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# Credit contagion in a network of firms with spatial interaction

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**Abstract.** In this contribution we carried out a wide simulation analysis in order to study the contagion mechanism induced in a portfolio of bank loans by the presence of business relationships among the positions. To this aim we jointly apply a structural model based on a factor approach extended in order to include the presence of microeconomic relationships that takes into account the counterparty risk, and a network model to describe the business connections among interdependent firms. The network of firms is generated resorting to an entropy spatial interaction model.

**Keywords:** credit risk, bank loan portfolios, contagion models, entropy spatial models.

**JEL Classification Numbers:**

**MathSci Classification Numbers:** 65C05, 90B99.

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## 1 Introduction

In this paper we consider the effects of the credit contagion induced by the presence of counterparty risk on the credit risk of a portfolio of positions.

The recent subprime credit crisis motivates models in which default events have repercussions on the other firms in the economy; in particular, this influence can be modeled through the introduction of counterparty risk and the contagion effect it induces.

Different approaches have been proposed in the literature in order to analyze the credit risk of a portfolio of bank loans: among others, we may cite for example Aguais, Forest and Roden [2], Kern and Rudolf [20], Westgaard, van der Wijst [29] and Lucas et al. [22].

Some recent approaches, proposed by Giesecke and Weber [17], Neu and Kühn [23], Egloff, Leippold and Vanini [14] introduce some models that take into account both the dependence on the business cycle and a direct contagion effect among the firms in the economic system. For a discussion on the relationships between business and default cycles see, for example, Koopman and Lucas [21]; see also Delli Gatti et al. [11], who analyze the role of credit interlinkages as a source of bankruptcy diffusion in a network economy with three sectors (downstream firms, upstream firms, and banks).

Along different lines, starting from an intensity-based context, other recent contributions analyze the problem of credit contagion in the context of multi-name credit derivatives; see e.g. [12], [13], [15], [16].

In this paper, starting from a previous work [3], we apply a discrete time model to analyze the role of counterparty risk in the contagion mechanism of defaults. The presence of business relations among the various firms in a portfolio entails a degree of connection which can translate into a mechanism of contagion of defaults in presence of financial distress. This contagion reflects also in the loss distribution of the portfolio of positions.

The model applied to study the contagion mechanism consists of two main components which describe the counterparty risk and the network of the business relations, respectively.

To describe the counterparty risk we use the discrete time model proposed in Barro and Basso [3], that models the asset value of a firm following a structural approach, a brief review of the model is presented in section 3. While to describe the connection among firms we resort to a network structure modelled using spatial interaction [4], [5]. For more details on the network structure we refer to [5].

In order to study the propagation and the dynamic behavior of the defaults in the system and their effects on the risk of a bank loan portfolio, we carry out a wide simulation analysis of the model proposed. More specifically, we apply a Monte Carlo simulation technique first to build a number of proper networks of firms and then to simulate the behavior of the model on these networks.

The networks of firms are simulated by taking into account different features, among which the economic sector and the geographical location. In particular, in order to simulate the location of the firms, we introduce an entropy spatial interaction model which considers both the distance among the different geographical areas and the economic weight of each area, thus including some interesting economic features that would be difficult to model otherwise.

The simulation analysis is carried out by generating networks that represent loan portfo-

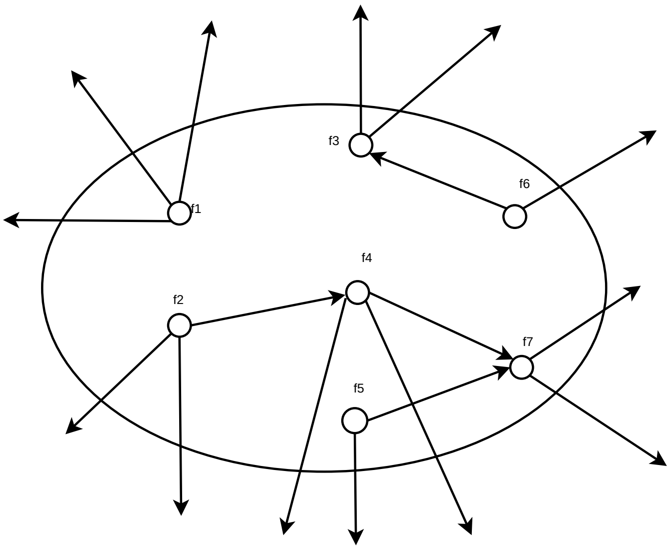



Figure 1: Example of network of business connections for $q > 0$.

lios of an Italian bank and sheds light on various features of the model. Among the features observed, we point out a fattening of the tails of the loss distribution of the portfolio due to the presence of the microeconomic component which models the counterparty risk. Hence, the contagion effect due to the counterparty risk seems to induce a higher level of credit risk.

