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Multidimensional Communication Mechanisms: Cooperative and Conflicting Designs *

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Abstract

This paper investigates optimal communication mechanisms with a two-dimensional policy space and no monetary transfers. Contrary to the one-dimensional setting, when a single principal controls two activities undertaken by his agent (cooperative design), the optimal communication mechanism never exhibits any pooling and the agent's ideal policies are never chosen. However, when the conflicts of interests between the agent and the principal on each dimension of the agent's activity are close to each other, simpler mechanisms that generalize those optimal in the one-dimensional case perform quite well. These simple mechanisms exhibit much pooling.

When each activity of the agent is controlled by a different principal (non-cooperative design) and enters separately into the agent's utility function, optimal mechanisms under private communication take again the form of simple delegation sets, exactly as in the one-dimensional case. When instead the agent finds some benefits in coordinating actions, a one-sided contractual externality arises between principals under private communication. Under public communication instead, there does not exist any pure strategy Nash equilibrium with continuous and piecewise differentiable communication mechanisms. Relaxing the commitment ability of the principals restores equilibrium existence under public communication and yields partitional equilibria. Compared with private communication, public communication generates discipline or subversion effects among principals depending on the profile of their respective biases with respect to the agent's ideal policies.

KEYWORDS: Communication; Delegation; Mechanism Design; Multi-dimensional Decision; Common Agency.

JEL CLASSIFICATION: C72; D82; D86.

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1 Introduction

Consider an informed party (the agent) who contracts with an uninformed receiver (his principal). When the principal's and the agent's interests are conflicting, the principal may exert some ex ante control on the agent by restricting the possible decision set from which this agent may pick actions. Such restriction help indeed to align objectives especially in circumstances where monetary transfers are not feasible.

Examples of such settings abound both in industrial organization and political science. Shareholders exert some form of ex ante control on the firm's CEO who is better informed on market conditions by restricting a priori the kind of business plans that the latter can implement. This CEO himself may control division managers by designing capital budgeting rules.[1] Other aspects of the firm's decisions related to the design, the quality and prices of its products or the quantity of its polluting emissions are instead controlled by regulators. Those regulators impose prices cap and minimal quality standard on those different aspects of the firm's activities. Congress Committees exert ex ante control on a better informed bureaucracy by designing various procedures and rules that put constraints on those regulatory agencies.[2]

Those examples have all in common that principals control their agents but can hardly use monetary transfers to do so. This limited ability to use transfers may be due to a legal ban as in the case of a regulatory agency legally prevented from using transfers towards regulated firms. It may also come from the limited response of agents to monetary incentives as in the case of a bureaucrat subject to Legislative control. Lastly, principals may not be able to credibly commit to use transfers to align their objectives with those of their agents. Following the seminal work of Melumad and Shibano (1991), those settings have been fruitfully analyzed as mechanism design problems without monetary transfers.[3]

In such instances, the Revelation Principle[4] states that the principal should commit ex ante to a communication mechanism that determines which actions should be taken by the agent as a response to any report that the latter can make on his private information. However, communication may be used strategically by the agent to promote his own interests instead of those of his principal. This imposes incentive compatibility constraints which limit the set of feasible allocations. An alternative interpretation relying instead on the Taxation Principle,[5] observes that everything happens as if the principal was offering a set of actions and let the agent choose from that set in accordance with his private information.[6]

Quite intuitively, the impossibility of using monetary transfers makes optimal communica-

[1] Harris and Raviv (1996).

[2] McCubbins et al. (1987).

[3] See Holmström (1984), Armstrong (1994), Baron (2000), Martimort and Semenov (2006), Goltsman et al. (2007), and Alonso and Matouschek (2008), among others.

[4] Myerson (1982).

[5] Rochet (1985) and Guesnerie (1995) for instance.

[6] This approach has for instance been recently pursued by Alonso and Matouschek (2008).

tion mechanisms look very crude when the set of actions controlled by the principal is one-dimensional. Indeed, a principal finds it hard to induce information revelation and align objectives with a single instrument. Roughly, in a one-dimensional setting, an optimal mechanism must thus balance the flexibility gains of letting the agent choose an action according to his own private information and the agency cost coming from the fact that the principal's and the agent's most preferred actions may conflict. There is a trade-off between *rules* and *discretion* in such environments. Inflexible rules involving much pooling over different realizations of the agent's information allow the principal to choose his most preferred policy although this choice is uninformed. Leaving discretion to the agent allows implemented policies to vary with the agent's information but this policy only reflects the agent's preferences and not those of the principal. In such one-dimensional contexts, the existing literature has uncovered properties of the optimal delegation sets, for instance, whether it is connected, capped from above or from below, and where those caps and floors are set, etc.[7]

Of course, a major theoretical issue is to assess the robustness of the previous findings. In particular, one may wonder whether the trade-off between rules and discretion still prevails in more complex models in which, even though transfers cannot be used, principals might control more than one of the agent's activities. If the answer is positive, it is also of great interest to assess whether optimal communication mechanisms turn out to be rather inflexible and involve much pooling. The present paper investigates the form of those optimal communication mechanisms when the agent's activity is two-dimensional. This question is highly relevant given that most of economic examples highlighted above involve indeed the control of more than one activity of his agent.

The investigation of multidimensional communication mechanisms raises several new theoretical problems of great interest. A first set of issues is related to the new contracting possibilities that arise in such contexts. Indeed, although the principal has still no transfers to better align his objectives with those of the agent, he may now trade off distortions on different aspects of the agent's activity to facilitate information revelation. The principal can recoup the cost of extra distortions on one dimension through the benefits in having lesser distortions on another one.

A second set of issues is instead related to the extra distortions that might arise when the various dimensions of the agent's activities are controlled by different non-cooperating principals. Examples of such non-cooperative designs again abound. A public bureaucracy is generally subject to the oversights of various legislative Committees. The prices of a regulated firm are capped by a Public Utility Commission whereas its polluting emissions must meet environmental standards set up by the Environmental Protection Agency. Different parent firms share control of the same downstream unit on retailing markets. In these contexts, we investigate whether

[7] Melumad and Shibano (1991), Martimort and Semenov (2006), Goltsman et al. (2007) and Alonso and Matouschek (2008).

and how and why competition between mechanism designers becomes an obstacle to trading off distortions on different dimensions to facilitate information revelation. We unveil also the consequences for the non-cooperative design of communication mechanisms of having either public or private communication in each bilateral relationship involving the agent and one of his principals.

Let us briefly review our main findings in each environment under scrutiny.

Cooperative Design: We characterize the optimal communication mechanism in a two-dimensional environment where the principal's biases with the agent's ideal point on each dimension of the latter's activities are different. This difference in biases makes it valuable for the principal to trade off distortions on each dimension. Quite surprisingly and in sharp contrast with the one-dimensional case, the optimal communication mechanism *never* exhibits any pooling in a simple environment with quadratic single-peaked utility functions and a uniform distribution of types. The agent always choose different action profiles as his private information changes. Moreover, the agent's ideal policies are never chosen at the optimal mechanism. This highlights again the gains for the principal of trading off distortions on each dimension.

The design of the optimal communication mechanism is made rather complex because of an unusual nonlinearity due to the absence of monetary transfers. We develop techniques to solve that difficult problem. Intuitively, what matters from the incentive viewpoint is, on the one hand, the average decision that the principal would like to implement and, on the other hand, its "variance", i.e., how apart levels of each activity should be. More variance may facilitate information revelation but it comes also at a cost from the principal's viewpoint.

More formally, the characterization of the optimal communication mechanism relies on the calculus of variations. The non-differentiability of the principal's objective function requires to use powerful results from that theory to ensure the existence of a solution and derive sufficient and necessary conditions for optimality.

However, we also show that quite *simple mechanisms* approximate well the performances of the optimal mechanism when the principal's biases on each dimension of the agent's activity are quite similar, i.e., when activities are sufficiently symmetric. Those simple mechanisms exhibit much pooling as in the one-dimensional case. These mechanisms are tractable generalizations of the optimal mechanisms used in the one-dimensional case.

Non-Cooperative Design: When non-cooperating principals control different aspects of the agent's two-dimensional activity, trading off distortions on each of those activities is no longer possible. The consequences of such non-cooperative design depend then on whether the agent's communication with each of those principals is either *private* or *public.*

Under private communication, the agent communicates privately with each principal. In that situation, the results of the one-dimensional model à la Melumad and Shibano (1991) carry over when the different activities enter separately into the agent's utility function. Hence, equilibrium mechanisms take the form of simple delegation sets and are the same as if principals were dealing separately with the agent. When instead the agent finds some benefits in coordinating actions, a one-sided contractual externality arises between principals. The one facing the highest bias with the agent still offers the same policy as if he was alone contracting with the agent, i.e., he imposes a floor on policy to avoid that the agent takes too low decisions. Instead, the principal having more congruence with the agent is affected by this floor constraint which, when binding, forces the agent to coordinate actions and implement an higher action.

Under public communication instead, the agent sends a message about his preferences which is publicly observable by both principals. Public communication opens new possibilities for competing principals because each of them may use the public report as another screening device complementary to the activity he controls. In other words, the public message can be used by either principal as another dimension of the agent's activity facilitating his own screening. Contrary to the case of a cooperative design, moving around that dimension is now costless for either principal. Since a given principal's payoff depend only on the activity he controls himself and not on how the public report affects the activity controlled by his rival, this new screening possibility comes at no cost. Hence, a principal has always some incentives to deviate and offer a communication mechanism that induces the agent to publicly lie. As a result, there does not exist any pure strategy equilibrium of that common agency game with continuous and piecewise differentiable mechanisms.

This non-existence suggests that the right institutional environment for studying public communication may instead have the agent move first before principals choose actions. In the corresponding private or public cheap talk games, partition equilibria generalize those found in the case of a single receiver.[8] In addition, generalizing Farrell and Gibbons (1989), we show how public communication generates discipline or subversion effects among principals depending on the sign and magnitude of the conflicts on each activity. We compare players' welfare under the two different communication protocols.

Section 2 reviews the relevant literature. Section 3 presents the model and the by-now standard results where a one-dimensional activity of the agent is controlled by a single principal. Section 4 derives the optimal mechanism when a single principal controls a two-dimensional vector of the agent's activities. Section 5 analyzes the case of a non-cooperative design with private and public communication. It also derives the partition equilibria and their welfare properties under both private and public cheap talk communication. Lastly, Section 6 concludes and points out a few alleys for further research. All proofs are relegated to the Appendix.

[8] Crawford and Sobel (1982).

2 Literature Review

Our paper is related to several strands of the literature. Since the work of Crawford and Sobel (1982), the literature on communication in settings with private information and conflicting interests has significantly spread with most analysis focusing on the cheap-talk timing where informed agents move first and sometimes imposing further constraints to compare alternative game forms.[9] Taking a more normative stance, Melumad and Shibano (1991) provided the first significant analysis of delegation problems in contexts with no transfers where the uninformed party (the principal) can move first and commit ex ante to offer a communication mechanism to the uninformed party. With quadratic payoffs and a uniform types distribution, Melumad and Shibano (1991) showed that the optimal mechanism is continuous provided the principal's and the agent's ideal points do not vary too differently. Alonso and Matouschek (2008) provide further characterization results showing how simple connected delegation sets are optimal;[10] a feature that was a priori assumed in Holmström (1984), Armstrong (1994) and Baron (2000) for instance. Focusing on dominant strategy to get sharp predictions on the set of incentive feasible allocations, Martimort and Semenov (2008) have extended this mechanism design literature to the case of multiple privately informed agents (lobbyists) dealing with a single principal (a legislature) in a political economy context where the principal chooses upon a one-dimensional policy. Battaglini (2002) has considered a cheap talk setting with multiple privately informed senders and a multidimensional policy.[11] However, none of these papers has addressed the design of multi-dimensional communication mechanisms with a single agent.

This multi-dimensionality issue was nevertheless investigated in a cheap-talk environment by Farrell and Gibbons (1989). Those authors have analyzed a bare-boned model (binary types and binary decisions) with a sender communicating with two audiences who may have different levels of conflict with the sender. A particular attention is given to the existence of a fully revealing equilibrium under private and public cheap talk. Our analysis of the mechanism design environment shows that even a tiny difference between the levels of conflict on each dimension is enough to generate fully separating equilibrium allocations under a cooperative design but it is also a significant obstacle to the existence of pure strategy equilibria in a non-cooperative context.

Through its analysis of the non-cooperative design of communication mechanisms, our paper contributes to the common agency literature in several respects. First, the applied literature in this field has only derived equilibrium properties in contexts where monetary transfers between principals and their agent are available.[12] This is a significant limitation especially in view of the importance of that paradigm to understand political organizations where such transfers

[9] See Gilligan and Krehbiel (1987) and Dessein (2002) for instance.
[10] See also Martimort and Semenov (2006) on that issue.
[11] See also Krishna and Morgan (2001).
[12] See Martimort (2006) for a survey.

are generally not available and shared control between competing principals is rooted in the principles of Checks and Balances.[13] Second, our specific concern on communication allows us to highlight significant differences in the equilibrium outcomes of the game depending on whether communication with the principals is either private or public. Although the common agency literature has sometimes stressed differences in the equilibrium outcomes when either all contracting variables are publicly available to all principals (public common agency) or not (private common agency),[14] the issue of private versus public communication investigated in this paper has not received any attention to the best of our knowledge.

