Department of Applied Mathematics, University of Venice

WORKING PAPER SERIES

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Asset Price Dynamics with Small World Interactions under Heterogeneous Beliefs

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March 2007

Abstract. We propose a simple model of a financial market populated with heterogeneous agents. The market represents a network with nodes symbolizing the agents and edges standing for connections between them, thus, embodying local interactions in the market. By local interactions we mean any kind of interplay between the decisions of the agents unaffected by the market mechanism and unrelated to the physical distance between the agents. Using the rewiring procedure we restructure a network from regular lattice to random graph by varying the probability of the agents to switch from one trading strategy to another. We study how the network structure influences the asset price dynamics. The results show that for some intermediate values of the probability to switch, corresponding to a small world network, the price dynamics become reminiscent to the real. While for the boundary values of the probability the dynamics lacks some typical features of the real financial markets.

Keywords: local interactions, networks, small world, heterogeneous beliefs, price dynamics, bifurcations, chaos.

JEL classification: C45, C62, C63, D84, G12.

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1. Introduction

Financial markets modelling is a constantly growing research area within the field of financial economics. It is highly attractive to the researchers who are armed with computational and numerical methods due to the structure of the real financial markets that are populated with a large number of various interacting traders. The impact of a single market participant on the dynamics of the price is usually negligibly small, however, she may influence the behavior of a small group of other traders who, in turn, can affect the behavior of others and, thus, influence the whole market to some extent. Financial markets, therefore, can be viewed as a "soup" of diverse agents who trade in attempt to maximize their profits and who interact intensively with each other making the market resemble a constantly boiling mixture.

Multi-agent modelling is a major tool to cope with such dramatically complex systems with intricate interactions among their constituents. It allows to employ the bottom-up approach, focusing on micro level of the agents interaction but aiming at studying macro effects of the asset price dynamics. Many of these systems are now studied with the help of computers that gained greatly in speed of computations in the recent years. One can tentatively distinguish two types of financial market models: heterogeneous agent models and agent-based models.

The models of the first type are relatively simple and analytically tractable. They posses an important feature of rigorous microeconomic foundation incorporated in their set-up while allowing for heterogeneity. Due to their analytical nature often closed-form solutions can be found, otherwise advanced numerical methods of nonlinear dynamics and bifurcation theory can be applied for the analysis. An excellent recent survey of the state of the art in heterogeneous agent modelling is written by Hommes (2006) in the Handbook of Computational Economics.

The initial steps on the way to the heterogeneous agent approach were taken by several scholars in the early ninetieth, among which are Day and Huang (1990), DeLong et al. (1990), Chiarella (1992), Kirman (1993) and Lux (1995). These early models focused mostly on the stylized analysis of the simple behavioral rules as a cause of endogenous fluctuations.

Following this line of research, Brock and Hommes (1997) and Brock and Hommes (1998) introduced the notion of Adaptive Belief System. In their set-up several types of agents have diverse beliefs about the future, which they adapt from the past history by switching from one strategy to another according to a certain fitness measure. With the help of methods of nonlinear dynamics the authors showed that a market populated with heterogeneous agents trading repeatedly in a Walrasian framework combined with evolutionary updating of their beliefs is able to replicate some of the stylized facts and reproduce chaotic behavior of asset prices.

Anufriev and Bottazzi (2004) supplemented the above model with heterogeneous horizons of the agents, while Anufriev and Panchenko (2006) investigated the changes in the model outcomes when different market mechanisms are introduced. de Fontnouvelle (2000) enriched the model with various information flow schemes about

the dividend payments, Brock et al. (2005) examined an extension to many trader types, Gaunersdorfer (2000) introduced heterogeneity of beliefs with respect to the returns volatility, Hommes et al. (2005) included a market maker into the market pricing mechanism, and Brock et al. (2006) studied how the presence of risk hedging instruments in form of Arrow securities affects market dynamics.

The second type of financial market models, the agent-based models, evolved from the first attempts to create real-proportion artificial stock markets simulated on computers. Santa Fe market (Arthur et al., 1997) is an example of such an approach. Some of the subsequent agent-based models are Großklags et al. (2000), Chen and Yeh (2001), Chen et al. (2001) and Duffy (2001). Such models are more computationally oriented and deal with a large number of artificial agents. Their major advantage is that they allow for richer behavioral assumptions and more realistic market architectures. They are generally implemented on computers and the results are almost always presented in the form of simulations. Closed-form solutions or numerical analysis of these models are practically unmanageable either due to their overwhelming complexity, or because their set-up could include units that may be difficult to tackle analytically, e.g. genetic algorithms or network formation modelling. A thorough overview of the filed of the agent-based modelling is written by LeBaron (2006) to which we refer the reader.

The communication effects in the multi-agent framework subject to different agents interaction arrangements are relatively unexplored. Commonly, the agents are either assumed to be randomly connected with each other or are conceived to interact on a completely regular basis, although many real-world complex systems possess some intermediate properties (Watts and Strogatz, 1998). We incorporate a network framework into the heterogeneous agent model of Brock and Hommes (1998) and transform it into an agent-based model by imposing additional structure on the agents interactions.

Our aim is to study asset price dynamics and its stability subject to different local interaction arrangements. We examine the effect of various network topologies on market behavior starting from a regular lattice, shifting through faintly random to completely stochastic structures, thus, exploring the whole range of possible interaction patterns. We implement the analysis of statistical properties of the simulated time series and conclude that networks possessing small-world properties engender the most realistic market dynamics.

The market in our model represents a network with nodes symbolizing the agents and edges standing for connections between them, thus, embodying local interactions in the market. A network is represented as a graph with many heterogeneous nodes connected by un-weighted and undirected links. By local interactions we mean any kind of interplay between the decisions of the agents unaffected by the market mechanism and unrelated to the physical distance between the agents. As a benchmark case we take a fully connected graph that represents a network implicitly modelled in most of the multi-agent models with agents possessing information about all the other traders in the market. We assume that the agents are passive with respect

to the network topology, thus, taking the network as given and not being able to modify it.

