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Is There a Real Estate Bubble in the Czech Republic?*

Petr ZEMČÍK – CERGE-EI (a joint workplace of the Center for Economic Research and Graduate Education, Charles University, and the Economics Institute of the Academy of Sciences of the Czech Republic) Prague
(petr.zemcik@cerge-ei.cz)

Abstract
Real estate prices more than doubled in many countries of Central and Eastern Europe from 2003 to 2008. In this paper, I provide one of the first assessments of whether housing prices in this region correspond to rents, i.e. to cash-flows related to an apartment purchase. State-of-the-art panel data stationarity and Granger causality techniques are employed to test the implications of the standard present value model using regional data from the Czech Republic. Apartment prices are only slightly overvalued. In addition, changes in prices are helpful in predicting changes in rents and vice versa.

1. Introduction

Increasing and then rapidly decreasing property prices around the world are often cited as the initial trigger of the on-going global recession. This raises the question of whether current real estate prices correspond to economic fundamentals and if not, how much (more) they are likely to fall. This question is highly relevant for the housing markets in Central and Eastern Europe (CEE), which have experienced a recent boom of the credit markets (see Egert and Mihajlek, 2007). Property prices more than doubled between 2003 and 2008 in majority of countries in this region. The present study is one of the first ones to employ panel data stationarity techniques to assess the possibility of a real estate bubble in CEE using regional data. The Czech Republic serves as a useful benchmark because its housing market is not very sensitive to the exchange rate fluctuations. The reason for this is a zero volume of mortgages denominated in foreign currency as compared with 80% in Estonia and 40% in Hungary.[1] In addition, the Czech Republic is fairly representative of a country in CEE with population of 10.4 mil., GDP growth of 3% in 2008,[2] and having entered the European Union in 2004.

From a theoretical perspective, a real estate bubble is usually defined as a discrepancy between property prices and fundamental property values. The fundamental value is based on an underlying economic model, which also determines explanatory variables that can be used in empirical tests. There are two main classes of theoretical models employed in the real estate literature and they both imply stationarity between house prices and selected factors. The first model is essentially a simple de-

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[1] See Table 2 in Egert and Mihajlek (2007), the data are from 2006.

[2] Czech Statistical Office is the source of this information.

mand and supply model. Demand determinants include income, housing cost, the user costs of owning a house, and other variables depending on data availability. Supply determinants are depreciation, construction costs, and other factors. For example, Gallin (2006) formulates a stock-flow housing model to characterize a relationship between house prices and income. A similar model is laid out in Mikhed and Zemčík (2009a) who employ a number of supply and demand shifters in addition to income. The second type of model for house prices is a present-value model, which relates asset prices to a stream of earnings connected with a particular asset. Campbell and Shiller (1987) demonstrate that asset prices and the corresponding cash-flows should be of the same order of integration in the presentvalue framework. In other words, under the null hypothesis of the present-value model to be the true model, if the order of integration is one for both variables,[3] they should be cointegrated. Wang (2000) and Mikhed and Zemčík (2009b) employ this methodology with a rent as the appropriate measure of cash-flow variable associated with a purchase of real estate. Finally, Mikhed and Zemčík (2009a) show that the present-value model can be re-written as a supply-demand model if a consumer is indifferent between renting and owning a house.[4]

Malpezzi (1999), Gallin (2006), and Mikhed and Zemčík (2009a,b) all use US regional data and panel data stationarity techniques, which have greater power than their univariate counterparts. One of the first panel data unit root test is described in Im, Pesaran, and Shin (2003). It is based on simply averaging single-series *t*-statistics from standard unit root regressions. Pesaran (2007) suggests an updated version of this test robust to cross-sectional correlation, which can be tested for by a test from Pesaran (2004). In this paper, I assess stationarity of the relationship between apartment prices and rents in the Czech Republic using the above-mentioned panel data unit root tests. Using the rent as the only fundamental factor has a number of obvious shortcomings. Malpezzi (1999) lists some reasons why the real estate markets are not as efficient as financial markets: low liquidity, high information and transactions costs, heterogeneity, and the view of a house purchase as an investment. On the other hand, there exist reasons why using rents is a sensible approach in general and in particular for the Czech Republic. Mikhed and Zemčík (2009a,b) study the US housing market using similar methodology and data for US Metropolitan Statistical Areas. The former paper uses a structural demand-supply model and the latter paper the present-value model. Empirically however, the only difference between the two papers is the use of several fundamental factors in the first paper and just the rent in the other paper. In spite of this fact, both papers demonstrate that there was a bubble in the US real estate market prior to 2006. This suggests that using either theoretical type of framework typically leads to a similar conclusion. Moreover, while it is possible to collect data for supply and demand factors for 14 Czech regions (‘kraje’ in Czech), it would be very difficult to gather such information for 57 Czech regions used in this paper. Hence there is a trade-off between using a larger cross-sectional dimension but only one fundamental factor vs. using a smaller cross-sectional dimension with several demand and supply factor. This paper opts for using the rents and a greater cross-

[3] Their levels are non-stationary but the first differences are stationary.

[4] This condition is satisfied if there are no frictions on the market for owners and on the rental market, respectively.

-sectional size of a panel dataset. Hlaváček and Komárek (2011) on the other hand do conduct the supply-demand analysis of regions.

The panel dataset consists of annual data for apartment prices and rents for major cities and towns of the Czech Republic from 2001 to 2008. I run cross-sectionally robust individual unit root tests for all the regions. If an apartment price has a unit root and the rent or the price-to-rent ratio are non-stationary, there is a bubble in the given region or district. Using this definition, 77% of the Czech regions exhibit bubble-like behavior. However, the null hypothesis of price series being non-stationary in some locations may be accepted only because of the small power of the univariate unit root tests. To rule out this possibility and to summarize the development of the housing markets overall, I conduct the joint panel data stationarity tests. These tests combine information from all of the individual unit root regressions. The results suggest that the overvaluation of apartments is fairly small. Both housing prices and rent series are not stationary. The null hypothesis of a unit root in the price-to-rent ratio is barely rejected at standard levels of significance. Therefore, technically there is a bubble but the degree of overpricing is fairly small as compared to the United States housing market in the early 2000s. Hence, a collapse of the real estate prices similar to the one in United States, Britain, Spain, Ireland, and other countries is not likely.

Finally, the question of mutual predictability of house prices and rents is addressed. The present-value model loosely implies that house prices should be useful in predicting rents and vice versa. One can test for Granger causality in both directions. Testing for causality is more complex due to the low time series dimension. I use the Generalized Method of Moments (GMM) Arellano-Bond (1991) estimator to show that lagged differences in rents predict differences in prices while controlling for differences in lagged prices and vice versa. This result is quantitatively and qualitatively similar to the results obtained for the US data in Mikhed and Zemčík (2009b). In other words, the Czech real estate market behaves in a manner resembling the developed markets in Western Europe, the Anglo-saxon world, and in Asia.

