Department of Applied Mathematics, University of Venice

WORKING PAPER SERIES

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Working Paper n. 180/2008
November 2008

ISSN: 1828-6887

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(November 2008)

Abstract. In this paper we analyze the stochastic model proposed by Galam in [2], for information spreading in a 'word-of-mouth' process among agents, based on a majority rule. Using the communications rules among agents defined in [2], we first perform simulations of the 'word-of-mouth' process and compare the results with the theoretical values predicted by Galam's model. Since some dissimilarities arise in particular when a small number of agents is considered, we suggest some enhancements by introducing a new parameter dependent model. We propose a modified Galam's scheme which is asymptotically coincident with the original model in [2]. Furthermore, for relatively small values of the parameter, we provide a numerical experience proving that the modified model often outperforms the original one, in terms of efficiency.

Keywords: Word-of-mouth process, information spreading, agent based simulation.

JEL Classification Numbers: C02, C40.

MathSci Classification Numbers: 90A.

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1 Introduction

The dynamics of social contagion, in particular of opinion spreading on a population, has been studied in many contexts, with many different approaches and several applications in social sciences, experimental psychology and consumers behavior. The significance of word-of-mouth information and rumors diffusion through social networks has been widely recognized as fundamental in all these contexts, see e.g. [9, 11, 3, 4, 6, 12].

The leverage effect of word-of-mouth information on consumer behavior, for example, improves the effectiveness of communications activities of a firm [10]. Word-of-mouth effects cannot be ignored as a powerful marketing tool, especially in case of new product introductions, when the aim is to reduce the probability of a post-launch failure [13].

We focus on a diffusion model which investigates opinion dynamics, driven by rumors in a population, proposed by Galam [2, 1]. He considers a population of agents who can have two opposite opinions, for example about political elections. Each of them can shift their opinion to the opposite one, due to repeated discussions within a group of people, following a majority rule which is biased in favor of one of the two opinions in case of parity. The model provides an explanation of the spreading of the rumor claiming that "No plane did crash on the Pentagon on September 11". Interestingly for this agent based model, Galam provides a closed form formula for the probability of one of the opinions to prevail, at a certain time instant.
Galam's model has been extended, for example considering agents having three possible opinions [8], and applied in several contexts, such as fashion industry [7] or political elections [4]. For a comprehensive review see [5].

Nevertheless, we observe that, considering a relatively small number of agents, when a simulation of the rumor spreading is run, Galam's model may possibly return inaccurate results. In order to cope with the latter drawback, we propose a modified Galam's scheme which is asymptotically coincident with the original model in [2]. When few agents are considered the modified model often turns out to be preferable to the original one.

This paper is organized as follows: Section 2 introduces Galam's model, detailing its peculiarities and some possible limits. Then, in Section 3 we describe our model, including a motivated numerical comparison with Galam's scheme. Finally, Section 4 completes the paper with suggestions for future work.

2 Galam's model

Galam [2] considers a population whose individuals can change opinion after discussing in groups. Let N be the overall number of people who meet into groups, in order to exchange their information. Each people either thinks '$+$' or '$-$' and $N = N_+(t) + N_-(t)$, where $N_+(t)$ $[N_-(t)]$ is the number of people who respectively think '$+$' ['$-$'] at time step t.
At time step t the N people gather into k-sized groups, $k = 1, \ldots, L$, with probability a_k ($a_1 + \cdots + a_L = 1$). Then, after a discussion in each group, at time $t+1$ people can shift their opinion to the opposite one, following a *majority rule* (in each group). The rule of shifting opinion is biased in favor of the opinion '$-$', in case of parity.

Let $P_+(0)$ be the probability to find people thinking '+' at time step '0', and let

$$P_+(0) = \frac{N_+(0)}{N}; \qquad\qquad P_-(0) = 1 - P_+(0).$$

Then, Galam's model is described by the following formula

$$P_+(t+1) = \sum_{k=1}^{L} a_k \sum_{j=\lfloor \frac{k}{2}+1 \rfloor}^{k} C_j^k P_+(t)^j \{1 - P_+(t)\}^{k-j}, \tag{1}$$

where $\lfloor x \rfloor$ indicates the largest integer which approximates x from below, and C_j^k is the binomial coefficient. We remark that observing (1), the quantity $P_+(t+1)$ does not depend explicitly on the number N of interacting people. As in [2], the dynamic expression (1) may be drawn in the space $(P_+(t), P_+(t+1))$, and identifies a fixed point addressed as the *killing point*. The killing point gives the theoretical threshold $\bar{P}_+$ so that