Our approach, therefore, is a side-step from an analytically tractable heterogeneous agent model of Brock and Hommes (1998) towards more computationally oriented agent based modelling. Our model retains attractive microeconomic foundation of heterogeneous agent models and, at the same time, attains more realistic structure of local interactions imposed on the traders in the market. Hence, we are able to replenish the model of Brock and Hommes (1998) with an even more credible framework of the agents interplay. We show that the level of connectivity and randomness of the network has a profound effect on the price dynamics.

The structure of the paper is the following. In the next section we provide an overview of the literature on the agents interaction modelling in financial markets, describe different structures of networks and examine their properties. Section 3 focuses on our model of network theory application to the model of Brock and Hommes (1998). In Section 4 we describe the artificial market, present and discuss the results of the simulations. Section 5 concludes the paper.

2. Local interactions and network design

To a large extent ideas and practices are adopted by communities through interpersonal communication. Popular ideas in financial markets, too, often spread through conversations (Shiller, 1995). Shiller and Pound (1989) surveyed 131 institutional investors in the United States. They found that money managers who invested in stocks that experienced extremely high growth of the price/earnings ratio were often participating in interpersonal communication with colleagues from within and outside of their institutions regarding the stocks they purchased. Arnswald (2001) surveyed fund managers in Germany and analyzed 275 completed questionnaires. He found that information exchange of fund managers with other financial and industry experts seconded only by conversations with their colleagues and reports from media. Madrian and Shea (2000) and Duflo and Saez (2002) show that workers are more likely to join an investment retirement scheme if their colleagues have done so. By reviewing data from the Health and Retirement Study, Hong et al. (2004) conclude that interaction with their neighbors or church attendance increases the likelihood of a household investing in stocks. A study of fund managers by Hong et al. (2005) also provides strong support for the importance of word-of-mouth effects.

In addition to the empirical evidence of significance of local interactions among market investors, many attempts were made to model this phenomenon explicitly. Among the first papers on the topic is Baker and Iyer (1992). The authors modelled financial market as a network of agents and examined the effect of network structure on price volatility and traded volume by considering several regular market arrangements. In their model the agents are homogeneous and only upon the introduction of a random signal in the network trade in the market could be achieved: the agents positioned in a certain neighborhood of the buy/sell inducing signals arrival blindly

copy them, thus, giving rise to different trading strategies. This set-up allowed to show that network structure affects market dynamics.

Another early contribution is Kirman (1993), who proposed a stochastic framework with the agents forming preferences based on purely random communication with the others. A later paper also based on random matching is Cont and Bouchaud (2000), who considered the market traders forming clusters through random interaction with no subsequent trade inside but only between such clusters. Lux (1995) studied mimetic contagion of the agents in the market in a similar to Kirman (1993) manner. However, in his set-up each agent can observe trading attitude (optimistic, pessimistic or neutral) of all the other traders and, depending on the predominant mood in the market, can switch her own trading strategy from an unpopular to the prevailing one. The above models exemplify the use of two extreme local interaction arrangements: a random network and a fully connected graph.

Some papers deal with financial markets formed as lattices, an example of another salient network structure with all the traders having the same number of neighbors. Specifically, Iori (2002) consider a market with the agents forming a square lattice, so that each agent is connected to precisely four neighbors. Such a set-up is a version of an often employed Ising model, whose limitation, as well as of any fully connected network, is the assumption that every agent has the same number of neighbors connected with him. This fact is at odds with some empirical findings. For example, Shiller and Pound (1989) found that the number of neighbors varied pretty much for different stock market investors, with the figure from their survey ranging between 7 and 21.

In our model we are not stick to any particular network structure, in contrast, we analyze how network topology affects the market. Our approach resembles that of Baker and Iyer (1992), however, we exploit the method of Watts and Strogatz (1998) to construct different random networks through rewiring procedure, thus, avoiding the necessity to pick only regular networks for the analysis. In essence, the rewiring procedure breaks off the tie of a randomly chosen edge and attaches it to another one as the stochastic connectivity parameter varies altering the homogeneous connectivity of the nodes. Hence, our model overcomes the shortcomings of fully connected and lattice-like networks. Moreover, we assume heterogeneity of the agents and employ an adapted version of a realistic market interaction mechanism of Brock and Hommes (1998), which allows to model local interactions in a trustworthy way.

Several works in the field of diffusion of knowledge advanced the topic of network effects in a very close manner to ours. In particular, Cowan and Jonard (2004) study the effects of the diffusion of a knowledge vector in various network topologies. Knowledge is bartered only between connected agents and only if there is a double coincidence of wants for the agents. Further, Morone and Taylor (2004) extend their model by allowing the network structure to evolve as a result of interactions and memory about previous interactions. Although we do not address the network formation issue, our model resembles the two above mentioned in the way the network topology is seen to influence information distribution in the market.

The way we model local interactions goes back to the chain-letter experiments of Stanley Milgram conducted in 1967. He showed that any two people are separated by about six degrees within their acquaintance network (Milgram, 1967). In addition, works by Wasserman and Faust (1994) and Valente and Davis (1999), who performed social network analysis studies, suggested that a typical social network has the following features (Watts, 1999): 1) there are many participants in the network; 2) each participant is connected to a small fraction of the entire network, in other words, the network is sparse; 3) even the most connected node is still connected only to a small fraction of the entire network, that is, the network is decentralized; 4) neighborhoods overlap, i.e. the network is clustered.

To capture these characteristics Watts and Strogatz (1998) introduced a network model called a "small world". Their framework assumes a shift from a regular lattice through small world networks to a random graph depending on the stochastic connectivity parameter. Such a model accounts for a wide range of organizational topologies that are neither regular nor completely random and has been used to simulate the properties of real-world networks.

By now, a long list of networks with small world properties has been discovered, among which are social networks of the US corporate elite (Davis et al., 2003), partnerships of investment banks in Canada (Baum et al., 2003), and many more. Small world networks emerge when participating agents form networks due to a mix of random and strategic interactions (Baum et al. (2003) and Morone and Taylor (2004)).