The rest of the paper is organized as follows. Section 2 discusses the implications of the present-value model for prices and rents, and Section 3 provides an overview of the relevant panel data econometrics. Section 4 describes in detail the used data and their sources, and Section 5 presents the results. Section 6 concludes the paper.

2. Present Value Model for the Real Estate Market

Here I follow closely Campbell and Shiller (1987). They express a present value model for variables $P_{i,t}$ and $R_{i,t}$ as:

$$P_{i,t}=\theta(1\text{-}\beta)\sum_{j=0}^{\infty}\beta^{j}E_{t}R_{i,t+j}+b_{i,t}+c, \quad i=1,\ldots,N \tag{1}$$

where $P_{i,t}$ is a local house price index and E_t is a mathematical expectation conditional on information at time t. $R_{i,t}$ is a rent i.e. a cash-flow from owning an apartment between the beginning of period t and the beginning of period t+1, and β is the discount factor. The discount factor can be written as $1/(1+D)$, where D is the constant discount rate.[5] θ is the coefficient of proportionality and c is the constant. $b_{i,t}$ is a random variable representing a rational bubble and satisfying $b_{i,t} = \beta E_t b_{i,t+1}$.[6] The summation in (1) represents the fundamental factors. Note that changing the timing convention for the rent between the beginning of period t and the beginning of period $t+1$ to $R_{i,t+j}$, setting $c = 0$, $\theta = 1/D$, and $b_{i,t} = 0$ yields the perhaps more familiar formula $P_{i,t} = \sum_{j=1}^{\infty} \beta^j E_t \frac{R_{i,t+j}}{(1+D)^j}$. The formula (1) holds for any asset in general, including stocks, bonds, etc. Here it captures the notion of a real estate purchase as an investment vehicle in a given location. Purchasing an apartment entitles the owner to a stream of future cash flows, namely the rents. This holds whether the owner lives in the apartment or not since by living there he/she saves on the rent he/she would otherwise pay elsewhere.

The present-value formula is not only very intuitive, it also is a solution to an optimization problem. In this problem, a consumer maximizes her expected life-time welfare subject to a budget constraint. Assuming a consumer is risk neutral and the discount rate is constant yields equation (1). Real estate prices and rents consistent with the present value formula are therefore consistent with consumers being rational. If $b_{i,t} \neq 0$, there are solutions to the stochastic difference equation (1), which contain housing prices growing much faster than rents. This indicates the presence of a bubble. If the random variable b_t satisfies the condition that $b_{i,t} = \beta E_t b_{i,t+1}$, then the bubble is rational since (1) is implied by first order conditions of the consumer optimization problem. In other words, if consumers expect real estate prices to be increasing for some time, it is in fact rational to pay a price higher than the one corresponding to the future streams of rents.[7] These represent the fundamentals in this case.

In the absence of the bubble $\left(b_{i,t} = 0\right)$, equation (1) relates real estate prices and rents. The equation is typically tested using some transformation of the variables present, which results in stationary series. However, there is a possibility that the series are not stationary and yet there is no bubble. The question of whether the series are stationary is empirical in its nature. Therefore, the next step is to test for unit roots in series for apartment prices and rents, with the following potential outcomes:

Case 1: $P_{i,t}$ stationary and $R_{i,t}$ stationary;

Case 2: $P_{i,t}$ stationary and $R_{i,t}$ non-stationary;

Case 3: $P_{i,t}$ non-stationary and $R_{i,t}$ stationary; and

Case 4: $P_{i,t}$ non-stationary and $R_{i,t}$ non-stationary.

[5] The assumption of constant discount rate is fairly strong, but in the case of the Czech Republic it is not unrealistic. The Czech National Bank reports monthly data on interest rates for fixed rate mortgages for the period from 5 to 10 years from January 2004. The minimum reported rate is 4.61% and the maximum rate is 5.63% with the majority of the rates being close to 5%.

[6] See for example Hamilton (1986) for a more detailed discussion of speculative bubbles.

[7] Note that Ponzi schemes are not rational in this sense since their participants are aware that they will end some time in the future – e.g. see the discussion in Cochrane (2001, Ch. 20.1.). Also, Campbell, Lo, and MacKinlay (1997, Ch. 7.1.2) analyze rational bubbles explicitly as arising from a standard linear present-value relation.

I focus only on Cases 3 and 4, which indicate run-away prices. Case 3 clearly violates the implications of the no-bubble condition. Case 4 is the most interesting one since nonstationary real estate prices do not necessarily indicate the presence of a bubble. To see if the bubble is in fact present, one needs to test for the cointegration between real-estate prices and cash-flows or test for the stationarity of *P/R*.

To illustrate why testing for cointegration between rents and prices is equivalent to both testing the present-value equilibrium model and to unit root testing of *P/R*, let us define a spread variable as the difference between an apartment price and a multiple of rents, i.e. $S_{i,t} \equiv P_{i,t} - \theta R_{i,t}$. Equation (1) implies that

$$\begin{aligned} S_{i,t} &= \theta\left(R_{i,t} + \beta E_t R_{i,t+1} + \ldots\right) - \beta\theta\left(R_{i,t} + \beta E_t R_{i,t+1} + \ldots\right) - \theta R_{i,t} + b_{i,t} + c = \\ &= \theta \sum\nolimits_{j=1}^{\infty} \beta^j E_t \Delta R_{i,t+j} + b_{i,t} + c \end{aligned} \tag{2}$$

Note that for any price given by (1),

$$\left(\frac{\beta}{1-\beta}\right) E_t \Delta P_{i,t+1} = \theta \sum\nolimits_{j=1}^{\infty} \beta^j E_t \Delta R_{i,t+j}$$

Since $b_{i,t} = \beta E_t b_{i,t+1}$. Therefore,

$$S_{i,t} = \left(\frac{\beta}{1-\beta}\right) E_t \Delta P_{i,t+1} + b_{i,t} + c \tag{3}$$

Note the two formulae relate the differences in rents to differences in prices. Also, the expected capitalization of a real estate purchase can be expressed in terms of the rent-to-price ratio. Setting $c = 0$, $\theta = 1/D$, and $b_{i,t} = 0$ and re-arranging equation (3) results in:

$$E_t \frac{\Delta P_{i,t+1}}{P_{i,t}} = D - \frac{R_{i,t}}{P_{i,t}} \tag{4}$$

Equation (4) can be viewed as a no arbitrage condition where the expected return on investment in an apartment equals the corresponding discount rate. The price--to-rent ratio serves as a predictor of capitalization. Equation (4) hence provides a theoretical prediction for the statistical relationship (but not for a direction of causality) between changes in rents and the rent-to-price ratio. This relationship has been tested for the US data for instance by Clark (1995) and Capozza and Seguin (1996).