$$\text{if } P_+(0) > \bar{P}_+ \text{ then } \lim_{t\to\infty} P_+(t) = 1,$$

$$\text{if } P_+(0) = \bar{P}_+ \text{ then } P_+(0) = P_+(t), \qquad \text{for each time step } t > 0,$$

$$\text{if } P_+(0) < \bar{P}_+ \text{ then } \lim_{t\to\infty} P_+(t) = 0.$$

In order to test Galam's model with a relatively small number of agents (N), in Figure 1 we provide the results of 500 independent simulations, which reveal the killing point. In these simulations, according to Galam we label each agent as belonging to a group of size k with probability a_k. If N is finite, the expected number of agents with label k is $N_k = a_k N$: in general N_k might not be a multiple of k. We will focus on the latter issue in Section 3. The example in Figure 1 was described in [2] (parameters of the simulations are detailed in the caption). Repeated simulations (see also [8]) reveal that the model (1) may be inaccurate when N is relatively small: an example of such a behavior is given in Table 1. We remark that results reported in the tables of this paper are averaged over 500 runs. Moreover, we set $P_+(0) = 80\%$, following the guidelines in [2]. Finally, our tables consider the following positions:

t-step	**Gal**	**simul**	**$\|\Delta\|$_Gal**
1	0.8000	0.8000	0.0000
2	0.9392	0.8823	0.0570
3	0.9891	0.9489	0.0402
4	0.9983	0.9817	0.0166
5	0.9997	0.9945	0.0052
6	1.0000	0.9986	0.0014
7	1.0000	0.9997	0.0003
8	1.0000	0.9999	0.0001
9	1.0000	0.9999	0.0001
10	1.0000	1.0000	0.0000

Table 1: Average results with $N = 100$, $P_+(0) = 80\%$, 500 runs, Conv-1= 500, Conv-0= 0, $L = 20$, $a_k = 1/20$, $k = 1, \ldots, L$. The column $|\Delta|$_Gal reveals that the model (1) may be imprecise.

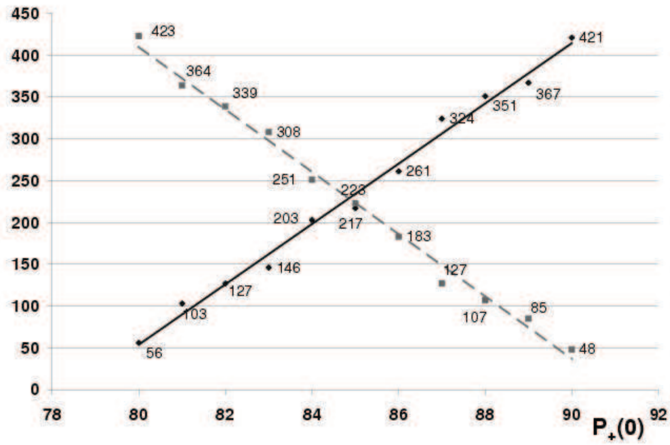

Figure 1: Results from 500 simulations. For $P_+(0) = 80\%, 81\%, \ldots, 90\%$, two points represent the number of simulations respectively converging to 1 (continuous regression line) and 0 (dashed regression line), within 20 time steps. Following the example in [2], we set in the simulations $N = 100$, $L = 6$, $a_1 = a_5 = a_6 = 0$ with $a_2 = a_3 = a_4 = 1/3$. The killing point $\bar{P}_+$ is evidently nearby 85%, as suggested in [2].

- numerical results are performed with the precision of 10^{-16}, though they are reported with just 4 exact digits. In those tables reporting comparisons (Tables 4-12), strictly better results are bolded;
- 'Conv-1' is the number of runs (out of 500) in which the final stationary point is '1', i.e. all the people think eventually '+';
- 'Conv-0' is the number of runs (out of 500) in which the final stationary point is '0', i.e. all the people think eventually '$-$';
- 't-step' is the time step (we allowed up to 20 time steps as in [2]);
- 'Gal' is $P_+(t)$ provided by the Galam model (1), at any time step;
- 'Gal$_{\mathcal{M}}$' (Tables 4-12) is $P_+(t)$ provided by the modified model (4), at any time step;
- 'simul' is the average ratio $N_+(t)/N$ obtained from the simulations, at time step t;
- '$|\Delta|$-Gal' is given by the quantity $|$Gal-simul$|$, and measures the *displacement* between 'Gal' and 'simul';
- '$|\Delta|$-Gal$_{\mathcal{M}}$' (Tables 4-12) is given by the quantity $|$Gal$_{\mathcal{M}}$-simul$|$, and measures the *displacement* between 'Gal$_{\mathcal{M}}$' and 'simul'.

