

$$P(\beta)U(\theta + m, \beta) - c > \bar{u}(\theta) \text{ for each } \theta + m \in [-\beta, \beta] \tag{13}$$

We have to show these inequalities imply

$$P(\beta)U(\theta, \beta) - c > \bar{u}(\theta) \text{ for each } \theta \in [-\beta, \beta]. \tag{14}$$

By symmetry of U and $\bar{u}$, it is sufficient to show that inequality (14) is met for positive θ. We consider separately θ in $[0, m]$ and θ in $[m, \beta]$.

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Let θ in $[0, m]$. The monotony of U (Proposition 1) and $\bar{u}$ implies the inequalities $U(\theta, \beta) > U(m, \beta)$ and $\bar{u}(0) \geq \bar{u}(\theta)$. Since taking $\theta = 0$ in (??) gives $P(\beta)U(m, \beta) - c > \bar{u}(0)$, we obtain that (14) holds for any θ with $0 \leq \theta \leq m$.

Let θ in $[m, \beta]$. Define $\theta' = \theta - m$. Note that both θ and θ' belong to $[0, \alpha]$ in the standing configuration and θ' is closer to the center than θ. Therefore $\bar{u}(\theta') > \bar{u}(\theta)$. Since $\theta' + m = \theta$ is in $[0, \beta]$, (13) applied to θ' gives $P(\beta)U(\theta, \beta) - c > \bar{u}(\theta') > \bar{u}(\theta)$, that is (14) holds for any θ with $0 \leq \theta \leq m$. ∎

Step 2. From Step 1, stability against reduction is checked by considering proposals centered at 0, $[-\beta, \beta]$, $\beta < \alpha$. For such proposals we have $\bar{u}(\theta) = P(\alpha)U(\theta, \alpha) - c$ for each $\theta \in [-\beta, \beta]$. Hence proposal $[-\beta, \beta]$ is accepted if

$$P(\beta)U(\theta, \beta) > P(\alpha)U(\theta, \alpha), \ \theta \in]-\beta, \beta[. \tag{15}$$

We shall distinguish between the two cases where B_+ or B_- holds.

Case 1 Assume B_+. Consider a community of size larger than α^0. A proposal of size α^0 is accepted. Conversely given a community of size less or equal than α^0, any proposal with a smaller size makes individuals close to the center worse off, hence is not accepted. Thus internal stability is satisfied for $\alpha \leq \alpha^0$.

Case 2 Assume B_-. Any size strictly below the leader's choice is internally stable: whether this choice is constrained or not, any peer prefers to increase the size.

By continuity, inequalities (15) imply $P(\beta)U(\beta, \beta) \geq P(\alpha)U(\beta, \alpha)$. Thus a sufficient condition for stability against reduction is

$$P(\alpha)U(\beta, \alpha) > P(\beta)U(\beta, \beta) \text{ for each } 0 < \beta < \alpha. \tag{16}$$

We show that, conversely conditions (16) are necessary for stability against reduction. By contradiction, let inequality $P(\beta^*)U(\beta^*, \beta^*) \geq P(\alpha)U(\beta^*, \alpha)$ be met for some β^* smaller than α. Proposal $[-\beta^*, \beta^*]$ makes β^*- or $-\beta^*$-individuals at least as well off. Conditions B_- then imply that all members in $]-\beta^*, \beta^*[$ are strictly better off. To see this, observe that the ratio $\frac{U(\theta, \alpha)}{U(\theta', \alpha)}$ decreases with α for $\theta < \theta'$ (because the derivative with respect to α of the log of the ratio is $\frac{U_\alpha(\theta, \alpha)}{U(\theta, \alpha)} - \frac{U_\alpha(\theta', \alpha)}{U(\theta', \alpha)}$, which is negative under B_-). Taking $\theta' = \beta^*$, this gives

$$\frac{U(\theta, \alpha)}{U(\beta^*, \alpha)} \leq \frac{U(\theta, \beta^*)}{U(\beta^*, \beta^*)} \text{ for } \theta < \beta^* < \alpha.$$

Using the assumption $P(\alpha)U(\beta^*, \alpha) \leq P(\beta^*)U(\beta^*, \beta^*)$, this gives

$$P(\alpha)U(\theta, \alpha) < P(\alpha)U(\theta, \beta^*)\frac{U(\beta^*, \alpha)}{U(\beta^*, \beta^*)} \leq P(\beta^*)U(\theta, \beta^*) \text{ for each } \theta < \beta^*$$

Thus, by symmetry, (15) is satisfied at $\beta = \beta^*$: proposal $[-\beta^*, \beta^*]$ is accepted.