$b_{it} = 0$ is a necessary condition for S_{it} to be stationary. For the stationary spread, it follows from (2) that $\Delta R_{i,t}$ is stationary and from (3) that $\Delta P_{i,t}$ is stationary, respectively. Hence if both prices and rents have unit root in levels and the spread is stationary, the present value model implies that first differences of both variables must be stationary as well. This satisfies the standard definition of cointegration, which requires that cointegrated series should be integrated of order 1, i.e. I(1) (see for example Hamilton, 1994, Ch. 19.1). Therefore, testing for stationarity of the spread is equivalent: (i) to testing for cointegration between our two variables because the spread is in fact their linear combination, and (ii) to testing of the present-value model with no bubbles. Finally, if $S_t = 0$ then $\frac{P_{i,t}}{R_{i,t}} = \theta$ i.e. a constant. Clearly, the stationary spread means that the price-to-rent ratio is also stationary.

3. Econometric Methodology

Real estate prices and rents tend to be correlated across regions. A general diagnostic test for cross section dependence in panels from Pesaran (2004) is used to find if this is in fact the case for the Czech Republic and in its capital. Testing for unit roots is then conducted using tests from Im, Pesaran, and Shin (2003) and from Pesaran (2007). Only the latter is robust to cross-sectional dependence. The former test is included because the robust test cannot be used in some cases due to the short time series span. The tests are executed both for individual series and jointly. They are applied to prices, rents, and price-to-rent ratios.[8] The tests are employed to find if there is a real estate bubble in the Czech Republic. In addition, the present value model (1) implies that prices should predict rents and vice versa. The time period for the annual Czech data is too short to conduct panel data causality tests, and therefore the GMM Arellano and Bond (1991) estimator provides substitution.

Pesaran (2004) diagnostic test for cross section dependence in panels uses residuals from the standard augmented Dickey Fuller (ADF) regression (e.g. Hamilton 1994, Ch. 17):

$$\Delta y_{it} = \mu_i + \omega_i t + \alpha_i y_{i,t-1} + \sum_{j=1}^{p_i} \lambda_{ij} \Delta y_{i,t-j} + \varepsilon_{it} \tag{5}$$

with ε_{it} being an error term. μ_i is an individual fixed effect, ω_i is an individual trend coefficient and $\rho_i = \alpha_i + 1$ is an autoregressive coefficient of a given series. $i = 1,...,N$ and $t = 1,...,T$. α_i and the lag order p_i may differ across regions. I will denote ADF(1) and ADF(0) the cases, where $p_i = 1$ and $p_i = 0$ for all $i = 1,\dots, N$, respectively. Peseran (2004) shows that the following statistic is asymptotically normally distributed:

$$CD = \sqrt{\frac{2T}{N(N-1)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} Corr\left(\hat{v}_i, \hat{v}_j\right) \right) \to N(0,1) \tag{6}$$

The $(T \times 1)$ vector of estimated residuals $\hat{v}$, $i = 1,\dots, N$ is estimated by Ordinary Least Squares (OLS) using the equation (5). A simple Gauss procedure is written to calculate the test statistic.

Pesaran (2007) proposes a panel data unit root test robust to correlation among regions. It is based on the cross sectionally augmented Dickey-Fuller regression (CADF):

$$\Delta y_{it} = \mu_i + \omega_i t + \alpha_i y_{i,t-1} + \sum_{j=1}^{p_i} \lambda_{ij} \Delta y_{i,t-j} + \upsilon_i \overline{y}_{t-1} + \sum_{j=0}^{p_i} \varpi_{ij} \Delta \overline{y}_{i,t-j} + \nu_{it} \tag{7}$$

[8] The stationarity tests for the ratios could be complemented by panel tests for cointegration between prices and cash-flows according to Pedroni (1999, 2004). However, the cointegration test often cannot be used because of the different order of integration in prices and rents. In addition, Mikhed and Zemčík (2009b) show that the results of testing for the cointegration relationship between prices and rents in the US are similar to the unit root test results.

Where v_{it} denotes an i.i.d. error term and $\overline{y}_t$ is the cross-section mean. Let $\tilde{t}_{i,T_i,N}\left(p_i\right)$ be the t-statistic for $\alpha_i = 0$ (a unit root) in the CADF regression. If $p_i = p$ for all *is* and the panel of data is balanced, $\tilde{t}_{i,T_i,N}\left(p_i\right) = \tilde{t}_i\left(T,N,p\right)$ CADF(0) and CADF(1) are defined similarly to ADF(0) and ADF(1), respectively. The test proposed in Pesaran (2007) averages t-tests from regressions (5):

$$\overline{t}* = \frac{1}{N}\sum_{i=1}^{N}\tilde{t}_i\left(T,N,p\right) \tag{8}$$

The null hypothesis is defined as

$$H_0 : \alpha_i = 0 \text{ for } i = 1,2,...,N \tag{9}$$

And the alternative as

$$H_1 : \begin{cases} \alpha_i = 0 \text{ for } i = 1,2,...,N_1 \\ \alpha_i < 0 \text{ for } i = N_1+1, N_1+2,...,N \end{cases} \tag{10}$$

Rejecting the null hypothesis means that at least one of the series is stationary.

A previous version of this test is described in Im, Pesaran, and Shin (2003). This version does not include the cross-sectional terms in the ADF regression. I will refer to it as the IPS test or the IPS statistic. The cross sectionally augmented version will be denoted as the CIPS test and statistic, respectively. The CIPS statistic is given in (8). The empirical analysis is conducted employing Gauss. Specifically, the IPS test is conducted using Nonstationary Panel Time Series Module 1.3 for Gauss (NPT 1.3) from Chihwa Kao. While I have written a code for the CADF and CIPS tests, I have also used a Gauss procedure from Pesaran (2007).[9] The number of regions for the available data is $N = 335$ for the Czech Republic as a whole and $N = 57$for a selected sub--sample. $T = 8$ (2001–2008). Critical values for these dimensions are not available in Pesaran (2007) and are generated by Monte Carlo simulations.

Finally, equations (2) and (3) suggest that changes in prices and changes in rents are related. This can be tested using the concept of Granger causality in panel data, which was pioneered in Hurlin (2004) and in Hurlin and Venet (2004). Let us consider two stationary variables y_i and x_i. In our case, these would refer to first differences of rents and prices. The following regression model captures a potential relationship between the two variables:

$$y_{it} = \mu_i + \sum_{l=1}^{L}\phi_i^{(l)} y_{i,t-l} + \sum_{l=1}^{L}\delta_i^{(l)} x_{i,j,t-l} + \xi_{it} \tag{11}$$

ξ_{it} follow a normal distribution with a zero mean and a finite variance, which differs across groups. They are i.i.d.. Their mean is zero and the variance is finite. The vectors $\xi_i = \left(\xi_{i1},...,\xi_{iT}\right)'$ are independent for $i \neq j$. L is the number of lags. The relevant null hypothesis is:

$$H_0 : \delta_i = 0, \forall i = 1,...,N \tag{12}$$

[9] Prof. Pesaran was kind enough to email me his code.