Observe from Table 1 that when N is relatively small, the model (1) possibly provides unsatisfactory results with respect to the simulation. We will give a possible explanation of the latter fact, after introducing our improvement of the model. On the other hand, for L small (say in the range $5 \le L \le 10$) and N relatively large, we observed that (1) possibly recovers pretty well the results

t-step	Gal	simul	$\lvert\Delta\rvert$_Gal
1	0.8000	0.8000	0.0000
2	0.8195	0.8195	0.0000
3	0.8418	0.8406	0.0013
4	0.8665	0.8626	0.0039
5	0.8921	0.8864	0.0057
6	0.9169	0.9096	0.0073
7	0.9392	0.9308	0.0084
8	0.9576	0.9483	0.0094
9	0.9717	0.9636	0.0082
10	0.9818	0.9742	0.0075
11	0.9885	0.9823	0.0062
12	0.9929	0.9878	0.0051
13	0.9957	0.9921	0.0036
14	0.9974	0.9948	0.0026
15	0.9984	0.9966	0.0018
16	0.9990	0.9978	0.0012
17	0.9994	0.9985	0.0010
18	0.9997	0.9990	0.0007
19	0.9998	0.9993	0.0005
20	0.9999	0.9995	0.0003

Table 2: Average results with $N = 500$, $P_+(0) = 80\%$, 500 runs, Conv-1= 458, Conv-0= 0, $L = 5$, $a_k = 1/5$, $k = 1,\dots,L$. The model (1) performs pretty well.

from simulation, as Tables 2-3 show. Numerical results suggest that the following drawbacks may arise for the model (1):

- as mentioned before, if N is finite (as in most of the applications), the rules to gather people into groups of size at most L, possibly generate incomplete groups. Thus, the expected number of people assigned to groups of size k, i.e. $N_k = a_k N$ (with $N_1 + \cdots + N_L = N$), is possibly not a multiple of k;
- the model (1) relies on the *strong law of large numbers*, which assumes $N \to \infty$. When N is finite the latter consideration, along with the previous item, suggests that for N small the model (1) may yield inaccurate results (see Table 1).

3 A model refinement

Here we propose the model $\mathrm{Gal}_{\mathcal{M}}$, a refinement of formula (1) which *partially* complies with the issues at the end of the previous section.
We highlight that if N is the size of the overall population, and a_k is the probability that a person is assigned to k-sized groups, then the quantity N_k given by

$$N_k = a_k N,$$

represents the expected number of people assigned to k-sized groups. As a consequence, the quantity $tail_k$ defined as

$$tail_k = \lfloor N_k \rfloor - \left\lfloor \frac{a_k N}{k} \right\rfloor k, \tag{2}$$

t-step	Gal	simul	$\lvert\Delta\rvert$_Gal
1	0.8000	0.8000	0.0000
2	0.8856	0.8848	0.0009
3	0.9514	0.9492	0.0022
4	0.9833	0.9818	0.0015
5	0.9947	0.9938	0.0009
6	0.9984	0.9980	0.0004
7	0.9995	0.9994	0.0001
8	0.9999	0.9998	0.0000
9	1.0000	0.9999	0.0000
10	1.0000	1.0000	0.0000
11	1.0000	1.0000	0.0000
12	1.0000	1.0000	0.0000
13	1.0000	1.0000	0.0000
14	1.0000	1.0000	0.0000
15	1.0000	1.0000	0.0000
16	1.0000	1.0000	0.0000
17	1.0000	1.0000	0.0000
18	1.0000	1.0000	0.0000
19	1.0000	1.0000	0.0000
20	1.0000	1.0000	0.0000

Table 3: Average results with $N = 500$, $P_+(0) = 80\%$, 500 runs, Conv-1= 500, Conv-0= 0, $L = 10$, $a_k = 1/10$, $k = 1,\dots,L$. The model (1) performs pretty well.

