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Sharing aggregate risks under moral hazard

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Abstract

This paper analyzes the efficient design of insurance schemes in the presence of aggregate shocks and moral hazard. The population is divided into groups, the labour force in different sectors for instance. In each group, individuals are ex ante identical but are subject to idiosyncratic shocks. Without moral hazard, optimality requires (1) full insurance against idiosyncratic shocks, which gives rise to a representative agent for each group and (2) optimal sharing of macro-economic risks between these representative agents. The paper investigates what remains of this analysis when the presence of moral hazard conflicts with the full insurance of idiosyncratic shocks. In particular, how is the sharing of macro-economic risks across groups affected by the partial insurance against idiosyncratic risks? The design of unemployment insurance schemes in different economic sectors, and the design of pension annuities in an unfunded social security system are two potential applications.

Keywords: moral hazard, insurance, mutuality principle, macro-economic risk

JEL : D82, D61, G20, G22

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1   Introduction

This paper discusses the efficient design of insurance schemes in the presence of aggregate shocks and moral hazard. Let us illustrate the situation with two examples.

Consider first insurance against unemployment risks in different sectors. Without moral hazard, optimal risk sharing requires two properties: first full insurance against idiosyncratic shocks within each sector, and second the pooling of macro-economic risks affecting employment in the different sectors (along the lines described by Borch 1960, Wilson 1968, and Malinvaud 1972 and 1973). In particular, individuals' consumption levels vary in the same direction, either all upward or all downward, as the state of the economy varies. Casual observation suggests that neither property holds in practice. Moral hazard explains why workers might not be fully insured against idiosyncratic shocks. The question investigated here is whether this may have a significant impact on the sharing of macro-economic risks across sectors .

Our second example bears on some aspects of the design of pension schemes in a pay as you go system. In several european countries, among them France, Germany, and Italy, the main part of pension benefits is paid by the current workers through a compulsory pay as you go system. There is some evidence that in the last thirty years the well being of retirees has significantly raised relative to that of the rest of the population. To a large extent, such a relative increase has resulted from a specific aspect of the design of pension benefits, namely the indexation of annuities on individual wages.[2] Consequently, annuities increase more than the average income of young individuals as unemployment increases. This raises the question of why the fluctuations in unemployment and wages are not shared with the retirees. The extent to which retirees should bear unemployment risk can be viewed as an optimal risk sharing problem. However 'generous' unemployment benefits might play a role in explaining the high level of unemployment. This leads to a second best approach. How the income of retirees and young individuals, employed and unemployed, should fluctuate in function of the state of the economy when the impact of unemployment benefits on the incentives to find a job is taken into account ?

The paper considers an economy composed of groups, each populated with a continuum of *ex ante* identical individuals. Each individual is subject to an idiosyncratic shock, possibly influenced by expanding a non contractible effort (moral hazard). In addition, the economic environment is uncertain, determined by the 'state' of the economy (the macro risk). We fstand consider the setting that is the more favorable to insurance. Both types of risk can be insured through contracts that are settled ex ante before the state of the economy and the idiosyncratic shocks materialize. Furthermore, individuals cannot freely combine different contracts in various quantities: contracts are exclusive and compulsory. Exclusive contracts allow for a better monitoring of effort and minimize the difficulties due to moral hazard. Without exclusivity, the incentives properties of a contract are modified by the other contracts that an individual may buy (this raises difficulties as first shown by Arnott and Stiglitz (1991) in a competitive environment). The domain of application is rich. In many countries, insurance against pervasive risks, unemployment and illness for instance, is mostly provided at the

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[2]In other words, the increase is not explained by active policies, although they played some role. There is a move to an adjustment with consumer price index but this move is too recent to have been significant yet.

country level through compulsory and (almost) exclusive schemes. These restrictions are justified by the presence of informational difficulties, adverse selection and moral hazard.

The paper takes a second best approach. Optimal contracts maximize a weighted sum of utilities accounting for the constraints due to moral hazard. As a benchmark, consider the first best optimal contracts, which can be reached if effort is contractible. Idiosyncratic risks are mutualized, leading to a representative agent for each group. Aggregate resources, which vary with the macro-economic state, are allocated to the groups according to some 'sharing rules' among their representative agents. Sharing rules provide a benchmark to assess the impact of moral hazard. They prescribe transfers *across* groups that are motivated by insurance against the macro-economic risks. The question is why moral hazard should affect these transfers. The intuition is the following. In a state for instance where a group is hurt and receives some transfers from other groups, its budget is eased. This modifies the trade-off between providing insurance and incentives *within* the group, and may result in a change in the level of effort. But then the overall resources are modified as well, calling for a change in the planned transfers. Optimal transfers must account of this feedback effect.

The paper develops some tools to analyze the distortionary impact of moral hazard on transfers across groups. A group's marginal utility for wealth is analyzed and a moral hazard premium is introduced. These tools allow us to describe the optimal share of aggregate resources received by each group in the presence of moral hazard and to compare them with the sharing rules. In particular, the externalities created by the moral hazard problem faced by other groups are assessed.

The analysis is conducted in two settings. In one setting, effort is irreversible chosen before the state of the economy is revealed and in the other one effort is flexible, adjusted once the state is revealed. Under a flexible effort, groups' shares may fail to be monotonic, meaning that some may benefit from a change in the state while others are hurt. More surprising, an improvement in a group's environment, an increase in its members' productivity for instance, may hurt the group. This suggests a difficulty in the implementation of insurance contracts, which may explain why macro-economic risks are poorly pooled, and mainly at a compulsory level.

Moral hazard has been recognized to induce inefficiencies in a competitive insurance market in which individuals can purchase several contracts (Prescott and Townsend 1984, Arnott and Stiglitz 1991 or more recently Bisin and Guaitoli 2002). Inefficiencies are the result of the non-convexities that are induced by moral hazard, more precisely by changes in the effort level. This type of difficulties does not arise here because contracts are exclusive. However, the discontinuities and non-monotonicity in the optimal contracts as the state varies are due to the same reason, namely non-convexities. Furthermore, these drawbacks are robust; in particular they arise even when the so-called 'first order' approach is valid (see Grossman and Hart 1983 and Rogerson 1985 for this approach).

The paper is organized as follows. Section 2 sets up the model. Section 3 describes the benchmark case of optimal insurance without moral hazard. Sections 4 analyzes the impact of moral hazard on the levels of the groups shares. Section 5 and 6 are devoted to the case of an irreversible effort and flexible effort respectively. Decentralization is investigated in Section 7. Proofs are gathered in the final section.

2   The model

We consider an economy with a single good. The population is partitioned into H observable classes or groups. A h-individual denotes an individual in group h, $h = 1, 2.., H$. There are two kinds of risks, at the macro-economic and individual levels.

Risks. The macro-economic environment is described by a state. A state s influences the resources and the moral hazard problem faced by all individuals as described below. There is uncertainty as to which state will occur. The state is distributed according to a probability distribution π on the set of possible states $\mathcal{S}$. The distribution π is exogenous and the set $\mathcal{S}$ may be infinite.

Within a group, individuals are identical before the realization of an idiosyncratic shock that determines their status. A simple framework is one in which individuals face a binomial risk: a status is 'employed' or 'unemployed', 'ill' or 'healthy', 'success' or 'failure' for instance. An individual influences the probability of his status by expanding some effort. Moral hazard arises if the level of effort is not contractible, in particular if it is not observable. The exact environment is determined by the state s of the economy. Specifically, denoting by θ a status and by e a level of effort, h's environment in state s is described by

  - the positive probability $p_h(\theta|e, s)$ for an h-individual to be in status θ if he exerts effort level e,
  - the output $\omega_h(\theta|s)$ of a h-individual whose status is θ.

The set of possible status for group h is finite and denoted by Θ_h. The higher the status is, the larger the output is: Θ_h can be ordered so that output $\omega_h(\theta|s)$ increases with θ in each state s.[3]

Preferences over income level and effort, c and e, are represented by a Von Neumann-Morgenstern utility function separable in effort : $u_h(c) - k_h(s)e$ for a h-individual. The function u_h is defined over $]\underline{c}_h, +\infty[$ ($\underline{c}_h$ is possibly $-\infty$), is strictly increasing, concave, and twice differentiable. Furthermore it satisfies an Inada condition, $\lim_{c \to \underline{c}_h} = \infty$, in order to avoid corner solutions. The cost of effort may be state dependent, $k_h(s)$, in the case of a flexible effort.

Individuals are risk averse. Most of the analysis assumes strict risk aversion, with each function u_h strictly concave. We also allow for a risk neutral group, which will be taken to be group 1: in that case $u_1(c) = c$ without bounds on consumption levels (allowing for at most one a risk neutral group simplifies the presentation by avoiding indeterminacy in optimal contracts).

Observe that a state s characterizes probability distributions, outputs and possibly cost of effort. Accordingly, the set $\mathcal{S}$ may be finite or infinite.

Information and timings. The *ex ante* distribution of macro-economic states π and the distributions p_h are common knowledge. We shall consider two different timings, as represented in Figures 1 and 2. Under both timings, contracts are settled before the macro-economic and idiosyncratic shocks are realized, thereby allowing for insurance.

Under the first timing, effort is exerted before the state is revealed:

$t = 1$: insurance contracts are designed,

$t = 2$: each individual chooses a level of effort,

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[3]The status has no intrinsic value: what matters for an individual is only his consumption. Thus one could identify status with output, i.e. take θ to be equal to ω, and set $p_h(\omega|e, s)$ to be the probability of receiving ω. We would have to work with a support for the status that varies with the state, which is not convenient.
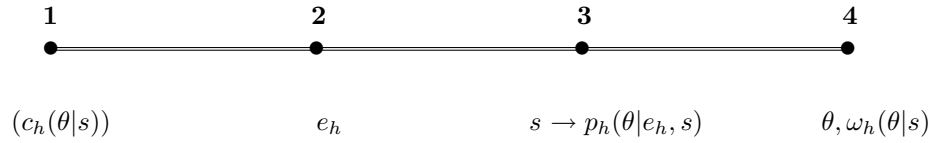
$$1 \qquad\qquad 2 \qquad\qquad 3 \qquad\qquad 4$$

$$(c_h(\theta|s)) \qquad\qquad e_h \qquad\qquad s \to p_h(\theta|e_h, s) \qquad\qquad \theta, \omega_h(\theta|s)$$

Figure 1: Timing for irreversible effort

$$1 \qquad\qquad 2 \qquad\qquad 3 \qquad\qquad 4$$

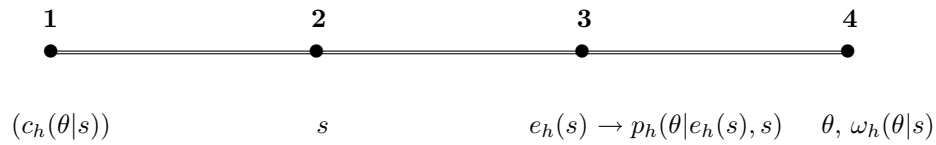$$(c_h(\theta|s)) \qquad\qquad s \qquad\qquad e_h(s) \to p_h(\theta|e_h(s), s) \qquad \theta, \omega_h(\theta|s)$$

Figure 2: Timing for flexible effort

$t = 3$: the macro-economic state is revealed,

$t = 4$: individuals' status are observed, and contracts are implemented.

An interpretation is that effort is a long term irreversible investment taken before the state materializes. For example, workers can take on their leisure time to invest say in formation in new technological skills that will be more or less useful depending on the evolution of the economy and technics.

Under the second timing, steps $t = 2$ and $t = 3$ are exchanged. For each macro-economic state, members of group h face a standard moral hazard problem: Effort is chosen after the state is revealed, hence can be adjusted in function of that state. Effort is said to be flexible.

Insurance contracts and insurance schemes. An *insurance scheme* specifies an insurance contract for individuals in each group. After describing an insurance contract for a group, we show how a scheme may provide some insurance against macro-economic risk through transfers across groups. The model is first presented for a flexible effort. The slight modifications to make for handling the case of an irreversible effort are explained afterwards.

I focus on insurance contracts that treat individuals within a group identically. An *insurance contract* for h specifies the income level (also called consumption level) $c_h(\theta|s)$ that a h-individual will receive at time $t = 4$ contingent on both the macro-economic state s and the realized status θ. It is convenient to write an insurance contract as the collection of income plans (contingent on status) faced by h-individuals in each possible state : $(\tilde{c}_h(s), s \in \mathcal{S})$, in which $\tilde{c}_h(s) = (c_h(\theta|s))_{\theta \in \Theta_h}$. (In the sequel, a $\tilde{\ }$ denotes a variable contingent on status θ.)

Effort is chosen at $t = 3$ knowing the state and the contract. Individuals in h will exert an identical level of effort in state s, denoted by $e_h(s)$. A non contractible effort is chosen optimally by

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each individual as follows. Let $U_h(\tilde{c}_h,|e,s)$ be the expected utility conditional on state s derived by a h-agent facing the contract $\tilde{c}_h(s)$ and choosing level e:

$$U_h(\tilde{c}_h|e,s) = \sum_\theta p_h(\theta|e,s)u_h(c_h(\theta|s)) - k_h(s)e. \tag{1}$$

An optimal effort level in state s, $e_h(s)$, satisfies

$$U_h(\tilde{c}_h|e_h(s),s) \geq U_h(\tilde{c}_h|e,s) \text{ each } e \in \mathcal{E}_h. \tag{2}$$

Under our assumptions (listed below), optimal effort is unique.