To build an example of a small world network we start with a regular lattice with 20 vertexes,[1] which is depicted in Figure 1. Each node is connected to two nodes on each side, that is, each node has 4 edges. With some probability p an edge is reconnected to a different, randomly chosen, node on the lattice (avoiding self- and double-connection). Such rewiring of the nodes continues until all the edges are processed. For the boundary values of the rewiring probability the network remains regular ($p = 0$) or becomes a random graph ($p = 1$).
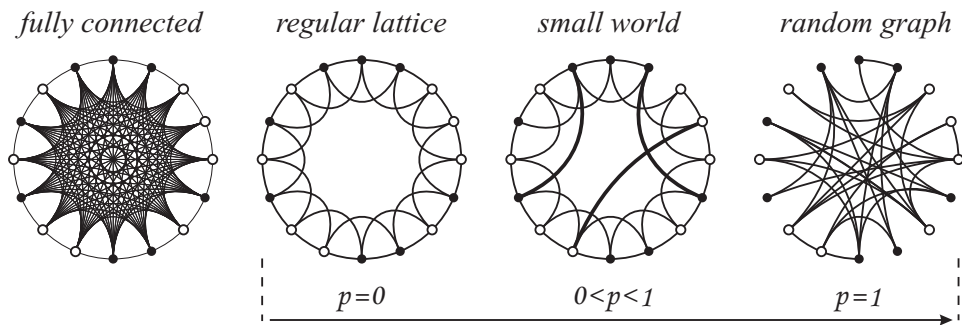

Figure 1: Network topologies (adapted from Watts and Strogatz (1998)).

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[1]This number of vertexes is used for an illustrative purpose only; in the model a considerably higher number of vertexes is used: 100 and 1000.

Different labelling of nodes of the graphs (black and white dots) is used to distinguish between different types of agents that constitute a network. This aspect will become apparent in Section 3 where we describe how to employ networks for financial markets modelling.

The properties of a network with N vertexes and K edges per vertex can be captured by the clustering coefficient $C(p)$ and characteristic path length $L(p)$ (Watts and Strogatz, 1998). The former is calculated by dividing the number of edges between certain neighboring nodes by the maximum possible number of edges K_{max} between them. Averaging over clustering coefficients for all the graph nodes gives the clustering coefficient of a graph $C(p)$. Characteristic path length $L(p)$ measures the average separation between two nodes and is defined as the average number of edges in the shortest path between two vertexes. Rewiring practically introduces shortcuts between the vertexes into the regular graph. The values of normalized clustering coefficients and characteristic path lengths for different rewiring probabilities p and network size N are depicted in Figure 2. Normalization is implemented over the corresponding characteristics of the regular lattice (for which $p = 0$).

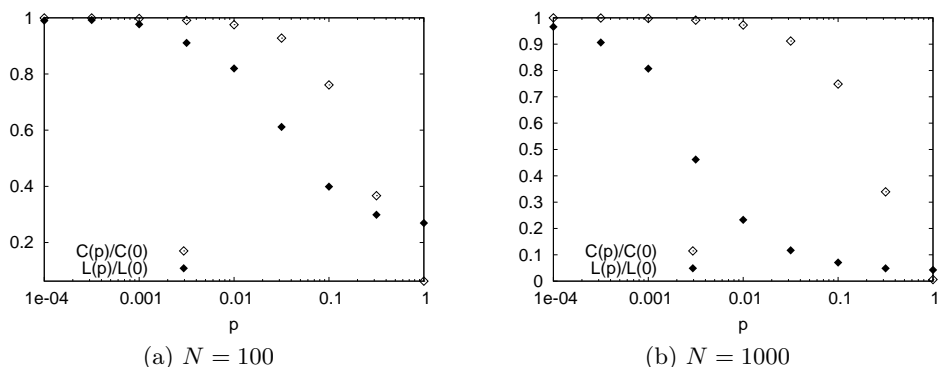(a) $N = 100$ (b) $N = 1000$

Figure 2: Clustering coefficient and characteristic path length for networks of different size (logarithmic scale is used in order to capture the small world phenomenon).

According to Watts (1999), a small world network can be defined in terms of the clustering coefficient and characteristic path length. Specifically, it is a decentralized ($N \gg K_{max} \gg 1$), sparsely connected graph with a large number of vertexes whose characteristic path length is small and close to an equivalent random graph's, i.e. $L(p) \approx L(1)$, but with a much greater clustering coefficient: $C(p) \gg C(1)$. Albert and Barabási (2002) suggest that emergence of small world properties depends on the network size, that is, the rewiring probability leading to a small world network is inversely proportional to the number of vertexes. Figure 2 supports this point.

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3. Heterogeneous belief model with networks

In this section we introduce the notion of network into the model of Brock and Hommes (1998).[2] As a benchmark we take a fully connected network which corresponds to the case with the traders being completely informed about the profits and the strategies of all the other investors in the market.

The set-up of the model is the following. There are two assets that are traded in discrete time: a perfectly elastically supplied risk-free asset paying a constant gross return $R_f = 1 + r_f$, and a risky asset paying a stochastic dividend y_t at the beginning of each trading period t, which is assumed to be independently and identically normally distributed (i.i.d.) with mean $\bar{y}$. The price p_t per-share (ex-dividend) of the risky asset in period t is obtained from the market clearing condition using Walrasian auctioneer scenario. The wealth dynamics reads as follows:

$$\boldsymbol{W}_{t+1} = R_f(W_t - p_t z_t) + (\boldsymbol{p}_{t+1} + \boldsymbol{y}_{t+1})z_t = R_f W_t + (\boldsymbol{p}_{t+1} + \boldsymbol{y}_{t+1} - R_f p_t)z_t, \quad (1)$$

where W_t and $\boldsymbol{W}_{t+1}$ are the wealth levels in period t and $t+1$ correspondingly, and z_t is the number of shares of the risky asset purchased at date t. Bold face type is used to denote random variables at date $t+1$.