## 2 Spatial interaction modelling for networks of firms

The model consists of two main parts which describe the counterparty risk and the network of the business relations, respectively. In this section we briefly present the model used to build the business relations network, for a more detailed discussion see [5], while the model used to describe the counterparty risk is presented in the next section. Afterwards, the two models will be used combined to study the effects of credit contagion in section 4.

The business connections are modelled using a weighted network in which the nodes represent the firms, directed edges connect each firm with its major clients and the weight associated to each edge is given by the percentage of sales to this client on the turnover of the firm.

The firms included in the network are given by the $N$ firms of the bank portfolio under consideration and, in addition, by the firms that are major clients of any of the firms in the portfolio. An example of network of business connections is shown in figure 1; where the firms in the bank portfolio are represented by the nodes inside the ellipse, while the arcs directed to points outside the ellipse represents the business connections with "major" clients that are not included in the bank portfolio.

In order to study the propagation of the defaults in the system and the effects on the risk of a bank loan portfolio, first we have simulated a number of networks of firms with a

Monte Carlo simulation technique and then we have simulated the behavior of the system in each of these networks.

These networks of firms are built by taking into consideration a spatial dimension and the firms are located in different areas by resorting to an entropy spatial interaction model which takes into account both the economic weight of the different areas considered and their distance, for the details of the entropy spatial interaction model used we refer to [5].

## 3 The model

In this section we briefly recall the discrete time model proposed in [3] to model counterparty risk. The model will be used in the simulation analysis to study the contagion effect among defaults in a portfolio of bank loans.

The model, based on structural-type approach, focuses on the asset value of firms which is modelled as the sum of a macroeconomic component, a microeconomic component and a residual idiosyncratic term.

The macroeconomic component, $F$, uses a factor model to describe the influence of the business cycle on the value of the firm; this component captures the systemic risk which affects each position in the portfolio. The microeconomic component, $M$, introduces the business connections with other firms in the economy thus allowing for potential contagion effects. The residual term, $\varepsilon$, is described as a random variable assumed to be independent of the factors in the macroeconomic component. The microeconomic component and the idiosyncratic component accounts for the firm-specific sources of risk.

We consider a portfolio of positions that we assume correspond to bank loans given to $N$ firms pertaining to $S$ economic sectors. We denote by $s(i)$ the sector of firm $i$, with $i = 1, 2, \dots, N$.

The value of the macroeconomic component $F_i(t)$ is described by a factor model (see, for example, Schönbucher [28], Saunders, Xiouros and Zenios [25], Jimenez and Mencia [19]; for a structural model with a jump-diffusion process for the risk factors see Schäfer et al [26]), as follows:

$$F_i(t) = \sum_{j=1}^{J} \beta_j^{s(i)} Y_j(t) \qquad t = 0, 1, \dots, \tag{1}$$

where $Y(t) = (Y_1(t), Y_2(t), \dots, Y_J(t))$ is the vector of the values at time $t$ of the driving factors and $\beta_j^s$ is the weight of factor $j$ for the firms of sector $s$.

The microeconomic component $M_i(t)$ takes into consideration the direct business connections among the firms represented in the weighted network defined in the previous section.

The business connections taken into consideration are the relations of a firm with its "important" clients, where the importance of client $k$ for firm $i$ is measured by the percentage $w_{ik}$ of the sales to client $k$ on the turnover of firm $i$.

The basic idea is that if a client which represents a significant percentage of the turnover of firm $i$, let us say above a given threshold $\theta$, defaults, this may result in a serious cause of distress for firm $i$, while if a "minor" client, with purchases below threshold $\theta$, defaults the repercussions may be negligible. Hence, we may take into consideration in the network only the connections with the clients with a percentage of turnover above a given threshold $\theta$.

We assume that the default of a "major" client affects the health of a firm with a one-period delay and that the effects are dampened according to an exponential decay in time.

More precisely, the microeconomic component $M_i(t)$ is modelled as a firm-specific term which depends on the distress undergone by firm $i$ due to the defaults observed among its clients in the previous periods. We define a distress measure $D_i(t)$ connected to the defaults observed at time $t$ among the clients of firm $i$, compared to the average default rate observed in the economy at time $t$, $p(t)$, as follows

$$D_i(t) = p(t) - \left[ \sum_{k \in C_i(t)} \delta_k(t)\, w_{ik}(t) + p(t)\, r_i(t) \right], \tag{2}$$

where $C_i(t)$ denotes the set of the major clients of firm $i$, and $\delta_k(t)$ is the indicator function of the default event at time $t$.

The per cent value of the turnover of firm $i$ sold to the remaining minor clients is defined as a residual term

$$r_i(t) = 1 - \sum_{k \in C_i(t)} w_{ik}(t). \tag{3}$$

For the residual part of the turnover, due to a large number of minor clients, a per cent amount equal to the average default rate $p(t)$ is assumed to default.

