Inflation Taxation and Welfare with Externalities and Leisure

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Abstract

This paper examines how inflation taxation affects resource allocation and welfare in a neo-classical growth model with leisure, a production externality and money in the utility function. Switching from consumption taxation to inflation taxation to finance government spending reduces real money balances relative to income, but increases consumption, labor, capital and output. The net welfare effect of this switch depends crucially on the strength of the externality and on the elasticity of intertemporal substitution: While it is always negative without the externality, it is likely to be positive with a strong externality and elastic intertemporal substitution.

How inflation taxation affects resource allocation and welfare has received a great deal of attention. So far, this debate has been controversial. Most studies claim that inflation taxation reduces welfare regardless of whether money demand is derived from a cash-in-advance constraint, from a transaction technology with money as an intermediate input, or from real money balances in the utility function (e.g. Friedman, 1969; Kimbrough, 1986; Prescott, 1987; Cole and Stockman, 1992; Schreft, 1992; Gillman, 1993; Gomme, 1993; Correia and Teles, 1996; Dotsey and Ireland, 1996; Aiyagari, Braun and Eckstein, 1998; Wu and Zhang, 1998, 2000; Lucas, 2000; and Erosa and Ventura, 2002). In many of these studies, the optimal rule of money growth is to generate a degree of deflation such that the nominal interest rate, i.e. the opportunity cost of holding money, equals zero as described by the Friedman rule.

On the other hand, some studies support a rate of money growth for a positive nominal interest rate by considering additional factors. In Phelps (1973), Braun (1994) and Palivos and Yip (1995), inflation taxation leads to higher welfare than income taxation as a means of public finance. Guidotti and Vegh (1993) derive optimal inflation taxation with increasing returns to scale in the transaction-cost technology. Shi (1999) supports an optimal money-growth rate in excess of the Friedman rule by assuming borrowing constraints. In Rebelo and Xie (1999), money does not affect production in the steady state but can alter it during the transition toward the steady state; and the transitional effect can be exploited by monetary policy to improve welfare if there is a production externality. Most of these studies have fully inelastic labor supply, and ignore externalities that are nevertheless prevalent in the real world. It remains to be seen how money growth affects welfare with both a production externality and a labor-leisure trade-off. As shown in Turnovsky (2000), the inclusion of an endogenous labor-leisure trade-off leads to fundamental changes in the economy's equilibrium structure as there is an equilibrium growth-leisure trade-off.

In this paper we examine how nominal money growth as a means of public finance affects resource allocation and welfare in a neoclassical growth model with leisure, a production externality and money in the utility function. For simplicity, we assume that government spending is a fixed fraction of output and is not valued by private agents. The government may also tax consumption to finance its spending, unlike Phelps (1973), Braun (1994) and Palivos and Yip (1995) where they

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compare an income tax with the inflation tax. Individuals allocate income to consumption and investment in both capital and real money balances, and allocate time to labor and leisure.

In this environment, switching from consumption taxation to inflation taxation drives up the cost of holding money, and thus reduces the demand for real money balances relative to income as is well known in the literature. Also well known is that economizing on money holdings accelerates the circulation of money. Further, the decrease in real money balances reduces the marginal benefits of consumption and leisure with a nonseparable utility function concerning these augments, which tends to reduce consumption and leisure. However, the rise in the nominal money growth rate has no direct effect on the cost of leisure (the real wage), and therefore leisure should decline and, accordingly, labor should increase. The increased labor in turn raises the marginal product of capital and stimulates capital accumulation, leading to higher output per capita.[1]

On the other hand, the accompanying decline in the consumption tax rate also lowers the cost of consumption, tending to raise consumption. Given the standard constant elasticity of intertemporal substitution (CEIS) form of the utility function in the literature on economic growth, the net effect on the ratio of consumption to output is zero. Thus, as output rises with the nominal money growth rate, so does consumption. The rise in output also works against the decline in the ratio of real money balances to output in the determination of the effect of faster nominal money growth on the level of real money balances. As a consequence, there are opposing effects of the switch from consumption taxation to inflation taxation on welfare: The increase in consumption tends to raise welfare but the decline in leisure tends to reduce welfare. How real money balances respond to the tax switch also affects welfare. The net effects of the tax switch on real money balances and welfare depend crucially on whether production externalities are taken into consideration.

First, consider the case that has no externality where the concern about monetary policy focuses mainly on the benefit and cost of holding money. In the spirit of the Friedman rule, since the social cost of producing money is zero, there should be a negative inflation tax such that the cost of holding money (the nominal interest rate) can be as close to zero as possible. As a result, the optimal inflation tax is negative along with a positive consumption tax, but the underlying nominal money growth rate should exceed the rate that corresponds strictly to the Friedman rule because

of the tax distortions on leisure, output and consumption.

Now consider the case with a production externality in the form of learning-by-doing and knowledge spillovers. In addition to the consideration above, we know that the private rate of return on capital investment must be lower than the social rate due to the externality, which implies that agents hold too little capital compared to the social optimum. Because of the under-investment in capital, the private rate of return on labor must also be lower than the social rate, which implies that agents have too much leisure and too little labor compared to the social optimum. The new consideration is that we want agents to hold more capital, which can be achieved by applying a higher inflation tax in order to induce them to switch out of real money balances into capital. The positive effects of the switch to inflation taxation on labor, capital investment and output help the economy correct the under-investment in capital and may thus improve welfare.

We find two key factors in the determination of the relative strength of the opposing effects of the tax switch on welfare. First, a stronger production externality strengthens the positive welfare effect, other things equal. Second, since the positive effect of the switch to inflation taxation on output takes time to reach its full potential through promoting capital accumulation, the relative strength of the positive welfare effect also depends on the elasticity of intertemporal substitution. More elastic intertemporal substitution accelerates growth and hence strengthens the positive welfare effect. We show that when output can fully adjust in the long run, the net welfare effect of switching from consumption taxation to inflation taxation can be positive so long as the externality is strong enough. However, when considering the entire equilibrium path in a tractable AK model with a strong enough externality for endogenous growth, a positive net welfare effect of this tax switch also requires the elasticity of intertemporal substitution to be sufficiently high.

The avenue through which inflation taxation affects production and welfare in this model differs from those in the related work mentioned above. By reducing the consumption tax, raising the inflation tax to finance a given government fiscal commitment can stimulate production in the long run in our model, as opposed to the long-run neutrality of money growth in Rebelo and Xie (1999) without a consumption tax. The difference originates mainly from the different assumptions concerning labor supply (elastic in our model but fully inelastic in their model). If labor supply

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were fully inelastic in our model, then switching from consumption taxation to inflation taxation would have no real effect on consumption and production in the long run.

The rest of the paper proceeds as follows. Section 1 introduces the model and characterizes the equilibrium. Section 2 presents results without sustainable growth in the absence of any externality or in the presence of a weak production externality. Section 3 derives results with a strong enough production externality for sustainable growth. We separate these cases because the analytical approaches differ between cases with or without sustainable growth. Section 4 discusses briefly what may happen with cash-in-advance constraints. The last section concludes. Proofs of results will be relegated to the Appendix.

1. THE MODEL

We consider an economy populated by identical infinitely-lived households with measure one. The representative household is endowed with one unit of time which is allocated to leisure l_t and labor $1 - l_t$. There is no uncertainty in the form of shocks in preferences and technology.[2]

1.1 Production

A single final good is produced by using capital K_t and labor $1 - l_t$ according to the following technology

$$Y_t = AK_t^\alpha (1 - l_t)^{1-\alpha} \bar{K}_t^\psi, \quad 0 < \alpha < 1, \quad 0 < \psi \leq 1 - \alpha, \tag{1}$$

where Y_t is final output, A is total factor productivity, and α measures the importance of capital relative to labor in production. Average capital $\bar{K}_t$ exhibits spillovers of degree ψ.[3] As is well known, when $\psi = 1 - \alpha$ the externality is strong enough to generate sustainable growth, whereas when $0 < \psi < 1 - \alpha$ the model becomes a neoclassical growth model without long-run growth.

Factors are compensated according to their marginal products:

$$w_t = A(1 - \alpha)K_t^{\alpha+\psi}(1 - l_t)^{-\alpha}, \tag{2}$$

$$r_t = A\alpha K_t^{\alpha+\psi-1}(1 - l_t)^{1-\alpha}, \tag{3}$$

where w_t is the real wage rate and r_t is the real interest rate. For simplicity we ignore capital depreciation until we consider numerical solutions with sustainable growth in Section 3.