The agents are assumed to be myopic expected utility maximizers, which is equivalent to the notion of mean-variance maximization as we assume that the risky asset return is normally distributed and the agent's utility function of wealth is negative exponential (constant absolute risk aversion). The demand for the risky asset at time t solves

$$\text{Max}_{z_t}\{\text{E}_t[\boldsymbol{W}_{t+1}] - \frac{a}{2}\text{V}_t[\boldsymbol{W}_{t+1}]\}, \quad (2)$$

where a denotes the coefficient of absolute risk aversion, while E_t and V_t denote conditional expectation and conditional variance based on publicly available information set $I_t = \{p_{t-1}, p_{t-2}, ...; y_{t-1}, y_{t-2}, ...\}$. E_t^h and V_t^h are the expectations (or predictors) of the trader of type h about, respectively, the mean and the variance. The demand for the risky asset of the type h agent is then given by:

$$z_t^h(p_t) = \frac{\text{E}_{t-1}^h[\boldsymbol{p}_{t+1} + \boldsymbol{y}_{t+1}] - R_f p_t}{a\text{V}_{t-1}^h[\boldsymbol{p}_{t+1} + \boldsymbol{y}_{t+1}]} = \frac{\text{E}_{t-1}^h[\boldsymbol{p}_{t+1} + \boldsymbol{y}_{t+1}] - R_f p_t}{a\sigma^2}. \quad (3)$$

The agents in the market hold heterogeneous beliefs in the sense of different conditional expectations but equal and constant conditional variances $\text{V}_t^h = \sigma^2$ among all the types at any period of time. Gaunersdorfer (2000) considered the model with variances changing over time and obtained similar results as in the case of constant variances.

Suppose that the supply of outside shares of the risky asset z^s is constant. Denote n_t^h be the fraction of type h at date t and H be the total number of trader types

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[2]While Brock and Hommes (1998) model is presented in terms of deviations from the fundamental price, we present the model in terms of absolute price.

in the market. The equilibrium of supply and demand then results in the following pricing equation:

$$\sum_{h=1}^{H} n_t^h \frac{\mathrm{E}_{t-1}^h[\boldsymbol{p}_{t+1} + \boldsymbol{y}_{t+1}] - R_f p_t}{a\sigma^2} = z^s. \tag{4}$$

Under the assumption of zero total supply of the risky asset and homogeneous ($H = 1$) beliefs of the agents, the fundamental price p^* is given by the discounted sum of the expected future dividends as a solution to the market-clearing equation (4), which is a well-known result. If, moreover, the dividend process is i.i.d. with constant mean $\bar{y}$, then $p^* = \bar{y}/r_f$.

We assume that there are two types of traders present in the market: *fundamentalists* and *chartists*, who are chosen to represent in a stylized way two distinct strategies employed in the real financial markets. Fundamentalist traders are assigned a predictor that forecasts the next period price p_{t+1} to be equal to the fundamental price p^*, that is

$$\mathrm{E}_{t-1}^f[\boldsymbol{p}_{t+1} + \boldsymbol{y}_{t+1}] = p^* + \bar{y}, \tag{5}$$

while chartists' predictor assumes persistent deviations from the fundamental value of the price in the following form:

$$\mathrm{E}_{t-1}^c[\boldsymbol{p}_{t+1} + \boldsymbol{y}_{t+1}] = a + g p_{t-1} + \bar{y}, \tag{6}$$

where $a > 0$ is a constant and $g > 0$ is the trend parameter. To stay within Brock and Hommes (1998) framework, we take $a = (1 - g)p^*$.

In Brock and Hommes (1998) the beliefs of the agents are updated over time through the co-evolution of the trader types fractions n_t^h and the market equilibrium price according to so-called Adaptive Belief System. The fractions of the agents following a particular trading strategy are updated every period depending on the values that the performance (fitness) measures of the strategies take on. In the capacity of a fitness measure Brock and Hommes (1998) propose past realized net profits:

$$U_t^h = \pi_t^h - C^h = (p_t + y_t - R_f p_{t-1}) \frac{\mathrm{E}_t^h[\boldsymbol{p}_t + \boldsymbol{y}_t] - R_f p_{t-1}}{a\sigma^2} - C^h, \tag{7}$$

which is essentially the difference of the excess return in the current period multiplied by the demand in the previous period and the cost of the strategy C^h. This cost is zero for the simple forecasting rule of chartists and strictly positive for the fundamentalists' predictor. In Brock and Hommes (1998) the fractions are determined by the discrete choice probability

$$n_t^h = \frac{\exp(\beta U_t^h)}{\exp(\beta U_t^h) + \exp(\beta U_t^{h^-})}, \tag{8}$$

where β is the intensity of choice parameter and h^- is the alternative strategy.

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This stochastic discrete choice model is derived within a random utility framework. Refer to e.g. Manski and McFadden (1990) for an exhaustive treatment of discrete choice models. To obtain this functional form one needs to assume that random utility or performance $\widetilde{U}_t^h$ is represented as

$$\widetilde{U}_t^h = U_t^h + \frac{1}{\beta}\varepsilon_t^h, \tag{9}$$

where ε_t^h is an i.i.d. random variable from the standard Gumbel (extreme value) distribution. With this representation (8) is equivalent to $n_t^h = \mathbb{P}(\widetilde{U}_t^h > \widetilde{U}_t^{h^-})$.

The parameter β determines the importance of the fitness measure U in decision to select a particular strategy. In the extreme case $\beta = 0$, the agent does not pay any attention to the fitness measure and picks a strategy at random with probability $1/2$. When $\beta \to \infty$, the agent selects the strategy with the highest fitness measure.

In our set-up the agents are located on the nodes of a network and can observe the fitness measure of the strategies employed only by those agents who reside on the nodes directly connected with them. Hence, they cannot observe the strategy performance of the traders located two or more edges away. Contrary to Brock and Hommes (1998) we do not assume that the performance of every strategy is available to all the agents. This is motivated by the fact that some strategies are costly and are not available to those agents who do not incur costs. Instead, we allow for local information exchange in the market.

