Department of Applied Mathematics, University of Venice

WORKING PAPER SERIES

Antonella Basso, Stefania Funari

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Working Paper n. 153/2007
June 2007

ISSN: 1828-6887

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Antonella Basso
<basso@unive.it>
Dept. of Applied Mathematics and SSAV
University of Venice

Stefania Funari
<funari@unive.it>
Dept. of Applied Mathematics
University of Venice

(November 2006)

Abstract. This paper tackles the problem of the presence of negative average rates of returns in the evaluation of the performance of mutual funds using a DEA approach. We present some extensions of DEA models for the evaluation of the performance of mutual funds that enable to compute the performance measure also in the presence of negative rates of returns. These extensions regard a model that can be used for investments in mutual funds which have profitability as main objective and two models specifically formulated for ethical mutual funds that include also the ethical objective among the outputs and differ in the way the ethical goal is pursued by investors.

The models proposed are applied to the European market of ethical mutual funds. In order to do so, a measure of the ethical level which takes into account the main socially responsible features of each fund is built.

Keywords: Data envelopment analysis, Performance evaluation, Mutual funds, ethical investment.

JEL Classification Numbers: C6, G1.

MathSci Classification Numbers: 90B50, 90B90.

Correspondence to:

Antonella Basso	Dept. of Applied Mathematics, University of Venice
	Dorsoduro 3825/e
	30123 Venezia, Italy
Phone:	[++39] (041)-234-6914
Fax:	[++39] (041)-522-1756
E-mail:	basso@unive.it

1 Introduction

In this paper we tackle the problem of the presence of negative average rates of returns in the evaluation of the performance of mutual funds using a data envelopment analysis (DEA) approach.

DEA models have been proposed in the literature in order to compare the performance of mutual funds by taking into consideration different aspects of the investment process: first of all, profitability and riskiness, but also initial and exit fees, and possible further objectives such as those which drive socially responsible investments. Along this line, we find the models proposed in Murthi et al. (1997), Morey and Morey (1999), Basso and Funari (2001, 2005a) and some generalizations to the measurement of the performance of ethical mutual funds proposed in Basso and Funari (2003).

However, a problem may arise with these models, due to the assumption, implicitly made in DEA approaches, that all the input and output values are non negative. As a matter of fact, in slump periods of the business cycle the average rate of return of most stocks is negative, and that of many mutual funds as well, so that one of the output variables may well take negative values.

In this contribution we present some extensions of the DEA models for the evaluation of the performance of mutual funds that enable to compute the performance measure also in the presence of negative rates of returns. These extensions regard a model that can be used for investments in mutual funds which have profitability as main objective and two models, specifically formulated for ethical mutual funds, that include the ethical objective among the outputs.

The two models for ethical funds differ in the way the ethical goal is assumed to be pursued by investors: the first model is appropriate in the case in which investors try to maximize both the return and the ethical level of the investment at the same time, whereas the second one is more appropriate when investors choose the ethical level a priori and try to maximize the return of their investment while satisfying the desired ethical level.

The models proposed are applied to the European market of ethical mutual funds. In order to do so, we have built a measure of the ethical level which takes into account the main socially responsible features of each fund. The analysis carried out concerns the main ethical equity funds of the Western European market, as well as a set of non ethical equity funds included for comparison. On average, in the period considered in the analysis the ethical funds turn out to perform somewhat better than the non ethical funds if a model that takes the ethical level into account is used, while they are overcome by the non ethical funds if the ethical goal is not explicitly considered.

The paper is organized as follows. Section 2 tackles the problem of the presence of negative mean returns in computing a performance indicator for mutual funds with a DEA model and presents an adjusted model that ensures the positivity of the output values. Section 3 presents a method to build an ethical measure for mutual funds starting from available information on the ethical features of the funds. Sections 4 and 5 propose two generalizations of the DEA models for ethical funds while Section 6 discusses the connections among the DEA performance measures obtained. In Section 7 we present the results of the analysis carried out on the European market. Finally, Section 8 gives some concluding remarks.

2 DEA performance evaluation of mutual funds in the presence of negative rates of returns

In order to measure the performance of mutual funds Murthi et al. (1997) and Basso and Funari (2001) propose some models which apply a DEA approach. Moreover, special DEA models have been proposed in Basso and Funari (2003) to evaluate the performance of ethical mutual funds.

Actually, it can be shown that the DEA technique can be used to define mutual fund performance indexes that take into account several inputs, such as different risk measures and the initial and exit fees of the investment, as well as several outputs, such as a return indicator and an ethical measure (Basso and Funari, 2001).

Let us consider a set of n mutual funds $j = 1, 2, \dots, n$ with risky rates of return R_j and assume to have to compare their performances. We denote by $E(R_j)$ the expected rate of return of fund j and by $\sqrt{Var(R_j)}$ the standard deviation of the rate of return, often used as a risk indicator for a fund investment.

It is usual to evaluate the performance of mutual funds over past periods and use this performance measure in order to assess the ability of the fund managers. This is often done by substituting the average rates of return

$$\overline{R}_j = \frac{1}{T}\sum_{t=1}^{T} r_{jt} \qquad j = 1, 2, \dots, n \tag{1}$$

obtained by the funds in the period considered and the historical volatilities of the returns

$$\sigma_j = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_{jt} - \overline{R}_j)^2} \qquad j = 1, 2, \dots, n \tag{2}$$

for the expected rates of return $E(R_j)$ and the standard deviations $\sqrt{Var(R_j)}$, respectively, where $r_{j1}, r_{j2}, \dots, r_{jT}$ denote the rates of return obtained by fund j in the periods $1, 2, \dots, T$.

For instance, let us consider the I_{DEA-1} index proposed in Basso and Funari (2001) to measure the performance of a set of mutual funds. The DEA performance index for fund j_0, $I_{j_0,DEA-1}$, with $j_0 = 1, 2, \dots, n$, is computed as the optimal value of objective function of the following optimization problem

$$\max_{\{u, v_i, w_i\}} \quad \frac{u o_{j_0}}{\sum_{i=1}^{h} v_i q_{ij_0} + \sum_{i=1}^{k} w_i c_{ij_0}} \tag{3}$$

subject to

$$\frac{u o_j}{\sum_{i=1}^{h} v_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \leq 1 \qquad j = 1, 2, \dots, n \tag{4}$$

$$u \geq \varepsilon \tag{5}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, \dots, h \tag{6}$$

$$w_i \geq \varepsilon \qquad i = 1, 2, \dots, k, \tag{7}$$

where

o_j	is a return measure of fund j, set equal to either $\overline{R}_j$ or the average excess return $\overline{R}_j - r$, where r is the riskless rate of return
$q_{1j}, \ldots, q_{hj}$	are h risk measures for fund j, such as the historical volatility σ_j and the β index
$c_{1j}, \ldots, c_{kj}$	are k initial and exit costs for fund j
u	is the weight assigned to the return measure o_j
v_i	is the weight assigned to the risk measure i $(i = 1, 2, ..., h)$
w_i	is the weight assigned to the cost i $(i = 1, 2, ..., k)$
$\varepsilon > 0$	is a non-Archimedean infinitesimal.

If we let an asterisk denotes the optimal values of the variables, we have

$$I_{j_0, DEA-1} = \frac{u^* o_{j_0}}{\sum_{i=1}^{h} v_i^* q_{ij_0} + \sum_{i=1}^{k} w_i^* c_{ij_0}}. \tag{8}$$

It is common in classical DEA models to assume that all the input and output values are non negative (see for example Cooper et al., 2000). This is indeed a crucial assumption in the measurement of performance with the DEA technique. On the other hand, when some output variables may take negative values, the DEA performance measure may give non satisfactory results; for some examples on this subject see Basso and Funari (2005b).

Actually, if we use in the DEA analysis as return indicator the average excess return observed in the period considered, its value is negative for all funds which obtain a rate of return lower than the riskless interest rate. On the other hand, if we use as return indicator the average rate of return, this often turns out to be negative for many mutual funds in the slump periods of the business cycle.

In order to ensure the positivity of all data, one might change the definition of the return indicator o_j in such a way as it is always positive under all circumstances and thus it can be directly used as an output variable in a DEA model.

To this purpose, it would be sufficient to use a suitable DEA model which is translation invariant. A model is said translation invariant if the optimal value of the objective function, which represents the DEA efficiency measure, is invariant for translations of the original input and output values consequent to an addition of a constant to the original data.

A DEA model which has such a property is the additive model (on additive DEA models see e.g. Cooper et al., 2000, Section 4.3), and actually this model is often used in order to tackle the problem of negative data in DEA analysis. In particular, it can be proved (see Ali and Seiford, 1990, and Lovell and Pastor, 1995) that the additive model is indeed translation invariant, while the basic CCR DEA model is not.

However, an additive DEA model discriminates between efficient and inefficient DMUs, but it cannot gauge the depth of eventual inefficiencies: indeed, the efficiency measure given by an additive model does not provide a radial efficiency measure such as that given by the basic CCR model.

Another approach, proposed in Silva Portela et al. (2003), treats the problem of negative data in DEA models by modifying the efficiency measure used, but neither this approach is directly connected to radial efficiency.

For this reason we prefer to take into consideration a return indicator which is financially meaningful and cannot take negative values: this can be found in the capitalization factor $\overline{U}_j = 1 + \overline{R}_j$, which gives the final value of a unit initial investment at the end of a unit period. This quantity cannot become negative, since in the worst case we may at most lose all the capital invested in a mutual fund.

Let us consider as risk measure the historical volatility σ_j and let us take into consideration among the inputs the initial and exit fees f_j^I and f_j^E, respectively. Moreover, since the output is represented by the final value of the investment, we include among the inputs also the initial capital invested in the mutual fund; in the comparison analysis, the same initial capital $C_0 = 1$ is assumed to be invested in all the funds under examination.

