

# Logical, mechanical and historical time in economics

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# Valeria Termini Logical, Mechanical and Historical Time in Economics Off-print from "Economic notes", vol. 10 no. 3 Published by *Monte Paschi di Siena*

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## Introduction

1.1 - In economics different notions of time imply alternative theories. This becomes crucial to explain the misunderstandings which arose between Keynes, Robertson and Ohlin in the debate on 'finance' in the late 1930's<sup>1</sup>. This debate offers an example of how the same terminology is used in different theories to stand for different concepts.

These are the key issues of this paper but some general conclusions on the use of "time" in economic models will also be attempted.

1.2 - Keynes, Robertson and Ohlin discussed the determination of income from a common macroeconomic perspective, but they utilized a set of aggregate categories which were only *apparently* similar. However, they had a basic element in common: all of them used "sequence analysis". By " sequence analysis" or "sequential construction" it is meant here a description of the economic process to be studied by a model in which antecedent component and sequential components are linked together in a sequence.

It follows that the whole process can be studied by stages and each component can be analyzed one at a time<sup>2</sup>. Models based on sequential structures may be opposed as a group to general equilibrium models, which solve simultaneously for the variables in a set of interdependent equations.

Beyond this similarity, however, it is usually overlooked that sequential structures may sharply differ; this is the case of the sequential schemes used by Keynes, Robertson and Ohlin. Their disagreements were far beyond the main issue of that debate – i.e. the determinants of the rate of interest – but involved their whole theoretical structure<sup>3</sup>. Therefore it is necessary to look into their methodologies in order to distinguish and compare the analytical features of their models. The classification which will emerge exemplifies the present scope of this essay.

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<sup>&</sup>lt;sup>1</sup> I am referring to the debate in the Economic Journal 1937-1939, where the determinants of the rate of interest were discussed in the light of the relation between investment and saving. See J.M. Keynes 1937b, 1937c, 1938; B. Ohlin 1937a, 1937b; D. Robertson 1937, 1938.

<sup>&</sup>lt;sup>2</sup> I use the terms 'sequence' and 'sequentiality' here with a broader meaning than is usually attributed to this concept by the Swedish literature. See, for instance, the definition of sequentiality in economics given by E. Lundberg 1937 p. 51; Svennilson 1938 p.3; B. Hansson 1980 pp 16-17.

<sup>&</sup>lt;sup>3</sup> In the recent literature in this area the neglect of methodological divergences has blurred the boundaries between them; along the lines of the 'methodological synthesis' the 'liquidity theory' of the rate of interest has been assimilated within the 'loanable funds' theory. See D. Patinkin 1958, C. Tsiang 1956, W. Fellner and H. Somers 1950. Alternative interpretations are given by H: Johnson 1951, F. Hahn 1955, K. Brunner 1950. By contrast, L. Klein 1950 stressed the divergences between the two theories.

1.3 - First, an absolute distinction between a logical (causal)<sup>4</sup> structure of sequences and a chronological structure of sequences has to be made. Certain economic models are strictly causal according to the definition that I am following here, but 'timeless', 'outside of time', or 'static' depending on the terminology used; their variables need not to be dated , nor adjustments considered through time. I propose to call this method '*logical time*' sequentiality and I will analyze it in section 2. (I will consider in detail an application of this method – Keynes's *General Theory* – in section 5). By contrast there are models which follow a temporal path but do not set up any causal logical structure to explain economic behaviour.

Furthermore, I will argue that even the concept of chronological time conceals two different methods. One introduces calendar time as an exogenous variable - i.e. 'time' is used as a 'vector' which helps to extend mechanically the theoretical relations of a model from period to period from the past to the future; I will call it '*mechanical time*' sequentiality. The other method deals with processes in which past, present and future are qualitatively different; I will call it '*historical time*' sequentiality. I will discuss the former method – i.e. Robertson's scheme – in section 3 and the latter – i.e. the Swedish School and Ohlin's scheme – in section 4.

The relevance of distinguishing time-based methods for analytical purposes is appreciated when these categories are used to interpret specific economic issues<sup>5</sup>. The debate on 'finance' shows how critical the assumptions of different *temporal co-ordinates* are in the definition of opposing theories. Many eminent economists became involved in that debate, including Harrod, Hawtrey, Lerner, J. Robinson, Shaw and Townsend<sup>6</sup>. I focus here on Keynes, Ohlin and Robertson because they conveniently illustrate the three methods I discuss in this paper.

The question posed by Robertson and Ohlin concerns the financing of new investment and the determinants of the rate of interest in an expanding economy. The question at issue, however, is not discussed in this paper<sup>7</sup>. Nevertheless my conclusion is that the concept of 'finance' is different in Keynes, Robertson and Ohlin because it is based on a different notion of investment, saving and income, which is consistent with their different sequential schemes<sup>8</sup>.

Furthermore, it follows that: (i) definitions which are based on a dychotomized classification of theoretical models – such as 'static' versus 'dynamic', 'equilibrium' versus 'disequilibrium' – and directly

<sup>&</sup>lt;sup>4</sup> No univocal definition of 'causality' is commonly accepted by the scientific community. Among the different interpretations the definition of causality proposed by Feigl in 1953, as "predictability according to a law or to a set of laws" is chosen here. This is mainly for two reasons. First, causality is defined according to inductive criteria which are appropriate to the interpretation of economic reality. (Predictability is based on inductive inference from the empirical data about the past and involves predictions of experience not yet observed). Furthermore, in the same definition, the need for theory in establishing any causal relation is made explicit (i.e. the 'law' or the 'set of laws' must be specified). It is interesting to note that Feigl (and Zellner who recently reiterated Feigl's definition of causality) have directed their fire specifically at the work of 'econometricians without theory'. (Zellner '79, p. 12).

Of course, the term 'predictability' must be qualified if it is to be applied in economics, since the concept of 'uncertain future' means in this context that the future vent 'has never happened before', and therefore conform to Georgescu-Roegen's definition of the unknown. A. Zellner 1979; H. Feigl 1953; N. Georgescu-Roegen 1971 Ch. V; H. Jeffereys 1967.

<sup>&</sup>lt;sup>5</sup> An early tentative discussion of different ways of introducing 'time' in economics is in P. Rosentein-Rodan 1934. Subsequent viewpoints of Amoroso, Masci, Shackle, J. Robinson, Georgescu-Roegen, Hicks ('76, '80) among others, will be considered in the next paragraphs.

<sup>&</sup>lt;sup>6</sup> See A. Lerner 1938; J. Hawtrey 1937; J. Robinson 1938; R. Shaw 1938; H. Townsend 1937.

<sup>&</sup>lt;sup>7</sup> I discussed this point in the paper "Keynes on 'finance': the 1937-1938 debate", presented at the London School of Economics, the 28th June 1978, mimeo.

<sup>&</sup>lt;sup>8</sup> The first view is that the amount of previously accumulated savings, which in Robertson's view assume a '*financial form*', determines the maximum investment growth available. The second viewpoint – that of Ohlin – is that the flow of new credit, also defined as '*finance*', can potentially match an increase of investment without limit, except that increase in investment decisions affects the level of the rate of interest. The third view, of Keynes, is that additional demand for cash should be added to the traditional demand for money in the liquidity theory, to account for the finance of new investment. The stock of cash is thus defined as '*finance*' by Keynes, and this also determines the rate of interest.

oppose a-temporal schemes to 'temporal'<sup>9</sup> schemes are inadequate. (ii) Models which make use of a mechanistic idea or time lack of any notion of causality. As a result, they can be in most cases assimilated to the a-temporal schemes of general equilibrium models, even though they may also accommodate lagged relations among the variables. (iii) The distinction of alternative sequential structures provides us with an additional methodological tool to understand some divergences in recent economic literature. A few remarks on this will he found in *section* 6.

### SECTION2. - Logical time

2.1 - By 'logical time' I mean a logical set of relations which links the variables in a *unique direction* implying a causal relationship between them<sup>10</sup>. Because this causality is framed within a logical scheme, logical precedence does not entail an chronological precedence – i.e. the variables need not be dated.

The task of this kind of analysis is to single out a *few fundamental relations* among a small defined group of basic variables in a causal ordering, which allows the *logical dynamics* of the system to be observed.

Three main features characterize this method. First, there is a unidirectional causality. *Second*, this scheme cannot cope with changing situations. *Third*, and most important, any temporal reference is absent from its laws. For instance, if we consider for the independent variables, the parameters and the dependent variables two sets of values – one at the point t' and the second at the point t'' – the theory tells us the *'logical law'* that links the different groups of values. The two equilibrium points t' and t'' do not correspond to any real position in time – i.e. the system itself is outside time. The points t' and t'' are *conventionally* defined only, according to the logical order which links them<sup>11</sup>.

This scheme cannot explain how the variables are actually changing at any intermediate point between t' and t'' nor does it enable us to study how the system actually responds to changes of parameters and functions. We can only compare the values of the dependent and independent variables registered at the equilibrium situations t' and t''.

These propositions appear clearly in Keynes's framework of the *General Theory*. Given the propensity to consume (the parameter) and the level of new investment (the independent variable) at any equilibrium point t', the 'logical law of the multiplier' (as Keynes himself defines it) ensures that a given increase in income and saving corresponds to that new level of investment. The analysis gives us the values of aggregate variables at the end of the process, where the final position of those variables is *conventionally* 'time t'''.

This method bas been attributed to Keynes's attitude of taking for granted the sufficiency and inevitability of confining strict analysis to equilibrium situations and treating the transitions between them as disorderly episodes, which by their nature defy any detailed explanation. Saving and net investment are determined independently of one another, yet they become equal. How? This question of *how* cannot be

<sup>&</sup>lt;sup>9</sup> The position held by L. Pasinetti, J. Robinson, J. Hicks, among few others, will be considered apart in the next sections.

<sup>&</sup>lt;sup>10</sup> For the definition of causality, appropriate to this argument, see n. 4.

<sup>&</sup>lt;sup>11</sup> This property of causal relations is emphasised for example by Simon. Simon specifies that "time sequence does indeed sometimes provide a basis for asymmetry between two elements" (A and B, where A is the cause of B) "but that *asymmetry* is the important thing, not the sequence". Simon 1953, p. 51.

Indeed Simon gives a deductive definition of causal law in terms of "logical properties of the model considered" (i.e. causal orderings of variables or groups of variables, on a model where the concepts to be defined belong to the model and not to the real world", ibidem p. 51). Even if Simon's idea of deductive logical causality may not be reconciled with the inductive definition employed in this essay, his argument is helpful in specifying Feigl's definition of causality, "that by putting asymmetry without necessarily implying a time sequence at the basis of our definition (of causality) we shall admit causal orderings where no time sequence appear s (and sometimes exclude causal orderings even where there is time sequence. We shall discover that causation does not imply time sequence, nor does time sequence imply causation". Simon 1953, p. 51.

satisfactorily answered within Keynes's own construction, for it belongs to the disequilibrium phases that intervene between one equilibrium point and another"<sup>12</sup>.

The logical sequentiality of Keynes' method enables him to focus attention on identifying *the forces* which determine any equilibrium level of income and employment, according to the causal relations that he specifies, totally neglecting the actual process of adjustment of the variables<sup>13</sup>.

To sum up, this method does not set out to supply any dynamic scheme which would enable us to follow the analysis of economic changes through time<sup>14</sup>. The shift of equilibrium from t' to t" only apparently indicates a dynamic movement of the system; in fact, it has to be understood as an exercise in comparative statics<sup>15</sup>.

From these three features several further points arise. *First*, one can see how this particular scheme of causality conventionally excludes the consideration of circular causality, in that the relations are linked in just one way  $(a \rightarrow b \rightarrow c)$  and not both ways  $(a \rightarrow b \rightarrow a)^{16}$ . If a circular flow of causality was to be assumed, the model would be formalized in terms of the simultaneous determination of the equilibrium values of the variables. However, the equilibrium system that one would get in this case must be sharply distinguished from a system framed according to 'logical time' sequentiality. For, even if the two are both static systems outside time, the former must actually neglect the sequential relations which on the contrary are essential to the latter<sup>1718</sup>.

Indeed, to express a circular flow of causality we can also resort to a system of temporal lags which order the occurrence of the feed-back effects ( $a10 \rightarrow b11 \rightarrow a12$ ) where t0, t1, t2 would be successive temporal symbols to indicate the temporal lags of the relations, and apparently showing a logical causal sequence. This scheme, however, may be assimilated to the '*mechanical time*' sequential method, which is discussed in the next section.

<sup>&</sup>lt;sup>17</sup> In algebraic terms, the scheme of circular causality is to be solved by means of a system of linear equations all interdependent. In order to formalize a causal ordering of events or variables one may refer to Simon's example. Simon shows the situation in which  $X_1 \rightarrow X_2 \rightarrow X_3$ : i.e.  $X_1$  is the direct cause of  $X_2$  and  $X_2$  of  $X_3$ , by means of a system of the following type:

$a_{11}x_1$	$= a_{10}$	1)
$a_{21}X_2 + a_{22}X_2$	$=a_{20}$	2)
$a_{32}x_2 + a_{33}x_3$	$=a_{30}$	3)
a: 1 · 1 · ·		

Simon explains that in this system (1) has a direct precedence over (2) and (2) over (3); and he specifies that not the temporal precedence is relevant here but the symmetry (i.e. the fact that one may go from (1) to (3) but not vice versa). Cfr. Simon '53, p. 58.

<sup>18</sup> On this point see L. Pasinetti '74 p. 46.