1.2 Household

We assume that the preferences of the representative household are given by

$$U = \int_0^\infty e^{-\rho t} \left[\frac{\left(C_t^\delta M_t^\theta l_t^\eta \right)^{1-\epsilon}}{1-\epsilon} \right] dt, \quad \delta > 0, \quad \theta > 0, \quad \eta \geq 0, \quad \epsilon > 0 \ (\neq 1), \quad \delta + \theta + \eta = 1, \quad (4)$$

where C_t is real consumption; M_t is real money balances; l_t is leisure; ρ is the constant rate of time preference; δ, θ and η measure the importance of consumption, real money balances, and leisure, respectively; and $1/\epsilon$ is the elasticity of intertemporal substitution.

With a consumption tax at a flat rate τ_c, the household budget constraint is given as:

$$(1 + \tau_c)C_t = w_t(1 - l_t) + r_t K_t - \dot{K}_t - \dot{M}_t - \pi_t M_t, \tag{5}$$

where K_t is the capital asset and $\pi_t \equiv \dot{P}_t/P_t$ refers to the inflation rate. A dot on the top of a variable represents its rate of change with respect to time. The product $\pi_t M_t$ is the amount of the inflation tax.[4] Denoting $B_t \equiv K_t + M_t$, we can rewrite (5) as

$$\dot{B}_t = w_t(1 - l_t) + r_t K_t - (1 + \tau_c)C_t - \pi_t M_t. \tag{6}$$

The household chooses consumption C_t, leisure l_t, and real money balances M_t to maximize utility in (4) subject to (6), taking $(w_t, r_t, \tau_c, \pi_t)$ as given. The current-value Hamiltonian function is formulated as

$$\mathcal{H} = \frac{\left(C_t^\delta M_t^\theta l_t^\eta \right)^{1-\epsilon}}{1-\epsilon} + \lambda_t[w_t(1 - l_t) + r_t K_t - (1 + \tau_c)C_t - \pi_t M_t] + \nu_t(B_t - K_t - M_t),$$

where λ_t is a co-state variable and ν_t is a multiplier. The first-order conditions are

$$\frac{\partial \mathcal{H}}{\partial C_t} = \delta C_t^{\delta(1-\epsilon)-1} \left(M_t^\theta l_t^\eta \right)^{1-\epsilon} - \lambda_t(1 + \tau_c) = 0, \tag{7}$$

$$\frac{\partial \mathcal{H}}{\partial M_t} = \theta M_t^{\theta(1-\epsilon)-1} \left(C_t^\delta l_t^\eta \right)^{1-\epsilon} - \lambda_t \pi_t - \nu_t = 0, \tag{8}$$

$$\frac{\partial \mathcal{H}}{\partial K_t} = \lambda_t r_t - \nu_t = 0, \tag{9}$$

$$\frac{\partial \mathcal{H}}{\partial l_t} = \eta l_t^{\eta(1-\epsilon)-1}(C_t^\delta M_t^\theta)^{1-\epsilon} - \lambda_t w_t = 0, \tag{10}$$

$$\frac{\partial \mathcal{H}}{\partial B_t} = \nu_t = \rho \lambda_t - \dot{\lambda}_t. \tag{11}$$

The transversality condition ruling out the Ponzi game is given by

$$\lim_{t\to\infty} e^{-\rho t} \lambda_t B_t = 0. \tag{12}$$

According to these conditions, the marginal gain in utility of each choice variable should equal its marginal loss. In particular, a higher inflation cost of holding money induces households to economize on real money balances in (8), other things equal.

These first-order conditions imply the following relationships:

$$[1 - \delta(1-\epsilon)]\frac{\dot{C}_t}{C_t} - \theta(1-\epsilon)\frac{\dot{M}_t}{M_t} - \eta(1-\epsilon)\frac{\dot{l}}{l} = r_t - \rho, \tag{13}$$

$$l_t = \frac{\eta(1+\tau_c)C_t}{\delta w_t}, \tag{14}$$

$$M_t = \frac{\theta(1+\tau_c)C_t}{\delta(r_t + \pi_t)}. \tag{15}$$

Equation (13) links consumption growth to real money growth, leisure growth, and the gap between the real interest rate and the rate of time preference. In (14) and (15), the optimal levels of consumption, real money balances, and the forgone wage income (leisure times the wage) are proportional to one another given (τ_c, π, r), due to the homothetic CEIS utility function.

1.3 Government

The government uses a consumption tax and money issuing (seignorage revenue or an inflation tax) to finance a government purchase G_t that is not valued by private agents. Throughout the paper, we abstract from income taxation because it is dominated by consumption taxation in this type of model. We assume that government spending is a fixed fraction β of final output, i.e. $G_t = \beta Y_t$ with $\beta > 0$, in the spirit of Ramsey. The task of the government is to select the welfare-maximizing rates of the consumption and inflation taxes to finance the required government spending subject to its budget constraint. Suppose that the government budget is balanced at each point in time:

$$\tau_c C_t + \frac{\dot{\bar{M}}_t}{P_t} = G_t = \beta Y_t, \quad \beta > 0, \tag{16}$$

where $\bar{M}_t$ is nominal money supply and $\dot{\bar{M}}_t/P_t$ is seignorage revenue.

1.4 Equilibrium

The equilibrium is characterized by the wage and interest rate equations, the first-order conditions of the household problem, the budget constraints of the household and the government, the time constraint between leisure and labor, and the overall resource constraint $C_t + G_t + \dot{K}_t = Y_t$. For analytical convenience, we transform the key variables (C, M, K) into their ratios to output $(\Gamma_C, \Gamma_M, \Gamma_K)$ in the system of equations determining the equilibrium. Also denote the growth rates $\dot{X}/X$ for $X = C, K, M, l$ as g_X, and the growth rate of nominal money supply as σ. We then have

$$[1 - \delta(1 - \epsilon)]g_C - \theta(1 - \epsilon)g_M - \eta(1 - \epsilon)g_l = r - \rho, \tag{17}$$

$$\Gamma_M = \frac{\theta(1 + \tau_c)\Gamma_C}{\delta(r + \pi)}, \tag{18}$$

$$l = \frac{\eta(1 + \tau_c)\Gamma_C(1 - l)}{\delta(1 - \alpha)}, \tag{19}$$

$$(1 + \tau_c)\Gamma_C = 1 - g_K\Gamma_K - g_M\Gamma_M - \pi\Gamma_M, \tag{20}$$

$$\tau_c\Gamma_C + \sigma\Gamma_M = \beta, \tag{21}$$

$$\Gamma_C + g_K\Gamma_K = 1 - \beta. \tag{22}$$

From (20)-(22), we have

$$g_M = \sigma - \pi, \tag{23}$$

which says that the growth rate of real money balances g_M equals the growth rate of nominal money supply σ minus the inflation rate π.

In what follows, we shall investigate three cases. The first case has no externality, i.e. $\psi = 0$, allowing us to link our work to the literature. In the other two cases we assume $0 < \psi < 1 - \alpha$ and $\psi = 1 - \alpha$, respectively, so that we can study the implications of the production externality. We call $0 < \psi < 1 - \alpha$ a *weak* externality case and $\psi = 1 - \alpha$ a *strong* externality case. In the cases with $0 \leq \psi < 1 - \alpha$ there is no sustainable growth, whereas in the case with $\psi = 1 - \alpha$ there is sustainable growth.

2. RESULTS WITHOUT SUSTAINABLE GROWTH

We first derive equilibrium solutions and results in the cases that have no externality or a weak production externality, i.e. $0 \leq \psi < 1 - \alpha$. In order to avoid the complexity in tracking down transitional dynamics, we only focus on the steady-state equilibrium in these cases. The steady-state equilibrium solution is derived in a few steps. First, it is obvious to have $\pi = \sigma$, that is, without long-run growth in output, the rate of inflation is just equal to the rate of nominal money growth.

Setting $\dot{C} = \dot{M} = \dot{l} = 0$ and substituting (3) into (17), we obtain $r = A\alpha K^{\alpha+\psi-1}(1-l)^{1-\alpha} = \rho$. As the real interest rate r is equal to the rate of time preference ρ without sustainable growth in output, the steady-state capital-labor ratio can be determined as

$$k^* = K^*/(1-l^*) = \left[\frac{A\alpha(1-l^*)^\psi}{\rho}\right]^{1/(1-\alpha-\psi)}. \tag{24}$$

According to this, the capital-labor ratio is increasing with (independent of) the quantity of labor in equilibrium with (without) the production externality. Also, nominal money growth can only affect the capital-labor ratio indirectly through the quantity of labor in the presence of the externality.