With these choices, the DEA model (3)–(7) is modified as follows

$$\max_{\{u,v_i\}} \quad \frac{u\overline{U}_{j_0}}{v_1 C_0 + v_2 \sigma_{j_0} + v_3 f_{j_0}^I + v_4 f_{j_0}^E} \tag{9}$$

subject to

$$\frac{u_1 \overline{U}_j}{v_1 C_0 + v_2 \sigma_j + v_3 f_j^I + v_4 f_j^E} \leq 1 \qquad j = 1, 2, \ldots, n \tag{10}$$

$$u \geq \varepsilon, \tag{11}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, 3, 4. \tag{12}$$

The DEA performance measure for fund j_0, $I_{j_0,DEA-S}$, is the optimal value of the objective function (9)

$$I_{j_0,DEA-S} = \frac{u^* \overline{U}_{j_0}}{v_1^* C_0 + v_2^* \sigma_{j_0} + v_3^* f_{j_0}^I + v_4^* f_{j_0}^E} \tag{13}$$

and lies in the interval $[0, 1]$.

It can be proved that the funds with a DEA performance measure equal to 1 exhibit the so called radial efficiency or technical efficiency, since we cannot reduce all inputs in the same proportion (thus maintaining the proportion of the input factors) without reducing also the outputs. If, in addition, it is not possible to reduce any input or increase any output without worsening the value of some other inputs or outputs, then the fund is said to be efficient in the sense of Pareto-Koopmans. In order to check the Pareto-Koopmans efficiency it is possible to solve a two-phase linear programming problem equivalent to the *DEA-S* problem (9)–(12) (see for example Cooper et al., 2000, 2006); indeed, from a computational point of view, this is the most convenient way to solve the DEA fractional programming problem (9)–(12).

Of course, a different optimization problem has to be solved for each fund $j_0 = 1, 2, \ldots, n$ in the set of funds compared, in turn.

3 How to define an ethical measure for mutual funds

Let us now turn our attention to the evaluation of the performance of ethical mutual funds.

First of all, in order to evaluate the performance of ethical mutual funds we need to build an ethical measure which can be used as an output variable to be taken into account together with the return indicator.

Various consultant agencies and research institutes analyze the ethical nature of mutual funds. For example, in the 'SRI Funds Service' the European Social Investment Forum (EUROSIF) together with Avanzi rating agency and Morningstar, give some basic information regarding the socially responsible profile of European ethical mutual funds. Such information is organized in various sections; in particular, the funds are analyzed on the basis of the most important questions taken into consideration in order to define negative and positive ethical screening.

Actually, one of the most important strategies applied by socially responsible mutual funds is ethical screening. According to such a strategy, the assets included in the mutual fund portfolios are selected on the basis of social and environmental grounds. The selection can be carried out either with a negative screening, by excluding from the portfolios the assets of the companies with a profile that is bad according to a socially responsible criterion, or with a positive screening, by including in the fund portfolio investments in companies which are selected on the ground of their ethically and socially responsible behaviour.

The most important information on the ethical screening used by the SRI Funds Service takes into consideration a set of features which can be either present or absent in the ethical profile of each fund:

a. **Negative screening issues**: 1. firearms; 2. weapons and military contracting; 3. nuclear energy; 4. tobacco; 5. gambling; 6. human rights and ELO fundamental conventions violations; 7. child labour; 8. oppressive regimes; 9. pornography; 10. alcohol; 11. animal testing; 12. factory farming; 13. furs; 14. excessive environmental impact and natural resources consumption; 15. GMO; 16. products dangerous to health/environment; 17. others.

b. **Positive screening issues**: 1. products beneficial for the environment and quality of life; 2. customers, product safety, advertisement competition; 3. environmental services and technologies; 4. environmental policies, reports, management systems; 5. environmental performances; 6. employees policies, reports, management systems; 7. employees performances; 8. suppliers and measures to avoid human rights violations; 9. communities and bribery; 10. corporate governance; 11. others.

Another important information on the ethical behaviour of mutual funds is the presence or absence of an ethical committee which has the function of defining the guidelines of the socially responsible investments and controlling the actions of the fund management in this respect.

We have used such information in order to define an ethical measure by assigning each ethical feature a weight and then computing their weighted sum.

More precisely, let us consider n mutual funds and let s^N and s^P be the number of negative and positive screening issues taken into account, respectively. Moreover, let s_j^N and s_j^P be the number of negative and positive screening features presented by fund j, with $j = 1, 2, \dots, n$. Then

$$N_j = \frac{s_j^N}{s^N} \qquad \text{and} \qquad P_j = \frac{s_j^P}{s^P} \tag{14}$$

represent the quota of the positive and negative screening issues which are present in the ethical profile of fund j, respectively. Moreover, let

$$C_j = \begin{cases} 1 & \text{if fund } j \text{ has an ethical committee with full powers} \\ 1/2 & \text{if fund } j \text{ has an ethical committee with partial powers} \\ 0 & \text{if fund } j \text{ does not have an ethical committee.} \end{cases} \tag{15}$$

An ethical measure defined in the real interval $[0, L]$ can be computed as follows:

$$e_j = \omega^N N_j + \omega^P P_j + \omega^C C_j \tag{16}$$

where ω^N, ω^P and ω^C are positive weights assigned to the negative and positive screening and to the ethical committee, respectively, and $L = \omega^N + \omega^P + \omega^C$.

By construction, fund j has a zero ethical measure if and only if it has no ethical profile, so that $e_j = 0$ for non ethical funds.

4 A DEA model for ethical funds with non negative outputs

In Section 3 we have defined a real measure of the ethical level for mutual funds; this measure can be used as an additional output variable in a DEA model. The manner in which the *DEA-S* model (9)–(12) can be extended to handle an ethical objective depends on the actual ethical goal pursuits by investors.

If investors choose the mutual fund in which to invest their money by trying to maximize both the return and the ethical level of the investment simultaneously, then we can resort to the following two-output *DEA-SE* model

$$\max_{\{u_r, v_i\}} \quad \frac{u_1 \overline{U}_{j_0} + u_2 e_{j_0}}{v_1 C_0 + v_2 \sigma_{j_0} + v_3 f_{j_0}^I + v_4 f_{j_0}^E} \tag{17}$$

subject to

$$\frac{u_1 \overline{U}_j + u_2 e_j}{v_1 C_0 + v_2 \sigma_j + v_3 f_j^I + v_4 f_j^E} \leq 1 \qquad j = 1, 2, \dots, n \tag{18}$$

$$u_r \geq \varepsilon \qquad r = 1, 2 \tag{19}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, 3, 4, \tag{20}$$

which is a direct extension of model (9)–(12).

According to this model, the DEA performance measure for fund j_0, $I_{j_0,DEA-SE}$, is the optimal value of the objective function (17)

$$I_{j_0,DEA-SE} = \frac{u_1^* \overline{U}_{j_0} + u_2^* e_{j_0}}{v_1^* C_0 + v_2^* \sigma_{j_0} + v_3^* f_{j_0}^I + v_4^* f_{j_0}^E}. \tag{21}$$

However, if investors choose the ethical level they desire a priori and then try to maximize the return of their investment by choosing the best mutual fund among all the funds that satisfy the required ethical level, model (17)–(20) is not appropriate. Actually, in this case the output e_j has to be considered as exogenously fixed, beyond the discretionary control of the managers of fund j.

On the other hand, it is known that the presence of an exogenously fixed output has a major consequence in the formulation of a DEA model, as pointed out in Banker and Morey (1986) and, as concerns the performance of ethical mutual funds, in Basso and Funari (2003). In next section we derive a more appropriate model for the case of non negative outputs when the ethical level is exogenously chosen by investors.

5 A DEA model for ethical funds with non negative outputs and exogenously fixed ethical levels

In order to see how the basic DEA model (17)–(20) has to be modified to take into account the presence of an exogenously fixed output, let us observe that this model is equivalent to the following linear programming problem (for the derivation see for example Cooper et al., 2000) in output-oriented form

$$\min_{\{u_r,v_i\}} \quad v_1 C_0 + v_2 \sigma_{j_0} + v_3 f_{j_0}^I + v_4 f_{j_0}^E \tag{22}$$

subject to

$$u_1 \overline{U}_{j_0} + u_2 e_{j_0} = 1 \tag{23}$$

$$-u_1 \overline{U}_j - u_2 e_j + v_1 C_0 + v_2 \sigma_j + v_3 f_j^I + v_4 f_j^E \geq 0 \quad j = 1, \ldots, n \tag{24}$$

$$u_r \geq \varepsilon \qquad r = 1, 2 \tag{25}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, 3, 4. \tag{26}$$

The DEA performance measure for the ethical fund j_0, $I_{j_0,DEA-SE}$, coincides with the reciprocal of the optimal value of the linear objective function (22).

The dual of this linear problem can be written as

$$\max \quad z_0 + \varepsilon s_1^+ + \varepsilon s_2^+ + \varepsilon \sum_{i=1}^{4} s_i^- \tag{27}$$

subject to

$$\overline{U}_{j_0} z_0 - \sum_{j=1}^{n} \overline{U}_j \lambda_j + s_1^+ = 0 \tag{28}$$

$$e_{j_0} z_0 - \sum_{j=1}^{n} e_j \lambda_j + s_2^+ = 0 \tag{29}$$

$$\sum_{j=1}^{n} C_0 \lambda_j + s_1^- = C_0 \tag{30}$$

$$\sum_{j=1}^{n} \sigma_j \lambda_j + s_2^- = \sigma_{j_0} \tag{31}$$

$$\sum_{j=1}^{n} f_j^I \lambda_j + s_3^- = f_{j_0}^I \tag{32}$$

$$\sum_{j=1}^{n} f_j^E \lambda_j + s_4^- = f_{j_0}^E \tag{33}$$

$$\lambda_j \geq 0 \qquad j = 1, 2, \ldots, n \tag{34}$$

$$s_r^+ \geq 0 \qquad r = 1, 2 \tag{35}$$

$$s_i^- \geq 0 \qquad i = 1, 2, 3, 4 \tag{36}$$

$$z_0 \quad \text{unconstrained}, \tag{37}$$

where z_0 is the dual variable associated with the equality constraint (23), λ_j (with $j = 1, 2, \ldots, n$) are the dual variables associated with the mutual funds constraints (24) and s_r^+ (with $r = 1, 2$) and s_i^- (with $r = 1, 2, 3, 4$) are the dual variables connected with the output and input weight constraints (25) and (26), respectively.