If the agent is surrounded by the agents of the same type (see Figure 3a), she does not switch as there is no information about the performance of the alternative strategy. If the agent has at least one neighbor of different type (see Figure 3b), she is able to compare her own strategy with the alternative one and to switch if the latter is better in terms of performance.
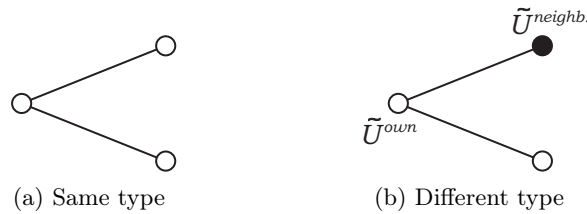

(a) Same type   (b) Different type

Figure 3: Different neighbor types.

In our setting the parameter β can be naturally interpreted as communication noise. Given the flexibility of the agent-based modelling, we can introduce noise directly into the performance (utility) function. We consider two instances: with symmetric noise and with asymmetric noise. In both cases noise term is distributed according to the standard Gumbel distribution. In the symmetric case noise with the same variance (determined by β) is added to both the profit measure of own strategy and the profit measure of the alternative strategy. In this situation we fully replicate the stochastic discrete choice model used in Brock and Hommes (1998). This set-up

may not be fully realistic and is used as a benchmark. In the asymmetric case the agent observes the performance of her own strategy without noise, which is added only to the performance of the alternative strategy that is communicated to the agent.

INITIALIZATION
50% of each type, price p_{-1} and p_0

NETWORK STRUCTURE CREATION
from lattice to random

EXPECTATION FORMATION
$E[p_{t+1}]$

DEMAND FUNCTION COMPUTATION
$z_t(p)$

WALRASIAN AUCTIONEER
$\Delta p=0$ and price p_t

PORTFOLIO UPDATING, DIVIDENDS

PROFIT BUILDUP
$\pi_t(z_{t-1})$

LOCAL SEARCH
neighbors' strategies

Different type neighbours | Same type neighbours

SWITCH with P
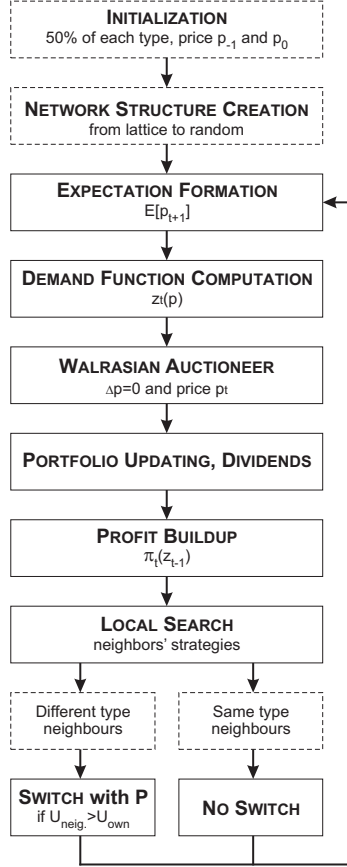if $U_{neig.}>U_{own}$ | **NO SWITCH**

Figure 4: Temporal flow.

Our market constitutes a complex adaptive dynamical system with co-evolving heterogeneous agents and equilibrium price. The model progresses in the following way. After the expectations of the agents are formed and their demand is ascertained the price is determined through the Walrasian auctioneer scenario. The profit of the agents is determined then and they switch to another strategy or remain with their own based on the fitness measure. Finally the agents form their expectations again and the cycle loops. The timing of the model is represented in Figure 4.

As we discussed in Section 2, many social studies recognized small world networks as the most prominent examples of local interactions in the real life. This fact is most likely due to the high speed of information transmission with relatively small number of connections that this network structure allows.

In the next section we investigate the implications of introducing small world information exchange into the Brock and Hommes (1998) model and compare it with other network topologies.

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4. Simulations and results

By introducing nontrivial communication structures into the Brock and Hommes (1998) model, we loose analytic tractability of the solution. Nevertheless, the complex behavior of the resulting models can still be analyzed by means of computer simulations.[3] We conduct simulations for four different network structures of local interactions, i.e. for a fully connected graph, for a regular lattice, for a small world graph, corresponding to the rewiring probability of 0.1, and for a random graph (see Figure 1). All the graphs are connected, that is, there are no vertexes that do not have any links. The fully connected graph is used as a benchmark corresponding to the finite number of agents implementation of the original Brock and Hommes (1998) model. We compare asset price dynamics for two different agent populations in the models: $N = 100$ and $N = 1000$. We also consider two cases discussed in the previous section: with symmetric and with asymmetric noise in the performance measure.
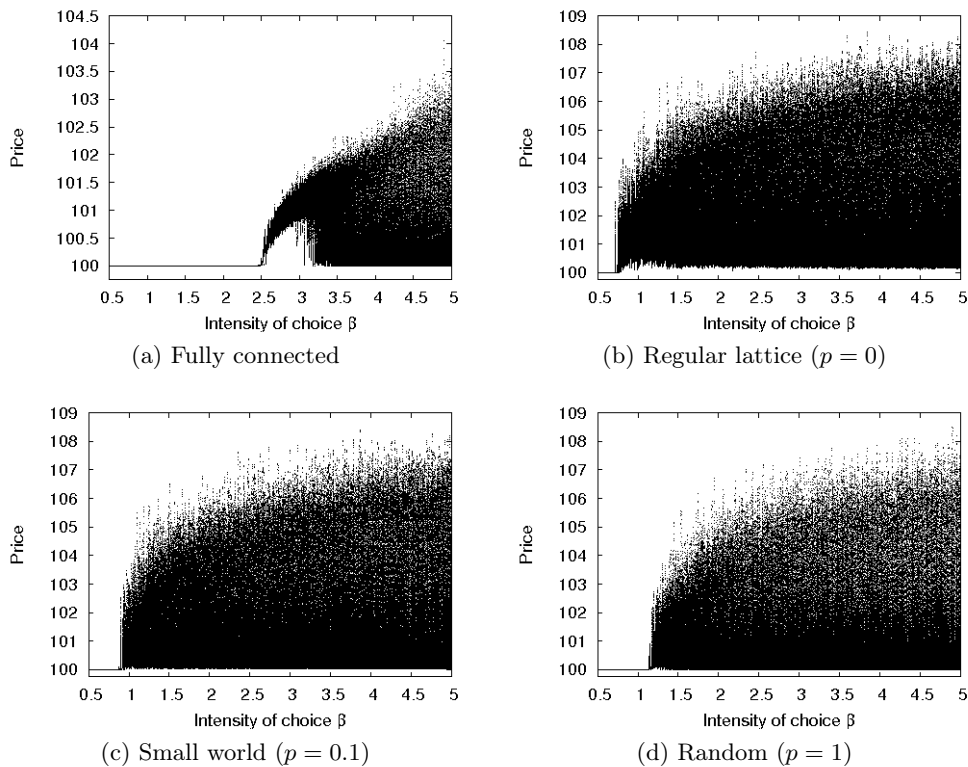

(a) Fully connected

(b) Regular lattice ($p = 0$)

(c) Small world ($p = 0.1$)

(d) Random ($p = 1$)

Figure 5: Bifurcation diagram ($N = 100$), symmetric noise.