Denote by A the set of sizes that satisfy inequalities (16). As just shown, the sizes that are viable and belong to A describe configurations that are stable against reduction. A is non-empty since it contains $]0, \alpha^0]$ (any peer prefers to increase the size). We show that A is an interval of the form $]0, \alpha^{red}[$. If (16) is satisfied for all viable sizes, $A =]0, \bar{\alpha}]$ and take $\alpha^{red} = \bar{\alpha}$. Otherwise, consider the smallest value, α^{red} for which (16) does not hold. We have $P(\alpha)U(\beta, \alpha) > P(\beta)U(\beta, \beta)$ for each $\alpha < \alpha^{red}$ and $\beta < \alpha$. Furthermore, since (16) does not hold at α^{red}, there is some $\alpha^* <$

α^{red} with $P(\alpha^{red})U(\alpha^*, \alpha^{red}) \leq P(\alpha^*)U(\alpha^*, \alpha^*)$. Therefore, by continuity, $P(\alpha^{red})U(\alpha^*, \alpha^{red}) = P(\alpha^*)U(\alpha^*, \alpha^*)$ and the function $\alpha \rightarrow P(\alpha)U(\alpha^*, \alpha)$ decreases at $\alpha = \alpha^{red}$. By logconcavity, it decreases for larger values than α^{red}, hence

$$P(\alpha)U(\alpha^*, \alpha) \leq P(\alpha^{red})U(\alpha^*, \alpha^{red}) = P(\alpha^*)U(\alpha^*, \alpha^*).$$

Inequality (16) does not hold for a size larger than α^{red}: the set A is the interval $]0, \alpha^{red}[$. ∎

PROOF OF PROPOSITION 7. Welfare W_Φ (7) writes:

$$W_\Phi(\alpha) = \frac{1}{\alpha} \int_0^\alpha \Phi[P(\alpha)U(\theta, \alpha)]d\theta. \tag{17}$$

Take the derivative with respect to α:

$$
\begin{aligned}
W_\Phi'(\alpha) &= \frac{1}{\alpha}\Big\{ \int_0^\alpha \Phi'[P(\alpha)U(\theta, \alpha)][PU_\alpha + P_\alpha U](\theta, \alpha)d\theta + \Phi[P(\alpha)U(\alpha, \alpha)] \\
&\quad - \frac{1}{\alpha}\int_0^\alpha \Phi[P(\alpha)U(\theta, \alpha)]d\theta \Big\}
\end{aligned}
\tag{18}
$$

Integration by parts of $\theta \rightarrow \theta\Phi[P(\alpha)U(\theta, \alpha)]$ over the interval $[0, \alpha]$ gives

$$\alpha\Phi[P(\alpha)U(\alpha, \alpha)] - \int_0^\alpha \Phi[P(\alpha)U(\theta, \alpha)]d\theta = \int_0^\alpha \theta\Phi'[P(\alpha)U(\theta, \alpha)]P(\alpha)U_\theta(\theta, \alpha)d\theta.$$

Hence

$$W_\Phi'(\alpha) = \frac{1}{\alpha}\int_0^\alpha \Phi'[P(\alpha)U(\theta, \alpha)][PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta](\theta, \alpha)]d\theta. \tag{19}$$

The term $[PU_\alpha + P_\alpha U]$ represents the size effect, and the term $[\frac{\theta}{\alpha}PU_\theta]$, which is always negative, reflects the position effect. We may restrict to viable sizes on which $PU(\theta, \alpha)$ is positive. Hence, writing $[PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta] = (PU)\left[\frac{P'}{P} + \frac{U_\alpha + \frac{\theta}{\alpha}U_\theta}{U}\right]$, Assumption $A2$ yields

$$\left[PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta\right](\theta, \alpha) \leq (PU)(\theta, \alpha)\left[\frac{P'}{P}(\alpha) + \frac{U_\alpha}{U}(0, \alpha)\right].$$

Consider the right hand side. For a size α larger than α^0, the term inside the square brackets is non-positive. Since PU is positive, the left hand side is non-positive as well. Using the expression (22) and the positivity of Φ', this implies the inequality $W_\Phi'(\alpha) < 0$ for $\alpha \geq \alpha^0$: surely $\alpha^\Phi < \alpha^0$.