It is known that the optimal solution of this dual problem enables to identify for each inefficient fund a composite unit made up of a linear combination of the efficient funds, i.e. the funds for which constraint (24) is satisfied as equality and which therefore get an efficiency value equal to 1. Actually, from the complementary conditions of duality in linear programming we have that only the optimal value of the dual variables λ_j associated to these efficient funds can have a strictly positive optimal value, the others being null.

A composite unit is defined as a linear combination of the set of funds $\{ F_1, F_2, \ldots, F_n \}$ considered in the analysis with coefficients given by the optimal values of the dual variables λ_j, namely $\sum_{j=1}^{n} \lambda_j^* F_j$. In a composite unit only the efficient funds can have a strictly positive coefficient, while the coefficients of the other funds are bound to be null.

If we analyze the constraints (28)–(29) of the dual problem when the variables take their optimal value, we can see that the composite units use a level of inputs which is not greater than that employed by fund j_0 and obtains a level of outputs that is not lower than that obtained by fund j_0. In particular, as concerns the output levels we have that

$$\sum_{j=1}^{n} \overline{U}_j \lambda_j^* \geq \overline{U}_{j_0} z_0^* \tag{38}$$

$$\sum_{j=1}^{n} e_j \lambda_j^* \geq e_{j_0} z_0^*, \tag{39}$$

so that the composite units have a level of both the capitalization factor and the ethical indicator that is not lower than that of fund j_0 multiplied by the optimal value z_0^* of the dual variable z_0.

If fund j_0 is radially efficient, then no other fund or combination of funds can increase both outputs without augmenting the value of the inputs and $z_0^* = 1$ (and all the more so for the Pareto-Koopmans efficiency). On the contrary, if fund j_0 is not radially efficient then the optimal value of the objective function of both primal and dual linear problems is greater than 1 and, given the non-Archimedean nature of ε (which is positive and smaller than any positive valued real number), z_0^* will be greater than 1, too. In such a case the composite units give for both the capitalization factor and the ethical indicator a value higher than that of fund j_0.

However, when investors choose the ethical level they desire a priori, we have that a constraint is actually imposed on the fund chosen; indeed, in this case investors choose the fund that maximizes the return of their investment among all the funds that satisfy the required ethical level. Formally, this entails that the ethical level has to be considered as an exogenously fixed output, so that a composite unit is required to have an ethical level not lower than that of fund j_0

$$\sum_{j=1}^{n} e_j \lambda_j^* \geq e_{j_0} \tag{40}$$

and constraint (29) of the dual problem has to be substituted by the following constraint

$$\sum_{j=1}^{n} e_j \lambda_j - s_2^+ = e_{j_0}. \tag{41}$$

Moreover, following the suggestion of Banker and Morey (1986), we relax the constraint on the weight u_2 in the primal problem to a pure non negativity constraint; this entails that the coefficient of the slack variable s_2^+ in the objective function of the dual problem vanishes.

The dual problem in the case of an exogenously fixed ethical level can therefore be written as follows

$$\max \quad z_0 + \varepsilon s_1^+ + \varepsilon \sum_{i=1}^{4} s_i^- \tag{42}$$

subject to

$$\overline{U}_{j_0} z_0 - \sum_{j=1}^{n} \overline{U}_j \lambda_j + s_1^+ = 0 \tag{43}$$

$$\sum_{j=1}^{n} e_j \lambda_j - s_2^+ = e_{j_0} \tag{44}$$

$$\sum_{j=1}^{n} C_0 \lambda_j + s_1^- = C_0 \tag{45}$$

$$\sum_{j=1}^{n} \sigma_j \lambda_j + s_2^- = \sigma_{j_0} \tag{46}$$

$$\sum_{j=1}^{n} f_j^I \lambda_j + s_3^- = f_{j_0}^I \tag{47}$$

$$\sum_{j=1}^{n} f_j^E \lambda_j + s_4^- = f_{j_0}^E \tag{48}$$

$$\lambda_j \geq 0 \qquad j = 1, 2, \ldots, n \tag{49}$$

$$s_r^+ \geq 0 \qquad r = 1, 2 \tag{50}$$

$$s_i^- \geq 0 \qquad i = 1, 2, 3, 4 \tag{51}$$

$$z_0 \quad \text{unconstrained.} \tag{52}$$

Let us observe that from the dual of the dual problem (42)–(52) we can reconstruct the equivalent fractional programming problem *DEA-SEef*

$$\max_{\{u_r, v_i\}} \quad \frac{u_1 \overline{U}_{j_0}}{v_1 C_0 + v_2 \sigma_{j_0} + v_3 f_{j_0}^I + v_4 f_{j_0}^E - u_2 e_{j_0}} \tag{53}$$

subject to

$$\frac{u_1 \overline{U}_j}{v_1 C_0 + v_2 \sigma_j + v_3 f_j^I + v_4 f_j^E - u_2 e_j} \leq 1 \qquad j = 1, 2, \ldots, n \tag{54}$$

$$u_1 \geq \varepsilon, \quad u_2 \geq 0 \tag{55}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, 3, 4. \tag{56}$$

The DEA performance measure for fund j_0, $I_{j_0,DEA-SEef}$, is the optimal value of the objective function (53) and coincides with the reciprocal of the optimal value of the objective function (42) of the linear dual problem (42)–(52).

If we compare the DEA model for ethical funds (42)–(52) with the exogenously fixed DEA model proposed in Basso and Funari (2003), it is apparent that the differences between the two models lie in the expedient used to tackle the case of negative average rates of returns, which has lead to a special choice for the return indicator and to the use of the initial capital as an additional input.

6 Connections among the efficiency measures

We may wonder which relation exists between the DEA scores obtained with the two models for ethical mutual funds (17)–(20) and (53)–(56), and the one obtained with the DEA model (9)–(12) which ignores the ethical objective. The following theorems 1 and 2 answer this question.

Theorem 1 *Let $I_{j_0,DEA-S}$, $I_{j_0,DEA-SEef}$ and $I_{j_0,DEA-SE}$ be the DEA performance measures for fund j_0 obtained by solving the DEA problems (9)–(12), (53)–(56) and (17)–(20), respectively. The following inequalities hold:*

$$I_{j_0,DEA-S} \leq I_{j_0,DEA-SEef} \leq I_{j_0,DEA-SE}. \tag{57}$$

Proof: Let us first prove the inequality $I_{j_0,DEA-S} \leq I_{j_0,DEA-SEef}$. If we compare the DEA fractional programming problems (9)–(12) and (53)–(56), we can observe that problem (9)–(12) can be obtained as a restriction of problem (53)–(56), since it can be obtained from the latter by considering the further constraint $u_2 = 0$. Hence, $I_{j_0,DEA-S}$, which is the optimal solution of the constrained problem (9)–(12), cannot be greater than $I_{j_0,DEA-SEef}$, that is the optimal solution of problem (53)–(56).

Let us now demonstrate the inequality $I_{j_0,DEA-SEef} \leq I_{j_0,DEA-SE}$. Let us consider the dual problem (27)–(37), which is the dual of the linear programming problem (22)–(26) equivalent to problem (17)–(20). Since in both problems (27)–(37) and (42)–(52) the optimal value of the dual variable z_0 is not lower than 1, if we cut the feasible regions of both problems by introducing the additional constraint $z_0 \geq 1$ we do not cut off the optimal solution. In the remaining part of the feasible region, we have $\sum_{j=1}^{n} e_j \lambda_j \geq e_{j_0} z_0 \geq e_{j_0}$ and therefore constraint (29) is more restrictive than constraint (44). As all the other constraints of the feasible regions of problems (27)–(37) and (42)–(52) are equal, we conclude that the feasible region of problem (27)–(37) is a subset of that of problem (42)–(52). Since the difference between the objective functions of the two dual problems is given by εs_2^+ which is lower than any positive real number, given the nature of non-Archimedean infinitesimal of ε, the optimal solution of problem (27)–(37) is lower than or equal to the optimal solution of problem (42)–(52). Hence, for their reciprocal values, which give the DEA performance measures $I_{j_0,DEA-SE}$ and $I_{j_0,DEA-SEef}$, respectively, the reverse inequality holds. $\diamond$

Another main issue concerns the efficiency measure of the non ethical mutual funds obtained with the models for ethical funds (17)–(20) and (53)–(56). Theorem 2 shows that if the ethical indicator of a fund is equal to 0 the use of these two models does not improve the fund efficiency score.

Theorem 2 *Let j_0 be a mutual fund with ethical measure $e_{j_0} = 0$ and let $I_{j_0,DEA-S}$, $I_{j_0,DEA-SEef}$ and $I_{j_0,DEA-SE}$ be the DEA performance measures obtained by solving the DEA problems (9)–(12), (53)–(56) and (17)–(20), respectively. The following equalities hold:*

$$I_{j_0,DEA-S} = I_{j_0,DEA-SEef} = I_{j_0,DEA-SE}. \tag{58}$$

Proof: The equality $I_{j_0,DEA-S} = I_{j_0,DEA-SEef}$ follows from the observation that when $e_{j_0} = 0$ the DEA fractional programming problem (53)–(56) coincides with problem (9)–(12).

The equality $I_{j_0,DEA-SEef} = I_{j_0,DEA-SE}$ can be proved by observing that when $e_{j_0} = 0$ the feasible regions of problems (27)–(37) and (42)–(52) coincide while their objective functions differ by the quantity εs_2^+ which is the product of a non-Archimedean infinitesimal and a real number and is therefore smaller than any positive real number. The DEA performance measures $I_{j_0,DEA-SE}$ and $I_{j_0,DEA-SEef}$, therefore, coincide. $\diamond$

Table 1: Empirical results of the analysis of the performance of the European ethical mutual funds.