Where $\delta_i = \left(\delta_i^{(1)},...,\delta_i^{(L)}\right)'$. H_0 captures the notion of Homogeneous Non Causality (HNC), where x is not useful for predicting y after controlling for lags of y. The alternative hypothesis is defined as

$$H_1 : \begin{cases} \delta_i = 0\,\forall i = 1,...,N_1 \\ \delta_i \neq 0, \forall i = N_1 + 1,...,N \end{cases} \tag{13}$$

Where $N_1 \in [0, N)$ is unknown. Let us define W_{it} as the Wald statistic from the individual test of H_0 for $i = 1,...,N$ and $W_{NT}^{HNC} = \left(1/N\right)\sum_{i=1}^{N} W_{iT}$. Hurlin (2004) demonstrates that

$$Z_{NT}^{HNC} = \sqrt{\frac{N}{2\times L} \times \frac{(T-2L-5)}{(T-L-3)}} \times \left[\frac{(T-2L-3)}{(T-2L-1)} W_{NT}^{HNC} - L\right] \to N(0,1) \tag{14}$$

for a fixed $T > 5 + 2L$ and $N \to \infty$.

The restriction $T > 5 + 2L$ implies that the Hurlin test cannot be used for the annual panel data with the number of effective observations $T = 6$ for differences and $T = 7$ for levels when $L = 1$. Therefore, a different approach needs to be adopted to test for Granger causality. One can estimate the equation (11) using dynamic panel data techniques if $\phi_i = \phi$ and $\delta_i = \delta$ for all i. The equation then can be estimated using first differences and the Generalized Method of Moments (GMM) Arellano and Bond (1991) *n*-step estimator with White correction for heteroskedasticity. Instruments are given by lags 2 and higher of first differences available at time *t*. The *t*-test with of $\delta = 0$ then can be loosely interpreted as a test of Granger causality in panel data.

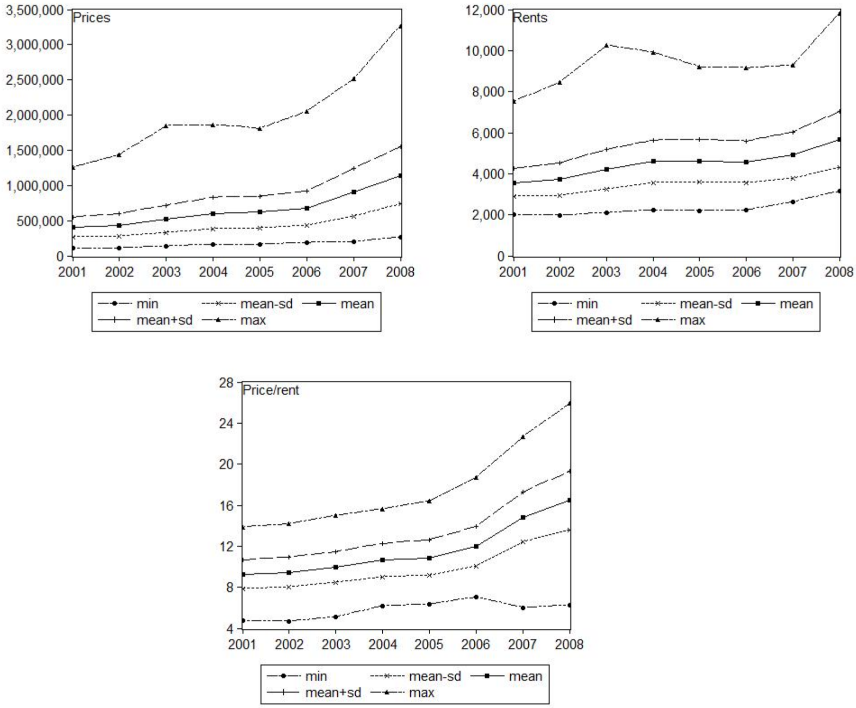
4. Data

I use data from the Institute of Regional Information at Brno (IRI). The IRI's data set contains advertised housing prices and rents for 335 locations from 2001 up to 2008. The regions are listed in the *Data Appendix*. The housing price and rent expresses price or rent for 1 m^2 for the standardized 68 m^2 existing (i.e. not newly constructed) apartment.

The housing prices for existing apartments are calculated as follows. First, if there are more than 30 data points in a given location, adjustment is made for size and for the type of all apartments. For example, the price of 1 m^2 in an 88 m^2 apartment was 0.9453 of the price of 1 m^2 in an 68 m^2 apartment in 2006. There are two main types: apartments in the socialist style panel buildings and apartments in typically nicer brick buildings.

IRI has also developed a simple algorithm to address a missing data problem, which occurs if there are less then 30 observations in a given location in some period. It divides locations into three categories, reflecting their size and relative importance. 'Fathers' are regional capitals ('krajskamesta' in Czech) and selected district capitals ('okresnimesta' in Czech). There are always 30 observations for cities in this group. Each father has a network of geographically close towns called 'Sons', some of which have 'Grandsons' i.e. close villages and small towns.

Figure 1 Annual Prices, Rents, and Price/Rent Ratios for a Standard 68 m² Apartment in the Czech Republic, Kč, 335 Regions, 2001–2008



Let us define P_t^F as the price of the standardized 68 m² apartment for one of the Fathers and P_t^S the price of the standardized 68 m² apartment for one of the Sons of this particular Father. Suppose we have an updated price P_{t+1}^F for the Father and $n < 30$ observations for the Son. Then

$$P_{t+1}^S = \frac{1}{30}\left[\sum_{i=1}^{n} P_{i,t+1}^S + \left(30 - n\right) c_t P_{t+1}^F\right]$$

where $P_{i,t+1}^S$ are observed prices for the Son. Also, $c_{t+1} = P_{t+1}^S / P_{t+1}^F$.

The prices are characterized in *Figure 1*. The average prices were gradually increasing until the Czech Republic joined the European Union in 2004. Then they stagnated in 2005 when a new, more rapid increase started. It continued up to the end of the sample in 2008. The mean prices almost tripled over the considered time interval, increasing from 407,725 Kč in 2001 to 1,142,836 Kč in 2008. The standard deviation was relatively small and stable over the whole period. While the minimum prices followed the average ones, their increase was somewhat less dramatic. The maximum prices on the other hand leaded the price increases, being about one year ahead of the mean prices. Clearly, the highest apartment prices are in Prague.

