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An efficient binomial approach to the pricing of options on stocks with cash dividends

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Abstract. In this contribution, we consider options written on stocks which pay cash dividends. Dividend payments have an effect on the value of options: high dividends imply lower call premia and higher put premia. While exact solutions to problems of evaluating both European and American call options and European put options are available in the literature, for American-style put options early exercise may be optimal at any time prior to expiration even in the absence of dividends. In this case numerical techniques, such as lattice approaches, are required. Discrete dividends produce a shift in the tree; as a result, the tree is no longer reconnecting beyond any dividend date. Methods based on non-recombining trees give consistent results, but they are computationally expensive. We analyze binomial algorithms and performed some empirical experiments.

Keywords: Options on stocks, discrete dividends, binomial lattices.

JEL Classification Numbers: C63, G13.

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1 Introduction

We consider options written on assets which pay dividends. Dividends are announced as a pure cash amount D to be paid at a specified ex-dividend date t_D. Empirically, one observes that at the ex-dividend date the stock price should drop by an amount equal to the size of the dividend. In order to exclude arbitrage opportunities, the jump in the stock price should be equal to the size of the net dividend. Since we cannot use the proportionality argument, the price dynamics depends on the timing of the dividend payment. As the stock price is known to jump downward due to the dividend payment, high dividends imply lower call premia and higher put premia.

Usually, derivative pricing theory assumes that stocks pay known dividends, both in size and timing. Moreover, new dividends are often assumed to equal former ones. Even if these assumptions might be too strong, in what follows we assume that we know both the amount of dividends and times of payment.

Valuation of options on stocks which pay discrete dividends is a rather hard problem which has received a lot of attention in the financial literature, but there is much confusion concerning the evaluation approaches. Different methods have been proposed for the pricing of both European and American options on dividend paying stocks, which suggest various model adjustments (as, for example, subtracting the present value of the dividend from the asset spot price). Nevertheless, all such approximations have some drawbacks and are not so efficient[1].

Haug and Haug [13], and Beneder and Vorst [2] propose a volatility adjustment which takes into account the timing of the dividend. A more sophisticated volatility adjustment to be used in combination with the escrowed dividend model is proposed by Bos *et al.* [4]. A slightly different implementation is suggested by Bos and Vandermark [5] which adjusts at the same time the stock price and the strike. Recently, de Matos *et al.* [8] derive arbitrarily accurate lower and upper bounds for the value of European options on a stock paying a discrete dividend. Haug *et al.* [14] provide an integral representation formula that can be considered the exact solution to problems of evaluating both European and American call options and European put options. Dai and Lyuu [7] propose a model for pricing European options in which discrete dividends are replaced with continuous dividend yields.

For American-style put options, it may be optimal to exercise at any time prior to expiration, even in the absence of dividends. Unfortunately, no analytical solutions for both the option price and the exercise strategy are available, hence one is generally forced to numerical solutions, such as binomial approaches. As well known (see Merton [15]), in the absence of dividends, it is never optimal to exercise an American call before maturity. If a cash dividend payment is expected to occur during the lifetime of the option, it might be optimal to exercise an American call option right before the ex-dividend date, while for an American put it may be optimal to exercise any time till maturity.

Lattice methods are commonly used for the pricing of both European and American options. In the binomial model (see Cox *et al.* [6]) the pricing problem is solved by backward induction along the tree. In particular, for American options, at each node of the

[1] See e.g. Haug [12] for a review.

lattice one has to compare the early exercise value with the continuation value.

In this contribution, we analyze binomial algorithms for the evaluation of options written on stocks which pay discrete dividends of both European and American type. In particular, we consider non-recombining binomial trees, hybrid binomial algorithms for both European and American call options (based on the Black-Scholes formula for the evaluation of the option after the ex-dividend date and up to maturity); a binomial method which implements the efficient continuous approximation proposed in [5]; and we propose a binomial method based on an interpolation idea of Vellekoop and Nieuwenhuis [18], in which the recombining feature is maintained. We also extend the model based on the interpolation procedure to the case of multiple dividends. We performed some empirical experiments and compare the results in terms of accuracy and speed.