Fund name	Country	Std. Dev.	Init. fee	Exit fee	Mean return	Ethic. level	Sharpe ratio	DEA S	DEA SE	DEA SEef
Aktie-Ansvar Europa	SE	16.08	0.00	0.00	-1.75	0.97	-0.19	0.939	0.951	0.949
Aktie-Ansvar Sverige	SE	21.00	0.00	0.00	5.28	0.97	0.20	0.986	**1.000**	**1.000**
Banco Etisk Global	SE	17.81	0.00	0.25	-11.47	2.19	-0.73	0.817	0.872	0.857
Banco Etisk Sverige	SE	26.51	0.00	0.25	-1.63	1.00	-0.05	0.891	0.915	0.912
Banco Hjalp	SE	25.49	0.00	0.25	-3.40	2.07	-0.13	0.875	0.928	0.923
Banco Human Pension	SE	24.97	5.00	5.00	-0.71	2.25	-0.03	0.846	0.894	0.888
Banco Ideell Miljo	SE	25.12	0.00	0.25	-1.56	2.25	-0.06	0.892	0.951	0.947
Banco Samarit Pension	SE	25.03	5.00	5.00	-0.84	2.25	-0.03	0.845	0.893	0.887
Banco Svensk Miljo	SE	24.37	0.00	0.25	1.65	1.91	0.06	0.921	0.968	0.966
Eldsjal 1	SE	26.75	0.00	0.00	2.15	0.35	0.10	0.945	0.953	0.953
Eldsjal 2	SE	26.78	0.00	0.00	2.34	0.47	0.10	0.947	0.958	0.958
Firstnordic SRI Europa	SE	22.06	0.50	0.50	-3.81	0.71	-0.18	0.849	0.859	0.858
Firstnordic SRI Global	SE	17.30	0.50	0.50	-10.68	0.71	-0.71	0.797	0.804	0.802
Firstnordic SRI Sverige	SE	24.32	0.50	0.50	2.29	0.71	0.08	0.903	0.913	0.912
Folksams Globala Aktiefond	SE	17.29	0.00	0.00	-6.53	2.67	-0.45	0.889	0.952	0.942
Folksams Idrottsfond	SE	21.07	0.00	0.00	-3.14	1.60	-0.19	0.907	0.939	0.936
Humanfonden	SE	25.11	0.00	0.25	-2.32	2.25	-0.09	0.885	0.944	0.940
KPA Etisk Aktiefond	SE	18.83	0.00	0.00	-2.93	3.64	-0.23	0.917	**1.000**	**1.000**
Lansforsakringar Hjarnfond	SE	21.24	0.00	0.00	-17.14	1.09	-0.89	0.776	0.795	0.789
Robur Gavofond	SE	20.56	0.00	1.00	-4.07	1.71	-0.25	0.818	0.899	0.890
Robur Miljofond	SE	24.46	0.00	1.00	1.04	1.91	0.04	0.861	0.952	0.947
Samaritfonden	SE	24.99	0.00	0.25	-1.93	2.25	-0.08	0.888	0.948	0.943
SEB Cancerfonden	SE	18.70	0.00	0.00	-3.95	0.47	-0.26	0.908	0.911	0.911
SEB Etisk Globalfond	SE	18.44	0.00	0.00	-9.67	0.94	-0.59	0.855	0.868	0.865
SEB Stiftelsefond Sverige	SE	24.82	0.00	0.00	0.92	0.82	0.03	0.934	0.953	0.952
SEB Stiftelsefond Utland	SE	17.15	0.00	0.00	-9.39	0.82	-0.63	0.862	0.872	0.869
SEB Ostersjofond/WWF	SE	21.72	0.00	0.00	1.16	0.59	0.02	0.945	0.953	0.952
Skandia Cancerfonden	SE	23.81	0.00	0.00	2.01	2.35	0.07	0.945	**1.000**	**1.000**
Skandia Ideer For Livet	SE	21.93	0.00	0.00	3.08	0.47	0.10	0.962	0.968	0.968
SPP Aktieindexfond Global Sustain.	SE	17.29	2.00	0.00	-8.03	3.41	-0.54	0.847	0.894	0.881
SPP Storebrand Miljofond	SE	18.96	0.00	0.00	-7.32	2.50	-0.44	0.875	0.929	0.924
Svenska Kyrkans Aktiefond MEGA	SE	20.16	5.00	1.00	-1.14	1.71	-0.11	0.843	0.879	0.874
Svenska Kyrkans Miljofond	SE	24.56	0.00	1.00	0.32	1.59	0.01	0.855	0.931	0.925
Svenska Kyrkans Vardepappersfond	SE	20.25	0.00	1.00	-2.15	1.71	-0.16	0.834	0.915	0.908
Ohman Etisk Index Europa	SE	18.27	0.00	0.00	-1.54	0.47	-0.13	0.932	0.935	0.935
Ohman Etisk Index Japan	SE	19.78	0.00	0.00	-1.78	0.47	0.00	0.925	0.929	0.928
Ohman Etisk Index Pacific	SE	13.96	0.00	0.00	3.79	0.59	0.23	**1.000**	**1.000**	**1.000**
Ohman Etisk Index USA	SE	19.13	0.00	0.00	-10.66	0.59	-0.56	0.843	0.849	0.848
Banco Offensiv	SE	20.79	0.00	0.25	-5.74	0.00	-0.43	0.860	0.860	0.860
Banco Smabolag	SE	29.78	0.00	0.25	9.37	0.00	0.20	0.991	0.991	0.991
Firstnordic Utland	SE	17.19	0.00	0.00	-4.15	0.00	-0.39	0.912	0.912	0.912
Firstnordic Global Index	SE	15.78	0.50	0.50	-5.27	0.00	-0.50	0.850	0.850	0.850
Firstnordic Sverige	SE	20.42	0.00	0.00	4.35	0.00	0.06	0.980	0.980	0.980
Folksams Tjanstemannafond Varlden	SE	17.47	0.00	0.00	-8.53	0.00	-0.65	0.869	0.869	0.869
Folksams Aktiefond Sverige	SE	22.96	0.00	0.00	5.50	0.00	0.10	0.981	0.981	0.981
Lansforsakringar Globalfond	SE	16.51	0.00	0.00	-8.92	0.00	-0.71	0.869	0.869	0.869
Robur IP Aktiefond	SE	18.95	0.00	1.00	-1.03	0.00	-0.22	0.844	0.844	0.844
Robur Nordenfond	SE	20.85	0.00	1.00	6.16	0.00	0.14	0.905	0.905	0.905
SEB Europafond	SE	17.63	0.00	0.00	-1.82	0.00	-0.25	0.932	0.932	0.932
SEB Globalfond	SE	16.65	0.00	0.00	-8.50	0.00	-0.68	0.872	0.872	0.872
SEB Sverige Aktiefond I	SE	23.37	0.00	0.00	3.39	0.00	0.01	0.960	0.960	0.960
SEB Lux Fund - Global Equity	SE	16.64	0.00	0.00	-9.26	0.00	-0.73	0.865	0.865	0.865
SEB Nordenfond	SE	20.29	0.00	0.00	4.90	0.00	0.08	0.986	0.986	0.986
Skandia Aktiefond Sverige	SE	21.88	0.00	0.00	4.48	0.00	0.06	0.976	0.976	0.976
Skandia Smabolag Sverige	SE	24.24	0.00	0.00	8.09	0.00	0.19	**1.000**	**1.000**	**1.000**
SPP Generation 60-tal	SE	14.84	2.00	0.00	-0.38	0.00	-0.23	0.925	0.925	0.925
SPP Generation 80-tal	SE	19.03	2.00	0.00	-3.68	0.00	-0.36	0.882	0.882	0.882
Robur Gavofond	SE	19.42	0.00	1.00	-2.44	0.00	-0.29	0.832	0.832	0.832
Robur Smabolagsfond Norden	SE	17.87	0.00	1.00	17.30	0.00	0.72	**1.000**	**1.000**	**1.000**
Robur Globalfond MEGA	SE	16.83	5.00	1.00	-7.78	0.00	-0.63	0.795	0.795	0.795
Ohman Europafond	SE	19.94	0.00	0.00	-4.02	0.00	-0.33	0.903	0.903	0.903
Ohman Varumarkesfond	SE	17.04	0.00	0.00	-8.99	0.00	-0.70	0.866	0.866	0.866
Aberdeen Ethical World Fund A Acc	UK	15.34	4.25	0.00	-2.17	2.09	-0.23	0.878	0.896	0.893
AEGON Ethical Fund A Acc	UK	17.03	5.50	0.00	0.35	2.76	-0.12	0.885	0.920	0.917
Allchurches Amity Fund A inc	UK	14.92	5.00	0.00	-0.56	4.05	-0.23	0.893	0.957	0.933
AXA Ethical Fund A Acc	UK	19.14	5.00	0.00	-4.05	2.03	-0.33	0.846	0.872	0.869
AXA Ethical Fund B Acc	UK	19.14	5.00	0.00	-4.56	2.03	-0.34	0.841	0.868	0.864
Banner Real Life Fund A Acc	UK	16.12	5.50	0.00	-2.06	2.64	-0.28	0.871	0.898	0.894
CIS Sustainable Leaders Trust Inc	UK	14.54	5.00	0.00	-3.47	3.50	-0.44	0.870	0.897	0.892
Credit Suisse Fellowship Fund R Inc	UK	16.03	5.25	0.00	-2.88	3.63	-0.35	0.864	0.906	0.899
F&C Stewardship Growth F.Sh.Cl.1	UK	15.21	5.00	0.00	1.18	3.98	-0.11	0.906	0.962	0.941
F&C Stewardship Income F.Sh.Cl.1	UK	10.94	5.00	0.00	7.48	3.98	0.35	0.999	**1.000**	**1.000**
F&C Stewardship Intern. F.Sh.Cl.1	UK	15.31	5.00	0.00	-5.60	3.98	-0.48	0.845	0.921	0.878
Family Charities Ethical Trust Acc	UK	15.12	5.00	0.00	-0.99	1.18	-0.24	0.888	0.889	0.888
Framlington Health Fund Acc	UK	21.02	5.50	0.00	-10.04	1.45	-0.51	0.791	0.811	0.807
Halifax Ethical Fund A Inc	UK	15.98	4.00	0.00	-5.44	3.13	-0.43	0.850	0.883	0.878