To focus on the sharing of macro-economic risks across groups, it is assumed that the frequency of an individual status within a group is exactly equal to its probability. Therefore, knowing the exerted effort level and the state of the economy, there is no uncertainty as to the level of a group's output: the aggregate output of group h in state s is risk-less equal to its mathematical expectation conditional on the effort level $e_h(s)$. Denoting this output by $\Omega_h(s)$, (keeping in mind that it depends on the effort level) this writes:

$$\Omega_h(s) = E[\omega_h|e_h(s),s] = \sum_\theta p_h(\theta|e_h(s),s)\omega_h(\theta|s) \tag{3}$$

Similarly, given $\tilde{c}_h(s)$ and $e_h(s)$, the aggregate income of group h in state s, denoted by $C_h(s)$, is risk-less equal to its mathematical expectation:

$$C_h(s) = E[c_h|e_h(s),s] = \sum_\theta p_h(\theta|e_h(s),s)c_h(\theta|s). \tag{4}$$

If each group designs its own contract independently, feasibility requires each one to satisfy the constraint $C_h(s) \leq \Omega_h(s)$ in each state s, preventing macro-economic risk to be shared. This calls for a joint design of the insurance contracts in which state-contingent transfers are implemented across groups. Such a design is described by an insurance scheme.

Definition 1 *An insurance scheme specifies an insurance contract for each group, $(\tilde{c}_h(s), s \in \mathcal{S})_{h=1,..,H}$. Under a flexible effort, let $(e_h(s), s \in \mathcal{S})_{h=1,..,H}$ be the optimal effort levels, i.e. those that satisfy (2) for each h in each state s. The scheme is feasible if, in each state, total aggregate income is not larger than total aggregate resources given the optimal effort levels:*

$$\sum_h C_h(s) \leq \sum_h \Omega_h(s) \text{ in each state } s. \tag{5}$$

Describing how macro-economic risks are shared amounts to describe how $C_h(s) - \Omega_h(s)$ varies with s. We are interested in how moral hazard distorts this sharing. For that, we consider a second-best optimality criterion, which accounts for the non-contractibility of effort.

Optimal insurance schemes. Positive weights are assigned to each group, λ_h to h. The welfare criterion is the weighted sum of the *ex ante* utilities of the groups. Given a scheme and effort levels $(\tilde{c}_h(s), e_h(s), s \in \mathcal{S})_{h=1,..,H}$ welfare is equal to

$$\sum_h \lambda_h E_\pi[U_h(\tilde{c}_h|e_h, s)] \tag{6}$$

where E_π denotes the expectation with respect to the states under the distribution π.

Definition 2 *Assume a flexible effort. An insurance scheme together with the optimal effort levels,* $(\tilde{c}_h(s), e_h(s), s \in \mathcal{S})_{h=1,..,H}$ *is said to be optimal if it maximizes the welfare criterion (6) over all schemes that satisfy in each state s the feasibility constraints (5) and the incentives constraints (2).*

The optimal scheme is determined by the assigned weights. However, our aim is to find properties that are independent on these weights, in particular to assess distortions due to the non contractibility of effort. For that purpose, next section is devoted to first best optimal schemes, that is the ones that maximize the same welfare criterion when effort is contractible, that is under the sole feasibility constraints (5).

Irreversible effort. The model is easily adapted. Contracts are contingent on both s and θ, as for a flexible effort. The difference is that an irreversible effort is chosen before the state is known, hence the incentive compatibility constraint bears on the ex ante utility level. Given a scheme $(\tilde{c}_h(s) s \in \mathcal{S})_{h=1,..,H}$, each h chooses effort e_h so as to maximize the ex ante expected utility level:

$$E_\pi[U_h(\tilde{c}_h|e_h, s)] \geq E_\pi[U_h(\tilde{c}_h|e', s)] \text{ each } e' \in \mathcal{E}_h. \tag{7}$$

The feasibility of a scheme $(\tilde{c}_h(s), e_h, s \in \mathcal{S})_{h=1,..,H}$ is defined as in definition 1 replacing the constraint on effort by (7). Hence the scheme is *optimal* if it maximizes the welfare criterion (6) over all schemes that satisfy the feasibility constraints (5) in each state s and the ex ante incentives constraints (7).

Examples Let us illustrate the framework with two examples.

Example 1. As an illustration of irreversible effort, consider wine production in different areas. Here, group h refers to wine production in a specific area. The output is the value of production and the status is the wine quality. Of course, the value of production depends on the wine quality, but it is also affected by market conditions, in particular by the price levels that will be realized after the production. These market conditions affect all producers in the same way in a given area, but possibly differently across different areas, say because of different weather conditions or different consumers' tastes,. One may refer to these market conditions as the 'state'. The effort of the winer along the season affects the chances for a good quality. Also these chances are influenced by the state in particular by weather conditions. To a large extent, the producer's effort can be assumed to be exercised before the state is known.

Example 2. As an illustration of flexible effort, consider the design of pensions in a pay as you go system. There are two groups, group 1 is the working age generation, the 'young', and group 2 is composed of the retired individuals. Group 2 does not face a moral hazard problem. Young individuals are either employed or unemployed with a probability that depends on their search effort and the state of the economy. This state influences also the wages (outputs). When searching, it is reasonable to assume that workers know the state. The retirees side is simplified by assuming that there are not subject to moral hazard. Excluding the benefits from Social Security they receive an exogenous possibly state-dependent resource (output in our terminology) that may come from previous investments. Such resource for instance is almost null in some European countries for a large part of the population.[4] Consider now a pay as you go system. The workers' contributions are directly

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[4]This excludes the interaction between the Social Security system and the incentives to invest for retirement. Our

paid to the retirees. Independently of their average levels, an important question is whether these contributions, or equivalently the pensions, should vary with the state. This is a risk sharing problem.

Assumptions We take assumptions under which the so-called 'first order' approach to moral hazard problems is valid (see Grossman and Hart 1983 and Rogerson 1985 for example). While these assumptions are not necessary, they allow us to contrast the impact of moral hazard. Recall that outputs are increasing with status θ.

The level of effort takes values in an interval, $[0, e_h^{max}]$. The probabilities $p_h(\theta|e, s)$ are continuous with respect to s, and differentiable with respect to e. Furthermore, in each state s, they satisfy

- (CDF) the cumulative probability for a status to be smaller than t, $\sum_{\theta \leq t} p_h(\theta|e, s)$, is convex in effort e

- (MLR) the ratio $(p'_{he}/p_h)(\theta|e, s)$ is nondecreasing in θ, where p'_{he} is the derivative of p_h with respect to e.

The role of the assumptions is the following. Thanks to the convexity of the distribution function[5](CDF), the function $e \rightarrow U_h(\tilde{c}|e, s)$ is concave with respect to e for a contract $\tilde{c}$ that is increasing in θ. The monotone likelihood ratio (MLR) ensures that an optimal contract is indeed increasing in θ. Hence, for a flexible effort, the incentive compatibility constraint (2) can be replaced by the first order condition associated with the maximization of the utility level with respect to effort::

$$\sum_\theta p'_{he}(\theta|e_h, s)u_h(c_h(\theta)) - k_h(s) \leq 0 \text{ with } = \text{ if } 0 < e_h \quad (8)$$

(the fact that (8) is satisfied as an equality if $e = e^{max}$ is a standard result).

The above assumptions ensure also the validity of the first order approach when effort is irreversible (see the proof of Proposition 3). Thus, the incentive compatibility constraint (7) can be replaced by the first order condition associated with the maximization of the ex ante utility level with respect to effort:

$$E_\pi[\sum_\theta p'_{he}(\theta|e_h, s)u_h(c_h(\theta|s)] - k_h] \leq 0 \text{ with an equality if } e_h > 0. \quad (9)$$

With two status levels, b and g, which represent 'failure' and 'success', both assumptions are met if the probability of success is increasing and concave in effort. An example with more than two status levels is obtained when the distributions are 'spanned' by the two probability distributions associated with the maximum and minimum effort levels. Dropping s and h to simplify notation, there are distributions p and q on Θ, and a nonnegative function ℓ on $[0, 1]$ with $\ell(0) = 0$, and $\ell(1) = 1$ such that

$$p(\theta|e) = \ell(e)p(\theta) + (1 - \ell(e))q(\theta)$$

Assumptions CDF and MLR are satisfied if the ratio $p(\theta)/q(\theta)$ increases with θ and ℓ is increasing and concave. For $\ell(e) = e$, the *linear effort model* is obtained. Since the maximum or the minimum level of effort is surely an optimal effort level, the properties of the linear effort model are close to a model with only two values for effort.

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aim is not to be realistic but simply to show that a pay as you go system should account of macro-economic risk in its design.

[5]More generally the CDF assumption implies that for any increasing function $\tilde{x} = (x(\theta))$ the function $e \rightarrow \sum_\theta p_h(\theta|e, s)x(\theta)$ is concave, and the MHR assumption implies that $\sum_\theta p'_{he}(\theta|e, s)x(\theta)$ is positive given e.

3   The benchmark case without moral hazard

This section recalls the basic features of optimal risk sharing in the absence of moral hazard and establishes some comparative statics results. By absence of moral hazard, we mean that either there is no effort or that it is contractible. A contractible effort is assumed to be implemented by the agent (the effort is observable and the agent would be severely punished in case of default). Hence, feasibility and optimality are defined as in section 2, with the difference that the incentive compatibility constraint is simply dropped.

Mutuality principle and sharing rules Assume first no effort. Individuals cannot influence the probability distribution of their status, which only depends on the state. Thus, the output $\Omega_h(s)$ generated by each group h in state s and the overall level $\Omega(s)$ are exogenous. To be optimal, a scheme must satisfy the *mutuality principle*, which asserts that individuals' income levels are identical across states with identical aggregate resources. Therefore, individuals bear risks only if it is unavoidable. In particular they are fully insured against idiosyncratic shocks. Furthermore, income levels are described by *sharing rules*,[6] S_h for a h-individual, that assign income in function of the aggregate level $\Omega(s)$ available in state s (since individuals within a group are treated equally, their sharing rules are identical). Sharing rules are defined as follows. Given a level Ω, there are unique consumption levels $(S_h(\Omega))$ that equalize the weighted marginal utilities across individuals and sum up to Ω:

$$(S_h(\Omega))_{1,..,H} \text{ is the unique } (c_h)_{1,..,H} \text{ s.t. } \lambda_h u'_h(c_h) = \alpha, h = 1,..,H \text{ for some } \alpha \text{ and } \sum_h c_h = \Omega. \quad (10)$$

Uniqueness holds if all agents are risk averse because consumption levels are strictly decreasing in α and if group 1 is risk neutral because the value α must be equal to λ_1. We have that *in the absence of effort level, the optimal insurance scheme allocates income according to the sharing rules* $(S_h)_{1,..,H}$:

$$\text{in each state } s, c_h(\theta|s) = S_h(\Omega(s)) \text{ for each } h.$$

Sharing rules depend on the groups' attitudes towards risk and the weights. Under risk neutrality of group 1, individuals in risk averse groups are fully insured against all risks, that is, their income level is constant equal to S_h that verifies $\alpha = \lambda_1 = \lambda_h u'_h(S_h)$. Under strict aversion for all, shares vary in proportion of risk tolerance coefficients (Wilson 1968). Denoting τ_h h's risk tolerance coefficient $(-u'_h/u''_h)$ evaluated at level $c_h = S_h(\Omega)$, we have

$$S'_h(\Omega) = \frac{\tau_h}{\sum_j \tau_j}, \text{ each } h \quad (11)$$

In particular, sharing rules are linear for constant absolute risk aversion $\rho_h = 1\tau_h$ for each group for instance (see Wilson for a characterization of the utility profile admitting linear sharing rules).

Contractible effort The analysis extends to the situation in which individuals exert an effort that can be contracted upon. However, the level of resources available in a state is no longer exogenous since it is affected by the chosen effort. As a result, comparative statics on the behavior of the contract

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[6]This is the terminology of Wilson (1968). In the principal agent literature, the term sharing rule is sometimes used differently to refer to the share that the agent receives in function of the observable characteristic he can influence, that is here the status.

as the state varies, in particular as output levels increase are unclear with a flexible effort. The reason is that an increase in output levels may induce a decrease in the effort level, which may lead to a decrease in overall resources. This leads us to distinguish the case for which states differ by changes in h-outputs that are *uniform* across status, that is where $\omega_h(\theta|s') - \omega_h(\theta|s)$ is constant equal to $\Delta\omega_h$ for each θ.

Proposition 1 *Assume effort to be contractible. An optimal insurance scheme is characterized by*

1. *(optimal risk insurance) aggregate output is distributed according to the sharing rules S_h:*

$$\text{in each state } s, \, c_h(\theta|s) = S_h(\Omega(s)) \text{ for each } h$$

2. *(optimal effort) Let $\alpha(s)$ be the common value of weighted marginal utilities $\lambda_h u'_h(S_h(\Omega(s)))$. Define the value to h of output net of effort as $(\alpha(s)/\lambda_h)\left[\sum_\theta p_h(\theta|e, s)\omega_h(\theta|s)\right] - k_h(s)e$.*

   – under an irreversible effort, e_h maximizes the expectation of the value to h of output net of effort under the distribution π

   – under a flexible effort, in each state s, $e_h(s)$ maximizes the value to h of output net of effort.

   Consumption, effort and utility levels are continuous in the state.

