Economic Growth, Technical Progress, and Social Capital: the Inverted U Hypothesis

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Economic Growth, Technical Progress, and Social Capital: the Inverted U Hypothesis[1]

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Abstract

We set up a theoretical framework to analyze the possible role of economic growth and technical progress in the erosion of social capital. Under certain parameters, the relationship between technical progress and social capital can take the shape of an inverted U curve. We show the circumstances allowing the economy to follow trajectories where the stock of social capital grows endogenously and unboundedly.

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1. Introduction

In his best-seller *Bowling Alone*, Robert Putnam (2000) documents how most indicators of social capital have followed an inverted U path in the United States during the twentieth century: during the first two-thirds of the century, "Americans took a more and more active role in the social and political life of their communities in churches and union halls, in bowling alleys and clubrooms, around committee tables and card tables and dinner tables", and they behaved in an increasingly trustworthy way toward one another (2000, p. 183). Then, beginning in the 1960s and 1970s and accelerating in the 1980s and 1990s, an inexorable erosion of the stock of American social capital started. Apparently, year by year Americans became less generous and trustworthy, less engaged in community problems and less inclined to meet their friends, neighbours, and acquaintances.

The author discusses three main explanations for this inverted-U trend: pressures on time and money, mobility and sprawl, and technology and mass media. Pressures on time and money, as well as mobility and sprawl, have been considered in the economic and sociological literature as channels through which economic growth can bring about a side effect of reduced social connectedness (Wellman 2001, Routledge and von Amsberg 2003, Hooghe 2003, Glanville 2004, van Ingen and Dekker 2011). An early account of the possible mechanism is given by Fred Hirsch (1976) in his book *Social Limits to Growth*: "As the subjective cost of time rises, pressure for specific balancing of personal advantage in social relationships will increase ... Perception of the time spent in social relationships as a cost is itself a product of privatized affluence. The effect is to whittle down the amount of friendship and social contact ... The huge increase in personal mobility in modern economies adds to the problem by making sociability more of a public and less of a private good. The more people move, the lower are the chances of social contacts being reciprocated directly on a bilateral basis" (p.80). However, as Putnam (2000) states, it is difficult to find evidence of an association between economic growth, pressure on time and money, and the decline in civic and social engagement experienced in the U.S. The author then points to technological progress as a possible reason for the erosion of the American social capital. Technology has in fact made news and entertainment increasingly individualized. In Putnam's words, people do not have to coordinate their "tastes and timing with others in order to enjoy the rarest culture or the most esoteric information … Electronic technology allows us to consume hand-tailored entertainment in private, even utterly alone" (2000, pp. 216-217). Putnam notes that it was throughout the 1970s—just as civic and social disengagement was gathering steam in the United States—that the time allocation of Americans "massively shifted toward home-based activities (especially watching TV) and away from socializing outside the home" (2000, p. 238). The author argues that the rise of electronic communications and entertainment has led agents to spend more and more time and money on goods and services consumed individually, rather than those consumed collectively.

In this paper, we develop a dynamic model integrating Putnam's and Hirsch's hypotheses, as well as other promising hints from the sociological and the economic empirical literature on social capital. The main objective of the analysis is to better understand how economic growth and technological progress may influence the accumulation of social capital.

We assume that the well-being of individuals depends on two kinds of goods: material and relational. Relational goods are a distinctive type of good that can only be enjoyed if shared with others. They are different from private goods, which are enjoyed alone (Uhlaner 1989). A peculiarity of relational goods is that it is virtually impossible to separate their production from consumption, since they coincide (Gui and Sugden 2005). For example, a football match with friends is enjoyed (consumed) in the very moment of its production (i.e. the 90 minutes spent on the sports field). Agents can choose how to allocate their time between the two types of production (of private vs. relational goods). Following Coleman (1988, 1990), we assume that social participation (i.e. the production/consumption of relational goods) incidentally generates social capital as a by-product. Social capital is here defined as the sum of durable ties that agents develop through their social participation. Following hints from political science (see for example Hochschild 1997), we assume that on-the-job interactions can stimulate the creation of interpersonal ties as well.

As sustained by Coleman (1988), the stock of social capital functions as a public good, which enters as an argument in the agents' utility functions and as an input in the production of both private and relational goods. In this way, we intend to model one of the most debated claims emerging from the empirical literature: the positive role of social capital in material production (Knack and Keefer 1997, Paldam and Svendsen 2000, Peri 2004, Beugelsdijk and van Schaik 2005, Akçomak and ter Weel 2009, Bjørnskov 2010).

Finally, we assume that private and relational goods can substitute for each other in the satisfaction of individual needs. For example, when the social environment is poor, people may be constrained to replace human interactions, such as playing football with friends, with private consumption, such as staying at home and watching a TV show or playing a virtual match against the computer. However, relational goods cannot satisfy primary needs such as food, security, clothing, and shelter.

The analysis of agents' time allocation choices accounts for the alternative cases of zero or positive Edgeworth substitutability between the two types of good. Under the more realistic assumption of positive Edgeworth substitutability, an increase in the stock of social capital can enhance the productivity of time spent on social interaction, thereby encouraging the consumption of relational goods. This process results in a positive level of participation which in turn creates and strengthens durable ties, further increasing the stock of social capital.

The analysis of dynamics shows that the economy may be attracted by alternative steady states, depending on the initial wealth of social capital and on exogenous parameters representing the importance of relational goods in well-being and production. The possibility exists for the economy to fall in a "social poverty trap", where agents devote all their time to private activities.

The introduction of exogenous technical progress in the production function of private goods causes interesting changes in social capital's accumulation dynamics. If there is no Edgeworth substitutability between private and relational goods, then the stock of social capital can grow indefinitely along any trajectories or alternatively trend to zero depending on the value of the model's parameters. Its trend relative to technological progress can be monotonic (always increasing or decreasing) or can experience an initial decline followed by growth, but not vice versa (growth followed by a decline is impossible). In case of positive Edgeworth substitutability, social capital may experience growth followed by a decline, so that its relationship with technological progress is described by an inverted-U shaped curve similar to the trends reported in Putnam's *Bowling Alone*. Along this curve, private activities infinitely expand at the expense of social interaction, thereby trapping the economy in a situation of social poverty. However, also in the context of substitutability, if social capital's initial endowments are high enough, the economy can follow a growth path along which both technological progress and social capital grow indefinitely.

The outline of the paper is as follows. Sections two and three present the model and analyze its dynamics. Section four studies the effect of exogenous technical progress on social capital's dynamics. The paper ends with a brief discussion of main results.

2. The model

We consider a population of size 1 constituted by a continuum of individuals. We assume that, in each instant of time t, the well-being of the individual $i \in [0,1]$ depends on the consumption of two goods: a private material good, $C_i(t)$, and a socially provided good, $B_i(t)$. We assume that $B_i(t)$ is produced through the joint action of the time devoted by agent i to social activities, $s_i(t)$, the average social participation $\bar{s}(t) = \int_0^1 s_i(t)di$, and the stock of social capital $K_s(t)$:

$$B_i(t) = F(s_i(t), \bar{s}(t), K_s(t)) \qquad (1)$$

In other words, since relational goods can be enjoyed only if shared with others, their production process depends on others' social participation and on the stock of networks existing in the surrounding environment. As outlined in the introduction, production and consumption of relational goods basically coincide.

The time agent i does not spend for social participation, $1-s_i(t)$, is used as input in the production of the output $Y_i(t)$ of the private good. As suggested by Antoci, Sacco and Vanin (2005, 2008) and Antoci, Sabatini and Sodini (2011), we assume that social capital also plays a role in the production process of the private good. In this way, we model the claim raised by many empirical studies that socio-cultural traits—often grouped together under the common label of social capital—are a factor enhancing the production of material goods (Knack and Keefer 1997, Paldam and Svendsen 2000, Peri 2004, Beugelsdijk and van Schaik 2005, Akçomak and ter Weel 2009, Bjørnskov 2010). In addition, for simplicity, we assume that $C_i(t) = Y_i(t)$, that is $Y_i(t)$ cannot be accumulated, and that the production process of $Y_i(t)$ requires only the inputs $1-s_i(t)$ and $K_s(t)$:

$$C_i(t) = Y_i(t) = G(1-s_i(t), K_s(t)) \qquad (2)$$

The functions F and G in (1) and (2) are assumed to be strictly increasing in each argument. Note that, in this context, $1-s_i(t)$ can be interpreted as the time spent both *to produce* and *to consume* $C_i(t)$.