Fund name	Country	Std. Dev.	Init. fee	Exit fee	Mean return	Ethic. level	Sharpe ratio	DEA S	DEA SE	DEA SEef
Henderson Ethical Fund A Inc	UK	16.67	5.00	0.00	-9.88	4.22	-0.69	0.797	0.916	0.866
Henderson Global Care Growth Fund	UK	16.89	4.50	0.00	-9.45	4.70	-0.65	0.806	1.000	0.933
Henderson Global Care Income Fund	UK	13.92	4.50	0.00	-2.93	4.70	-0.33	0.879	**1.000**	**1.000**
Insight Investment Europ. Ethical F.	UK	14.39	5.25	0.00	0.22	3.19	-0.19	0.904	0.925	0.922
Insight Investment Evergreen Fund	UK	15.12	5.25	0.00	-3.98	2.89	-0.35	0.861	0.884	0.879
Jupiter Ecology Fund Inc	UK	19.10	5.00	0.00	-7.17	4.70	-0.43	0.818	1.000	0.956
Jupiter Environmental Opportunities F.	UK	16.33	5.25	0.00	0.53	4.35	-0.12	0.892	0.990	0.984
Legal & General Ethical Trust Acc	UK	18.03	0.00	0.00	-2.03	2.03	-0.23	0.929	0.969	0.968
Morley Sustain.Future Abs.Growth F.1	UK	19.73	4.00	0.00	-5.59	3.62	-0.32	0.840	0.890	0.883
Morley Sustain.Future Eur.Growth F.1	UK	15.89	4.00	0.00	-1.84	3.62	-0.20	0.883	0.922	0.918
Morley Sustain.Fut. Global Growth F.1	UK	15.47	4.00	0.00	-10.74	3.62	-0.82	0.804	0.859	0.835
Morley Sustain.Future UK Growth F.1	UK	16.52	4.00	0.00	-2.27	3.62	-0.30	0.877	0.918	0.914
Old Mutual Ethical Fund A Acc	UK	16.30	4.00	0.00	-0.85	2.95	-0.21	0.890	0.922	0.918
Prudential Ethical Trust Acc	UK	16.09	4.75	0.00	-0.65	1.29	-0.19	0.883	0.895	0.893
Scottish Widows Environ. Investor	UK	17.68	5.00	0.00	-10.82	2.85	-0.76	0.786	0.824	0.817
Scottish Widows Ethical Fund Acc	UK	13.76	5.00	0.00	-4.03	2.24	-0.50	0.870	0.875	0.872
Sovereign Ethical Fund Inc	UK	16.74	5.50	0.00	-3.56	2.41	-0.36	0.853	0.881	0.877
Standard Life UK Ethical Fund R Acc	UK	15.08	4.00	0.00	0.24	3.92	-0.16	0.904	0.952	0.941
AEGON UK Equity Fund A Acc	UK	14.07	5.50	0.00	1.26	0.00	-0.18	0.916	0.916	0.916
Allchurches UK Equity Growth Fund	UK	14.16	5.00	0.00	5.47	0.00	0.10	0.953	0.953	0.953
AXA UK Equity Income A Acc	UK	15.36	5.00	0.00	6.32	0.00	0.15	0.951	0.951	0.951
CIS UK Growth Trust Inc	UK	13.86	5.00	0.00	1.38	0.00	-0.19	0.919	0.919	0.919
Credit Suisse UK Mid 250 Fund R	UK	16.94	5.25	0.00	5.39	0.00	0.08	0.931	0.931	0.931
F&C UK Growth & Income F.Sh.Cl.1	UK	14.57	5.00	0.00	2.59	0.00	-0.10	0.924	0.924	0.924
F&C UK Equity Income Share Cl.1	UK	13.82	5.00	0.00	3.10	0.00	-0.07	0.935	0.935	0.935
F&C UK Global Growth F.Share Cl.1	UK	15.23	5.00	0.00	-5.14	0.00	-0.52	0.850	0.850	0.850
Family Asset Trust Acc	UK	14.20	6.00	0.00	2.14	0.00	-0.13	0.923	0.923	0.923
Halifax International Growth Fund A	UK	16.67	4.00	0.00	-1.04	0.00	-0.22	0.888	0.888	0.888
Henderson International Fund A Acc	UK	16.95	5.00	0.00	-3.34	0.00	-0.34	0.854	0.854	0.854
Insight Invest.UK Equity Inc.Port.F.A	UK	12.55	5.25	0.00	7.15	0.00	0.24	0.982	0.982	0.982
Insight Investment Global Equity F.A	UK	17.82	5.25	0.00	-2.27	0.00	-0.27	0.859	0.859	0.859
Jupiter Global Opportunities Fund Inc	UK	14.44	5.25	0.00	8.51	0.00	0.39	0.978	0.978	0.978
Jupiter UK Smaller Companies Fund	UK	17.46	5.25	0.00	13.75	0.00	0.52	**1.000**	**1.000**	**1.000**
Legal & General Equity Trust E	UK	14.52	0.00	5.00	0.91	0.00	-0.20	0.954	0.954	0.954
Morley European Focus Fund 1 Acc	UK	18.75	5.00	0.00	5.51	0.00	0.09	0.930	0.930	0.930
Morley Global Theme Fund 1 Acc	UK	14.49	5.00	0.00	-2.73	0.00	-0.36	0.877	0.877	0.877
Morley UK Focus Fund 1 Acc	UK	18.01	5.00	0.00	7.59	0.00	0.20	0.948	0.948	0.948
Old Mutual UK Equity Acc	UK	13.63	3.00	0.00	4.66	0.00	0.04	0.962	0.962	0.962
Prudential UK Growth Trust Inc	UK	14.85	5.50	0.00	-0.02	0.00	-0.27	0.898	0.898	0.898
Scottish Widows UK Equity Income	UK	12.01	5.00	0.00	5.13	0.00	0.10	0.968	0.968	0.968
Scottish Widows UK Growth Fund	UK	12.53	5.00	0.00	3.86	0.00	0.00	0.952	0.952	0.952
Sovereign UK Growth Fund Inc	UK	15.71	5.50	0.00	2.12	0.00	-0.12	0.911	0.911	0.911
Standard Life UK Equity Growth F.R	UK	14.34	4.00	0.00	3.12	0.00	-0.05	0.932	0.932	0.932
CA-AM Activaleurs Durables	FR	20.12	0.00	0.00	-7.94	3.04	-0.43	0.865	0.935	0.929
CLAM Euro Develop. Durable C	FR	20.74	2.00	0.00	-5.36	2.46	-0.29	0.861	0.892	0.889
Credit Mutuel Valeurs Ethiques C	FR	19.82	2.75	0.00	-4.77	1.70	-0.28	0.860	0.881	0.878
Ecureuil 1 2 3Futur D	FR	19.48	2.00	0.00	-5.31	1.59	-0.31	0.865	0.881	0.878
Epargne Ethique Actions	FR	19.16	2.00	0.00	-6.31	3.33	-0.37	0.857	0.897	0.892
Ethiciel C	FR	19.72	2.50	0.00	-2.36	1.91	-0.15	0.885	0.907	0.905
Ethique et Partage C	FR	16.92	0.00	0.00	-7.21	2.86	-0.50	0.883	0.968	0.961
Etoile Environnement C	FR	22.22	2.00	0.00	-6.48	1.05	-0.30	0.848	0.863	0.861
Etoile Partenaires	FR	21.50	2.00	0.00	-4.20	1.05	-0.20	0.869	0.883	0.882
Euro-Mid-Cap Active Investors C	FR	20.02	5.00	0.00	-8.93	2.51	-0.49	0.803	0.836	0.830
Europe Croissance Develop.Durable	FR	19.20	5.00	0.25	-3.90	2.09	-0.25	0.838	0.867	0.863
Europe Gouvernance C	FR	19.07	3.00	0.50	-4.52	1.18	-0.29	0.826	0.843	0.840
EuroSocietale C	FR	21.67	3.00	0.50	-3.36	1.64	-0.16	0.837	0.859	0.856
Federal Actions Ethiques	FR	22.40	3.00	0.00	-5.85	0.55	-0.27	0.846	0.853	0.852
Generation Ethique C	FR	22.42	3.00	0.00	-7.22	2.40	-0.34	0.833	0.866	0.861
HSBC AM Valeurs Responsables	FR	19.16	3.00	0.00	-3.66	1.95	-0.24	0.869	0.892	0.889
Insertion-Emplois D	FR	17.54	0.00	0.00	-6.27	1.48	-0.42	0.890	0.917	0.914
IXIS Euro 21	FR	21.89	2.00	0.00	-5.87	2.35	-0.28	0.854	0.886	0.882
Macif Croissance Durable C	FR	18.24	4.00	2.00	-2.77	2.82	-0.21	0.829	0.888	0.881
Macif Croissance Durable Europe C	FR	19.49	2.00	0.00	-6.12	2.82	-0.36	0.858	0.891	0.886
MAM Actions Environnement C	FR	15.16	2.00	1.00	-4.69	0.66	-0.40	0.837	0.839	0.838
MAM Actions Ethique C	FR	27.35	2.00	1.00	-6.24	2.31	-0.19	0.799	0.857	0.847
MG Croissance Durable Europe	FR	19.48	2.00	0.00	-6.78	2.82	-0.40	0.852	0.885	0.880
Objectif Ethique Socialement Resp.	FR	17.13	4.00	1.00	-2.57	1.77	-0.21	0.837	0.868	0.863
Robeco Hommes Terre Expansion	FR	15.68	3.00	0.50	-10.14	4.02	-0.76	0.791	0.903	0.856
Sarasin Euro Mid-Caps Exp.Dur.C	FR	22.29	2.00	0.00	4.11	1.91	0.19	0.944	0.971	0.970
SGAM Invest Develop. Durable C	FR	18.65	5.00	0.00	-5.59	2.82	-0.36	0.832	0.869	0.864
CA-AM Actions Euroland C	FR	19.11	2.50	0.00	-4.80	0.00	-0.39	0.865	0.865	0.865
CLAM Actions Euro C	FR	19.49	5.00	0.00	-3.18	0.00	-0.29	0.853	0.853	0.853
Credit Mutuel Europe Actions C	FR	16.64	2.75	0.00	-5.13	0.00	-0.47	0.866	0.866	0.866
Ecureuil Actions Europeennes D	FR	18.83	2.00	0.00	-3.47	0.00	-0.32	0.884	0.884	0.884
Ecofi European Stocks C	FR	16.26	4.00	0.00	-5.15	0.00	-0.48	0.852	0.852	0.852
Amplitude Europe C	FR	18.33	2.50	0.00	-5.40	0.00	-0.44	0.862	0.862	0.862
MAM Actions Europe C	FR	18.50	2.00	1.00	-3.48	0.00	-0.32	0.823	0.823	0.823
Etoile Index Euro C	FR	20.87	2.00	0.00	-4.44	0.00	-0.33	0.869	0.869	0.869
Groupama Euro Stock C	FR	18.25	2.75	0.00	-0.82	0.00	-0.19	0.900	0.900	0.900
Federal Epargne Actions	FR	16.62	1.00	0.00	0.91	0.00	-0.10	0.945	0.945	0.945