3. *(Comparative statics) Let state s' be obtained from s by an increase in some outputs (i.e. for some h and some status). Under an irreversible effort, consumption levels of all risk averse groups are all strictly larger in s' than in s.*

   Let state s' be obtained from s by uniform changes in groups' outputs, $\Delta\omega_h$ for h. Under a flexible effort, if $\sum_h \Delta\omega_h \geq 0$ (resp. > 0), then all groups are at least as well off in s' as in state s (resp. all are strictly better off if all are risk averse).

According to point *1*, the mutuality principle still holds, and furthermore each individual consumes the same share of aggregate resources as without effort. To understand why, observe that, given effort levels, the groups face the problem of sharing optimally a *given* amount of risky aggregate resources as without effort. The result follows thanks to the separability of preferences in effort and consumption. Point *2* then easily follows. Decentralization of an optimal scheme under some informational requirements will be considered in section 7. The Arrow-Debreu contingent price of one unit of consumption deliverable in state s will be defined as $\pi(s)\alpha(s)$. In the sequel, $\alpha(s)$ is called the income *price* of state s.

The monotonicity property stated in point *3* is straightforward under an irreversible effort. Since the effort is identical in all states, the aggregate output $\Omega(s')$ is larger than $\Omega(s)$, and it suffices to use the monotony of the sharing rules. The same result does not hold with a flexible effort because an increase in the output of one group for instance results not only in a change in its members' effort but also in the resources of the other groups due to risk sharing hence in their effort levels. Without further assumptions, the overall impact is difficult to assess and possibly counterintuitive, as illustrated below. Under uniform changes, however, a global increase ($\sum_h \Delta\omega_h \geq 0$) is welfare improving for all. As shown in the proofs, effort levels decrease but the impact is moderate enough for overall resources nevertheless to increase: welfare increases owing both to a reduction in effort levels and an increase in

consumption levels. Without uniform changes, monotonicity may fail. As a simple example, consider two groups. Group 1 is risk neutral so that the price α and 2's consumption levels are kept constant across states. Group 2 is faced with two status, failure and success. An increase in 2's output in case of success without change in case of failure induces an increase in 2's effort level.[7] Thus 2's individuals are made worse off by a (non uniform) increase in output because their consumption do not change but they exert a higher effort.

Example 2 (Pensions) Let us illustrate the proposition with Example 2 on pensions. The individuals in the working age generation, group 1, are either employed with probability $p(e,s)$, generating output $\omega_1(s)$ (dropping unnecessary index θ), or unemployed with probability $1 - p(e,s)$ generating no output. Retirees receive an exogeneous output $\Omega_2(s)$. Thus aggregate output in state s is equal to $\Omega(s) = p(e,s)\omega_1(s)$. By the mutuality principle, young workers are fully insured against unemployment, and income levels of both young and old agents are functions of $\Omega(s)$. Take for example $\Omega_2(s)$ to be negligible, meaning that retirees' income comes mainly from Social Security (as we said before, a reasonable assumption in some countries). The above result says that pension annuities should not be indexed on individual wages, but rather on aggregate wages. In particular they should decrease as unemployment raises, and conversely. Such a result supports a *notional* pay as you go system, as has been recently implemented in Sweden, in which annuities are indexed on growth.

4   Impact of moral hazard on consumption levels

With a contractible effort, the optimal scheme can be described as involving two types of risk sharing: a full insurance contract against idiosyncratic shocks within each group, and a contract sharing the macro-economic risks across groups. The latter dictates contingent and balanced transfers across groups that are given by the difference between the share received by a group and its aggregate output, $S_h(\Omega(s)) - \Omega_h(s)$. Our aim is to investigate how these transfers are affected by moral hazard. In the first place, why should moral hazard have any impact?

When effort is neither observable nor verifiable, an individual who faces a contract that provides full insurance against status risk exerts the minimal level of effort. Hence, typically, the schemes described in Proposition 1 are not feasible. Furthermore, it may be optimal to insure idiosyncratic shocks only partially so as to induce individuals' effort. There is a trade-off between the insurance of idiosyncratic risks on one hand and the 'powerfulness' (in terms of incentives) on the other hand. It is this trade-off that is the source of distortion in macro-economic risk sharing. As we have seen, macro-economic risk sharing dictates income transfers across groups. In the presence of moral hazard, a transfer received by a group modifies the trade-off between the insurance and powerfulness of contracts. With a flexible effort, providing some supplementary income to a group results in a rather complex modification of the contract and the effort level. With an irreversible effort, even though the effort is exerted before the state is known, contracts must nevertheless be risky so as to provide incentives ex ante, and the

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[7]Let success and failure be denoted respectively by 1 and 0. According to proposition 1, α is constant equal to λ_1 and 2's effort maximizes $\frac{\alpha}{\lambda_2}p_2(1|e,s)[\omega_2(1|s) - \omega_2(0|s)] - k_2(s)e$. By assumption, $p_2(1|e,s)$ increases and is concave in e, and the term in square brackets is positive. This gives that the first order condition, $\frac{\alpha}{\lambda_2}p'_{2e}(1|e,s)[\omega_2(1|s) - \omega_2(0|s)] = k_2(s)e$ is sufficient and that the optimal effort increases with $\omega_2(1|s)$, the output in case of success.

trade-off arises as well. Hence how useful it is to give income to a group in a given state interacts with the incentives in that state (with a flexible effort) or ex ante (with an irreversible effort). In other words, the group's marginal benefit of wealth is affected by the need to use an incentive contract. In view of extending the Borch conditions, we first develop some tools for assessing this effect.

4.1   An auxiliary intra-group problem and moral harard risk premium

The impact of moral hazard for a risk averse group will be determined by an effort level and a 'price' for effort, which is given by the multiplier of the incentives constraint. With an irreversible effort, the effort level and multiplier are determined ex ante once for all, and with a flexible effort they vary with the state, as detailed respectively in sections 5 and 6. This leads us to analyze the marginal benefit of wealth for a group in a given state when both the effort level (e) and its price (β) are fixed, taken as parameters. To simplify notation, we drop any reference to a specific group h or to a state. Let v denote the members utility, which is assumed strictly concave, e a positive effort and β a positive scalar. Consider the following problem parameterized by a level R that determines the income level of the group.

$$\mathcal{P}(R) : \max_{\tilde{c}} \sum_{\theta} p(\theta|e)v(c(\theta)) - ke + \beta \left[\sum_{\theta} p'_e(\theta|e)v(c(\theta)) - k \right]$$

$$\text{under the budget constraint } \sum_{\theta} p(\theta|e)[c - \omega](\theta) \leq R \qquad (12)$$

β is the price of the incentives constraint because the net marginal benefit of increasing effort given contract $\tilde{c}$ is equal to $\sum_{\theta} p'_e(\theta|e)v(c(\theta)) - k$.

Lemma 1 *Let v be strictly concave. The optimal solution of problem $\mathcal{P}(R)$ is characterized by an income price $\alpha > 0$ such that*

$$\frac{1}{v'(c(\theta))} = \frac{1}{\alpha} + \frac{\beta}{\alpha} \frac{p'_e}{p}(\theta|e) \ \ \text{for each } \theta \qquad (13)$$

and the budget constraint (12) is satisfied as an equality. The value function $V(.)$ is concave and differentiable with a derivative given by

$$V'(R) = 1/E[\frac{1}{v'(\tilde{c})}|e] = \alpha. \qquad (14)$$

 The value α is the multiplier associated with the budget constraint. For β positive, consumption levels depend on status, preventing full insurance. Conditions (13) are well known in the principal agent literature as the optimality conditions on a contract that implements a given effort level. This is not surprising since the dual of a principal-agent problem[8] is of the form $\mathcal{P}(R)$ for an appropriate value of β.

    Formula (14) is easily proved, thanks to the envelope theorem. The group's marginal utility for wealth $V'(R)$ is equal to α. Taking the expectation over θ of (13) and using that $\sum_{\theta} p'_e(\theta)$ is null gives the result. One may understand this as follows. Consider a marginal change in the group's income

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[8]The principal maximizes the surplus, $\sum_{\theta} p(\theta|e)(\omega - c)(\theta)$, under a minimum utility level for the agent and given an effort to be implemented.

ΔR to the group. At an optimal solution, the induced marginal change in utility is given by $V'(R)\Delta R$ *independently* of how ΔR is used to modify the optimal contract. Modify the contract at the margin so as to equalize the marginal change in utility across various status θ. One has $v'(c(\theta))\Delta c(\theta) = \Delta v$ where Δv is chosen so as to satisfy the budget equation (12) : $E[\frac{1}{v'(c(\tilde{\theta}))}|e]\Delta v = \Delta R$. The marginal benefit for providing effort, that is the value $\sum_\theta p'_e(\theta|e)v(c(\theta)) - k$, is unchanged since $\sum_\theta p'_e(\theta)$ is null. Hence $V'(R)\Delta R$ is equal to Δv, which yields formula (14).

Moral hazard premium In general the group's marginal utility for wealth differs from the marginal utility of a representative individual who would consume the aggregate consumption and had the same preferences. For a log utility however, there is no difference. Applying (14) to $v(c) = \ln c$, the linearity of $1/v'(c) = c$ yields

$$1/V'(R) = E[\frac{1}{v'(\tilde{c})}] = E[\tilde{c}] = (1/v')(E[\tilde{c}]).$$

More generally the concavity or convexity of the reciprocal of individual marginal utility determines how group's marginal utility for income is distorted in the presence of moral hazard. To make this more precise it is useful to introduce the notion of moral hazard premium.

Definition 3 *Define the moral hazard premium at contract $\tilde{c}$ as the value ψ for which*

$$v'(E\tilde{c} - \psi) = 1/E[\frac{1}{v'}(\tilde{c})]. \tag{15}$$

An individual with utility v is said to be prudent (resp. imprudent) against moral hazard if $1/v'$ is concave (resp. convex). Under prudence, the moral hazard premium ψ is positive, and under imprudence it is negative.

The moral hazard premium is the sure decrease in expected consumption that has the same effect on marginal utility than the incentive contract $\tilde{c}$. It has a similar interpretation as the precautionary saving premium (Kimball 1990).[9]Assume that the group could save a period ahead at $t = 0$. At that period, it would equalize its marginal utility to the expected marginal utility at $t = 1$. The elimination of the risk in date 1's consumption profile at the cost of the premium would leave saving behavior unchanged. (This interpretation holds if the elimination of the risk has no impact on the effort level). From (15), ψ is the Arrow-Pratt risk premium associated to the function $1/v'$. This immediately gives that the sign of the premium is determined by the concavity property of $1/v'$, which motivates the definitions above.

Imprudence against moral hazard obtains for constant absolute risk aversion ρ. For a constant relative risk aversion γ ($v = c^{1-\gamma}/(1-\gamma)$) prudence obtains for γ smaller than 1 and imprudence for γ larger than 1. (Hence, imprudence may be thought as the more plausible assumption since γ is usually assumed to be larger than 1). One may approximate the premia by using the Arrow-Pratt indices of the functions $1/v'$. The Arrow-Pratt index of $1/v'$ is expressed in terms of that of v. Letting $\rho_v(c) = [-v''/v'](c)$, and $\tau_v(c) = [-v'/v''](c)$ the risk tolerance coefficient of v at level c simple computation gives:

$$\rho_{1/v'}(c) = \frac{-\rho'_v}{\rho_v}(c) - \rho_v(c) = \frac{\tau'_v - 1}{\tau_v}(c)$$

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[9]In the precautionary saving premium, $\tilde{c}$ is exogenous, and the premium is defined by $v'(E\tilde{c} - \psi) = E[v'(\tilde{c})]$.

Under constant absolute risk aversion ρ, one has $\rho_{1/v'}(c) = -\rho$. This gives a moral hazard premium that is independent of R (or equivalently of α) and only depends on β and the probability given e:

$$\psi = \tau E_\theta \ln(1 + \beta \frac{p'_e}{p}). \tag{16}$$

For a constant relative risk aversion γ one has $\rho_{1/v'}(c) = (1-\gamma)/c$. From the optimal conditions (13), we have $c(\theta)^\gamma = \alpha^{-1}(1 + \beta \frac{p'_e}{p}(\theta))$ and, by definition of the premium, $(E[\tilde{c}] - \psi)^\gamma = \alpha^{-1}$ which gives

$$\frac{\psi}{E[\tilde{c}]} = 1 - \frac{1}{E[(1 + \beta \frac{p'_e}{p}(\theta))^{\frac{1}{\gamma}}]}$$

4.2   Extended Borch conditions and their implications

The optimality of income exchanges between groups in a state takes a similar form under both timings (with a qualification for a flexible effort, as shown in Section 6). Optimality requires the equalization across groups of their weighted marginal utilities for income properly adjusted to account of incentives, as stated as follows.

Extended Borch conditions *At an optimal scheme, in a given state s, there is $\alpha(s)$ for which*

$$\lambda_h \frac{1}{E[\frac{1}{u'_h}|e_h(s), s]} = \alpha(s) \quad any\ h. \tag{17}$$

As we have seen, for each group h, the left hand side is the weighted group's marginal utility for wealth adjusted to account of incentives, that is given their effort level and effort price. Hence, conditions (17) extends Borch conditions (10) to situations with moral hazard by requiring the equalization across groups of their adjusted weighted marginal utilities for wealth.