The accumulation of social capital is path-dependent: first, it improves the technology of the production of relational goods; second, greater social participation taking the form of higher levels of relational goods' production and consumption fosters the consolidation of ties and trust among people, thereby increasing the stock of social capital as a by-product. Of course, we cannot exclude the possibility that agents engage in social activities for instrumental purposes (for example, to achieve a better job). However, following hints from rational choice sociology (Coleman, 1990), we assume that most of the time the creation of interpersonal ties does not depend on rational investment decisions. Thus, as in Antoci, Sacco and Vanin (2005, 2007, 2008), social capital is accumulated as a by-product of social participation.

Moreover, we assume that the process of private goods production exerts a positive spillover on social capital's accumulation. On-the-job interactions can in fact stimulate the creation of durable ties among workers. Friendships often start in the workplace, both spontaneously and as a result of precise human resources management strategies. In political science, several schools of thought claim that citizens can develop their relational and political attitudes at the workplace. Putnam stresses how "professional and blue-collars workers alike are putting in long hours together, eating lunch together, travelling together, arriving early and staying late ... Work is where the heart is for many solitary souls" (2000, p. 86). Even for the minority who live with a spouse and children, argues Hochschild (1997), the workplace increasingly serves as a sanctuary from the stresses of marriage, children and housework. Work structures are a generator of face-to-face interactions that stimulate the sharing of social norms and the creation of interpersonal ties (Goul Andersen and Hoff 2001; Karasek 1976; Peterson 1992: Schur 2003; Smith 1985). The workplace can thus be considered as a training ground where people may improve those communication and organizational abilities which are crucial for the production and consumption of relational goods. In other words, such skills can raise the productivity of time spent on social participation[5].

Finally, since human relations need care to be preserved, we introduce a positive social capital's depreciation rate to account for their possible cooling over time:

$$\dot{K}_s(t) = H\left[\overline{B}(t), \overline{Y}(t)\right] - \eta K_s(t) \qquad (3)$$

where $\dot{K}_s(t)$ indicates the time derivative of $K_s(t)$, the parameter $\eta > 0$ is the depreciation rate of $K_s(t)$, $\overline{B}(t) = \int_0^1 B_i(t)di$ and $\overline{Y}(t) = \overline{C}(t) = \int_0^1 Y_i(t)di$ are the average production/consumption of the socially provided good and the average production/consumption of the private good, respectively. The resulting stock is a public resource, which enters as an argument in every agent's utility function due to its ability to contribute to the production of both private and relational goods.

For simplicity, we consider the following specifications for (1),(2),(3):

$$Y_i(t) = \left[1 - s_i(t)\right] \cdot K_s^{\alpha}(t)$$

$$B_i(t) = \bar{s}^{\varepsilon}(t) \cdot s_i^{1-\varepsilon}(t) \cdot K_s^{\gamma}(t) \qquad (4)$$

$$\dot{K}_s(t) = \left[\overline{Y}(t)\right]^{\beta} \cdot \left[\overline{B}(t)\right]^{\delta} - \eta K_s(t)$$

where $\varepsilon \in (0,1)$ is the productivity of time spent on social interaction in the individuals' production process of relational goods, and $\alpha, \beta, \gamma, \delta > 0$.

Note that a positive average social participation $\bar{s}(t) > 0$ is essential for the production/consumption of $B_i(t)$, that is $B_i(t) = 0$ if $\bar{s}(t) = 0$ whatever the values of $s_i(t)$ and $K_s(t)$ are. If no one participates, single agents have no possibility to enjoy relational goods, even in the presence of a positive stock of social capital. If $\gamma > \alpha$, then the role of social capital is more relevant in the production/consumption of relational goods than in the production/consumption of private goods.

According to (4), $\overline{B}(t)$ and $\overline{Y}(t)$ are both essential factors for the accumulation of social capital, that is the stock of social capital $K_s(t)$ decreases (that is $\dot{K}_s(t) < 0$) if $\overline{B}(t) = 0$ or $\overline{Y}(t) = 0$.

Finally, we assume that the instantaneous utility function of individual i is:

$$U_i\left[C_i(t), P_i(t)\right] = \ln C_i(t) + b \ln P_i(t) \qquad (5)$$

where $P_i(t)$ represents the whole of social needs, $C_i(t)$ are the agents' private needs, and $b > 0$ measures the relative importance of social needs in respect to private ones. We assume that private goods can satisfy both private and social needs. If the surrounding environment is socially poor, agents may choose to replace human interactions with private consumption (e.g. they may play a virtual match against the computer instead of meeting friends on a sports field, or chat with unknown and distant

[5] Those who possess well-developed relational skills are likely to find social and political participation less daunting and costly (Brady et al. 1995; Burn and Konrad 1987; Elden 1981; Greenberg 1986; Verba et al. 1995). Some authors claim that on the job interactions foster the development of democratic attitudes (Paterman 1970; Verba et al. 1995) and active political participation (Adman 2008; Greenberg et al. 1996; Mutz and Mondak 2006; Sobel 1993) as well.

people through the web instead of talking with neighbours). By contrast, relational goods cannot satisfy primary needs such as food, security, clothing and shelter. A useful way to describe this issue is to consider the following linear specification:

$$P_i(t) = B_i(t) + d \cdot C_i(t) \qquad (6)$$

where the parameter d measures the degree of Edgeworth substitutability between $B_i(t)$ and $C_i(t)$ with respect to the production of $P_i(t)$. If $d = 0$, then there is no Edgeworth substitutability between the two goods. Note that if $d > 0$, the mixed partial derivative of U_i with respect to $C_i(t)$ and $B_i(t)$ is strictly negative:

$$\frac{\partial^2 U_i}{\partial C_i \partial B_i} = -\frac{bd}{(dC_i + B_i)^2} < 0$$

This means that the lower the value of $B_i(t)$ is, the greater the marginal utility of private consumption $C_i(t)$ will be. As argued by Antoci, Sacco and Vanin (2007), it is more rewarding to interact with people in a context that offers many options for socially enjoyed leisure. On the contrary, if the social environment is poor, and people have few chances to meet and enjoy relational goods, private consumption is more rewarding. For brevity's sake, from now the term "substitutability" will mean "Edgeworth substitutability".
Letting r be the discounting rate of future utility, the $i-$agent's maximization problem is:

$$\max_{s_i(t)} \int_0^{+\infty} \{\ln C_i(t) + b \ln P_i(t)\} e^{-rt} dt \qquad (7)$$

subject to the dynamic constraint (4). The agent i solves problem (7) taking as exogenously given the value of $K_s(t)$ and the average values $\bar{s}(t)$, $\bar{B}(t)$ and $\bar{Y}(t)$ because the choice of $s_i(t)$ by agent i does not modify the average values, being economic agents a continuum. As a consequence, by applying the *Maximum Principle* to problem (6) we obtain that the choices of individual i do not depend on the co-state variable associated to $K_s(t)$ (that is the "price" of $K_s(t)$) in the maximization problem (7). Consequently, to solve problem (7), agent i chooses in each instant t the value of $s_i(t)$ maximizing the value of the instantaneous utility function (5). This implies that the dynamics of $K_s(t)$ we study do not represent the social optimum. However, since agent i plays the best response $s_i(t)$, given the others' choices, the trajectories followed by $K_s(t)$ represent Nash equilibria. In fact, along these trajectories, no agent has incentive to modify his choices if the other agents do not revise theirs as well.
To simplify our analysis, in this paper we focus on *symmetric Nash equilibria*. In particular, we assume that individuals are identical and make the same choices. This assumption allows us to study the choices of a *representative agent*. Thus we can omit the subscript i in the variables $s_i(t)$, $B_i(t)$, $Y_i(t)$ and $C_i(t)$ writing simply $s(t)$, $B(t)$, $Y(t)$ and $C(t)$. In this symmetric Nash equilibrium context, we have that ex ante average values $\bar{s}(t)$, $\bar{B}(t)$ and $\bar{Y}(t)$ are considered as exogenously given by the representative agent. However, once $s(t)$ is chosen, ex post it holds:

$$\bar{s}(t) = s(t)$$

$$\bar{B}(t) = \bar{s}^{\varepsilon}(t) \cdot \bar{s}^{1-\varepsilon}(t) \cdot K_s^{\gamma}(t) = \bar{s}(t) \cdot K_s^{\gamma}(t) = s(t) \cdot K_s^{\gamma}(t)$$

$$\bar{Y}(t) = \left[1 - \bar{s}(t)\right] \cdot K_s^{\alpha}(t) = \left[1 - s(t)\right] \cdot K_s^{\alpha}(t)$$