Fund name	Country	Std. Dev.	Init. fee	Exit fee	Mean return	Ethic. level	Sharpe ratio	DEA S	DEA SE	DEA SEef
Ina Actions Europeennes C	FR	17.18	3.00	1.00	-5.51	0.00	-0.48	0.812	0.812	0.812
HSBC AM Actions Europe C	FR	18.03	5.00	0.00	-2.79	0.00	-0.30	0.857	0.857	0.857
IXIS AM Europe Large Cap I D	FR	17.73	4.00	0.00	-2.84	0.00	-0.30	0.868	0.868	0.868
IXIS Euro Actions C	FR	22.35	2.20	0.00	-6.18	0.00	-0.40	0.849	0.849	0.849
MAM Euro Indice C	FR	22.27	2.00	1.00	-3.48	0.00	-0.27	0.823	0.823	0.823
Objectif Actions Euro	FR	16.28	4.00	1.00	-0.81	0.00	-0.20	0.861	0.861	0.861
Robeco Global Equties (EUR) D	FR	15.26	1.00	0.00	-8.71	0.00	-0.76	0.859	0.859	0.859
SGAM Invest Euro Actions C	FR	17.91	2.00	0.00	-0.13	0.00	-0.16	0.918	0.918	0.918
ABN AMRO Socially Resp.Equity F.	LU	17.38	5.25	0.00	-14.14	3.39	-0.93	0.755	0.817	0.792
Activest Eco Tech C Inc	LU	14.77	5.00	0.00	-2.88	0.89	-0.29	0.873	0.874	0.873
Aviva F. Eur. Socially Resp.Equity F.	LU	16.72	5.00	0.00	-2.94	3.62	-0.24	0.859	0.905	0.898
Credit Swiss Equity F.LUX Glob.Sust.B	LU	15.47	5.00	0.00	-10.32	3.64	-0.78	0.802	0.862	0.830
Dexia Equities L World Welfare C	LU	15.30	6.00	0.00	-6.86	3.17	-0.56	0.834	0.863	0.856
Henderson Horiz.Glob.Sust.Invest.F.A2	LU	15.94	5.00	0.00	-10.66	3.99	-0.77	0.795	0.891	0.832
Ing (L) Invest Sustainable Growth P	LU	16.42	3.00	0.00	-10.48	4.58	-0.74	0.815	**1.000**	**1.000**
Ing (L) Invest Sustainable Growth X	LU	16.42	5.00	0.00	-10.96	4.58	-0.77	0.790	0.975	0.901
Meridion Green Balance Acc	LU	18.73	4.00	0.00	-10.79	2.13	-0.64	0.795	0.822	0.817
Pioneer F.Global Ethical Equity E Eur	LU	17.13	4.75	0.00	-8.27	3.64	-0.56	0.813	0.875	0.852
Pioneer Funds Global Ethical Equity F	LU	17.06	0.00	0.00	-8.93	3.64	-0.61	0.867	**1.000**	**1.000**
Pioneer F. Global Ethical Equity G	LU	16.97	2.50	0.00	-8.27	3.64	-0.57	0.839	0.893	0.875
Post Bank Dynamic Vision Acc	LU	14.14	3.75	0.00	-3.44	1.82	-0.35	0.877	0.891	0.889
SAM Sustainable Leaders Fund Acc	LU	16.55	5.00	0.00	-7.86	2.82	-0.57	0.816	0.849	0.843
SAM Sustainable Pioneer Fund Acc	LU	21.93	5.00	0.00	-10.78	3.29	-0.53	0.786	0.830	0.822
SAM Sustainable Water Fund Acc	LU	13.29	5.00	0.00	0.36	2.17	-0.11	0.914	0.914	0.914
Sarasin New Energy Fund Acc	LU	20.50	5.00	0.00	-6.46	0.95	-0.35	0.824	0.837	0.835
Sarasin ValueSar Equity Inc	LU	17.02	5.00	0.00	-8.80	4.11	-0.60	0.806	0.913	0.864
SEB Etisk Europafond-Lux Acc	LU	18.73	0.00	0.00	-0.26	0.82	-0.06	0.943	0.952	0.951
SEB Etisk Globalfond-Lux Acc	LU	18.39	0.00	0.00	-10.74	0.94	-0.66	0.845	0.858	0.855
SEB Etisk Sverigefond-Lux Inc	LU	24.86	0.00	0.00	0.40	0.82	0.01	0.929	0.949	0.948
SEB Invest OkoLux Acc	LU	16.86	4.50	0.00	-13.62	2.99	-0.93	0.769	0.805	0.797
Swisscanto LU Port.F.Green Inv.Eq.B	LU	16.26	5.00	0.00	-7.02	3.76	-0.53	0.826	0.893	0.862
UBS LUX Equity F.Eco Performance CHF	LU	17.67	4.00	0.00	-8.30	3.93	-0.54	0.820	0.901	0.861
Oko-Altienfonds Acc	LU	21.05	5.00	0.00	-13.51	3.07	-0.70	0.762	0.803	0.794
Okovision Acc	LU	13.27	5.00	0.00	-1.30	4.05	-0.22	0.899	0.953	0.926
ABN AMRO Global Equity Fund Acc	LU	16.44	5.25	1.00	-11.41	0.00	-0.88	0.767	0.767	0.767
Aviva Funds European Equity Fund	LU	20.06	5.00	0.00	1.60	0.00	-0.04	0.895	0.895	0.895
Dexia Equities L World C Acc	LU	15.16	6.00	0.00	-9.29	0.00	-0.81	0.813	0.813	0.813
Henderson Horizon Glob.Equity F.A2 C	LU	16.88	5.00	0.00	-7.24	0.00	-0.59	0.820	0.820	0.820
ING (L) Invest Global High Dividend P	LU	15.27	5.00	0.00	0.71	0.00	-0.12	0.902	0.902	0.902
Pioneer Funds - Global Equity E Acc	LU	16.67	5.00	0.00	-7.39	0.00	-0.61	0.820	0.820	0.820
Pioneer Funds - Global Equity F Acc	LU	16.17	0.00	0.00	-8.29	0.00	-0.69	0.876	0.876	0.876
Pioneer Funds - Global Equity G Acc	LU	16.84	2.50	0.00	-7.52	0.00	-0.61	0.847	0.847	0.847
Sarasin EquiSar (Lux) Inc	LU	15.74	5.00	0.00	-1.58	0.00	-0.26	0.878	0.878	0.878
UBS (Lux) Strategy Fund Equity CHF B	LU	13.80	6.00	2.00	-3.88	0.00	-0.47	0.858	0.858	0.858
Meridio Standard Vision (t)	LU	21.56	5.00	0.00	-4.13	0.00	-0.31	0.845	0.845	0.845
Postbank Dynamik Global (t)	LU	15.20	3.75	0.00	-8.53	0.00	-0.75	0.827	0.827	0.827
Julius Baer Glob.Megatrend Stock F.B	LU	21.16	5.00	0.00	-13.09	0.00	-0.78	0.766	0.766	0.766
SEB Invest Global Players (a)	LU	18.41	3.75	0.00	-12.28	0.00	-0.84	0.785	0.785	0.785
ACATIS Champions Select Champ.Glob.	LU	22.82	5.50	0.00	1.70	0.00	-0.04	0.894	0.894	0.894
Athena World Equity C Acc	BE	14.31	2.00	0.00	-8.84	3.29	-0.75	0.848	0.927	0.909
Dexia Allocation Sustainable Europe	BE	15.69	1.00	0.00	-4.81	2.32	-0.40	0.895	0.931	0.925
Dexia Allocation Sustainable World	BE	15.76	1.00	0.00	-8.27	2.32	-0.62	0.862	0.900	0.891
Dexia Sustainable Accent Earth Acc	BE	15.68	3.00	0.00	-7.44	3.17	-0.58	0.845	0.877	0.871
Dexia Sustainable Accent Social Acc	BE	15.45	3.00	0.00	-5.33	3.17	-0.45	0.865	0.897	0.892
Dexia Sustainable EMU Acc	BE	18.90	3.00	0.00	-5.18	3.17	-0.33	0.856	0.895	0.890
Dexia Sustainable Europe Acc	BE	16.88	3.00	0.00	-5.40	3.17	-0.40	0.860	0.893	0.888
Dexia Sustainable North America Acc	BE	16.99	3.00	0.00	-9.75	3.17	-0.60	0.820	0.854	0.847
Dexia Sustainable Pacific Acc	BE	16.70	3.00	0.00	0.67	3.17	0.04	0.916	0.948	0.945
Dexia Sustainable World Large Caps	BE	15.25	3.00	0.00	-7.41	3.17	-0.60	0.846	0.878	0.873
KBC ECO Fund Acc	BE	18.42	3.00	0.00	-5.82	2.82	-0.38	0.852	0.884	0.880
KBC ECO Fund Ethi Equity Euroland	BE	22.18	3.00	0.00	-5.74	2.82	-0.28	0.847	0.885	0.880
Athena Global Opportunities C	BE	8.50	2.00	0.00	5.31	0.00	0.32	**1.000**	**1.000**	**1.000**
Dexia Allocation European Equities	BE	15.48	1.00	0.00	-3.35	0.00	-0.38	0.909	0.909	0.909
Dexia Allocation World Equities Acc	BE	15.22	1.00	0.00	-6.75	0.00	-0.63	0.878	0.878	0.878
Dexia Invest EMU Acc	BE	17.93	3.00	0.00	-3.03	0.00	-0.31	0.878	0.878	0.878
Dexia Invest Europe Acc	BE	15.92	3.00	0.00	-5.40	0.00	-0.50	0.863	0.863	0.863
Dexia Invest US Acc	BE	16.56	3.00	0.00	-12.26	0.00	-0.87	0.798	0.798	0.798
Dexia Invest Japan Acc	BE	20.61	3.00	0.00	-1.07	0.00	-0.06	0.889	0.889	0.889
Dexia Index World Acc	BE	15.42	2.00	0.00	-5.30	0.00	-0.52	0.878	0.878	0.878
KBC Equity Fund - World Acc	BE	17.16	3.00	0.00	-10.07	0.00	-0.77	0.817	0.817	0.817
KBC Equity Fund - Europe Acc	BE	18.96	3.00	0.00	-3.02	0.00	-0.30	0.875	0.875	0.875
ABN AMRO Duurzame Wereld Fonds	NL	17.98	0.50	0.50	-12.88	3.57	-0.80	0.776	0.889	0.867
ASN Aandelenfonds	NL	21.60	0.40	0.40	-9.90	1.32	-0.48	0.803	0.825	0.819
ASN Milieufonds	NL	20.65	0.40	0.40	-12.76	1.32	-0.67	0.779	0.799	0.793
ING Duurzaame Rendement Fonds	NL	16.74	0.50	0.50	-9.32	4.70	-0.64	0.811	**1.000**	**1.000**
Postbank Duurzaame Aandelenfonds	NL	15.01	0.40	0.40	-9.46	4.58	-0.75	0.825	**1.000**	**1.000**
Robeco DuurzaameAandelen	NL	16.21	0.40	0.40	-11.32	3.46	-0.81	0.804	0.902	0.883
SNS Duurzaame Aandelenfonds	NL	17.32	0.50	0.50	-5.63	1.85	-0.38	0.842	0.878	0.871
Triodos Meerwaarde Aandelenfonds	NL	12.66	0.50	0.50	-3.91	4.32	-0.45	0.920	**1.000**	**1.000**
ABN AMRO Global Fund	NL	18.14	0.50	0.50	-9.97	0.00	-0.71	0.801	0.801	0.801