These extended Borch conditions allow us to derive some implications on the impact of moral hazard on the income levels of each group. The questions we address are the following ones. Consider an outside observer who observes aggregate levels of income in each group and aggregate resources. Under which conditions is the share of aggregate resources received by a group at an optimal scheme smaller or larger than without moral hazard? How this share is affected by the moral hazard problem faced by other groups? In other words, given the observed aggregate resources $\Omega(s)$, we want to compare group h income, $C_h(s) = E_\theta[c_h(\theta|s)]$, with the share $S_h(\Omega(s))$.

If each group has a log utility function (or is not subject to moral hazard), then the optimality conditions (17) coincide with the Borch conditions applied to expected consumption levels. Hence, at an optimal insurance scheme, aggregate resources are shared between groups as without moral hazard: $C_h = S_h(\Omega)$, for $h = 1, .., H$. For general utility functions, moral hazard has an impact but sharing rules can still be used to describe the groups income levels. The shares are computed at the aggregate output adjusted by the premia of all groups, as stated in the following proposition.

Proposition 2 *At an optimal scheme, let $\psi_h(s)$ be the moral hazard premium of group h in state s, as defined by (15). Denoting by $\psi(s)$ the sum of the premia, $\psi(s) = \sum_h \psi_h(s)$, one has*

$$C_h(s) = S_h(\Omega(s) - \psi(s)) + \psi_h(s). \tag{18}$$

PROOF From the extended Borch conditions (17) and the definition of a moral hazard premium, we have

$$\lambda_h u'_h(C_h(s) - \psi_h(s)) = \alpha(s).$$

The weighted marginal utilities $\lambda_h u'_h$ are equalized at $C_h(s) - \psi_h(s)$, that is the Borch conditions (10) are satisfied with an aggregate wealth given by the sum $\sum_h (C_h(s) - \psi_h(s))$. The sum is equal to $\Omega(s) - \psi(s)$, this gives that each $C_h(s) - \psi_h(s)$ is given by the share $S_h(\Omega(s) - \psi(s))$. ∎

As a result of risk sharing across groups, a group is affected by the environment, in particular by the moral hazard problem faced by other groups. Formula (18) allows us to assess these externalities. Assume first the presence of a risk neutral group. Since the sharing rule S_h of a risk averse group h is constant, we get the following corollary.

Corollary 1 *Assume group 1 to be risk neutral. Then S_h is constant and the expected income of a risk averse group varies with its own premium only*

$$C_h(s) = S_h + \psi_h(s), h > 1.$$

This can be explained as follows. Since exchanges across groups are due to insurance purpose against maroeconomic risks, they are channeled through the risk neutral group. As a result, the variation of income of a risk averse group varies only with its own environment and the severity of its moral hazard problem. Group 1's income is deduced by feasibility : $C_1(s) = \Omega(s) - \sum_{h \neq 1}(S_h + \psi_h(s))$.

When all groups are risk averse, each sharing rule is increasing. Hence, given a level for total resources Ω, a group's income increases with the moral hazard premium of another group. Using that the sign of the premium depends on prudence and that the slope of S_h is positive and smaller than 1, the corollary follows.

Corollary 2 *Assume all groups risk averse. Let h be prudent against moral hazard (or not be subject to moral hazard in state s). At the optimal scheme*

$$C_h(s) \geq S_h(\Omega(s) - \sum_{k \neq h} \psi_k(s)).$$

(The inequality is reversed if h is imprudent.) If in addition all other groups are imprudent against moral hazard then

$$C_h(s) \geq S_h(\Omega(s)).$$

(The inequality is reversed if h is imprudent and all other groups are prudent.)

Corollary 3 immediately applies to two groups, one of which being not subject to moral hazard: the sure share received by the group not subject to moral hazard is increased if the other group is imprudent (and decreased if it is prudent).

We have drawn some consequences from the extended Borch conditions and the sign of the moral hazard premia. To go further, we need to examine separately each timing.

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5 Irreversible effort

This section considers the first timing, in which an 'investment' effort has to be exerted before the macro-economic state is revealed without any possible adjustment afterwards. An insurance scheme together with the optimal effort levels, $(\tilde{c}_h(s), e_h)_{h=1,..,H}$ is optimal if it maximizes the welfare criterion (6) over all schemes that satisfy the feasibility constraints (5) in each state s and the ex ante incentives constraints (9). The optimal problem to be solved is :

$$\mathcal{I}: \quad \max_{(\tilde{c}_h(s), e_h)_{h=1,..,H}} \sum_h \lambda_h E_\pi \left[U_h(\tilde{c}_h | e_h, s) \right] \text{ under}$$

$$
\begin{cases}
\sum_h \sum_\theta p_h(\theta | e_h, s)[c_h - \omega_h](\theta | s) \leq 0 \text{ in each state } s & (5) \\
E_\pi[\sum_\theta p'_{he}(\theta | e_h, s) u_h(c_h(\theta | s)] - k_h] \leq 0 \text{ with } = \text{ if } 0 < e_h \ h = 1,..,H & (9)
\end{cases}
$$

Denote by $\beta_h \geq 0$ the value of the multiplier associated to the incentives constraint (9) and by $e_h \geq 0$ the effort level at the optimal scheme. Given these values, the problem to be solved writes as

$$\max (\tilde{c}_h(s), e_h)_{h=1,..,H} E_\pi \left[\sum_h \left\{ \lambda_h [\sum_\theta U_h(\tilde{c}_h | e_h, s) + \beta_h(\sum_\theta p'_{he}(\theta | e_h, s) u_h(c_h(\theta | s)) - k_h)] \right\} \right]$$

under the feasibility constraints (5). This problem is separable across states. Thus, given the correct values for β_h and e_h, we can analyze the optimal scheme state by state. In order to assess the impact of moral hazard on macro-economic risk sharing, we make clear the transfers across groups that take place in each state. This leads to a decomposition of the problem to be solved in each state into two sub-problems, one intra-group and the other inter-group.

Decomposition into intra and inter-group problems In what follows, take the values for β_h and e_h that obtain at the optimal scheme. Consider a state s. Given a contract, define the implicit net transfer to group h in that state as the expected surplus of its income over output:

$$R_h = \sum_\theta p_h(\theta | e_h, s)[c_h - \omega_h](\theta | s). \quad (19)$$

The feasibility constraint (5) in state s writes as $\sum_h R_h \leq 0$. Using the expression of U_h given by (1), the problem to to be solved in state s can be written as

$$\max_{(\tilde{c}_h(s), e_h)_{h=1,..,H}} \quad \sum_h \lambda_h \left[\sum_\theta p_h(\theta | e_h, s) u_h(c_h(\theta | s)) - k_h e_h + \beta_h(\sum_\theta p'_{he}(\theta | e_h, s) u_h(c_h(\theta | s)) - k_h) \right]$$

$$\text{under } \sum_h R_h \leq 0.$$

Both the objective and the constraint are separable across h. Hence denoting by $R_h(s)$ the transfer at the optimal solution, the optimal contract for group h maximizes the h-term in brackets under the budget constraint given by $R_h(s)$. Observe that the h-term is the objective associated to an auxiliary problem $\mathcal{P}_h(R, s)$ for appropriate values for utility ($v = \lambda_h u_h$) and probabilities, and given the values β_h and e_h. This implies that the optimal contract solves $\mathcal{P}_h(R_h(s), s)$. The decomposition into intra and inter-group problems follows. The inter-group problem analyzes how income should be distributed

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across groups, knowing that this income will be optimally used within each group, i.e. will solve an intra-group problem $\mathcal{P}_h(., s)$. Specifically, let us denote by $V_h(R, s)$ the value function of $\mathcal{P}_h(R, s)$. The inter-group problem looks for transfers that solve

$$\max_{(R_h)_{1,..,H}} \sum_h V_h(R_h, s) \text{ over the } (R_h)_{1,..,H} \text{ under the feasibility constraint } \sum_h R_h = 0. \qquad (20)$$

Given the optimal transfers $(R_h(s))_{1,..,H}$, the contract for group h solves the intra-group problem $\mathcal{P}_h(R_h(s), s)$.

The decomposition has the following implications, using Lemma 1. Firstly, since the V_h are derivable with a derivative equal to $\lambda_h / E[\frac{1}{u'_h}|e_h, s]$, the optimality of the transfers requires the equalization of these derivatives to a common value $\alpha(s)$, the multiplier of the scarcity constraint: this gives the extended Borch conditions. Secondly, optimal contracts chosen by h satisfies the optimality conditions (13) associated to intra-group problem $\mathcal{P}_h(R_h(s), s)$. These conditions can be written in the following form that is more appropriate for our purpose here:

$$\lambda_h[1 + \beta_h \frac{p'_{he}}{p_h}(\theta|e_h, s)]u'_h(c_h(\theta|s)) = \alpha(s) \text{ for each } \theta \text{ in } \Theta_h. \qquad (21)$$

Consider states that differ according to outputs only, that is the probability distribution is constant over these states. The above expression (21) implies that marginal utilities weighted by a status-adjusted weight are equalized across individuals : the weight assigned to a h-individual with status θ is the value $\lambda_h(1 + \beta_h \frac{p'_{he}}{p_h}(\theta|e_h, s))$ which is constant along these states. Further properties can be derived, as stated in the following proposition.

Proposition 3 *Assume effort to be irreversible. Status-contingent income levels, $c_h(\theta|s)$, are continuous with respect to the state. Along states with identical probability distributions for each group*

1. *resources are distributed according to some sharing rules that differentiate individuals according to their status within a group. As a result, status-contingent income levels $c_h(\theta|s)$ depend on s through the aggregate level Ω only and increase with it.*

2. *If group 1 is risk neutral, then other groups face the same contract whatever the state. If all groups are risk averse, a variation in the aggregate level, $d\Omega$, is distributed to (θ, h)-agents in proportion of their risk tolerance and to groups in proportion of their average risk tolerance $E_\theta \tilde{\tau}_h$:*

$$dc_h(\theta) = \frac{\tau_h(c_h(\theta))}{\sum_h E_\theta \tilde{\tau}_h} d\Omega \text{ each } \theta, h \text{ and } dC_h = \frac{E_\theta \tilde{\tau}_h}{\sum_h E_\theta \tilde{\tau}_h} d\Omega \text{ each } h. \qquad (22)$$

Continuity is a direct implication of the decomposition presented before the proposition. Since neither the effort level nor the price of effort vary with the state, the value functions V_h are concave. Hence the transfers across groups, which solve the inter -group problem (20), are continuous in the state and the optimal consumption levels, which solve the intra-group problems $\mathcal{P}_h(R_h(s), s)$, are continuous as well.

Point *1* states that the mutuality principle holds in a weak form: along the states with the same distribution probabilities, the income level of h-individuals only depend on the level of aggregate

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resources and their status, but not on their own output (which may differ across states with identical aggregate resources). Also, they all vary in the same direction, either up or down with the state. The mutuality principle does not fully hold however since idiosyncratic shocks are not insured.

Furthermore, in the presence of a risk neutral group, even though risk averse individuals may bear risks for incentives purpose, they are 'insured' against macro economic risk since they face the same contract. The shares received by each group in general differ from S_h according to their prudence or imprudence, as we have seen in the previous section. This is made precise by point 1. However, their behavior is rather similar. Expression (22) gives that a variation of total resources $d\Omega$ is distributed to groups in proportion to their adjusted risk tolerance. Under linear risk tolerance for instance, the adjusted risk tolerance of a group is the risk tolerance of an individual whose consumption is the group expected consumption. Thus, even though the levels of the shares differ, their variation are the same as without moral hazard.

As an example, consider a constant risk aversion function for each group, with risk tolerance coefficient τ_h for h, and define τ as the sum $\sum_{h=1,..,H} \tau_h$. Shares are linear in Ω. The moral hazard premium is given by $\psi_h(s) = \tau_h E_\theta \ln(1 + \beta_h \frac{p'_{he}}{p_h}(\theta|e_h, s))$ (see (16) in section 4.1). Simple computation gives

$$c_h(\theta|s) = C_h(s) - \psi_h(s) + \tau_h \ln(1 + \beta_h \frac{p'_{he}}{p_h}(\theta|e_h, s)) \text{ and } C_h(s) = S_h(\Omega(s)) + \psi_h(s) - \frac{\tau_h}{\tau}(\sum_k \psi_k(s)).$$

Under constant probability distributions, contracts take a very simple form. The part in the contract which is status dependent as well as the moral hazard premium are constant. Hence groups' income are up to a constant given by the sharing rules S_h. This means that they are optimal for different weights (this is of course specific to constant risk aversion).