2 Continuous time option pricing with known dividends

Dividends affect option prices through their effect on the underlying stock price. In a continuous time setting, the underlying price dynamics depends on the timing of the dividend payment and is assumed to satisfy the following stochastic differential equation

$$\begin{aligned} dS_t &= rS_t dt + \sigma S_t dW_t \qquad t \neq t_D \,, \\ S_{t_D}^+ &= S_{t_D}^- - D_{t_D} \,, \end{aligned} \tag{1}$$

where r is the risk-free interest rate, $\sigma > 0$ is the volatility, $(W_t)_{t\geq 0}$ is a standard Wiener process, D_{t_D} is the dividend paid at time t_D, $S_{t_D}^-$ and $S_{t_D}^+$ denote the stock price levels right before and after the jump at time t_D, respectively.

Due to this discontinuity, the solution to equation (1) is no longer lognormal but in the form

$$S_t = S_0 e^{(r-\sigma^2/2)t+\sigma W_t} - D_{t_D} e^{(r-\sigma^2/2)(t-t_D)+\sigma W_{t-t_D}} I_{\{t\geq t_D\}} \,, \tag{2}$$

where I_A denotes the indicator function of A.

Valuation of options on stocks which pay discrete dividends is a rather hard problem which has received a lot of attention in the financial literature. In the next subsections we will consider European and American options. It is worth noting that most traded options on stocks are of American type.

2.1 European-style options

The simplest evaluation approach to price options on stocks consists in adjusting the well known generalized Black-Scholes-Merton formula (BSM) by replacing the stock price S_0 with the stock price minus the present value of the dividend

$$S_0 - De^{-rt_D} \,; \tag{3}$$

this is called the *escrowed dividend model* and typically leads to too low absolute price volatility σS in the period before the dividend is paid. Since the initial stock price is lowered under the true observed price, this approach typically undervalues call options, and the mispricing is larger the later the dividend is paid during the lifetime of the option. In order to overcome such a problem, several corrections of volatility have been suggested in literature. The following adjustment is popular among practitioners:

$$\sigma_2 = \frac{\sigma S}{S - De^{-rt_D}} \,. \tag{4}$$

Such an adjustment increases the volatility relative to the basic escrowed divided process, but yields too high volatility if the dividend is paid early in the option's lifetime. The approach typically overprices call options in this situation.

Another volatility adjustment takes into account the timing of the dividend (see [13] and [2]). The idea behind the approximation is to leave volatility unchanged in the time before the dividend payment, and to apply the volatility σ_2 after the dividend payment. This method performs particularly poorly in the presence of multiple dividends. A more sophisticated volatility adjustment to be used in combination with the escrowed dividend model is proposed by Bos *et al.* [4]. The method is quite accurate for most cases. For very large dividends, or in the case of multiple dividend payouts, the method can yield significant mispricing, however. A slightly different implementation (see Bos and Vandermark [5]) adjusts at the same time the stock price and the strike. The dividends are divided into two parts, "near" and "far", that are used for the adjustments to the spot and the strike price respectively. This approach seems to work better than the approximation mentioned above. Recently, de Matos *et al.* [8] derive arbitrarily accurate lower and upper bounds for the value of European options on a stock paying a discrete dividend.

A different approach is proposed by Haug *et al.* [14] (henceforth HHL). The authors derive an integral representation formula for the fair price of a European call option on a dividend paying stock. The basic idea is that after the dividend payment, option pricing reduces to simple Black-Scholes formula for a non-dividend paying stock. Before t_D one considers the discounted expected value of the BS formula adjusted for the dividend payment. In the geometric Brownian motion setup, the HHL formula is given by

$$C_{HHL}(S_0, D, t_D) = e^{-rt_D} \int_d^{\infty} c_E(S_x - D, t_D) \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx \,, \tag{5}$$

where

$$d = \frac{\log(D/S_0) - (r - \sigma^2/2)t_D}{\sigma\sqrt{t_D}} \,, \tag{6}$$

$$S_x = S_0 e^{(r-\sigma^2/2)t_D + \sigma\sqrt{t_D}x} \,, \tag{7}$$

and $c_E(S_x - D, t_D)$ is simply the BS formula with time to maturity $T - t_D$. The integral representation (5) can be considered as the exact solution to the problem of valuing a European call option written on stock with a discrete and known dividend. Let us observe that the well known put-call parity relationship allows to calculate immediately the theoretical price of a European put option with a discrete dividend.