<sup>&</sup>lt;sup>12</sup> See G. L Shackle, 1967b, p. 142.

The analytical features of the *General Theory* which make it possible to classify it within the logical time sequential method are specified in section 5.

<sup>&</sup>lt;sup>13</sup> Cfr. J.M. Keynes, 1936, P. VII.

<sup>&</sup>lt;sup>14</sup> Quite different meaning have been given to the term 'dynamics' in the economic literature on the subject. By the term 'dynamic', I mean here the features of historical time sequentiality that I consider in section 4. Of that method, I am emphasising here the study of adjustment of variables *in time* according to a set of laws.

<sup>&</sup>lt;sup>15</sup> A definition of the conceptual and analytical differences which distinguish this method from a method which analyzes changes of the parameters and the functions is attempted for instance by Lindahl. See Lindahl 1951, p. 31. See also J. Robinson, 1964; J. Hicks, 1965; F. Hayek, 1941.

<sup>&</sup>lt;sup>16</sup> From the logical relations of the kind  $a \rightarrow b \rightarrow c$  it does not necessarily follow, of course, that the author excludes any link of the kind  $c \rightarrow a$ . It only follows that the link ( $c \rightarrow a$ ) is considered as a weak relation and may be neglected for the purpose of analysis as an approximation of the second order.

It may also be recalled here that even Lindahl, lacking a specific consideration of causal relations within a static field, does not recognize the causal structure of the G.T. and he formally resettles it according to a system of simultaneous and interdependent equations. The analytical fallacy of formalizing in this way a logical structure of sequences is considered in section 5.

*Secondly*, I should point out that I am using the term 'logical time' here with a different broader meaning from J. Robinson<sup>19</sup>. She describes as 'logical time' only the logical sequence of equilibrium positions that occur in a process of balanced growth (the "golden age"), in the sense that each equilibrium position implies all the preceding and all the successive ones<sup>20</sup>. Accordingly, she uses the scheme of the 'golden age' to describe situations of simultaneous equilibrium, in order to distinguish them from the whole set of sequential schemes that she labels, together, as 'historical models'. It is then proposed here to enlarge a bit upon the distinction drawn by J. Robinson.

*Thirdly*, no significance is attributed to the speed of adjustment of the variables, nor to the actual development of the process. *The unit of time is irrelevant* for the analytical purpose of the logical time scheme<sup>21</sup>. *The process can be analytically squeezed at will* so that the relations can be assumed to work out their effects instantaneously, provided that the *logical priorities* are respected.

Again referring to the *General Theory*, one can see that Keynes, founding his analysis on these features, consistently neglected to consider the whole set of adjustment mechanisms (among others the financial adjustments) that are implied in the working of the multiplier<sup>22</sup>. It is precisely the neglect of these mechanisms that lies at the core of the criticism Robertson makes of Keynes's theory of the multiplier. In doing so he considers it from the different methodological view point of *'mechanical time'* sequentiality.

#### SECTION 3. – Mechanical time

3.1 – Analysis based on 'mechanical time' sequences exhibits very different features from 'logical time'. The primary feature of 'mechanical time' sequences is that they implicitly assume the mechanical extension through time of a set of unchanging relations. This is because this method is neither used to enquire into the forces that determine a process, or to isolate its logical structure. Instead, the scope of this method is to describe the process step by step and give the values of the variables at any intermediate point between two specified positions in time.

Calendar sequences are followed and *the variables need to be dated* in order to follow the sequence of their values. The analytical scheme is divided into uniform intervals corresponding to the unit of time that has been chosen<sup>23</sup>. However, the progressive dates by which the variables are ordered in the analysis do not show that the future values of the variables are *qualitatively* different from the present and the past values. They therefore cannot show that an irreversible break has occurred at each moment in time, which logically and analytically separates each moment from the previous and the following one.

What the method does enable us to follow is the step by step development of a process, precisely because a mechanical notion of time is assumed. For instance, in economic theory it is implicitly assumed that a given ratio of consumption to income is a fixed constant at any point of 'time' as if a long period relationship is always present. Of course this assumption excludes by definition the possibility of

<sup>&</sup>lt;sup>19</sup> On this point cfr. section 6.

<sup>&</sup>lt;sup>20</sup> This corresponds to the definition by which Nagel labels any kind of model as "deterministic", i.e. "a theory (where)... the theoretical state of a system at one instant logically determines a unique state of that system for any other instant". See E. Nagel 1961, p. 285.

<sup>&</sup>lt;sup>21</sup> Cfr. J.M. Keynes, letter to Ohlin, 27<sup>th</sup> January 1937, vol. XIV, C.W., p184.

 $<sup>^{22}</sup>$  Cfr. the analysis of this issue in section 5.

<sup>&</sup>lt;sup>23</sup> The choice of the unit if time is determined, of course, by the assumptions made for analytical purposes. D. Robertson's "day", for instance, the unit of time according to which he scans the process of the generation of income, helps to show that: first, "the income a man receives on a given "day" cannot be allocated during its course to any particular use" (a lag is assumed between  $Y_t$  and  $C_{t+1}$  pr  $S_{t+1}$ ); and second, that there is a unity velocity for the circulation of money. D. Robertson, 1933 (p. 399).

For the definition of Robertson's "daily" method see also D. Robertson, 1940, in particular "Mr. Keynes..." and table 1, p. 119.

For the critical arguments addressed by Keynes and Hawtrey on this point, see E.J. 1933 Dec.

interpreting in the same theory situations where changing behaviours of economic units have to be explained, in which case change of the parameters and functions during the process of the generation of income should itself be the object of explanation. The parameters cannot be assumed to be unchanging. The typical case of situations which cannot be studied in terms of fixed parameters and functions is the cyclical path of the economy.

A second feature of this method is that the absence of any unidirectional law ensures the symmetry of the process itself: for instance in the following sequence  $\dots a_{t\rightarrow} b_{t+1\rightarrow} c_{t+2\rightarrow} a_{t+3}$  one may not infer any law of asymmetry.

In other words, the sequences of the '*mechanical time*' method are repetitive (i.e. they consider the occurrence of reiterated processes) and *reversible* (i.e. they are 'invariant in relation to the inversion of the temporal vector')<sup>24</sup>. The same kind of sequences which enable us to follow the values of the variables from time  $t_o$  to time  $t_z$ , may be used in reverse to go back from the values registered at time  $t_z$  to the earlier values of time  $t_o$ . And these two features of reversible and repetitive relations, cancel out any criterion of causality from the method itself.

The similarity of this method with the laws of dynamics in classical physics in obvious. The law of motion continues to hold mechanically, irrespective of actual conditions: it can be repeated, unchanged, again and again.

In economic literature, the working of these processes 'in time' is generally formalized in terms of systems of differential equations. The pattern of *the whole process* and the full development of its different stages may thus be specified at the beginning of the process, when certain values are attributed to the variables and the parameters of the system. This means, however, that to scan the economic process according to a 'mechanical time' method amounts to a quite peculiar notion of dynamic relations<sup>25</sup>.

If one agrees with B. Thomas, that 'the aim of the method of economic dynamics is to analyze the probable course of events on different assumptions relating to producers expectations'<sup>26</sup>, then the hypothesis of perfect foresights should be ruled out altogether from the study of dynamic relations<sup>27</sup>. However, the '*mechanical time*' method disregards this view and thus to divide the economic process into a sequence of subperiods it relies on the assumption that no unexpected alterations in the values of the variables will occur during any period, which would affect the values of the subsequent period; or, at least, no uncertainty about the future disturbs the decisions of economic units, nor is any need to revise plans envisaged.

Within the 'mechanical time' method, for instance, the cyclical path of the economy is interpreted by the mechanical extension or given economic behaviours, which are assumed to be homogeneous and symmetric, and it is these that explain the ups and downs of the economy. Neither the changing behaviours of economic units (the parameters and the functions) which determine and are determined by the different phases of the cycle, nor the analysis of uncertain plans, revisions and reactions to changes of expectations are the object of the theory.

Thus we have the main reasons that separate '*mechanical time*' sequences from the temporal sequences of the '*historical time*'.

3.2 - Robertson's analysis corresponds well to the features of *'mechanical time'* sequentiality. The process of determination of income is assumed by Robertson to be a *circular process* in relation to causality - i.e. there isn't an a-priori variable which is selected as the original cause of the process itself. The process can be induced by the initial stimulus of *any* variable.

<sup>&</sup>lt;sup>24</sup> This criterion is used by M. Hesse, when she describes in her book the different features of causal and not causal relations. See. M. Hesse, (25), See n. 11.

<sup>&</sup>lt;sup>25</sup> See n. 11.

<sup>&</sup>lt;sup>26</sup> See B. Thomas, 1936, p. 104.

<sup>&</sup>lt;sup>27</sup> Indeed, not only the assumption of certain expectations but also the assumption of inertial behaviour of economic units should be ruled out from an analysis of the economic process which is framed in time, where both expectations and economic decisions change according to the changing conditions of the process itself.

The different stages of the process are singled out and the model is thus mechanically extended through time, from time t<sub>o</sub> to any time t<sub>n</sub>, according to the sequence  $\rightarrow I_t \rightarrow Y_{t+1} \rightarrow S_{t+2} \rightarrow I_{t+3} \rightarrow Y_{t+4}^{28}$ . Robertson's sequence method obviously does not entail logical precedences.

Equilibrium values of the variables are determined by Robertson according to the traditional rules of interdependent models in accordance with the scheme followed, for instance, by Pigou. In fact it is interesting to note that both Robertson and Pigou, to whom Robertson refers at length, give an almost identical interpretation of the General Theory, by neglecting the logical causal ordering which is the core of the principle of effective demand in the *General Theory*, and is examined in further detail in section  $5^{29}$ .

The equilibrium levels of investment and saving are simultaneously determined mutually adjusting to each other according to an equilibrium level of the rate of interest<sup>30</sup>.

Robertson's own contribution concerns the equality I(i) = S(i) as a process, by dividing the period of the generation of income into many 'subperiods'. Thereby he focuses on conditions which must be met in each of the 'subperiods' to obtain, at the end of the whole period, the equality of I(i) = S(i). In other words, an interdependent equilibrium must be met in each subperiod of time<sup>31</sup>.

Robertson also attempts to apply this method to a different conceptual scheme, i.e. to Keynes's reasoning of the General Theory. "For those who prefer a more explicit temporal method of analysis", he 'redefines' Keynes's scheme of the generation of income according to his own temporal 'daily' intervals<sup>3233</sup>. It is interesting to outline his analytical exercise here to show the theoretical misunderstandings which arise when one neglects differing features and analytical objectives which correspond to different methods.

Robertson follows Keynes's assumption that an increase of investment gives the initial stimulus to the economy, but he splits the total increase of investment (of period  $t_1 - t_n$ ) into n equal shares  $I_1, I_2, I_3, \dots I_n$ , and he assumes that these shares are progressively undertaken by the economy at time  $t_1, t_2, t_3, \dots, t_n$ . In the same way, he mechanically extends in the consecutive periods  $t_1, t_2, t_3, \ldots t_n$  the increase of income which should follow the investment I according to the given parameter of the multiplier. This is an attempt to apply his own method to Keynes logical law of the multiplier, and thereby prove that the multiplier cannot guarantee a final value of income that should result from Keynes's reasoning. According to Robertson, in the new scheme provisional conditions of equilibrium have to be settled for all the n-2 intermediate positions of the variables, between the initial value of income  $(Y_1 \text{ in } t_1)$  and the final value of equilibrium  $(Y_n \text{ in } t_n)$ , so that the process can take place. Most important, Robertson maintains that at all the intermediate points (t<sub>k</sub>) at which a share of investment has to be realized, this must be met by a corresponding amount of savings ( $S_{tk} = sY_{tk-1}$ ) in order to be undertaken. As this is not warranted, he posits the possibility that the equality I(i) = S(i) can be turned into the equality  $S(i) + \Delta M(i) = I(i)(I)$ , following the theory of the "loanable funds" to which he implicitly refers. Lacking an increased money supply, the rate of interest is still the crucial variable which ensures the fulfilment of the final value of equilibrium, when I = S. Robertson himself emphasizes this in several passages. "Mr Keynes is impressed by the possibility that the desire to save is not responsive to change in the rate of interest... But the fact that the rate of interest measures the marginal convenience of holding idle money need not prevent it from measuring also the marginal inconvenience of abstaining from consumption"<sup>34</sup>.

<sup>30</sup> See D. Robertson, 1937.

 $<sup>^{28}</sup>$  In the same way, of course, this sequence may start by S<sub>t</sub> or Y<sub>t</sub> alternatively.

<sup>&</sup>lt;sup>29</sup> Cfr. D. Robertson, 1940, pp. 5-6 in particular. Cfr. also Robertson's quotations of Pigou and Lavington in relation to the equilibrium determination of investment, saving and the rate of interest, ibidem, p. 17, and D. Robertson, 1937, n. 4, p 431. The same point is in Pigou, 1950, see for instance n.1. p. 27. Cfr. also Pigou 1923 p. 181.

<sup>&</sup>lt;sup>31</sup> This point contrasts with those interpretations of Robertson's analysis in terms of a dynamic reformulation of the General Theory, which are expressed for instance by J. Presley and M. Danes. See D. Robertson, 1937.

<sup>&</sup>lt;sup>32</sup> See also D. Robertson, 1940, p. 2 and pp. 149-150.