6   Optimal schemes under flexible effort

We shall proceed as in the previous section by first noticing that the problem to be solved in each state is separable across states, and then by analyzing the transfers to implement in each state. Recall that an insurance scheme is *optimal* if it maximizes the welfare criterion (6) over all schemes that satisfy in each state s the feasibility and incentives constraints. With a flexible effort, these constraints are independent across states. Observe furthermore that the welfare criterion is separable across states since exchanging the sums over h and the states, we have

$$\sum_h \lambda_h E_\pi[U_h(\tilde{c}_h|e_h(s), s)] = E_\pi[\sum_h \lambda_h U_h(\tilde{c}_h|e_h(s), s)].$$

Thus, the optimization problem can be solved state by state by maximizing in each state the weighted sum, $\sum_h \lambda_h U_h(\tilde{c}_h|e_h, s)$, under the constraints prevailing in that state. Replacing the incentive constraint by the first order condition, the problem $\mathcal{F}(s)$ to be solved in state s writes as:

$$\mathcal{F}(s): \quad \max_{(\tilde{c}_h, e_h)_{h=1,..,H}} \sum_h \lambda_h U_h(\tilde{c}_h|e_h, s) \text{ under}$$

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$$\begin{cases} \displaystyle\sum_h \sum_\theta p_h(\theta|e_h, s)[c_h - \omega_h](\theta|s) \leq 0 & (5) \\ \sum_\theta p'_{he}(\theta|e_h, s)u_h(c_h(\theta)) - k(s) \leq 0 \text{ with } = \text{ if } 0 < e_h \ h = 1, .., H & (8) \end{cases}$$

Decomposition into intra and inter-group problems As for an irreversible effort, making explicit the transfers across groups leads to a decomposition of problem $\mathcal{F}(s)$ into two sub-problems, one intra-group and the other inter-group. To see this, given an optimal scheme and effort levels in state s, $(\tilde{c}_h(s), e_h(s))_{h=1,..,H}$, let us consider $R_h(s)$ the implicit net transfer to group h as defined by (19) (given the chosen level of effort). Surely, the insurance contract $\tilde{c}_h(s)$ for group h is optimal given the net transfer $R_h(s)$ to that group. In other words, if group h is given $R_h(s)$ and can choose a contract freely under the associated budget constraint, it cannot do better than choosing $\tilde{c}_h(s)$. This leads us to consider the *intra-group* problem that finds an optimal insurance contract for h *given* a net transfer R_h. The problem writes

$$\mathcal{F}_h(R_h|s) : \max_{\tilde{c}_h, e_h} \lambda_h U_h(\tilde{c}_h|e_h, s) \text{ under}$$

$$\begin{cases} \displaystyle\sum_\theta p(\theta|e_h, s)[c_h - \omega_h](\theta|s) \leq R_h \\ \sum_\theta p'_{he}(\theta|e_h, s)u_h(c_h(\theta)) - k(s) \leq 0 \text{ with } = \text{ if } 0 < e_h \ h = 1, .., H & (8) \end{cases}$$

Let us denote by $F_h(R_h|s)$ the value of the program $\mathcal{F}_h(R_h|s)$. Problem $\mathcal{F}(s)$ amounts to solve optimally the *inter-group* problem of allocating net transfers across groups, that is to find transfers $(R_h(s))_{1,..,H}$ that maximize the weighted sum $\sum_h F_h(R_h|s)$ over the balanced transfers. The following proposition summarizes this decomposition.

Proposition 4 *An optimal insurance scheme satisfies the following conditions in each state s:*

1. *(optimal contract within a group) For each h, the contract $(c_h(\theta|s))$ and the effort level $e_h(s)$ are optimal for group h given the state s and the net transfer $R_h(s)$, that is, they solve $\mathcal{F}_h(R_h(s)|s)$.*

2. *(optimal risk sharing across groups) The transfers $(R_h(s))_{1,..,H}$ maximize $\sum_h F_h(R_h|s)$ over $(R_h)_{1,..,H}$ that satisfy the feasibility constraint $\sum_h R_h = 0$, where $F_h(R_h|s)$ is the value of the program $\mathcal{F}_h(R_h|s)$.*

Surely, the contract that solves $\mathcal{F}_h(R_h(s)|s)$ solves also the auxiliary problem $\mathcal{P}_h(R_h(s)|s)$ where the values e_h and β_h are fixed at the optimal value of effort and at the multiplier of the incentive constraint. This implies that the optimality conditions (13) are satisfied by the contract for some $\alpha_h(s)$ (there is an additional optimality condition with respect to effort; this will be considered in more details in section 6.2). Furthermore we have the following lemma.

Lemma 2. *At a point R where the value function $F_h(.|s)$ is differentiable, its derivative is given by*

$$F'_h(R|s) = 1/E[\frac{1}{\lambda_h u'_h(\tilde{c})}|e, s] = \alpha_h(s) \qquad (23)$$

in which $(\tilde{c}, e)$ solves the problem $\mathcal{F}_h(R|s)$.

This formula allows us to assess the cost of moral hazard if the increase ΔR in the transfer is used to increase consumption levels uniformly across the various status. Since a uniform consumption

increase, $\Delta c(\theta) = \Delta c$, typically reduces effort, Δc has to differ from ΔR so as to ensure feasibility.[10] The difference, the 'marginal cost' of moral hazard, is derived as follows. By the envelope theorem, the marginal change of utility is equal to $\Delta c E[u']$ so that (23) gives $\Delta c E[u'] = \Delta R / E[1/u']$. The extra amount of consumption that can be given to each θ-agent is smaller than ΔR by Jensen inequality (applied to the reciprocal function $1/x$). More precisely the decrease in the level of effort induces a loss of resources equal to $\Delta R(1 - 1/(E[u']E[1/u']))$.

Remark. Problem $\mathcal{F}_h(R)$ is the dual of a standard principal-agent problem with a risk neutral principal. In a repeated moral hazard problem, Rogerson (1985) shows that the reciprocal of the marginal utility plays a role. More precisely, a risk neutral principal always benefits from equalizing the agent's reciprocal marginal utility in a first period to the discounted expected reciprocal marginal utility of the following period. Lemma 2 helps us to understand this result: the agent's marginal utility for income is equalized across periods.

6.1 Discontinuities and non monotonicity

Optimal schemes enjoy continuity and monotonicity properties with respect to the macro-economic state under both a contractible or an irreversible effort (Propositions 1 or 3). These properties may fail under a non-contractible flexible effort. The reason is that the endogenous level of effort induces the non convexity of the program faced by a group, even if the first approach is valid, as we show now.

Define the indirect utility for a contingent income $\tilde{c}_h$ in state s as the value derived under optimal effort: $\hat{U}_h(\tilde{c}_h|s) = max_{e \in \mathcal{E}_h} U(\tilde{c}_h|e, s)$. Since $\hat{U}_h$ integrates the effort choice, the intra-group program $\mathcal{F}_h(R|s)$ amounts to choose the contingent income that maximizes $\hat{U}_h$ under the feasibility constraint. Function $\hat{U}_h$ is not necessarily concave in $\tilde{c}_h$.[11] To understand why, consider two status, success ($\theta = 1$) and failure ($\theta = 0$). Using the envelope theorem, the marginal indirect utility for consumption contingent on a status θ, $(\frac{\partial \hat{U}}{\partial c(\theta)})$ is equal to $p(\theta|e)u'(c(\theta))$ where e is the optimal level of effort (omitting index h and s for simplification). Consider an increase in $c(0)$, the income level contingent on failure. It may induce a lower effort hence an increase in the probability of failure, which, if large enough, outweighs the decrease in $u'(c(0))$ and results in an increase of the product $p(0|e)u'(c(0))$. In that case the marginal indirect utility for consumption in case of failure increases with the consumption level: $\hat{U}$ is not concave. This surely occurs when probabilities are linear in effort. Let $p = p(1|e^{max}$ and $q = p(1|e^{min}$ be respectively the probabilities of success if the maximal and minimum levels are exerted with $p > q$. The linearity implies that the optimal effort is typically at the minimum or maximum level. Starting with a contract for which e^{max} is optimal, increase consumption in case of failure, $c(0)$, keeping $c(1)$ fixed. There is a threshold value $c^*(0)$ for which the optimal level of effort

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[10] Assuming e to be interior, the derivation of the first order condition of the incentives constraint $\sum_\theta p_e'(\theta|e,s)u(c(\theta)) = k$ gives $\Delta c[\sum_\theta p_e'u'(c(\theta))] + [\sum_\theta p_{ee}''u(c(\theta))]\Delta e = 0$. Both terms in brackets are negative: the first because of increasing likelihood ratio, the second because of the second order condition.

[11] The non concavity of the indirect utility function has been identified by Prescott and Townsend (1984) and Arnott and Stiglitz (1991). The non concavity of the utility over contracts may justify the use of lotteries. The use of lotteries do not seem an appealing solution in real life situations and we have not investigated whether the discontinuities in contracts as the state varies would be eliminated.

jumps to the minimum. At this point, the marginal utility for consumption in case of failure jumps upward from $(1-p)u'(c^*(0))$ to $(1-q)u'(c^*(0)) : \hat{U}$ is not concave.

The non concavity of $\hat{U}_h$ has two consequences on the behavior of the intra-group program $\mathcal{F}_h(R|s)$ faced by h as R varies (recall that the program chooses the contingent income that maximizes $\hat{U}_h$ under the feasibility constraint associated to R). First a marginal change in R may induce discontinuous changes in the optimal contract and effort level. Second, the value function $F_h(R|s)$ may not be concave in R. The nonconcavity of F_h has in turn consequences on the optimal transfers across groups, since these transfers maximize the sum of the F_h (proposition 4). The non concavity of the objective may generate discontinuities in the optimal transfers and in the insurance contracts as the sate varies. Discontinuities and non monotonicity in contracts are illustrated by the following example.

Example There are two groups. Only individuals in group 1 are subject to idiosyncratic risk with two values for status and probabilities are linear in effort. The state is characterized by (ω_1, ω_2), where ω_1 is the output of a 1-individual in case of success, (0 in case of failure) and ω_2 is the output of a 2-individual. Utility functions are log in both groups. As we have seen, shares of aggregate resources are not distorted by moral hazard with log utility. Hence the discontinuity and non-monotonicity in the contracts are independent of any distortion in aggregate resources. Weights are taken to be equal.

Given a state $s = (\omega_1, \omega_2)$, the optimal transfer is obtained by maximizing the welfare function $F_1(R|s) + F_2(-R|s)$ in which R is the transfer from group 2 to group 1. One has $F_2(-R|s) = \log(\omega_2 - R)$. As for group 1 easy computation gives $F_1(R|s)$ as:

$log(p\omega_1 + R) - a$ if $R \leq R^*$ (1's effort level is maximal)

$log(q\omega_1 + R)$ if $R \geq R^*$ (1's effort level is minimal)

for two scalars a and R^*. The value function F_1 is continuous[12] at R^* with an upward jump for its derivative.

Total welfare, $F_1 + F_2$, is concave in R on each interval where 1's effort is constant, but not globally concave. As a result, welfare may have two local maxima.[13] In Figure 6.1 welfare as a function of the transfer from group 2 to group 1 is drawn in two states with identical value for ω_1 but two distinct values for ω_2. For the low value of ω_2, the dashed line, welfare is maximized at a low enough transfer so as to induce maximal effort. For the high value, the plain line, effort is minimal. The second graph depicts the set of states for which effort is maximal, below the line, or minimal, above the line. The two graphs in the middle represent contracts for a fixed value of ω_2 as ω_1 increases and similarly for a fixed value of ω_1 as ω_2 increases. The last graph gives the value of the income price as a function

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[12]With probabilities linear in effort, it suffices to consider contracts for which either the minimum or the maximum effort level is optimal. For $e = 0$, the contract is risk-less : $c(1) = c(0) = q\omega_1 + R$. For $e = 1$, the contract is determined by the incentive constraints $(p - q)[\ln c(1) - \ln c(0)] = k$ (the first order condition is satisfied as an equality at the maximum effort level) and the budget constraint $pc(1) + (1 - p)c(0) = R$. This yields the utility level $\ln(p\omega + R) - a$ with $a = q\frac{k}{p-q} - \ln[p\exp(\frac{k}{p-q}) + 1 - p]$. The necessary first order conditions of program $\mathcal{P}_1(R)$ have two distinct solutions, corresponding to these two effort values. The risk less contract is better for $R \geq R^*$, where R^* is defined by $\ln(p\omega_1 + R) - a = \ln(q\omega_1 + R)$.

[13]Total welfare $F_1 + F_2$ is maximized on $\{R/R \leq R^*\}$ (the interval of transfers smaller than R^*) at $\min(R_1, R^*)$ where $R_1 = (\omega_2 - p\omega_1)/2$, and on $\{R/R \geq R^*\}$ at $\max(R_0, R^*)$ where $R_0 = (\omega_2 - q\omega_1)/2$. In a state for which $R_1 < R^* < R_0$, welfare has two local maxima at R_1 and R_0 with respective values $2\ln([p\omega_1 + \omega_2]/2) - a$ and $2\ln([q\omega_1 + \omega_2]/2)$. The global maximum is R_1 (with maxima effort) if $\omega_2 \leq \omega_1 K$ for some constant K.

of ω_2.

At ω^* utility of group 2 jumps downward. Since maximal welfare (maximum of $F_1 + F_2$) is continuous in the state by standard arguments, the jump is exactly compensated by a positive jump of 1's utility. The reason why an increase in 2's output hurts group 2 is that it triggers an increase in the transfer to group 1 that shifts 1's effort level to its minimum and results in a decrease in overall resources. This can also be explained by the behavior of the Arrow-Debreu (AD) prices supporting the optimal scheme. The AD prices are discontinuous. To illustrate this point, take a fixed value for ω_1 so that a state s is identified with ω_2. As we shall see in the decentralization section 7, the AD price for consumption contingent on ω_2 is equal to the common value marginal utility for revenue in that state ω_2, multiplied by the probability of the state. These AD prices are depicted in figure 6.1 under a uniform probability for ω_2. Their shapes can be explained as follows Group 2's demand for contracts contingent on ω_2 decreases with ω_2 (price unchanged). As for group 1, if it exerts the same level of effort, its demand is constant with respect to ω_2 and decreasing in the price. This explains the behavior of prices up for contracts contingent on ω_2 smaller than some value $\omega^* = 1.23$: Because group 2 's demand decreases, the AD price decreases. Group 1 sells AD contracts contingent on ω_2 smaller than 0.8 and buys for ω_2 between 0.8 and ω^* but in a moderate amount so that it still provides effort if these states realize. At ω^*, the price is sufficiently low so that group 1 is as well off by buying a large amount of AD securities contingent on ω^* and exerting no effort if ω^* materializes. If the price was to decrease further with ω_2, 1's demand would jump upward and the market would be unbalanced: The AD price has to increase. This explains why the AD price jumps upward at ω^*. Due to this high price, it is ex ante in the interest to group 2 to sell a large quantity of good contingent on the endowment ω being larger than ω^* because it allows for a larger income when the endowment is small. Ex post, if indeed $\omega^* + \epsilon$ realizes for $\epsilon > 0$ small enough, group 2 individuals finally end up with a smaller income than if $\omega^* - \epsilon$ realizes.