The solution for the ratio of consumption to output is given by

$$\Gamma_C = 1 - \beta. \tag{25}$$

For a given β, increasing the rate of nominal money growth, accompanied by decreasing the consumption tax rate for a balanced government budget, will have opposing effects on the ratio of consumption to output. According to (25), the net effect is zero.

The solution for the ratio of real money balances to output is

$$\Gamma_M = \frac{\theta}{\delta(\rho+\sigma) + \sigma\theta}. \tag{26}$$

As expected, the ratio of real money balances to output is certainly decreasing with the rate of nominal money growth because a higher nominal money growth rate raises the cost of holding money (the nominal interest rate) by increasing the inflation rate.

The solution for leisure is found to be

$$l^* = \frac{\eta(\rho+\sigma)}{\eta(\rho+\sigma) + (1-\alpha)[\delta(\rho+\sigma) + \sigma\theta]}. \tag{27}$$

It is easy to verify that leisure is decreasing with the rate of nominal money growth. Consequently, labor supply is increasing with the ratio of nominal money growth.

According to (24) and (27), the steady-state level of capital $K^* = k^*(1 - l^*)$ should also be increasing with the rate of nominal money growth. The steady-state quantity of output is then equal to

$$Y^* = \left[A(1 - l^*)^{1-\alpha} \left(\frac{\alpha}{\rho} \right)^{\alpha+\psi} \right]^{\frac{1}{1-\alpha-\psi}}, \quad \psi \in [0, 1 - \alpha). \tag{28}$$

In this equation, the level of output is an increasing function of the quantity of labor, after we substitute capital stock out using $A\alpha K^{\alpha+\psi-1}(1 - l)^{1-\alpha} = \rho$. Since labor is increasing with the rate of nominal money growth, so must be the steady-state level of output regardless of whether the externality is absent ($\psi = 0$) or present ($0 < \psi < 1 - \alpha$). The case with sustainable growth ($\psi = 1 - \alpha$) will be analyzed separately later.

Using (21), (25) and (26), we have the budget-balance restriction on the policy parameters:

$$\beta = \frac{\tau_c \delta(\rho + \sigma)}{(1 + \tau_c)[\delta(\rho + \sigma) + \sigma\theta]} + \frac{\sigma\theta}{\delta(\rho + \sigma) + \sigma\theta}, \tag{29}$$

which implies that $\tau_c = \{\beta[\delta(\rho + \sigma) + \sigma\theta] - \sigma\theta\}/\{(1 - \beta)[\delta(\rho + \sigma) + \sigma\theta]\}$.

Further, the velocity of money V is positively related to the rate of nominal money growth or inflation. Specifically, by the identity $\bar{M}V = PY$, the income velocity of money can be expressed as $V = Y/(\bar{M}/P) = Y/M = 1/\Gamma_M$ in this model. Since the ratio of real money balances to output Γ_M is decreasing with the rate of nominal money growth, the velocity of money, $V = 1/\Gamma_M$, must be increasing with the rate of nominal money growth.

We give the effect of inflation taxation on resource allocation below and put the proof in the Appendix.

PROPOSITION 1: *In the steady state with $\beta > 0$ and $0 \leq \psi < 1 - \alpha$, switching from consumption taxation toward inflation taxation reduces leisure and the ratio of real money balances to income, but increases consumption, labor, capital, output, and the velocity of money. Also, the tax switch reduces real money balances for $\psi = 0$ but may increase it for a large $\psi \in (0, 1 - \alpha)$.*

The results in Proposition 1 can be explained as follows. Holding government spending as a fixed fraction of output, a reduction in the consumption tax rate goes hand-in-hand with a rise

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in the nominal money growth rate for a balanced government budget. The consequent rise in the inflation rate with $\pi = \sigma$ drives up the cost of holding money in (8), and thus reduces the demand for real money balances relative to income, as is well known in the literature. Also well known is that economizing on money holdings accelerates the circulation of money. Moreover, the decrease in real money balances reduces the marginal benefits of consumption and leisure with a nonseparable utility function concerning these augments, which tends to reduce consumption and leisure as can be seen in (7) and (10). However, this rise in the rate of nominal money growth has no direct effect on the cost of leisure (the real wage), and therefore, leisure should decline and, accordingly, labor should increase. The increased labor in turn raises the marginal product of capital, and hence stimulates capital accumulation and raises output. Since consumption is equal to output net of government spending in the steady state with $\dot{K} = 0$, the switch from consumption taxation to inflation taxation also raises steady-state consumption.

When the rate of nominal money growth rises along with a falling consumption tax, there are two opposing effects on the *level* of real money balances: a positive one through the rise in output and a negative one through the fall in the ratio of real money balances to output. In the absence of any externality, the positive effect through the rise in output is weak and is dominated by the negative one, resulting in a net decline in real money balances. With a sufficiently strong production externality, however, the rise in output caused by faster nominal money growth can be substantial and may dominate the negative effect on real money balances, leading to a possible net increase in the level of real money balances.

It is important to note that the real effect of a higher nominal money growth rate (accompanied by a lower consumption tax rate) on resource allocation in this model originates from endogenous labor supply. If labor supply were fully inelastic, then the real effect of switching from consumption taxation to inflation taxation on consumption and output would disappear.

To examine the welfare effect of inflation taxation, we now derive the solution for the welfare level in the steady state. Since in the steady state $C_t^\delta M_t^\theta l_t^\eta = Y^{\delta+\theta} \Gamma_C^\delta \Gamma_M^\theta l^\eta$, we obtain the solution

for the welfare level by substituting (25)-(28) into (4): $U^* = \Phi \left[F(\tau_c, \sigma) \right]^{1-\epsilon} / (1 - \epsilon)$, where

$$F(\tau_c, \sigma) \equiv \frac{(\rho + \sigma)^{\delta + \eta}(1 + \tau_c)^{-\delta}[\delta(\rho + \sigma) + \theta\sigma]^{\frac{\psi(\delta + \theta)}{1 - \alpha - \psi}}}{\{\eta(\rho + \sigma) + (1 - \alpha)[\delta(\rho + \sigma) + \theta\sigma]\}^{\frac{1 - \alpha - \eta\psi}{1 - \alpha - \psi}}}, \tag{30}$$

$$\Phi \equiv \frac{1}{\rho} \left\{ \delta^\delta \theta^\theta \eta^\eta \left[A(1 - \alpha)^{1 - \alpha} \left(\frac{\alpha}{\rho} \right)^{\alpha + \psi} \right]^{\frac{\delta + \theta}{1 - \alpha - \psi}} \right\}^{1 - \epsilon} > 0.$$

The welfare level U^* is a function of the rates of the consumption tax and nominal money growth via the function F, while $\Phi > 0$ is a constant and independent of the consumption tax and nominal money growth. The welfare level U^* refers to the steady state equilibrium when capital and output fully adjust to a permanent policy change in the long run. Also, note that U^* is monotonically increasing with F. We thus focus on F in the welfare analysis below, with or without the production externality.

2.1 No Externality

In the absence of any externality ($\psi = 0$), the welfare level in (30) reduces to

$$F^o(\tau_c, \sigma) \equiv \frac{(\rho + \sigma)^{\delta + \eta}(1 + \tau_c)^{-\delta}}{\eta(\rho + \sigma) + (1 - \alpha)[\delta(\rho + \sigma) + \sigma\theta]}. \tag{31}$$

The government chooses the tax rates (τ_c, σ) to maximize welfare in (31) subject to (29), holding government spending as a fixed fraction β of output. We proceed in two stages. First, we look at regimes where the government uses either consumption taxation or inflation taxation but not both. Second, we consider their mix.

In the case with consumption taxation only, we have $\tau_c = \beta/(1 - \beta)$ from (29) and

$$F^o(\tau_c) = \frac{\rho^{-\theta}(1 - \beta)^\delta}{\eta + \delta(1 - \alpha)},$$

where the assumption $\theta + \eta + \delta = 1$ has been used to simplify the expression.