<sup>&</sup>lt;sup>33</sup> On the different definitions of the 'period of the generation of income' (an ambiguous term unless it is clearly defined) see F. Machup, 1976. D. Robertson 1940, p. 117. The process is reformulated in Table 1 p. 119. <sup>34</sup> D. Robertson, Sept. 1937, p. 431; cfr also n. 29.

The critical point is that if we follow Robertson's interpretation of the temporal sequential phases of the generation of income, no autonomous increases of investment can be conceived. This is because his model has a lagged structure which by definition shows at least a temporal divergence between new investment and saving. Since, however, Robertson does *not* consider any *causal relation* ensuring that an increase of investment will be followed by a corresponding increase of saving, consequently he interprets the problem of *financing* an autonomous increase in investment in terms of the problem of stating the sufficient availability of past and present saving as a preliminary condition for any increase of investment.

In other words, consistent with the features of his own method, Robertson does not grasp the causal nature of the theory of effective demand which underlies Keynes's logical theory of the multiplier. In so doing, he approaches the problem from the opposite perspective to Keynes.

Robertson's exercise has been stressed here, as it shows several features. First, his 'mechanical time' method enables Robertson to raise problems which are related to the adjustment mechanisms, while remaining within a 'quasi-static' framework; for instance, to identify the problem of the financial adjustments required by the multiplier. The need to find finance for any increase of investment that may be undertaken by the economy is hidden by Keynes's method; it becomes evident only if the different stages of the process of the multiple increase of income are singled out as Robertson does.

*Second*, this example shows that it was critical in the mutual misunderstanding of Keynes and Robertson that they did not explain their basic analytical divergences at a methodological level and the different objects which corresponded to their different methods<sup>35</sup>. Instead, whereas Keynes's work was concerned with identifying the forces which determine the dynamic of the system, Robertson concentrated on pinpointing the stages of the process itself.

*Third*, and finally, the '*mechanical time*' sequentiality shows that the attempt to transform the logical relations of a timeless scheme into a set of temporal elations may not be accomplished without redefining the variables used and the relations among them. "There are inevitably difficulties", Robertson writes, "in expressing in statically framed terms the situation existing *at a moment of time during a period of change*"; "it is precisely for this among other reasons, that Mr. Keynes's photographic formulation seems to me to need supplementing by a cinematographic one"<sup>36</sup>.

Employing this analogy, Robertson is merely replacing "Keynes's photographic formulation" with a sequence of photographic formulations. He then fits together all the sequences, each individually considered, and obtains the final values of the variables as by calculus of the definite integrals.

3.3. – The objection might be raised that the sequences of 'mechanical time' may conceive the same causal sequences as the 'logic time' method on the one hand with respect to the relations of asymmetry which belong to the a-temporal causality of 'logical time' (of the kind  $a \rightarrow b \rightarrow c$ ), but on the other hand projecting them *into time*. I maintain that one may not truly frame 'in time' those causal relations by means of the relations of 'mechanical time'.

This methodological mixture can be seen by extending by sequences the multiplier 'through time'. This case, however, shows that to follow different phases of the multiple increase of income assumed to be determined by a given increase of investment, has nothing to do with time: it is merely an expository expedient of static equilibrium analysis. If we follow the different phases of the generation of income through time, we necessarily find disequilibrium points of the variables. One is then faced with the choice of ignoring them or dealing with them. In the first case, the analysis corresponds totally to the logical scheme which is a-temporally framed, and *the same* 'final' values of variables in equilibrium are reached here for all variables as in the other scheme. In the second case, it is important to acknowledge that responses become unpredictable and this in turn implies that expectations are uncertain; one can no longer relate  $I_0$ , at time  $t_0$ , with  $Y = Y_1$  at time  $t_1$  according to parameters quantitatively determined a priori.

<sup>&</sup>lt;sup>35</sup> See n. 3. Robertson thus incited Keyne's famous reaction rejecting "Mr. Robertson's incorrigible confusion between the revolving fund of money in circulation and the flow of new saving, which causes all his difficulties". J.M. Keynes, 1938.

<sup>&</sup>lt;sup>36</sup> D. Robertson, 1940, p. 17.

On the contrary, in *'mechanical time'* sequentiality the future is never uncertain. Deterministic relations are implied, even when facing 'disequilibrium' points. (Similar difficulties arise in relation to those models which follow Keynes's reasoning more closely i.e. where investment determines saving. These include those models which analyze the relations of the *General Theory* in cyclical paths)<sup>37</sup>.

The expedient of dating the values of the variables according to the method of 'mechanical time' sequentiality does not offer alternative basis for framing economic relations in time. Like the form of analysis ruled by 'logical time', these 'mechanical time' models do supply only the two end positions of the variables. The main difference between them is that 'mechanical time' sequences apparently enable to stop the process at any given point between the two end positions of the interval to observe the values being studied. They do not however supply any general abstract law which would identify the determinants of changes in the structure of the system as the causal logical method does. Neither do they supply any rule of adjustment to identify the dynamics of these changes in the manner of the third method, that of 'historical time' sequentiality.

If then we leave the field of equilibrium, where causal laws may be generally inferred to explain the asymmetric relations which rule the economic structure, and instead set out to study the *actual* development of these relations in time, quantitative kinds of relations must be substituted for those qualitative relations<sup>38</sup>. This means using the actual values that the variables assume during the process, in relation to the values which have been forecast by the operators when plans were formulated and decisions taken. "The concept of dynamic in economics", Hayek wrote in 1941, "when it is used in contrast to equilibrium analysis in general, it refers to an explanation of the economic process as *it proceeds in time*, an explanation in terms of causation which must necessarily be treated as a chain of historical sequences. What we find here is not mutual interdependence between all phenomena but a unilateral dependence of the succeeding event on the preceding one"<sup>39</sup>.

Indeed, the marriage of a *logical sequentiality* with *temporal sequentiality* to infer the laws which rule the structure of the system (e.g. the law of effective demand) and to fit these laws into the dynamic field of adjustment of uncertain plans was the aim of the Swedish School method. It is the scope of *'historical time'* sequentiality.

#### SECTION 4. – Historical time

The core of '*historical time*' is that past, present and future are qualitatively different, linked by expectations and plans.

To extend the analogy with physical laws, '*mechanical time*' sequentiality resembles the laws of the classical mechanics, while '*historical time*' borrows from the progressive law of 'cosmological time', to which Georgescu-Roegen<sup>40</sup> refers in recalling the features of the law of entropy<sup>41</sup>.

<sup>&</sup>lt;sup>37</sup> Cfr. section 5.

<sup>&</sup>lt;sup>38</sup> A similar conclusion is expressed by Lindahl, who deals with different numerical examples of the working of the multiplier according to the lagged Robertsonian scheme, or to temporal (ex-ante) formulation of it, reaching different results, see E. Lindahl, 1953, pp 11-18.

<sup>&</sup>lt;sup>39</sup> F. Hayek, (1941) reprint <sup>7</sup>76 p.17 (italics added). Here one may recall that Hayek's own use of dynamic method has nothing to do with his own definition of dynamics; it relates to 'the intermediate field', as he himself defined it, "which refers to phenomena which still come within the scope of equilibrium analysis. All that the use of the term dynamics meant here", Hayek further specified, "is that we do not postulate the existence of a stationary state", pp. 17-18.

It is interesting to note that Hayek's awareness of the methodological differences which separate his method from the notion of 'historical time' has disappeared with his followers, who pretend to follow a dynamic method just by formalizing and specifying Hayek's notion of intemporal equilibrium.

The same criticism had been addressed by Lundberg and Myrdal to Lindahl's early attempt at dealing with dynamic problems using the method of intemporal equilibrium (cfr. Lindahl, 1929 ('39). See n. 42 and n. 56 below. <sup>40</sup> This distinction between mechanical time and historical time has been envisaged by Georgescu-Roegen, who notes the

<sup>&</sup>lt;sup>40</sup> This distinction between mechanical time and historical time has been envisaged by Georgescu-Roegen, who notes the problem but does not analyze it in details. Cfr. Georgescu-Roegen, 1971, pp129-140 in particular.

That the future is qualitatively different from the past is not expressed only by a generic condition of uncertainty in which the economy works out its processes; uncertainty of future events invokes analytical relations to account for it<sup>42</sup>.

Functional relations can never recur unchanged for they "leave traces" in so far as they cumulatively alter the values of the variables upon which decisions are based. In other words, the *parameters* are not to be presumed constant or predetermined, as they are in the two methods discussed above, neither may the functions be mechanically extended from one interval to the next one as in the case of '*mechanical time*' sequentiality.

Three features characterize *historical time* method. *First*, because the variables refer to an uncertain future, their future values and *definitions* are subject to and must take account of the subjective evaluations by individuals.

*Second*, the *functions* must explain the responses of economic units to the actual outcomes of the economic process *and* to *changes of confidence* with changes of future expectations. This is because today's decisions are determined by the interplay of yesterday events and the subjective assessment of the probability that today's expectations will be realized tomorrow. As a consequence, the theory should supply a set of functions that explain how a given situation in the present diverges from expectations that preceded it as well as how this divergence influences expectations and decisions and thereby the setting of the values of the variables, for the following period.

*Third, a cumulative process* has to be studied: the last value of any variable includes the path of previous ones (the expected and the realized ones) but does *not* mechanically imply the trend determination of any future value of the variables. This deeply separates the schemes which are founded on this method from those founded on the '*mechanical time*' sequentiality. In particular, it raises a barrier between the alternative ways of formalizing these schemes in mathematical language.

Historical time relations cannot easily be translated into elegant and synthetic formulas. They lend themselves better to an approach which explains the different reactions of the variables in a range of conditions, and analyzes the different adjustments that may prevail in each situation. It is clear (for instance) that within the sequences of historical time the future values of variables may *not* be calculated by the deterministic features of differential equations, which otherwise are typically used to estimate the values of '*mechanical time*' temporal sequences.

It emerges that the '*historical time*' approach focuses the attention on the *changes* which occur during the adjustment processes of the variables more than on the *equilibrium levels* of these variables measured as final outcomes at the end of the process.

4.2 – Overall, the methodological problem cannot be properly discussed unless we have previously pointed out that the same terms which are commonly employed to analyze the generation of income, such

Schumpeter in his later writings also stresses the difference of *historical* from *dynamic* time, where by the term "dynamic" he means a mechanical notion of time relations corresponding to "mechanical time" sequentiality. He underlines this difference in the History of Economic Analysis, 1954, n. 5, p. 965.

<sup>&</sup>lt;sup>41</sup> See Bridgman, 1928. For the definition of the entropy law see Max Plank, 1932, p. 81 and A. Eddington, 1943.

<sup>&</sup>lt;sup>42</sup> This is why I maintain that the *General Theory*, though it underlines many features that are linked to the problem of uncertainty and focuses on an object which is definitely 'in time', nevertheless does not offer any analytical scheme of relations which are framed in 'historical time'. (Hints of this kind of relations are given only in the liquidity preferences theory and in the article of the Q.J.E, 1937). Cfr. on this point section 5.

In a different way, the models of intertemporal equilibrium do not deal properly with uncertainty and temporal relations, although they give great emphasis to the difference between plans and results. Uncertainty of the future totally disappears, as soon as they assume that plans for all the future periods are all made consistent at the beginning of the first period and then realized with certainty, i.e. as soon as one introduces complete future markets uncertainty disappears, and with it the difference between past, present and future also disappears. Since not other causal law rule the relations among variables in these models, the analysis may thus assume the form of systems of interdependent equations to determine the values of the variables, precisely like those general equilibrium models which do not deal with problems of time. See sections 5 and 6.

as investment, saving, and income itself, in fact amount to different concepts coming from different sequential schemes. To show this, it may be useful to recall part of the complex body of theory which developed into the 'Swedish School'43 44.

Let us recall, first, the well-known distinction between ex-ante and ex-post values of economic variables which is drawn by Myrdal<sup>45</sup>. In the *ex-ante* category Myrdal classifies values based on past experience, those based on mere conjectures about the future and the values of plans of entrepreneurs which are still to be realized. These latter may be incompatible with each other or may underrate the degree of economic expansion and profit achievable in the current period. Within the category of *ex-post* values, Myrdal classifies the result of plans and the re-adjustment made in the process.

I have already mentioned two points which necessarily follow from this distinction: one, the need to embody the subjective evaluation of future data, uncertainty and risk, in the *definitions* of the variables<sup>46</sup>; and two, the need to consider the economic process in quantitative terms. Together, these mean calculating the numerical values of the variables according to the subjective anticipation of individual of future data, and revising these values as the process proceeds and the actual results confirm or falsify earlier evaluations.

These two needs are well interpreted by Lindahl in the Studies, as well as in the continuous research that he pursued in his work; in particular, by his definition of the concept of income $4^{47}$ .

Lindahl founds the idea of 'anticipated income' (i.e. 'income as interest' in the widest sense which refers to a certain period forward) upon those features of subjective evaluation of uncertain future data, that enable him to calculate its changes with the passing of time. So that the whole set of concepts which are based on the notion of income (consumption, saving, etc.) show the same temporal features of subjective anticipation. "This concept" (of income as interest) Lindahl writes, refers to "the continuous appreciation of capital goods owing to the time-factor, that is to say, the current interest on the *capital value* which the good represents". The notion of capital value also refers to anticipation of an imperfectly foreseen future: "the expected future services of the capital goods are the basic factor in the estimation of capital value, for the

<sup>&</sup>lt;sup>43</sup> The major contributions are considered. E. Lindahl (1939), reprint 1970, G. Myrdal (1939), reprint 1965; E. Lundberg (1937) reprint 1955.