represents the number (integral) of people (i.e. a *tail*) assigned to an *incomplete* k-sized group, i.e. $tail_k < k$. Observe that for any k there is at most one incomplete k-sized group, which includes exactly $tail_k$ people. Moreover, at any time step the probability Q_k that an individual is assigned to the incomplete k-sized group containing $tail_k$ elements, is given by

$$Q_k = \frac{tail_k}{a_k N}. \tag{3}$$

Therefore, we can consider a new model which encompasses both complete k-sized groups, and incomplete groups with $tail_k$ elements. Based on (2)-(3), we propose the following refinement of Galam's formula (1)

$$\begin{aligned} P_+(t+1) \quad = \quad & \sum_{k=1}^{L} a_k \left[(1-Q_k) \sum_{j=\left\lfloor \frac{k}{2}+1 \right\rfloor}^{k} C_j^k P_+(t)^j \{1-P_+(t)\}^{k-j} \right. + \\ & + \left. Q_k \sum_{i=\left\lfloor \frac{tail_k}{2}+1 \right\rfloor}^{tail_k} C_i^{tail_k} P_+(t)^i \{1-P_+(t)\}^{tail_k - i} \right]. \end{aligned} \tag{4}$$

The first term in square brackets takes into account only the contribution of k-sized groups (as Galam's formula). On the other hand, the second term in square brackets only considers the contribution of incomplete groups with $tail_k$ elements.
Observing (4) we note that when $N \to \infty$ our proposal (4) coincides with Galam's formula (1), since $Q_k \to 0$. In addition, unlike (1) the model (4) *explicitly* depends on the number of people N, since both Q_k and $tail_k$ depend on N. Thus, if N changes, the probabilities $\{P_+(t)\}$ computed by (4) are

affected accordingly (see also Tables 4-12).
From (2) and (3), if the quantity $a_k N/k$ is integral, then $tail_k = 0$ and $Q_k = 0$. Thus, models (1) and (4) coincide (an example is given in Table 4). This may be disappointing, since $tail_k = 0$ *does not mean* that no incomplete k-sized groups are formed. Indeed, simulations reveal (see the last column of Table 4) that the average number 'av_tail' of people in the 'tails' (i.e., in incomplete k-sized groups generated by the simulation, with $k = 1,\dots,L$), may be significant.
Obviously, this drawback appears only in special cases when $a_k N/k$ is integral. In order to avoid the drawback just described, for our model we can simply consider N such that $a_k N/k$ is not integral. On this purpose, in order to carry on a comparison with the results of Table 4, we set $N = 90$ in Table 5. As a result, observe that our proposal is evidently preferable (in Table 5 we could not choose $81 \le N \le 89$, since otherwise 80% of N would have not been integral, and a comparison with Table 4 could be possibly meaningless).

Table 6 completes the results reported in Table 1 (they are obtained by setting the same parameters). Results show that our model often yields smaller errors (bolded results). Again, the column 'av_tail' in Table 6 indicates that there are large tails, which explain our efficiency.

t-step	Gal	Gal$_{\mathcal{M}}$	simul	$\lvert\Delta\rvert$_Gal	$\lvert\Delta\rvert$_Gal$_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.8358	0.8358	0.8400	0.0042	0.0042	3.1
3	0.8797	0.8797	0.8788	0.0010	0.0010	3.0
4	0.9238	0.9238	0.9076	0.0162	0.0162	3.1
5	0.9578	0.9578	0.9312	0.0266	0.0266	3.1
6	0.9783	0.9783	0.9467	0.0316	0.0316	3.2
7	0.9891	0.9891	0.9496	0.0395	0.0395	3.0
8	0.9946	0.9946	0.9501	0.0445	0.0445	2.9
9	0.9973	0.9973	0.9476	0.0497	0.0497	3.1
10	0.9986	0.9986	0.9473	0.0513	0.0513	3.0
11	0.9993	0.9993	0.9467	0.0526	0.0526	3.1
12	0.9997	0.9997	0.9471	0.0525	0.0525	2.9
13	0.9998	0.9998	0.9476	0.0522	0.0522	2.9
14	0.9999	0.9999	0.9474	0.0526	0.0526	2.9
15	1.0000	1.0000	0.9476	0.0524	0.0524	2.9
16	1.0000	1.0000	0.9478	0.0522	0.0522	3.1
17	1.0000	1.0000	0.9480	0.0520	0.0520	3.0
18	1.0000	1.0000	0.9480	0.0520	0.0520	3.0
19	1.0000	1.0000	0.9480	0.0520	0.0520	3.1
20	1.0000	1.0000	0.9480	0.0520	0.0520	3.1

Table 4: Average results with $N = 80$, $P_+(0) = 80\%$, 500 runs, Conv-1= 473, Conv-0= 26, $L = 6$, $a_1 = a_3 = a_4 = a_5 = 0$, $a_2 = 1/4$, $a_6 = 3/4$. The performance of models (1) and (4) coincide.