Analogously, when inflation taxation is the only option, we have $\sigma = \beta\delta\rho/[\theta(1 - \beta) - \beta\delta]$ and

$$F^o(\sigma) = \frac{(\rho\theta)^{-\theta}(1 - \beta)^{\delta + \eta}[\theta(1 - \beta) - \beta\delta]^\theta}{\eta + \delta(1 - \alpha)}.$$

Intuitively, in order to use seignorage revenue to finance the required government spending, the taste parameter for real money balances θ must be large enough relative to the ratio of government

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spending to output β and to the taste parameter for consumption δ. The exact restriction on these parameters can be found as follows. First, the government budget balance with the inflation tax as the sole means is $\sigma\Gamma_M = \beta$. Using the solution for Γ_M, this budget balance restriction becomes $\sigma\theta/[\delta(\rho + \sigma) + \sigma\theta] = \beta$. In this equation, the ratio of seignorage revenue to output (the left-hand side) is increasing with the rate of nominal money growth σ and achieves its maximum at $\sigma = \infty$: $\theta/(\delta+\theta)$. This maximum ratio of seignorage revenue to output less the ratio of government spending to output equals $[\theta(1-\beta)-\beta\delta]/(\delta+\theta)$. Therefore, $\theta > \beta\delta/(1-\beta)$ is essentially a necessary condition for inflation taxation alone to finance the required government spending. We thus make this as an assumption. Under this assumption, there is an upper limit on the ratio of government spending to output β which in turn sets an upper limit on the rate of nominal money growth or on the rate of inflation by $\pi = \sigma$.

We compare the two regimes in terms of welfare below. The proof is in the Appendix.

PROPOSITION 2: *In the steady state without any externality and with $\beta > 0$ and $\theta > \beta\delta/(1-\beta)$, using pure consumption taxation to finance government spending obtains a higher welfare level than using pure inflation taxation.*

By Proposition 2, inflation taxation leads to a lower welfare level than does consumption taxation as a sole means of financing government spending in the long run. What happens when both instruments are used together? The answer is given below, with the proof in the Appendix.

PROPOSITION 3: *In the steady state without any externality and with $\beta > 0$, if both consumption taxation and inflation taxation are used to finance government spending, their optimal mix to maximize welfare has the following features: $\tau_c^* > \beta/(1 - \beta)$ and $\sigma^* = \pi^* < 0$.*

By Proposition 3, when there is no production externality, the rate of nominal money growth should be negative and the consumption tax should be positive. The intuition is as follows. Since real money balances and consumption enter the utility "symmetrically", there is a "uniform taxation principle" saying that the government should tax both at the same rate in order to avoid distorting the margin between consumption and real money balances. See Atkinson and Stiglitz (1972) for more discussions on the uniform taxation principle. This consideration implies that the rates of

consumption and inflation taxes should be equal. However, real money balances are also an asset, and ideally the government also does not want to distort the return on money relative to the return on capital so as to avoid distorting the margin between real money balances and capital. Since capital income is not taxed in our model, this consideration implies that the inflation tax should be zero. Combining the consumption-money consideration with the capital-money consideration suggests that, in the absence of any externality, the consumption tax should exceed the inflation tax. Moreover, in the spirit of the Friedman rule, because the social cost of producing money is zero, there should be a negative inflation tax such that the cost of holding money (the nominal interest rate) can be as close to zero as possible. As a result, the optimal inflation tax is negative along with a positive consumption tax. However, the underlying nominal money growth rate should exceed the rate that corresponds strictly to the Friedman rule, because of the distortions of the consumption tax and the negative inflation tax on leisure and consumption.

Further, can inflation taxation improve on the case without any government intervention (with a zero nominal-money growth rate, no government spending, and no taxes) in welfare terms? We design a template to answer this question as follows. Suppose that we start with the no-government case, i.e. $\beta = \tau_c = \sigma = 0$. Then, we allow for inflation taxation and spend seignorage revenue on subsidizing consumption. That is, there exists a function $\tau_c(\sigma)$ such that $\tau_c \Gamma_C = -\sigma \Gamma_M$, which reduces to $\tau_c = -\sigma\theta/[\delta(\rho + \sigma) + \sigma\theta]$. Inserting this relationship that balances the government budget into $F^o(\tau_c, \sigma)$ to obtain $F^o(\tau_c(\sigma), \sigma)$, we increase the rate of nominal money growth from zero, leading to a subsidy on consumption $\tau_c < 0$, and see what happens to welfare $F^o(\tau_c(\sigma), \sigma)$. The result is given below and the proof is in the Appendix.

PROPOSITION 4: *In the steady state without any externality and with $\beta = 0$, raising the inflation tax from zero to subsidize consumption reduces welfare from the level of a competitive equilibrium with no government intervention. The optimal monetary policy should generate deflation ($\pi^* < 0$).*

As stated in Propositions 2-4, in the absence of the production externality, inflation taxation always reduces welfare, whether it is used alone or along with consumption taxation, or whether there is a required public purchase or a consumption subsidy. In essence, as the rate of nominal money growth rises along with a falling consumption tax rate, the losses in welfare arising from the

decreases in leisure and real money balances dominate the gain in welfare arising from the increase in consumption.

The quantitative implications of the results are illustrated in Table 1, using numerical solutions based on the parameterization $\alpha = 0.3$, $A = 0.3$, $\rho = 0.03$, $\theta = 0.1$, $\epsilon = 3.0$, $\eta = 0.3$, and $\psi = 0$. The values of α, ρ and ϵ are in line with those widely used in the literature, while the values of other parameters are chosen such that the numerical results for the key variables of interest are plausible. To make a welfare comparison across all cases in Table 1, we first select a benchmark case and then compute the percentage change in income in the benchmark case such that we reach the same welfare level in each of the other cases (see the note in Table 1 for this equivalent payment).

The first case (the benchmark case) in Table 1 has no government intervention. In the second case, either or both of consumption taxation and inflation taxation may be used to finance government spending as 10% of output ($\beta = 0.1$). When both instruments are used, their optimal mix is a positive consumption tax and a negative inflation tax (a deflation transfer at a rate of nominal money growth -2.04%), implying higher real money balances than in the benchmark. Compared to the benchmark case regarding real allocation and welfare, this optimal mix has higher leisure but lower levels of steady-state capital and output, and achieves a welfare gain in the magnitude of 2.87% of the benchmark income. The reason for this welfare gain is a reduced nominal interest rate (the opportunity cost of holding money) under the negative inflation tax, $r + \sigma = \rho + \sigma = 3\% - 2.04\% = 0.96\%$. Note that this nominal interest rate exceeds what the Friedman rule suggests (i.e. a zero nominal interest rate), because of the distortion on leisure and consumption. When the consumption tax is used alone to finance government spending as 10% of output (at a tax rate 11.11%), there is no real effect on the allocation of time and output. This is because with the CEIS utility function the positive substitution effect of the consumption tax on leisure is fully offset by its negative income effect when government spending is not valued by private agents.[6] Since government spending is wasted, the consumption tax reduces welfare from the no-intervention benchmark case in the magnitude of 8.64% of income. When the inflation tax is used alone, there is a greater loss in welfare compared to the no-intervention case (21.86% of income) because the inflation tax (at a 6% nominal money growth rate) reduces both leisure and

real money balances.

2.2 A Weak Production Externality

With a weak production externality $\psi \in (0, 1 - \alpha)$, we first compare pure inflation taxation with pure consumption taxation as in Proposition 2. The proof is in the Appendix.

PROPOSITION 5: *In the steady state with $\beta > 0$ and $\theta > \beta\delta/(1 - \beta)$, if $\psi \in (0, 1 - \alpha)$ is large enough, then using pure inflation taxation to finance government spending obtains a higher welfare level than using pure consumption taxation.*

In addition, we investigate whether inflation taxation can improve on pure consumption taxation as in Proposition 3. The proof is in the Appendix.

PROPOSITION 6: *In the steady state with $\beta > 0$, when consumption taxation is used to finance government spending, inflation taxation ($\pi^* = \sigma^* > 0$) should also be used together if $\psi \in (0, 1 - \alpha)$ is large enough.*

Further, we explore whether inflation taxation can improve on the case without any government intervention as in Proposition 4. The proof is in the Appendix.

PROPOSITION 7: *In the steady state with $\beta = 0$, if $\psi \in (0, 1 - \alpha)$ is large enough, then increasing the inflation tax rate from zero to subsidize consumption increases welfare from the level of a competitive equilibrium without any government intervention.*

Unlike the results in Propositions 2 to 4, inflation taxation can raise welfare in Propositions 5 to 7 with a strong enough production externality, whether it is used alone or along with consumption taxation, or whether there is a required government purchase or a consumption subsidy. This result differs from that in Rebelo and Xie (1999) in two aspects. First, in their study it is unclear whether the rate of nominal money growth should be positive. Second, with the same type of production externality, the optimal monetary policy arises from its real effects on the steady-state equilibrium in our model, but it emerges from its transitional effect in their model.