We assume that all the distresses undergone by the firm in the past periods affect the current health of the firm, with an exponentially decaying in time. The overall distress influencing the health of firm $i$ at time $t$ is therefore measured as the sum of the effects of all the past defaults of its clients as follows

$$\begin{aligned} \sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} D_i(t-\tau) = \sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} \Bigg[ p(t-\tau) - \Bigg( & \sum_{k \in C_i(t-\tau)} \delta_k(t-\tau)\, w_{ik}(t-\tau) + \\ & + p(t-\tau)\, r_i(t-\tau) \Bigg) \Bigg], \end{aligned} \tag{4}$$

where the parameter $\lambda_s$, with $0 \leq \lambda_s < 1$, is the dampening factor which determines the distress memory of firms in sector $s$. We may notice that if, as usual, the value of $\lambda_s$ is sufficiently small, only the first terms in the infinite summation in equation (4) have a non negligible value.

The microeconomic component, $M_i(t)$, takes into account the effects of the past distresses on the health of firm $i$ and is defined as follows

$$M_i(t) = \mu_{s(i)} \sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} D_i(t-\tau), \tag{5}$$

where $\mu_s \in \mathbb{R}_+$ is a real parameter, possibly dependent on the economic sector of the firm. As it can be seen, the microeconomic component $M_i(t)$ is a firm-specific additive term which brings about a rise or a decrease in the value of firm $i$ with respect to macroeconomic component $F_i(t)$, according to the fact that the overall financial distress due to the past defaults of clients is lower or higher than the average distress undergone by the sector of the firm. As a result, a contagion mechanism is introduced in the model.

In addition to the macro and microeconomic components we have a residual idiosyncratic term $\varepsilon_i(t)$, which is assumed to be normally distributed with zero mean and standard deviation $\sigma_{\varepsilon_i}$. Moreover, we assume that the residual idiosyncratic terms $\varepsilon_1(t), \varepsilon_2(t), \dots, \varepsilon_N(t)$ are both mutually independent and independent of the factors $Y_1, Y_2, \dots, Y_J$.

Therefore, the asset value of firm $i$, $V_i(t)$, is defined as the sum of the macroeconomic, microeconomic and residual terms, as follows

$$\begin{aligned} V_i(t) &= F_i(t) + M_i(t) + \varepsilon_i(t) = \\ &= \sum_{j=1}^{J} \beta_j^{s(i)} Y_j(t) + \mu_{s(i)} \sum_{\tau=1}^{\infty} \lambda_{s(i)}^{\tau} D_i(t-\tau) + \varepsilon_i(t). \end{aligned} \tag{6}$$

For more details on the model we refer to [3].

# 4 An empirical investigation of credit contagion with Monte Carlo simulation

In this section we present the simulation analysis carried out in order to study the effects of default contagion on a portfolio of bank loans. This study can be considered as the continuation of a first basic analysis carried out in [4]. On the basis of the previously obtained result we devised a set of simulation to better tackle the issue of contagion analysis considering different values for the parameters and different time horizons.

In the following we present the choices of the parameters and the simulation settings. We consider large portfolios of exposures, such as the portfolios of bank loans to small and medium enterprises; in particular, the number of firms in the portfolios is set equal to $N = 10000$. The number of clients of each firm and the volume of sales for each client are randomly generated according to a normal distribution with mean 50 and standard deviation 25 and to a lognormal distribution with parameters $(5, 2)$, respectively.

For the macroeconomic component we consider one factor simulated according to a mean-reverting process with drift 0.5, volatility 0.08 and long-run mean 1.

We consider $S = 10$ sectors according to the Global Industry Classification Standard (GICS) developed by Morgan Stanley Capital International (MSCI) and Standard & Poor's (Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financial, Information Technology, Telecommunications Services, and Utilities). The GICS methodology has been widely accepted as a framework in industry analyses for investment research, portfolio management and asset allocation. For a more detailed description of the sectors and of the classification methodology we refer to the documentation which is available at the MSCI web site (www.msci.com/equity/gics.html).

In order to determine the sector of each firm and of each major client of the firms in the portfolio we use an Input/Output table which quantify the relationships between different sectors. We consider the Input/Output table for the Italian economy for the year 2001 and aggregate it according to the GICS classification. The resulting table is normalized in such a way that the generic element of the matrix $a_{ij}$ gives the probability that a firm in sector $i$ has a client in sector $j$. Therefore, each row of the table represents a vector of probabilities describing the relations among the sector considered and all the sectors in the economy.