Indeed, different labels have been attached to the authors whom I would place within the 'Swedish School' group. On the one hand the group has been divided, in particular by Palander, into those authors who have been "primarily concerned with sequence analysis", such as Hammarskjold (1933), A. Johansson (1954) and E. Lundberg (1937), and those who framed their approaches more directly on Wicksell's and Davidson's ideas, whom Palander label 'neo-wicksellian'. Myrdal (1932), Lindahl (1930) and Ohlin (1934) should thus belong to this second group.

On the other hand, authors like B. Thomas and B. Hannson have emphasised the collective research of these authors towards common definition of a "macroeconomic dynamic theory". See T. Palander, 1953, B. Thomas, 1936, B. Hannson, 1980.

<sup>&</sup>lt;sup>44</sup> At the beginning of the 1980's O. Steiger, C. uhr and D. Patinkin dealt further with the formal problem of deciding whether the stream of thought expressed by these authors was a proper school to be counterposed to Keynes and the English approach, or, on the contrary, whether the two streams of thought were mutually interdependent. The point of these discussions seems to be mainly referred to "the possible anticipation of the G.T. by the Stockholm School" (as Patinkin puts it). More recently, after the publication of Ohlin's 1933 article in an English translation, that debate has been reopened by D. Patinkin, Yohe, H. Brems, O. Steiger. On these specific issues, Keynes private correspondence is particularly interesting (see for instance the letters to and from Ohlin on the *ex-ante* method of the Swedish School, at the time of the debate in the Economic Journal. (Vol. XIV, p. 183). C. Uhr 1973; O. Steiger 1976; C. Uhr 1977; D. Patinkin 1978.

Ohlin's article in the Economisk Tidscrift 1933 has been translated in "On the formulation of monetary theory", H.P.E. Marche 1978; on the same number of the review is the debate between O. Steiger, W. Yohe, D. Patinkin, H. Brems.

<sup>&</sup>lt;sup>45</sup> The emphasis on the 'backwards' effects of expectations and on the role of anticipation of uncertain future events has been put forward first by Myrdal, in his doctoral dissertation (unfortunately not yet completely translated by the Swedish), even though its influence on the development of the Swedish method has been enormous). See G. Myrdal, The problem of *pricing and change*, in T. Palander 1953, p. 9, n. 5. <sup>46</sup> See n. 42; Cfr. Knight's definitions of uncertainty and risk in F. Knight (1921), 1971.

<sup>&</sup>lt;sup>47</sup> E. Lindahl, 1933, see pp. 400-405 in particular.

latter can be considered equal to the sum of the anticipated value of these services, discounted at the current rate of interest, due reduction also having made for the *risk factor*".

The appreciation rises when the discounted future services come nearer and nearer (this corresponds to an increase in value which for a given period forward is regarded by Lindahl as "the product of the capital value" and the rate of interest (income) applying to the period. As the services are consumed, capital value is reduced.

"During a given period of time, this reduction in value due through consumption may be less or greater than the contemporaneous appreciation due to the time-factor. These differences between interest and consumption anticipated for a certain period can be regarded as the *saving*, positive or negative as the case may be, which takes place during the period".

From these features, a picture emerges of a *speculative world*, which is essentially founded on the uncertain features which result from the unidirectional passing of time.

"When the period is finished, Ohlin wrote, new investment is equal to saving. But (he stressed) how does this equality come about? The answer is that the inequality of saving *ex-ante* and *ex-ante* investment sets in motion a process which makes realized income differ from expected income. (...) The discrepancy between planned saving and planned investment, Ohlin concluded, can be regarded as the cause of the process"<sup>48</sup>.

We see clearly here that the concepts are differently defined in the debate. Keynes's definition of saving, for instance (i.e. the residual entity which is not consumed out of current income, nor directly affected by the rate of interest) has nothing to do with Ohlin's *ex-ante* definition of the same category (i.e. "people disposition to save (...) in relation to income expectations for many future periods")<sup>49 50</sup>. Nor does Ohlin's view correspond to Robertson's concept of saving, which relies on individual decisions to save part of disposable income that has been earned in the previous period, in relation to a given level of the rate of interest.

If one uses the Swedish categories, Keynes's approach falls into the *ex-post* group: his concepts are consistently defined in terms of *ex-post* notions of investment, saving, income. This, of course, does not imply that uncertainty, ignorance of the future and risk do not play a role in the description of the actual decision-making process in the *General Theory*).

However, it is easy to understand why Ohlin was led to maintain that either Keynes's reasoning in the debate is *ex-post*, and then it does not explain anything, or it is *ex-ante* and then is entirely wrong<sup>51</sup>. Indeed, Ohlin does not perceive that different methods correspond to different objects of analysis: they focus attention on different problems of the economic process, and, therefore, can only illuminate the appropriate questions.

It is obvious that the Swedish concern for the adjustment process (which also implies each time the determination of the value of the multiplier) is far removed from Keynes's own concerns, which are

<sup>&</sup>lt;sup>48</sup> B. Ohlin, 1937, p. 5. In relation to the multiplier, Ohlin further emphasised: "the income which has causal significance (for consumption) is not Keynes's ex-post concept, the realized income during the last period, but the *expected* income". Ibidem, p. 66. Ohlin's method is discussed by W. Yohe, 1962, pp. 274-280.

<sup>&</sup>lt;sup>49</sup> See B. Ohlin 1937 a.

 $<sup>^{50}</sup>$  Lindahl's definition have to be recalled (see n. 47): i.e. that "income as interest (...) is the total *sum of the consumption and the saving expected* to take place during a certain period", but the element of saving must be expressed in the "increase in value of the capital, exclusive of gains and losses" (emphasis added). E. Lindahl 1933, pp 400-403.

The concepts of gains and losses, as they were defined by Myrdal and used by Lindahl, need some consideration too. "When an investment (for instance) is made, its value is determined by the owner's expectations at that moment. If, in a year's time, the demand for his product has fallen and his receipts are below *what he anticipated*, he will then change his views about the future". I.e. the capital value of the equipment falls by the amount on the 'loss'. Thus Myrdal concluded: "when something happens which alters the basis of capitalising future incomes and costs, whether it be a weakening or a strengthening of the *owner's confidence in his estimates* (emphasis added), a net 'profit' or 'loss' occurs and the value of the capital will rise or fall correspondingly. Profits and loss are, therefore, not actual values, but estimated adjustments in capital value". G. Myrdal, pp. 59-60.

<sup>&</sup>lt;sup>51</sup> B. Ohlin, 1937 b.

focussed on the final values of income and employment with the emphasis on identification of *the forces* which sooner or later bring about equilibrium levels of income and employment.

It is interesting to note that the concepts which have been defined according to the requirements of *historical time* may all be reduced to their corresponding *ex-post* categories, if the effects of uncertainty are removed from the scheme. Having defined 'income as earnings' as one of the two theory's components of 'income obtained' (*ex-post*) which is perfectly compatible with Keynes's own definition of income, for instance, Lindahl in fact explained: "If the future could he completely foreseen, so that the future streams of services and the rates of interest at which they should be capitalized were known beforehand, the total value of the capital stock could only be changed by the elements of savings as defined above, that is to say, the difference between interest and consumption. On this supposition *income as earnings would correspond to income as interest*"<sup>52</sup>.

On the contrary, it also is clear, but has to be underlined, that the reverse operation it not so straightforward: to embody the uncertain features of *historical time* in a theoretical scheme requires more than merely mentioning uncertainty among the determinants of individual decisions<sup>53</sup>.

4.3 - The *historical scheme* deals with plans and anticipations, with reaction functions and similar volatile concepts which may *not* be framed outside of time, and *quantitative* relations must therefore be substituted for the qualitative laws of the logical scheme<sup>54</sup>. As a consequence, that nice 'division of labour'<sup>55</sup> among economists that well corresponds to the classical notions of 'dynamic' and 'static' fields, is difficult to maintain.

This point is a crucial one: in order to combine the logical causalities of the former method (as found, for example, in the law of effective demand) with the temporal features of the second method, a *unique* complex scheme is required, *different from both the previous ones*<sup>56</sup>. In introducing the role of anticipations of an uncertain future as independent data within his theory, Myrdal was implicitly recognizing the need for a new method to deal with economic problems from a temporal (historical) point of view. Myrdal was in fact questioning precisely the proposition that "the static forces set the standards and the dynamic forces produce the variations", by which Clark had interpreted the current thought of neoclassical economists in relation to dynamic theory<sup>57</sup>.

<sup>&</sup>lt;sup>52</sup> E. Lindahl, 1933, p. 403

<sup>&</sup>lt;sup>53</sup> See n. 42 and section 5.

<sup>&</sup>lt;sup>54</sup> By underlining the quantitative aspects of these relations I mean that one must specify, numerically, the values that are actually attributed to expectations and plans together with the realized values of the variables. The discrepancies between them, which are also numerically calculated, allow us to quantify future plans and to develop the actual process in time. As it is clearly expressed by Yohe, "the emphasis is on the numerical sequence and not on the general properties of the system". W. Yohe, 1959, p. 165.

<sup>&</sup>lt;sup>55</sup> See F. Hayek (1941), p. 17.

<sup>&</sup>lt;sup>56</sup> Unfortunately, Lindahl himself generated a theoretical misunderstanding in his 1954 model, where he analyzed the process of generating income of the Keynesian type by means of short run reaction functions, and concluded that Keynes's equilibrium conditions of comparative statics (as he defined Keyne's scheme) should be complementary to his own system of reaction functions, which is a "good starting point for the determination of the dynamic process which is induced by the disturbance of equilibrium" (Ibidem, p. 31). However, while the reaction functions appear consistent with the Swedish method, Lindahl assumed that the process of expansion continues, until investment per unit of time has reached a value which can be expected to remain stable (see ibidem, fig 4, p. 29). But he was then led explicitly to declare the need for supplementing this set of function (which determine the directions of movement from an equilibrium towards a future which should be truly uncertain) with another scheme which determines "the conditions for an equilibrium" as he did not have any rule which links one period to the next. Cfr. E. Lindahl 1954.

<sup>&</sup>lt;sup>57</sup> Cfr. J.B. Clark, 1899, p. 32. "Actual prices are now above the standards and now below them, as a pendulum is now on one side of an imaginary vertical line and now on the other. This vertical line coincides with the position that the pendulum would hold, if it were under the influence of static forces only. The oscillations are due to dynamic forces... The same thing is true of natural wages and interest, (...). Static forces set the standard and dynamic forces produce the variations": (p. 32). On Myrdal's critique to this issue cfr. B. Hannson, 1980, p. 34.

The point is that if we follow the economic process according to the rules of the *historical time* method, even if we assume the same logical causalities of a *logical time* scheme, we may *not* assume at the end of the process that the system will have reached *those* final values of equilibrium as predetermined in comparative statics<sup>58 59</sup>.

If we consider, for instance, a 'keynesian' process of the generation of income, where  $i\rightarrow I\rightarrow Y$ , in a *logical time* scheme the value of the multiplier is the relevant given element which determines the level of income. But in a scheme of *historical time*, one has instead to study *how* the value of the multiplier is changing in time. Since this scheme follows the *actual* development of the process in time, it cannot neglect the obstacles which impair the decisions affecting economic units (e.g. investment decisions) and prevent them from being realized, nor the eventual changes of income which continuously follow on from the process of revision of current decisions.

But the problem of analyzing 'how', 'how quickly' and 'if' investment planned is actually realized in a defined period is analytically irrelevant to Keynes's approach in the General Theory where attention is focussed on the autonomous features of investment and on the logical law which ensures that any level of investment generates 'its own' level of saving<sup>60</sup>. On the contrary, if the same process is analyzed within an historical time approach, the adjustment phases must also be emphasised. For example, we may find investment plans inconsistent with the possibility of financing them from the banking system, or with the current monetary policy, at the end of the period, and therefore we would find that the *ex-post* results are different from the *ex-ante* plans of the aggregate variables.

The relation between consumption and income offers another example of the three methods types. The proportion of income which is devoted to consumption changes with the shifting direction of the path taken by the economy. Thus, there may occur a level of income different from the previously expected level, upon which expenditure plans were formulated. The process of growth may have met for instance a set of 'unexpected' problems which were not fully anticipated, thus enforcing the need to revise expectations and make new decisions.

If we focus attention on the levels of income and consumption realized at one moment in time, we find of course that the consumption expenditure amounts to a certain proportion of the level of income which is actually earned. The method of 'logical time' sequentiality underlines the fact that the causal links run from income to consumption, projects the values of the parameter in a 'logical time', in an exercise of comparative statics.

If we follow the method of *mechanical time* sequentiality, the period of the generation of income is divided into several intervals of 'time', during which income and consumption progressively realize the values that have been predetermined according to given parameters as in the *logical time* method, and are projected forward 'in time'. In *historical time*, as Lundberg emphasises, the theory must explain the changes of the multiplier instead of assuming that the latter is given<sup>61</sup>.