Fund name	Country	Std. Dev.	Init. fee	Exit fee	Mean return	Ethic. level	Sharpe ratio	DEA S	DEA SE	DEA SEef
ING Global Fund	NL	17.43	0.50	0.50	-8.18	0.00	-0.62	0.819	0.819	0.819
Postbank Wereldmerken Fonds	NL	17.01	0.40	0.40	-4.66	0.00	-0.43	0.862	0.862	0.862
Robeco Global Equities (EUR) D Acc	NL	15.26	1.00	0.00	-8.71	0.00	-0.76	0.859	0.859	0.859
SNS Euro Aandelenfonds	NL	16.91	0.50	0.50	-4.30	0.00	-0.40	0.855	0.855	0.855
3 Banken Global Oko-Mix Acc	AT	16.46	5.00	0.00	-8.93	0.36	-0.63	0.807	0.809	0.808
ESPA Stock Umwelt T Acc	AT	13.95	4.00	0.00	-5.84	0.36	-0.54	0.853	0.855	0.855
Hypo Global Value Acc	AT	17.30	3.50	0.00	-8.87	2.58	-0.59	0.821	0.849	0.844
KEPLER Sustainability Aktienfonds A	AT	16.08	4.00	0.00	-9.83	3.41	-0.72	0.810	0.848	0.841
KEPLER Sustainability Aktienfonds T	AT	16.09	4.00	0.00	-9.81	3.41	-0.71	0.810	0.848	0.841
Klassik Oko Trends T Acc	AT	15.88	5.00	0.00	-8.15	2.91	-0.61	0.818	0.848	0.842
ESPA Stock Global T (t)	AT	15.57	5.00	0.00	-6.53	0.00	-0.59	0.835	0.835	0.835
KEPLER Global Aktienfonds A (a)	AT	16.18	4.00	0.00	-6.11	0.00	-0.54	0.843	0.843	0.843
KEPLER Global Aktienfonds T (t)	AT	16.18	4.00	0.00	-6.10	0.00	-0.54	0.844	0.844	0.844
Klassik Aktien A (a)	AT	13.89	5.00	0.00	-0.25	0.00	-0.20	0.904	0.904	0.904
Gutmann Europa-Portfolio (t)	AT	16.05	4.00	0.00	-1.60	0.00	-0.26	0.884	0.884	0.884
Pictet (CH) Sustainable Equities P Inc	CH	14.45	5.00	0.00	-1.18	3.99	-0.05	0.891	0.949	0.921
Raiffeisen-Fonds Futura Global Stock A	CH	17.34	1.50	0.00	-8.62	4.23	-0.57	0.847	**1.000**	**1.000**
Raiffeisen-Fonds Futura Swiss Stock A	CH	21.11	1.50	0.00	3.22	4.23	0.23	0.945	**1.000**	**1.000**
Swisscanto CH Equity F. Green Invest	CH	16.05	5.00	0.00	-6.27	3.76	-0.48	0.834	0.897	0.869
Raiffeisen-Fonds Global Invest 100 A a	CH	15.63	5.00	0.00	-6.35	0.00	-0.59	0.836	0.836	0.836
Raiffeisen-Fonds SwissAc B (t)	CH	16.71	5.00	0.00	-2.86	0.00	-0.22	0.859	0.859	0.859
Swisscanto (LU) Portfolio F.Equity B t	CH	13.63	5.00	0.00	-3.27	0.00	-0.44	0.878	0.878	0.878
Gerling Select 21 (t)	DE	16.93	4.50	0.00	-10.67	3.03	-0.72	0.795	0.831	0.824
INVESCO Umwelt Nachhaltigkeits F.t	DE	17.41	4.50	0.00	-13.45	2.75	-0.88	0.769	0.803	0.795
Gerling Ivera Fonds (t)	DE	15.26	4.50	0.00	-2.67	0.00	-0.34	0.872	0.872	0.872
INVESCO Global Dynamik Fonds	DE	16.38	5.00	0.00	-3.19	0.00	-0.35	0.859	0.859	0.859
Ducato Etico Geo Acc	IT	17.31	3.00	0.00	-10.53	1.99	-0.70	0.812	0.831	0.826
Sanpaolo Azionario Internaz. Etico	IT	16.11	0.00	0.00	-11.45	2.97	-0.82	0.846	0.975	0.965
Sanpaolo Azioni Internazionali Acc	IT	15.55	4.00	0.00	-12.15	0.00	-0.91	0.791	0.791	0.791
Ducato Geo Globale Acc	IT	14.75	3.00	0.00	-9.30	0.00	-0.75	0.830	0.830	0.830
GreenEffects NAI-Wertefonds Acc	DB	16.39	4.00	0.00	-5.04	3.48	-0.38	0.852	0.892	0.886
Mellon Eur.Ethical Index Tracker F.A	DB	19.38	5.00	0.00	-3.66	2.27	-0.23	0.849	0.879	0.875
Mellon Continental Eur.Equity Portfolio	DB	16.61	5.00	0.00	-1.79	0.00	-0.26	0.870	0.870	0.870

7 An analysis of the European market of ethical mutual funds

We have used the DEA models proposed in the previous sections for the evaluation of the performance of ethical mutual funds with non negative outputs in order to analyze the European market of ethical mutual funds.

The analysis refers to the the three-year period 31/01/2002 to 31/01/2005 and takes into consideration a large number of ethical funds from western European countries in which the phenomenon of ethical investing is significant.

We have used the 'SRI Funds Service' data base and have included in the analysis all the ethical equity funds for which the data on input and output variables were available during the period investigated. In such a way, a total of 159 ethical equity funds were obtained, domiciled in 11 different countries. The number of ethical mutual funds of equity typology comprised in the study is considerable for Sweden (38 funds), United Kingdom (32 funds), France (27 funds) and Luxembourg (26 funds), while it is less substantial for the other European countries: Belgium (12 funds), The Netherlands (8 funds), Austria (6 funds), Swiss (4 funds), Germany (2 funds), Italy (2 funds) and Dublin (2 funds).

In addition, in order to compare the performance obtained by ethical and non ethical funds, we have included in the set of funds analyzed also a non ethical fund with analogous features for each ethical fund considered, each time one such non ethical fund was offered by the same fund company (source: Morningstar Europe).