The important point is not the discontinuity in the contract but the discontinuity in utility levels and the fact that a group may be hurt by an increase in its output (other elements unchanged). Since maximal welfare (maximum of $F_1 + F_2$) is continuous in the state by standard arguments, the jump downward in 2's utility level is exactly compensated by a positive jump in 1's utility level. For sake of comparison, consider the situation in which there is no insurance against macro-economic risks across groups. Each group 'solves' $\mathcal{F}_h(0|s)$ in each state s. Whereas the policies, that is the contract and the effort, may be discontinuous in the state, the value $F_h(0|s)$ is nevertheless continuous in s and is increasing in each h-output. Thus a group is never hurt by an increase in its own output. Comparing with the case where effort is contractible or irreversible, or where there is no insurance across groups, it is the implementation of insurance transfers at the same time as noncontractible effort is chosen that generates discontinuous and non monotone groups' welfare levels.

6.2   An analysis of the moral hazard premium

To analyze further the impact of moral hazard, we establish some comparative static exercises on how an optimal contract and the associated moral hazard premium vary as the state varies. An optimal contract for a group in state s solves an intra-group problem of the type $\mathcal{F}(R|s)$ for some value of R

Figure 3: $k = 0.5$, $p = 0.8$, $q = 0.3$
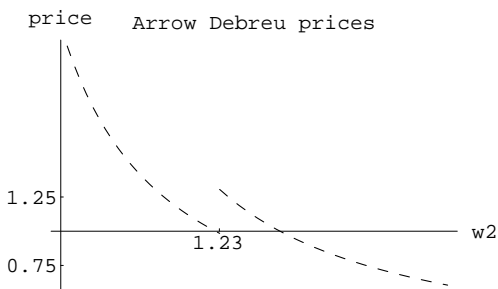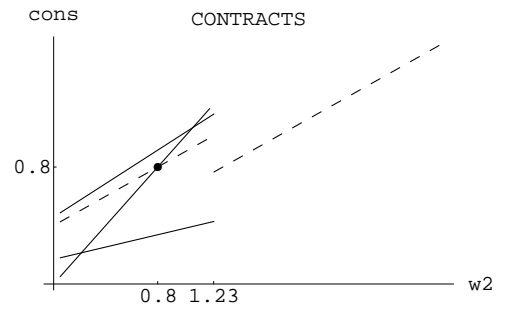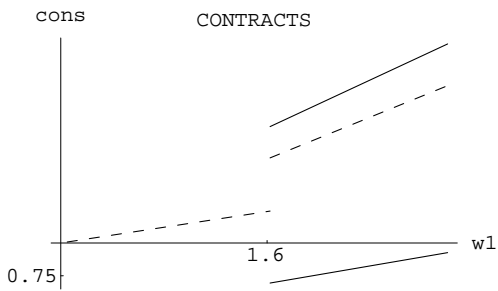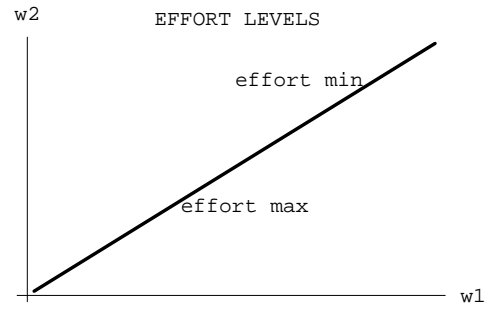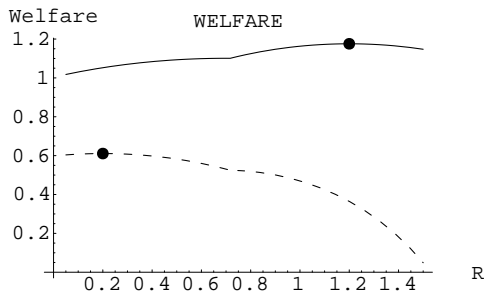
(where we drop any reference to a specific group h). The analysis is carried out by observing that optimal contracts can be parameterized by the effort level e and the multipliers α and β associated with the budget and incentive compatibility constraints as given in Lemma 1. Some comparative static results on how the optimal contract and moral hazard premium vary are obtained when the effort level is kept fixed so as to avoid discontinuities. We focus on a positive effort since with a null effort the contracts vary with a state as without moral hazard.

We first recap all the conditions satisfied by a contract $(c(\theta))$ and a positive effort level e that solve an intra-group problem with flexible effort $\mathcal{F}(R)$.

$$\sum_\theta p(\theta|e)\left[c(\theta) - \omega(\theta)\right] \leq R \tag{12}$$

$$\sum_\theta p'_e(\theta|e)v(c(\theta)) = k \tag{24}$$

$$\text{for each } \theta, \quad \frac{1}{v'(c(\theta))} = \frac{1}{\alpha} + \frac{\beta}{\alpha}\frac{p'_e}{p}(\theta|e) \tag{13}$$

$$\alpha\sum_\theta p'_e(\theta|e)(\omega - c)(\theta) - \beta\left(\sum_\theta -p''_{ee}(\theta|e)u(c(\theta))\right) \geq 0 \text{ with } = \text{ if } e < e^{max}. \tag{25}$$

Equations (12) and (24) are the budget and incentive compatibility constraints and (13) and (25) are the first order optimality conditions on consumptions and effort respectively. Recall that the marginal utility for income α and the moral hazard premium ψ satisfy: $\alpha = 1/E[\frac{1}{v'(\tilde{c})}] = v'(E\tilde{c} - \psi)$.

Consider some positive values for α, β and e. Conditions (13) uniquely determine a contract as a function of these values. Let us denote the solution by $\mathbf{c}(\alpha, \beta, e) = (\mathbf{c}(\alpha, \beta, e; \theta))$. Observe that this contract satisfies the other constraints (12), (24), and (25) for appropriate values for k, and $(\omega(\theta))$.[14] In particular, the marginal benefit to provide effort, $\sum_\theta p'_e(\theta|e)v(c(\theta))$, gives the value of the cost k supporting the contract by (24). Thus, instead of considering contracts as a function of the state (that is of ω and k), we shall consider them as a function of α, β and e. We analyze how contracts vary as α or β varies, keeping e fixed.

Consider first a variation in income price α for fixed positive values of β and e. Observe that this is the situation of an irreversible effort. In that case contracts are of the form $\mathbf{c}(., \beta, e)$, since they satisfy the optimal conditions (13) for fixed value of β and e. As for an irreversible effort, contingent income levels vary as in a standard risk sharing problem without moral hazard in which agents are treated differently according to their status:

$$d\mathbf{c}(\theta) = \tau_v(\mathbf{c}(\theta))(-d\alpha/\alpha)$$

where $\tau_v(\mathbf{c}(\theta))$ is the risk tolerance coefficient computed at the consumption level of a θ-agent.[15] With a flexible effort however, the variation in the contract typically changes the marginal benefit to

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[14] Adjust the cost of effort k so as to satisfy the incentive compatibility constraint (24) and adjust outputs $(\omega(\theta))$ so as to satisfy the condition for optimality of effort (25) and the budget constraint (12). With more than two status, there is much flexibility. Also one does not need to adjust R, since it is equivalent to a uniform increase in ω.

[15] This is obtained by differentiation of (13) or by arguing as in proposition 3 and rewriting (24) as $(1 + \beta\frac{p'_e}{p})(\theta|e)v'(c(\theta)) = \alpha$. For fixed e and β, these are the Borch conditions applied to θ-individuals with a weight equal to $(1 + \beta\frac{p'_e}{p})(\theta|e)$. It suffices then to apply (11).

provide effort, hence changes the value of the cost k supporting the contract. Whether this results in a decrease or an increase in this value and in the moral hazard premium depends on prudence, as stated in the following proposition.

Proposition 5 *As the price α varies, contingent income levels vary in proportion of the risk tolerance of each θ-agent and group's income satisfies : $dE\mathbf{c} = E[\tau_v(\tilde{c})](-d\alpha/\alpha)$. The marginal benefit to provide effort increases as α decreases under prudence, and decreases under imprudence. Assume in addition linear risk tolerance. The moral hazard premium decreases in absolute value as α decreases: it decreases (resp. increases) under prudence (resp. imprudence).*

The proof is given in the appendix. Changing the consumption in proportion of risk tolerance changes the marginal benefit according to the monotony of $\tau_v v'$. As $\tau_v v'$ is equal to $v'^2/(-v'')$, which is the reciprocal of the derivative of $1/v'$, this monotony in turn depends on prudence as indicated in the table below.

Proposition 5 compares states for which the price of effort is constant. Hence as the price of income is decreased, conditions are improved better. With a flexible effort, the same level of effort is optimal only if the cost for effort k along these states vary in a precise way as α varies. This adjustment differs according to the prudence of individuals: the cost increases for prudent individuals to exert some effort level, meaning that it becomes easier to incite them but it decreases for imprudent ones, making it more difficult to incite them. As for the distortion on marginal utility for wealth as measured by the absolute value of the premium, it always diminishes as the price α becomes lower. In particular , when the adjusted price for income α is low, group income levels should be close to the values given by the shares. These results may be interpreted as saying that for prudent individuals the distortions due to moral hazard becomes unambiguously less severe in good states from both points of view: premium (distortion on marginal utility of wealth) and effort (changing the contract as in a standard risk sharing problem in proportion of risk tolerance improves their incentives to make effort). For imprudent ones, this is not true because they become reluctant to provide effort as conditions are eased.

We investigate now the impact of a change in the multiplier β associated with the incentives constraint, α and e being fixed. From (13), the distribution of the reciprocal of marginal utility, $1/v'(\tilde{c})$, have all the same expectation, namely $1/\alpha$, across states with same α. These distributions can be compared in the sense of second order stochastic dominance. They decrease with[16] β, and in particular, they are all dominated by the sure distribution $1/\alpha$. The sure distribution is obtained for the constant consumption level with a null effort : the level $\mathbf{C}(\alpha)$ defined by $v'(c) = \alpha$. With a fixed value for α, the expected consumption diminished of the premium, $\mathbf{C}(\alpha, \beta, e) - \psi$, is fixed as well equal to $\mathbf{C}(\alpha)$, hence the expected consumption varies in opposite direction of the moral hazard premium.

Proposition 6 *The marginal benefit to provide effort increases with β. Furthermore, the moral hazard premium increases in absolute value with β: Under prudence, the premium, which is positive,*

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[16]To see this, observe that for θ for which $p'_e > 0$, $1/v'(c(\theta))$ is larger than the mean $E[1/v'(\tilde{c}(\theta))]$ and conversely for θ for which $p'_e < 0$, it is smaller. Thus, from (13), as β increases, the values larger than the mean all increase (since $p'_e > 0$), and those smaller than the mean all decrease: this is a mean preserving spread.

increases with β, hence expected income decreases:

$$\mathbf{C}(\alpha, \beta', e) \geq \mathbf{C}(\alpha, \beta, e) \geq \mathbf{C}(\alpha), \beta' \geq \beta$$

with strict inequalities under strict prudence. Under imprudence, the moral hazard premium, which is negative, decreases and the above inequalities are reversed.

Next table summarizes Propositions 5 and 6. The value k refers to the value of the cost supporting the optimal contract.

	prudent	imprudent
example	$1/v'$ concave $\Leftrightarrow \tau_v v'$ is increasing $\frac{c^{1-\gamma}}{1-\gamma}$ for $\gamma \leq 1$	$1/v'$ convex $\Leftrightarrow \tau_v v'$ is decreasing $\frac{c^{1-\gamma}}{1-\gamma}$ for $\gamma \geq 1$, $-e^{-\rho c}$
premium ψ	$\psi \geq 0$	$\psi \leq 0$
$\alpha \downarrow$	$k \uparrow$ HARA $\psi \downarrow$	$k \downarrow$ HARA: $\psi \uparrow$
$\beta \uparrow$	$k \uparrow, \psi \uparrow$	$k \uparrow \psi \downarrow$

A risk neutral group Under risk neutrality of individuals in group 1, the price α is constant across states, equal to λ_1. With a flexible effort, there is no reason for the effort and its price to remain constant for risk averse groups. This means that risk averse individuals are not insured against macro-economic shocks: the contract thyy face and their expected conditional utility level differ according to the state of the economy.

This result translates into the framework of a risk neutral principal facing a risk averse agent in various situations or 'states'. Assume that a contract is signed before the state is known, and must be honored thereafter whatever the state that materializes. Thus, the agent must be given a minimum ex ante utility level in order to get his participation. The contract that maximizes the profit of the principal under the agent's participation constraint is optimal (that is there are some weights λ_h supporting it). From the previous result, the principal is likely to choose a contract in which the agent's conditional utility level vary across states. In other words, the optimal contract cannot be solved state by state by setting a constant reservation utility level across states.