More generally, on a wide range of numerical tests, the model (4) is quite often more accurate than (1), in particular when $tail_k$, $1 \le k \le L$, is relatively large. The latter result confirms the theory described. Observe that by simply setting

$$N = 40, 60, 80, 100, 300, 500; \qquad L = 6; \qquad a_k = 1/L, \quad k = 1,\dots,L,$$

we obtained relatively large values of the tails in our simulations (see Tables 7-12).

t-step	Gal	$\text{Gal}_{\mathcal{M}}$	simul	$\lvert\Delta\rvert$_Gal	$\lvert\Delta\rvert$_$\text{Gal}_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.8358	0.8347	0.8406	**0.0048**	0.0059	2.9
3	0.8797	0.8773	0.8812	**0.0015**	0.0039	3.1
4	0.9238	0.9206	0.9147	0.0091	**0.0059**	2.9
5	0.9578	0.9548	0.9382	0.0196	**0.0167**	3.0
6	0.9783	0.9762	0.9524	0.0259	**0.0238**	3.0
7	0.9891	0.9878	0.9576	0.0315	**0.0301**	2.9
8	0.9946	0.9938	0.9594	0.0352	**0.0344**	3.0
9	0.9973	0.9968	0.9604	0.0368	**0.0364**	3.1
10	0.9986	0.9984	0.9609	0.0377	**0.0374**	3.1
11	0.9993	0.9992	0.9615	0.0378	**0.0377**	3.0
12	0.9997	0.9996	0.9615	0.0381	**0.0380**	3.0
13	0.9998	0.9998	0.9616	0.0382	**0.0381**	3.0
14	0.9999	0.9999	0.9615	0.0384	**0.0384**	2.9
15	1.0000	0.9999	0.9615	0.0384	**0.0384**	3.0
16	1.0000	1.0000	0.9616	0.0384	**0.0384**	2.9
17	1.0000	1.0000	0.9618	0.0382	**0.0382**	3.0
18	1.0000	1.0000	0.9619	0.0381	**0.0381**	3.0
19	1.0000	1.0000	0.9619	0.0381	**0.0381**	3.0
20	1.0000	1.0000	0.9620	0.0380	**0.0380**	3.1

Table 5: Average results with $N = 90$, $P_+(0) = 80\%$, 500 runs, Conv-1= 480, Conv-0= 19, $L = 6$, $a_1 = a_3 = a_4 = a_5 = 0$, $a_2 = 1/4$, $a_6 = 3/4$. The model (1) is almost outperformed by the model (4).

4 Conclusions

From the results described in the present paper, some issues arise and deserve to be analyzed in future works.
Suppose the number N of people is assigned. How should we choose the integer L and the vector $a \in \mathbb{R}^L$ such that the stationary point is reached as soon as possible? We conjecture that both L and a may play a key role.

Moreover, we experienced that in the runs converging to '0', the average number of time steps performed is relatively smaller than in the cases of convergence to '1'. In this regard we already know that the rule of shifting opinion is slightly biased in favor of the opinion '$-$' (i.e. towards the stationary point 0), in case of parity. However, we cannot exclude that other specific reasons may yield the latter result.

Finally, suppose N is given: how can we modify our model in (4), in order to possibly drive the solution towards either the stationary point '1' or '0' ? This question turns to be relevant, for example, in marketing, where the successful spreading of a product is a crucial issue.

Acknowledgements The authors wish to thank Paolo Pellizzari for his valuable advice and for the careful reading of the manuscript. Giovanni Fasano also thanks *Progetto di Ricerca VISIR*, and *Programma PRIN 20079PLLN7 "Nonlinear Optimization, Variational Inequalities, and Equilibrium Problems"*, for the support received.