Indeed, in spite of what is usually asserted, Keynes himself was aware of his own methodological choice, as he declared in a letter to Ohlin: "As regards the *ex-post* and *ex-ante* method, I shall certainly give further thought to its advantages (...). My reason for giving it up was my failure to establish any definite unit

<sup>&</sup>lt;sup>58</sup> Interesting evidence of awareness of these problems, which have been ignored in the recent literature, is given for instance by L. Amoroso. "The fundamental supposition of equilibrium theory" Amoroso wrote in 1924, "is that the functions do not change in time. From this follows that the economic system must necessarily reach a certain foreseen equilibrium position (B), as a result of the interaction of forces which persist for a greater or lesser interval of time. (...) But before B is reached the forces change from S to S' so that the system itself alters its direction and, now, diverging from B, tends towards another equilibrium position B'. (...) The study of the successive changes in these forces, and the corresponding adjustments of the system to them, is the object of economic dynamics". L. Amoroso, 1924, pp. 88-94. <sup>59</sup> See also G. Masci in F. Caffè 1975, F. Hahn 1973, J. Hicks 1974 and B. Hamsson 1980, p. 733.

<sup>&</sup>lt;sup>60</sup> Cfr. section 5.

<sup>&</sup>lt;sup>61</sup> E. Lundberg (1937) 1955, P. 37.

of time (...). So, after writing out many chapters along what were evidently the Swedish lines, I scrapped the lot and felt that my new treatment was safer and sounder from the logical point of view<sup>362</sup>.

After forty years, we can legitimately wonder whether the contemporary field of economic analysis could better receive attempts at formalizing a methodological synthesis of this kind.

#### SECTION 5. – Causality and time in the General Theory: an example

5.1 – In this section, I will support my claim that the *General Theory* is framed irrespective of *'mechanical time'* and *'historical time'* sequences.

My aim is to show *firstly*, that in the *General Theory* the logical precedence of the variables does not imply any calendar precedence<sup>63</sup>.

Secondly, I try to show that the *General Theory* cannot he interpreted in 'historical' terms, unless all the variables are redefined in *ex-ante* terms. However, even once this was done, the whole set of problems would arise in relation to the definition of 'equilibrium' which have been hinted at in paragraph 2.

Thirdly, once the static features of the *General Theory* are singled out, the logical precedence of some variables over the others emerges, as Pasinetti noticed<sup>64</sup>. This means that the logical structure of the *General Theory* cannot be represented as a general system of relations *wholly interdependent with each other*.

Two analytical features characterise the logical frame of the *General Theory*. 1) It is well known that the rate of interest is determined within a set of stock relations, while investment and saving are analyzed according to *flow* considerations. 2) The rate of interest is a strictly *monetary* phenomenon, while the relationship between investment and saving concerns *real* variables. Because Keynes stresses that investment units are separated from saving units, and that investment and saving are ruled by different behavioural laws, it appears crucial to him to show that investment flows do logically precede the corresponding saving flows – i.e. that *no stock concepts of savings constraints make sense within this scheme*. In the same way, it is crucial to him to determine the rate of interest by its monetary components, so that its level may be established *logically precedent* to the decisions to invest<sup>65</sup>.

It should also be recognised however, that all the investment plans which arise from comparison of the rate of interest and the marginal efficiency of capital are *wholly undertaken* by the economy. Since no financial constraints (and even less real constraints) are assumed to hinder the undertaking of any increase of investment plans, it follows that the latter (*ex-ante I*) need not be logically and analytically distinguished from realized investment (*ex-post I*). Within this simplifying framework, as I will show, their aggregate levels are equal by definition.

This strong simplifying assumption is nowhere explicitly stated by Keynes (nor does it seem to have been appreciated by most of Keynes's interpreters). But I will show here that it is strictly dependent on some basic assumptions that Keynes associates with the working of the multiplier.

Indeed, the mechanism of the multiplier creates much of the ambiguity in the different interpretations of Keynes framework, and it needs careful consideration. He employs a double edged concept. For it is a *logical expedient* that enables him to maintain his logical causal chain irrespective of the temporal relations that should be accounted for in the analysis; however, the implicit assumptions supporting it encompass in my view most of the misconceptions in different interpretations of his approach. To recall, the type of

<sup>&</sup>lt;sup>62</sup> J. M. Keynes, letter to Ohlin, C.W. vol. XIV, p. 184.

<sup>&</sup>lt;sup>63</sup> This amounts to saying that *the variables need to be dated*, and this is different from Hick's interpretation which implies that the theory incorporated the contemporaneous occurrence of cause and effect *in a period of time*. See Hick's definition of "*contemporaneous causality*" in J. Hicks 1979, (Ch. V and Ch. VI).

<sup>&</sup>lt;sup>64</sup> Cfr. L. Pasinetti, 1974.

<sup>&</sup>lt;sup>65</sup> These two distinctions are also at the core of the contrasting results which emerge from the debate on 'finance' between Keynes, Ohlin and Robertson. Here, however, I consider the, only as far as they support the relations which underlie the logical sequences of the *General Theory*.

*logical time* which underlies the *General Theory* enables Keynes to squeeze out of his analysis any temporal process, and to neglect the intervals of time which are required by the adjustment mechanisms of the variables; this holds in primis for the law of the multiplier<sup>66</sup>.

Once this point has been established, two basic features can be associated with it. First, if comparative statics is defined as the "study of the changes in production and prices which we should expect to occur when the data are changed"<sup>67</sup>, then the analysis of the *General Theory* proves to be wholly framed in *static* terms<sup>68</sup>. Secondly, and most important, the *ex-ante* values of the variables are equal to the corresponding values that are registered *ex-post*<sup>69</sup>. Accordingly, the analysis ca be considered *as if* the aggregate variables were defined only in *ex-post* terms.

Once this twofold aspect of the *General Theory* is grasped, it is clear that Keynes's causal relations are framed outside o f both '*mechanical time*' and '*historical time*' sequences.

5.2. – Let us consider these two points separately and analyze first the proposition that Keynes's multiplier must be 'instantaneous' and encompass static relations among variables.

As is well known, Keynes bases his analysis of the investment multiplier on Kahn's own model<sup>70</sup>. However, Keynes attributes a quite different role to it. For, Kahn's aim is that of creating an analytical tool that will enable him to *calculate retrospectively* the realized (*ex-post*) ratio between the values of certain variables in an aggregate framework (i.e. to calculate retrospectively the level of secondary employment which is induced by an original act of investment under certain conditions)<sup>71</sup>.

It is obvious that, if Kahn's multiplier is expressed in terms of income instead of employment – and the same assumptions on the parameters and the functions are maintained – the result gives us the ratio of realized *ex-post* incremental flow of investment to the incremental output flow associated with  $it^{72}$ . Kahn seems to be well aware of the analytical limits entailed by the set of assumptions under which the relation holds, as he specifies in a footnote: "I am here considering the final position of equilibrium when everything has settled down. I do not enter into the question of the time-lag"<sup>73</sup>.

<sup>&</sup>lt;sup>66</sup> It follows, *inter alia*, that the disequilibrium positions of aggregate variables are thus neglected and that the problem of financing the increase of investment does not enter into the picture. I analyze in detail these issues in a forthcoming paper, (see n. 7). Here I merely underline that this point is strictly linked to the methodological propositions that I have considered. <sup>67</sup> J. Hicks 1965, chapter VI pp. 67-68. Hicks further specifies in chapter VII the features of a "fixed-price" method that he

attributes to the General Theory.

<sup>&</sup>lt;sup>68</sup> On this concept of '*ceteris paribus* dynamics' see G.L. Shackle 8'58) 1967, chapter III, p. 67 and following; and G.L. Shackle, 1972; see also E. Lundberg, 1937; J. Hicks, 1965, chapter II. Within this field of dynamic theory – i.e. "the analysis of the process in time" – Hicks singles out the 'static method' as the analysis drawn by stages: "the process is divided into stages which are analyzed separately and then fitted together".

<sup>&</sup>lt;sup>69</sup> See n. 49. Divergences between planned and realized values of certain variable have been introduced in the so-called 'dynamic' versions of Keynes's framework of the *General Theory*. Changes in the level of stocks – i.e. 'unplanned' increases or decreases of investment – are analyzed in the cyclical schemes of the generation of income drawn by Godwin, Hicks, Minsky, (see section 6) etc., and framed in a system of differential equations. I will consider elsewhere the critical arguments that can be addressed to this way of 'dynamizing' the analytical process, by merely resorting to a system of differential equations (i.e. to the introduction of time-lags into the analysis). Hints of this kind of critique were addressed by Lindahl to Samuelson's method in the Foundations of Economic Analysis: see E. Lindahl, ibidem 1954, n. 1, p. 27.

<sup>&</sup>lt;sup>70</sup> "The conception of the multiplier was first introduced into economic theory by Mr Hahn" J.M. Keynes, 1936, p 113. See A. Wright 1956. Wright dates back the origin of the multiplier mechanism inquiring about it in the analysis drawn by Pigou (1927), Mund and Giblin (1930). He then concludes recognizing that Kahn's originality consists in establishing the complete formulation of the mechanisms together with its implications. See Also g.l. Shackle 1951; Shackle 1967, cfr. 14.

<sup>&</sup>lt;sup>71</sup> Cfr. F. Kahn 1931. Among the other assumptions Kahn stresses that "money wages are not raised as a consequence of the reduction of unemployment; (...) no account is taken of any increase of productive efficiency", etc. (p. 173 and fll.). <sup>72</sup> See G.L. Shackle 1967 (1968), p. 78.

<sup>&</sup>lt;sup>73</sup> See R.F. Kahn, 1951, n.2, p. 183. The *retrospective* features that are involved by Kahn's analysis of the multiplier have been fully appreciated by Robertson, for instance, as it emerges from the following sentence: "If an Authoritarian act of investment of money amount N generates a series of increments of money income – qN  $q^2N$ , etc. – and a series of

By contrast, and this is a crucial point, Keynes's aim is a prospective one. It is fundamentally different from Kahn's retrospective and quantitative task. For Keynes pretends to synthesise by means of the multiplier a *whole process in being*. He has to account, *prospectively*, for the chain of logical priorities and causality that he has set in motion by defining the functional relations among variables (and, most important, to account for the logical causality from I flows to S flows).

However obvious the theoretical difference, which separates a retrospective study of relations among aggregate variables from a prospective one, may appear, some of the consequent methodological features need to be clarified. Analytical understanding of the difference between them, as is well known, derives from the analysis by Myrdal in the late 1930's. To state the problem in Myrdal's words, "an important distinction exists between prospective and retrospective methods of *calculating* economic quantities"<sup>74</sup>. Quantities defined in terms of measurement made at the end of the period in question are referred to as *expost*; quantities defined in terms of action planned at the beginning of the period in question are referred to as *ex-ante*.

Lundberg further specifies that "every economic theory" – understanding it as a 'prospective' theory – "must be built upon fixed, individual functions of reactions, more or less simple, which determine how a group of individuals will act when taking certain expectations into consideration. *In what degree these expectations are realised is the subject of explanation*" (emphasis added)<sup>75</sup>.

Nevertheless, it is legitimate to question whether macroeconomic theories have fully appreciated the issue raised by Myrdal and whether they have taken account of the analytical implications which flow from it.

According to this distinction, the task of the economist should be to inquire how the *ex-ante* values of the variables reach the correspondent *ex-post* values that are actually registered at the end of any period, the differences between the two sets of values being the objective of the *same analysis*<sup>76</sup>.

In the *General Theory*, the only mechanism which is supposed to account for the causal chain of prospective relations among aggregate variables is the multiplier. And the multiplier is assumed to work out all its effects until the process of the growth of income has attained *its full extent*. It is clear then that the relevant parameters and the functions are implicitly assumed as unchanging during the period to which the prospective analytical process refers. They must be unchanging, otherwise nothing could guarantee that the final results (that have been calculated according to the value of the multiplier) will actually be reached, or that they will even exist.

Now, in the *General Theory*, the relevant parameter – i.e. the propensity to consume – is explicitly assumed constant by Keynes<sup>77</sup>. And for the problem of establishing functional relations which will be unchanging through time, Keynes resorts to the *logical assumption* that the investment multiplier works out *instantaneously* the correspondent changes of aggregate income and saving. By assuming an 'instantaneous' multiplier at work Keynes is assuming that the functions remain constant during the period of generation of income<sup>78</sup> or, even more, that they remain constant during the whole period between an

increments of saving -(1-q) N, (1-q)qN, - at later dates, we can regard the latter series as adding up to and, as it were, *balancing retrospectively* (emphasis added) the original act of investment". D. Robertson, 1933, p. 399. It again appears that Robertson, while misleading the logical sequence of Keynes's framework, is clear in singling out the

It again appears that Robertson, while misleading the logical sequence of Keynes's framework, is clear in singling out the problem that are by a process of temporal sequences.

From a completely different analytical objective, one can quote Shackle's similar description of Kahn's multiplier, that he synthesises as follows: "the picture shown us a mechanical, time extended process of *calculating* the numerical value if the Multiplier by treating the growth of income as an infinite convergent series", G.L. Shackle ('58) 1967.

A similar argument is addressed by Lundberg (1937) 1955, p. 37.

<sup>&</sup>lt;sup>74</sup> G. Myrdal, 1939, p. 143.

<sup>&</sup>lt;sup>75</sup> E. Lundberg, 1937, p. 145.

<sup>&</sup>lt;sup>76</sup> Lundberg further specifies that "the motives for investing and saving must be related to certain expected and realized changes in time". E. Lundberg, 1937, p. 143.

<sup>&</sup>lt;sup>77</sup> J.M. Keynes, 1936, p. 115.

<sup>&</sup>lt;sup>78</sup> Cfr. the definition of the 'equilibrium adjustment period' as it is stated by F. Machlup, 1976, p. 268.

investment decision and the complete increase of income that the latter brings forth. "The logical theory of the multiplier", Keynes stresses, "holds good continuously, without time lag, at all moments of time"<sup>79</sup>.