6.3 Shares variations

The previous section has considered states along which only one price α or β varies. Along these states, simultaneous variations in both the output and cost of effort are necessary for keeping one of the prices and the effort level constant. Consider now variations in output only. Along states for which the group provides a constant level of effort, both prices α and β must vary simultaneously. Next proposition states the impact on expected consumption as function of the variation in the price of wealth.

Proposition 7 *Consider states that differ in their output only and for which the group provides a constant level of effort at an optimal scheme. The variation of expected consumption with respect to*

price α satisfies

$$dC = dE_\theta \tilde{c} = T \frac{-d\alpha}{\alpha} \ \text{ where } T = E_\theta[\tau_v(\tilde{c})] - \frac{(E_\theta[\tau_v(\alpha - v')(\tilde{c})])^2}{E_\theta[\tau_v(\alpha - v')^2(\tilde{c})]} \tag{26}$$

T is a risk tolerance coefficient adjusted for moral hazard. For a log function, the second term is null since $\tau_v v'$ is constant. Hence T is equal to the average risk tolerance coefficient across status which is also the risk tolerance coefficient at the expected consumption (by linearity of τ_v). This is not surprising since the group behaves as a representative agent as we have seen even in the presence of moral hazard. For a general utility function, coefficient T is always smaller than the group's average risk tolerance coefficient. This can be understood by Propositions 5 and 6 as follows. Along states for which only the price α varies, the group's risk tolerance coefficient can be defined as the average risk tolerance coefficient across status. With a prudent individual, decreasing α however increases the incentive to provide effort. To keep the effort level constant, the value of β must decrease, which decreases the moral hazard premium, reducing the original increase in consumption. Under imprudent behavior, decreasing α decreases the incentive to provide effort so that the value of β must be decreased. Under imprudence this is achieved by decreasing the premium : again keeping the effort level constant induces a lower increase in expected consumption than the sole decrease in price α.

Proposition 7 is useful to derive how a marginal variation in aggregate resources is allocated to the groups. In the absence of moral hazard, it is allocated in proportion to the groups' risk tolerance (which is that of an individual since status risk is fully insured). In the presence of moral hazard a similar expression holds, as given by (27).

Proposition 8 *Consider states that differ in their output only and for which each group provides a constant level of effort at an optimal scheme. Then groups' income levels vary with a marginal change in aggregate resources as follows :*

$$dC_h = \frac{T_h}{\sum T_h} d\Omega \tag{27}$$

where coefficient T_h is given by (26) computed for group h.

PROOF OF PROPOSITION 6. Applying (26) for each group, we have $dC_h = T_h \frac{-d\alpha}{\alpha}$. Using that the sum over h of dC_h is equal to $d\Omega$ gives the result. ∎

Corollary 3 *Consider two groups. Group 1 exhibits linear risk tolerance and imprudence. Group 2-individuals are not subject to moral hazard and have a risk aversion coefficient non increasing in income. Then along states in which 1-individuals provide a constant positive level of effort, moral hazard decreases the variation of 1's share with respect to aggregate resources.*

Examples of utility functions that exhibit linear risk tolerance and imprudence are those with constant risk aversion or with constant relative risk aversion larger than 1.

PROOF OF COROLLARY 3 Since one group is not subject to moral hazard, we can apply Corollary 2. Group 1 being imprudent, its share is diminished by moral hazard and 2's share is increased: $C_1 \leq S_1(\Omega)$ and $C_2 \geq S_2(\Omega)$). Hence T_2 is larger than $\tau_2(S_2(\Omega))$: T_2 is the tolerance coefficient at the sure consumption C_2, which is larger than $S_2(\Omega)$. At the opposite 1's tolerance coefficient is decreased:

We know from (26) that T_1 is smaller than the group's average risk tolerance. By the assumption of linearity of tolerance, the group's average risk tolerance is also the tolerance at the expected consumption C_1. Since C_1 is smaller than the share $S_1(\Omega)$, $T_1 \leq \tau_1(S_1(\Omega))$. Hence, T_1/T_2 is smaller $\tau_1(S_1(\Omega))/\tau_2(S_2(\Omega))$, which gives that group 1's consumption varies less than without moral hazard according to (27). ∎

The statement of corollary is not reversed when group 1 is assumed to be prudent. Moral hazard has two effects on the risk tolerance coefficient of group 1 that are in opposite direction. On one hand, moral hazard diminishes 1's tolerance coefficient relative to the tolerance at the expected consumption C_1 (still assuming linear tolerance), but on the other hand C_1 is larger than the share $S_1(\Omega)$. As a result, we cannot compare T_1 with $\tau_1(S_1(\Omega))$.

Pension example Let us apply our findings to example 2 on pensions (section 2). Since retirees are not subject to moral hazard, the impact of workers' moral hazard on the shares of aggregate resources can be assessed if workers are either prudent or imprudent (thanks to Corollary 2). Under imprudence, the expected share received by the working age generation is lowered to account for moral hazard, and as a result the pensions are increased. The opposite result holds under prudence. Consider now the sensitivity of the shares to aggregate resources at a given effort level. Corollary 3 gives an answer when workers exhibit linear risk tolerance and imprudence. Inducing a given effort level results in net wages that are less variable and pensions that are more variable with aggregate resources than without workers' moral hazard. Because the distortions due to moral hazard increases with wealth for imprudent workers, it is optimal that pensions increase more in good time (so as not to give to much to workers) and less in bad time. With prudent workers, these findings no longer hold. The following example illustrates the various effects.

Let both groups have the same utility function with relative risk aversion constant equal to γ. Group 1 faces two outcomes. The equalization of weighted marginal utility with respect to income (17) writes as $\lambda_1 c_2^\gamma = \lambda_2 E_\theta[c_1^\gamma]$ and adjusted tolerance coefficient T_1 is equal to[17] $T_1 = \gamma \frac{[E_\theta \tilde{c}_1^\gamma]^2}{E_\theta[\tilde{c}_1^{2\gamma-1}]}$ This gives

$$\frac{T_2}{T_2} = [\frac{\lambda_2}{\lambda_1}]^{\frac{1}{\gamma}} \frac{E_\theta[\tilde{c}_1^{2\gamma-1}]}{[E_\theta \tilde{c}_1^\gamma]^{2-1/\gamma}} = [\frac{\lambda_2}{\lambda_1}]^{\frac{1}{\gamma}} E_\theta \left[(\frac{\tilde{c}_1^\gamma}{E_\theta[\tilde{c}_1^\gamma]})^{\frac{2\gamma-1}{\gamma}} \right].$$

The impact of moral hazard is reflected by the expectation term on the right (which is equal to 1 without moral hazard): If this term is larger than 1, then the ratio of risk tolerance coefficients is increased and the share received by group 1 varies less with aggregate resources than without moral hazard. The reverse properties hold if the expectation term is less than 1. Observe now that the expectation term is less than 1 for a coefficient γ between 1/2 and 1 (the function $x^{\frac{2\gamma-1}{\gamma}}$ is concave) and it is larger than 1 if either γ is less than 1/2 or larger than 1. For γ larger than 1, as expected from the previous corollary, the ratio T_2/T_1 is increased. For γ smaller than 1, we know that there

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[17] With two status levels, an easy computation is available because the incentive constraint and the feasibility constraint fully determine how the optimal contract adjusts. Keeping a constant level effort level implies $\sum p_e'(\theta) v'(c(\theta)) dc(\theta)) = 0$. The identity $\sum p_e'(\theta) = 0$ implies that $v'(c(\theta)) dc(\theta)$ is equalized at $\theta = 0, 1$. Since effort is constant, $Ev'(c)dc = d\alpha$. Hence $v'(c(\theta)) dc(\theta)) = d\alpha$. $dc(\theta) = dC/[v'(c(\theta)E[1/v'(\tilde{c})]]$. This gives

$$T = E[\frac{1}{v'}]^2 / E[-\frac{v''}{v'} \cdot \frac{1}{v'^2}].$$

It can be checked that it coincides with (26).

are two effects in opposite directions. For $1/2 < \gamma < 1$ the effect due to an increase in share levels which increases risk tolerance dominates. To sum up, moral hazard has the following impact on the workers share :

if $\gamma < 1/2$, it increases the share and lowers its variation with resources

if $1/2 < \gamma < 1$, it increases the share and its variation

if $\gamma > 1$, moral hazard decreases both the share of group 1 and its variation.

7 Decentralization

Lump sum transfers are typically needed to decentralize an optimal scheme. Given an optimal scheme and the associated level of efforts $(\tilde{c}_h^*(s), e_h^*(s), s \in \mathcal{S})_{h=1,2}$ let define the lump sum transfer T_h to h as

$$T_h = \sum_s \pi(s)\alpha(s)\{\sum_\theta p_h(\theta|e_h^*(s), s)[c_h^* - \omega_h)(\theta|s)]\} \tag{28}$$

in which $\alpha(s)$ is the price of state s. By feasibility of the scheme, transfers are balanced.

Consider first that there is no effort level. We fall back on a standard economy under uncertainty. By standard arguments, the optimal allocation can be reached by trading a complete set of contingent securities on shocks (θ, s) at $t = 1$. Set the price of one unit of consumption given θ and s at $\pi(s)\alpha(s)p(\theta, s)$. The lump sum transfer T_h precisely allows a h-individual to buy the optimal consumption levels at these prices.

Since the macro-economic state and the idiosyncratic shocks are revealed at different dates, an alternative organization in which markets open sequentially can be also considered. As shown by Radner (1972), sequential markets allow for the same allocation to be decentralized provided future prices are correctly expected. In a setting with contractible effort or with moral hazard, it is this organization that is appropriate for decentralization (under additional informational assumptions). The sequential structure is as follows.

- At $t = 1$, markets contingent on the macro-economic states[18] are open. The price of an Arrow-Debreu (AD) security contingent on s is $\pi(s)\alpha(s)$. A h-agent faces the budget constraint

$$\sum_s \pi(s)\alpha(s)r_h(s) \leq T_h, \tag{29}$$

in which $r_h(s)$ is the trade in the security contingent on s.

- At $t = 2$, after the state s has been revealed, an insurer, say a group syndicate, offers contracts contingent on idiosyncratic shocks to h-members at prices $p_h(\theta|s)$. The budget constraint of a h-agent is

$$\sum_\theta p_h(\theta|s)[c_h(\theta|s) - \omega_h(\theta|s)] \leq r_h(s). \tag{30}$$

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[18]Thanks to the specific form of the allocation due to the mutuality principle, Arrow-Debreu securities can be replaced by options on the level of aggregate resources (see Breeden and Litzenberger (1978) in a situation in which individual risky endowments are issued as securities and Demange and Laroque (1999) for an extension to incomplete exchanges).

Assume at $t = 1$ that individuals form correct expectations on the prices that will be set at $t = 2$. A h-agent faces exactly the same optimization problem with the same contingent prices as if the full set of contingent markets were open at date 1. Thus, he is able to reach the same consumption levels by buying the correct amount of AD securities and cannot do better.

With a contractible effort, the sequential organization extends as follows. At $t = 1$, nothing is changed. At $t = 2$, once the state is known, agents choose an effort level e, which is observable, and buy status-contingent income. The price of one unit of consumption given θ and s now depends on e, equals to $p_h(\theta|e, s)$ for a h-agent. Since prices are fair (given e and s), a h-agent chooses full insurance, that is a consumption level independent of the idiosyncratic shock, $c_h(s)$. Effort level is chosen to maximize $u_h(c_h(s)) - k(s)e$ under the budget constraint (30). At step 1, he trades AD securities so as to maximize his expected utility under (29). Under correct expectations, this is equivalent to choose ex ante $c_h(s)$ and $e(s)$ so as to maximize $\sum \pi(s) [u_h(c_h(s)) - k(s)e(s)]$ under $\sum_s \pi(s)\alpha(s) [c_h(s) - \sum_\theta p_h(\theta|e, s)\omega_h(\theta|s)] \le T_h$. Under the assumption CDF, the program is convex. The optimal solution as given in Proposition 1 satisfies the first order condition of this program (setting the multiplier of the constraint to be equal to $1/\lambda_h$) hence it is the choice of the h-agent.

With a noncontractible effort, an optimum can be decentralized under the condition that the whole individual's demand of contingent income is observed (see Kocherlakota (1998) for a similar argument). Observing the demand allows the group syndicate to infer the chosen effort level and to set fair prices given that effort level : In state s the price of demand $(\tilde{x}_h)$ is set equal to $\sum_\theta p_h(\theta|e, s)x_h(\theta)$ where e is the effort that maximizes $\hat{U}(\tilde{\omega} + \tilde{x}|e, s)$. The crucial point is that under this pricing rule a h-individual who has bought $r_h(s)$ units of the AD security contingent on s faces exactly the problem $\mathcal{P}_h(r_h(s)|s)$. Hence, under correct expectations at step 1, he expects the utility level $\pi(s)F_h(r_h(s)|s)$. Since the price of an AD security contingent on s is $\pi(s)\alpha(s)$, the agent can afford the optimal quantities thanks to (29) and this is optimal for him. Of course, the observability of the whole demand $(x(\theta))$ is a strong requirement.

Finally, observe that in all cases, the same result holds under a somewhat more realistic organization in which, at step 1, the group syndicate intervenes on the market of AD securities on behalf of its members and, at step 3, distributes the revenues of the AD securities to its members and prices their contracts.

8   Concluding remarks

This paper has investigated how moral hazard distorts the sharing of macro-economic risks. To interpret the distortions, the analysis makes clear the transfers across groups and extends the Borch conditions for optimality of risk sharing across groups. We introduce a moral hazard risk premium and a notion of (im)prudence against moral hazard. These tools may be useful in other contexts.