t-step	Gal	$\mathrm{Gal}_{\mathcal{M}}$	simul	$\lvert\Delta\rvert$_Gal	$\lvert\Delta\rvert$_$\mathrm{Gal}_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.9392	0.9077	0.8823	0.0570	**0.0255**	73.5
3	0.9891	0.9750	0.9489	0.0402	**0.0261**	73.4
4	0.9983	0.9950	0.9817	0.0166	**0.0133**	73.7
5	0.9997	0.9990	0.9945	0.0052	**0.0045**	73.6
6	1.0000	0.9998	0.9986	0.0014	**0.0012**	74.1
7	1.0000	1.0000	0.9997	0.0003	**0.0003**	73.7
8	1.0000	1.0000	0.9999	0.0001	**0.0001**	73.8
9	1.0000	1.0000	0.9999	0.0001	**0.0001**	73.4
10	1.0000	1.0000	1.0000	**0.0000**	0.0000	74.3

Table 6: Average results with $N = 100$, $P_+(0) = 80\%$, 500 runs, Conv-1= 500, Conv-0= 0, $L = 20$, $a_k = 1/20$, $k = 1,\dots,L$. The modified model (4) is preferable to (1) in the early time steps.

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t-step	Gal	Gal$_{\mathcal{M}}$	simul	$\lvert\Delta\rvert$_Gal	$\lvert\Delta\rvert$_Gal$_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.8331	0.8206	0.8269	**0.0062**	0.0063	7.1
3	0.8704	0.8445	0.8500	0.0204	**0.0055**	7.3
4	0.9080	0.8707	0.8659	0.0420	**0.0048**	7.4
5	0.9407	0.8976	0.8815	0.0593	**0.0161**	7.6
6	0.9651	0.9229	0.8885	0.0766	**0.0344**	7.3
7	0.9808	0.9447	0.8908	0.0899	**0.0538**	7.3
8	0.9899	0.9619	0.8915	0.0984	**0.0704**	7.4
9	0.9948	0.9746	0.8873	0.1075	**0.0873**	7.4
10	0.9974	0.9834	0.8824	0.1149	**0.1010**	7.4
11	0.9987	0.9894	0.8779	0.1208	**0.1115**	7.4
12	0.9993	0.9932	0.8743	0.1251	**0.1190**	7.3
13	0.9997	0.9957	0.8711	0.1285	**0.1246**	7.6
14	0.9998	0.9973	0.8681	0.1317	**0.1292**	7.3
15	0.9999	0.9983	0.8650	0.1349	**0.1333**	7.3
16	1.0000	0.9989	0.8642	0.1357	**0.1347**	7.4
17	1.0000	0.9993	0.8632	0.1367	**0.1361**	7.2
18	1.0000	0.9996	0.8622	0.1378	**0.1374**	7.4
19	1.0000	0.9997	0.8624	0.1376	**0.1374**	7.2
20	1.0000	0.9998	0.8620	0.1380	**0.1378**	7.1

Table 7: Average results with $N = 40$, $P_+(0) = 80\%$, 500 runs, Conv-1= 427, Conv-0= 67, $L = 6$, $a_k = 1/6$, $k = 1,\ldots,L$. The model (1) is almost outperformed by its modified version.

[12] Kamins, M., A., Folkes, V., S., and Perner, L., *Consumer Responses to Rumors: Good News, Bad News,* Journal of Consumer Psychology 6 (1997) 165–187.

[13] Meade, N., and Islam, T., *Modelling and forecasting the diffusion of innovation – A 25-year review,* International Journal of Forecasting 22 (2006) 519–545.

t-step	Gal	Gal$_{\mathcal{M}}$	simul	$\|\Delta\|$_Gal	$\|\Delta\|$_Gal$_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.8331	0.8200	0.8290	**0.0041**	0.0089	7.7
3	0.8704	0.8436	0.8576	**0.0128**	0.0139	7.5
4	0.9080	0.8700	0.8836	0.0244	**0.0136**	7.4
5	0.9407	0.8974	0.9029	0.0378	**0.0055**	7.7
6	0.9651	0.9237	0.9145	0.0506	**0.0091**	7.6
7	0.9808	0.9464	0.9230	0.0578	**0.0234**	7.3
8	0.9899	0.9644	0.9315	0.0584	**0.0329**	7.7
9	0.9948	0.9773	0.9328	0.0620	**0.0445**	7.7
10	0.9974	0.9860	0.9339	0.0634	**0.0521**	7.7
11	0.9987	0.9915	0.9335	0.0652	**0.0580**	7.5
12	0.9993	0.9950	0.9347	0.0646	**0.0603**	7.6
13	0.9997	0.9970	0.9342	0.0655	**0.0628**	7.6
14	0.9998	0.9983	0.9349	0.0649	**0.0634**	7.5
15	0.9999	0.9990	0.9337	0.0662	**0.0653**	7.4
16	1.0000	0.9994	0.9335	0.0665	**0.0659**	7.4
17	1.0000	0.9997	0.9331	0.0668	**0.0665**	7.4
18	1.0000	0.9998	0.9330	0.0670	**0.0668**	7.3
19	1.0000	0.9999	0.9333	0.0667	**0.0665**	7.4
20	1.0000	0.9999	0.9335	0.0665	**0.0664**	7.5