These are, unquestionably, the essential features of static analysis<sup>80</sup>. In insisting on the timeless, continual, ever perfect operation (of the multiplier) – Shackle points out – Lord Keynes was allowing the mathematician in him to opt for the concise elegance of an essentially static method<sup>381</sup>.

5.3 - Indeed, since the methodological distinction between logical sequences and the two different types of calendar sequences is not generally understood, this conclusion about Keynes's static method often seemed paradoxical, as it refers to a theoretical framework that apparently more than anything stressed the role of expectations of future events in conditioning present economic behaviour<sup>82</sup>.

Interestingly enough, Keynes himself does not seem to appreciate the analytical limits of the assumption of an instantaneous multiplier. He seems to confine the concept of 'static' analysis to those theories which assume a given level of output (i.e. that follow the neoclassical assumption of full employment of resources). Keynes is then prepared to consider his own framework of the *General Theory* as a dynamic analysis, as far as it accounts for the "forces" that determine changes of the level of income. He then writes in the introduction of the *General Theory*: "the outstanding fault of the theoretical parts of that work [the Treatise on Money] is that I failed to deal thoroughly with the effects of *changes* in the level of output. (...) But the dynamic as distinct from the instantaneous picture was left incomplete and extremely confused. This book [the *General Theory*], on the other hand, has evolved into what is primarily a study of *the forces which determine the scale of output*<sup>83</sup>.

It is of course clear that to single out the determinants of a process of change does not imply the study of the dynamics of the changing process itself.

5.4 - It has been maintained (by Keynes himself among the others) that the *General Theory* does account for an *ex-ante* definition of investment<sup>84</sup>. I will try to show here that this is not so.

<sup>&</sup>lt;sup>79</sup> J.M. Keynes, ibidem, p. 122.

<sup>&</sup>lt;sup>80</sup> The static method which is implied by Keynes's multiplier has not been fully appreciated by Keynes interpreters. O. Steiger, for instance, drawing a comparison between the G.T. and the analytical achievements of the Swedish School, explicitly states his doubts on the static interpretation of the G.T.; see O. Steiger 1978, p. 44 and n. 26 p. 442.

Opposed to Steiger is Hicks' own argument, as he states: "the multiplier theory (and indeed the theory of production and prices which is somehow wrapped up in the multiplier theory) is outside time. It runs in terms of demand curves, supply curves and costs curves – just the old tools of equilibrium economics. A state of equilibrium, by definition, is a state in which something, something relevant, is *not* changing; *so* the use of an equilibrium concept is a signal that time in some respect at least, has been put on one side". J. Hicks, 1976, p. 140.

From a different position Leijonhuvfud reaches a similar conclusion; see also P. Davidson, "Postscript to Money and the Real World", 1972, p. 380.

<sup>&</sup>lt;sup>81</sup> G:L: Shackle, 1967, p. 76.

<sup>&</sup>lt;sup>82</sup> J.M. Keynes, 1936, p. 155.

<sup>&</sup>lt;sup>83</sup> J.M. Keynes, 1936, pp. VI-VII, Preface.

<sup>&</sup>lt;sup>84</sup> For an *ex-ante* interpretation of Keynes's definition of investment see Vandenborre 1958. Vandenborre's arguments, however, are convincingly refuted by Neisser 1959. Neisser shows that only one concept has been defined by Keynes in *exante* terms, namely the aggregate demand -D -, that Keynes defines in chapter 3 as "the proceeds that entrepreneurs expect to receive from the employment of *N* men". Neisser shows that Keynes substitutes for it in the analysis the concept of income, which is treated as a proxy variable of *D*. And income, as is well known, is defined in the G.T. only in *ex-post* terms. Neisser argues, as a conclusion, that "the qualitative distinction between *D* and income is overlooked (by Keynes). There cannot be any doubt that the *ex-post* interpretation eventually prevails". Neisser, 1959, p. 292.

The same point is stressed by Lindahl, who goes further in questioning "whether the system displayed in the G.T. with its static formulation and its limitation to *ex-post* magnitudes is the best base for making economic theory dynamic. In the beginning of the work, Lindahl emphasises, Keynes introduces a distinction between 'aggregate demand' and 'aggregate supply', but the concepts are unfortunately not rigidly defined. (...) If Keynes had grasped this idea in a more consistent way, he would probably have had greater understanding of such concepts as purchasing *plans* and sales *expectations* and, on

Indeed, Keynes's '*ex-ante*' definition of investment is relevant, but it has to be considered as such only in the *logical* sense of underlining he autonomous occurrence of *decisions* to invest – i.e. to specify the features of the act of volition to invest, which is logically precedent and actually separated from that of abstaining from consumption. On the contrary, *the value* of investment planned (*ex-ante* I) is exactly equal to the value of the correspondent investment realized (*ex-post* I). To maintain the distinction between *ex-ante* and *ex-post* investment in the General Theory, 'ex-ante' I must be seen as nothing but the value of net investment which is registered, *with perfect certainty*, before it is realized<sup>85</sup>.

Moreover, since calendar sequences do not enter into Keynes's picture, planned investment 'instantaneously' becomes realized investment, i.e. *ex-ante* I is coincident with *ex post* I not only in value terms, but also in temporal terms. Keynes's intuition of a possible *ex-ante* interpretation of investment – as I planned, or decisions to invest – which is the basis of his understanding of the concept of effective demand, is not followed by any analytical account of the distinction between the *ex-ante* and the *ex-post* viewpoints<sup>86</sup>.

Two arguments, I think, may be given as proof of this a-temporal definition of Keynes 'ex-ante investment'. First, Keynes always counterposes the concept of ex-ante I to that of 'saving-induced I', and not to the correspondent ex-post value of the same investment. Second, and most relevant, the whole set of the other aggregate variables is analyzed in the General Theory in ex-post terms<sup>87</sup>.

Indeed, if an *ex-ante* definition of investment could be envisaged in the analysis, the same should be true for the consumption variable<sup>88</sup>. Now, as is well known, in the *General Theory* the consumption function links an actual level of consumption with an actual realized level of income, given the propensity to consume. An *ex-ante* definition of consumption, on the contrary, would explain the definition of consumption *plans* (at any time  $t_0$ ) in relation to a share of *expected* earnings that the consumers would decide to consume at time, say,  $t_1^{89}$ . This would imply, within Keynes's scheme, an *ex-ante* definition of the concept of expected income. The subjective evaluations of individuals on the value of income that they could conceivably earn at a given future date should also be considered<sup>90</sup>. And this is clearly not the case of the *General Theory*, where all the aggregate variables are explicitly considered at their current realized values<sup>91</sup>.

The *ex-ante* definition of investment which was framed by the Swedish School is based on a quite different framework: *all* the functions and *all* the variables they employed are framed in time. The point is

<sup>87</sup> Cfr. n. 84.

<sup>88</sup> Only once, in a passage of his article in the Economic Journal 1937, in the discussion with Ohlin, Keynes admits that C should be also considered in *ex-ante* terms.

<sup>89</sup> For a clear analysis of this point, see H. Neisser 1959.

<sup>90</sup> For an extensive discussion of the alternative ways of defining the expected values of aggregate variables see G.L. Shackle (1958), pp. 38-66 in particular.

The problem of weighing individuals expectations was first faced by Knight (1921). Knight distinguished among a) the individual estimate of the objective probability, b) the degree of faith he has in his own judgment, and c) his gambling preference. See Knight, ibidem, p. 225. Keynes critique to Knight's arguments is well known.<sup>91</sup> In the discussion with Ohlin, Keynes appears to be consistent with his own framework in rejecting Ohlin's concept of *ex*-

<sup>91</sup> In the discussion with Ohlin, Keynes appears to be consistent with his own framework in rejecting Ohlin's concept of *exante* saving; indeed, according do Keynes's definition, saving is a residual entity, that can only be calculated as the difference between *current realized income* and *current realized consumption*. On the contrary, Keynes's argument in defence of his own definition of *ex-ante* investment appears nebulous, and even misleading within the set of assumptions that he has framed in the G.T.

the whole, of the system of concepts, including ex-ante and ex-post magnitudes, as well as the differences between them – which are worked with in the Swedish Theory". E. Lindahl, 1954, part. II, p. 168, Lindahl is here going too far in denying the distinction between aggregate demand and aggregate supply, which is indeed crucial to Keynes framework of the G.T.. Lindahl's argument, however, is valid so far as it highlights Keynes's neglect of the *ex-ante* definitions of the relevant relations.

<sup>&</sup>lt;sup>85</sup> Cfr. n. 42.

<sup>&</sup>lt;sup>86</sup> G.L. Shackle, pp. 74-75.

relevant, as it involves a more general issue i.e. that no framework can account for the *ex-ante* definition of any variable (letting it to be counterposed to its correspondent *ex-post* value) unless *the whole* analytical process is framed *in time*<sup>92</sup>. Or, in other words, that if one variable can be defined in its *ex-ante* value, then it must possible to state all the other variables in *ex-ante* terms as well, within the same framework<sup>93</sup>.

5.5 - Finally, it is obvious that even if the *General Theory* is framed without calendar sequences it embodies strong logical sequentiality. Nonetheless, the static features that have been pointed out so far and associated with the mechanism of the instantaneous multiplier, unfortunately have created space for those kinds of interpretation which definitely neglect the causality which is essential to Keynes's scheme<sup>94</sup>.

A set of functional relations which are drawn in a static framework is usually represented in algebraic terms by a system of simultaneous equations. The values of the variables are simultaneously obtained by solving the system of equations according to a given set of parameters: the variables turn out to be *all interdependent on each other*.

We could cite a huge number of 'keynesian' models that are built by means of a system of simultaneous equations<sup>95</sup> it has to be stressed that the great majority of them does not respect the logical causality which is the crucial methodological point underlying the whole framework of the *General Theory*.

As Hicks, the natural father of these pseudo-keynesian approaches has recently recognized, subjecting his most popular article in Econometrica to vigorous self critical scrutiny<sup>96</sup>, "the equilibrists thought that

<sup>94</sup> On this issue, cfr. L. Pasinetti, 1974, pp. 45-50.

<sup>&</sup>lt;sup>92</sup> An interesting, although partial, discussion of the different ways of introducing 'time' in economics is to be found in P. Rossenstein-Rodan 1934. See also J. Hicks, 1976, pp. 135-155.

<sup>&</sup>lt;sup>93</sup> If the methodological distinctions between '*logical time*', '*mechanical time*' and '*historical time*' sequentiality are followed, it is interesting to note that even the second type of analysis, which is sequentially framed following calendar units of time, cannot account for these concepts of *ex-ante* variables. Robertson himself notes this point, as he specifies in a passage of his Essay: "I have a difficulty in assimilating my terminology completely to that of Prof. Ohlin (...): expected income is necessarily a somewhat nebulous concept". D. Robertson, 1940, pp. 6-7.

Robertson is quite correct in emphasising that 'expected income' as any other expected variable is a 'nebulous concept', as it cannot be clearly defined within his own framework. For Robertson's '*mechanical time*' method also does not account for temporal relations which cannot be endogenously fitted into his analysis.

<sup>&</sup>lt;sup>95</sup> After Hicks' '*ISLM*' diagram (1937), and Modigliani's approach which is similarly framed (1944), one finds a huge number of these 'keynesian' models in the more recent literature. One important example of such extensions which definitely distort Keynes's framework of the G.T can be seen in M. Friedman (1974). In what Friedman defines as the 'common model' – which should include the basic analytical features of the G.T – saving is assumed to be affected by the level of the rate of interest (!): the level of consumption is altered by change in the rate of interest (i.e. substitution is assumed among cash, financial assets and consumption goods, where the supply of money determines the shift from one expenditure level to another).

The same objection holds for the analysis drawn by K. Brunner and A. Meltzer (1974). The critical argument the address to Friedman's 'common model' is based on the role of the rate of interest within the real balance effect. Brunner and Meltzer, in fact, resort to a mechanism of changing relative prices – between cash, financial assets and consumption goods – to explain how an increase of money supply may alter the flow of expenditure in consumption goods. It is obvious that the issue they raise does not alter the essential features, of Friedman's model as far as it does violence to the methodological framework of the G.T.

Among the other well known models, even Lindahl gives a misleading picture of the *G.T.* in his article in 1954. Lindahl utilizes in his formulation of the model some assumptions that are crucial to Keynes's analysis – e.g.  $W = W^0$  (real wage); Y = Y(S), etc. Lindahl goes even further, as he explains, having depicted Keynes's analysis by means of his own model of equations: "It is true that Keynes would lay heavy stress on the fact that the causation goes (from increased investment) and not in the opposite direction, which would perhaps be more in keeping with traditional theory. But as we are discussing an equilibrium with simultaneous interdependence of the various magnitudes the distinction seems rather artificial" (!). E. Lindahl, 1954, part I, p. 258.

<sup>&</sup>lt;sup>96</sup> J. Hicks, 1976, p. 141. In relation to his old '*ISLM*', diagram, Hicks stresses: "I must say that the diagram is now much less popular with me than I think it still is in many other people. It reduces the G.T. to equilibrium economics; it is not really in time. That, of course, is why it has done so well". Instead of stressing the temporal sequences, I think, Hicks would have

what Keynes had said could be absorbed into their equilibrium systems: all that was needed was that the scope of their equilibrium systems should be extended". But what inevitably happens, as one can observe with Pasinetti, is that " behind the formal façade of a simultaneous equations system a substitution of interpretative models takes place"<sup>97</sup>.