3 The Model

Two principals, P_1 and P_2, control and are respectively interested by two actions, x_1 and x_2, undertaken by a common agent A on their behalf. We denote by $(x_1, x_2) \in \mathbb{R}^2$ the bi-dimensional vector of those actions. Utility functions for these players are single-peaked, quadratic and respectively given for the principals and their agent by:

$$V_i(x_i, \theta) = -\frac{1}{2}(x_i - \theta - \delta_i)^2, \ i = 1, 2, \text{ and } U(x_1, x_2, \theta) = -\frac{1}{2}\sum_{i=1}^{2}(x_i - \theta)^2.$$

With those preferences, the agent's ideal point on each dimension is θ whereas principal P_i has an ideal point located at $\theta + \delta_i$. Our formulation entails no externality be they direct or indirect between the principals' controlled actions; i.e., neither principal P_i's marginal utility with respect to x_i nor that of the agent depends on x_{-i}.[15] Principals are biased in the same direction but their own ideal points may be more or less distant from that of the agent, i.e., $0 < \delta_1 \leq \delta_2$. In the sequel and to avoid trivial situations where principals offer pooling allocations whatever the agent's type, we will assume $\delta_2 < \frac{1}{2}$. For further references, we denote $\Delta \equiv \delta_2 - \delta_1$ the degree of polarization between principals and $\delta \equiv \frac{\delta_1+\delta_2}{2}$ the average bias between those principals and their agent.

• **Information:** The agent has private information on his ideal point θ, that is drawn from a uniform distribution on $\Theta = [0, 1]$.[16] Principals are not informed about the agent's type.

• **Mechanisms:** Under a *cooperative design*, principals merge into a single entity who controls the whole vector of the agent's activities (x_1, x_2). From the Revelation Principle (Myerson, 1982), there is no loss of generality in restricting the analysis to direct communication mechanisms stipulating (maybe random) decisions as a function of the agent's report on his type. Following

[13] We thank Thomas Palfrey for pointing out this issue. Semenov (2008) developed a model along these lines.

[14] See again Martimort (2006) for a discussion and related references.

[15] Martimort (2006) provides a definition for those externalities and more discussion in a framework where monetary transfers are allowed. Section 5.2 analyzes such externalities when we introduce the possibility that the agent benefits from coordinating actions.

[16] The characterization of some contractual outcomes below, especially in the case of a cooperative design, would be untractable if we were assuming other distributions.

Melumad and Shibano (1991) and Kovac and Mylovanov (2007), we will further restrict the analysis to deterministic communication mechanisms. Any such deterministic communication mechanism is thus a mapping $x(\cdot) = \{x_1(\cdot), x_2(\cdot)\} : \Theta \to \mathbb{R}^2$.

On the contrary, under *non-cooperative design*, principals do not cooperate in designing communication mechanisms with the agent. We will be more explicit on the nature of competition between mechanism designers below but for the time being it is already useful to distinguish two non-cooperative institutions in terms of the degree of transparency of the communication protocol.

- ⋆ **Private communication:** Principal P_i chooses a communication space $\mathcal{M}_i$ with the agent and a mapping $x_i(\cdot) : \mathcal{M}_i \to \mathbb{R}$. The message m_i sent by A is privately observed by principal P_i.
- ⋆ **Public communication:** Each principal chooses a communication mechanism $x_i(\cdot) : \mathcal{M} \to \mathbb{R}$ where $\mathcal{M}$ is a common communication space which is given at the outset. The agent's message $m \in \mathcal{M}$ is observed by both principals.

• **Timing:** The communication game between principals and their agent unfolds as follows:

- ⋆ First, the agent learns the state of nature θ.
- ⋆ Second, principals simultaneously offer their communication mechanisms $\{x_i(\cdot), \mathcal{M}_i\}$ either cooperatively or not (non-cooperative communication being then either public or private).
- ⋆ Third, the agent communicates with the principals under the chosen communication modes (cooperative or non-cooperative and public/private), and the requested actions are implemented.

• **Benchmark: The one-principal case.** For further references, let us consider the case where a single principal, say P_i, controls the decision x_i taken by the privately informed agent. Incentive compatibility requires:

$$\theta \in \arg\max_{\hat{\theta}} -\frac{1}{2}\left(x_i(\hat{\theta}) - \theta\right)^2. \tag{1}$$

Focusing on continuous and piecewise differentiable mechanisms,[17] incentive compatibility implies:

$$\dot{x}_i(\theta)\left(x_i(\theta) - \theta\right) = 0. \tag{2}$$

[17] It is well-known since Melumad and Shibano (1991) that discontinuous mechanisms may be incentive compatible provided those discontinuities are conveniently designed so that, at the type where the discontinuity arises, the agent is just indifferent between moving up or down. Continuous mechanisms are nevertheless often sufficient to characterize the optimum as shown by Melumad and Shibano (1991) in the case of a uniform distribution and quadratic payoffs, Martimort and Semenov (2006) for more general distributions, and Alonso and Matouschek (2008) for more general payoffs and distributions. Moreover, continuous and piecewise differentiable mechanisms are admissible in the sense of the calculus of variations that will be used in the sequel to characterize the optimal mechanism.

It is straightforward to check that such a mechanism is of the form:[18]

$$x_i(\theta) = \min\left\{\max\left\{\theta_i^*, \theta\right\}, \theta_i^{**}\right\}. \tag{3}$$

Within this class of communication mechanisms, one can easily find the optimal one as follows.

Proposition 1 (One Principal) *Assume that a single principal P_i communicates with the agent. The optimal communication mechanism $x_i^*(\theta)$ entails:*

$$x_i^*(\theta) = \max\left\{\theta, \theta_i^*\right\}, \quad \textit{where } \theta_i^* = 2\delta_i < 1. \tag{4}$$

The optimal communication mechanism that a single principal would offer had he being alone contracting with the agent has a simple structure: It corresponds to the agent's ideal point if the latter is large enough and is otherwise independent of the agent's preferences. This outcome can be easily achieved by means of a simple *Delegation Set*. Instead of using a direct revelation mechanism and communicating with the agent, the principal could as well offer a menu of options $\mathcal{D}_i = [\theta_i^*, 1]$ and let the agent freely choose within this set. When the floor θ_i^* is not binding, the agent is not constrained by the principal's choice of a delegation set and everything happens as if he had full discretion in choosing his own ideal point. When the floor is binding, the agent is constrained and cannot choose a policy which is too low compared with what the principal would implement himself.

The optimal communication mechanism trades-off the benefits of flexibility (the agent choosing sometimes a state-dependent action) against the loss of control it implies (this state-dependent action being different from the principal's ideal point). Setting a floor θ_i^* limits the agent's discretion and alleviates the loss of control. Clearly, θ_i^* increases with δ_i meaning that a more rigid rule is chosen when the conflict of interests between the principal and the agent is more pronounced. The optimal communication mechanism entails thus much pooling. We will see later how this trade-off between rules and discretion is modified when principals can control more than one decision taken by the agent.

The approach above in terms of delegation sets relies on the so-called *Taxation Principle* to find out an optimal mechanism.[19] From Martimort and Stole (2002), we know that this approach is also particularly useful in the case of a non-cooperative design analyzed below.

4 Cooperative Design

We first consider the case where principals cooperate in designing a communication mechanism with the agent. We assume that, when merging into a single entity, principals have equal bargaining powers.

[18] See Melumad and Shibano (1991), and Martimort and Semenov (2006) for instance.
[19] See Rochet (1985) and Guesnerie (1995).

4.1 Incentive Compatibility

Under cooperative design, incentive compatibility constraints can be written as:

$$\theta \in \arg\max_{\hat{\theta}} -\frac{1}{2}\sum_{i=1}^{2}\left(x_i(\hat{\theta})-\theta\right)^2.$$

Using revealed preferences arguments, it is straightforward to show that $\sum_{i=1}^{2} x_i(\theta)$ is non-decreasing in θ and thus almost everywhere differentiable with, at any differentiability point,

$$\sum_{i=1}^{2}\dot{x}_i(\theta) \geq 0. \tag{5}$$

Such incentive compatible mechanism must also satisfy the following first-order condition of the agent's revelation problem:

$$\frac{\partial}{\partial\hat{\theta}}\left(\sum_{i=1}^{2}(x_i(\hat{\theta})-\theta)^2\right)\Bigg|_{\hat{\theta}=\theta} = \sum_{i=1}^{2}\dot{x}_i(\theta)(x_i(\theta)-\theta) = 0. \tag{6}$$

The optimal communication mechanism within the class of continuous and piecewise differentiable ones must maximize the expected payoff of the merged principal:

$$(\mathcal{P}^c): \quad \min_{\{x_1(\cdot),x_2(\cdot)\}} \int_0^1 \left(\sum_{i=1}^{2}(x_i(\theta)-\theta-\delta_i)^2\right)d\theta$$

subject to (5) and (6).

The merged principal can now use both $x_1(\cdot)$ and $x_2(\cdot)$ to better screen the agent's preferences. To better understand how it can be so, it is useful first to observe that, when merging and designating cooperatively a communication mechanism, principals could at least offer the optimal communication mechanisms they would offer separately, namely the pair of mechanisms described in Proposition 1. Although communication mechanisms that would satisfy (2) for $i = 1, 2$ also satisfy (6), more communication mechanisms become now available. By trading-off distortions on x_1 against distortions on x_2, the merged principal could for instance introduce countervailing incentives which might facilitate information revelation.[20]

Example 1 Consider indeed the linear communication mechanism $\{x_1^\alpha(\theta), x_2^\alpha(\theta)\}_{\theta\in\Theta}$ such that $x_1^\alpha(\theta) = \theta - \alpha$ and $x_2^\alpha(\theta) = \theta + \alpha$ where α is a fixed number. This mechanism is incentive compatible since it satisfies both (5) and (6). The best of such communication mechanisms maximizes the merged principal's profit, i.e., α should be optimally chosen so that any concession made by the merged principal on x_1 by moving this decision closer to the agent's own ideal point

[20] See Lewis and Sappington (1989) and Laffont and Martimort (2002, Chapter 3) for models with monetary transfers and countervailing incentives.

is compensated by an equal shift in x_2 in the direction of the merged principal's ideal point. Typically, a uniform distortion $\alpha = \frac{\Delta}{2}$ does the trick since

$$\arg\min_{\alpha} \int_0^1 \left((x_1^{\alpha}(\theta) - \theta - \delta_1)^2 + (x_2^{\alpha}(\theta) - \theta - \delta_1)^2 \right) d\theta = \arg\min_{\alpha} (\alpha + \delta_1)^2 + (\alpha - \delta_2)^2 = \frac{\Delta}{2}.$$

Still such decision rules may be "on average" too close to the agent's ideal point. The simple mechanism $\{x_1^{\alpha}(\theta), x_2^{\alpha}(\theta)\}_{\theta\in\Theta}$ can be improved upon by introducing a pooling area as in the one-dimensional case.

Example 2 Consider thus the new incentive compatible mechanism $\{\tilde{x}_1(\theta), \tilde{x}_2(\theta)\}_{\theta\in\Theta}$ obtained as follows:

$$\tilde{x}_1(\theta) = \begin{cases} \theta - \frac{\Delta}{2} & \text{if } \theta \geq \tilde{\theta} \\ \tilde{\theta} - \frac{\Delta}{2} & \text{otherwise} \end{cases} \text{ and } \tilde{x}_2(\theta) = \begin{cases} \theta + \frac{\Delta}{2} & \text{if } \theta \geq \tilde{\theta} \\ \tilde{\theta} + \frac{\Delta}{2} & \text{otherwise.} \end{cases} \tag{7}$$

This new mechanism is obtained by piecing together a floor on policy for $\theta \leq \tilde{\theta}$ and a mechanism trading off distortions on each dimension for $\theta \geq \tilde{\theta}$. The optimal such mechanism, denoted thereafter $\{\tilde{x}_1^*(\theta), \tilde{x}_2^*(\theta)\}_{\theta\in\Theta}$, is such that

$$\begin{aligned} \tilde{\theta} = & \arg\min_{\tilde{\theta}} \int_0^1 \left(\sum_{i=1}^2 (\tilde{x}_i(\theta) - \theta - \delta_i)^2 \right) d\theta \\ = & \arg\min_{\tilde{\theta}} \int_0^{\tilde{\theta}} (\tilde{\theta} - \theta - \delta)^2 d\theta + \int_{\tilde{\theta}}^1 \delta^2 d\theta = 2\delta, \end{aligned}$$

which corresponds to a non-trivial pooling area since $2\delta < 1$. This mechanism is such that cooperating principals limit the pooling area to an average between the pooling areas that they would individually choose.

The above example is instructive because it stresses two aspects of optimal mechanisms that our more general analysis will confirm: First, cooperating principals trade off distortions on the upper tail of the distribution; second, decision rules should be rather flat for types in the lower tail of that distribution.