In this context, the representative agent, in each instant of time t, choses $s(t)$ solving the following static optimization problem:

$$\max_s \left\{ \ln\left[(1-s) \cdot K_s^{\alpha}\right] + b \ln\left[s^{\varepsilon} \bar{s}^{1-\varepsilon} K_s^{\gamma} + d \cdot (1-s) \cdot K_s^{\alpha}\right] \right\} \tag{8}$$

taking as exogenously given the values of $\bar{s}$ and K_s. The solution $s(t)$ of the problem (8) has to be substituted to $\bar{s}(t)$ in the equation (4) which, under our symmetric Nash equilibria assumption, can be written as follows:

$$\begin{aligned} \dot{K}_s(t) &= \left[\bar{Y}(t)\right]^{\beta} \cdot \left[\bar{B}(t)\right]^{\delta} - \eta K_s(t) = \\ &= \left[\left[1 - \bar{s}(t)\right] \cdot K_s^{\alpha}(t)\right]^{\beta} \cdot \left[\bar{s}^{\varepsilon}(t) \cdot \bar{s}^{1-\varepsilon}(t) \cdot K_s^{\gamma}(t)\right]^{\delta} - \eta K_s(t) = \\ &= \left[1 - \bar{s}(t)\right]^{\beta} \left[\bar{s}(t)\right]^{\delta} \cdot K_s^{\alpha\beta+\gamma\delta}(t) - \eta K_s(t) \end{aligned} \tag{9}$$

where β and δ are strictly positive parameters. Note that under dynamics (9), social capital accumulation is negative if $\bar{s}(t) = 0$ (no social participation) or if $\bar{s}(t) = 1$ (the production/consumption of the private good is equal to zero). As noted above, $\bar{B}(t)$ and $\bar{Y}(t)$ are in fact both essential factors for the accumulation of social capital. Thus, even if people devote all their time to social participation, the stock of social capital is doomed to erosion if there is no private production so that primary needs cannot be satisfied.

According to (9), the value of $\bar{s}(t)$ which, given $K_s(t)$, maximizes the rate of growth of $K_s(t)$ is:

$$\bar{s}(t) = s^g := \frac{\delta}{\beta + \delta}$$

The latest expression can be interpreted as the "golden rule" for the accumulation of social capital. Note that $s^g \to 0$ if $\delta \to 0$, and $s^g \to 1$ when the value of β is negligible in relation to that of δ.

3. Analysis of the model

3.1 The time allocation choice

For simplicity, we limit our analysis to robust cases only, that is those not corresponding to equality conditions on parameters' values. The following result concerns the choice of $s(t)$ by the representative agent (due to space constraints, propositions' proofs are omitted if straightforward).

Lemma 1 *Problem (8) admits solution and the time allocation choice $s^*(t)$ of the representative agent* *is:*

1) if $\gamma - \alpha > 0$

$$s^*(t) = \begin{cases} 0, & \text{if } K_s(t) \le \left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}} \\ \frac{b\varepsilon K_s^{\gamma-\alpha}(t) - d(b+1)}{(1+b\varepsilon)K_s^{\gamma-\alpha}(t) - d(b+1)}, & \text{if } K_s(t) > \left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}} \end{cases} \quad (10)$$

2) if $\gamma - \alpha < 0$

$$s^*(t) = \begin{cases} \frac{b\varepsilon K_s^{\gamma-\alpha}(t) - d(b+1)}{(1+b\varepsilon)K_s^{\gamma-\alpha}(t) - d(b+1)}, & \text{if } K_s(t) \le \left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}} \\ 0, & \text{if } K_s(t) > \left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}} \end{cases} \quad (11)$$

Social capital produces contrasting pressures on the representative agent's time allocation choices. Remember that γ and α are the exponents of K_s in the production functions of $B(t)$ and $Y(t)$, respectively, $b > 0$ represents the weight of social needs, and d measures the degree of Edgeworth substitutability between $B_i(t)$ and $C_i(t)$ in the satisfaction of social needs $P_i(t)$.

If $\gamma - \alpha > 0$ then (ceteris paribus) an increase in the stock of social capital K_s has the effect of raising the productivity of time spent on social participation $s(t)$ relative to that of time spent on the production and consumption of the private good. In this case, if the stock of social capital is "high" enough, then social participation will be positive ($s(t) > 0$). If social capital's initial endowments are "low", then agents will devote all their time to private production: despite the improvement in the productivity of time devoted to social participation, the agents' allocation choice will be guided by the necessity of meeting private needs $C_i(t)$ first.

If $\gamma - \alpha < 0$, an increase in the stock of social capital K_s raises the productivity of time spent on private production more than the productivity of time devoted to social participation. Then, if the initial stock of social capital is "high", agents will prefer to exploit its eventual increases to raise the production/consumption of private goods, and we will have $s(t) = 0$. Otherwise, if the stock of social capital is low, individuals will devote time to social interaction leading to a positive level of participation.

An implication of this result is that, according to equation (4), the stock of social capital cannot grow indefinitely when $\gamma - \alpha < 0$. In this case, an increase in the stock of social capital raises the productivity of time spent on private production, therefore leading to a restriction of social participation, which in turn hampers the accumulation of social capital in the long run. By contrast, when $\gamma - \alpha > 0$, an increase in the stock of social capital can trigger a self-feeding process resulting in an increase in the production/consumption of relational goods and the formation and of new ties.

If there is no substitutability between private consumption C and the socially provided good B, that is $d = 0$ in (5), it holds $d(b+1) = 0$; therefore $\left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}} = 0$ if $\gamma - \alpha > 0$. In this context, by (10)-(11), the following proposition holds.

Proposition 2 *Under the assumption* $d = 0$*, problem (8) gives the following time allocation choice* $s^*(t)$ *of the representative agent:*

$s^*_{d=0}(t) = \dfrac{b\varepsilon}{1+b\varepsilon}$, whatever the value of $K_s(t)$ is;

When the two goods are not Edgeworth substitutes in the satisfaction of social needs, the reduction in the production/consumption of relational goods is not accompanied by an increase in the marginal utility of private goods. For example, the reduction of opportunities to go to the cinema with friends will not raise the marginal utility of devices to watch movies alone. In this case, private goods will not replace socially provided goods for the satisfaction of social needs, and participation will be constant, such that $1 > s^*(t) > 0$, whatever the value of $K_s(t)$ is.

Notice that social participation $s^*(t)$ in (10)-(11) is (ceteris paribus) a strictly decreasing function of the parameter d, which measures the degree of substitutability between B and C. Therefore, it always holds $s^*(t) < s^*_{d=0}(t)$. If agents admit the possibility to satisfy social needs (or to compensate for their deprivation) through the production/consumption of private goods (e.g. a playstation is considered as a perfect substitute of a match played on the tennis field), then the level of social participation will be lower, whatever the value of $K_s(t)$ is.

3.2 Dynamics of social capital accumulation and well-being analysis

Even if we have considered very simple specifications of functions F, G and H, there may exist multiple steady states and poverty traps.

The following result concerns the evolution of representative agents' well-being along the trajectories under dynamics (9).

Proposition 3. *Along the trajectories of (9), the values of the utility function U and of K_s are positively correlated. This implies that if there exist two steady states K_s^1 and K_s^2 such that $K_s^2 > K_s^1$, then K_s^2 Pareto-dominates K_s^1; that is K_s^1 is a poverty trap.*

The following proposition defines social capital dynamics resulting from the time allocation choices of the representative agent described in Proposition (1).

Proposition 4. *If $\gamma - \alpha > 0$, social capital dynamics are given by:*

$$\dot{K}_s = \begin{cases} -\eta K_s \\ \left[\frac{K_s^{\gamma-\alpha}}{(1+b\varepsilon)K_s^{\gamma-\alpha} - d(b+1)}\right]^\beta \cdot \left[\frac{b\varepsilon K_s^{\gamma-\alpha} - d(b+1)}{(1+b\varepsilon)K_s^{\gamma-\alpha} - d(b+1)}\right]^\delta \cdot K_s^{\alpha\beta+\gamma\delta} - \eta K_s \end{cases} \tag{12}$$

for, respectively, $K_s \leq \left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}}$ and $K_s > \left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}}$.

If $\gamma - \alpha < 0$, they are given by:

$$\dot{K}_s = \begin{cases} \left[\frac{K_s^{\gamma-\alpha}}{(1+b\varepsilon)K_s^{\gamma-\alpha} - d(b+1)}\right]^\beta \cdot \left[\frac{b\varepsilon K_s^{\gamma-\alpha} - d(b+1)}{(1+b\varepsilon)K_s^{\gamma-\alpha} - d(b+1)}\right]^\delta \cdot K_s^{\alpha\beta+\gamma\delta} - \eta K_s \\ -\eta K_s \end{cases} \tag{13}$$

for, respectively, $K_s \leq \left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}}$ and $K_s > \left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}}$.