A similar argument yields $\alpha^\Phi > \alpha^{ext}$ where α^{ext} is the maximum of $PV(\alpha)$, the payoff to an extreme individual. For $\alpha \leq \alpha^{ext}$, we have $\frac{P'}{P}(\alpha) + \frac{V'}{V}(\alpha) \geq 0$ (by concavity of $\log(PV)$). Under $A2$

$$[PU_\alpha + P'U + \frac{\theta}{\alpha}PU_\theta](\theta, \alpha) \geq (PU)(\theta, \alpha)\left[\frac{P'(\alpha)}{P(\alpha)} + \frac{V'}{V}\right] \geq 0.$$

This implies $W_\Phi'(\alpha) > 0$ for $\alpha \leq \alpha^{ext}$, which gives $\alpha^{ext} < \alpha^\Phi$. ∎

APPENDIX

BINARY FUNCTION. Given a binary function, set $k = \frac{(b+g)}{4gd^*}$. As computed below the utility levels for peers inside a community of size $\alpha \leq d^*$ are given by

$$U(\theta, \alpha) = pg[1 - k\alpha(1 + \frac{\theta^2}{\alpha^2})] \text{ for } \theta \in [-\alpha, \alpha] \tag{20}$$

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and for those outside the community at a distance less than d^*

$$U(\theta, \alpha) = pg[1 - 2k\theta] \text{ for } \alpha < |\theta| \leq d^*. \tag{21}$$

Let us check that assumptions *A0*, *B_−*, and *A2* (which implies *A1*) are satisfied. Equation (20) gives $V(\alpha) = pg(1 - 2k\alpha)$ and $\alpha^{max} = 1/2k$. Hence *A0* is met since U and V are concave in α.

B_−. We have

$$-\frac{U_\alpha}{U}(\theta, \alpha) = k(1 - \frac{\theta^2}{\alpha^2})/[1 - k(\alpha + \frac{\theta^2}{\alpha})].$$

Hence the function is decreasing in θ^2 if $\frac{k}{\alpha} < \frac{1}{\alpha^2}[1 - k\alpha]$ or equivalently if $1 - 2k\alpha > 0$, that is $\alpha \leq \alpha^{max} = 1/2k$. Consider now *A2*. One has:

$$U(\theta, \alpha) = U(0, \alpha) - pgk\frac{\theta^2}{\alpha}, \ U_\alpha(\theta, \alpha) = U_\alpha(0, \alpha) + pgk\frac{\theta^2}{\alpha^2}, \ U_\theta(\theta, \alpha) = -2pgk\frac{\theta}{\alpha}.$$

This gives $-[U_\alpha + \frac{\theta}{\alpha^0}U_\theta] = -U_\alpha(0, \alpha) + pgk\frac{\theta^2}{\alpha^2}$, which is increasing and positive with θ in $[0, \alpha]$. Since U decreases, *A2* is met.

Computation of (20). We shall use repeatedly that for t_1, t_2 in the (positive) disagreement zone one has

$$\int_{t_1}^{t_2} f^\alpha(t)dt = \frac{-1}{8d^*\alpha}(\alpha + d^* - t)^2|_{t_1}^{t_2} = \frac{1}{8d^*\alpha}(t_2 - t_1)(2\alpha + 2d^* - (t_2 + t_1)) \tag{22}$$

Let us compute the utility derived from buying conditional on a *yes*, for a θ-individual in the community, i.e., U divided by p. For objects $t \geq 0$, he achieves:

in the acquiescence zone $[0, -\alpha + d^*]$: g,

in the disagreement zone $[-\alpha + d^*, \alpha + d^*]$: g for any object in $[-\alpha + d^*, \theta + d^*]$, and $-b$ on $[\theta + d^*, \alpha + d^*]$. No object in the refusal zone receives a *yes*. This gives using (22)

$$g\frac{d^* - \alpha}{2d^*} + \frac{1}{8d^*\alpha}[g(\alpha + \theta)(3\alpha - \theta) - b(\alpha - \theta)^2].$$