Asset price dynamics for a range of values of the parameter β are shown by means of bifurcation diagrams in Figures 5 and 6 for the population size of 100 and in Figures

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[3]The C++ code for our simulations is partially adopted from the code of Bottazzi et al. (2005).

(a) Fully connected   (b) Regular lattice ($p = 0$)

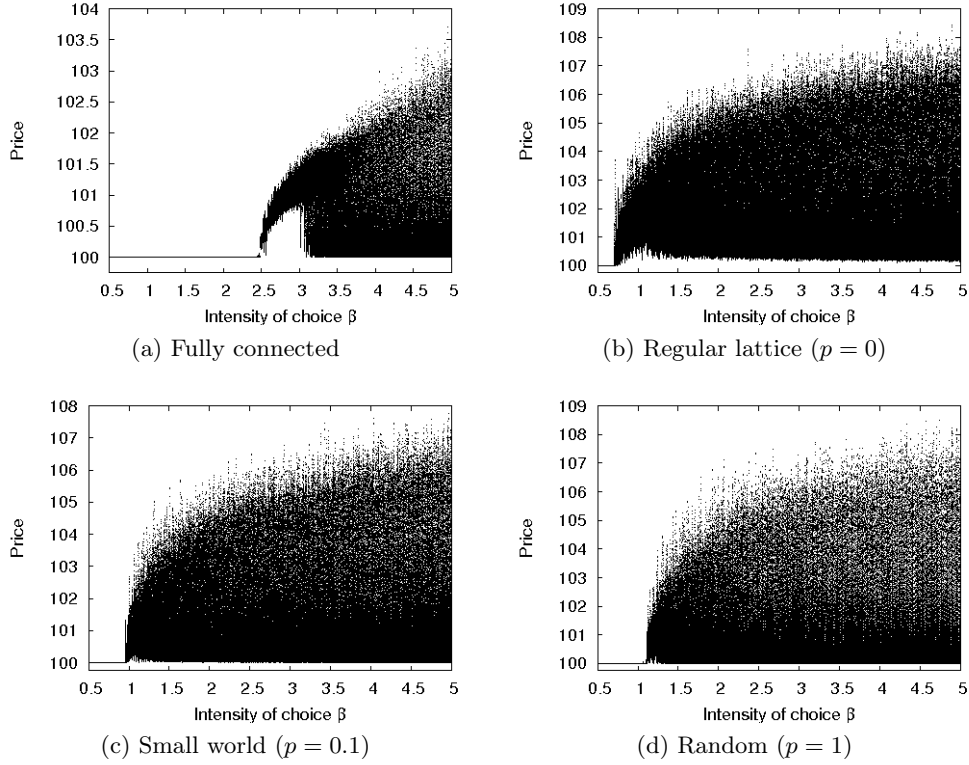(c) Small world ($p = 0.1$)   (d) Random ($p = 1$)

Figure 6: Bifurcation diagram ($N = 100$), asymmetric noise.

7 and 8 for the population size of 1000. For the both cases the networks in terms of the occurrence of the bifurcation with respect to the parameter β can be arranged in the following order (in decreasing value of β): the fully connected graph (the benchmark), the random network, the small world network and the regular lattice. This suggests that elimination of links in the fully connected graph induces price instabilities, while the introduction of the shortcuts in the regular lattice promotes greater price stability. The amplitude of price fluctuation depends on the network topology in a similar way: it is the highest for the regular lattice, it reduces with the increase of p through the small world network to the random graph, and then attains the smallest value for the fully connected network. This difference is more evident for the bigger population size. Moreover, the amplitudes of price fluctuations are substantially greater for $N = 1000$ in comparison to $N = 100$. Surprisingly enough, we find practically no difference in the case of symmetric noise added to the fitness measure and the case of asymmetric noise, that is when the noise is added only to the fitness measure received though the communication.

Since some discrepancy in bifurcation analysis was found for different population sizes, we continue the analysis for the more realistic networks of size 1000. Figure 9 depicts time series of the price for two values of the intensity of switching parameter:
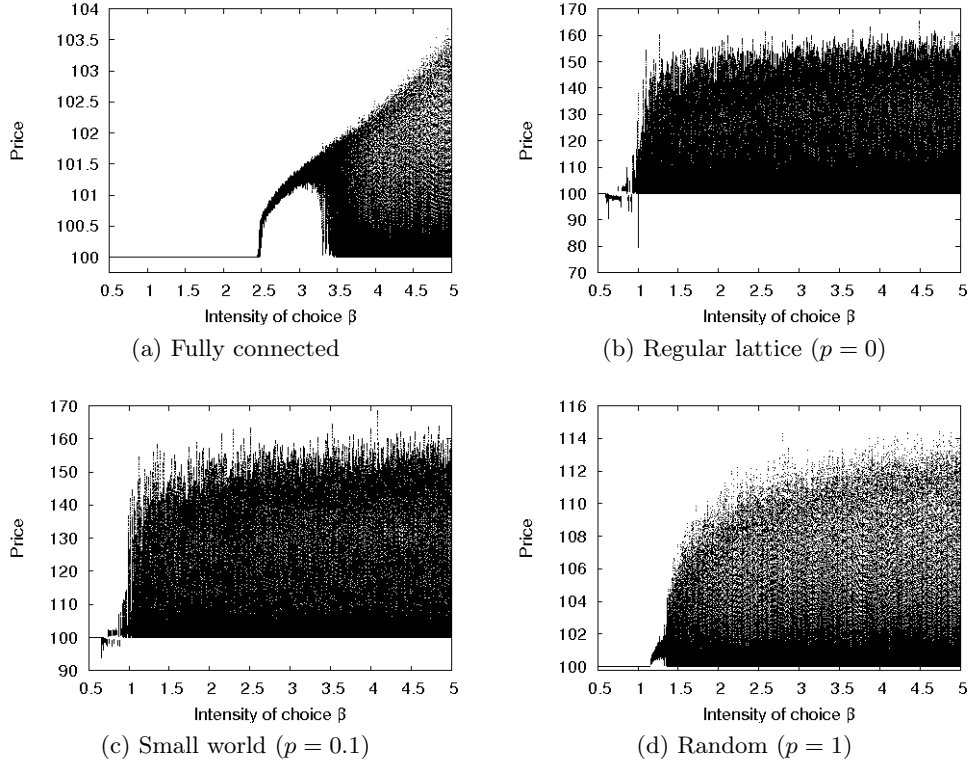
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Figure 7: Bifurcation diagram ($N = 1000$), symmetric noise.