In the simulation we consider a bank, which we assume to be located in area 1, and a portfolio of loans. For each obligor in the portfolio one of the feature considered is its geographical location. In more detail, we consider 15 different areas. The areas which surround the area of the bank are smaller while the areas which are more far-away are wider; this allows to obtain a higher degree of detail in the classification of the loans in the portfolio. We assume that the first area, that is the area of the bank, corresponds to the province of Venezia, areas from 2 to 5 covers the Veneto region while areas from 6 to 10 correspond to the other regions in the North of Italy. The remaining areas from 10 to 14 cover the Central and South part of Italy, while area 15 is a wide generic area which includes all the foreign countries.

To determine the area of each firm we apply the entropy spatial interaction model described in section 4. As for the weights associated to each destination in the entropy spatial interaction model, we assume that the population of an area represents an adequate proxy for the economic relevance of the area. For the foreign countries area the economic weight is computed proportionally to the relative weight of the exports over the Italian GNP. The weights are then normalized in such a way as they sum to one.

Table 1: Matrix of the distances among the 15 geographical areas considered.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 41 | 46 | 70 | 120 | 167 | 165 | 159 | 284 | 415 | 265 | 567 | 803 | 1540 | 1600 |
| 2 | | 0 | 70 | 98 | 145 | 146 | 166 | 176 | 296 | 426 | 283 | 565 | 770 | 1453 | 1600 |
| 3 | | | 0 | 43 | 90 | 139 | 186 | 122 | 247 | 378 | 228 | 530 | 765 | 1503 | 1600 |
| 4 | | | | 0 | 54 | 151 | 206 | 155 | 214 | 345 | 261 | 563 | 799 | 1536 | 1600 |
| 5 | | | | | 0 | 100 | 256 | 141 | 164 | 295 | 247 | 549 | 790 | 1534 | 1600 |
| 6 | | | | | | 0 | 279 | 233 | 218 | 349 | 339 | 641 | 882 | 1626 | 1600 |
| 7 | | | | | | | 0 | 308 | 420 | 551 | 414 | 715 | 951 | 1689 | 1600 |
| 8 | | | | | | | | 0 | 218 | 338 | 106 | 408 | 660 | 1415 | 1600 |
| 9 | | | | | | | | | 0 | 139 | 324 | 626 | 878 | 1633 | 1600 |
| 10 | | | | | | | | | | 0 | 442 | 702 | 972 | 1743 | 1600 |
| 11 | | | | | | | | | | | 0 | 302 | 600 | 1345 | 1600 |
| 12 | | | | | | | | | | | | 0 | 367 | 1043 | 1600 |
| 13 | | | | | | | | | | | | | 0 | 774 | 1600 |
| 14 | | | | | | | | | | | | | | 0 | 1600 |
| 15 | | | | | | | | | | | | | | | 0 |

Table 1 presents the distance matrix used in the simulation. Distances are expressed in kilometers and are computed with reference to the most representative city in each area, with the exception of the distances to the foreign countries area 15, which have been chosen roughly, in a subjective manner. Table 2 describes the different areas considered, the weights $W_j$ assigned to each area, and the probabilities $\pi_{1j}$ obtained with the entropy spatial interaction model by choosing $\beta = 0.08$.

As for the dampening factor of the microeconomic component, we choose a value $\lambda =$

Table 2: The geographical areas, the economic weights associated to each area and the probabilities obtained applying the entropy spatial interaction model with $\beta = 0.08$.

| | Area | $W_j$ | $\pi_{1j}$ (%) |
|---|---|---|---|
| 1 | Venezia | 0.01096 | 92.16148 |
| 2 | Treviso-Belluno | 0.01372 | 4.34255 |
| 3 | Padova-Rovigo | 0.01486 | 3.15215 |
| 4 | Vicenza | 0.01079 | 0.33569 |
| 5 | Verona | 0.01115 | 0.00635 |
| 6 | Trentino Alto Adige | 0.01258 | 0.00017 |
| 7 | Friuli Venezia Giulia | 0.01624 | 0.00025 |
| 8 | Emilia Romagna | 0.05399 | 0.00136 |
| 9 | Lombardia | 0.12265 | 2.66E-12 |
| 10 | North-West Italy | 0.08301 | 4.15E-07 |
| 11 | Central Italy 1 | 0.07939 | 1.57E-17 |
| 12 | Central Italy 2 | 0.09339 | 1.82E-25 |
| 13 | South Italy | 0.17192 | 2.43E-51 |
| 14 | Sicilia-Sardegna | 0.09260 | 3.38E-67 |
| 15 | Foreign countries | 0.12340 | 1.4E-171 |

0.15; this means that the effects of the default of a client becomes negligible from the fourth period on. To initialize the model we assume that no firm specific information is available for the past and thus the lagged terms, before the initial time $t = 0$, have been set equal to 0; of course this choice will influence the behavior of the system in the first periods.