By symmetry, the utility for a θ-individual on negative t is equal to that of a ($-\theta$)-individual on positive t. Thus the utility for t-objects with $t \leq 0$ is equal to:

$$g\frac{d^* - \alpha}{2d^*} + \frac{1}{8d^*\alpha}[g(\alpha - \theta)(3\alpha + \theta) - b(\alpha + \theta)^2]$$

Collecting terms gives that the overall utility of a θ-peer receiving a *yes* is equal to:

$$g(\frac{d^* - \alpha}{d^*}) + \frac{1}{4d^*\alpha}[g(3\alpha^2 - \theta^2) - b(\alpha^2 + \theta^2)]$$

Rearranging and multiplying by p gives (20). ∎

Consider now an individual outside the community whose distance to the center is less than d^* and let us prove (21). We compute the utility derived on objects with positive t for θ-individual and distinguish positive and negative θ.

A θ-individual with $\alpha < \theta < d^*$, gets the benefit g for all positive objects that he buys: the individual likes all t-objects with $0 \leq t \leq \theta + d^*$ and a t-object can be recommended only if $t < \alpha + d^*$ which is smaller than $\theta + d^*$. Thus he achieves $g/2$ ($1/2$ is the probability of an object being in the positive zone knowing that it is recommended). An individual located at $-\theta$, $\alpha < \theta < d^*$, achieves in the acquiescence zone g for objects with $0 \leq t \leq -\theta + d^*$ and $-b$ for $-\theta + d^* \leq t \leq -\alpha + d^*$. In the

disagreement zone he dislikes all objects hence achieves $-b$. This gives a total of $g\frac{d^*-\theta}{2d^*} - b\frac{\theta-\alpha}{2d^*} - b\frac{\alpha}{2d^*}$, or $g/2 - (b+g)\frac{\theta}{2d^*} = g(1/2 - 2k\theta)$. Using the symmetry property again, collecting terms and multiplying by p gives (21). The utility is positive for $\theta \leq d^*(2g/(b+g)) \leq d^*$, the last inequality because $g \leq b$. ∎

QUADRATIC FUNCTION. Let $u(d) = d^{*2} - d^2$. We first show that $U(\theta, \alpha) = \frac{d^*}{\pi}[\frac{d^{*2}}{3} - \frac{2}{3}\alpha^2 - \theta^2]$.

Note that $d(t, \theta) = |t - \theta|$ for all t when this expression is smaller than π. This is surely the case for t not in the refusal zone, that is $|t| \leq \alpha + d^*$ and $\theta \in [-\alpha, \alpha]$ (for such t we have $|t - \theta| \leq 2\alpha + d^*$ and it suffices to use $\alpha \leq d^*$ and the assumption $d^* < \pi/4$ to obtain $2\alpha + d^* < \pi$). Hence, since f^α is null in the refusal zone, the value of U writes $U(\theta, \alpha) = \frac{d^*}{\pi}[\int_{-\pi}^{\pi}(d^{*2} - (\theta - t)^2)f^\alpha(t)dt$.

This implies that $U(0, \alpha) - U(\theta, \alpha)$ is independent of α. To see this write $U(0, \alpha) - U(\theta, \alpha) = \frac{d^*}{\pi}\int_{-\pi}^{\pi}[(\theta - t)^2 - t^2]f^\alpha(t)dt$, and developing $(\theta - t)^2 - t^2 = \theta^2 - 2\theta t$, the integral of the term associated to the product θt is null by symmetry of f^α. Using $\int_{-\pi}^{\pi} f^\alpha(t) = 1$ gives $U(0, \alpha) - U(\theta, \alpha) = \frac{d^*}{\pi}\theta^2$.

Thus $U_\alpha(\theta, \alpha)$ is independent of θ. Hence the ratio $[-U_\alpha/U](\theta, \alpha)$ increases with θ because $-U_\alpha$ is positive and U decreases with θ: B_+ is satisfied.