Why does the welfare consequence of inflation taxation depend on whether we take the production externality into account? Intuitively, when there is a production externality, the private

rate of return on investment in capital is lower than the social rate, leading to under-investment in capital. When the level of capital is below its socially optimal level, the private rate of return on labor must also be lower than the social rate, leading to a suboptimal solution with too little labor and too much leisure. Therefore, the positive effects of the inflation tax on labor and capital accumulation help the economy correct the under-investment in capital and the under-supply of labor. This possibility is further enhanced by another interesting finding in Proposition 1: With a strong enough externality, the rise in the inflation tax may raise real money balances, rather than reduces it as in the no-externality case. Thus, the rise in the inflation tax can improve welfare when the externality is strong enough such that the welfare gain from increasing consumption and possibly real money balances dominates the welfare loss from decreasing leisure. This mechanism through which we support inflation taxation differs from those in the related literature.

For quantitative insights, we conduct a numerical analysis of the model with a weak production externality $\psi \in (0, 1 - \alpha)$ and report the results in Table 2. The degree of the externality is set at $\psi = 0.5$ first and then 0.58 (below the value of $1 - \alpha = 0.7$). In the former case, the welfare ranking in a descending order is: the mix of consumption taxation and inflation taxation, consumption taxation alone, and inflation taxation alone. In the latter case with a stronger externality, the mix remains the best, while the ranking of the other two is reversed.

Table 3 reports the numerical results with no government purchase, i.e. $\beta = 0$, using otherwise the same parameterization as in Table 2. The mixed case has a higher welfare level than the no-intervention case in Table 3. Also, the inflation tax is positive and the consumption tax is negative in their optimal mix. In other words, with a strong enough production externality, using an inflation tax to subsidize consumption obtains a higher welfare level than having no intervention at all. When $\psi = 0.58$, for example, the inflation tax (at a rate 3.169% of nominal money growth) obtains a gain in welfare by a magnitude of 6.11% of income and raises long-run output by almost 25% (from 40.79 to 49.57), both of which are substantial.

3. RESULTS WITH SUSTAINABLE GROWTH

With $\psi = 1 - \alpha$, the model becomes the well known Romer's model (1986) in which the externality is so strong that growth is sustainable even in the long run. As is also well known, this

type of model has no transitional dynamics. That is, the growth rate is constant at all points in time and shared by output, capital, consumption, and real money balances, while labor, leisure and the ratios of consumption and investment to output are all time-invariant. With these features, we can obtain the solution for the system of equations implicitly on the entire equilibrium path as follows.

Setting $g_l = 0$ in (17) and $g_C = g_M = g$, we have:

$$g = \frac{r - \rho}{1 - (1 - \eta)(1 - \epsilon)}, \tag{32}$$

which implies

$$r = \rho + [1 - (1 - \eta)(1 - \epsilon)]g \equiv R(g). \tag{33}$$

Note that the coefficient on g is positive since $\eta \in (0, 1)$ and $\epsilon > 0$. Thus, the function $R(g)$ is increasing with the growth rate g.

Substituting (33) into (3) gives

$$l = 1 - \left[\frac{R(g)}{\alpha A}\right]^{1/(1-\alpha)}. \tag{34}$$

Since $R'(g) > 0$, there is an equilibrium growth-leisure trade-off as in Turnovsky (2000).

In addition, setting $g_M = g$ in (23) leads to

$$g = \sigma - \pi. \tag{35}$$

Combining (18)-(22), (32), (35) and $\Gamma_K = K/Y = 1/[A(1 - l)^{1-\alpha}] = \alpha/R(g)$ together yields,

$$\left[\frac{\alpha A}{R(g)}\right]^{\frac{1}{1-\alpha}} - 1 = \frac{\eta[\rho + \sigma - (1 - \eta)(1 - \epsilon)g][1 - \alpha g/R(g)]}{(1 - \alpha)\{\theta\sigma + \delta[\rho + \sigma - (1 - \eta)(1 - \epsilon)g]\}}, \tag{36}$$

which determines the growth rate g implicitly. Once the growth rate is obtained, we can determine the inflation rate, $\pi = \sigma - g$, as well as the following ratios:

$$\Gamma_C = 1 - \beta - \frac{\alpha g}{R(g)}, \tag{37}$$

$$\Gamma_M = \frac{\theta}{\theta\sigma + \delta[\rho + \sigma - (1 - \eta)(1 - \epsilon)g]}\left[1 - \frac{\alpha g}{R(g)}\right]. \tag{38}$$

The ratio of capital investment to output is $g\Gamma_K = \alpha g/R(g)$.

According to (33), the factor $g/R(g)$ can be rewritten as $1/[1 - (1 - \eta)(1 - \epsilon) + \rho/g]$, which is obviously increasing with the growth rate g. With this observation, the growth rate of output is positively associated with the ratio of capital investment to output $g\Gamma_K = \alpha g/R(g)$, and negatively associated with the ratio of consumption to output Γ_C in (37). These relationships reflect a typical trade-off between current and future consumption. According to (38), the relationship between the growth rate g and the ratio of real money balances to output Γ_M is ambiguous in general. Also, the solution for leisure l is given by (34), while the policy parameters (τ_c, σ) satisfy (21).

We now demonstrate how a switch from consumption taxation to inflation taxation under a balanced government budget affects resource allocation and the growth rate of output below. The proof is in the Appendix.

PROPOSITION 8: *Given $\beta > 0$ and $\psi = 1 - \alpha$, if ρ is sufficiently small, then switching from consumption taxation with σ near zero to inflation taxation to finance government spending reduces leisure and the ratio of consumption to output and raises the growth rate of output.*

As mentioned earlier, the growth rate is decreasing with both leisure and the fraction of output spent on consumption. Therefore, when the switch from consumption taxation to inflation taxation reduces both leisure and the fraction of output for consumption as in the previous case, it accelerates economic growth in the present case with sustainable growth.

Further, given the solution for $(\Gamma_C, \Gamma_M, l, g)$ and the initial capital stock K_0, the solution for the welfare level may be rewritten as

$$U = \frac{\left[(AK_0)^{1-\eta} \Gamma_C^\delta \Gamma_M^\theta l^\eta (1 - l)^{(1-\alpha)(1-\eta)} \right]^{1-\epsilon}}{(1 - \epsilon)[\rho - (1 - \eta)(1 - \epsilon)g]}. \tag{39}$$

The expression of U in (39) measures the lifetime utility of the representative household on the entire equilibrium path from the moment that a permanent policy change is implemented. That is, a change in U captures both the short-run and long-run effects of a policy change on welfare. Unfortunately, because there is no reduced-form solution for $(g, l, \Gamma_C, \Gamma_M)$, it is difficult to conduct a welfare analysis of government policy in the same way as in the previous section. We will instead use a numerical approach to investigate the welfare effect of inflation taxation below.

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The numerical analysis here differs from the ones in Section 2 in at least two aspects. First, since the present case with sustainable growth ($\psi = 1 - \alpha$) does not display transitional dynamics, the entire equilibrium path (from the moment that the policy change is implemented) can be studied numerically, by solving the nonlinear equations that characterize the equilibrium. By contrast, in the previous case without sustainable growth ($\psi \in (0, 1 - \alpha)$), we only analyzed the long-run steady-state equilibrium to avoid the complexity in tracking down the transitional dynamics. When considering the entire equilibrium path, there is a difference in time at which the various effects of a policy change reach their full strength. For example, leisure and the ratio of consumption to output fall immediately to their new solutions after a permanent rise in the inflation tax (a fall in the consumption tax) in this model with sustainable growth. In contrast to the immediate decline in leisure and in the ratio of consumption to output, the levels of capital and output will only rise *gradually* over time after this policy switch, which will eventually raise future consumption. In this adjustment process after the switch to inflation taxation, there is a trade-off between a current welfare loss from the decline in leisure and current consumption and a future welfare gain from the rise in future consumption. Therefore, when we consider the entire equilibrium path, we expect that the net welfare effect of switching from consumption taxation to inflation taxation will depend on the elasticity of intertemporal substitution measured by $1/\epsilon$. This is because more elastic intertemporal substitution (lower ϵ) typically leads to faster consumption growth $\dot{C}/C$ in (32), other things equal, as households are more willing to trade a current welfare loss for a future welfare gain.

Second, in order to generate realistic values for the output growth rate and the real interest rate, we need to introduce capital depreciation at a rate ξ. With capital depreciation, the real interest rate equation becomes $r = \alpha A(1-l)^{1-\alpha} - \xi$, and the resource constraint becomes $C + G + \dot{K} + \xi K = Y$. The equilibrium analysis with capital depreciation is similar to the one with no depreciation.