2.2 American-style options

Most traded options are of American-style. The effect of a discrete dividend payment on American option prices is different than for European options. While for European-style options the pricing problem basically arises from mis-specifying the variance of the underlying process, for American options the impact on the optimal exercise strategy is more important. As well known, it is never optimal to exercise an American call option on non-dividend paying stocks before maturity. As a result, the American call has the same value as its European counterpart. In the presence of dividends, it may be optimal to exercise the American call and put before maturity. In general, early exercise is optimal when it leads to an alternative income stream, i.e. dividends from the stock in case of a call and interests on cash in case of a put option. In the case of discrete cash dividends, the call option may be optimally exercised early instantaneously prior to the ex-dividend date t_D^-. Note that after the dividend date t_D, the option is a standard European call which can be priced using the BS formula; we have implemented this idea in a hybrid BS-binomial model. While for an American put it may be optimal to exercise at every time until maturity. Simple adjustments like subtracting the present value of the dividend from the asset spot price make little sense for American options.

The first approximation to the value of an American call on a dividend paying stock has been suggested by Black in 1975 [3]. This is basically the escrowed dividend method, where the stock price in the BS formula is replaced by the stock price minus the present value of the dividend. In order to take into account for early exercise, one also computes an option value just before the dividend payment, without subtracting the dividend. The value of the option is considered to be the maximum of these values.

A model which is often used and implemented in many commercial softwares was proposed, simplified and correct respectively by Roll [16], Geske [10] and [11], and Whaley [19] (henceforth RGW model). These authors construct a portfolio of three European call options which replicates an American call and accounts for the possibility of early exercise right before the ex-dividend date. The portfolio consists of two long positions with exercise prices X and $S^* + D$ and maturities T and t_D^-, respectively. The third option is a short call on the first of the two long calls with exercise price $S^* + D - X$ and maturity t_D^-. The stock price S^* makes the holder of the option indifferent between early exercise at time t_D and continuing with the option. Formally, we have

$$C(S^*, T - t_D, X) = S^* + D - X \,. \tag{8}$$

This equation can be solved if the ex-dividend date is known. The two long positions follow from the BS formula, while for the compound option Geske [9] provides an analytical solution.

The RGW model was considered for more than twenty years as a brilliant solution in closed form to the problem of evaluating American call options on equities that pay a discrete dividend. Although some authoritative authors still consider the RGW formula as the exact solution, the model does not yield good results in many cases of practical interest. Moreover, it is possible to find situations in which the use of the formula RGW allows for arbitrage. Whaley, in its recent monograph [20], presents an example that shows the limits of the RGW model. The data reported by Whaley for the American call option in the presence of a discrete dividend are: $S_0 = X = 50$, $T = 90$ days, $\sigma = 0.36$, $D_{t_D} = 2$, $t_D = 75$ days, $r = 0.05$. With these data, the critical price is $S^* = 49.060$ and the value of the American call computed in the model RGW is $C_{RGW} = 3.445$, while the correct value is $C_{HHL} = 3.57041$. The BMS price of a European call option with the same features, but with maturity the day before ex-dividend date ($t = 74$ days) is 3.47193 and therefore higher than the price obtained in the RGW model.

Haug *et al.* [14] derived an integral representation formula for the American call option fair price in the presence of a single dividend D paid at time t_D. Since early exercise is only optimal instantaneously prior to the ex-dividend date, in order to obtain the exact solution for an American call option with a discrete dividend one can merely replace relation (5) with

$$C_{HHL}(S_0, D, t_D) = e^{-rt_D} \int_d^{\infty} \max\{S_x - X, c_E(S_x - D, t_D)\} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx \,. \tag{9}$$

For American-style put options early exercise may be optimal at any time prior to expiration, even in the absence of dividends. So, in this case, one is generally forced to a numerical methods, such as lattice approaches, which are discussed in the next section.