It remains to compute $U(0, \alpha) = \frac{d^*}{\pi}[\int_{-\pi}^{\pi}(d^{*2} - t^2)f^\alpha(t)dt] = \frac{d^*}{\pi}[d^{*2} - \int_{-\pi}^{\pi} t^2 f^\alpha(t)dt]$. Simple but tedious computation gives successively

$$\int_{-\pi}^{\pi} t^2 f^\alpha(t)dt = \frac{1}{d^*}(\int_0^{d^*-\alpha} t^2 dt + \int_{d^*-\alpha}^{d^*+\alpha} t^2 \frac{\alpha + d^* - t}{2\alpha}dt)$$

$$= \frac{1}{12\alpha d^*}[(d^* + \alpha)^4 - (d^* - \alpha)^4] = \frac{2}{3}[d^{*2} + \alpha^2]$$

Hence $U(0, \alpha) = \frac{d^*}{\pi}[\frac{d^{*2}}{3} - \frac{1}{3}\alpha^2]$ and $U(\alpha, \alpha) = \frac{d^*}{\pi}[\frac{d^{*2}}{3} - \frac{5}{3}\alpha^2]$. Thus U is concave in both α and θ. $A0$, B_+ are satisfied hence also $A1$ and one checks that $A2$ is satisfied. The value α^{max} that satisfies $U(\alpha, \alpha) = 0$ is equal to $d^*/\sqrt{5}$. ∎

COORDINATION FAILURE EXAMPLE. Given proposals $[-\alpha, \beta]$ and $[-\beta, \alpha]$, let C_1 be the individuals whose characteristics belong to $(-\alpha, -\beta) \cup (0, \beta)$, and C_2 to $(-\beta, 0) \cup (\beta, \alpha)$. We claim that for $\beta = 2\alpha/3$ the two communities C_1 and C_2 form an equilibrium. The expected utility levels derived from choosing C_1 and C_2 under the expectations that they form are given respectively by (see footnote 10)

$P(\frac{\alpha}{2})[\frac{(\alpha-\beta)}{\alpha}U(\theta, [-\alpha, -\beta]) + \frac{\beta}{\alpha}U(\theta, [0, \beta])]$ and $P(\frac{\alpha}{2})[\frac{(\alpha-\beta)}{\alpha}U(\theta, [\beta, \alpha]) + \frac{\beta}{\alpha}U(\theta, [-\beta, 0])]$.

We shall use the following property. From the expressions (20) and (21) the utility derived from an arc is concave with respect to the distance ($|\theta|$) to the center for the peers inside the arc and it is linear in that distance for the peers outside. Overall the function is concave since it is differentiable at the boundary point when $\theta = \alpha$.

We first show that peers at the boundaries, $-\beta$, 0, and β are indifferent between C_1 and C_2. Given the symmetry, it is obvious for 0 and it suffices to show it for $-\beta$ for instance. Observe that $-\beta$ is outside or at the extreme of any of the arcs considered. Hence the utility from either community is derived by computing the weighted distance of $-\beta$ to the center of each arc in the community. One checks that for $\beta = 2\alpha/3$, the weighted distance to the arcs in C_2 is $\frac{2}{3}\frac{\beta}{2} + \frac{1}{3}(\frac{\alpha+3\beta}{2})$, and that to C_1 is $\frac{2}{3}\frac{3\beta}{2} + \frac{1}{3}(\frac{\alpha-\beta}{2})$ which are both equal to $\frac{\alpha+5\beta}{6}$: the $-\beta$-peer is indifferent between the two communities.

It remains to show that the peers in the interior of $(-\beta, 0)$ or (β, α) prefer C_2 to C_1. As a function of θ in $(-\beta, 0)$, the utility for C_2 is concave and the utility for C_1 is linear (since such a θ is

outside the intervals forming C_1). Hence the difference in the utility levels is a concave function of θ in $(-\beta, 0)$ and is null at the extreme $-\beta$ and 0: all peers in $(-\beta, 0)$ prefer C_2 to C_1. The same result holds for θ in the arc (β, α) since a β-peer is indifferent between C_2 to C_1 and an α-peer clearly prefers C_2 to C_1. ∎