The numerical solutions with sustainable growth and capital depreciation are reported in Table 4. In this table, we increase the rate of nominal money growth exogenously and let the consumption tax rate fall to balance the government budget. In so doing, the consumption tax rate and the amount of leisure decline quickly, while the inflation rate and the growth rate of output all increase.

The net welfare effect of this acceleration in nominal money growth is indeed dependent on the elasticity of intertemporal substitution $1/\epsilon$. When $\epsilon = 0.40$ (high elasticity), the welfare-maximizing nominal money growth rate is equal to 2.2%, leading to a positive inflation rate. When $\epsilon = 3.0$ (low elasticity), the welfare-maximizing nominal money growth rate is equal to -3.2%, leading to deflation. Because the nominal interest rate in the latter case is still positive, $r + \pi = 7.01\% - 4.87\% > 0$, the welfare-maximizing nominal money growth rate is still greater than the one that follows the Friedman rule.

4. DISCUSSION OF EXTENSION

Finally, our present model does not include cash-in-advance constraints. If money is introduced into our model through cash-in-advance constraints instead of through money in the utility function, we have obtained the following results.[5] First, when there is no sustainable growth with $\psi \in [0, 1 - \alpha)$, the steady-state equilibrium level of labor or leisure is independent of both the level of government spending and the division between consumption taxation and inflation taxation. Second, when there in no sustainable growth with $\psi \in [0, 1 - \alpha)$, a switch from consumption taxation to inflation taxation reduces the steady-state capital stock and welfare if both consumption and investment are subject to the cash-in-advance constraint, but has no effect on them if the cash-in-advance constraint is only imposed on consumption. This result is consistent with the finding in Stockman (1981) that assumes fixed labor supply and treats seignorage revenue as lump-sum transfers. Third, when there is sustained growth ($\psi = 1 - \alpha$), switching from consumption taxation to inflation taxation will have no effect on resource allocation and welfare if the cash-in-advance constraint is only imposed on consumption as in the case without sustainable growth. However, when both consumption and capital investment are subject to the cash-in-advance constraint, we find numerically that the switch to inflation taxation reduces labor, the output growth rate and welfare.

Overall, whether the results will change when we take cash-in-advance constraints into account depends crucially on whether investment is subject to this cash-in-advance constraint. If this cash-in-advance constraint only restricts consumption spending, then there is no additional effect of a switch from consumption taxation to inflation taxation. However, if the cash-in-advance

constraint also applies to capital investment spending, the additional effects of the tax switch on real allocation and welfare are of the opposite signs compared to what we have found with money in the utility function. The key reason for this result is that when the cash-in-advance constraint applies to the purchase of capital, the complementarity between money and capital investment built in this constraint implies that the effects of an inflation tax on real money balances and capital accumulation are both negative. By contrast, with money in the utility function, there exists a trade-off between real money balances and capital. Thus, one should interpret our results with caution.

5. CONCLUSION

This paper considers a neoclassical growth model with leisure and money in the utility function and with government spending as a fixed share in output and financed by seignorage revenue and/or consumption taxation. We study the effects of switching from consumption taxation to inflation taxation on resource allocation and welfare. Concerning resource allocation, we find that the switch to inflation taxation decreases leisure and the ratios of consumption and real money balances to income, but increases the levels of consumption, capital and output in the long run. With a strong production externality, the positive output effect of the switch to inflation taxation may lead to a positive net effect on the level of real money balances. In the case with sustainable growth originating from a strong enough production externality, the switch from consumption taxation to inflation taxation is likely to promote economic growth.

The welfare effect of switching from consumption taxation to inflation taxation is conditional on the strength of the production externality and on the elasticity of intertemporal substitution. In the absence of the externality, the switch to inflation taxation is always welfare reducing. With a strong enough production externality and with inelastic intertemporal substitution, the welfare-maximizing policy is a negative inflation tax and a positive consumption tax, but the underlying nominal money growth rate exceeds the rate that follows the Friedman rule due to tax distortions on labor and consumption. With a strong enough production externality and with elastic intertemporal substitution, however, switching from consumption taxation to inflation taxation may raise welfare by correcting the under-investment of capital and the under-supply of labor.

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APPENDIX PROPOSITION PROOF OF PROPOSITION 1: First, from $M = \Gamma_M Y$, there are opposing

effects of a higher rate of nominal money growth on the level of real money balances (a decline in

Γ_M and a rise in Y). Using this equation and (26)-(28) to substitute out (Γ_M, Y) and differentiating

it with respect to σ, we obtain

$$\text{sign}\frac{\mathrm{d}M}{\mathrm{d}\sigma} = \text{sign}\left\{\frac{\psi(\delta+\theta)}{\delta(\rho+\sigma)+\sigma\theta} - \frac{(1-\alpha)[\eta+(1-\alpha)(\delta+\theta)]}{\eta(\rho+\sigma)+(1-\alpha)[\delta(\rho+\sigma)+\theta\sigma]}\right\},$$

which is negative for $\psi = 0$. But it is positive if ψ is close enough to $1 - \alpha$ from below:

$$\lim_{\psi\to1-\alpha}\left\{\frac{\psi(\delta+\theta)}{\delta(\rho+\sigma)+\sigma\theta} - \frac{(1-\alpha)[\eta+(1-\alpha)(\delta+\theta)]}{\eta(\rho+\sigma)+(1-\alpha)[\delta(\rho+\sigma)+\theta\sigma]}\right\}$$

$$= \frac{(1-\alpha)\rho\theta\eta}{[\delta(\rho+\sigma)+\theta\sigma]\{\eta(\rho+\sigma)+(1-\alpha)[\delta(\rho+\sigma)+\theta\sigma]\}} > 0,$$

that is, $\mathrm{d}M/\mathrm{d}\sigma > 0$. Second, from $C = \Gamma_C Y$, knowing that $\mathrm{d}Y/\mathrm{d}\sigma > 0$ and $\mathrm{d}\Gamma_C/\mathrm{d}\sigma = 0$ in our

earlier discussion, we have $\mathrm{d}C/\mathrm{d}\sigma > 0$. All the other results in this proposition follow our earlier

discussion.

PROOF OF PROPOSITION 2: Using the F^o's defined in Section 2.1, we have

$$\frac{F^o(\tau_c)}{F^o(\sigma)} = \frac{\theta^\theta}{(1-\beta)^\eta[\theta(1-\beta)-\beta\delta]^\theta} \equiv \mathcal{B}(\beta).$$

Obviously, we have: (i) $\mathcal{B} = 1$ at $\beta = 0$, and (ii) $\mathcal{B}' > 0$. These two results establish $\mathcal{B} > 1$ for

$\beta > 0$. Therefore, the claim holds.

PROOF OF PROPOSITION 3: Using the welfare function $F^o(\tau_c, \sigma)$ in (31), we define the La-

grangian of the government maximizing welfare by choosing (τ_c, σ) subject to its budget constraint

(29) as:

$$\mathcal{L} = F^o + \mu\left\{\frac{\tau_c\delta(\rho+\sigma)}{(1+\tau_c)[\delta(\rho+\sigma)+\sigma\theta]} + \frac{\sigma\theta}{\delta(\rho+\sigma)+\sigma\theta} - \beta\right\},$$

where μ is the multiplier. The first-order condition with respect to τ_c is:

$$\frac{\partial\mathcal{L}}{\partial\tau_c} = -\frac{\delta F^o}{1+\tau_c} + \frac{\delta\mu(\rho+\sigma)}{(1+\tau_c)^2[\delta(\rho+\sigma)+\theta\sigma]} = 0. \tag{A1}$$

In order to see whether inflation taxation can contribute additionally to the value of the problem, i.e. whether sign $\partial\mathcal{L}/\partial\sigma > 0$, it is convenient to start with pure consumption taxation where $\tau_c^* = \beta/(1-\beta)$ and $\sigma^* = 0$. If $\partial\mathcal{L}/\partial\sigma < 0$ at $\tau_c^* = \beta/(1-\beta)$ and $\sigma^* = 0$, then there should be deflation $\sigma^* = \pi^* < 0$ and accordingly $\tau_c^* > \beta/(1-\beta)$. At this starting point $\tau_c^* = \beta/(1-\beta)$ and $\sigma^* = 0$, we have $F^o/\mu = (1-\beta)/\delta$ and $F^o > 0$ by using the definition of F^o and (A1).