Notice that if $d = 0$ (that is, if relational and private goods are not substitutes), the dynamics of social capital accumulation become:

$$\dot{K}_s = \frac{(b\varepsilon)^\delta}{(1+b\varepsilon)^{\beta+\delta}} \cdot K_s^{\alpha\beta+\gamma\delta} - \eta K_s \tag{14}$$

and the corresponding dynamic regimes are described by the following proposition.

Proposition 5. *Under the assumption of no Edgworth substitutability ($d = 0$), the basic features of dynamics are the following (whatever the sign of the expression $\gamma - \alpha$ is):*

a) If $\alpha\beta + \gamma\delta < 1$, there exist two steady states:

$$\overline{K}_s = \left[\frac{\eta(1+b\varepsilon)^{\beta+\delta}}{(b\varepsilon)^{\delta}} \right]^{\frac{1}{\alpha\beta+\gamma\delta-1}} \text{ and } K_s = 0$$

The economy approaches $\overline{K}_s$ (the steady state $K_s = 0$ is repulsive) whatever the initial value of $K_s > 0$ is.

b) If $\alpha\beta + \gamma\delta = 1$, then the steady state $K_s = 0$ is the unique steady state. In particular,

b.1) if $\frac{(b\varepsilon)^{\delta}}{(1+b\varepsilon)^{\beta+\delta}} < \eta$, the economy reaches the steady state $K_s = 0$;

b.2) if $\frac{(b\varepsilon)^{\delta}}{(1+b\varepsilon)^{\beta+\delta}} > \eta$, the economy follows a trajectory along which the value of K_s grows indefinitely (that is $K_s \to +\infty$) at the constant rate $\frac{(b\varepsilon)^{\delta}}{(1+b\varepsilon)^{\beta+\delta}} - \eta$.

If private goods cannot substitute relational ones in the satisfaction of social needs, then the economy can follow a virtuous trajectory where the stock of social capital unboundedly grows, raising social participation and consolidating interpersonal ties. When there is some degree of substitutability between C and B, that is $d > 0$, dynamics become more complicated and are characterized by the following propositions.

Proposition 6. *Under the assumption of substitutability $d > 0$ and if $\gamma - \alpha > 0$, the dynamics are characterized by the following properties:*

1) *The steady state $K_s = 0$ is always locally attractive (whatever the value of the expression $\alpha\beta + \gamma\delta$ is).*
2) *If $\alpha\beta + \gamma\delta < 1$ then the number of steady states with $K_s > 0$ is (generically) zero (see Figure 1) or two (see Figure 2); if two steady states K_s^1 and K_s^2 ($K_s^1 < K_s^2$) exist, then K_s^2 is attractive while K_s^1 is repulsive.*
3) *If $\alpha\beta + \gamma\delta = 1$ then there exists at least a steady state with $K_s > 0$; furthermore, the number of steady states with $K_s > 0$ is one (see Figure 3) or three; if an unique steady state exists, then it is repulsive; if three steady states K_s^1 , K_s^2 and K_s^3 ($K_s^1 < K_s^2 < K_s^3$) exist, then K_s^2 is attractive while K_s^1 and K_s^3 are repulsive. Finally, if $\frac{(b\varepsilon)^{\delta}}{(1+b\varepsilon)^{\beta+\delta}} > \eta$ and the initial value of K_s is greater than K_s^*, where K_s^* is the steady state with the highest value of K_s, then there exists an unbounded endogenous growth path with increasing well-being along which $s \to \frac{b\varepsilon}{1+b\varepsilon}$.*

If agents tend to replace relational goods with private ones for the satisfaction of social needs (or to compensate for the deprivation of human interactions), and if an increase in the stock of social capital K_s has the effect of raising the productivity of time spent on social participation $s(t)$ relative to that of

time spent on the production and consumption of the private good, then a steady state where $K_s = 0$ is always locally attractive: an erosion of the entire stock of social capital is possible.
Under the assumption of substitutability ($d > 0$) and if $\gamma - \alpha < 0$, the dynamics are characterized by the following properties:

Proposition 7.

1) *The value of* K_s *always approaches a steady state value lower than the upper bound* $\left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}}$ *(see proposition 4), whatever the value of the expression* $\alpha\beta + \gamma\delta$ *is. Consequently, the stock of social capital* K_s *cannot grow indefinitely.*
2) *If* $\alpha\beta + \gamma\delta < 1$*, then the steady state* $K_s = 0$ *is always repulsive; there exists at least a steady state with* $K_s > 0$*; the number of steady states with* $K_s > 0$ *is (generically) one (see Figure 4) or three; steady states with an odd index are attractive and those with an even index are repulsive. Whatever the initial value of* K_s *is, the economy approaches a steady state with* $K_s > 0$.
3) *If* $\alpha\beta + \gamma\delta = 1$*, then the steady state* $K_s = 0$ *is always locally attractive; the number of steady states with* $K_s > 0$ *is zero (see Figure 5) or two (see Figure 6); the steady states with an odd index are repulsive and those with an even index are attractive*[6].

High initial endowments are a necessary but insufficient condition for an endogenous and unbounded growth of the stock of social capital. Necessary and sufficient conditions are as follows:

$$\gamma - \alpha > 0$$
$$\frac{(b\varepsilon)^{\delta}}{(1+b\varepsilon)^{\beta+\delta}} > \eta$$
$$\alpha\beta + \gamma\delta = 1$$

When these conditions hold, the poverty trap $K_s = 0$ is always locally attractive. However, when the initial value of K_s is high enough, the economy follows a trajectory along which $K_s \to +\infty$. As stated above, along this trajectory we have that:

$$s \to \frac{b\varepsilon}{1+b\varepsilon}$$

and the equation (12) tends (for $K_s \to +\infty$) to the equation:

[6] Values of parameters in Figure 4: $\alpha = 0.9$, $\beta = 0.5$, $\gamma = 0.4$, $\delta = 1$, $\varepsilon = 0.7$, $\eta = 0.04$, $b = 4$, $d = 0.05$; Figure 5: $\alpha = 0.9$, $\beta = 0.8$, $\gamma = 0.4$, $\delta = 1$, $\varepsilon = 0.7$, $\eta = 0.24$, $b = 4$, $d = 0.05$; Figure 6: $\alpha = 0.9$, $\beta = 0.8$, $\gamma = 0.38$, $\delta = 1$, $\varepsilon = 0.5$, $\eta = 0.12$, $b = 4$, $d = 0.4$. For $\alpha = 0.9$, $\beta = 0.5$, $\delta = 0.55$, $\gamma = 1$, $\varepsilon = 0.5$, $\eta = 0.11$, $b = 4$, $d = 0.05$ we obtain three positive fixed points of coordinates $K_s^* = 1.27$ (attractive), $K_s^{**} = 1.41$ (repulsive), $K_s^{***} = 1.52$ (attractive).

$$\dot{K}_s = \frac{(b\varepsilon)^{\delta}}{(1+b\varepsilon)^{\beta+\delta}} \cdot K_s - \eta K_s$$

which coincides with the equation (14) describing social capital dynamics under the assumption $d = 0$ (no substitutability between C and B). This allows us to say that when the stock of social capital becomes "high enough", then the dynamics under the assumption $d = 0$ and those under the assumption $d > 0$ become very similar. This result is more likely if b and $\varepsilon \in (0,1)$ exhibit high values and if the social capital's depreciation rate η is low. As pointed out above, b measures the weight of social needs in determining agents' satisfaction. A high value of b indicates that agents are not so prone to sacrifice their relational sphere for private needs. Such parameters may be determined by cultural factors acknowledging the importance of non-market relations in respect to material consumption. For example, a culture exalting the prominence of cooperation and solidarity in social life is a good starting point. ε represents the productivity of social participation in the individual production of relational goods. A high value of this parameter indicates that agents are more capable of influencing their relational sphere with their own efforts, independent of others' current levels of social participation and of the stock of social capital. This ability may be due to human and social factors. As regards human factors, it is remarkable that people have diverse attitudes towards social interactions. Charismatic agents may be able to carry away other people in interpersonal relationships even if the surrounding environment is not particularly rich of social participation opportunities. It is noteworthy that sociological studies claim that charismatic agents behaving as leaders in social networks may act as catalysts for the creation of social capital (Burt 1999; Renshon 2000; Roch 2005). However, aggregative behaviours need a certain degree of generalized trust to take place. In a socially poor environment, agents may use their relational skills to produce relational goods, and to carry away other people in social participation, only if there is a reasonable likelihood that their effort will not be wasted and will be repaid. The diffusion of social norms of reciprocity is thus a crucial precondition for these aggregative and pro-social behaviours. It is noteworthy that, starting from Coleman (1987, 1988), part of the literature considers norms of trust and reciprocity as integral parts of the definition of social capital. Following such an approach, it is possible to argue that the parameter ε incorporates human and social factors whose development in the long run is influenced by the stock of social capital.
The following six figures show Solow-like graphs where the intersection points between $H\left[\overline{B}(t), \overline{Y}(t)\right]$ and $\eta K_s(t)$ are fixed points of the differential equation. The black dot is used for attractive stationary points, the grey dot for repulsive stationary points[7].