3 Binomial models

The evaluation of options using binomial methods is particularly easy to implement and efficient at standard conditions, but it becomes difficult to manage in the case in which the underlying asset pays one or more discrete dividends, due to the fact that the number of nodes grows considerably and entails huge calculations. In the absence of dividends or when dividends are assumed proportional to the stock price, the binomial tree reconnects in the sense that the price after an up-down movement coincides with the price after a down-up movement. As a result, the number of nodes at each step grows linearly.

If during the life of the option a dividend of amount D is paid, at each node after the ex-dividend date a new binomial tree has to be considered, with the result that the total number of nodes increases to the point that it is practically impossible to consider trees with an adequate number of stages. To avoid such a drawback, often it is assumed that the underlying dynamics is characterized by a dividend yield which is discrete and proportional to the stock price. Formally,

$$\begin{cases} S_0 u^j d^{i-j} & j = 0, 1, \dots i \\ S_0(1-q) u^j d^{i-j} & j = 0, 1, \dots i \,, \end{cases} \tag{10}$$

where the first law applies in the period preceding the ex-dividend date and the second applies after the dividend date; S_0 denotes the initial price, q is the dividend yield, and u and d are respectively the upward and downward coefficients, defined by

$$u = e^{\sigma\sqrt{T/n}} \qquad\qquad d = 1/u \,. \tag{11}$$

The hypothesis of a proportional dividend yield can be accepted as an approximation of dividends paid in the long term, but it is not acceptable in a short period of time during

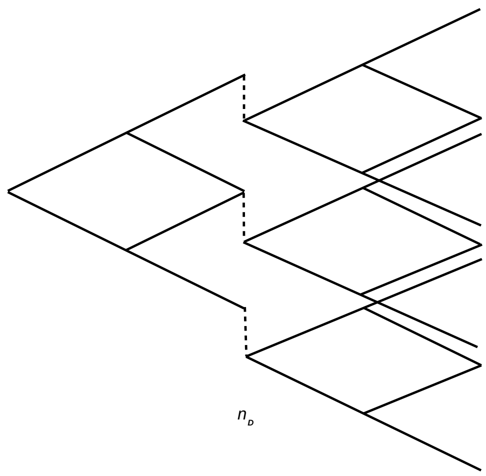

Figure 1: Non-recombining binomial tree

which the stock pays a dividend in cash and its amount is often known in advance or estimated with appropriate accuracy.

If the underlying asset is assumed to pay a discrete dividend D at time $t_D < T$ (which in a discrete time setting corresponds to the step n_D), the dividend amount is subtracted at all nodes at time point t_D. Due to this discrete shift in the tree, as already noticed, the lattice is no longer recombining beyond the time t_D and the binomial method becomes computationally expensive, since at each node at time t_D a separate binomial tree has to be evaluated until maturity (see figure 1). Also in the presence of multiple dividends this approach remains theoretically sound, but becomes unattractive regarding the computational intensity.

Schroder [17] describes how to implement discrete dividends in a recombining tree. The approach is based on the *escrowed dividend* process idea, but the method leads to significant pricing errors.

The problem of the enormous growth in the number of nodes that occurs in such a case can be simplified if it is assumed that the price has a stochastic component $\tilde{S}$ given by

$$\tilde{S} = \begin{cases} S - De^{-r(t_D - i)} & \text{if } i \leq t_D \\ S & \text{if } i > t_D \,, \end{cases} \tag{12}$$

and a deterministic component represented by the discounted value of the dividend or of dividends that will be paid in the future. Note that the stochastic component gives rise to a reconnecting tree. Moreover, one can build a new tree (which is still reconnecting) by adding the present value of future dividends to the price of the stochastic component in correspondence of each node. Hence the tree reconnects and the number of nodes in each period i is equal to that at step $i+1$.