Differentiating the Lagrangian with respect to σ, we get

$$\frac{\partial\mathcal{L}}{\partial\sigma} = -\frac{F^o[\eta + (\delta+\theta)(1-\alpha)]}{\eta(\rho+\sigma) + (1-\alpha)[\delta(\rho+\sigma) + \theta\sigma]} + \frac{F^o(\delta+\eta)}{\rho+\sigma} + \frac{\mu\delta\rho\theta}{[\delta(\rho+\sigma) + \theta\sigma]^2(1+\tau_c)}.$$

Using $F^o/\mu = (1-\beta)/\delta$ and $F^o > 0$ in this equation and rearranging terms, we have:

$$\text{sign } \frac{\partial\mathcal{L}}{\partial\sigma} = \text{sign } \{-\theta(1-\alpha)\} \ < 0 \text{ at } \tau_c^* = \beta/(1-\beta) \text{ and } \sigma^* = 0.$$

The claim follows.

PROOF OF PROPOSITION 4: As discussed earlier, we start with $\sigma = \tau_c = \beta = 0$ and then increase the inflation tax rate (the nominal money growth rate) subject to $\tau_c = -\sigma\theta/[\delta(\rho+\sigma)+\theta\sigma]$. Under this constraint of the government budget balance without government spending ($\beta = 0$), the solution for the welfare level in (31) becomes

$$F^o(\tau_c(\sigma),\sigma) = \frac{\delta^{-\delta}(\rho+\sigma)^\eta[\delta(\rho+\sigma) + \theta\sigma]^\delta}{\eta(\rho+\sigma) + (1-\alpha)[\delta(\rho+\sigma) + \theta\sigma]}.$$

The sign of $dF^o/d\sigma$ at $\sigma = \tau_c = 0$ is given by: $[\eta + (1-\alpha)\delta](\eta + \delta + \theta) - [\eta + (1-\alpha)(\delta+\theta)] = -\theta(1-\alpha) < 0$. Thus, the optimal rate of the inflation tax should be negative.

PROOF OF PROPOSITION 5: Recall that using pure consumption taxation to finance government spending means $\tau_c = \beta/(1-\beta)$ and $\sigma = 0$. Substituting these into the definition of $F(\tau_c,\sigma)$ in (30), we have

$$F(\tau_c) = \frac{\rho^{-\theta}(1-\beta)^\delta\delta^{\frac{\psi(\delta+\theta)}{1-\alpha-\psi}}}{[\eta + \delta(1-\alpha)]^{\frac{1-\alpha-\eta\psi}{1-\alpha-\psi}}}.$$

Similarly, with pure inflation taxation, $\sigma = \beta\delta\rho/[\theta(1-\beta) - \beta\delta]$ and $\tau_c = 0$, we have

$$F(\sigma) = \frac{(\rho\theta)^{-\theta}(1-\beta)^{\delta+\eta}[\theta(1-\beta) - \beta\delta]^\theta\delta^{\frac{\psi(\delta+\theta)}{1-\alpha-\psi}}}{[\eta(1-\beta) + \delta(1-\alpha)]^{\frac{1-\alpha-\eta\psi}{1-\alpha-\psi}}}.$$

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The ratio of the two welfare levels is defined as

$$\frac{F(\tau_c)}{F(\sigma)} = (1-\beta)^{-\eta} \left[\frac{\theta}{\theta(1-\beta) - \beta\delta}\right]^{\theta} \left[\frac{\eta(1-\beta) + \delta(1-\alpha)}{\eta + \delta(1-\alpha)}\right]^{\frac{1-\alpha-\eta\psi}{1-\alpha-\psi}} \equiv H(\beta).$$

Here, it is obvious that $H(0) = 1$. The remaining proof of $H < 1$ is to show $H' < 0$ for $\beta > 0$ and for a large enough ψ. The sign of H' is determined by $\eta/(1-\beta) + \theta(\theta+\eta)/[\theta(1-\beta) - \beta\delta] - \eta(1-\alpha-\eta\psi)/\{(1-\alpha-\psi)[\eta(1-\beta) + \delta(1-\alpha)]\}$. Clearly, as $\psi \to 1-\alpha$ from below (sufficient but not necessary), sign $H' \to -\infty < 0$ under $\beta > 0$ and $\theta > \beta\delta/(1-\beta)$.

PROOF OF PROPOSITION 6: Paralleling the proof of Proposition 3, we use the welfare function (30). We also define the Lagrangian of the government maximizing welfare by choice of (τ_c, σ) subject to its budget constraint (29) as:

$$\mathcal{L} = F + \mu\left\{\frac{\tau_c\delta(\rho+\sigma)}{(1+\tau_c)[\delta(\rho+\sigma) + \sigma\theta]} + \frac{\sigma\theta}{\delta(\rho+\sigma) + \sigma\theta} - \beta\right\},$$

where μ is the multiplier. The first-order condition with respect to τ_c is:

$$\frac{\partial\mathcal{L}}{\partial\tau_c} = -\frac{\delta F}{1+\tau_c} + \frac{\mu\delta(\rho+\sigma)}{(1+\tau_c)^2[\delta(\rho+\sigma) + \theta\sigma]} = 0. \tag{A2}$$

Starting with pure consumption taxation where $\tau_c^* = \beta/(1-\beta)$ and $\sigma^* = 0$, we have $F/\mu = (1-\beta)/\delta$ and $F > 0$ by using the definition of F and (A2). At this point, we ask whether accelerating nominal-money growth can contribute additionally to the value of the problem, i.e. whether sign $\partial\mathcal{L}/\partial\sigma > 0$ as in the proof of Proposition 3. If $\partial\mathcal{L}/\partial\sigma > 0$ at $\tau_c^* = \beta/(1-\beta)$ and $\sigma^* = 0$, then $\sigma^* = \pi^* > 0$.

Differentiating the Lagrangian with respect to σ, we have

$$\frac{\partial\mathcal{L}}{\partial\sigma} = -\frac{F(1-\alpha-\eta\psi)[\eta + (\delta+\theta)(1-\alpha)]}{(1-\alpha-\psi)\{\eta(\rho+\sigma) + (1-\alpha)[\delta(\rho+\sigma) + \theta\sigma]\}} + \frac{F(\delta+\eta)}{\rho+\sigma}$$
$$+ \frac{F\psi(\delta+\theta)^2}{(1-\alpha-\psi)[\delta(\rho+\sigma) + \theta\sigma]} + \frac{\mu\delta\rho\theta}{[\delta(\rho+\sigma) + \theta\sigma]^2(1+\tau_c)}.$$

Using $F/\mu = (1-\beta)/\delta$ and $F > 0$ in this equation and rearranging terms, we have:

$$\text{sign }\frac{\partial\mathcal{L}}{\partial\sigma} = \text{sign } \{(1-\eta)\psi\,\theta\,[\eta + \delta(1-\alpha)] - \delta\theta(1-\alpha)(1-\alpha-\eta\psi)\} \text{ at } \tau_c^* = \beta/(1-\beta) \text{ and } \sigma^* = 0.$$

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Note that $\partial\mathcal{L}/\partial\sigma$ is increasing in ψ. If ψ approaches $1 - \alpha$, then

$$\text{sign } \frac{\partial\mathcal{L}}{\partial\sigma} \to \text{sign } \theta\eta(1-\alpha)(1-\eta) > 0.$$

Clearly, $\psi \to 1 - \alpha$ is sufficient (but not necessary) to generate $\partial\mathcal{L}/\partial\sigma > 0$.

PROOF OF PROPOSITION 7: Paralleling the proof of Proposition 4, we start with $\sigma = \tau_c = \beta = 0$ and then increase the inflation tax rate subject to the government budget balance $\tau_c = -\sigma\theta/[\delta(\rho + \sigma) + \theta\sigma]$. Consequently, the solution for the welfare level $F(\tau_c, \sigma)$ in (30) becomes

$$F(\tau_c(\sigma), \sigma) = \frac{(\rho + \sigma)^\eta \delta^{-\delta}[\delta(\rho + \sigma) + \theta\sigma]^{\frac{\psi\theta + \delta(1-\alpha)}{1-\alpha-\psi}}}{\{\eta(\rho + \sigma) + (1 - \alpha)[\delta(\rho + \sigma) + \theta\sigma]\}^{\frac{1-\alpha-\eta\psi}{1-\alpha-\psi}}}.$$

The sign of $dF/d\sigma$ at $\sigma = \tau_c = 0$ is given by: $[\eta + (1 - \alpha)\delta]\psi\theta(1 - \eta) - (1 - \alpha - \eta\psi)\delta\theta(1 - \alpha)$ $\to \theta\eta(1-\alpha)(1-\eta) > 0$ as $\psi \to 1 - \alpha$ (sufficient but not necessary).