[7] Values of simulations in Figure 1: $\alpha = 0.01$, $\beta = 0.4$, $\gamma = 0.82$, $\delta = 1$, $\varepsilon = 0.12$, $\eta = 0.15$, $b = 4$, $d = 0.03$; Figure 2: $\alpha = 0.01$, $\beta = 0.4$, $\gamma = 0.82$, $\delta = 1$, $\varepsilon = 0.12$, $\eta = 0.04$, $b = 4$, $d = 0.03$; Figure 3: $\alpha = 0.4$, $\beta = 0.9$, $\gamma = 0.64$, $\delta = 1$, $\varepsilon = 0.7$, $\eta = 0.3$, $b = 4$, $d = 0.3$. For $\alpha = 0.9$, $\beta = 0.5$, $\delta = 0.55$, $\gamma = 1$, $\varepsilon = 0.5$, $\eta = 0.11$, $b = 4$, $d = 0.05$ we obtain three positive fixed point of coordinates $K_s^{*} = 0.005$ (repulsive), $K_s^{**} = 0.02$ (attractive), $K_s^{***} = 93$ (repulsive).

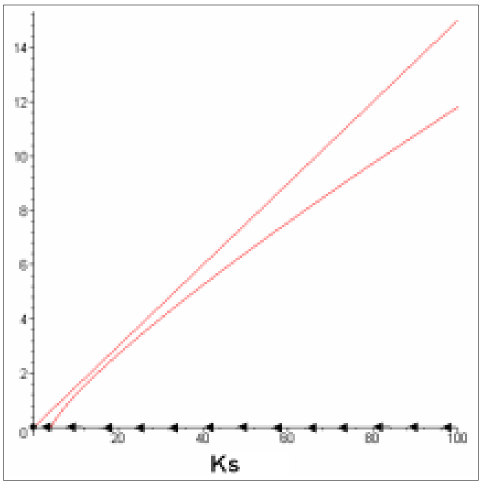

Figure 1

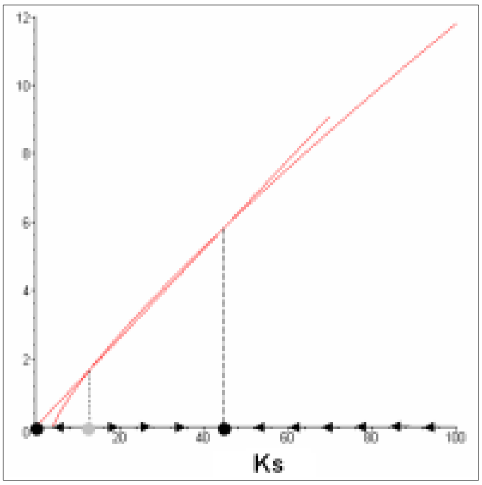

Figure 2

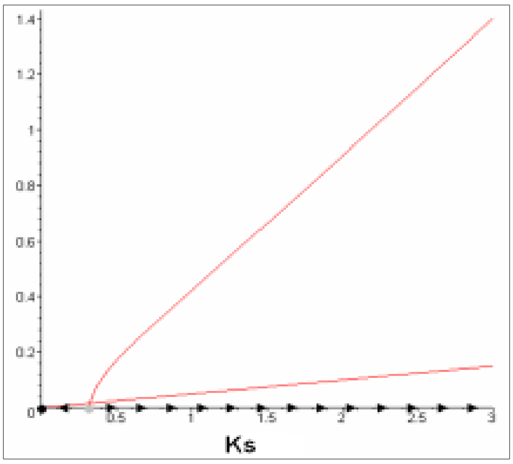

Figure 3

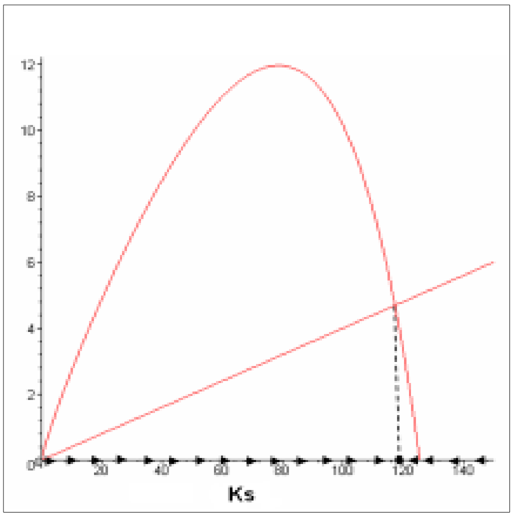

Figure 4

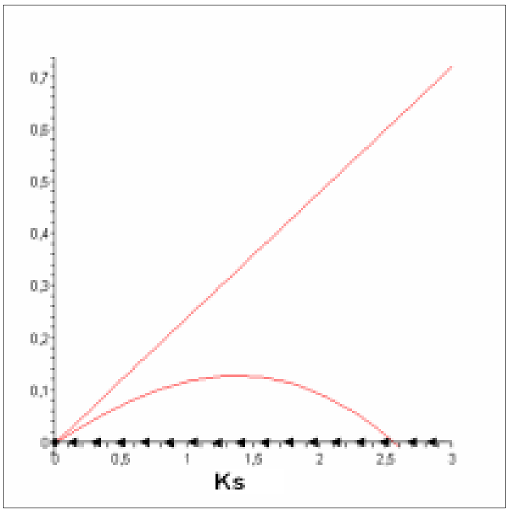

Figure 5

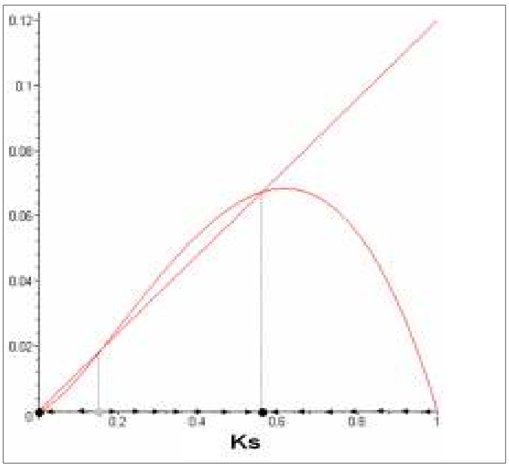

Figure 6

4. The effects of exogenous technical progress

In the introduction, we briefly reported how, according to Putnam (2000), the main channel leading from economic growth to a reduction in social connectedness may be related to technology.

In this section we study the dynamic effects generated by the introduction of exogenous technical progress $T(t)$ in the production function of the private good:

$$Y(t) = T(t) \cdot [1 - s(t)] \cdot K_s^{\alpha}(t) \qquad (15)$$

where the growth rate of T is assumed to be given by the equation:

$$\dot{T}(t) = \sigma T(t) \qquad (16)$$

where σ is a strictly positive parameter representing the growth rate of T. In this context, the time allocation choice $s^*(t)$ by the representative agent is described by the following proposition.