The recombining technique described above can be improved through a procedure which preserves the structure of the tree until the ex-dividend time and that will force the recombination after the dividend payment. For example, one can force the binomial tree to recombine by taking, immediately after the payment of a dividend, as extreme nodes

$$S_{n_D+1,0} = (S_{n_D,0} - D)\, d \qquad\qquad S_{n_D,n_D} = (S_{n_D,n_D} - D)\, u\,, \tag{13}$$

and by calculating the arithmetic average of the values that are not recombining. This technique has the characteristic of being simple from the computational point of view.

Alternatively, one can use a technique that we call "*stretch*" that calculates the extreme nodes as in the previous case; in such a way, one forces the reconnection at the intermediate nodes by choosing the upward coefficients as follows

$$u(i,j) = e^{\lambda\sigma\sqrt{T/n}}\,, \tag{14}$$

where λ is chosen in order to make equal the prices after an up and down movement. This technique requires a greater amount of computations as to each stage both the coefficients and the corresponding probabilities change.

3.1 The interpolated binomial method

A method which performs very efficiently and can be applied to both European and American call and put options is a binomial method which maintains the recombining feature and is based on an interpolation idea proposed by Vellekoop and Nieuwenhuis [18].

For an American option, the method can be described as follows: a standard binomial tree is constructed without considering the payment of the dividend (with $S_{ij} = S_0 u^j d^{i-j}$, $u = e^{\sigma\sqrt{T/n}}$, and $d = 1/u$), then it is evaluated by backward induction from maturity until the dividend payment; at the node corresponding to an ex-dividend date (at step n_D), the continuation value V_{n_D} is approximated using the following linear interpolation

$$V(S_{n_D,j}) = \frac{V(S_{n_D,k+1}) - V(S_{n_D,k})}{S_{n_D,k+1} - S_{n_D,k}}\,(S_{n_D,j} - S_{n_D,k}) + V(S_{n_D,k}) \qquad j = 0, 1, \ldots, n_D\,, \tag{15}$$

for $S_{n_D,k} \leq S_{n_D,j} \leq S_{n_D,k+1}$; then continue backward along the tree. The method can be easily implemented also in the case of multiple dividends (which are not necessarily of the same amount).

We have implemented a very efficient method which combines this interpolation procedure and the binomial algorithm for the evaluation of American options proposed by Basso *et al.* [1].

We performed some empirical experiments and compare the results in terms of accuracy and speed.

4 Numerical experiments

In this section, we briefly report the results of some empirical experiments related to European calls and American calls and puts. In table 1, we compare the prices provided by the HHL exact formula for the European call, with those obtained with the 2 000-step non-combining binomial method and the binomial method based on interpolation (15). The second method is accurate and very fast: for a European call, the non-recombining binomial method requires a couple of seconds, while the calculations with a 2 000-step binomial interpolated method are immediate. Outcomes of the recombining methods based on the approximation proposed by [5], the average value and the stretching procedure are also reported. In particular, these last two methods provide results that entail too much error to be used in practice.

Table 1: European calls with dividend $D = 5$ ($S_0 = 100$, $T = 1$, $r = 0.05$, $\sigma = 0.2$).

t_D	X	HHL	non-rec. bin. ($n = 2\,000$)	interp. bin. ($n = 2\,000$)	Bos-Vand.	av. value	stretch
	70	28.7323	28.7323	28.7324	28.7387	28.9063	28.2240
0.25	100	7.6444	7.6446	7.6446	7.6456	7.9890	7.6203
	130	0.9997	0.9994	1.0000	0.9956	1.1349	1.0540
	70	28.8120	28.8120	28.8121	28.8192	28.9291	28.7682
0.5	100	7.7740	7.7742	7.7742	7.7743	8.0001	7.9199
	130	1.0501	1.0497	1.0506	1.0455	1.1371	1.1212
	70	28.8927	28.8927	28.8928	28.8992	28.9517	28.8746
0.75	100	7.8997	7.8999	7.8999	7.9010	8.0112	7.9738
	130	1.0972	1.0969	1.0977	1.0934	1.1392	1.3121

Table 2 shows the results related to American options. For the American call we have compared the results yielded by a 5 000-step non-recombining hybrid binomial method (in which we have used the BS formula in order to calculate the continuation value at the ex-dividend node) and the 10 000-step binomial method based on the interpolation procedure (15). For the American put, we have compared the 2 000-step non-recombining binomial method, the 10 000-step interpolated binomial method, and the approximation proposed by [5]. This last method provides higher pricing errors for American put options, as in some cases it overprices the true option value, while the interpolated binomial approach turned out to be accurate.