Table 8: Average results with $N = 60$, $P_+(0) = 80\%$, 500 runs, Conv-1= 461, Conv-0= 33, $L = 6$, $a_k = 1/6$, $k = 1,\ldots,L$. The model (1) is almost outperformed by the model (4).

t-step	Gal	Gal$_{\mathcal{M}}$	simul	$\|\Delta\|$_Gal	$\|\Delta\|$_Gal$_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.8331	0.8306	0.8304	0.0027	**0.0002**	7.7
3	0.8704	0.8652	0.8607	0.0097	**0.0045**	7.5
4	0.9080	0.9005	0.8910	0.0170	**0.0095**	7.6
5	0.9407	0.9325	0.9153	0.0254	**0.0171**	7.7
6	0.9651	0.9577	0.9334	0.0317	**0.0243**	7.5
7	0.9808	0.9752	0.9477	0.0331	**0.0276**	7.5
8	0.9899	0.9861	0.9552	0.0347	**0.0309**	7.6
9	0.9948	0.9924	0.9607	0.0342	**0.0318**	7.4
10	0.9974	0.9960	0.9628	0.0346	**0.0332**	7.7
11	0.9987	0.9979	0.9644	0.0343	**0.0335**	7.6
12	0.9993	0.9989	0.9639	0.0354	**0.0350**	7.5
13	0.9997	0.9994	0.9635	0.0362	**0.0359**	7.5
14	0.9998	0.9997	0.9630	0.0369	**0.0367**	7.5
15	0.9999	0.9998	0.9624	0.0375	**0.0374**	7.6
16	1.0000	0.9999	0.9615	0.0385	**0.0384**	7.6
17	1.0000	1.0000	0.9608	0.0392	**0.0392**	7.5
18	1.0000	1.0000	0.9600	0.0399	**0.0399**	7.4
19	1.0000	1.0000	0.9600	0.0400	**0.0400**	7.6
20	1.0000	1.0000	0.9599	0.0401	**0.0401**	7.7

Table 9: Average results with $N = 80$, $P_+(0) = 80\%$, 500 runs, Conv-1= 478, Conv-0= 18, $L = 6$, $a_k = 1/6$, $k = 1,\ldots,L$. The model (1) is outperformed by its modified version (4).

t-step	Gal	Gal$_{\mathcal{M}}$	simul	$\|\Delta\|$_Gal	$\|\Delta\|$_Gal$_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.8331	0.8274	0.8293	0.0038	**0.0019**	7.6
3	0.8704	0.8591	0.8608	0.0095	**0.0017**	7.8
4	0.9080	0.8928	0.8923	0.0156	**0.0004**	7.5
5	0.9407	0.9248	0.9167	0.0241	**0.0081**	7.4
6	0.9651	0.9515	0.9375	0.0276	**0.0140**	7.5
7	0.9808	0.9709	0.9523	0.0285	**0.0186**	7.5
8	0.9899	0.9835	0.9628	0.0270	**0.0207**	7.7
9	0.9948	0.9910	0.9690	0.0258	**0.0220**	7.4
10	0.9974	0.9952	0.9732	0.0242	**0.0220**	7.5
11	0.9987	0.9975	0.9763	0.0224	**0.0212**	7.5
12	0.9993	0.9987	0.9776	0.0217	**0.0210**	7.7
13	0.9997	0.9993	0.9791	0.0205	**0.0202**	7.7
14	0.9998	0.9996	0.9793	0.0205	**0.0203**	7.6
15	0.9999	0.9998	0.9796	0.0203	**0.0202**	7.5
16	1.0000	0.9999	0.9802	0.0198	**0.0197**	7.5
17	1.0000	0.9999	0.9801	0.0199	**0.0198**	7.5
18	1.0000	1.0000	0.9799	0.0200	**0.0200**	7.4
19	1.0000	1.0000	0.9797	0.0203	**0.0203**	7.4
20	1.0000	1.0000	0.9797	0.0203	**0.0203**	7.4