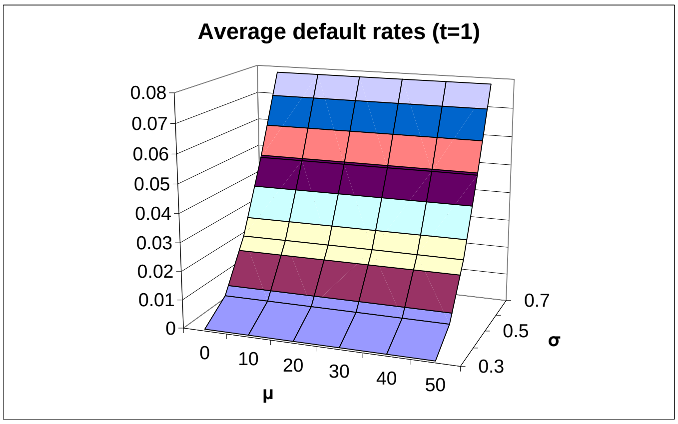
The idiosyncratic term $\varepsilon_i(t)$ is generated according to a normal random variable $N(0, \sigma)$.

In the simulations the default barriers are set equal to zero for all firms, the exposure of the bank with each obligor is held constant and all portfolio losses are measured as a percentage of the overall exposure of the bank portfolio. The recovery rate is set equal to 50 % and held constant.

We have carried out different sets of simulations in order to investigate the characteristics of the model. In each simulation, for every set of parameters we randomly generated 10 portfolios with 10 000 positions each, and for each of them we simulated 10 000 time paths of the system.

In a first set of experiments we have studied the behavior of the average default rate of the obligors in the portfolio as the coefficient $\mu$ of the microeconomic component and the standard deviation $\sigma$ of the residual idiosyncratic term vary. In this set of trials we have fixed the value of the parameter $q$, which determines the density of the business connections in the network, to the value 0.3.

The results are shown in table 3, which reports the average default rate for the first periods ($t = 1, \ldots, 5$). Notice that by introducing the assumption of a constant recovery rate for all the exposures we can immediately obtain the expected loss distribution of the portfolio. In the experiments the recovery rate is set equal to 50 % so that the expected loss turn out to be half the default rate.



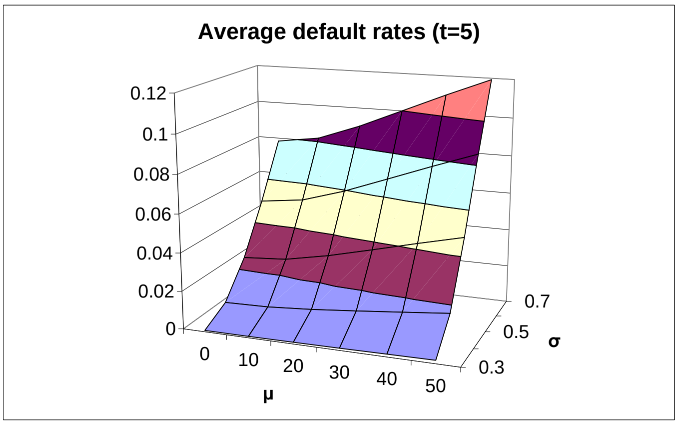




Figure 2: Average default rates of the obligors in the simulated bank portfolios for $t = 1$ and $t = 5$ for different values of $\mu$ and $\sigma$.

As can be seen, the parameter $\mu$, which determines the relative importance of the microeconomic component on the firm's value $V(t)$, has the effect of raising the default rate. Indeed, for any fixed value of $\sigma$, the average default rate increases with the value of $\mu$. The only exception is for time $t = 1$, in which the value of $\mu$ has no effect due to the assumption of a null initial value of the microeconomic component (no prior information on the clients' defaults in the past). The difference between the first and the fifth periods is pointed out in figure 2 which enables to compare the average default rate in these two periods. Figure 3 shows the effects of the initial conditions as time goes on; it is apparent that this effect vanishes very soon and after two periods is negligible.

Let us observe that in the case $\mu = 0$ the microeconomic component is not present in the model and the observed average default rates are entirely due to the macroeconomic and idiosyncratic effects. From the comparison between the case $\mu = 0$ and the cases with $\mu > 0$ it is clear that the addition of the microeconomic component entails a raising in the average default rates due to the contagion mechanism among the defaults, and the higher the value of the microeconomic parameter $\mu$ is, the more substantial this effect is.

As for the effect of the residual idiosyncratic term on the default rates, it is not surprising that it depends on the magnitude of the standard deviation $\sigma$ and that the average default rates increase with the value of $\sigma$.