Table 2: American call and put options with dividend $D = 5$ ($S_0 = 100$, $T = 1$, $r = 0.05$, $\sigma = 0.2$).

		American call		American put		
t_D	X	non-rec. hyb. bin. ($n = 5\,000$)	interp. bin. ($n = 10\,000$)	non-rec. bin. ($n = 2\,000$)	interp. bin. ($n = 10\,000$)	Bos-Vand.
	70	30.8740	30.8744	0.2680	0.2680	0.2630
0.25	100	7.6587	7.6587	8.5162	8.5161	8.5244
	130	0.9997	0.9998	33.4538	33.4540	35.0112
	70	31.7553	31.7557	0.2875	0.2876	0.2901
0.5	100	8.1438	8.1439	8.4414	8.4412	8.5976
	130	1.0520	1.0522	32.1195	32.1198	35.0112
	70	32.6407	32.6411	0.3070	0.3071	0.2901
0.75	100	9.1027	9.1030	8.2441	8.2439	8.6689
	130	1.1764	1.1767	30.8512	30.8515	35.0012

We also extended the model based on the interpolation procedure to the case of multiple dividends. Table 3 shows the results for the European call with multiple dividends. We have compared the non-reconnecting binomial method with $n = 2\,000$ steps (only for the case with one and two dividends) and the interpolated binomial method with $n = 10\,000$

steps (our results are in line with those obtained by Haug *et al.* [14]). Different maturities (in years) have been considered, with dividends paid every year at $t_D = 0.5$, 1.5, and so on. In table 4 we have reported the results for the at-the-money American options written on an asset with multiple dividends. Also in the case of more dividends, the interpolated binomial method proved to be efficient.

Table 3: European call option with multiple dividends $D = 5$ paid at times $t_D \in \{0.5, 1.5, 2.5, 3.5, 4.5, 5.5\}$ ($S_0 = 100$, $X = 100$, $r = 0.05$, $\sigma = 0.2$).

T	non-rec. bin. ($n = 2\,000$)	interp. bin. ($n = 10\,000$)
1	7.7742	7.7741
2	10.7119	10.7122
3		12.7885
4		14.4005
5		15.7076
6		16.7943

Table 4: American options with multiple dividends in the interpolated 10 000-step binomial method (with parameters $S_0 = 100$, $X = 100$, $r = 0.05$, $\sigma = 0.2$) a cash dividend $D = 5$ is payed at the dates $t_D \in \{0.5, 1.5, 2.5, 3.5, 4.5, 5.5\}$ for different maturities.

T	American call	American put
1	8.1439	8.4412
2	11.2792	11.5904
3	13.3994	13.7399
4	15.0169	15.3834
5	16.3136	16.7035
6	17.3824	17.7938

5 Concluding remarks

The evaluation of options on stocks that pay discrete dividends was the subject of numerous studies that concern both closed-form formula and numerical approximate methods. Recently, Haug *et al.* [14] have proposed an integral expression that allows to calculate in precise terms the values of European call and put options and American call options. Such an integral representation is particularly interesting because it can be extended to the case of non-Brownian dynamics and to the case of multiple cahs dividends. Moreover, it is worth noting that the HHL formula requires the numerical evaluation of an integral that, for certain values of the parameters, presents hard difficulties from the computational point of view.

The pricing of American put options written on stocks which pay discrete dividend can be obtained with a standard binomial scheme that produces very accurate results, but it leads to non-recombining trees and therefore the number of nodes does not grow linearly with the number of steps.

In this contribution, we analyzed alternative methods to the classical binomial approach for European and American options; in particular, the binomial method based on an interpolation procedure, in which the recombining feature is maintained, turned out to be very efficient, easy to implement and to be applied also to the case of multiple dividends.

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