On the whole, the set consists of 269 equity funds, 159 ethical and 110 non ethical funds. The list of all funds considered is displayed in table 1. This table lists the funds grouped by country (SE=Sweden, UK=United Kingdom, FR=France, LU=Luxembourg, BE=Belgium, NL=The Netherlands, AT=Austria, CH=Switzerland, DE=Germany, IT=Italy, DB=Dublin), arranging the ethical funds first (those with a strictly positive value for the ethical level).

Table 1 reports also the relevant data for each fund and the results of the analysis carried out. In particular, the table reports the rate of return and the volatility of each fund, computed as per cent values on an annual base, the initial and exit fees and the ethical measure (16), computed by using the weights $\omega^N = \omega^P = 2$ and $\omega^C = 1$ (notice that this choice of the weights stresses the screening activity of the ethical funds). In the analysis, the initial capital invested has been set equal to 1 for each fund.

The last columns of table 1 show the performance measure obtained using the Sharpe ratio (see Sharpe, 1966)

$$I_{j,Sharpe} = \frac{\overline{R}_j - r}{\sigma_j} \tag{59}$$

and the DEA performance measures I_{DEA-S}, I_{DEA-SE} and $I_{DEA-SEef}$.

From table 1 it can be seen that 79 % of the funds analyzed exhibit a negative average rate of return in the period considered in our analysis, and for no less that 86 % of the funds the observed excess return is negative. This clearly shows that it is necessary to use a DEA model which is able to cope with such cases.

On the other hand, the negativity of the observed excess return entails the negativity of the Sharpe ratio and this can be misleading. Indeed, only when the excess return is positive, the (positive) value of the Sharpe ratio decreases with the risk indicator σ_j, as we would expect for a performance indicator; on the contrary, when the excess return is negative, the (negative) value of the Sharpe ratio increases with the value of the standard deviation.

The values stressed in boldface in the last three columns of table 1 indicate the funds that are efficient in the sense of Pareto-Koopmans, namely the funds with a DEA score equal to 1 and for which it is not possible to reduce any input or increase any output without worsening the value of some other inputs or outputs; they can be identified by solving a convenient two-phase linear programming problem equivalent to the original DEA problem (see for example Cooper et al., 2006). The efficient funds are 5 for the *DEA-S* model, 4 non ethical funds and only 1 ethical fund (Ohman Etisk Index Pacific). The number of efficient ethical funds increases considerably when the ethical objective is taken into account; actually, in the ethical models *DEA-SEef* and *DEA-SE* a good 13 ethical funds turn out to be efficient.

Of course, as stated by theorem 2, the non ethical funds do not change their efficiency score when the ethical models are used and neither does the number of efficient funds.

In order to analyze how the efficiency measure changes with the DEA model used, figure 1 compares the empirical cumulative distribution function of the DEA efficiency measures I_{DEA-S}, I_{DEA-SE} and $I_{DEA-SEef}$ obtained for the ethical funds analyzed with the three models considered. By theorem 1, the cumulative distribution function of model *DEA-S* lies above that of model *DEA-SEef*, and the latter lies above that of model *DEA-SE*. Figure 1 shows that the more considerable increase in the DEA score, which corresponds to the more notable shift rightwards in the cumulative distribution function, takes place when the ethical objective is taken into account, while the difference is much slighter between the

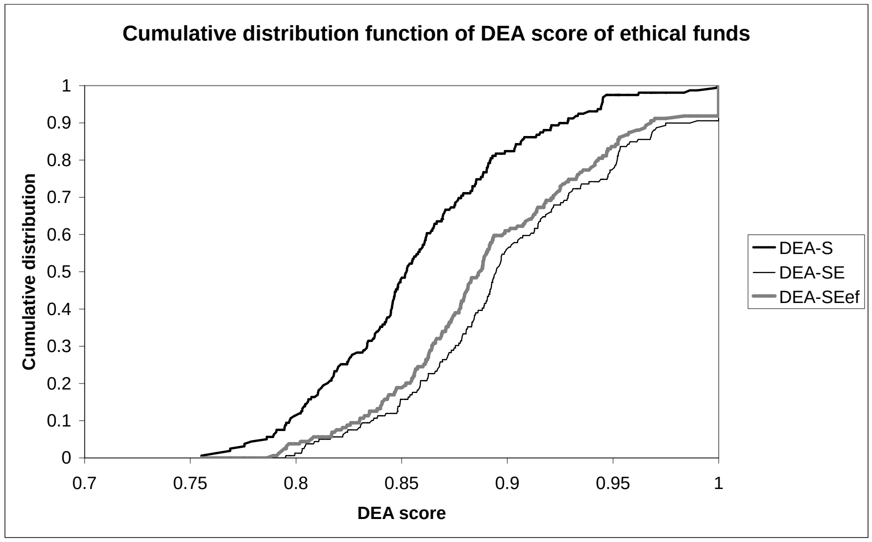


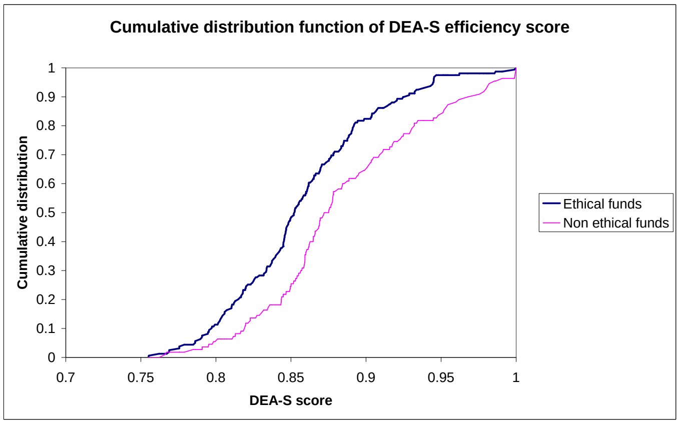
Figure 1: Empirical cumulative distribution function of the DEA efficiency measures I_{DEA-S}, I_{DEA-SE} and $I_{DEA-SEef}$ for the European ethical funds analyzed.

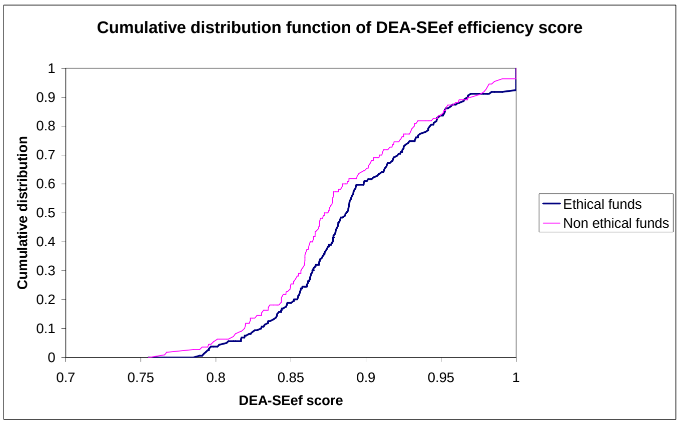
two ethical models. Actually, the average DEA score of the ethical funds is equal to 0.857 for the *DEA-S* model, 0.894 for the *DEA-SEef* model and 0.903 for the *DEA-SE* one. As regards the non ethical funds, their average DEA score is equal to 0.884, which is higher than the average score of the ethical funds when the ethical measure is not considered, while it is lower than that of the ethical funds in the models that take the ethical measure into account.

The comparison of the DEA efficiency measures between ethical and non ethical funds is highlighted in figure 2, which compares the empirical cumulative distribution functions of the DEA efficiency measures I_{DEA-S}, I_{DEA-SE} and $I_{DEA-SEef}$ for the ethical and non ethical mutual funds. It can be noticed that the non ethical funds obtain a sensibly higher efficiency score than the ethical funds when the only output variable taken into consideration is the capitalization factor, i.e. in the *DEA-S* model. The improvement in the efficiency score obtained by the ethical funds when the ethical indicator is included in the set of outputs causes the dominance to reverse in the ethical models, so that with these models the highest efficiency scores are obtained by the ethical funds.

8 Conclusions

In this paper we have proposed an extension to some DEA models for the evaluation of mutual fund performance which enables to tackle the problem of negative data that often occurs with the DEA models suggested by the literature in slump periods of the business cycle.





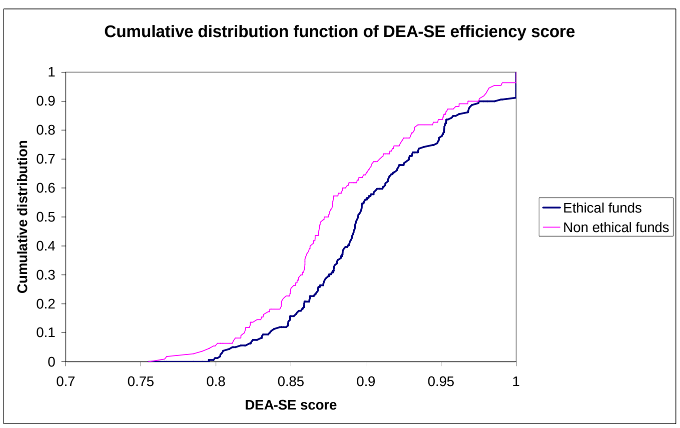


Figure 2: Comparison of the empirical cumulative distribution functions of the DEA efficiency measures I_{DEA-S}, $I_{DEA-SEef}$ and I_{DEA-SE} for the ethical and non ethical European mutual funds.

The first extension leads to a basic model for the measurement of the performance of general mutual funds with a pure investment goal. The second extension considers a model, applicable to ethical investments, which includes also an ethical objective among the outputs of the mutual funds. The third extension is applicable to ethical mutual funds in the case in which investors fix a priori the ethical level desired and maximize the investment return by considering an ethical level as an exogenously fixed output.

The investigation carried out on data from the European market of ethical mutual funds shows that the highest values of the performance measure are generally obtained by the ethical funds if we use a model that takes the ethical level into account. On the contrary, the performance score obtained by the ethical funds is generally lower than that of the non ethical funds when the only output variable taken into consideration is the capitalization factor.

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