Table 3: Average default rates of the obligors in the simulated bank portfolios for $t = 1, \ldots, 5$ for different values of $\mu$ and $\sigma$.

| | $\sigma$ | | | | |
|---|---|---|---|---|---|
| $\mu$ | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
| | | | **t = 1** | | |
| 0 | 0.00064 | 0.00713 | 0.02416 | 0.04926 | 0.07787 |
| 10 | 0.00064 | 0.00714 | 0.02417 | 0.04929 | 0.0779 |
| 20 | 0.00064 | 0.00713 | 0.02416 | 0.04927 | 0.07788 |
| 30 | 0.00064 | 0.00713 | 0.02421 | 0.04927 | 0.07788 |
| 40 | 0.00064 | 0.00712 | 0.02414 | 0.04932 | 0.07791 |
| 50 | 0.00064 | 0.00712 | 0.02409 | 0.04929 | 0.07792 |
| | | | **t = 2** | | |
| 0 | 0.0007 | 0.00736 | 0.02446 | 0.04963 | 0.07826 |
| 10 | 0.00074 | 0.00788 | 0.02616 | 0.05255 | 0.08177 |
| 20 | 0.00088 | 0.00946 | 0.03075 | 0.05995 | 0.09088 |
| 30 | 0.00109 | 0.01163 | 0.03668 | 0.06904 | 0.10189 |
| 40 | 0.00134 | 0.01393 | 0.0427 | 0.07839 | 0.11299 |
| 50 | 0.00161 | 0.01636 | 0.04873 | 0.08747 | 0.12362 |
| | | | **t = 3** | | |
| 0 | 0.00071 | 0.00741 | 0.02459 | 0.04968 | 0.07829 |
| 10 | 0.00076 | 0.00792 | 0.02625 | 0.05259 | 0.08172 |
| 20 | 0.00092 | 0.00961 | 0.03086 | 0.05977 | 0.09034 |
| 30 | 0.00115 | 0.01182 | 0.03661 | 0.0684 | 0.1006 |
| 40 | 0.00142 | 0.01412 | 0.04253 | 0.07717 | 0.11072 |
| 50 | 0.00173 | 0.01661 | 0.04834 | 0.08577 | 0.12042 |
| | | | **t = 4** | | |
| 0 | 0.00072 | 0.00742 | 0.02462 | 0.04975 | 0.07831 |
| 10 | 0.00076 | 0.00795 | 0.02623 | 0.05266 | 0.08174 |
| 20 | 0.00093 | 0.00961 | 0.03084 | 0.05962 | 0.09022 |
| 30 | 0.00117 | 0.01182 | 0.03658 | 0.06825 | 0.10017 |
| 40 | 0.00145 | 0.01418 | 0.04253 | 0.07694 | 0.11018 |
| 50 | 0.00175 | 0.01662 | 0.04816 | 0.08534 | 0.11969 |
| | | | **t = 5** | | |
| 0 | 0.00072 | 0.00744 | 0.02461 | 0.04969 | 0.07834 |
| 10 | 0.00077 | 0.00797 | 0.02624 | 0.05251 | 0.08172 |
| 20 | 0.00093 | 0.00962 | 0.03083 | 0.05951 | 0.09009 |
| 30 | 0.00117 | 0.0118 | 0.03654 | 0.06812 | 0.09998 |
| 40 | 0.00146 | 0.01418 | 0.04243 | 0.0768 | 0.10991 |
| 50 | 0.00175 | 0.01661 | 0.04804 | 0.08508 | 0.11931 |

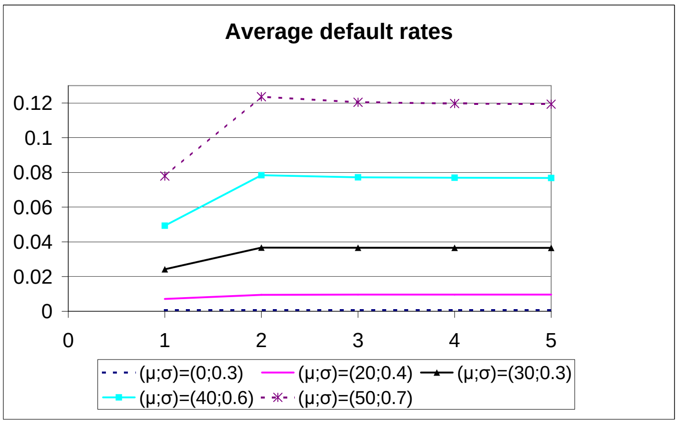



Figure 3: Average default rates of the obligors in the simulated bank portfolios as $t$ varies for some pairs $(\mu, \sigma)$.