$\beta = 1$, and $\beta = 3.5$. For $\beta = 1$ the price dynamics corresponding to the fully connected graph and the random network converge to a steady state, while the regular graph and the small world network lead to highly irregular chaotic asset price fluctuations. The latter observation is due to the probabilistic nature of the model as the discrete choice framework with the finite number of agents (in contrast to the original model with $N \to \infty$) is no more deterministic but rather stochastic. For $\beta = 3.5$ chaotic behavior is observed for all the network topologies, however, the regularity and the amplitudes of fluctuations vary considerably among them.

The analysis reveals that according to the price dynamics the topology properties of the random network is close to the fully connected network. The price dynamics produced by the regular lattice is the most distinct from the fully connected network. The small world network produces price dynamics with the properties somewhere in between of those of the random graph and the regular lattice.

To provide more insights into the effects of different network topologies on market behavior we analyze the evolution of every agent in time. In Figure 10 we show the type of each out of 1000 agents (Agent's ID axis) at every time step from 0 to 1000 (Time axis). Each dot on every vertical line represents an agent: black dots stand for fundamentalists, while white dots represent chartists (white space in the figures,
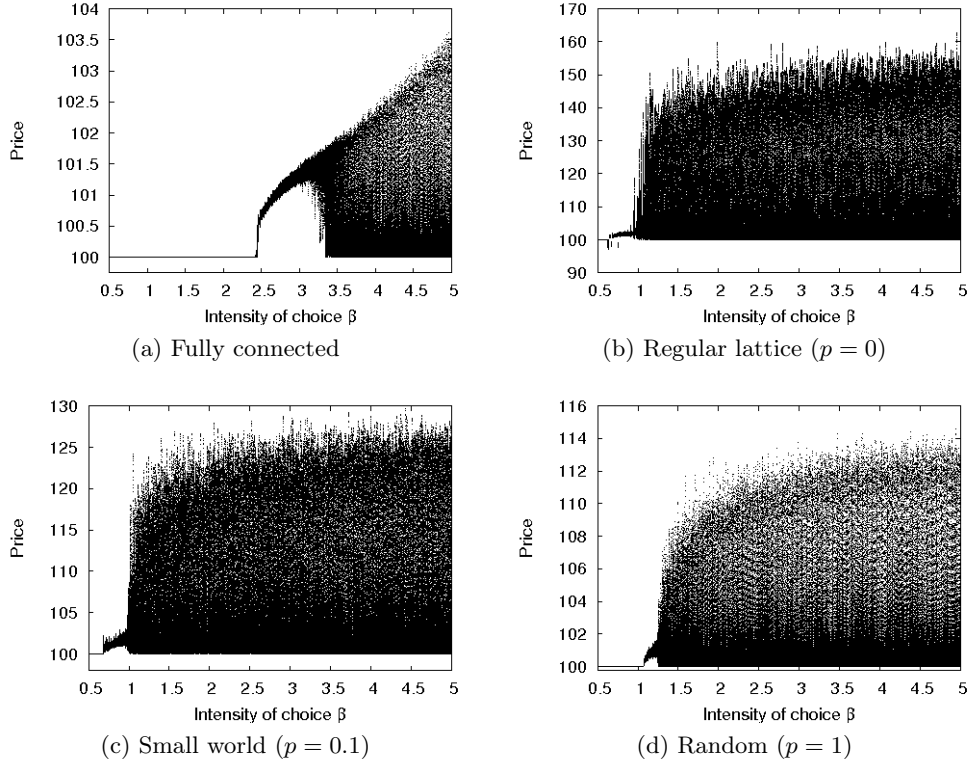
14

(a) Fully connected   (b) Regular lattice ($p = 0$)

(c) Small world ($p = 0.1$)   (d) Random ($p = 1$)

Figure 8: Bifurcation diagram ($N = 1000$), asymmetric noise.

thus, denotes a mass of chartists sticked together). The figures have a torus-like layout, that is, every agent situated on one edge (ID 0) is connected to the agent situated on the other edge (ID 1000). The highest concentration of fundamentalist corresponds to the fall of price to fundamental level on the time series plot, while its lowest concentration corresponds to the highest deviation from the fundamental value of the price. Therefore, the spatio-temporal pattern figures and the time series plots are intimately interrelated since a change in concentration of one type leads to immediate change in the asset price dynamics.

The time series plots of the analyzed network topologies suggest that the asset price dynamics generated by a regular lattice and a small world network bear closer resemblance to reality than the time series plots inherent to the other two network structures.

We also provide time series plots corresponding to the case of stochastic dividends. We assume that the dividends are independent and identically normally distributed with the mean set equal to 10 and the variance equal to 25. The negative realizations of the dividend are truncated at zero.