Table 10: Average results with $N = 100$, $P_+(0) = 80\%$, 500 runs, Conv-1= 486, Conv-0= 10, $L = 6$, $a_k = 1/6$, $k = 1, \ldots, L$. The model (1) is outperformed by the model (4).

t-step	Gal	Gal$_{\mathcal{M}}$	simul	$\|\Delta\|$_Gal	$\|\Delta\|$_Gal$_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.8331	0.8284	0.8320	**0.0011**	0.0036	7.5
3	0.8704	0.8610	0.8681	**0.0022**	0.0072	7.6
4	0.9080	0.8950	0.9026	**0.0053**	0.0076	7.5
5	0.9407	0.9268	0.9335	0.0073	**0.0067**	7.5
6	0.9651	0.9527	0.9572	0.0079	**0.0045**	7.2
7	0.9808	0.9714	0.9734	0.0074	**0.0021**	7.5
8	0.9899	0.9834	0.9843	0.0055	**0.0009**	7.1
9	0.9948	0.9907	0.9905	0.0043	**0.0001**	7.6
10	0.9974	0.9949	0.9944	0.0029	**0.0004**	7.5
11	0.9987	0.9972	0.9963	0.0024	**0.0009**	7.5
12	0.9993	0.9985	0.9974	0.0020	**0.0011**	7.7
13	0.9997	0.9992	0.9978	0.0019	**0.0014**	7.4
14	0.9998	0.9996	0.9980	0.0018	**0.0016**	7.5
15	0.9999	0.9998	0.9980	0.0019	**0.0018**	7.4
16	1.0000	0.9999	0.9980	0.0020	**0.0019**	7.6
17	1.0000	0.9999	0.9980	0.0020	**0.0019**	7.5
18	1.0000	1.0000	0.9980	0.0020	**0.0020**	7.4
19	1.0000	1.0000	0.9980	0.0020	**0.0020**	7.7
20	1.0000	1.0000	0.9980	0.0020	**0.0020**	7.6

Table 11: Average results with $N = 300$, $P_+(0) = 80\%$, 500 runs, Conv-1= 499, Conv-0= 1, $L = 6$, $a_k = 1/6$, $k = 1, \ldots, L$. The model (1) is almost outperformed by the model (4).

t-step	Gal	Gal$_{\mathcal{M}}$	simul	$\|\Delta\|$_Gal	$\|\Delta\|$_Gal$_{\mathcal{M}}$	av_tail
1	0.8000	0.8000	0.8000	0.0000	0.0000	0.0
2	0.8331	0.8330	0.8321	0.0010	**0.0009**	7.6
3	0.8704	0.8700	0.8694	0.0010	**0.0006**	7.5
4	0.9080	0.9072	0.9042	0.0037	**0.0030**	7.4
5	0.9407	0.9398	0.9354	0.0054	**0.0044**	7.5
6	0.9651	0.9642	0.9597	0.0054	**0.0044**	7.7
7	0.9808	0.9800	0.9765	0.0042	**0.0035**	7.6
8	0.9899	0.9894	0.9869	0.0030	**0.0025**	7.3
9	0.9948	0.9945	0.9928	0.0020	**0.0017**	7.5
10	0.9974	0.9972	0.9961	0.0012	**0.0010**	7.6
11	0.9987	0.9985	0.9980	0.0007	**0.0006**	7.6
12	0.9993	0.9993	0.9989	0.0005	**0.0004**	7.4
13	0.9997	0.9996	0.9994	0.0003	**0.0002**	7.7
14	0.9998	0.9998	0.9997	0.0002	**0.0002**	7.6
15	0.9999	0.9999	0.9998	0.0001	**0.0001**	7.5
16	1.0000	1.0000	0.9999	0.0000	**0.0000**	7.5
17	1.0000	1.0000	1.0000	**0.0000**	0.0000	7.5
18	1.0000	1.0000	1.0000	**0.0000**	0.0000	7.4
19	1.0000	1.0000	1.0000	**0.0000**	0.0000	7.6
20	1.0000	1.0000	1.0000	**0.0000**	0.0000	7.8

Table 12: Average results with $N = 500$, $P_+(0) = 80\%$, 500 runs, Conv-1= 500, Conv-0= 0, $L = 6$, $a_k = 1/6$, $k = 1,\dots,L$. The model (1) is almost outperformed by its modified version (4).