To better understand how distortions on each variable can be traded one against the other more generally, it is useful to transform a little bit the problem. Define the agent's *information rent* $U(\theta)$ as:

$$U(\theta) \equiv \max_{\hat{\theta}} -\frac{1}{2} \left(\sum_{i=1}^2 \left(x_i(\hat{\theta}) - \theta \right)^2 \right).$$

Let also introduce two extra auxiliary variables

$$x(\theta) \equiv x_1(\theta) + x_2(\theta) \text{ and } t(\theta) \equiv \frac{x_1^2(\theta)}{2} + \frac{(x(\theta) - x_1(\theta))^2}{2}. \tag{8}$$

Clearly, $x(\theta)$ is related to the "average" decision whereas $t(\theta) - \frac{x^2(\theta)}{4} = \frac{1}{2}(\frac{x(\theta)}{2} - x_1(\theta))^2 + \frac{1}{2}(\frac{x(\theta)}{2} - x_2(\theta))^2$ is a measure of the "variance" of the decisions.

Inserting the expression of $x_1(\theta)$ obtained from the first equation in (8) into the second one and solving a second-degree equation yields:[21]

$$x_1(\theta) = \frac{x(\theta)}{2} - \sqrt{t(\theta) - \frac{x^2(\theta)}{4}} \quad \text{and} \quad x_2(\theta) = \frac{x(\theta)}{2} + \sqrt{t(\theta) - \frac{x^2(\theta)}{4}}. \tag{9}$$

Using these expressions helps to rewrite $U(\theta)$ as:

$$U(\theta) = \max_{\hat{\theta}} -\frac{1}{2}\left(\left(\frac{x(\hat{\theta})}{2} - \sqrt{t(\hat{\theta}) - \frac{x^2(\hat{\theta})}{4}} - \theta\right)^2 + \left(\frac{x(\hat{\theta})}{2} + \sqrt{t(\hat{\theta}) - \frac{x^2(\hat{\theta})}{4}} - \theta\right)^2\right),$$

which finally yields

$$U(\theta) = \max_{\hat{\theta}} \theta x(\hat{\theta}) - t(\hat{\theta}) - \theta^2. \tag{10}$$

The agent's utility depends only on the decisions rule through the aggregate decision $x(\theta)$ and the quantity $t(\theta)$. It becomes "quasi-linear" with the "transfer" $t(\theta)$ measuring the cost for the agent in choosing different decisions on each dimension. The technical difficulty that we will face in the sequel comes from the fact that this transfer does not enter linearly into the principal's objective. This will force us to develop specific techniques tailored to the environment under scrutiny.

The aggregate decision $x(\theta)$ has an impact on the agent's marginal utility which depends on his realized type. It can thus be used as a screening variable as in standard screening models. Clearly, an agent with type θ may be tempted to lie downward to move the aggregate decision closer to his own ideal point. The merged principal can make that strategy less attractive by putting more "risk" on the agent, increasing the variance of the decision for the lowest types.[22]

The incentive compatibility conditions (5) and (6) give us that $U(\theta) = \theta x(\theta) - t(\theta) - \theta^2$ is absolutely continuous with a first derivative defined almost everywhere and given by the Envelope Theorem as

$$\dot{U}(\theta) = x(\theta) - 2\theta, \tag{11}$$

with a second-order derivative which, to satisfy the second-order condition (5), is written as

$$\ddot{U}(\theta) = \dot{x}(\theta) - 2 \geq -2. \tag{12}$$

Note also that $t(\theta) \geq \frac{x^2(\theta)}{4}$ implies

$$\dot{U}(\theta) \leq 2\sqrt{-U(\theta)}, \tag{13}$$

with an equality only when $x_1(\theta) = x_2(\theta) = x(\theta)/2$, i.e., when decisions on each dimension are the same.[23]

[21]Observe that $x_i^2(\theta) - x(\theta)\,x_i(\theta) - t(\theta) + x^2(\theta)/2 = 0$. Since $t(\theta) \geq \frac{x^2(\theta)}{4}$ and $x_2(\theta) \geq x_1(\theta)$ (recall that $\delta_2 \geq \delta_1$), this equation has two solutions given in (9).

[22]The principal-agent literature has already stressed that a principal can use the agent's risk-aversion to ease incentives (see, e.g., Arnott and Stiglitz, 1988) by for instance using stochastic mechanisms. Introducing some variance in the agent's decisions in a model with quadratic payoffs has a similar flavor.

[23]This is typically the case in the one-dimensional world. The fact that (13) is not an equality reflects the multi-dimensionality aspect of the screening problem.

4.2 Optimal Multi-Dimensional Mechanisms: Design and Properties

With the new set of variables, we rewrite the merged principal's payoff in each state of nature θ as (modulo a constant):

$$U(\theta) + \sum_{i=1}^{2} \delta_i x_i(\theta) = U(\theta) + \delta x(\theta) + \Delta\sqrt{t(\theta) - \frac{x^2(\theta)}{4}}$$

$$= U(\theta) + \delta(\dot{U}(\theta) + 2\theta) + \Delta\sqrt{-U(\theta) - \frac{\dot{U}(\theta)^2}{4}},$$

where the second equality follows from (9) and the third uses the expression of $t(\theta)$ and $x(\theta)$ in terms of $U(\theta)$ and $\dot{U}(\theta)$ coming from (11). This yields the following expression of the cooperating principals' relaxed problem:

$$(\mathcal{P}^c_\Delta): \quad \max_{U(\cdot)} \int_0^1 L_\Delta(U(\theta), \dot{U}(\theta))d\theta,$$

where

$$L_\Delta(U(\theta), \dot{U}(\theta)) = U(\theta) + \delta\dot{U}(\theta) + \Delta\sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}}.$$

Let us denote by $U^*(\cdot)$ the solution to $(\mathcal{P}^c_\Delta)$. Provided this schedule satisfies also the second-order condition (12) it is indeed the solution to the original problem $(\mathcal{P}^c)$. This solution allows then to recover the pair of decision rules $\{x_1^*(\cdot), x_2^*(\cdot)\}$ using the formulae:

$$x_1^*(\theta) = \theta + \frac{\dot{U}^*(\theta)}{2} - \sqrt{-U^*(\theta) - \frac{(\dot{U}^*(\theta))^2}{4}} \text{ and } x_2^*(\theta) = \theta + \frac{\dot{U}^*(\theta)}{2} + \sqrt{-U^*(\theta) - \frac{(\dot{U}^*(\theta))^2}{4}}. \quad (14)$$

For the time being, we proceed as follows. First, we derive a second-order Euler-Lagrange equation satisfied by any solution to the so relaxed problem $(\mathcal{P}^c_\Delta)$. Second, a first quadrature tells us that such solution solves a first-order differential equation known up to a constant. Finally, we shall impose conditions on that constant so that the second-order condition (12) holds.

In the parlance of the calculus of variations, $(\mathcal{P}^c_\Delta)$ is actually a Bolza problem with free end-points.[24] However, it is non-standard because the functional $L_\Delta(s, v)$ even though it is continuous, and strictly concave in (s, v) is not everywhere differentiable (or even Lipschitz), especially at points where $-s - \frac{v^2}{4} = 0$ if any such point exists on an admissible curve where $v(\theta) = \dot{U}(\theta)$ and $s(\theta) = U(\theta)$. Nevertheless, we first prove the following existence result:

Lemma 1 *A solution to* $(\mathcal{P}^c_\Delta)$ *exists.*

[24] Clarke (1990).

Putting aside the difficulties linked to the non-differentiability of the functional at some points, the following Euler-Lagrange equation must hold at any interior point of differentiability:

$$\frac{\partial L_\Delta}{\partial U}(U(\theta),\dot U(\theta)) = \frac{d}{d\theta}\left(\frac{\partial L_\Delta}{\partial \dot U}(U(\theta),\dot U(\theta))\right). \tag{15}$$

On top of that, our Bolza problem admits also the following free end-points necessary conditions on the boundaries of the interval $[0,1]$:

$$\frac{\partial L_\Delta}{\partial \dot U}(U(\theta),\dot U(\theta))|_{\theta=0} = \frac{\partial L_\Delta}{\partial \dot U}(U(\theta),\dot U(\theta))|_{\theta=1} = 0. \tag{16}$$

A priori, the Euler-Lagrange condition is a second order ordinary differential equation. The next Lemma investigates further the nature of the solution to this equation by obtaining a first quadrature parameterized by some integration constant $\lambda \in \mathbb{R}$. This constant must be non-positive to ensure that the second-order condition (12) holds.

Lemma 2 *For each solution $U(\theta,\lambda)$ to (15) which is everywhere negative and satisfy (12), there exists $\lambda \in \mathbb{R}_-$ such that*[25]

$$\dot U(\theta,\lambda) = 2\sqrt{-U(\theta,\lambda) - \Delta^2\left(\frac{U(\theta,\lambda)}{U(\theta,\lambda)+\lambda}\right)^2}, \tag{17}$$

and

$$(U(\theta,\lambda)+\lambda)^2 - \Delta^2 U(\theta,\lambda) > 0 \text{ for all } \theta. \tag{18}$$

We are now ready to characterize the optimal mechanism under a cooperative design.

Proposition 2 (Cooperative design) *: Assume that $\Delta > 0$ and that principals cooperate in designing a communication mechanism. The optimal communication mechanism is such that:*

- *The rent profile $U^*(\theta)$ is everywhere negative, strictly increasing and satisfies $U^*(\theta) = U(\theta,\lambda^*)$ with the boundary conditions:*

$$U^*(0) = -\lambda^* - \frac{1}{2}\left(\Delta^2 + 4\delta^2 + \sqrt{(\Delta^2+4\delta^2)^2 + 4\lambda^*(\Delta^2+4\delta^2)}\right) \tag{19}$$

  and

$$U^*(1) = -\lambda^* - \frac{1}{2}\left(\Delta^2 + 4\delta^2 - \sqrt{(\Delta^2+4\delta^2)^2 + 4\lambda^*(\Delta^2+4\delta^2)}\right); \tag{20}$$

[25] It is important to note that the differential equation (17) may a priori have a singularity and more than one solution going through a given point, namely when, for such solution, there exists θ_0 where $U(\theta_0,\lambda) + \Delta^2\left(\frac{U(\theta_0,\lambda)}{U(\theta_0,\lambda)+\lambda}\right)^2 = 0$. Indeed, the right-hand side of (17) fails to be Lipschitz at such a point. It turns out that this possibility does not arise for the optimal mechanism described below because a careful choice of λ ensures that the condition (18) holds everywhere on the optimal path.

- $\lambda^* \in (-\frac{\Delta^2}{4} - \delta^2, -\frac{\Delta^2}{4})$ *is defined implicitly as a solution to:*

$$U^*(1) - U^*(0) = \int_0^1 \dot{U}(\theta, \lambda) d\theta; \tag{21}$$

- *Optimal decisions on each dimension are respectively given by*

$$x_1^*(\theta) = \frac{x^*(\theta)}{2} - \Delta\left(\frac{U^*(\theta)}{U^*(\theta) + \lambda^*}\right) < x_2^*(\theta) = \frac{x^*(\theta)}{2} + \Delta\left(\frac{U^*(\theta)}{U^*(\theta) + \lambda^*}\right) \tag{22}$$

with

$$x^*(\theta) = 2\theta + \dot{U}^*(\theta);$$

- *There is no pooling area. Second-order conditions are satisfied everywhere*

$$\dot{x}^*(\theta) = -\frac{4\Delta^2\lambda^* U^*(\theta)}{(U^*(\theta) + \lambda^*)^3} > 0. \tag{23}$$

Several features of this optimal communication should be stressed. First, and in sharp contrast with the case of a one-dimensional decision, the agent's ideal point is never chosen at the optimal mechanism. By trading off distortions on each dimension, the cooperating principals are always able to induce truthtelling without having to let the agent be residual claimant for those decisions. Second, even when the agent's ideal point is on the lower tail of the distribution, there is no need to offer a pooling contract; $\dot{x}^*(\theta) > 0$ everywhere. Again, there is always a better option than a pooling contract which is to trade off distortions on each decision even if it is marginally so.

For further references, let define the image of the aggregate decision rule $x^*(\cdot)$ as $Im\, x^* = [x^*(0), x^*(1)]$. The next corollary compares this aggregate decision in the optimal mechanism with the aggregate decision induced by the simpler communication mechanism $\{\tilde{x}_1^*(\theta), \tilde{x}_2^*(\theta)\}_{\theta\in\Theta}$ introduced at the beginning of the section, i.e.,

$$\tilde{x}^*(\theta) = \tilde{x}_1^*(\theta) + \tilde{x}_2^*(\theta) = \max\{4\delta, 2\theta\}.$$

Notice that in the limit case where $\Delta = 0$, this mechanism coincides with the optimal mechanism described in Proposition 2 for the one-dimensional problem, with

$$U^*(\theta) = -(\min\{\theta - 2\delta, 0\})^2 \quad \text{and} \quad \lambda^* = 0.$$

Corollary 1 *For any $\Delta > 0$, we have $[4\delta, 2] \subsetneq Im\, x^*$. More precisely, there exists $\theta^*(\Delta) \in (0, 2\delta)$ such that:*

$$x^*(\theta) < 4\delta = \text{ if and only if } \theta \le \theta^*(\Delta).$$

Moreover, we have also:

$$x^*(\theta) > 2\theta \text{ for all } \theta \in [0,1].$$

This corollary first shows that the aggregate decision is systematically spread over a greater interval than if the principals were restricted to offer the simple mechanism $\{\tilde{x}_1^*(\theta), \tilde{x}_2^*(\theta)\}_{\theta\in\Theta}$. There would be a systematic bias in the analysis one would restrict the modeler to use only those simple mechanisms.