Two settings are investigated, one where effort is irreversible chosen before the state of the economy is revealed and the other one where it is flexible adjusted once the state is revealed. Our results suggest that it not moral hazard per se that may hamper macro-economic risks to be shared, but rather the timing at which effort is exerted. With an irreversible effort, the impact of moral hazard exists but is not dramatic. In contrast, with a flexible effort, optimal contracts may have undesirable

properties. Utility levels may be discontinuous in the state of the economy. Furthermore, in some non pathological circumstances, individuals are penalized by an increase in their own outputs. Such phenomena does not occur in the absence of macro-economic risk sharing. This may raise difficulties in the implementation of the insurance scheme and may support the view that moral hazard explains why macro-economic risks are poorly shared.

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9   Proofs

Proof of Proposition 1   Assume that effort is contractible. The proof is given only in the more complex case of a flexible effort. Observe that both the welfare criterion and the feasibility constraints are separable across sates. Dropping index s the lagrangean of the optimization problem in a state writes as

$$\sum_h \lambda_h [E_\theta u_h(\tilde{c}_h) - k_h e_h] + \alpha(\sum_h \sum_{\theta \in \Theta_h} p_h(\theta|e)[\omega_h - c_h](\theta))$$

where α is the multiplier associated with the feasibility constraint (5). The lagrangean has to be maximized with respect to $\tilde{c}_h$ and e_h, $h = 1,..,H$. Thanks to the assumptions, the problem is concave in the variables, hence the solution is continuous with respect to the parameters of the problem, hence with the state s.

The maximization with respect to consumption levels gives Borch conditions (10): $\lambda_h u'_h(c_h(\theta)) = \alpha$. Hence consumption levels are given by the sharing rules. Furthermore, $Eu_h(\tilde{c}_h)$ and $E\tilde{c}_h$ do not depend on e_h since $\tilde{c}_h$ is certain. Thus, the maximization with respect to effort e_h is simplified into

$$\max_{e_h} F_h(\alpha, \omega_h, e_h) \text{ where } F_h(\alpha, \omega_h, e_h) = \alpha \sum_\theta p_h(\theta|e_h)\omega_h(\theta) - \lambda_h k_h e_h \tag{31}$$

which says that effort e_h maximizes the value to h of output net of effort, as stated in point *2*. It remains to prove the monotonicity properties on consumption and welfare under uniform changes in groups outputs. Let $\Delta\omega_h$ denote the uniform change in h output. The proof is divided into two steps.

Step 1. We show that changes in α and effort levels are in the same direction: in all groups, individuals exert less effort as the price of the feasibility constraint decreases, and conversely.

To simplify, let us consider only the case where e_h is interior. Function F_h defined in (31) is concave thanks to CDC (using footnote 5 and that outputs are increasing in status) and is twice differentiable. The first order condition on effort, which is sufficient, is $(F_h)'_e(\alpha, \omega_h, e) = \alpha \sum_\theta p'_{he}(\theta|e_h)\omega_h(\theta) - k_h\lambda_h = 0$. By the implicit function theorem, the marginal changes in α, effort, and outputs, $\Delta\alpha$, Δe_h, and $\Delta\omega_h$ satisfy

$$\sum_\theta p'_{he}(\theta|e_h)\omega_h(\theta)\Delta\alpha + \alpha(F_h)''_{ee}\Delta e_h + \alpha \sum_\theta p'_{he}(\theta|e_h)\Delta\omega_h = 0.$$

The last term is null since the sum $\sum_\theta p'_{he}$ is identically null. Since outputs are increasing in status, CDC implies that the term multiplying $\Delta\alpha$ is positive. $(F_h)''_{ee}$ is nonpositive by concavity of F. Hence Δe_h and $\Delta\alpha$ are of the same sign, the desired result.

Step 2. To establish the monotonicity of consumption levels and welfare, let us first consider the case where all groups are risk averse. In that case shares strictly increase as α decreases (by Borch condition). Since, from step 1, individuals exert less effort as α decreases, they are all better off. In the opposite case where $\Delta\alpha$ is positive, all are worse off. To determine which case occurs, notice that by the envelope theorem the marginal change of the weighted sum of the utilities due to an exogenous change in output is equal to $\alpha \sum_h \Delta\omega_h$. Hence, if the sum $\sum_h \Delta\omega_h$ is positive, then each group is better off. Hence α decreases, individuals exert less effort and consume more: the decrease in effort does not offset the exogenous increase since overall resources increase. Conversely, if the sum $\sum_h \Delta\omega_h$ is negative, then each group is worse off.

When there is a risk neutral group, the price α is constant. The consumption levels of the risk averse groups are constant (by Borch condition) as well as their effort levels (by step 1). Change in outputs only affect the risk neutral groups. ∎

Proof of Lemma 1. The program is well behaved, with a differentiable objective and a linear constraint. Denoting by α the Lagrange multiplier of the budget constraint, the first order conditions with respect to consumption levels $c(\theta)$ for each θ are $p(\theta|e)v'(c(\theta)) = \alpha + \beta p'_e(\theta|e)v'(c(\theta))$ for each θ which can be rewritten as (13). Observe that these conditions determine $c(\theta)$ as a strictly decreasing function of α. Hence there is a unique value of α for which the constraint is satisfied an an equality. This implies that there is unique solution to the program, that this solution varies continuously with respect to R and that the value function is derivable. The envelope theorem states that the derivative with respect to R is equal to α. Since $\sum_\theta p(\theta|e) = 1$, one has $\sum_\theta p'_e(\theta|e) = 0$. Multiplying (13) by $p(\theta|e)$ and summing over θ gives (14): $\frac{1}{V'(R)} = \frac{1}{\alpha} = E_\theta[\frac{1}{u'(c(\theta))}]$. As we have just seen, the multiplier α is adjusted so as to satisfy the budget constraint, hence is decreasing in R. This implies that $V'(R)$ is decreasing with R, hence V is concave. ∎

Proof of Proposition 3. We first check that the first order approach is valid under the assumptions CDF and MLR. Facing a contract, an individual chooses effort so as to maximize the ex ante expected utility level $E_\pi[U_h(\tilde{c}_h|e_h, s)]$ in which the expectation is taken over the set of states $\mathcal{S}$ and the function U_h is defined by (1) : $U_h(\tilde{c}_h|e_h, s) = \sum_\theta p_h(\theta|e_h, s)u_h(c_h(\theta|s)) - k_h(s)e_h$. Given a state s and a contingent income $\tilde{c}_h(s) = (c_h(\theta|s))$ that is non decreasing in θ, $U_h(\tilde{c}_h|e_h, s)$ is concave in e_h. Thus the expectation $E_\pi[U_h(\tilde{c}_h|e_h, s)]$ is also concave in e_h if each contract $\tilde{c}_h(s) = (c_h(\theta|s))$ is non decreasing in θ for each s. As we have seen in the text (or see below), contingent income plans satisfy the first order conditions (21) : in state s, for some $\alpha(s)$, e_h and $\beta_h \geq 0$ we have that for each $\theta \in \Theta_h$, $\frac{1}{\alpha(s)} = \frac{1}{u'_h(c_h(\theta|s))} - \frac{\beta_h}{\alpha(s)}\frac{p'_{he}}{p_h}(\theta|e_h, s)$. Hence income level is nondecreasing in θ.

Most of the results have been proved in the text by using the transfers across groups. We provide here a formal and more technical proof. The lagrangean of the problem with irreversible effort writes:

$$E_\pi\left[\sum_h \left\{\lambda_h U_h(\tilde{c}_h|e_h, s) + \beta_h(\sum_\theta p'_{he}(\theta|e_h, s)u_h(c_h(\theta|s)) - k_h)\right] + \alpha(s)(\sum_\theta p_h(\theta|e_h, s)[\omega_h - c_h](\theta|s)\}\right]$$

where β_h the multiplier associated with the incentive constraint of group h and $\pi(s)\alpha(s)$ is the multiplier associated with the resource constraint in state s. Given effort levels (e_h), the problem is strictly concave in consumption levels. Hence consumption levels and utility levels are continuous in the state. The first order conditions on consumption levels give (21). For given (e_h) and (β_h), the maximization of the lagrangean is solved state by state. Thus consumption levels $(\tilde{c}_h(s))$ solve

$$\max_{\tilde{c}_h}\sum_h \lambda_h p_h[1 + \beta_h\frac{p'_{e,h}}{p_h}(\theta|e_h, s)]u_h(c_h(\theta)) \text{ over the constraint} \tag{32}$$

$$\sum_h\sum p_h(\theta|e_h, s)(\omega_h(\theta|s) - c_h(\theta)] \geq 0.$$

Along states with output risk only, the probabilities $p_h(\theta|e_h, s)$ and $p'_{he}(\theta|e_h, s)$ are constant for each status θ. Consider individuals indexed by (θ, h), in number $p_h(\theta|e_h, s)$. The criterion in (32) can be

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considered as the weighted sum of their utilities with a weight equal to $\lambda_h[1 + \beta_h \frac{p'_{e,h}}{p_h}]$ for each (θ, h) individual, and the constraint as the standard resource constraint for these individuals. Thus the optimal scheme solves a standard optimal risk sharing problem without moral hazard applied to these individuals. Points 1 and 2 follow. ∎

Proof of proposition 5. Set $c(\theta) = \mathbf{c}(\alpha, \beta, e; \theta)$. A marginal change $d\alpha$ induces a variation $d\mathbf{c}(\theta) = \tau_v(\mathbf{c}(\theta))\frac{-d\alpha}{\alpha}$. Hence the variation in the marginal benefit to effort $(\sum_\theta p'_e(\theta|e)v(\mathbf{c}))$ is equal to

$$dk_1 = \{\sum_\theta p'_e(\theta|e)[\tau_v v'](c(\theta))\}\frac{-d\alpha}{\alpha}. \tag{33}$$

We show that under prudence the term in brackets is positive. By CDF (see footnote 5), this holds if the function $\tau_v v'$ is increasing in θ or equivalently (since consumption levels are increasing in θ) if the function $\tau_v v'$ is an increasing function. Observe that $\tau_v v'$ is equal to $v'^2/(-v'')$, which is the reciprocal of the derivative of $1/v'$. By definition of prudence, $1/v'$ is concave, hence $\tau_v v'$ is increasing, the desired property.

By the definition of the premium (15), $\lambda v'(E\mathbf{c} - \psi) = \alpha$. Log-differentiation gives $d[E\mathbf{c} - \psi] = \tau_v(E\mathbf{c} - \psi).\frac{-d\alpha}{\alpha}$. Since $d\mathbf{c}(\theta) = \tau_v(\mathbf{c}(\alpha, \beta, e; \theta))\frac{-d\alpha}{\alpha}$, we obtain

$$d\psi = \{E[\tau_v(\mathbf{c})] - \tau_v(E\mathbf{c} - \psi)\}\frac{-d\alpha}{\alpha}$$

Let τ_v be linear and increasing. The term in brackets is equal to $\tau_v(\psi)$ and is of the same sign than ψ. Under prudence, ψ is positive, which gives that $d\psi$ is of opposite sign to $d\alpha$: moral hazard premium decreases with α. Results under imprudence are obtained similarly. ∎

Proof of proposition 6. Set $c(\theta) = \mathbf{c}(\alpha, \beta, e; \theta)$. Differentiation of (13) gives $dc(\theta) = [\tau_v v'](c)(\theta)\frac{p'_e}{p}\frac{d\beta}{\alpha}$. Hence the marginal variation of $\sum_\theta p'_e(\theta|e)v(c(\theta))$ with respect to a marginal change $d\beta$ is given by

$$dk_2 = \sum_\theta \tau_v v'^2 \frac{p'^2_e}{p}\frac{d\beta}{\alpha} \tag{34}$$

The variation in average consumption is $E_\theta[d\tilde{c}] = \{\sum_\theta p'_e(\theta|e)[\tau_v v'](c(\theta))\}\frac{d\beta}{\alpha}$. It is of the same form as dk_1, hence we know that $E_\theta[d\tilde{c}]$ is positive under prudence and negative under imprudence. ∎

Proof of Proposition 7. We have computed the variation in the cost supporting a contract along states where α or β varies and the effort level is constant as given by dk_1 or dk_2 respectively. We consider here states for which outputs vary but the cost does not change. The level of effort can be constant only if α and β vary in such a way that $dk_1 + dk_2$ is null. We shall write expressions in expectation form by using that $\beta p'_e v' = p(\alpha - v')$ thanks to the conditions (13). This gives $dk_1 = E_\theta[\tau_v(\alpha - v')(\tilde{c})]/\beta$, $dk_2 = E_\theta[\tau_v(\alpha - v')^2(\tilde{c})]/\beta^2$, and a variation in consumption

$$E[d\tilde{c}] = E_\theta[\tau_v(\tilde{c})]\frac{-d\alpha}{\alpha} + E_\theta[\tau_v(\alpha - v')(\tilde{c})]\frac{d\beta}{\alpha\beta}.$$

Hence the condition $dk_1 + dk_2 = 0$ is

$$E_\theta[\tau_v(\alpha - v')(\tilde{c})]\frac{-d\alpha}{\alpha} + E_\theta[\tau_v(\alpha - v')^2(\tilde{c})]\frac{d\beta}{\alpha\beta} = 0.$$

Plugging the value of $d\beta$ as a function of $d\alpha$ into the variation in consumption gives formula (26). ∎