The resulting asset price dynamics is depicted in Figure 11. For $\beta = 1$, the time series of the small world network and the regular lattice exhibit fluctuations with
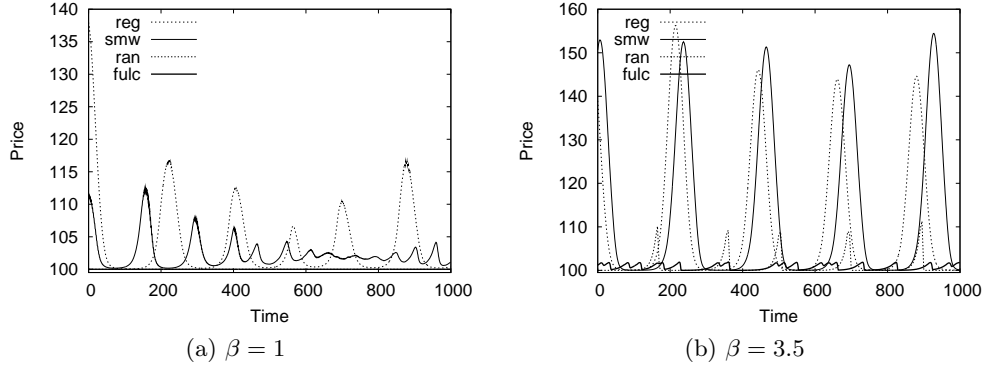
15

(a) $\beta = 1$ (b) $\beta = 3.5$

Figure 9: Time series of price ($N = 1000$).

(a)   Fully connected (b)   Regular lattice   (c)   Small world   (d)   Random graph
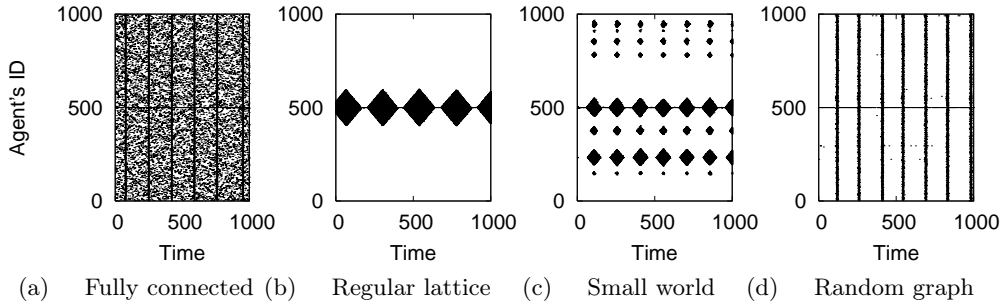
Figure 10: Spatio-temporal pattern of agents type evolution.

booms and crashes close to those observed in the real financial markets, while the random graph and the fully connected network exhibit virtually no fluctuations. For $\beta = 3.5$ the regular lattice and the small world network produce large scale irregular single-peak deviations from the fundamental price, while the fully connected and the random networks produce relatively regular fluctuation of much smaller order.

It becomes evident if one compares Figure 9 with Figure 11 that the impact of the stochastic dividend is the strongest on the price dynamics resulting from the small world interactions arrangement, while its effect is relatively weaker on the price dynamics corresponding to the other network structures. It is also worth noticing that the fluctuation amplitude of the price conforming to the small world network is smaller than the amplitude of the regular network price for the case of no stochastic dividends, while the situation becomes almost reversed after the introduction of the stochastic dividends. This is particularly visible for $\beta = 3.5$. Thus, we can conclude that exogenous noise affects small world networks more than any other network structure, which is most likely a result of the fastest information transmitting abilities of the local interaction arrangements exhibiting small world properties.
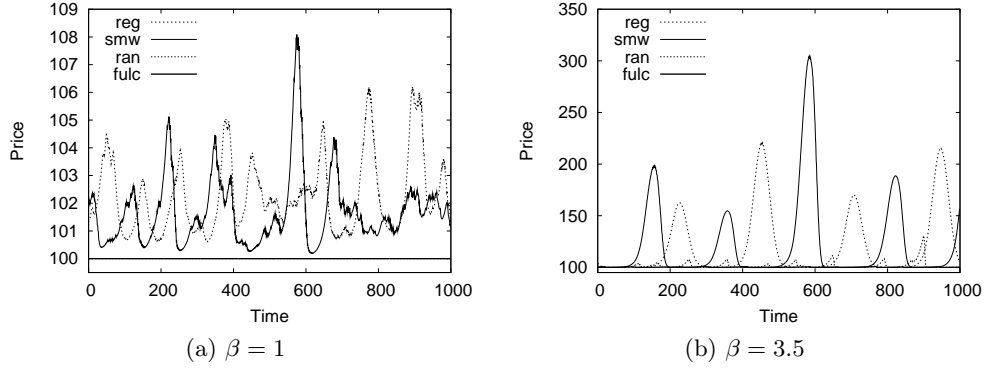
16

Figure 11: Time series of price with the stochastic dividend process ($N = 1000$).

5. Concluding remarks

In this paper we introduced local information exchange in the form of networks into the model of Brock and Hommes (1998). While the major qualitative dynamics are retained, the dynamics of the asset price are enriched by the introduction of local interactions between the agents. We studied how different network structures affect asset price dynamics. Upon the analysis of the time series generated by different networks with both stochastic and deterministic dividends we could conclude that the asset price dynamics produced by a small world network exhibit the properties closest to those observed in real financial markets. Thus, we conjecture that a small world network is the most suitable description of the information distribution arrangement.

In many networks there is a feedback between network performance and network formation. This means that performance of the agents is affected by the network topology they are active in. But at the same time, network topology may be at least partially affected by the links created by the participating agents. Therefore, future work may include preferential attachment. In general, any network formation mechanism applied to our model can be analyzed as it would bring in more realistic picture of a market, which is randomly formed in our case. A reader interested in endogenous network formation can consult for example, Bala and Goyal (2000), Morone and Taylor (2004), and Jackson and Wolinsky (1996).

Another future perspective could be the use of heterogeneous β parameter. We employ the same value of β for every agent from the consideration of tractability. But it would be interesting to study how price dynamics changes when different intensity of switching to another strategy among the agents is introduced. We refer the reader to our discussion of this issue in Section 3.

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