Second, the corollary also shows that the benefits of fine-tuning the distortions on decisions allows cooperating principals to move up the aggregate decision further away from the agent's ideal points. These features are illustrated in Figure 1, which compares the aggregate decision $\tilde{x}^*$ with the optimal aggregate decision x^* for a fixed average bias δ and different values of Δ. The associated agent's information rent under the optimal mechanism is represented in Figure 2.

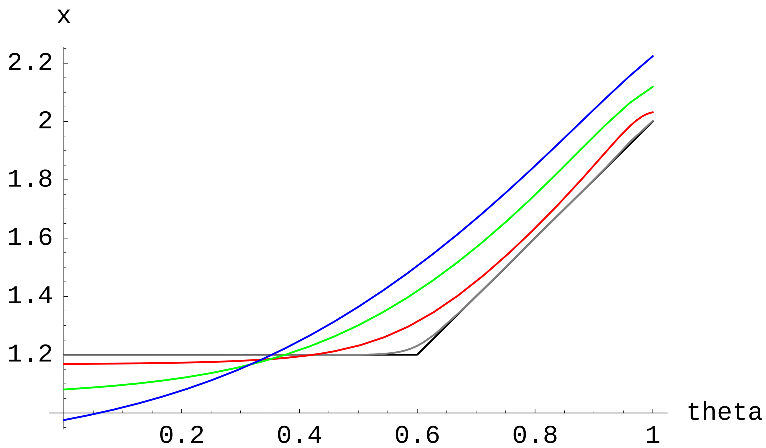

Figure 1: Aggregate decisions $\tilde{x}^*(\theta)$ and $x^*(\theta)$ when $\delta = 0.3$, and $\Delta = 0.6$, $\Delta = 0.4$, $\Delta = 0.2$ and $\Delta = 0.05$ ($\tilde{x}^*(\theta) = x^*(\theta)$ when $\Delta = 0$).

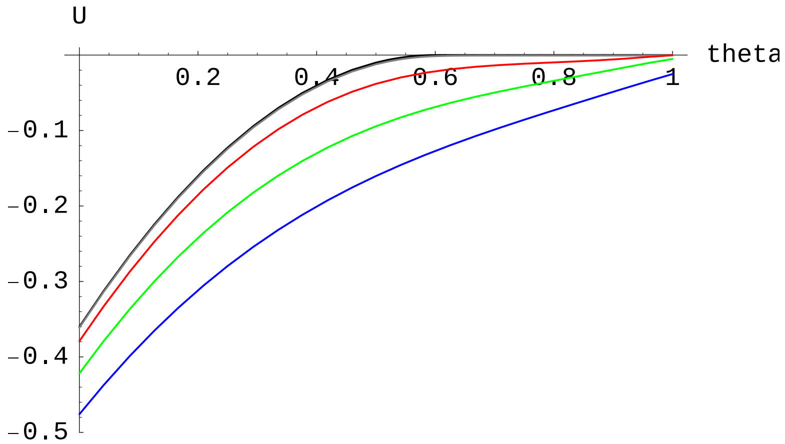

Figure 2: Agent's information rent $U^*(\theta)$ when $\delta = 0.3$, and $\Delta = 0.6$, $\Delta = 0.4$, $\Delta = 0.2$, $\Delta = 0.05$ and $\Delta = 0$.

The distortions on each dimension are more complex, as illustrated by Figure 3 for a fixed value of δ_1, δ_2 and Δ. While $x_2^*(\theta)$ is always strictly larger than θ, it is not always increasing, while $x_1^*(\theta)$ is strictly increasing over $[0, 1]$ but not always higher than θ. In addition, for the incentive compatibility constraint (6) to be satisfied, $x_2^*(\theta)$ should be strictly decreasing if and only if $x_1^*(\theta)$ is larger than θ. This feature of the optimal mechanism is general, and is summarized in the next corollary.

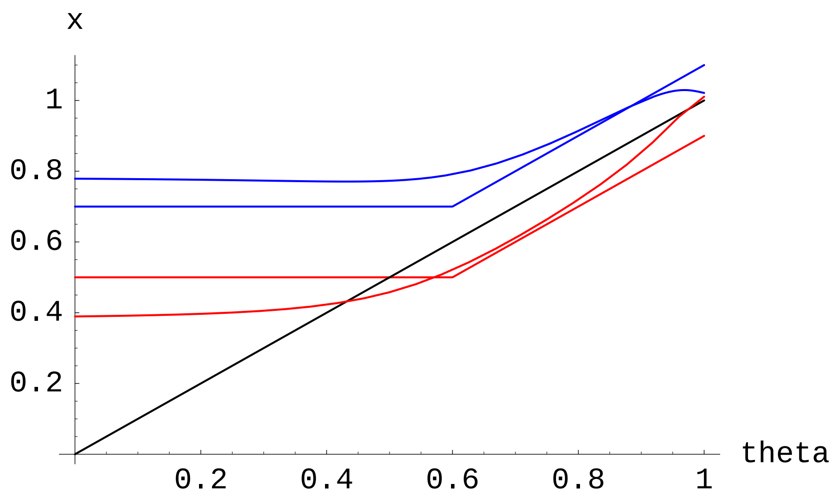

Figure 3: Individual decisions $\tilde{x}_i^*(\theta)$, $x_i^*(\theta)$ and the agent's ideal point θ, for $i = 1, 2$, when $\delta_1 = 0.2$, $\delta_2 = 0.4$ and $\Delta = 0.2$.

Corollary 2 *Assume that* $\Delta > 0$.

1. *For every* $\theta \in [0, 1]$, *we have* $\dot{x}_1^*(\theta) > 0$ *and* $x_2^*(\theta) > \theta$;
2. *For every* $\theta \in [0, 1]$, *we have* $x_1^*(\theta) > \theta$ *if and only if* $\dot{x}_2^*(\theta) < 0$;
3. $x_1^*(0) > 0$ *and* $\dot{x}_2^*(0) < 0$;
4. $x_1^*(1) > 1$ *and* $\dot{x}_2^*(1) < 0$.

It is useful for the rest of our analysis to recast those results in terms of delegation sets. Clearly, and contrary to the findings of Proposition 1 for the one-dimensional case, we now have:

Corollary 3 *When* $\Delta > 0$ *and principals cooperate in designing a communication mechanism, delegation sets are never optimal.*

Indeed, the optimal communication mechanism must allow a joint control by this merged principal on how the decisions x_1 and x_2 should be tied together. For instance, suppose that principals offer the mechanism $\{\tilde{x}_1^*(\theta), \tilde{x}_2^*(\theta)\}_{\theta\in\Theta}$ which, as we will see below, is approximately optimal when Δ is small. When $\theta > 2\delta$, the agent's freedom in choosing a pair (x_1, x_2) is limited to those pairs on a straight line satisfying $x_1 - \delta_1 = x_2 - \delta_2$. When $\theta \leq 2\delta$, the agent's is forced to choose the point $(2\delta - \frac{\Delta}{2}, 2\delta + \frac{\Delta}{2})$. Nevertheless, it is still true that a version of the Taxation Principle holds. The merged principal can implement the optimal communication mechanism by offering a specific curve $x_2(x_1)$ (this curve can be reconstructed from the parametrization $\{x_1^*(\theta), x_2^*(\theta)\}_{\theta\in\Theta}$) and letting the agent picks a point on this curve.

Since the design of the optimal mechanism is rather complex, one may wonder whether simpler mechanisms perform well and under which circumstances. The intuition is that, although the optimal mechanism requests full separation of types, it is only marginally so on the lower tail of the type distribution.

Our next proposition shows that the simple linear mechanism $\{\tilde{x}_1^*(\theta), \tilde{x}_2^*(\theta)\}_{\theta\in\Theta}$ performs quite well when the polarization between principals is not too important, i.e., when Δ is small enough.

Proposition 3 *The merged principal's loss from using the mechanism $\{\tilde{x}_1^*(\theta), \tilde{x}_2^*(\theta)\}_{\theta\in\Theta}$ instead of the optimal mechanism $\{x_1^*(\theta), x_2^*(\theta)\}_{\theta\in\Theta}$ is of order at most 2 in Δ:*

$$\int_0^1 L_\Delta(\tilde{U}^*(\theta), \dot{\tilde{U}}^*(\theta))d\theta = \frac{4\delta^3}{3} + \frac{\Delta^2}{4} < \int_0^1 L_\Delta(U^*(\theta), \dot{U}^*(\theta))d\theta < \frac{4\delta^3}{3} + \Delta^2.$$

We will show in the sequel how non-cooperative communication in the case of competing principals changes the pattern of delegation and might either reintroduce some bunching and justify again the use of simple delegation sets or opens the possibility of non-existence of a pure-strategy equilibrium.

5 Non-Cooperative Design

We now turn to the analysis of the non-cooperative design of communication mechanisms. We distinguish below between the case of private and public communications stressing thereby different sorts of conflicts between principals.

5.1 Private Communication

Under a non-cooperative design and private communication, principals independently choose their communication spaces with the agent and design their own mechanism. We should first observe that there is no loss of generality in looking for principal P_i's best-response within the class of continuous and piecewise differentiable direct revelation mechanisms $\{x_i(\hat{\theta}_i)\}_{\hat{\theta}_i\in\Theta}$, which satisfy (2). Indeed, in the absence of any externalities between the principals (i.e., in our context, each principal P_i's utility only depends on the decision x_i and the agent's utility function is separable in the decisions controlled by each principal), the Revelation Principle applies and there is no loss of generality in restricting the principals to offer such direct revelation mechanisms and requesting that the agent adopts a truthful strategy vis-à-vis each principal.[26]

[26]This observation is directly related to the findings of Peters (2003).

Equivalently, principal P_i offers the delegation set $\mathcal{D}_i =$ range $x_i(\cdot)$ which is a connected set when $x_i(\cdot)$ is continuous. That is, instead of focusing on communication per se, principals could as well offer the range of relevant decisions to the agent and let the latter chooses from that set.[27] Given this observation, the analysis of the Nash equilibrium between principals is straightforward. From Proposition 1, equilibrium delegation sets are again given by

$$\mathcal{D}_i^N = [2\delta_i, 1].$$

Proposition 4 *With conflicting principals and private communication, the unique equilibrium has each principal offering the same communication mechanism as if he was alone.*

Compared with the cooperative outcome, two new distortions arise under private communication between competing principals. First, because principals can no longer jointly restrict the agent's decisions, if they both let some freedom to the agent, the latter chooses $x_1(\theta) = x_2(\theta) = \theta$. This corresponds roughly to a too low decision x_2 given the optimal solution when principals cooperate. Second, principals choose now to impose floors on the agent's decision as if they were alone, i.e., irrespectively of the impact of their own communication mechanism on the overall cost of providing incentives.

5.2 Private Communication with Coordination

In Section 5.1, principals offer the same communication mechanism as if they were alone because there is no interaction between the two activities controlled by the principals; they enter separately into the agent's utility function. Consider instead the following utility function for the agent:

$$U(x_1, x_2, \theta) = -\frac{1}{2}\sum_{i=1}^{2}(x_i - \theta)^2 - \frac{\mu}{2}(x_1 - x_2)^2, \text{ where } \mu \geq 0.$$

Although the agent's ideal point remains unchanged, choosing different decisions is now costly for the agent. There are gains from coordinating decisions from the agent's viewpoint. The principals' utility functions on the other hand remain unchanged. This setting will introduce an interesting one-sided externality between principals.

The analysis is similar in spirit to that made in common agency with private contracting and direct externalities in the agent's utility function performed in Stole (1991) and Martimort (1992) although the latter papers were developed in environments with transfers. Those papers argue that, under private communication, each principal, say P_i, finds his best-response at any pure strategy equilibrium by designing a communication mechanism with an agent whose indirect utility function takes into account how a change in that mechanism affects his communication

[27] See Martimort and Stole (2002).

with the other principal P_{-i}.[28]

Formally, we will be interested in looking for equilibria in delegation sets. From Martimort and Stole (2002) such delegation sets are the right strategy spaces to describe equilibria of our common agency game. Suppose thus that P_{-i} chooses a connected delegation set of the form $\mathcal{D}_{-i} = [\underline{x}_{-i}, +\infty)$ where $\underline{x}_{-i}$ is a floor on any decision x_{-i} that the agent may choose.[29] To compute a best-response by P_i in the set of delegation sets, we shall use the Revelation Principle to find a direct mechanism $\{x_i(\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta}$ whose range is itself a delegation set $\mathcal{D}_i$ which, as we will see, is connected and has a floor $\underline{x}_i$. This direct mechanism should induce the agent to reveal the truth on his type once it is taken into account how a change in the decision x_i that principal P_i would like to implement influences the action choice x_{-i}. Note that a continuous communication mechanism has a connected range and corresponds thus to a connected delegation set.