The monthly advertised rents are collected annually for the same sample of 335 locations. The rents are again shown in *Figure 1*. The pattern is similar to that of the housing prices but with two important differences. First, the mean rents in 2006 were still lower than mean rents in 2004. Second, the maximum Prague rents were declining much faster and longer than prices, from 2003 to 2006, and still stagnating in 2007. Further analysis is needed by constructing price to rent ratios to see if the relationship between prices and rents in the Czech republic was stable in the sample 2001–2008.

Addressing the missing data problem is somewhat more problematic than in the case of the housing prices. The adjustment relies on a relationship between housing prices and rents. Whenever the number of observations was lower than 30 for a given period, the rent was calculated as follows:

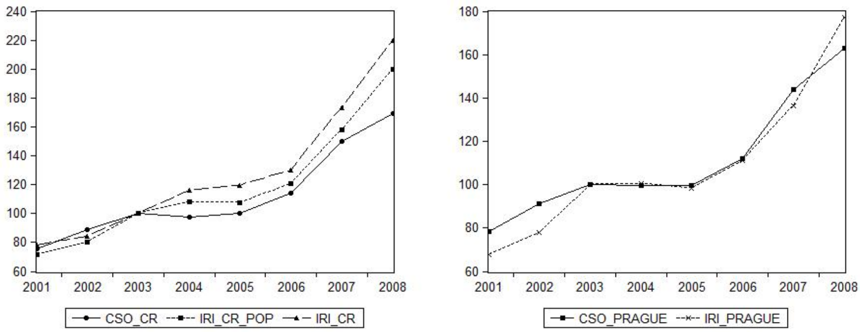
$$R_t = P_t \frac{k_t}{12} + f \tag{15}$$

where R_t is a monthly rent and P_t is the housing price, respectively. f is the monthly fixed cost of 1,550 Kč. k_t is the annual expected return on investment in housing estimated by IRI to be 7% from 2003 to 2005 and 6% from 2006 to 2007. IRI does not provide information which rate it uses for years 2001, 2002, and 2008. I use integer interest rates with the largest number of observations, for which the difference between rent per squared meter calculated using equation (15) and the reported rents is less than 1 Kč. This is plus or minus almost 2% from the average monthly rate of 52.77 per one squared meter i.e. plus or minus 68 Kč of 3,589 for the standard 68 m^2 apartment. Interest rates calculated in this way are k_{2001} = 6%, k_{2002} = 6%, and k_{2008} = 4%, respectively. There were 256 of such cases in 2001 and 202 in 2002. The numbers were at most 32 since then. However, using such rents is clearly not appropriate when one is testing the relationship between prices and rents. I therefore exclude all reported rents, for which the difference between rent calculated using equation (15) and the reported rents is less than 1 Kč in any year. The resulting sample has 57 cities left, mainly major regional and national centers. The cities in the sample are in the *Data Appendix*. Only this reduced sample will be used in the subsequent econometric analysis.

Finally, price-rent ratios are calculated using annualized rents for all 335 regions and displayed in *Figure 1*. The highest ratios in 2008 were in Prague, Olomouc, and Hradec Kralove, all large cities by Czech standards. The population size is above 1 million inhabitants in Prague and slightly above 100,000 in Olomouc and Hradec Kralove. The lowest ratios in 2008 were recorded for small, fairly unknown towns such as Duchcov, Dubi, and Krupka. With the exception of the minimum ratios, the summary statistics were rising at the end of the sample, indicating a potential real estate price bubble. It is subject to formal econometric tests to see if this increase was statistically significant using panel data stationarity tests. Due to the lack of data for rents for small regions, the focus will be mainly on larger cities, effectively eliminating the line for minimum in *Figure 1*.

The IRI price index is a supply-price index and can differ from a transactions based index. The transactions index is available in the Czech Republic only for 14 large regions ('kraje') and in an aggregate form. It is computed quarterly. Here I use the fourth observation in each year. The index is calculated by the Czech Statistical

Figure 2 Comparison of Apartment Price Indexes from the Czech Statistical Office (CSO) and the Institute of Regional Information (IRI)



Notes: (1) POP stands for a population-weighted index for each of the 335 regions. CR is the Czech Republic. (2) Index=100 in 2003.

Office (CSO), which receives source data from the Ministry of Finance. The Ministry of Finances collects this information because any real estate purchase is subject to a 3% sales tax. Unfortunately, this detailed dataset is not available to researchers due to privacy violation concerns. *Figure 2* compares the two types of indexes both for the Czech Republic and for Prague. Both CSO and IRI indexes differ somewhat for the Czech Republic and are very similar for Prague, respectively. The difference between the country-level data is in part due to the fact that the overall IRI index is a simple mean of prices. If each region is weighted by its population, Prague's weight is 19%. The Prague Index is increases at a slower rate and hence pulls the overall index down. A large portion of the difference between the two indexes is still left unexplained. One can only speculate that the remaining difference may be due to expectations of the public for prices outside of Prague to have a tendency to get closer to Prague's prices over time. However, the overall pattern is visible in both types of price indexes: the apartment prices were rising prior to 2003/4, then the growth slowed down at about the time when the Czech Republic entered the European Union. After a brief stagnation period, the prices again started increasing, peaking in 2008. It therefore seems reasonable to use the supply-price based index for an in-depth statistical analysis.

5. Empirical Results

In this section, I discuss results of the econometric tests over-viewed in Section 3 using data described in Section 4 in the light of theory from Section 2. First, there is the question of whether the prices and rents are correlated across regions of the Czech Republic. Common sense suggests that there should be regional dependence because the Czech Republic is rather small (comparable to smaller states in the United States). Therefore all considered locations are geographically close to each other, and more importantly, they are a part of a small economy with a common fiscal and monetary policy and a legal system. *Table 1* confirms this prediction. The Pesaran (2004) CD statistic strongly rejects the null hypothesis of no correlation both at levels and first differences. These test results call for the use of cross sectionally robust stationarity tests of the type suggested in Pesaran (2007).

Table 1 Diagnostic Tests for Cross Section Dependence in Panels

price-level CD	price-diff. CD	rent-level CD	rent-diff. CD
109.76***	79.95***	96.61***	48.52***

Notes: (1) ADF regression employed in the CD calculation: intercept, trend, and the first lag of the dependent variable.
(2) Under the null of no cross section dependence: $CD \rightarrow N(0,1)$.
(3) *** – significant at the 1% level, ** – significant at the 5% level,* – significant at the 10% level.
(4) The frequency is annual, and the sample period is 2001–2008.