Lemma 8. *If the production function of the private good is given by (15), then problem (8) admits solution and the time allocation choice* $s^*(t)$ *of the representative agent is:*

If $\gamma - \alpha > 0$

$$s^*(t) = \begin{cases} 0 & \text{if } K_s(t) \le \left(\frac{d \cdot (b+1) \cdot T(t)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}} \\ \frac{b\varepsilon K_s^{\gamma-\alpha}(t) - d \cdot (b+1) \cdot T(t)}{(1+b\varepsilon) K_s^{\gamma-\alpha}(t) - d \cdot (b+1) \cdot T(t)} & \text{if } K_s(t) > \left(\frac{d \cdot (b+1) \cdot T(t)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}} \end{cases} \qquad (17)$$

If $\gamma - \alpha < 0$

$$s^*(t) = \begin{cases} \frac{b\varepsilon K_s^{\gamma-\alpha}(t) - d\cdot(b+1)\cdot T(t)}{(1+b\varepsilon)K_s^{\gamma-\alpha}(t) - d\cdot(b+1)\cdot T(t)} & \text{if } K_s(t) \le \left(\frac{d\cdot(b+1)\cdot T(t)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}} \\ 0 & \text{if } K_s(t) > \left(\frac{d\cdot(b+1)\cdot T(t)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}} \end{cases} \qquad (18)$$

4.1 The case without substitutability between *C* and *B*

If there is no substitutability between private consumption C and relational goods B (i.e. if $d = 0$), social participation is constant and strictly positive $s^*(t) = b\varepsilon/(1+b\varepsilon)$. In this context, the dynamic of social capital's accumulation is given by the equation:

$$\dot{K}_s = \frac{(b\varepsilon)^\delta}{(1+b\varepsilon)^{\beta+\delta}} \cdot T^\beta K_s^{\alpha\beta+\gamma\delta} - \eta K_s \qquad (19)$$

where the evolution of T is described by the differential equation (16). Note that $\dot{K}_s = 0$ for $K_s = 0$ and, if $\alpha\beta + \gamma\delta < 1$, along the graph of the function:

$$K_s = \left[\frac{(b\varepsilon)^\delta}{\eta(1+b\varepsilon)^{\beta+\delta}}\right]^{\frac{1}{1-\alpha\beta-\gamma\delta}} \cdot T^{\frac{\beta}{1-\alpha\beta-\gamma\delta}} \qquad (20)$$

which is increasing in T. Note that it holds $\dot{K}_s < 0$ above the curve (20) and $\dot{K}_s > 0$ below it. The basic features of dynamics under the assumption of no substitutability are described by the following Proposition.

Proposition 9. *Dynamics (19) have the following properties:*

1) *If $\alpha\beta + \gamma\delta < 1$, then both T and K_s grow without bound (i.e. $\lim_{t\to+\infty} T(t) = +\infty$ and $\lim_{t\to+\infty} K_s(t) = +\infty$) along any trajectory starting from a strictly positive initial value of K_s. Among these trajectories, there exists a trajectory represented by the equation:*

$$K_s = \left[\frac{(1-\alpha\beta-\gamma\delta)(b\varepsilon)^\delta}{[\beta\sigma + \eta(1-\alpha\beta-\gamma\delta)](1+b\varepsilon)^{\beta+\delta}}\right]^{\frac{1}{1-\alpha\beta-\gamma\delta}} \cdot T^{\frac{\beta}{1-\alpha\beta-\gamma\delta}} \qquad (21)$$

along which the growth rates of T and K_s are given by:

$$\frac{\dot{K}_s}{K_s} = \frac{\beta}{(1-\alpha\beta-\gamma\delta)} \frac{\dot{T}}{T}. \qquad (22)$$

where $\dot{T}/T = \sigma$ by assumption and $\frac{\dot{K}_s}{K_s} > \frac{\dot{T}}{T}$ if and only if $\frac{\beta}{1-\alpha\beta-\gamma\delta} > 1$. Along the remaining trajectories, the growth rate of K_s approaches the value given in (rates1) as $t \to +\infty$ (see Figure7[8]).

2) *If $\alpha\beta + \gamma\delta = 1$, then an explicit solution of (19) can be calculated:*

[8] Values of parameters: $\alpha = 0.3$, $\beta = 0.21$, $\gamma = 0.2$, $\delta = 0.4$, $\varepsilon = 0.4$, $\eta = 0.02$, $\sigma = 0.4$, $b = 0.1$.

$$K_s(t)=\frac{K_s(0)e^{\frac{A\cdot[T(0)]^{\beta}e^{\beta\sigma t}-\eta t\beta\sigma}{\beta\sigma}}}{e^{\frac{A\cdot[T(0)]^{\beta}}{\beta\sigma}}}$$

where $A=\frac{(b\varepsilon)^{\delta}}{(1+b\varepsilon)^{\beta+\delta}}$ *and* $K_s(0)$ *and* $T(0)$ *are the initial conditions on social capital and technology (see Figure 8*[9]*).*

Proof. The first part of Proposition can be proved by defining a new variable:

$$x:=\frac{T^{\beta}}{K_s^{1-(\alpha\beta+\gamma\delta)}} \quad (23)$$

and by calculating its time derivative:

$$\dot{x}=\frac{T^{\beta}}{K_s^{1-(\alpha\beta+\gamma\delta)}}\left(\beta\frac{\dot{T}}{T}-(1-(\alpha\beta+\gamma\delta))\frac{\dot{K}_s}{K_s}\right)= \quad (24)$$

$$=x\left[[\beta\sigma+\eta(1-\alpha\beta-\gamma\delta)]-\frac{(b\varepsilon)^{\delta}}{(1+b\varepsilon)^{\beta+\delta}}(1-\alpha\beta-\gamma\delta)x\right] \quad (25)$$

Equation (25) has two stationary states:

$$x^{*}=0 \text{ and } x^{**}=\frac{[\beta\sigma+\eta(1-\alpha\beta-\gamma\delta)](1+b\varepsilon)^{\beta+\delta}}{(1-\alpha\beta-\gamma\delta)(b\varepsilon)^{\delta}}$$

Notice that, since $\alpha\beta+\gamma\delta<1$, then $x^{**}>0$ is globally attractive in the positive x-axis; since, by (24), $\dot{x}=0$ if and only if (22), point 1) of Proposition is proved. The second part follows by a direct calculation.

It is interesting to note that:

1) If there is no substitutability between C and B, then K_s can grow indefinitely, whatever the sign of the expression $\gamma-\alpha$ is (we will show later that this result no longer hold when there is substitutability).
2) Along the trajectories under dynamics (19), the evolution of K_s is monotonic (always increasing or decreasing) or follows a U-shaped path, according to which K_s is initially decreasing and then becomes definitively increasing. It is worthwhile to stress this result in that, as we will see, if there is substitutability between C and B, then the evolution of K_s can take the shape of an inverted U

[9] Time-evolution of K_s. Values of parameters: $\alpha=0.3$, $\beta=0.9$, $\gamma=0.73$, $\delta=1$, $\varepsilon=0.4$, $\eta=0.18$, $\sigma=0.9$, $b=0.1$. Initial condition: $K_s(0)=3$, $T(0)=0.02$.

curve.

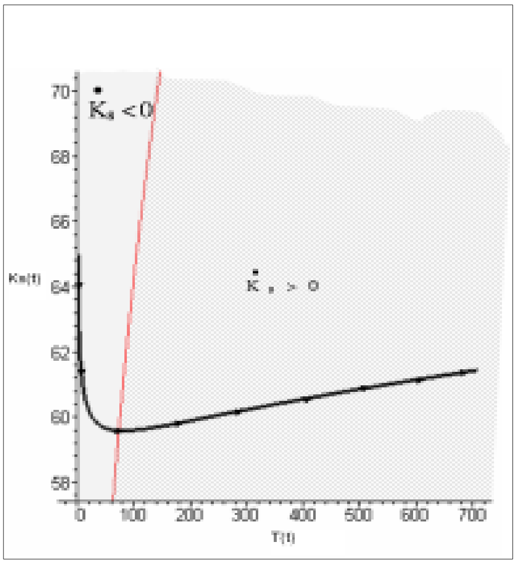

Figure 7

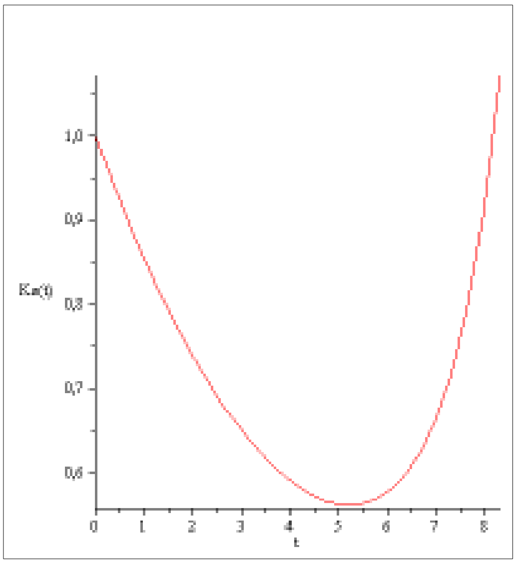

Figure 8

4.2 The case with substitutability between *C* and *B*

If $d > 0$, then social participation s^* depends on the values of T and K_s and can assume the value 0. In particular, the graph of the function:

$$K_s = \left(\frac{d(b+1)T}{b\varepsilon} \right)^{\frac{1}{\gamma-\alpha}} \tag{26}$$

separates, in the plane (T, K_s), the region where $s^* = 0$ (below the curve) from that where $s^* > 0$ (above it). Note that the function (26) is increasing (decreasing) in T if $\gamma - \alpha > 0$ (respectively, if $\gamma - \alpha < 0$).