Proposition 5 *Among the class of continuous mechanisms, there exists a unique equilibrium between principals under private communication. Each principal P_i offers the delegation set $\mathcal{D}_i = [\underline{x}_i, +\infty)$, where principal P_2 always offers the same delegation set as if he was alone, $\underline{x}_2 = 2\delta_2$, and:*

- *When $\mu \leq \frac{\delta_1}{2\delta_2 - \delta_1}$, i.e., for small benefits of coordination,*

$$\underline{x}_1 = \frac{2}{1-\mu}(\delta_1 - \mu\delta_2) < 2\delta_1 < \underline{x}_2; \tag{24}$$

- *When $\mu \geq \frac{\delta_1}{2\delta_2 - \delta_1}$, i.e., for strong benefits of coordination,*

$$\underline{x}_1 = \frac{2\mu\delta_2}{1+\mu} < \underline{x}_2. \tag{25}$$

The intuition behind this proposition is the following. Because principal P_2 offers a floor on decision $\underline{x}_2 = 2\delta_2$ which is rather high, and principal P_1 is instead interested in giving more freedom to the agent even when θ is lower, it is very likely that the floor $\underline{x}_2$ binds. When it is so, the agent's ideal point vis-à-vis decision x_1 is shifted upwards to benefit from coordination. When μ is small relatively to δ_1, this facilitates information revelation with principal P_1 who leaves more freedom to the agent and decreases the floor of his own delegation set. The equilibrium mechanisms in that situation are illustrated by Figure 4.

Actually, principal P_1 gains from the existing coordination for small benefits of coordination if his expected payoff given by Equation (A.24) is larger than

$$-\frac{1}{2}\left(\int_0^{2\delta_1} (\delta_1 - \theta)^2 d\theta + \int_{2\delta_1}^1 \delta_1^2 d\theta\right),$$

[28] Details are provided in the Appendix. Of course, such changes only arise because there is now a non-separability in the impact of each principal's requested action on the agent's utility function: The level of activity requested by one principal affects now the agent's marginal utility with respect to the other activity.

[29] We simplify presentation and computation of the best-response mapping by considering that this delegation set has no upper bound although, at equilibrium, only a bounded set of actions are taken by the agent.

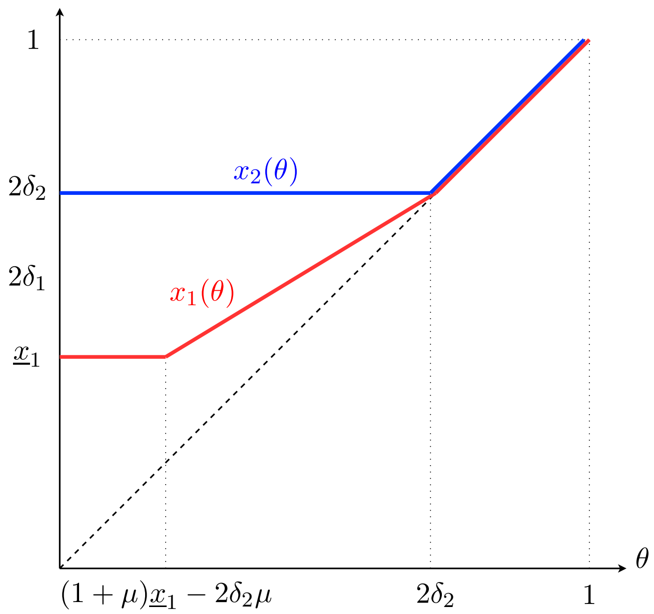

Figure 4: Equilibrium communication mechanisms under private communication for small benefits of coordination.

i.e., $\frac{2\mu(\delta_1-\delta_2)^2(\delta_1(3-\mu)-2\delta_2\mu)}{3(1-\mu)^2} > 0$, which is always satisfied when $\mu \leq \frac{\delta_1}{2\delta_2-\delta_1}$. On the contrary, when μ is large relatively to δ_1, principal P_1 may be worse off than in the standard setup. As an extreme example, when $\delta_1 = 0$ principal P_1 gets his first best without coordination externalities but incurs a loss otherwise since in that case $\underline{x}_1 = \frac{2\mu\delta_2}{1+\mu} > 0$. Note that this externality between principals is only one-sided: Principal P_2 is not affected by the presence of principal P_1 because the latter wants to implement decisions which are too small from his own point of view.

5.3 Public Communication

In this section, we assume that principals use the same public message space although they do not cooperate in designing the decision rule they respectively want to implement. Suppose that this given common communication space is equal (or at least diffeomorphic) to the set of possible types, i.e., $\mathcal{M} = \Theta$. Consider then two *direct public communication mechanisms* $\{x_i(m)\}_{m\in\Theta}$ $(i = 1, 2)$. Turning again to our basic model without coordination externalities $(\mu = 0)$, the agent's optimal strategy $m^*(\cdot)$ from Θ onto satisfies:

$$m(\theta) \in \arg\max_{m\in\Theta} -\sum_{i=1}^{2} \frac{1}{2}(x_i(m) - \theta)^2. \quad (26)$$

Following the logic of the Revelation Principle, we may construct from the original pair of mechanisms $\{x_i(m)\}_{m\in\Theta}$ and the continuation strategy $m(\cdot)$ a new pair of direct public communication mechanisms $\{x_i^*(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ such that

$$x_i^*(\hat{\theta}) = x_i(m(\hat{\theta})), \quad \forall\, \theta \in \Theta \text{ and } i = 1, 2.$$

This means that, at any putative equilibrium with pure strategy we may consider, the pair of direct public communication mechanisms $\{x_1^*(\cdot), x_2^*(\cdot)\}$ offered at this equilibrium is jointly incentive compatible and satisfies:

$$\theta \in \arg\max_{\hat{\theta}\in\Theta} -\sum_{i=1}^{2} \frac{1}{2}(x_i^*(\hat{\theta}) - \theta)^2. \tag{27}$$

This is a rather weak statement of the Revelation Principle. Indeed, off the equilibrium path, each principal may want to offer another communication mechanism which corresponds to a non-truthful public communication strategy by the agent. This is a general point of the common agency literature (see Martimort and Stole, 2002 for instance) that applies here also.

Formally, for any possible deviation by principal i, say a communication mechanism $\tilde{x}_i(\cdot) : \Theta \to \mathbb{R}$, the agent's communication strategy in the continuation satisfies now:

$$m^*(\theta) \in \arg\max_{m\in\Theta} -\frac{1}{2}(\tilde{x}_i(m) - \theta)^2 - \frac{1}{2}(x_{-i}^*(m) - \theta)^2. \tag{28}$$

For each of those deviations, the principal may alternatively offer the direct communication mechanism $\{x_i(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ defined as

$$x_i(\theta) = \tilde{x}_i(m^*(\theta)) \quad \forall\, \theta \in \Theta.$$

With this observation, (28) implies also that the following incentive constraints hold:

$$\theta \in \arg\max_{\hat{\theta}\in\Theta} -\frac{1}{2}(x_i(\hat{\theta}) - \theta)^2 - \frac{1}{2}(x_{-i}^*(m^*(\hat{\theta})) - \theta)^2. \tag{29}$$

Those incentive constraints can be interpreted as follows. Everything happens as if principal P_i could indeed deviate by offering an *extended direct mechanism* $\{x_i(\hat{\theta}), m^*(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ to the agent. The agent then *privately* communicates his type to that deviating principal by sending a message $\hat{\theta}$ who then recommends first which decision $x_i(\hat{\theta})$ the agent should adopt and, second which public message $m^*(\hat{\theta})$ the agent should sent to the non-deviating principal $-i$.

If there exists any pure strategy equilibrium with direct public communication mechanisms $\{x_1^*(\cdot), x_2^*(\cdot)\}$ between the principals, with (without loss of generality) a truthful strategy being played by the agent in the continuation, it must be that none of these principals may want to deviate to an alternative extended direct mechanism inducing possibly a non-truthful continuation. It turns out that such a condition is highly constraining. Indeed, the public message $m^*(\cdot)$ can be used by the deviating principal, say P_i, as a second screening variable on top of the decision rule $x_i(\cdot)$. This second variable facilitates information revelation towards principal i but possibly comes at the cost of inducing *"public lies"*. Following the logic of Section 4, adding more screening instruments may facilitate deviations compared with the case of private communication.

Example 3 As an illustration, consider the optimal pair of private mechanisms when $\delta_1 = 0 < \delta_2 < 1/2$, i.e., $x_1^*(\theta) = \theta$ and $x_2^*(\theta) = \max\{\theta, 2\delta_2\}$. Since this pair of mechanisms induces truthtelling to both principals under private communication, it also induces truthtelling under public communication. Consider the deviation by principal P_2 to the mechanism $\tilde{x}_2(\theta) = \theta + \delta_2$. This deviation induces the agent to underreport his type, and the associated extended direct mechanism is:

$$m^*(\theta) = \arg\min_{m\in\Theta}(m-\theta)^2 + (m+\delta_2-\theta)^2 = \max\{\theta - \delta_2/2, 0\}$$

and $x_2(\theta) = \tilde{x}_2(m^*(\theta)) = \theta + \delta_2/2$. This deviation is always profitable for principal P_2 because

$$\int_0^1 (x_2^*(\theta)-\theta-\delta_2)^2 d\theta > \int_0^1 (\tilde{x}_2(m^*(\theta))-\theta-\delta_2)^2 d\theta \iff \delta_2 < 9/16.$$

We will show below that, more generally, no pair of (continuous and piecewise differentiable) mechanisms can prevent a profitable deviation by one of the principals.

Remark 1 The set of extended direct mechanisms $\{x_i(\hat{\theta}), m^*(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ may allow more deviations than what is available with direct communication mechanisms. In other words, although (28) implies (29) the reverse is not true. Think indeed of a deviation in extended direct mechanisms that would require the agent to send a fixed public message $m^*(\theta) = m^* \in \Theta$ independently of his own type θ but that would require full separation in terms of the decisions, i.e., $x_i(\theta)$ being now injective. Clearly, there would not exist a direct communication mechanism $\{\tilde{x}_i(m)\}_{m\in\Theta}$ that could replicate such outcome. This is only possible when the $m^*(\cdot)$ induced by the deviation is itself injective. This points at the fact that the set of deviations with direct communication mechanisms leads to an open set. Our response to this difficulty is to consider first deviations in the larger set defined by (29) and then show that such a deviation can be approximated as closely as wanted by using a injective recommendation $m^*(\cdot)$.

From now on, we adopt an approach close to that performed in Section 4. We first look for the incentive properties of the information rent profile that principal i may induce by deviating within the class of extended direct mechanisms. Second, we optimize within that set. This will lead us to our impossibility result.

Fix the direct public communication mechanism $\{x_{-i}^*(m))\}_{m\in\Theta}$ offered at a putative pure strategy equilibrium by principal $-i$. We assume that this mechanism is continuous and piecewise differentiable.[30] For a given extended mechanism $\{x_i(\hat{\theta}), m^*(\hat{\theta})\}_{\hat{\theta}\in\Theta}$, define the agent's information rent $U(\theta)$ as:

$$U(\theta) = \max_{\hat{\theta}\in\Theta} -\frac{1}{2}(x_i(\hat{\theta})-\theta)^2 - \frac{1}{2}(x_{-i}^*(m^*(\hat{\theta}))-\theta)^2 = -\frac{1}{2}(x_i(\theta)-\theta)^2 - \frac{1}{2}(x_{-i}^*(m^*(\theta))-\theta)^2. \quad (30)$$

Using the Envelope Theorem, we immediately obtain

$$\dot{U}(\theta) = x_i(\theta) + x_{-i}^*(m^*(\theta)) - 2\theta, \quad (31)$$

[30] A remark is in order here. In the case of a cooperative design, we derive the differentiability properties of the communication mechanisms from the incentive compatibility constraints. This is no longer possible to proceed similarly jointly for both principals since the differentiability properties of one mechanism may follow from those of the other. We just look for such differentiable equilibria right away.

with the second-order condition for incentive compatibility being now expressed as

$$\ddot{U}(\theta) = \dot{x}_i(\theta) + \dot{m}^*(\theta)\dot{x}^*_{-i}(m^*(\theta)) - 2 \geq -2. \tag{32}$$

From (30) and (31), we immediately derive a second order equation in $x^*_{-i}(m^*(\theta))$, namely:

$$U(\theta) = -\frac{1}{2}(\dot{U}(\theta) - (x^*_{-i}(m^*(\theta)) - \theta))^2 - \frac{1}{2}(x^*_{-i}(m^*(\theta)) - \theta)^2. \tag{33}$$