#### SECTION 6. - The use of time in the economic models. Some examples

The purpose of this paper is to argue that methodological differences alter the meanings of apparently similar relations among important macro-economic variables. The methodologies are not capable of combination. They perforce answer different analytical questions.

The findings in relation to the Keynes, Ohlin and Robertson debate show us a classification of time vector types and causal relations which allows for a more general classification of theoretical models in economics. The models selected are mere examples to be taken as an indication of further study on this area.

probably been better employed in emphasising the logical causality and sequences which the diagram completely misses in the G.T.

A vigorous critical review of the so-called Keynesian extended models is given by H. Minsky 1977; see also A. Leujonhuvfud, 1973.

<sup>&</sup>lt;sup>97</sup> L. Pasinetti, 1974, pp. 47-48.

Time structure	$B_1$	 B <sub>2</sub>	B <sub>3</sub>
Causal structure	Timeless	MECHANICAL TIME (section 3)	HISTORICAL TIME (section 4)
A <sub>1</sub>	$A_1 B_1$	 $A_1 B_2$	$A_1 B_3$
NO CAUSALITY	e.g.: - general equilibrium models - stationary states	e.g.: - Robertson - intertemporal equilibrium models - temporary equilibrium models	e.g.: - Swedish School 1920's – 1930's
A <sub>2</sub>	$A_2 B_1$	$A_2 B_2$	
LOGICAL CAUSALITY (SECTION 2)	e.g.: - Keyne's General Theory - Sraffa - Classical theories of production	e.g.: - Keynesian theories of growth - Keynesian theories of cycle	

CASE A<sub>1</sub> B<sub>1</sub> has been separately drawn as it indicates the method of non sequential models.

Examples of *mechanical time* sequentiality, in which no logical causality is embodied, are given by *international equilibrium* models and *temporary equilibrium* models (see *case*  $A_1 B_2$  and *section 3*).

Classification of these two methods within *'mechanical time'* requires special consideration, because it is usually claimed by their authors that these methods are appropriate to deal with dynamic 'temporal' problems without any further specification.

For the *intemporal equilibrium* I refer, first, to the particular notion of time which one finds at first in Hayek<sup>98</sup>; second, to Lindahl's use of it to attempt to cope with expectations and plans over time<sup>99</sup>; third to the mathematical formalization of the 1950's by Arrow (1971) and Debreu (1959); and, fourth, to the more

 $<sup>^{98}</sup>$  F. Hayek, 1931, pp. 29-30; 1941, definition pp. 22-25. It has to be remembered, however, that Hayek was conscious of methodological differences which separate his method from an historical notion of time. Cfr. F. Hayek, 1941, reprint 1976, pp. 17-18. See n. 3, p. 11. In Hayek 1931 (n. 1 p. 30) one finds Hayek's reference to his own definition of the method on intertemporal equilibrium as early as in 1928. For a discussion of this issue – i.e. on the origin of this method – cfr. Millgate, 1979.

<sup>&</sup>lt;sup>99</sup> E. Lindahl, 1929, 1939. See n. 42 and n. 39. But Lindahl abandoned this method in 1930, instead moving towards a method of temporary equilibrium, because he recognized that otherwise he could not study any real succession of events, where uncertain expectations affect plans and behaviours. Cfr. E. Lindahl, 1954, p. 27 fll.

recent attempts at dealing with uncertain plans by models of quantity rationing such as Malinvaud (1971) and Benassy (1975) although they label their models of "neo-keynesian disequilibrium".

All these models posit a sequence of periods different from each other; plans are made mutually consistent at the beginning of the first period and are perfectly realized during the following periods; and, in fact, no new contracts are made in the periods.

The mechanical notion of time underlying the set of these models becomes apparent in three ways. 1) If one follows these methods, the only implied alternative to perfect foresights is the assumption of perfect future markets, since no uncertainty of future events enters in the definition of the variables in these models<sup>100</sup>; 2) no role of money is defined or analyzed even by those models of rationing schemes (e.g. Benassy 1975) where monetary contracts are introduced<sup>101</sup>; in the absence of real time and uncertainty, money is just a 'numeraire'<sup>102</sup>. Finally, and most important for our purposes of classification, 3) the outcomes of all periods - i.e. the equilibrium values of the variables - are all simultaneously determined at the beginning of the first period<sup>103</sup>. Accordingly, not only do these models show a mechanical notion of time: they are general equilibrium models where no logical sequentiality is considered, so that they enter in case A<sub>1</sub> B<sub>2</sub>.

A mechanical notion of time is also inherent in the models of *temporary equilibrium*. The interval of time is divided into several periods, different from each other, but the theory focuses on one period at a time, and determines the equilibrium values of the variables for that period. At the beginning of each period, data are changed by changes in the previous period and are considered as newly given<sup>104</sup>. The models I am referring to here are the early models by Lindahl 1930 (in 1939) and Hicks (1939)<sup>105</sup>, which were formulated to deal with imperfect foresights over time<sup>106</sup>. In a similar way, however, time is treated mechanically ill those recent models of temporary equilibrium developed by Grandmont (1971), Grandmont - Younes (1973), Grandmont - Laroque (1976), which try to overcome the critiques addressed to the method of intertemporal equilibrium in the 1970's. In particular, they do not deal properly with monetary problems<sup>107</sup>: this is self-evident as "money only matters when we analyze the problems of the real world in which expectations are liable to disappointment"<sup>108</sup> 109.

The assumption that plans are consistent over one period only, is an expedient used there to overcome the assumption of perfect foresights of previous methods. But this is not sufficient to fit these models within a truly temporal framework; the equilibrium levels of the variables which this method helps to define are only determined for one period, by a set of simultaneous and interdependent equations. For the following periods there are, it is true, imperfect foresights<sup>110</sup>, but at the beginning of each period new plans will be formed, and the model is not concerned with the adjustment between different periods. As a consequence, the sequence of these different equilibria does not amount to the analysis of a process in time, but leads us to rely instead on "the comparison of alternative temporary equilibria"<sup>111</sup>.

<sup>105</sup> Cfr. J. Hicks, ibidem, pp. 65-66.

<sup>&</sup>lt;sup>100</sup> Cfr. Leijonhufvud, 1973; P. Davidson, 1981.

<sup>&</sup>lt;sup>101</sup> Cfr. F. Hahn, 1973, pp. .15-16; P. Davidson, 1981, p. 61.

<sup>&</sup>lt;sup>102</sup> See F. Hayek (1941), 1976, p. 29; Debreu, 1959, p. 28.

<sup>&</sup>lt;sup>103</sup> See F. Hayek 1941, p. 17; E. Lindahl, 1953, pp. 11-18; E- Lindahl, 1954.

<sup>&</sup>lt;sup>104</sup> Cfr. J. Hicks, 1965, Ch. III and Ch. IV.

<sup>&</sup>lt;sup>106</sup> Cfr. P. Davidson, 1981, and A. Leijonhufvud, 1973, p. 35.

<sup>&</sup>lt;sup>107</sup> Cfr. S. Fisher, A.E.R., 1975, p. 162.

<sup>&</sup>lt;sup>108</sup> J.M. Keynes, 1936, p. 293, cited by P. Davidson, 1981.

<sup>&</sup>lt;sup>109</sup> Money holdings are introduced, for instance, by Grandmont (1971), to cope with unexpected situations of the following periods. <sup>110</sup> J. Hicks, 1965.

<sup>&</sup>lt;sup>111</sup> Cfr. Section 3 above. Steady growth models are also included within mechanical time sequentiality, even if this classification is not wholly univocal. The model by Harrod (Domar) (1948, 1946), for instance, shows a mechanical notion of time.

Examples of *mechanical time* sequentiality are also given by the *neokeynesian model of growth* and the *aggregate cyclical* models (see case  $A_2 B_2$  and *section 3*).

*Neokeynesian models of growth* (e.g. J. Robinson 1964) clearly identify a set of causal relations which should account for the generation of income. But they assume an exogenous income growth rate which requires simultaneous determination of the distribution of income, consumption and relative prices. Thus this complies with *mechanical time* sequentiality.

The same notion of time underlies the *aggregate cyclical models*. These include those which follow Keynes's framework (Hicks 1950), Minsky (1957) and respect the causal logical sequentiality of the *General Theory* by setting the relations of the model in a time-lagged structure. But they do not, however, frame these relations among variables in *historical time*; a determined path for the economy is assumed which is interrupted only by the occurrence of some obstacles<sup>112</sup>. Accordingly, they link a mechanical notion of time to a logical sequential structure (*Case A*<sub>2</sub> *B*<sub>2</sub>).

Other macroeconomic cyclical models (such as Metzler (1951)), which introduce lagged relations in a logically non causal framework, of the kind  $\ldots a_t \rightarrow b_{t+1} \rightarrow a_{t+2} \ldots^{113}$  belong to *Case A*<sub>1</sub>*B*<sub>2</sub>.

Examples of *logical time* sequentiality are given by the *classical theories of production*, *Sraffa's* model and *Keynes's General Theory* (see sections 2 and 5 and case  $A_2 B_1$ ). It must be remembered, however, that though this method can be formally 'out of time', all models which are concerned with forces determining structural changes of the economy necessarily refer to the actual movements of the economy in time. They therefore can be said to tackle historical time problems using 'static', logically sequential, method.

The method of *classical theories of production* is not discussed here; its similarity is with the method considered in section 2, from the analytical perspective of this essay.

The listing of Sraffa's model into the same case may appear less obvious; however, the core of his analysis it that production, distribution and accumulation are logically separated, and must be sequentially considered.

Finally, definitions of 'disequilibrium' relations are framed according to an irreversible notion of time (Case *B3* section 4), which is not similar to 'disequilibrium conditions' which are only included to test the local stability of the solution of a model and which do not allow for the subjective evaluations by individuals of future events.

#### SECTION 7. – Concluding remarks

We have seen that 'time', or temporal structure, is of central importance in classifying and interpreting the different analytical applications of models which initially appear similar but are theoretically very different.

Three broad conclusions follow from our argument.

1) The choice of method used in economic theories is strictly constrained by what the theories are appropriate to analyze; and it therefore follows that no direct comparisons may be drawn between analytical relations (i.e. pieces of theories) founded on different methods: the same variables are perforce defined by incomparable terms.

<sup>&</sup>lt;sup>112</sup> For instance, Minsky's financial model of 1957, which makes an important contribution in the direction of integrating real and financial variables in a framework of effective demand, does not in fact define aggregate variables in *historical time*. However, when almost twenty years later the author was dealing with some of the truly temporal aspects of the generation of income (still though following Keynes's laws and sequentiality), he did not strictly rely on the relations of the General Theory, but felt the need to base his considerations on the new version of the General Theory, in terms of *ex-ante* evaluations, which was formulated by Keynes in the Q.J.E. Feb. 1937. See H. Minsky, 1975.

<sup>&</sup>lt;sup>113</sup> It is important to note here that there is a difference between stating that expectations of event 'a' influence event 'b', and stating that event  $a_t$  determines event  $b_{t+1}$ .

2) The different notions of time singled out in this paper show the inadequacy of usual definitions which are based on a dychotomized classification of theoretical models, such as "static" versus "dynamic", "equilibrium" versus "disequilibrium", etc.

Examples of dychotomized categorizing are found for instance in Frish' definition of statics and dvnamics<sup>114</sup> where a-temporal schemes are counterposed to temporal schemes (which fit together the models founded on a mechanical notion of time with those founded on an historical notion of time)<sup>115</sup>.

In a similar way, a dychotomized classification has been introduced by J. Robinson, who ignored any explicit role for 'mechanical time' sequentiality. J. Robinson's distinction between 'logical time' and 'historical time' sequentiality would seem to be close to the one proposed here. But she considered any theory to come within 'logical time' which embodies time as a logical construction in equilibrium conditions. She fitted, for instance, Keynes's General Theory within 'historical time' models<sup>116</sup>, apparently including within 'historical time' any set of relations aiming to describe how the system works, that is, models where initial conditions are given as " historical accidents" and decisions are taken on the base of an uncertain future. In this, she failed to appreciate that these features may be differently reproduced by a disequilibrium model and by the use of logical causality in a model out of (historical) time.

3) The idea has to be rejected too that a 'nice division of labour' exists between economists using 'equilibrium' (static) models which are intended to determine the final equilibrium values of variables, and those using 'disequilibrium' (dynamic) models which set out to study the stages of adjustment of the variables towards equilibrium values (see Section 4). This is because, first of all, the two groups of equilibrium values (which should be defined according to different methods) are different. Secondly, the field of dynamic analysis is thus restricted to a mechanical notion of the adjustment processes which should not necessarily characterize this method.

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<sup>&</sup>lt;sup>114</sup> R. Frish, 1936, pp. 100-105. Frish's definition has been followed, among others, by Hicks (1963), see. P. 65.

A similar definition of statics and dynamics in terms of chronological features is followed by Harrod, though he introduces the need to determine the annual rate of change of variables in dynamic analysis, while static analysis is thought to deal solely with the determinants of annual value of the same variables. Cfr. R. Harrod, 1948.

<sup>&</sup>lt;sup>115</sup> In this direction, Hicks' specification (1965) is a peculiar one; he distinguished between  $C_t = f(Y_t)$  where he sets out a static relation between variables of the same date, and  $C_t = f(Y_{t-1})$  where he defined a dynamic relation referring to lagged periods.