Table 2 Cross Sectionally Augmented Dickey-Fuller Tests 1/2

	Prices	Rents	P/R	Rank
Bilina	-6.42**	1.70	-2.91	36
Breclav	-0.25	-2.67	-2.89	35
Brno	-4.07	0.23	-2.06	21
Bruntal	0.88	-1.00	-2.60	32
Cesky Tesin	-0.84	-1.24	0.44	4
Cheb	-1.47	-2.02	-1.82	17
Chomutov	-3.49	0.06	-2.94	38
Decin	-5.73*	-3.53	-3.88	45
Dubi	-1.58	1.70	-3.43	41
Havirov	0.16	-0.51	-0.76	5
Hodonin	-1.02	-3.88	-1.16	12
Hradec Kralove	-3.86	-3.16	-1.02	10
Jablonec nad Nisou	-6.42**	-0.80	-3.65	42
Jihlava	1.70	0.78	-2.27	28
Jirkov	-6.42*	0.41	-2.11	23
Kadan	-0.31	1.70	-5.66*	54
Karvina	0.54	0.47	-0.78	6
Kladno	-2.72	-0.39	-2.92	37
Klasterec nad Ohri	-1.80	1.70	-2.16	25
Kolin	-3.00	-6.42**	-4.59	50
Koprivnice	-0.57	-6.42**	-4.11	46
Krupka	-2.30	-1.31	-3.79	44
Liberec	-2.15	0.20	-2.65	33
Litomerice	-2.41	-6.22*	-2.42	30
Litvinov	-0.95	1.70	-0.88	9
Louny	-0.50	-1.82	-0.83	8
Mlada Boleslav	-1.33	-2.74	-6.42**	57
Most	-1.43	1.70	-2.09	22
Nechanice	-6.42**	1.70	-6.42**	56

Notes: (1) The cross sectionally augmented Dickey-Fuller tests with an intercept, a trend, the difference of the cross section mean and the first lag of the cross-section mean.
(2) No lags of the differenced dependent variable are included. The test is denoted CADF(0).
(3) The critical values are generated using the procedure from Pesaran (2007). ** – significant at the 6.19% level (i.e. 6.19% of 10,000 generated *t*-statistics has the truncated value of -6.42), * – significant at the 10% level.
(4) Annual data, the sample period is 2001–2008, and there are 57 regions.

In the next step, I analyze prices, rents, and price-to-rent ratios using the cross-sectionally augmented Dickey-Fuller regression CADF(0). There are no lagged terms in the regression equation due to a reduced number of time series observations.

Table 3 Cross sectionally Augmented Dickey-Fuller Tests 2/2

	Prices	Rents	*P/R*	Rank
Novy Jicin	-2.45	-2.69	-0.82	7
Olomouc	-4.27	-1.09	-4.58	49
Opava	-2.23	-2.76	-3.31	39
Orlova	1.70	-1.33	-4.49	48
Pardubice	-0.41	-0.87	-2.66	34
Pisek	-2.43	-5.75*	-1.42	14
Plzen	-1.09	-0.25	-4.92	53
Prague	-1.94	-1.25	1.54	2
Prerov	-2.86	-3.45	-3.71	43
Pribram	-5.43*	-2.00	-4.31	47
Prostejov	-6.42**	0.11	-2.17	26
Sokolov	-2.03	0.52	-2.16	24
Spindleruv Mlyn	-0.93	-1.72	-2.00	18
Sumperk	-2.14	-1.80	-2.22	27
Tabor	-3.36	1.48	-6.42**	55
Teplice	-6.15*	-0.70	-1.82	16
Trebic	-4.13	-1.74	-2.06	20
Trutnov	-6.42**	-2.84	-2.38	29
Uherske Hradiste	-1.81	-5.14*	0.79	3
Unhost	-3.14	-0.62	-4.63	51
Usti nad Labem	-6.00*	-0.62	-3.33	40
Valasske Mezirici	-3.58	-1.41	-2.06	19
Varnsdorf	-1.96	-0.99	-4.78	52
Vejprty	-1.44	0.44	-2.60	31
Vsetin	-3.09	-1.74	-1.70	15
Zatec	-0.56	0.65	1.70	1
Zlin	-2.97	-1.65	-1.21	13
Znojmo	-1.19	-0.27	-1.08	11

Notes: (1) The cross sectionally augmented Dickey-Fuller tests with an intercept, a trend, and the difference of the cross section mean, and the first lag of the cross-section mean.
(2) No lags of the differenced dependent variable are included. The test is denoted CADF(0).
(3) The critical values are generated using the procedure from Pesaran (2007). ** – significant at the 6.19% level (i.e. 6.19% of 10,000 generated *t*-statistics has the truncated value of -6.42), * – significant at the 10% level.
(4) Annual data, the sample period is 2001–2008, and there are 57 regions.

The lags can only be included at the cost of dropping the cross-sectional terms. Results presented in *Tables 2* and *3* are based only on one of the specifications considered, albeit arguably the one closest to the actual data generating process. Cross--sectional dependence in the data requires the cross-sectional terms and a casual look at the graphs in *Figure 1* identifies the presence of a trend and intercept in all the series. Robustness to autocorrelation using the lagged terms does not seem to matter in all specifications used but one. This issue will be discussed later when the panel data test results will be analyzed. Note that there are only two levels of significance, one star denotes the standard 10% level and two stars denote a 6.19% level of significance. This is because 6.19% of the estimated *t*-statistics are equal to -6.42, the point at which the test values are truncated. Prices are stationary in 10, rents in 5, and price-to-rent ratios in 0 out of 57 regions. This hints the possibility of overvalued apartments in the majority of the tested areas. The highest *P/R* is reported for Zatec, Prague, and Uherske Hradiste. The lowest *P/R* can be seen in Mlada Boleslav, Nechanice,

Table 4 Panel Data Unit Root Tests

Test	Specification	Prices	Rents	*P/R*
CIPS	no int., no trend, CADF(0)	-1.44*	-0.92	-1.07
CIPS	no int., no trend, CADF(1)	-1.56**	-0.83	-1.12
CIPS	int., no trend, CADF(0)	-2.61**	-1.42	-1.00
CIPS	int., no trend, CADF(1)	-3.03***	-2.03	-1.56
CIPS	int., trend, CADF(0)	-2.51	-1.22	-2.59
IPS	int., no trend, ADF(1)	16.28	7.12	14.41
IPS	int., trend, ADF(1)	1.84	-14.16***	7.56

Notes: (1) The IPS test is based on the Augmented Dickey-Fuller regressions with the first lag of the dependent variable (ADF(1)). The specification may include an intercept and a trend. The specification for the CIPS test uses the cross sectionally augmented version of these regressions. CADF (0) does not include any lags of the differenced dependent variable while CADF(1) includes the first lag. CADF(0) also includes the difference of the crosssection mean and the first lag of the cross--section mean. The CADF(1) specification adds the first lag of the difference of the cross section mean. For both tests, the null hypothesis is that of a unit root and the alternative hypothesis is that at least one of the series is stationary.