Having a real solution to this equation requires $U(\theta) \leq 0$ and, keeping the lowest such solution,[31] we get:

$$x^*_{-i}(m^*(\theta)) - \theta = \frac{\dot{U}(\theta)}{2} - \sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}} \text{ and } x_i(\theta) - \theta = \frac{\dot{U}(\theta)}{2} + \sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}}. \tag{34}$$

This equation in fact implicitly defines the public message $m^*(\theta)$ that helps principal i to implement a rent profile $U(\theta)$ when considering a deviation. Given the range of $x^*_{-i}(\cdot)$, $Im\, x^*_{-i}$, this implementability condition can as well be written as:

$$\frac{\dot{U}(\theta)}{2} - \sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}} + \theta \in Im\, x^*_{-i}. \tag{35}$$

Using (34), we may rewrite principal i's payoff in each state of nature θ as:

$$-\frac{1}{2}\delta_i^2 - \frac{1}{2}\left(\frac{\dot{U}(\theta)}{2} + \sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}}\right)^2 + \delta_i\left(\frac{\dot{U}(\theta)}{2} + \sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}}\right).$$

Neglecting the second-order condition (32), we can express principal i's problem in looking for an optimal deviation as:

$$(\mathcal{P}_i^{pc}): \quad \max_{U(\cdot)} \int_0^1 L_i(U(\theta), \dot{U}(\theta))d\theta, \text{ subject to (35)},$$

where

$$L_i(U(\theta), \dot{U}(\theta)) = -\frac{1}{2}\left(\frac{\dot{U}(\theta)}{2} + \sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}}\right)^2 + \delta_i\left(\frac{\dot{U}(\theta)}{2} + \sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}}\right).$$

Note that $L_i(s, v)$ is a concave function of (s, v) although it may not be differentiable everywhere especially at points where $U(\theta) + \frac{\dot{U}^2(\theta)}{4} = 0$ if any such θ exists. Equipped with this formulation, we can now show the following proposition:

Proposition 6 *With conflicting principals and public communication, there does not exist any pure strategy equilibrium of the public communication game with continuous and piecewise differentiable direct mechanisms.*

[31] We could keep the highest such solution by permuting the role of principal i and $-i$.

5.4 Cheap-Talk with Multiple Audiences

One possibility to restore the existence of an equilibrium is to consider the situation in which the principals can no longer commit to a decision rule. More precisely, assume that communication takes place via unilateral cheap talk, from the agent to the principals. The timing of this communication game is as follows:

- ⋆ First, the agent learns his type $\theta \in \Theta$.
- ⋆ Second, the agent sends a (public or private) message to the principals.
- ⋆ Third, each principal chooses an action as a function of the message he received.

A strategy for principal P_i is a mapping $\sigma_i : \Theta \to \mathbb{R}$, where $\sigma_i(\theta)$ is the action chosen by principal P_i when he has received the message θ. Under private communication, a strategy for the agent is a pair (σ_A^1, σ_A^2), where $\sigma_A^i : \Theta \to \Theta$ and $\sigma_A^i(\theta)$ is the message privately observed by principal P_i when the agent's type is θ. A strategy for the agent under public communication is defined similarly, with the restriction that principals P_1 and P_2 observe the same public message $\sigma_A^1(\theta) = \sigma_A^2(\theta)$ when the agent's type is θ.

Since there is a one-to-one correspondence between the message received by principal P_i and his optimal action, every equilibrium is outcome equivalent to a partitional equilibrium, in which the agent's strategy takes the following form for some $K \in \mathbb{N}_+^*$: $\sigma_A^i(\theta) = m_1$ if $\theta \in [0, y_1^i)$, ..., $\sigma_A^i(\theta) = m_k$ if $\theta \in [y_{k-1}^i, y_k^i)$, ..., $\sigma_A^i(\theta) = m_K$ if $\theta \in [y_{K-1}^i, 1]$, with $0 < y_1^i < \cdots < y_{K-1}^i < y_K = 1$, $m_k \neq m_{k'}$ for $k \neq k'$, and the restriction $\sigma_A^1 = \sigma_A^2$ under public communication.

The following proposition shows that whatever the communication protocol (public or private), equilibrium outcomes take exactly the same form as in Crawford and Sobel (1982), with the bias parameter being δ_i in the private communication game with principal P_i, and $\delta = \frac{\delta_1+\delta_2}{2}$ in the public communication game. Notice that for this characterization we do not impose any assumption on δ_1 and δ_2.

Proposition 7 (Cheap Talk: Equilibrium Characterization)

- *Under private communication, there is a partitional equilibrium with K_i different messages sent privately to principal P_i and*

$$y_k^i = \frac{k}{K_i} + 2k\delta_i(K_i - k), \quad i = 1, 2, \quad k = 1, \ldots K_i, \tag{36}$$

  if and only if $|\delta_i| \leq \frac{1}{2K_i(K_i-1)}$.

- *Under public communication, there is a partitional equilibrium with K different messages sent publicly to both principals and*

$$y_k = \frac{k}{K} + 2k\delta(K - k), \quad k = 1, \ldots K, \tag{37}$$

if and only if $|\delta| \leq \frac{1}{2K(K-1)}$.

A generalization of the phenomenon already identified by Farrell and Gibbons (1989) appears here. Indeed, in a model with binary actions and states, these authors show that full revelation of information to both principals under private communication implies full revelation of information in public, but that the reverse in not true. More specifically, if there is a K_i-partitional equilibrium with each principal P_i in private, with $K_1 = K_2 = K$, then there is also a K-partitional equilibrium in public, but the reverse is not true. As an extreme example, take $\delta_1 = -\delta_2 > 1/4$. Then, the unique equilibrium outcome under private communication is non-revealing while there is a K-partitional equilibrium under public communication for all $K \in \mathbb{N}^*_+$.

The following proposition characterizes players' (ex-ante) preferences over the two communication protocols depending on the configuration of the biases δ_1 and δ_2. To allow this comparison, we consider the "most informative" equilibria in both cases, i.e., those equilibria with the largest number of different messages sent by the agent.[32]

Proposition 8 (Cheap Talk: Welfare) *Consider the most informative K_i-partitional equilibrium outcome with principal P_i under private communication, for $i = 1, 2$, and the most informative K-partitional equilibrium outcome under public communication. For every $i = 1, 2$, we have:*

If $|\delta| < |\delta_i|$ and $\max\{K_i, K\} > 1$, then principal P_i is strictly better off under public communication than under private communication.

If $|\delta| > |\delta_i|$ and $\max\{K_i, K\} > 1$, then principal P_i is strictly worse off under public communication than under private communication.

If $|\delta| < |\delta_i|$ for $i = 1, 2$ and $\max\{K_1, K_2, K\} > 1$, then the agent is strictly better off under public communication than under private communication.

If $|\delta_1| < |\delta| < |\delta_2|$, then the agent may be strictly better off or strictly worse off under public communication than under private communication.

In particular, players are indifferent between public and private communication only when principals are perfectly symmetric ($\delta_1 = \delta_2$) or when there is no communication ($K_1 = K_2 = K = 1$). Notice also that the ex ante expected social welfare, defined as the sum of all three players' expected utilities, can always be identified with the agent's expected utility.

[32] This equilibrium selection can be justified with several perturbations of the cheap talk game. See, e.g., Chen et al. (2008).

6 Conclusion

Let us briefly wrap the main findings of our analysis and point out a few directions for further research.

First, multi-dimensional communication mechanisms exhibit quite different features than the simple delegation sets found in the one-dimensional case when they are jointly designed by co-operating principals. The possibility of trading off distortions on each dimension of the agent's activity leads to full separation and makes simple delegation sets suboptimal. However, the loss of using simple mechanisms that generalize those delegation sets might be quite small when the principals' objectives are not too different. It would be worth investigating the cooperative design of communication mechanisms in more complex environments allowing more general utility functions or type distributions. This research program is likely to meet strong technical difficulties coming from the impossibility to learn much from the Euler-Lagrange equation beyond the quadratic utility function/uniform type distribution investigated here. These difficulties will have to be overcome by unattractive numerical methods. Nevertheless, we feel quite confident about the robustness of our findings and especially on the increased scope for full separating allocations.

Second, we have found that delegation sets remain optimal in a non-cooperative environment as long as communication between the agent and each of his principals remains private. However, such non-cooperative design of delegation sets might have to take into account any existing coordination benefit from having the actions controlled by the principals jointly performed by the agent. Such settings lead to one-sided externality between principals. Other forms of externalities between principals (be they only indirect through the agent's utility function or also more direct) could certainly be handled with the same techniques as the ones we used to get this result.

Third, public communication between competing principals opens possibilities for strategic gaming where each principal tries to affect the publicly communicated report to his own advantage. Far from helping screening as in the cooperative setting, the mere possibility of enlarging the screening abilities of each principal by having him play on what is publicly communicated by the agent makes impossible to sustain any pure strategy equilibrium with continuous mechanisms. Again, there is no longer any scope for delegation sets to emerge in such contexts. An open and extraordinarily difficult question is the form that any mixed strategy equilibria might have in such context. This non-existence result suggests that the alternative cheap talk timing where principals only react to the agent's announcement may have quite an appeal. We investigated equilibria of the public and private communication games in such environments.

Appendix

Proof of Proposition 1. Given the class of communication mechanisms we consider, the optimization problem of principal P_i can be written as

$$\min_{(\theta_i^*,\theta_i^{**})} \int_0^{\theta_i^*} (\theta_i^* - \theta - \delta_i)^2 d\theta + \int_{\theta_i^*}^{\theta_i^{**}} (\delta_i)^2 d\theta + \int_{\theta_i^{**}}^1 (\theta_i^{**} - \theta - \delta_i)^2 d\theta,$$

when $\theta_i^* < \theta_i^{**} \leq 1$, and

$$\min_{\theta_i^*} \int_0^1 (\theta_i^* - \theta - \delta_i)^2 d\theta,$$

when $\theta_i^* = \theta_i^{**}$. This yields $\theta_i^* = 2\delta_i$ and $\theta_i^{**} = 1$ when $\delta_i \leq 1/2$. ■

Proof of Lemma 1. We proceed along the lines of Clarke (1990, Chapter 4) and especially Theorem 1.1.3 therein. We first observe that:

1. $L_\Delta(s,v)$ is $\mathcal{B}$-measurable where $\mathcal{B}$ denotes the $\sigma-$algebra of subsets of $\mathbb{R}_- \times \mathbb{R}$.

2. $L_\Delta(s,v)$ is continuous and thus lower-hemi continuous.

3. $L_\Delta(s,v)$ is concave in (s,v).

Define now the Hamiltonian $H(s,p)$ as:

$$H(s,p) = \max_{v\in\mathbb{R}}\{pv + L_\Delta(s,v)\} = \max_{v\in\mathbb{R}}(p+\delta)v + s + \Delta\sqrt{-s - \frac{v^2}{2}}.$$

The maximum above is achieved for

$$v^* = 2(p+\delta)\sqrt{\frac{-2s}{2(p+\delta)^2 + \Delta^2}},$$

which yields

$$H(s,p) = s + \sqrt{-2s(2(p+\delta)^2 + \Delta^2)}$$

This implies the following inequality :

$$H(s,p) \leq -s + \Delta\sqrt{-2s} + \sqrt{-4s(p+\delta)^2} = |s| + \Delta\sqrt{2|s|} + 2|p+\delta|\sqrt{|s|}.$$

Using now that $\sqrt{|s|} \leq 1 + \frac{|s|}{2}$ and that $|p+\delta| \leq |p| + \delta$, we obtain finally:

$$H(s,p) \leq \Delta\sqrt{2} + 2\delta + |s|\left(1 + \delta + \frac{\Delta}{\sqrt{2}} + |p|\right). \tag{A.1}$$

This is a "growth" condition on the Hamiltonian as requested in Clarke (1990, Theorem 1.1.3).