Indeed, Hicks' most recent work and important book on the problems of causality in economics overcomes the notion of a dychotomized distinction between theoretical models. But, in so doing, he does not isolate the features of a purely logical (causal) sequentiality. I have discussed elsewhere the relation of Hicks' categories to the classification proposed here. <sup>116</sup> Cfr. J. Robinson (1977), in C.W., Vol. I, p. 12.

#### REFERENCES

AMOROSO L., La Meccanica Economica, in "Giornale degli Economisti", 1924.

- ARROW K. HAHN F., General Competitive Analysis, Oliver and Boyd, 1971.
- BENASSY J. P., *Neo-Keynesian Desequilibrium Theory in a Monetary Economy*, in "Review of Economic Studies", October 1975.
- BREMS H., What was new in Ohlin's 1933-34 macroeconomics?, in "History of Political Economy", March 1978.

BRIDGMAN P., The Logic of Modern Physics, MacMillan N.Y., 1928.

BRUNNER K., Stock and Flow Analysis: Discussion, in "Econometrica", July 1950.

BRUNNER K. - MELTZER A., Friedman's Monetary Theory, in Friedman 1974.

CLARK J.B., The Distribution of Wealth, MacMillan, London 1899.

DAVIDSON P., Money and the Real World, MacMillan, London (72), 2d ed. 1978.

- DAVIDSON P., Alfred Marshall is Alive and Well in Post-Keynesian Economics, I.H.S. Journal 1981.
- DEBREU G., *Theory of Value; an Axiomatic Analysis of Economic Equilibrium*, Yale Univ. Cowles Found. for Research in Economics, N.Y. 1959.

DOMAR E., Capital Expansion, Rate of Growth on Employment, in "Econometrica", April 1946.

EDDINGTON A, The Nature of Physical World, New York, 1943.

- FEIGL H., Notes on Causality, in "Readings in the Philosophy of Science", Feigl and Brodbeck Eds., N.Y., 1953.
- FELLNER W. SOMERS H., Stock and Flow Analysis: Comment, in "Econometrica", July 1950.

FISHER S., Recent Developments in Monetary Theory, A.E.R., May 1975.

- FRIEDMAN M., A Theoretical Framework for Monetary Analysis, in "Milton Friedman's Monetary Framework", R. Gordon Ed., Chicago Univ. Press 1974.
- FRISCH R., On the Notion of Equilibrium and Desequilibrium, in "Review of Economic Studies, Vol. 3, 1936.
- GAREGNANI. A., Note su Consumi, Investimenti e Domanda Effettiva, in "Economia Internazionale", Nov. 1964.65.
- GEORGESCU-ROEGEN N., The Entropy Law and the Economic Process, Harvard Univ. Press, Cambridge Mass. (1971) 1976.
- GRANDMONT J.M., On the Short Run Equilibrium in Monetary Economy, in Drèze J. Ed., Allocations under Uncertainty Equilibrium and Optimality, MacMillan, London, 1971.
- GRANDMONT J.M. LAROQUE G., On Temporary Keynesian Equilibria, in "Review of Economic Studies", Feb. 1976.
- GRANDMONT J.M. YOUNES Y., On the Efficiency of Monetary equilibrium, in "Review of Economic Studies", April 1973.
- HAHN F., The Rate of Interest and General Equilibrium Analysis, in "Economic Journal", March 1955.
- HAHN F., On the Notion of Equilibrium in Economics, Cambridge Univ. Press, 1973.
- HANSSON B., *The Stockholm School and the Development of Dynamic Method*, manuscript for Ph.d. dissertation, Cambridge Univ. 1980.
- HARROD R., Towards a Dynamic Economics, MacMillan, 1948.
- HAWTREY J., Mr. Robertson on 'Saving and Hoarding', in "Economic Journal", December 1933.
- HAWTREY J., Alternative Theories of the Rate of Interest, in "Economic Journal", Sept. 1937.

HAYEK F., Prices and Production, G. Rouletge and Sons Ltd., London (1931), 1935.

- HAYEK F., The Pure Theory of Capital, (1941) Rouletge and Kegan Publ 1976.
- HICKS J., Mr Keynes and the 'Classics'; a Suggested Interpretation, in "Econometrica" 1937.
  - A Contribution to the Theory of the Trade Cycle, Oxford Univ. Press, 1950.
    - Capital and Growth, Oxford Univ. Press (1965) 1972.
    - The Crisis of Keynesian Economics, N.Y. Basic Books, 1974.

- Some Questions of Time in Economics, in "Evolution, Welfare, and Time in Economics: Essays in Honour of N. Georgescu-Roegen", A. Tang, F. Westfield and J. Worley Eds., Lexington Mass, 1976
- Causality in Economics, B. Blackwell Publ., London 1979.

KAHN F., The Relation of Home Investment to Unemployment, in "Economic Journal", June 1931.

- KEYNES J.M., Mr. Robertson and 'Saving and Hoarding, in "Economic Journal", December 1933.
  - The General Theory of Interest and Money, (1936) C.W. MacMillan, 1973.
  - The Collect Writings of J.M. Keynes, D. Moggridge Ed. Macmillan, London (Vol. XIV, XXIX).
  - Alternative Theories of the Rate of Interest, in "Economic Journal", June 1937 b.
  - The Ex-Ante Theory of the Rate of Interest, in "Economic Journal", 1937 c.
  - Mr. Keynes and Finance: Comment in "Economic Journal", June 1938.
- KLEIN L., Stock and Flow Analysis in Economics, in "Econometrica", July 1950.
- KNIGH F., Risk, Uncertainty and Profit (1921), Univ. of Chicago Press, 1971.
- KORNAI J., Anti-Equilibrium, N.Y. American Elsevier, 1971.
- KREGEL J., Economic Methodology in the Face of Uncertainty: the Modelling Method of Keynes and the Post-Keynesian, in "Economic Journal", June 1976.
- JEFFREYS H., Theory on Probability, MacMillan, London, 1967.
- JOHNSON H., Some Cambridge Controversies in Monetary Theory, in "Review of Economic Studies", 1951-1952.

LEIJONHUFVUD A., *Effective Demand Failures*, in "Swedish J. of Economics", March 1973.

- On Keynesian Economics and the Economics of Keynes, Oxford Univ. Press, N.Y. 1968.

LERNER A.P., Alternative Formulations of the Theory of Interest, in "Economic Journal", June 1938.

LINDAHL E., The Concept of Income, in "Economic Essays in Honor of G. Cassel", (1933).

- Studies in the Theory of Money and Capital, (1939), A.M. Kelley, Publ.. London. 1970
  - Sulla Teoria del Moltiplicatore, in "L'industria", 1953.
  - On Keynes's Economic System, in "Economic Record", May 1954, Nov. 1954.
  - · Introduction to Wicksell's Selected Papers, London 1958.
- LUNDBERG E., *Studies in the Theory of Economic Expansion*, (1937), B. Blackwell, Publ., 1955. MACHLUP F., *Selected Economic Writings of F. Machlup*, G. Bitros Ed., N.Y. Univ. Press, N.Y. 1976. MALINVAUD E., *Lectures on Microeconomic Theory*, North Holland, London 1972.
  - The Theory of Unemployment reconsidered, B. Blackwell, Pub., 1977.
- MASCI G. in F. CAFFE', Frammenti per lo Studio del Pensiero Economico Italiano, Giuffrè, 1975.
- METZLER L.A., Wealth Saving and the Rate of Interest, in "Journal of Political Economy", April 1953.

MILLGATE M., On the Origin of the Notion of 'Intemporal Equilibrium', in "Economica", Feb. 1979.

- MINSKY H., *Monetary Systems and Accelerator Models*, in "American Economic Review", Dec. 1957. John Maynard Keynes, Columbia Univ. Press, N.Y. 1975.
- MODIGLIAN F., Liquidity Preference and the Theory of Interest and Money, in "Econometrica", Jan. 1944.
- MYRDAL G., Monetary Equilibrium, (1939) A.M. Kelley, Publ., London 1965.
- NAGEL E., The Structure of Science, (1961), Rouletge and Kegan Publ., London, 1974.

NEISSER H., Ex-Ante and Ex-Post Propensity to Consume, in "Oxford Economic Paper", 1959.

- OHLIN B., On the Formulation of Monetary Theory, (E.T. 1933, in "History of Political Economy", March 1978.
  - Some Notes on the Stockholm Theory of Saving and Investment, part I in "Economic Journal", March 1937; part II in "Economic Journal", Sept. 1937 a.
    - Alternative Theories of the Rate of Interest, in "Economic Journal", Sept. 1937 b.
- PALANDER T., On the Concepts and Methods of the Stockholm School, in "International Economic Papers, n. 3 1953.
- PASINETTI L., Growth and Income Distribution, Cambridge Univ. Press, 1974.

PATINKIN D., *Liquidity Preference and Loanable Funds: Stock and Flow Analysis*, in "Economica", Nov. 1958.

- On the Relation between Keynesian Economics and the Stockholm School, in "Scandinavian J. of Ec.", 1978.
- Some Observations on Ohlin's 1933 Article, in "History of Political Economy", March 1978
  and Leiths Eds., *Keynes, Cambridge and the General Theory*, MacMillan, 1977.
- PIGOU A., *The Exchange Value of Legal Tender Money*, in "Essays in Applied Economics", F. Cass and Co. Ltd., London (1923) 1965 reprint.
- Keynes's General Theory. A Retrospective View, MacMillan, London, 1950.
- PLANK MAX, Theory of Heat, London, 1932.
- ROBERTSON D., Saving and Hoarding, in "Economic Journal", Sept. 1933.
  - Mr Robertson on 'Saving and Hoarding', in "Economic Journal", Dec. 1933.
  - Alternative Theories of the Rate of Interest, in "Economic Journal", Sept. 1937.
  - Mr. Keynes and Finance, in "Economic Journal", June 1938.
  - Essays in Monetary Theory, P. King and Son Ltd., London, 1940.
- ROBINSON J., The Concept of Hoarding, in "Economic Journal", June 1938.
  - Essays in the Theory of Economic Growth, MacMillan, London, 1964.
  - *History Versus Equilibrium*, Thames Papers in Political Economy, Thames Polytechnic, London, 1974 (CEP V).
  - What are the Questions?, in "Journal of Economic Literature", Dec. 1977 (CEP V).
- ROSENSTEIN- RODAN P., The Role of Time in Economics, in "Economica", Feb. 1934.

SCHUMPETER J., History of Economic Analysis, (1954.), Oxford Univ. Press N.Y., 1976:

- SHACKLE G., Twenty Years on: A Survey of the Theory of the Multiplier, in Economic Journal", 1951
  - Time in Economics, (1958), North Holland Publ. Amsterdam, 1967.
  - *The Years of the High Theory*, Cambridge Univ. Press, 1967 b.
  - Epistemics and Economics, Cambridge Univ. Press, 1972.

SHAW E., False Issues in the Interest Theory Controversy. in "Journal of Political Economy", Dec. 1938.

- SIMON H ., Causal Ordering and Identifiability, in "Studies in Econometric Method", Cowles Commission Monograph, n. 14, Hood W. – Coopmans T. Eds., J. Wiley and Sons Inc., N.Y., 1953. SOLOW R., A Contribution to the Theory of Economic Growth, in "Quarterly Journal of Economics", Feb. 1956.
- SRAFFA P., Production of Commodities by Means of Commodities, Cambridge Univ. Press, 1960.
- STEIGER O., B. Ohlin and the Origin of the Keynesian Revolution, in "History of Political Economy", March 1976.
  - Prelude to the Theory of a Monetary Economy: Origins and Significance of B. Ohlin's 1933 Approach, in "History of Political Economy", 1978.
- TERMINI V., Keynes on "finance": the 1937-1938 debate, paper presented at the London School of Economics (28th June 1978) mimeo.
- THOMAS B., Monetary Policy and Crises, Rouletge and Sons Ltd., London, 1936.
- TOWNSEND H., Liquidity Premium and the Theory of Value, in "Economic Journal", March 1937.
- TSIANG C., Liquidity Preference and Loanable Funds Theories: Multiplier and Velocity Analyses: A Synthesis, in "American Economic Review", Sept. 1956.
- UHR C., The *Emergence of the 'New Economics' in Sweden: a Review of a Study by O. Steiger*, in "History of Political Economy", 1973.
  - Economists and Policy Making 1930-193,: Sweden's Experience, in "History of Political Economy", Sept. 1977.
- VANDENBORRE H., An Integration of Employment Economics Within the Keynesian Theory of Money Flows, in "Oxford Economic Papers", 1958.
- VICARELLI F., Introduzione a: La Controversia Keynesiana, Il Mulino, 1974.
  - Keynes. L'instabilità del Capitalismo, Etas Libri, Milano, 1977.

WEINTRAUB E., Microfoundations, Cambridge Univ. Press, 1979.

WEINTRAUB S., *The Microfoundation of Aggregate Demand and Supply*, in "Economic Journal", March 1957.

WRIGHT A., The Genesis of the Multiplier, in "Oxford Economic Papers", n. 8, 1956.

- ZELLNER A, *Causality and Econometrics*, Brunner K. and Meltzer A., Eds. Carnegie Rochester Conference Series on Public Policy, Vol. 10, 1979.
- YOHE W., The Lesser Known Works of Professor Lindahl, in "Canadian J. of Economic and Political Sciences", 1962.
  - Ohlin's 1933 Reformulation of Monetary Theory, in "History of Political Economy, March 1978.