(2) IPS has an asymptotic standardized normal distribution. Critical values for the CIPS statistic are from Pesaran (2007), Table II. For $T = 6$ (i.e. 8 data points), the critical values are generated using the procedure from Pesaran (2007).

(3) *** – significant at the 1 % level, ** – significant at the 5 % level,* – significant at the 10 % level.

(4) The frequency is annual, the sample period is 2001–2008, and there are 57 regions.

and Tabor. A formal definition of a bubble corresponds to our four combinations of stationarity of prices and rents in Section 2. One first looks if the price is stationary. A stationary price indicates no bubble in a given region. If the price is not stationary we check the test for rents. Stationary rents and non-stationary prices imply a rational bubble. Finally, in the case where both rents and prices have unit roots, we turn to the price-to-rent ratio. If it is stationary, there is no bubble. Using this algorithm, 44 local bubbles can be identified in the Czech Republic, out of 57 districts. Bubbles are present mainly in bigger cities, such as Prague, Olomouc, or Hradec Kralove. The question is if this will translate into a discrepancy between prices and rents at the national level using the *t*-statistic averaged across regions.

Finally, the joint panel data tests are conducted with the results reported in *Table 4*. Using the annual data is somewhat complicated due to the fact that there are only 8 years of data, which does not leave one with a sufficient number of degrees of freedom in the CADF(p) and ADF(p) type of regressions. To address this issue, all major types of plausible specifications are considered – see *Table 4*. The price-to-rent ratio is again non-stationary in all cases. Common regularities appearing in the results for rents and prices are as follows. First, adding lagged terms does not make a difference for any specification where the comparison is possible. For prices, inclusion of a trend seems to matter. Including the trend in the regression equation can be justified visually since the trend in *Figure 1* is apparent. From a theoretical perspective, Mikhed and Zemčík (2009a) illustrate that the present value model can be under certain conditions shown to be equivalent to a structural supply and demand model where the real estate price is a function of various supply and demand shifters such as construction costs, personal income, and mortgage rates. In this case, the use of a trend variable is justified since many of the mentioned variables contain trends as well. Therefore, prices are more likely to be non-stationary. With respect to rents,

Table 5 Tests for Causality in Panel Data

Coefficient	Estimate	S.e.	*t*-stat.	Prob.
		Price as the dep. var.		
φ	1.8528	0.0313	59.11	0
δ	-19.9272	0.8458	-23.56	0
		Rent as the dep. var.		
φ	0.1305	0.0304	4.30	0
δ	0.0059	0.0006	10.43	0

Notes: (1) Estimated equation: $\Delta y_{it} = \varphi \Delta y_{i,\, t-l} + \delta \Delta x_{i,\, j,\, t-l} + \Delta \xi_{it}$.
(2) The data for CR, 2001–2008, 57 regions.
(3) GMM Arellano-Bond *n*-step estimator with White correction for heteroskedasticity.
(4) Instruments: lags 2 and higher of first differences in prices and rents available at time *t*.

the evidence of a unit root presence is overwhelming. The CIPS test with CADF(0) with the intercept and the trend is the relevant result. However, the present value model is rejected in any case since the price-to-rent ratios are not stationary. I also calculate the bubble indicator according to the definition suggested in Mikhed and Zemčík (2009b). If both prices and rents have a unit root, the indicator is equal to the p-value of the one-sided CIPS test statistic. The statistic in our case is -2.59 (see *Table 4*), and the corresponding p-value is 0.20. For comparison, Mikhed and Zemčík (2009b) report values around 0.90 for years 2003–2005 for the US data right before the US bubble started collapsing in 2006 with a bubble indicator of 0.32. This suggests that while prices are somewhat higher in the Czech Republic as compared to the fundamentals represented by rents, the overpricing is much milder as compared with the United States. Consequently, one can expect the stagnation of real estate prices or perhaps a small decline, but a collapse of the apartment prices is not very likely.

As a final exercise, I focus on predictability of changes in prices using changes in rents, and vice versa. Potential predictability is based on equation (4) and it can be tested using the Hurlin Homogeneous Granger Causality test. The Hurlin test statistic (14) can only be used for $T > 5 + 2L$, where L is the number of lags in the regression equation (11). If $L = 1$, the minimum number of effective observations needed is $T = 8$. This precludes the possibility of using the dataset, where the effective number of observations is 8–2 = 6 for differences and 8–1 = 7 for levels. The Hurlin causality tests are therefore replaced by tests of the coefficient for the lagged explanatory variable in equation (11) being equal to 0. *Table 5* presents the results of these tests, where the null hypothesis of $\delta = 0$ is strongly rejected using the t-statistic from the dynamic panel data estimation procedure. This result corresponds to results of the Hurlin tests in the working paper version of Mikhed and Zemčík (2009b), which made use of 273 Metropolitan Statistical Areas in the United States. Overall, the outcome of coefficient tests can be interpreted as a confirmation of the theory represented by equation (4) with the rents being somewhat more useful in predicting prices than vice versa.

6. Summary

In this paper, the relationship between Czech real estate prices and rents is evaluated. This is one of the first studies to analyze a particular housing market in Central and Eastern Europe using panel data stationarity techniques. The analysis is now possible due to newly available panel data unit root tests and due to the availability of panel data in the Czech Republic. The objective is to find if apartment prices are overvalued as compared with the stream of future cash-flows. The relationship between the two variables is captured using the present-value model. The IRI data used in this paper span the period from 2001 to 2008. The used methodology consists of a set of various panel data techniques, which have been developed only recently. First, I test for potential regional cross-sectional dependence. Second, a panel data unit root test robust to regional cross-dependence is conducted to test for stationarity of the price-to-rent ratios. Finally, the present value model predicts that changes in prices should predict changes in rents and vice versa. This prediction is tested using panel data causality tests.

The results are as follows. There is regional inter-dependence, which calls for the use of the cross sectionally robust unit root tests. While the results are somewhat sensitive to the used specification, it appears that prices and rents are non-stationary for the whole Czech Republic. The data spans the period from 2001 to 2008, which includes the rapidly increasing prices and rents in the early 2000s. The outcome of unit root tests is much more robust for price-to-rent ratios. Regardless of the employed specification, the price-to-rent ratios are not stationary. This indicates evidence of overpriced real estate in the Czech Republic. However, the degree of overpricing seems small judged by the *p*-values of the unit root tests for the price-to-rents ratios. Therefore, one can expect a minor decline of prices in some locations and stagnation in others and not the type of collapse observed in the United States, Spain, or Britain. Finally, the changes in rents predict changes in prices and vice versa, confirming the implications of the present-value model.