Lemma 3 *Clarke (1990). Assume that $L_\Delta(\cdot)$ satisfied conditions [1.] to [3.] above, that $H(\cdot)$ satisfies the "growth" equation (A.1) and that $\int_0^1 L_\Delta(U_0(\theta), \dot{U}_0(\theta))d\theta$ is finite for at least one admissible arc $U_0(\theta)$. Then, problem $\mathcal{P}_\Delta^c$ has a solution.*

It remains to show that $\int_0^1 L_\Delta(U_0(\theta), \dot{U}_0(\theta))d\theta$ is finite for at least one admissible arc $U_0(\theta)$. Take $U_0(\theta) = 0$ which corresponds to decisions $x_{10}(\theta) = x_{20}(\theta) = \theta$. This arc does the job and yields $\int_0^1 L_\Delta(U_0(\theta), \dot{U}_0(\theta))d\theta = 0$. ■

Proof of Lemma 2. Since the functional $L_\Delta(\cdot)$ does not depend on θ, we can obtain a first quadrature of (15) using Gelfand and Fomin (1991, Chapter 1, p. 19) on any interval where $U(\theta) + \frac{\dot{U}^2(\theta)}{4} < 0$ as:

$$L_\Delta(U(\theta,\lambda), \dot{U}(\theta,\lambda)) - \dot{U}(\theta,\lambda)\frac{\partial L}{\partial \dot{U}}(U(\theta,\lambda), \dot{U}(\theta,\lambda)) = -\lambda \tag{A.2}$$

where a priori $\lambda \in \mathbb{R}$ and where we make explicit the dependence of the solution on this parameter. We obtain immediately:

$$U(\theta,\lambda) + \delta\dot{U}(\theta,\lambda) + \Delta\sqrt{-U(\theta,\lambda) - \frac{\dot{U}^2(\theta,\lambda)}{4}} - \dot{U}(\theta,\lambda)\left(\delta - \frac{\Delta\dot{U}(\theta,\lambda)}{4\sqrt{-U(\theta,\lambda) - \frac{\dot{U}^2(\theta,\lambda)}{4}}}\right) = -\lambda.$$

Simplifying yields:

$$U(\theta,\lambda)\left(1 - \frac{\Delta}{\sqrt{-U(\theta,\lambda) - \frac{\dot{U}^2(\theta,\lambda)}{4}}}\right) = -\lambda.$$

Solving for $\dot{U}(\theta,\lambda)$ yields

$$\dot{U}^2(\theta,\lambda) = -4\left(U(\theta,\lambda) + \Delta^2\left(\frac{U(\theta,\lambda)}{U(\theta,\lambda)+\lambda}\right)^2\right), \tag{A.3}$$

which requires $-\Delta^2\left(\frac{U(\theta,\lambda)}{U(\theta,\lambda)+\lambda}\right)^2 \geq U(\theta,\lambda)$ or $(U(\theta,\lambda)+\lambda)^2 + \Delta^2 U(\theta,\lambda) \geq 0$ given that $U(\theta,\lambda) \leq 0$ since by definition the agent's information rent is the sum of two negative terms. Solving the second-order equation (A.3) and keeping the positive root only,[33] we get (17). When $\dot{U}(\theta,\lambda) > 0$, differentiating (17) with respect to θ yields

$$\ddot{U}(\theta,\lambda) + \frac{\dot{U}(\theta,\lambda)\left(1 + 2\lambda\Delta^2\frac{U(\theta,\lambda)}{(U(\theta,\lambda)+\lambda)^3}\right)}{\sqrt{-U(\theta,\lambda) - \Delta^2\left(\frac{U(\theta,\lambda)}{U(\theta,\lambda)+\lambda}\right)^2}} = \ddot{U}(\theta,\lambda) + 2\left(1 + 2\lambda\Delta^2\frac{U(\theta,\lambda)}{(U(\theta,\lambda)+\lambda)^3}\right) = 0 \tag{A.4}$$

Hence, on any interval where $\dot{U}(\theta,\lambda) > 0$, the second-order condition (12) can be written as

$$0 \leq \ddot{U}(\theta,\lambda) + 2 = -4\lambda\Delta^2\frac{U(\theta,\lambda)}{(U(\theta,\lambda)+\lambda)^3}. \tag{A.5}$$

Since $U(\theta,\lambda) \leq 0$ holds, $\lambda \leq 0$ implies also $U(\theta,\lambda) + \lambda \leq 0$ and then (A.5) holds which puts the requested restriction on the admissible solutions to (17). Finally, note that the second-order condition (12) holds obviously on any interval where instead $\dot{U}(\theta,\lambda) = 0$. ■

[33]Since it corresponds to a aggregate decisions $x(\theta)$ biased towards the merged principal, namely $x(\theta) \geq 2\theta$ (see equation (11)).

Proof of Proposition 2. The structure of the proof is as follows. First, we derive the necessary free end-point conditions (16) and the characterization given in Lemma 2. Sufficiency follows.

Necessity: Define the function

$$P(x) = \frac{-x((x+\lambda)^2 + \Delta^2 x)}{(x+\lambda)^2}.$$

For $x < 0$, $P(x) > 0$ if and only if the second degree polynomial $(x+\lambda)^2 + \Delta^2 x$ is everywhere positive. This is so when

$$\lambda < -\frac{\Delta^2}{4}.$$

When that condition holds, the differential equation (17) is Lipschitz at any point where $U(\theta, \lambda) < 0$ and thus it has a single solution at any such point. Moreover, a solution $U(\theta, \lambda)$ is then everywhere increasing on the whole domain where $U(\theta, \lambda) < 0$. As a result, the differential equation (17) is everywhere Lipschitz when $U(1, \lambda) < 0$ which turns out to be the case for the path we derive below.

The necessary free end-points conditions (16) can be rewritten for an optimal path as:

$$\left(-\delta + \Delta \frac{\dot{U}^*(\theta)}{4\sqrt{-U^*(\theta) - \frac{(\dot{U}^*(\theta))^2}{4}}} \right) |_{\theta=0,1} = 0.$$

Using (17) to express $\dot{U}^*(\theta)$, those conditions can be simplified so that $U^*(0)$ and $U^*(1)$ solve indeed the following second-order equation in U:

$$(U+\lambda)^2 = -(\Delta^2 + 4\delta^2)U \tag{A.6}$$

Assume now that

$$\lambda > -\frac{\Delta^2}{4} - \delta^2.$$

Then (A.6) admits two solutions $U(0, \lambda)$ and $U(1, \lambda)$ which are respectively given by

$$U^*(0, \lambda) = -\lambda - \frac{1}{2}\left(\Delta^2 + 4\delta^2 + \sqrt{(\Delta^2 + 4\delta^2)^2 + 4\lambda(\Delta^2 + 4\delta^2)} \right) \tag{A.7}$$

and

$$U^*(1, \lambda) = -\lambda - \frac{1}{2}\left(\Delta^2 + 4\delta^2 - \sqrt{(\Delta^2 + 4\delta^2)^2 + 4\lambda(\Delta^2 + 4\delta^2)} \right). \tag{A.8}$$

Equations (19) and (20) follow when $\lambda = \lambda^*$. Note in particular that (A.6) implies that both solutions are negative.

The last step is to show that there exits $\lambda^* \in (-\frac{\Delta^2}{4} - \delta^2, -\frac{\Delta^2}{4})$ such that the corresponding path $U^*(\theta) = U(\theta, \lambda^*)$ solving (17) and starting from $U(0, \lambda^*) = U^*(0, \lambda^*) = U^*(0)$ given by (19) reaches $U(1, \lambda^*) = U^*(1, \lambda^*) = U^*(1)$ given by (19). This requires to find a solution λ^* to the equation

$$\varphi(\lambda) = \psi(\lambda),$$

where

$$\varphi(\lambda) = U^*(1,\lambda) - U^*(0,\lambda) = \sqrt{(\Delta^2+4\delta^2)^2 + 4\lambda(\Delta^2+4\delta^2)},$$

and

$$\psi(\lambda) = \int_0^1 \dot{U}(\theta,\lambda)d\theta = \int_0^1 2\sqrt{-U(\theta,\lambda) - \Delta^2\left(\frac{U(\theta,\lambda)}{U(\theta,\lambda)+\lambda}\right)^2}d\theta,$$

where the path $U(\theta,\lambda)$ starts from the initial condition $U(0,\lambda) = U^*(0,\lambda)$. Note that both $\varphi(\cdot)$ and $\psi(\cdot)$ are continuous in λ. It is clear that $\varphi(\lambda)$ is strictly increasing in λ with, for $\lambda_1 = -\frac{\Delta^2}{4} - \delta^2$ and $\lambda_2 = -\frac{\Delta^2}{4}$,

$$\varphi(\lambda_1) = 0 < 2\delta\sqrt{\Delta^2+4\delta^2} = \varphi(\lambda_2).$$

On the other hand, note that

$$\psi(\lambda_1) > 0 = \varphi(\lambda_1), \tag{A.9}$$

since the path $U(\theta,\lambda_1)$ starting from $U^*(0,\lambda_1)$ is strictly increasing. Moreover, for λ_2, (17) can be rewritten as:

$$\dot{U}(\theta,\lambda_2) = 2\sqrt{-U(\theta,\lambda_2)}\frac{|U(\theta,\lambda_2)-\lambda_2|}{|U(\theta,\lambda_2)+\lambda_2|}. \tag{A.10}$$

The path solving (A.10) and starting at $U^*(0,\lambda_2)$ (note that $U^*(0,\lambda_2) < \lambda_2 < U^*(1,\lambda_2)$) is strictly increasing everywhere and cannot cross the boundary $U = \lambda_2$ because the only solution to (A.10) such that $U(\theta_1) = \lambda_2$ for a given $\theta_1 > 0$ is such that $U(\theta) = \lambda_2$ for all θ since the right-hand side of (A.10) satisfies a Lipschitz condition at any point $U(\theta,\lambda_2)$ away from zero; a contradiction with $U^*(0,\lambda_2) < \lambda_2$. From that, we deduce $U(\theta,\lambda_2) < \lambda_2$ for all θ. Hence, the following sequences of inequalities holds:

$$\psi(\lambda_2) = \int_0^1 \dot{U}(\theta,\lambda_2)d\theta < \lambda_2 - U^*(0,\lambda_2) = \lambda_2 - U^*(1,\lambda_2) + \varphi(\lambda_2).$$

Finally, we get:

$$\psi(\lambda_2) < \varphi(\lambda_2). \tag{A.11}$$

Gathering Equations (A.9) and (A.11) yields the existence of $\lambda^* \in (\lambda_1,\lambda_2)$ such that $\varphi(\lambda^*) = \psi(\lambda^*)$.

Sufficiency: Sufficiency follows from Clarke (1990, Chapter 4, Corollary p. 179). Indeed, as we have shown above in the proof of Lemma 1, $L_\Delta(\cdot)$ satisfies the convexity assumption. Moreover, the Hamiltonian

$$H(s,p(\theta)) = s + \sqrt{-2s(2(p(\theta)+\delta)^2+\Delta^2)},$$

is a concave function of s where we denote $p(\theta)$ an arc satisfying the necessary conditions:

$$-\dot{p}(\theta) = \frac{\partial H}{\partial U}(\theta,U^*(\theta),p(\theta)) = 1 - \sqrt{-\frac{(2(p(\theta)+\delta)^2+\Delta^2)}{2U^*(\theta)}}, \tag{A.12}$$

$$\dot{U}^*(\theta) = \frac{\partial H}{\partial p}(\theta,U^*(\theta),p(\theta)) = -(p(\theta)+\delta)\sqrt{\frac{U^*(\theta)}{-2(2(p(\theta)+\delta)^2+\Delta^2)}}, \tag{A.13}$$

$$p(0) = p(1) = 0. \tag{A.14}$$

It can be checked that conditions (A.12) and (A.13) are altogether equivalent to (15) whereas (A.14) is equivalent to the free-end points conditions (16). ■

Proof of Corollary 1. First observe that

$$x^*(0) = \dot{U}^*(0) = 2\frac{\sqrt{-U^*(0)((U^*(0)+\lambda^*)^2 + \Delta^2 U^*(0))}}{|U^*(0)+\lambda^*|}.$$

Using (19), we get:

$$x^*(0) = 4\delta\frac{|U^*(0)|}{|U^*(0)+\lambda^*|} < 4\delta.$$

Finally, we obviously have $x^*(\theta) - 2\theta = \dot{U}^*(\theta) > 0$ for all θ. ■

Proof of Corollary 2. The first property directly follows from (22). Now, from the incentive constraint (6) and the first property of the corollary we get the second property. Next, using (22) we have $x_1^*(0) > 0$ if and only if

$$\frac{\dot{U}^*(0)}{2} > \Delta\left(\frac{U^*(0)}{U^*(0)+\lambda^*}\right).$$

Using (17) and simplifying we get

$$2\Delta^2 U^*(0) > -(U^*(0)+\lambda^*)^2,$$

i.e., by (A.6), $2\Delta^2 < \Delta^2 + 4\delta^2$, which is always satisfied. $x_2^*(0) < 0$ follows now from the second property of the corollary. The last property is proved similarly. ■

Proof of Corollary 3. The proof is immediate and is omitted. ■

Proof of Proposition 3. Consider the mechanism $\{\tilde{x}_1^*(\theta), \tilde{x}_2^*(\theta)\}_{\theta\in\Theta}$ which can be written as:

$$\tilde{x}_1^*(\theta) = \begin{cases} \theta - \frac{\Delta}{2} & \text{if } \theta \geq 2\delta \\ 2\delta - \frac{\Delta}{2} & \text{otherwise} \end{cases} \text{ and } \tilde{x}_2^*(\theta) = \begin{cases} \theta + \frac{\Delta}{2} & \text{if } \theta \geq 2\delta \\ 2\delta + \frac{\Delta}{2} & \text{otherwise.} \end{cases} \tag{A.15}$$

The corresponding information rent for the agent satisfies:

$$\tilde{U}^*(\theta) = \begin{cases} -\frac{\Delta^2}{4} & \text{if } \theta \geq 2\delta \\ -(2\delta-\theta)^2 - \frac{\Delta^2}{4} & \text{otherwise.} \end{cases} \tag{A.16}$$

Note in particular that we have:

$$-\tilde{U}^*(\theta) - \frac{(\dot{\tilde{U}}^*(\theta))^2}{4} = \frac{\Delta^2}{4} \text{ for all } \theta.$$