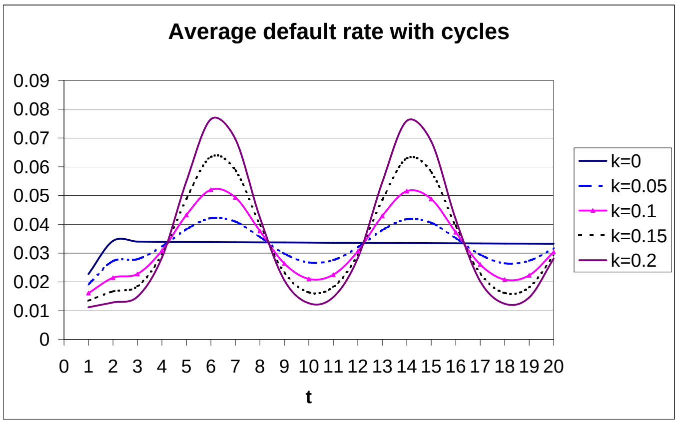



Figure 4: Dynamic behavior of the average default rate for different values of the amplitude $k$ of the cycle of the macroeconomic factor, with $(\mu, \sigma) = (30, 0.5)$.

In addition we have carried out a second sets of experiments with the aim of analyzing the influence of the macroeconomic component on the average default rate. More precisely, we have considered a macroeconomic factor with a given cyclical behavior and studied how much this cyclical behavior reflects on the value of the default rate.

The results are summarized in figure 4 which shows the dynamic behavior of the average default rate for the case $(\mu, \sigma) = (30, 0.5)$, for different amplitudes of the cycle of the macroeconomic factor. If we compare the results obtained with a constant value of the macroeconomic factor, for an amplitude of the cycle $k = 0$, with those obtained with an amplitude $k > 0$, we observe that the presence of a cycle in the macroeconomic factor induces a cyclical behavior in the default rate. Besides, the more pronounced the amplitude of the cycle is, the higher the amplitude of the cycle of the default rate is. In addition we note that the effect of a slump period on the default rate is more marked than that of a boom.

Furthermore we have carried out a third set of experiments in order to investigate the portfolio losses and in particular the behavior of the loss distribution, the Value at Risk (VaR) and the Expected Shortfall (ES) of the portfolio. These simulations, too, have been

Table 4: Simulated VaR of the bank loan portfolio at the 99 % and 99.9 % confidence levels as $\mu$ and $\sigma$ varies.

| | $\sigma$ | | | | |
|---|---|---|---|---|---|
| $\mu$ | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
| | | | **VaR 0.99** | | |
| 0 | 0.002315 | 0.012385 | 0.029445 | 0.04823 | 0.065795 |
| 10 | 0.002555 | 0.013265 | 0.030715 | 0.050365 | 0.068015 |
| 20 | 0.00286 | 0.015495 | 0.03546 | 0.054745 | 0.072765 |
| 30 | 0.003445 | 0.01859 | 0.04013 | 0.060475 | 0.07835 |
| 40 | 0.004075 | 0.02189 | 0.04455 | 0.065795 | 0.084325 |
| 50 | 0.005035 | 0.024495 | 0.049175 | 0.07137 | 0.088945 |
| | | | **VaR 0.999** | | |
| 0 | 0.004735 | 0.018645 | 0.038775 | 0.06041 | 0.07646 |
| 10 | 0.004685 | 0.019065 | 0.03975 | 0.060795 | 0.08043 |
| 20 | 0.00532 | 0.021935 | 0.04614 | 0.0656 | 0.085885 |
| 30 | 0.006325 | 0.02715 | 0.051375 | 0.07175 | 0.090615 |
| 40 | 0.007825 | 0.02994 | 0.05655 | 0.07892 | 0.09474 |
| 50 | 0.009245 | 0.03495 | 0.061995 | 0.08281 | 0.099685 |

Table 5: Simulated Expected Shortfall of the bank loan portfolio as $\mu$ and $\sigma$ varies at the 99 % and 99.9 % confidence levels.

| | $\sigma$ | | | | |
|---|---|---|---|---|---|
| $\mu$ | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 |
| | | | **ES 0.99** | | |
| 0 | 0.00237 | 0.012505 | 0.02963 | 0.048685 | 0.066465 |
| 10 | 0.00259 | 0.01334 | 0.030755 | 0.05055 | 0.068195 |
| 20 | 0.002895 | 0.01573 | 0.035635 | 0.05522 | 0.072885 |
| 30 | 0.003585 | 0.018705 | 0.040425 | 0.06084 | 0.078375 |
| 40 | 0.004185 | 0.02217 | 0.044575 | 0.06686 | 0.08479 |
| 50 | 0.005165 | 0.02482 | 0.049325 | 0.07177 | 0.089725 |
| | | | **ES 0.999** | | |
| 0 | 0.00487 | 0.019185 | 0.03936 | 0.061335 | 0.07724 |
| 10 | 0.0055 | 0.01917 | 0.039775 | 0.063815 | 0.081635 |
| 20 | 0.00562 | 0.022145 | 0.04705 | 0.065625 | 0.087595 |
| 30 | 0.00636 | 0.027175 | 0.051635 | 0.071775 | 0.093345 |
| 40 | 0.00814 | 0.030375 | 0.056575 | 0.08011 | 0.09662 |
| 50 | 0.00969 | 0.034975 | 0.065385 | 0.083655 | 0.100675 |