In the region where $s^* = 0$, social capital dynamics are given by $\dot{K}_s = -\eta K_s < 0$ and the slope of trajectories $\frac{dK_s}{dT} = -\frac{\eta K_s}{\sigma T}$ is negative; this implies that along the trajectories below the curve (26), T increases and K_s decreases.

Above the curve (26), social capital dynamics are given by:

$$\dot{K}_s = \left[\frac{K_s^{\gamma-\alpha}}{(1+b\varepsilon)K_s^{\gamma-\alpha} - d(b+1)T} \right]^{\beta} \cdot \left[\frac{b\varepsilon K_s^{\gamma-\alpha} - d(b+1)T}{(1+b\varepsilon)K_s^{\gamma-\alpha} - d(b+1)T} \right]^{\delta} \cdot T^{\beta} K_s^{\alpha\beta+\gamma\delta} - \eta K_s \tag{27}$$

where T evolves according to the differential equation (16).

The basic features of dynamics (27) are described in the following Proposition.

Proposition 10. *If* $\gamma-\alpha>0$ *, then the region under the curve (26) is positively invariant under dynamics: every trajectory entering this region cannot leave it. Along the trajectories under the curve (26), the value of* K_s *approaches 0 for* $t\to+\infty$ *(see Figure 9).*
If $\gamma-\alpha<0$*, then the region above (under) the curve (26) is positively invariant if* $\sigma/\eta(\alpha-\gamma)\geq 1$ *(respectively, if* $\sigma/\eta(\alpha-\gamma)<1$ *); in any case, along every trajectory the value of* K_s *approaches 0 for* $t\to+\infty$ *(see Figure*[10] *10 and 11).*

Proof. To check the results on positive invariance of the sets under or above the curve (26), we simply have to compare the slope of the trajectories $\frac{dK_s}{dT}=-\frac{\eta K_s}{\sigma T}$, evaluated along the curve (26), and the slope of (26). To prove the results about the evolution of K_s, notice that in the set where $s^*=0$ it holds $\frac{dK_s}{dt}=-\eta K_s$ and consequently $K_s(t)=K_s(0)\cdot e^{-\eta t}$, where $K_s(0)$ is the initial value of K_s. Finally, note that in case $\gamma-\alpha<0$, every trajectory lies definitively in the set under the curve (26) or in the set above it; in any case, since $T\to+\infty$, the value of K_s approaches 0.
Whatever the sign of the expression $\gamma-\alpha$ is, along the trajectories crossing the curve (26) we will show that the evolution of K_s can take an inverted U-shape, different from the case without substitutability.
According to the above Proposition, a necessary condition to have unbounded growth of K_s is $\gamma-\alpha>0$; that is, the importance of K_s as an input in the production process of the private good (measured by α) must be lower than its importance in the production process of the relational good (measured by γ).
To analyze the behaviour of K_s in case $\gamma-\alpha>0$ we introduce the following definition.

Definition. *A Regular Growth Curve (RGC) is a curve in the plane* (T,K_s) *along which the rate of growth of* T *is equal to the exogenously given value* σ *while the rate of growth of* K_s *is equal to a constant strictly positive value* g *, possibly different from* σ *.*

Notice that along a *RGC*, being $\frac{\dot T}{T}=\sigma$ and $\frac{\dot K_s}{K_s}=g$, it holds $\frac{dK_s}{dT}=\frac{\dot K_s}{\dot T}=\frac{gK_s}{\sigma T}$; consequently a *RGC* is the graph of a function $K_s(T)=CT^{\frac{g}{\sigma}}$, where C is a positive arbitrary constant. *RGCs* are not trajectories under our dynamics. However, we aim to show that, for T high enough, there exist values of g and C , that we will denote by $\overline{g}$ and $\overline{C}$ respectively, such that the corresponding *RGC* $K_s(T)=\overline{C}T^{\frac{\overline{g}}{\sigma}}$ is asymptotically approached, for $T\to+\infty$, by the trajectories starting "near" it. This implies that along such trajectories $\frac{\dot K_s}{K_s}\to\overline{g}$ as $t\to+\infty$.
Notice that, for T high enough, a *RGC* can be approached by the trajectories only if it lies above the curve (separatrix); this requires that g must satisfy the necessary condition: $\frac{g}{\sigma}>\frac{1}{\gamma-\alpha}$, where $\frac{1}{\gamma-\alpha}>1$.

---

[10] Values of parameters in Figure 9: $\alpha=0.7$, $\beta=0.2$, $\gamma=0.2$, $\delta=0.3$, $\varepsilon=0.4$, $\eta=0.06$, $\sigma=0.04$, $b=3$, $d=0.3$. Figure 10: $\alpha=0.3$, $\beta=0.2$, $\gamma=0.71$, $\delta=0.3$, $\varepsilon=0.4$, $\eta=0.06$, $\sigma=0.04$, $b=3$, $d=0.3$.

That is, it must hold $g > \frac{\sigma}{\gamma-\alpha}$ (where $\frac{\sigma}{\gamma-\alpha} > \sigma$), so we introduce a further definition.

Definition. *A Reachable Regular Growth Curve (RRGC) is a RGC satisfying the condition* $g > \frac{\sigma}{\gamma-\alpha}$ *(see Figure 11).*

Looking at (17), it is easy to check that, along a *RRGC*, the social participation choice s^* approaches the value $\frac{b\varepsilon}{(1+b\varepsilon)}$ as $t \to +\infty$. Remember that $\frac{b\varepsilon}{(1+b\varepsilon)}$ is the value of social participation in the context without substitutability. In other words, if T and K_s grow following a *RRGC*, then for T and K_s high enough, social participation is almost equal to the level achieved under the assumption of no substitutability. Now the problem is: are there values of g and C such that the associated *RRGC* may be approached by some trajectories of the dynamics? To solve this problem, we analyze the behaviour of the variable:

$$x = \frac{T^{\beta}}{K_s^{1-\alpha\beta-\gamma\delta}}$$

previously defined (see (23)) and limit our analysis to the case $1-\alpha\beta-\gamma\delta > 0$. Remember that in the context in which $s^* = \frac{b\varepsilon}{(1+b\varepsilon)}$ always (i.e. in the context without substitutability), it holds $\dot{x} = 0$ (see (24)) along the curve (see (21)):

$$K_s = \overline{C} \cdot T^{\frac{\overline{g}}{\sigma}}$$

where $\overline{C} \equiv \left[\frac{(1-\alpha\beta-\gamma\delta)(b\varepsilon)^{\delta}}{[\beta\sigma+\eta(1-\alpha\beta-\gamma\delta)](1+b\varepsilon)^{\beta+\delta}}\right]^{\frac{1}{1-\alpha\beta-\gamma\delta)}}$ and $\overline{g} \equiv \frac{\beta}{1-\alpha\beta-\gamma\delta}$. So, if $\frac{\beta\sigma}{1-\alpha\beta-\gamma\delta} > \frac{\sigma}{\gamma-\alpha}$ (i.e. $\frac{\beta}{1-\alpha\beta\gamma\delta}(\gamma-\alpha) > 1$), we have that, as $T \to +\infty$, along $K_s = \overline{C} \cdot T^{\frac{\overline{g}}{\sigma}}$ the value of $\dot{x}$ approaches 0 while $\dot{x}$ becomes (see (24)) strictly positive (respectively, strictly negative) along the *RRGCs* corresponding to values of $g < \overline{g}$ (respectively $g > \overline{g}$), with g and C near enough to $\overline{g}$ and $\overline{C}$, respectively. This implies that all trajectories starting (for T high enough) sufficiently near to $K_s = \overline{C} \cdot T^{\frac{\overline{g}}{\sigma}}$ approach $K_s = \overline{C} \cdot T^{\frac{\overline{g}}{\sigma}}$ as $T \to +\infty$. Notice that, by Proposition 3, all trajectories in the plane (T, K_s) can be Pareto-ranked; in particular, we have that given two trajectories $\tilde{K}_s(T)$ and $\hat{K}_s(T)$, with $\tilde{K}_s(T) < \hat{K}_s(T)$, then $\hat{K}_s(T)$ Pareto-dominates $\tilde{K}_s(T)$. Furthermore, well-being may be decreasing when the economy follows a trajectory along which $K_s \to 0$ (see Figures[11] 12, 13, 14).

[11] Values of parameters in Figures 12, 13, 14: $\alpha = 0.7$, $\beta = 0.2$, $\gamma = 0.2$, $\delta = 0.3$, $\varepsilon = 0.4$, $\eta = 0.01$, $\sigma = 0.04$, $b = 3$, $d = 0.3$.