Using these expression, we get:

$$\int_0^1 L_\Delta(\tilde{U}^*(\theta), \dot{\tilde{U}}^*(\theta))d\theta = \int_0^{2\delta} (2\delta(2\delta-\theta) - (2\delta-\theta)^2)d\theta + \frac{\Delta^2}{4} = \frac{4\delta^3}{3} + \frac{\Delta^2}{4}. \tag{A.17}$$

We also have

$$\int_0^1 L_\Delta(U^*(\theta), \dot{U}^*(\theta))d\theta \leq \int_0^1 \left(L_0(U^*(\theta), \dot{U}^*(\theta)) + \Delta\sqrt{-U^*(\theta) - \frac{\left[\dot{U}^*(\theta)\right]^2}{4}} \right) d\theta$$

$$= \frac{4\delta^3}{3} + \Delta^2 \int_0^1 \frac{|U^*(\theta)|}{|U^*(\theta) + \lambda^*|} d\theta$$

where the last equality follows from (17). The last inequality implies:

$$\int_0^1 L_\Delta(U^*(\theta), \dot{U}^*(\theta))d\theta \leq \frac{4\delta^3}{3} + \Delta^2. \tag{A.18}$$

Putting together (A.17) and (A.18) yields the result. ■

Proof of Proposition 4. The proof is immediate and is omitted. ■

Proof of Proposition 5. Following the methodology of Martimort and Stole (2007), let us define the agent's indirect utility function $\tilde{U}(x_i, \theta)$ as:

$$\tilde{U}(x_i, \theta) = \max_{x_{-i} \in \mathcal{D}_{-i}} U(x_i, x_{-i}, \theta).$$

Similarly, we denote $\tilde{x}_{-i}(x_i, \theta)$ the agent's best choice in $\mathcal{D}_{-i}$ given his type and the decision that P_i would like to induce as

$$\tilde{x}_{-i}(x_i, \theta) = \arg \max_{x_{-i} \in \mathcal{D}_{-i}} U(x_i, x_{-i}, \theta).$$

By concavity of $U(\cdot)$, $\tilde{x}_{-i}(x_i, \theta)$ is uniquely defined as

$$\tilde{x}_{-i}(x_i, \theta) = \max\left\{ \frac{\theta + \mu x_i}{1 + \mu}, \underline{x}_{-i} \right\}. \tag{A.19}$$

Defining $x_i^*(\theta) = \frac{1}{\mu}((1 + \mu)\underline{x}_{-i} - \theta)$, we have

$$\tilde{U}(x_i, \theta) = \begin{cases} -\frac{1+2\mu}{2(1+\mu)}(x_i - \theta)^2 & \text{if } x_i \geq x_i^*(\theta) \\ -\frac{1}{2}(x_i - \theta)^2 - \frac{1}{2}(\underline{x}_{-i} - \theta)^2 - \frac{\mu}{2}(x_i - \underline{x}_{-i})^2 & \text{otherwise.} \end{cases} \tag{A.20}$$

Note that $\tilde{U}(x_i, \theta)$ is continuous in x_i although not differentiable everywhere.

Incentive compatibility at differentiability points: With these definitions, we can rewrite the incentive compatibility conditions vis-à-vis principal P_i as:

$$\theta \in \arg\max_{\hat{\theta}} \tilde{U}(x_i(\hat{\theta}), \theta) = \arg\max_{\hat{\theta}} U(x_i(\hat{\theta}), \tilde{x}_{-i}(x_i(\hat{\theta}), \theta), \theta). \tag{A.21}$$

The first-order condition for the agent's incentive problem can thus be written at any differentiability point as:

$$0 = \begin{cases} \dot{x}_i(\theta)\,(x_i(\theta) - \theta) & \text{if } x_i \geq x_i^*(\theta) \\ \dot{x}_i(\theta)\,\big((1 + \mu)x_i(\theta) - \theta - \mu\underline{x}_{-i}\big) & \text{otherwise.} \end{cases} \tag{A.22}$$

The corresponding second-order condition for the agent's incentive problem remains at any differentiability point

$$\dot{x}_i(\theta) \geq 0. \tag{A.23}$$

Observe that (A.22) shows that any continuous $x_i(\theta)$ has either flat parts, or is such that $x_i(\theta) = \theta$ when $x_i(\theta) = \theta \geq x_i^*(\theta)$ (i.e., when $\theta \geq \underline{x}_{-i}$) and $x_i(\theta) = \frac{1}{1+\mu}(\theta + \mu \underline{x}_{-i})$ when $x_i(\theta) \leq x_i^*(\theta)$ (i.e., when $\theta \leq \underline{x}_{-i}$).

Best-response in the class of continuous mechanisms: We will use the characterization of continuous decision rules given above to describe an equilibrium in which $\underline{x}_2 \geq \underline{x}_1$. That equilibrium turns out to be unique.

When $x_1(\theta)$ is continuous and $(1+\mu)\underline{x}_1 - \mu\underline{x}_2 \in [0, \underline{x}_2]$, principal P_1 expected payoff's can be written as:

$$-\frac{1}{2}\left(\int_0^{(1+\mu)\underline{x}_1-\mu\underline{x}_2}(x_1-\theta-\delta_1)^2 d\theta \right. \\ \left. + \int_{(1+\mu)\underline{x}_1-\mu\underline{x}_2}^{\underline{x}_2}\left(\frac{1}{1+\mu}(\theta+\mu\underline{x}_2)-\theta-\delta_1\right)^2 d\theta + \int_{\underline{x}_2}^1 \delta_1^2 d\theta\right). \tag{A.24}$$

Optimizing with respect to $\underline{x}_1$ yields the following first-order condition:

$$\int_0^{(1+\mu)\underline{x}_1-\mu\underline{x}_2}(\underline{x}_1-\theta-\delta_1)d\theta = 0.$$

Simplifying, we obtain the following best-response:

$$\underline{x}_1 = \frac{1}{1-\mu}(2\delta_1 - \mu\underline{x}_2), \tag{A.25}$$

as long as $\mu\underline{x}_2 \leq (1+\mu)\delta_1 \leq \underline{x}_2$.

Turning now to principal P_2's best-response. Given the characterization of incentive compatibility given above and the fact that we are looking for an equilibrium such that $\underline{x}_2 \geq \underline{x}_1$, principal P_2 expected payoff's can be written as:

$$-\frac{1}{2}\left(\int_0^{\underline{x}_2}(\underline{x}_2-\theta-\delta_2)^2 d\theta + \int_{\underline{x}_2}^1 \delta_2^2 d\theta\right).$$

Optimizing yields the following best-response in the class of continuous mechanisms:

$$\underline{x}_2 = 2\delta_2. \tag{A.26}$$

Hence, $Imx_2(\cdot) = \mathcal{D}_2 = [2\delta_2, +\infty)$.

Equilibrium: Gathering (A.25) and (A.26) yields finally (24). Conditions (25) are then obtained when $\underline{x}_1$ is at a corner, i.e., $(1+\mu)\underline{x}_1 - \mu\underline{x}_2 = 0$. ■

Proof of Proposition 6. We first prove that there does not exist any equilibrium with one principal, P_{-i}, offering a separating allocation $x_{-i}(\cdot)$ over an interval $[\theta_0, \theta_1] \subseteq [0, 1]$. Suppose the contrary. Since each principal finds optimal to induce a message $m^*(\theta) = \theta$ for all $\theta \in \Theta$, on such an interval of separation it must necessarily be that the implementability condition (35) is slack for principal P_i. This implies that principal P_i would like to implement a rent profile $U(\theta)$ that maximizes pointwise the maximand in $(\mathcal{P}_i^{pc})$ over the interval $[\theta_0, \theta_1]$, i.e., such that:

$$\frac{\dot{U}(\theta)}{2} + \sqrt{-U(\theta) - \frac{\dot{U}^2(\theta)}{4}} = \arg\max_{\phi} -\frac{\phi^2}{2} + \delta_i \phi = \delta_i, \tag{A.27}$$

which is achieved with $U(\theta) = \delta_i^2$ and decision rules $x_i^*(\theta) = \theta + \delta_i$ and $x_{-i}^*(m^*(\theta)) = \theta - \delta_i$. The mechanism $x_i^*(\theta)$ being separating over that interval, a symmetric reasoning yields $x_{-i}^*(\theta) = \theta + \delta_{-i}$ and $x_i^*(m^*(\theta)) = \theta - \delta_{-i}$. This is impossible when $\delta_2 \neq -\delta_1$ and when $m^*(\theta) = \theta$ as requested in a pure strategy equilibrium.

Since there is no interval of separation, each principal must offer a pooling mechanism over the whole interval $[0, 1]$. But in that case, principal P_i offers as a best response his optimal private mechanism. This mechanism is separating over the interval $[2\delta_i, 1]$, which yields a contradiction. ∎

Proof of Proposition 7. We show the proposition under public communication (the proof is similar and well known under private communication). In a K-partitional equilibrium, the optimal action of principal P_i after message m_k is $\sigma_i(m_k) = \frac{y_{k-1}+y_k}{2} + \delta_i$. The best response of the agent is such that for all $k = 1, \ldots, K-1$, he is indifferent between sending message m_k and m_{k+1} when his type is $\theta = y_k$. After some simplifications, this yields $y_{k+1} - y_k = y_k - y_{k-1} - 4\delta$, so $y_k = ky_1 - 2k(k-1)\delta$. In particular, $y_K = Ky_1 - 2K(K-1)\delta = 1$, i.e., $y_1 = \frac{1}{K} + 2(K-1)\delta$, which gives the threshold of Equation (37). This K-partitional equilibrium is feasible if $y_1 > 0$ and $y_{K-1} < 1$, i.e., $-\frac{1}{2K(K-1)} < \delta < \frac{1}{2K(K-1)}$. ∎

Proof of Proposition 8. When for $i = 1, 2$, a K_i-partitional communication strategy σ_A^i is played with principal P_i in private, his ex-ante expected equilibrium payoff is given by

$$\begin{aligned} EV(K_i, \delta_i) &= -\sum_{k=1}^{K_i} \int_{y_{k-1}^i}^{y_k^i} (\sigma_i(m_k) - \theta - \delta_i)^2 \, d\theta = -\frac{1}{12} \sum_{k=1}^{K_i} (y_k^i - y_{k-1}^i)^3 \\ &= -\frac{1}{12} \sum_{k=1}^{K_i} (1/K_i - 2\delta_i(2k - K_i - 1))^3 = -\frac{1}{12K_i^2} - \frac{\delta_i^2(K_i^2 - 1)}{3}, \end{aligned} \tag{A.28}$$

and the ex-ante expected equilibrium payoff of the agent is given by

$$EV(K_1, \delta_1) + EV(K_2, \delta_2) - \delta_1^2 - \delta_2^2. \tag{A.29}$$

Similarly, when a K-partitional communication strategy σ_A is played in public, the ex-ante expected equilibrium payoff of principal P_i is given by $EV(K, \delta)$, and the ex-ante expected equilibrium payoff of the agent is given by

$$2EV(K, \delta) - \delta_1^2 - \delta_2^2. \tag{A.30}$$

If $|\delta| < |\delta_i|$, then by Proposition 7 the most informative K_i-partitional equilibrium under private communication with principal P_i and the most information K-partitional equilibrium under public communication are such that $K_i \leq K$. Principal P_i is strictly better off under public communication if and only if $EV(K, \delta) > EV(K_i, \delta_i)$, i.e.,

$$\frac{1}{4K^2} + \delta^2(K^2 - 1) < \frac{1}{4K_i^2} + \delta_i^2(K_i^2 - 1).$$

This inequality is immediate when $K = K_i \neq 1$. When $K_i < K$, a sufficient condition for the inequality to be satisfied is

$$\frac{1}{4K^2} + \delta^2(K^2 - 1) < \frac{1}{4K_i^2} + \delta^2(K_i^2 - 1),$$

i.e., $|\delta| < \frac{1}{2KK_i}$. Since $K_i \leq K - 1$, this condition is implied by $|\delta| < \frac{1}{2K(K-1)}$, which is exactly the condition for a K-partitional equilibrium to exist under public communication.

Similarly, if $|\delta| > |\delta_i|$, then $K_i \geq K$ and principal P_i is strictly worse off under public communication if and only if

$$\frac{1}{4K^2} + \delta^2(K^2 - 1) > \frac{1}{4K_i^2} + \delta_i^2(K_i^2 - 1),$$

which is immediate when $K = K_i \neq 1$. When $K_i > K$, a sufficient condition for the inequality to be satisfied is $|\delta_i| < \frac{1}{2K_i(K_i-1)}$, which is exactly the condition for a K_i-partitional equilibrium to exist under private communication with principal P_i.

If $|\delta| < |\delta_i|$ for $i = 1, 2$ it is obvious from the analysis above and the expressions of the agent's expected payoff given by Equations (A.29) and (A.30) that he is strictly better off under public than under private communication.

If $|\delta_1| < |\delta| < |\delta_2|$, then it is easy to construct examples in which the agent is strictly better off under public communication (e.g., with $K = K_1$), or in which he is strictly worse off (e.g., with $K = K_2$). ■

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