Data Appendix

335 Regions

As, Bechyn, Bila pod Bezdezem, Benatky nad Jizerou, Benesov, Beroun, Blina, Bilovec, Blansko, Blatna, Blovice, Bohumin, Bor, Boskovice, Brandys nad Labem-Stara Boleslav, Brno, Broumov, Brumov-Bylnice, Bruntal, Breclav, Breznice, Bystrice nad Pernstejnem, Bystrice pod Hostynem, Caslav, Celakovice, Cernosice, Cerveny Kostelec, Ceska Kamenice, Ceska Lipa, Ceska Skalice, Ceska Trebova, Ceske Budejovice, Ceske Velenice, Cesky Brod, Cesky Krumlov, Cesky Tesin, Dacice, Decin, Dobruska, Dobrany, Dobris, Doksy, Domazlice, Dubi, Dubnany, Duchcov, Dvur Kralove nad Labem, Frantiskovy Lazne, Frenstat pod Radhostem, Frydek-Mistek, Frydlant, Frydlant nad Ostravici, Fulnek, Golcuv Jenikov, Hanusovice, Havirov, Havlickuv Brod, Hermanuv Mestec, Hlinsko, Hluboka nad Vltavou, Hlucin, Hodonin, Holesov, Holice, Holysov, Horazdovice, Horni Plana, Horni Slavkov, Horsovsky Tyn, Horice, Horovice, Hostinne, Hostivice, Hradec Kralove, Hredek nad Nisou, Hranice, Hronov, Hrusovany nad Jevisovkou, Humpolec, Hustopece, Cheb, Chlumec nad Cidlinou, Chocen, Chodov, Chomutov, Chotebor, Chrastava, Chrudim, Ivancice, Ivanovice na Hane, Jablonec nad Nisou, Jablonne v Podjestedi, Jablunkov, Jaromer, Jaromeriice nad Rokytnou, Javornik, Jemnice, Jesenik, Jicin, Jihlava, Jilemnice, Jilove u Prahy, Jindrichuv Hradec, Jirkov, Kadan, Kamenice nad Lipou, Kaplice, Karlovy Vary, Karolinka, Karvina, Kdyne, Kladno Klasterec nad Ohri, Klatovy, Kolin, Konice, Kopidlno, Koprivnice, Kostelec nad Cernymi Lesy, Kostelec nad Orlici, Kraliky, Kralovice, Kralupy nad Vltavou, Kraluv Dvur, Kraslice, Kravare, Krnov, Kromeriz, Krupka, Kunovice, Kurim, Kutna Hora, Kyjov, Kynsperk nad Ohri, Lanskroun, Lazne Belohrad, Lazne Bohdanec, Ledec nad Sazavou, Letohrad, Letovice, Liberec, Libochovice, Lipnik nad Becvou, Lisov, Litomerice, Litomysl, Litovel, Litvinov, Lomnice nad Popelkou, Louny, Lovosice, Luhacovice, Lysanad Labem, Marianske Lezne, Melnik, Mestec Kralove, Mesto Albrechtice, Mikulov, Milevsko, Mimon, Miroslav, Mlada Boleslav, Mlada Vozice, Mnichovo Hradiste, Mnisek pod Brdy, Mohelnice, Moravska Trebova, Moravske Budejovice, Moravsky Beroun, Moravsky Krumlov, Most, Nachod, Nemest nad Oslavou, Napajedla, Nechanice, Nejdek, Neratovice, Netolice, Nova Bystrice, Nova Paka, Nove Mesto na Morave, Nove Mesto nad Metuji, Nove Mesto pod Smrkem, Nove Straseci, Novy Bor, Novy Bydzov, Novy Jicin, Nymburk, Nyrsko, Nyrany, Odolena Voda, Odry, Olomouc, Opava, Orlova, Ostrava, Ostrov, Otrokovice, Pacov, Pardubice, Pelhrimov, Petrvald, Pisek, Plana, Plzen, Pocatky, Podborany, Podebrady, Pohorelice, Police, nad Metuji, Policka, Polna, Postoloprty, Prague, Prachatice, Prostejov, Protivin, Prelouc, Prerov, Prestice, Pribor, Pribram, Pribyslav, Rakovnik, Rokycany, Rokytnice nad Jizerou, Rokytnice v Orlickych horach, Rosice, Roudnice nad Labem, Rousinov, Roztoky, Rozmital pod Tremsinem, Roznov pod Radhostem, Rumburk, Rychnov nad Kneznou, Rychvald, Rymarov, Ricany, Sazava, Sedlcany, Semily, Sezimovo Usti, Skutec, Slany, Slavicin, Slavkov u Brna, Sobeslav, Sobotka, Sokolov, Stankov, Stare Mesto, Stary Plzenec, Stochov, Strakonice, Straznice, Stribro, Studenka, Suchdol nad Luznici, Susice, Svetla nad Sazavou, Svitavy, Senov, Slapanice, Sluknov, Spindleruv Mlyn, Sternberk, Steti, Sumperk, Tabor, Tachov, Tanvald, Telc, Teplice, Tisnov, Touzim, Trhove Sviny, Trutnov, Trebechovice pod Orebem, Trebic, Trebon, Tremosna, Tremosnice, Trest, Trinec, Turnov, Tyn nad Vltavou, Tynec nad Sazavou, Tyniste nad Orlici, Uherske Hradiste, Uhersky Brod, Unhost, Unicov, Upice, Usti nad Labem, Usti nad Orlici, Ustek, Uvaly, Valasske Klobouky, Valasske Mezirici, Vamberk, Varnsdorf, Vejprty, Velka Bites, Velka nad Velickou, Velke Mezirici, Velvary, Veseli nad Moravou, Vimperk, Vitkov, Vizovice, Vlasim, Vodnany, Volary, Votice, Vratimov, Vrbno pod Pradedem, Vrchlabi, Vsetin, Vysoke Myto, Vyskov, Vyssi Brod, Zabreh, Zbiroh, Zlate Hory, Zlin, Znojmo, Zruc nad Sazavou, Zubri, Zacler, Zamberk, Zatec, Zdar nad Sazavou, Zelezna Ruda, Zelezny Brod, Zidlochovice.

Reduced Sample, 57 Regions

Bilina, Brno, Bruntal, Breclav, Cesky Tesin, Decin, Dubi, Havirov, Hodonin, Hradec Kralove, Cheb, Chomutov, Jablonec nad Nisou, Jihlava, Jirkov, Kadan, Karvina, Kladno, Klasterec nad Ohri, Kolin, Koprivnice, Krupka, Liberec, Litomerice, Litvinov, Louny, Mlada Boleslav, Most, Nechanice, Novy Jicin, Olomouc, Opava, Orlova, Pardubice, Pisek, Plzen, Prague, Prostejov, Prerov, Pribram, Sokolov, Spindleruv Mlyn, Sumperk, Tabor, Teplice, Trutnov, Trebic, Uherske Hradiste, Unhost, Usti nad Labem, Valasske Mezirici, Varnsdorf, Vejprty, Vsetin, Zlin, Znojmo, Zatec.

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