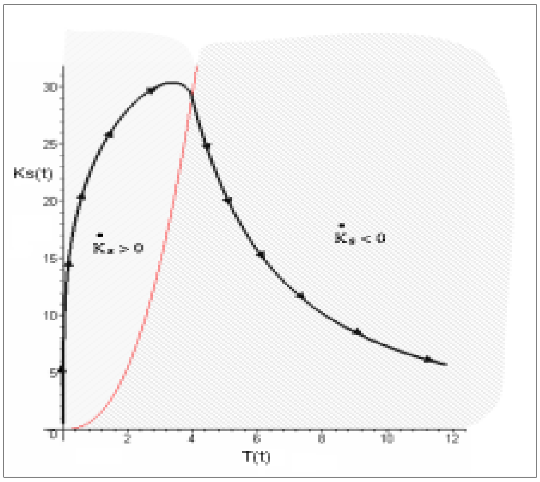

Figure 9

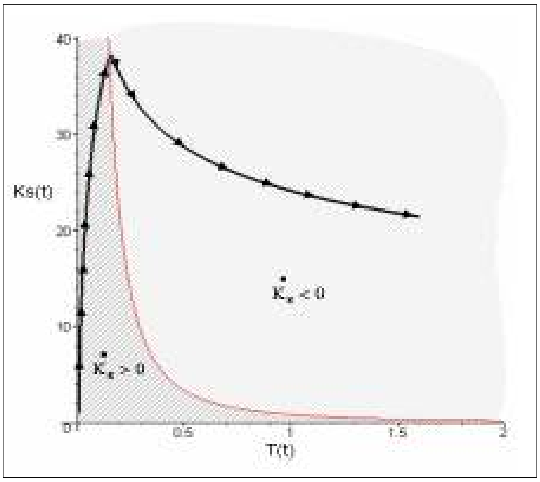

Figure 10

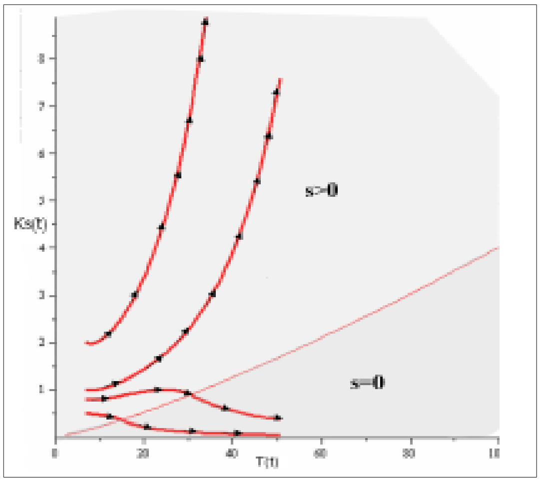

Figure 11

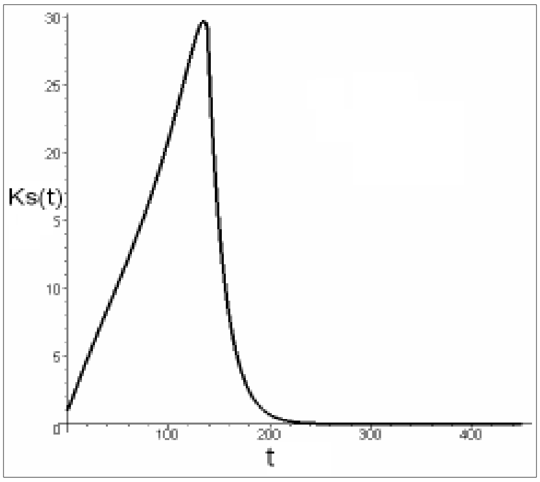

Figure 12

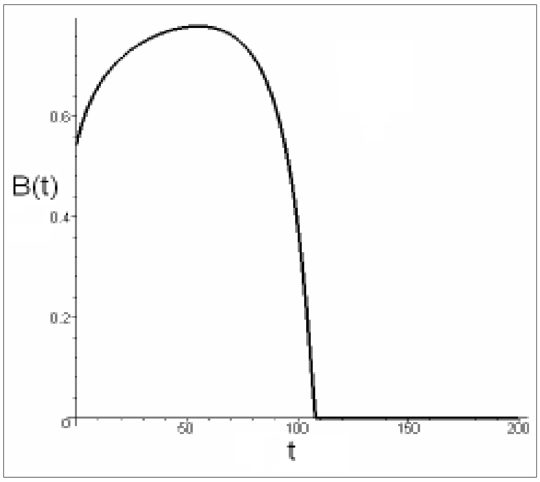

Figure 13

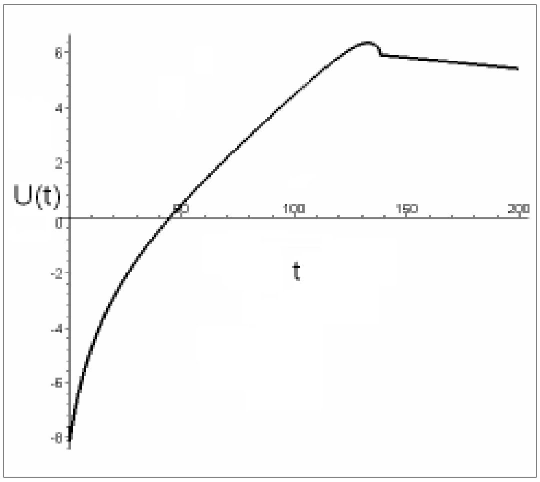

Figure 14

6. Concluding remarks

Our framework addresses a number of hypotheses drawn from the sociological and political science literature that have never jointly been taken into account within a theoretical model. Agents allocate their time between labour aimed at the production of private goods and social participation activities. Private consumption and relational goods are substitutable: if the environment is poor in participation opportunities and social participation is perceived as costly and frustrating, people can disengage from relational activities and devote more time and resources to private consumption. Following hints from Hochschild (1997) and Putnam (2000), we account for the possibility of positive spillovers from private to relational production, due to the ability of on-the-job interactions to stimulate the creation of durable ties. Following Coleman (1988, 1990), we assume that most of the time the creation of interpersonal ties does not depend on rational investment decisions. Rather, it is an incidental, unnecessary by-product of social participation. The resulting stock is a public resource, which enters as an argument in an agent's utility function and as an input in both private and relatonal goods' production functions. The main results of the study can be summarized as follows.

In the framework without technological progress, an unbounded growth of social capital can be observed only in the following two cases:

a) The case in which private goods and relational goods are not Edgeworth substitutes.

b) The case in which the two types of goods are Edgeworth substitutes but the relevance of the impact exerted by the stock of social capital on the production/consumption process of relational goods is greater than that exerted on the production process of private ones (i.e. $\gamma - \alpha > 0$).

In both cases (a) and (b), social capital dynamics are path-dependent. In a "favourable" configuration of the model's parameters, starting from a high enough initial endowment of social capital, the economy follows a virtuous trajectory where the stock of social capital endogenously and unboundedly grows, raising social participation and consolidating interpersonal ties. On the other hand, the reverse process may be self-feeding as well: if the initial endowment of social capital is low, then the economy experiences a simultaneous decline in social participation and social capital, leading to "social poverty traps" where the time spent on relational activities becomes more expensive (in terms of opportunity cost) and less productive (in terms of relational goods).

If private and relational goods are substitutes but (b) doesn't hold (i.e. $\gamma - \alpha < 0$), then no trajectory exists along which social capital grows without bound.

In the context without technological progress, the evolution of social capital is always monotonic, always increasing or decreasing. Introducing exogenous technical progress in the production function of private goods leads to interesting modifications in social capital's accumulation dynamics. In this context, social capital's trend relative to technological progress can be non-monotonic. In particular, it may experience an initial decline followed by growth, but not vice versa, if relational and private goods are not substitutes. Under the assumption of positive substitutability, the stock of social capital may exhibit a growth followed by a decline, so that its relationship with technological progress is described by an inverted U-shaped curve.

This result is consistent with the inverted U-shaped trends that several indicators of social participation followed in the twentieth century.

Also under the assumption of exogenous technical progress, there exist trajectories along which social capital can grow without bound only in the contexts (a) and (b) described above. The possibility to find the path to a sustainable growth of social capital crucially depends on the initial endowment of its stock: an environment rich in participation opportunities, a culture acknowledging the importance of non-market relations, and the diffusion of moral norms of reciprocity and cooperation certainly constitute a good "starting point".

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