PROOF OF PROPOSITION 8: We begin with consumption taxation and raise the nominal money growth rate from a starting value near zero. Differentiating (36) yields $\partial g/\partial\sigma = \Omega_1/\Omega_2$, where

$$\Omega_1 \equiv \frac{\theta[\rho - (1 - \eta)(1 - \epsilon)g]}{[\rho + \sigma - (1 - \eta)(1 - \epsilon)g]\{\theta\sigma + \delta[\rho + \sigma - (1 - \eta)(1 - \epsilon)g]\}},$$

$$\Omega_2 \equiv \frac{1 - (1 - \eta)(1 - \epsilon)}{(1 - \alpha)R(g)\, l} - \frac{\alpha\rho}{R(g)[R(g) - \alpha g]}$$

$$+ \frac{\theta\sigma(1 - \eta)(\epsilon - 1)}{[\rho + \sigma - (1 - \eta)(1 - \epsilon)g]\{\theta\sigma + \delta[\rho + \sigma - (1 - \eta)(1 - \epsilon)g]\}}.$$

Since the transversality condition implies that $\rho - (1 - \eta)(1 - \epsilon)g > 0$, we have $\Omega_1 > 0$. As a result, the sign of $\partial g/\partial\sigma$ is the same as that of Ω_2. The first term of Ω_2 is positive since $\eta \in (0, 1)$, $\epsilon > 0$ and $R(g) = r > 0$; the second term is negative since $R(g) - \alpha g = R(g)(\Gamma_C + \beta) > 0$ according to (37); and the last term is close to zero when σ starts with a value near zero (either positive or negative). Thus, the sum of these three terms is positive if ρ is sufficiently small (sufficient but not necessary). From (33), (34), (37), and (38), a positive growth effect of a higher nominal money growth rate (accompanied by a lower consumption tax rate) implies negative effects of the same policy change on both leisure and the ratio of consumption to output.

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1. This positive long-run relationship between inflation and output is in line with some previous predictions (e.g., Tobin, 1965; van der Ploeg and Alogoskoufis, 1994; Espinosa-Vega and Yip, 1999). Empirical evidence in this regard is mixed. In low inflation countries, a permanent rise in the inflation rate permanently raises the level of output in the postwar era as found by Bullard and Keating (1995). However, there is no such a permanent positive effect of inflation on output in high inflation countries in both Bullard and Keating (1995) and Bruno and Easterly (1998).

2. In other words, our model abstracts from how optimal monetary policy should respond to such shocks, which has been examined in some studies; see e.g. Williamson (1996) and Rebelo and Xie (1999, Section 4).

3. The rationale for the externality is that firms and workers enhance their knowledge through learning-by-doing, which also benefits other firms through spillovers to some extent as knowledge is partly a public good in nature (e.g. Arrow, 1962). Moreover, Arrow's idea of linking learning-by-doing to investment builds on the evidence of strong positive effects of experience on productivity, and on evidence that patents — a proxy for learning — closely follow investment in some industries; see Barro and Sala-i-Martin (1995, p. 147). More recent evidence also supports the idea of the spillovers both within and across industries (e.g. Bernstein and Nadiri, 1989; Nakanishi, 2002).

4. With endogenous labor supply, government debt is welfare reducing as shown in Burbidge (1983), and is hence omitted for simplicity.

5. The results are derived from the same CEIS utility function nesting a Cobb-Douglas relationship between leisure and consumption (excluding money by setting $\theta = 0$), the same production function, and a cash-in-advance constraint on consumption expenditure, or on investment expenditure as well. Also, government spending is assumed as a fixed fraction of output.

6. From (27), we get $l^* = \eta/(\eta + \delta(1 - \alpha))$ when $\sigma = 0$. However, if tax revenue is made as lump-sum transfers to households, rather than wasted, the consumption tax raises leisure and reduces output.

TABLE 1
Numerical Results: No Externality

Parameters: $\alpha = 0.3$, $A = 0.3$, $\rho = 0.03$, $\theta = 0.1$, $\epsilon = 3.0$, $\eta = 0.3$, $\psi = 0.0$

Cases	Consumption tax rate %	Money growth %	Leisure	Capital	Output	Welfare	Equivalent payment %
	τ_c	σ	l	K	Y	U	
No Government Spending: $\beta = 0.0$							
No intervention	0.000	0.000	0.417	2.802	0.280	-118.715	Benchmark
With Government Spending: $\beta = 0.1$							
Mix	71.66	-2.04	0.524	2.284	0.228	-114.100	+2.87
Cons tax only	11.11	0.000	0.417	2.802	0.280	-134.714	-8.64
Money only	0.000	6.000	0.391	2.924	0.292	-167.678	-21.86

Note: Equivalent payment refers to Hicksian-equivalent payment, measured by the percentage change in income we should add to (+) or subtract from (−) the benchmark income to reach the same welfare level as in any other concerned case.

TABLE 2
Numerical Results: Weak Externality with Government Spending

Parameters: $\alpha = 0.3$, $\beta = 0.1$, $A = 0.3$, $\rho = 0.03$, $\theta = 0.1$, $\epsilon = 3.0$, $\eta = 0.3$

Cases	Consumption tax rate %	Money growth %	Leisure	Capital	Output	Welfare	Equivalent payment %
	τ_c	σ	l	K	Y	U	
$\psi = 0.50$							
1. Mix	8.310	0.550	0.411	38.229	3.823	-3.64244	Benchmark
2. Cons tax only	11.11	0.000	0.417	36.840	3.684	-3.65677	-0.28
3. Money only	0.000	6.000	0.391	42.757	4.276	-3.92165	-5.14
$\psi = 0.58$							
1. Mix	2.350	3.170	0.397	495.7	49.57	-0.11619	Benchmark
2. Cons tax only	11.11	0.000	0.417	407.8	40.79	-0.12624	-5.76
3. Money only	0.000	6.000	0.391	522.8	52.28	-0.11781	-0.99

Note: Equivalent payment is defined in the note of Table 1.

TABLE 3
Numerical Results: Weak Externality without Government Purchase

Same parameterization as in Table 2 except $\beta = 0.0$

Cases	Consumption tax rate %	Money growth %	Leisure	Capital	Output	Welfare	Equivalent payment %
	τ_c	σ	l	K	Y	U	
			$\psi = 0.50$				
No intervention	0.00	0.000	0.417	36.84	3.685	-3.22247	Benchmark
Mix	-2.52	0.550	0.410	38.23	3.823	-3.20984	+0.28
			$\psi = 0.58$				
No intervention	0.00	0.000	0.417	407.9	40.79	-0.11125	Benchmark
Mix	-7.89	3.169	0.397	495.7	49.57	-0.10239	+6.11

Note: Equivalent payment is defined in the note of Table 1.

TABLE 4
Numerical Results with Sustainable Growth $(\psi = 1 - \alpha)$

Parameters: $\alpha = 0.3$, $\beta = 0.1$, $\rho = 0.03$, $\theta = 0.1$, $\xi = .06$, $\eta = 0.3$, $\psi = 1 - \alpha$

Money growth rate %	Consumption tax rate %	Inflation rate %	Leisure	Output growth %	Interest rate %	Welfare	Equivalent payment %
σ	τ_c	π	l	g	r	U	
		Strong intertemporal substitution $\epsilon = 0.40$ $(A = 0.45)$					
0.000	14.92	-1.61	0.355	1.608	3.93	35.8822	-4.93
1.000	9.36	-0.84	0.342	1.841	4.07	36.5018	-0.97
2.200*	6.15	0.22	0.335	1.978	4.15	36.6526	Benchmark
4.000	3.80	1.92	0.329	2.079	4.21	36.5279	-0.81
6.000	2.47	3.86	0.326	2.137	4.24	36.2732	-2.44
		Weak intertemporal substitution $\epsilon = 3.00$ $(A = 0.65)$					
-3.40	71.28	-4.84	0.470	1.437	6.51	- 47.9902	-0.74
-3.20*	51.69	-4.87	0.439	1.669	7.01	- 47.5013	Benchmark
0.000	13.75	-2.12	0.371	2.121	8.09	- 54.2081	-9.01
2.000	9.22	-0.18	0.362	2.181	8.24	- 57.2055	-12.44
5.000	5.81	2.77	0.355	2.228	8.35	- 60.7703	-16.14

Notes: a. "*" indicates the welfare-maximizing nominal money growth rate.
b. Equivalent payment is defined in the note of Table 1.

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