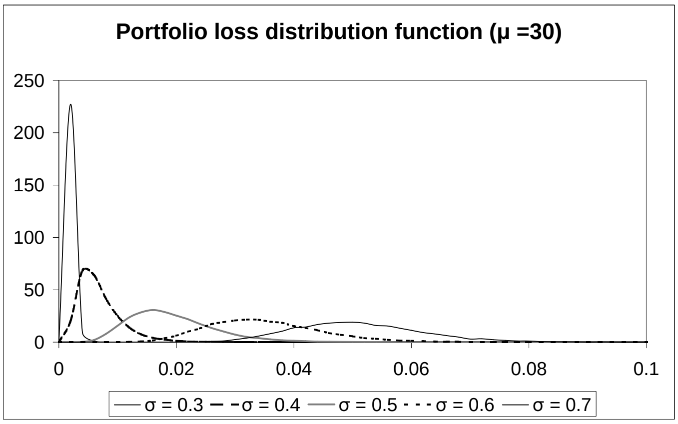



Figure 5: Simulated pdf of the portfolio losses, measured as a fraction of the overall value of the portfolio, for $\mu = 30$ as $\sigma$ varies.

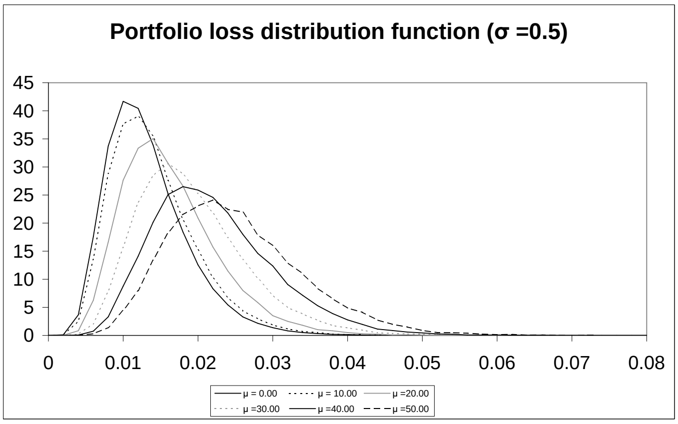



Figure 6: Simulated pdf of the portfolio losses, measured as a fraction of the overall value of the portfolio, for $\sigma = 0.5$ as $\mu$ varies.

performed by assuming that the recovery rate is constant for all the exposures and equal to 50 %. In these trials we have simulated one portfolio and increased the number of simulations to 200 000.

Figures 5 and 6 show the behavior of the loss distribution of the bank loan portfolio for different values of the parameters $\mu$ and $\sigma$.

As can be seen from figure 5, the loss distribution is very sensitive to $\sigma$, i.e. to the weight of the idiosyncratic term. In particular, the loss distribution turns out to be highly concentrated on small losses for the lower values of $\sigma$ and more and more dispersed on higher loss values as $\sigma$ increases. An analogous shift of the loss distribution can be observed with respect to changes in the weight of the microeconomic component $\mu$. As a matter of fact, as $\mu$ increases the loss distribution moves to the right while its tails fatten.

The fattening of the tails as $\mu$ and $\sigma$ increase leads to higher values of the VaR and ES for all confidence levels, as can be seen in tables 4 and 5.

The results point out the effect of the introduction of the microeconomic component on the portfolio loss distribution: it entails a shift of the entire loss distribution to the right and, at the same time, a fattening of the right tail of the distribution. Therefore, the probability of the occurrence of a loss greater than or equal to a given value increases considerably, as it increases the Value at Risk. And the higher the relative importance of the microeconomic component, i.e. the value of $\mu$, is, the more marked this effect is. Thus the introduction of the contagion mechanism in the model results in an increase of the overall riskiness of the portfolio.

The effects of an increase in the value of the standard deviation of the idiosyncratic term on the risk profile of the portfolio are similar, with an even more pronounced shift of the loss distribution to the right.

## 5 Concluding remarks

In this contribution we have applied a dynamic model that takes into account the counterparty risk in a network of interdependent firms linked by business connections to analyze the contagion effects on the credit risk of a portfolio of bank loans.

In order to model the firms in the network we have considered different features, such as the economic sector and the geographical location. To simulate the location of the firms, we have applied an entropy spatial interaction model which allows to take into account both the distance among the different geographical areas and the economic weight of each area.

We have carried out a wide simulation study to investigate the main features of the model proposed for different values of the model parameters. In particular, we have analyzed in detail the behavior of the average default rate of the system, the probability density function of the portfolio losses, and the Value at Risk and the Expected Shortfall of the portfolio.

The calibration of the model on real data and an extension of the model which enables to introduce a set of ratings classes and to study the downgrade/upgrade transition probabilities are open issues and represent possible directions for further research.

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