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Portfolio Allocation and International Risk Sharing

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Abstract
Recent contributions have shown that it is possible to account for the so-called consumption-real exchange anomaly in models with goods market frictions where international asset trade is limited to a riskless bond. In this paper, we consider a more realistic international asset market structure and show that as soon as we depart from the single bond economy, we can no longer account for the consumption-real exchange anomaly. Our central result holds for a simple asset market structure in which two nominal bonds are traded across countries. We explore the role of demand shocks such as news shocks in generating meaningful market incompleteness. We show that only under specific settings news shocks can improve the performance of the model in matching the portfolio positions and consumption-real exchange rate correlations that we observe in the data.

Keywords: Portfolio choice, incomplete financial markets, international risk sharing, consumption-real exchange rate anomaly
JEL Classifications: F31, F41

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1 Introduction

Last two decades have witnessed a dramatic increase in international capital flows. Lane and Milesi-Ferretti (2001, 2006) have documented the increase in gross holdings of cross-country bond and equities for various countries. Their analysis show that gross external financial positions now exceed 100% of GDP for major industrialised countries.

Despite this massive wave of financial globalisation, international risk sharing remains low. Efficient risk sharing requires that consumption should be higher in the country where it is cheaper to consume, implying a positive correlation between relative consumption and real exchange rate (RER)[1]. However, as first shown by Backus and Smith (1993) and Kollmann (1995), this is strongly rejected in the data. More recently, Obstfeld (2006) measures the degree of risk-sharing by looking at averages of consumption growth and real exchange rates for various countries as in the original Backus and Smith (1993) paper. Using this metric, he finds a distinct negative relationship (i.e. faster consumption growth is associated with a real appreciation) in the data for the period going from 1991 to 2006 - the period of financial integration- suggesting a worsening rather than an improvement in international risk-sharing. Table 1 displays data on international portfolios and international risk sharing (measured by the correlation of relative consumption and real exchange rate) for industrialised countries for 1991 and 2006.

While recent contributions (Benigno and Thoenissen, 2008 and Corsetti *et al* 2008[2]) have successfully replicated the low degree of international risk sharing in the context of DSGE models, their analysis is based on a simple international financial market structure in which a riskless bond is traded, a structure that is far from reflecting the recent trend in international financial integration.

The contribution of this paper is to examine the extent to which a more plausible asset market structure is compatible with low international risk sharing as current evidence suggests. We find that even in the case where we only allow for international trade in two nominal bonds, the so-called consumption real exchange anomaly is back.

It is well-known in international risk sharing literature that specifying a model with incomplete financial markets is not sufficient to generate a negative correlation between relative consumption and real exchange rates even when international asset trade is restricted to a non-contingent bond (see Baxter and Crucini, 1995 and Chari *et al* , 2002). More importantly, Cole and Obstfeld (1991) show that terms of trade movements can provide considerable insurance against supply shocks

[1] We define real exchange rate as the price of foreign consumption basket in home consumption units, i.e. an increase implies a real depreciation of home currency.

[2] Throughout the paper we frequently refer to these papers as BT and CDL, respectively.

irrespective of the asset market structure. Therefore, it is important to start from a model which can account for the anomaly when there is trade in a single bond and analyse the implications of introducing a second internationally traded bond to this set-up.

We use a two-country, two-sector model with shocks to tradable and non-tradable sector productivity in each country along the lines of Benigno and Thoenissen (2008) and Corsetti *et al* (2008). We first solve the model under the assumption that international asset trade is restricted to a non-contingent bond and review the mechanisms that can account for the anomaly within this framework. Both of these mechanisms rely on the strong wealth effects generated by uninsured country-specific supply shocks. In Benigno and Thoenissen (2008), a favourable supply shock in the domestic tradables sector increases the relative wealth of domestic agents and leads to higher consumption demand in the domestic country, which in turn raises the prices of domestic non-tradable goods relative to foreign, resulting in a real exchange rate appreciation. On the other hand, Corsetti *et al* (2008) emphasise the role of low-substitutability between home and foreign goods. They show that the relative increase in domestic wealth following a favourable supply shock leads to a stronger increase in consumption of home goods due to home bias in consumption and increases the relative price of home goods. Since trade elasticity is very low, a rise in the relative price of home goods cannot generate substitution away from home goods to foreign goods, thus the income effect dominates the substitution effect and terms of trade appreciates.

When we allow for international trade in domestic and foreign currency bonds, the above-mentioned wealth effect disappears and the anomaly returns. Why does a seemingly small move away from one-bond to two-bonds brings the model much closer to complete consumption risk sharing despite the fact that markets are incomplete?[3]

First of all, relative consumption risk is affected more by tradable sector shocks than by non-tradable sector shocks. This is because the country that enjoys a rise in non-tradable sector productivity also experiences a fall in the price of non-tradable goods relative to the other country, which in turn ensures high risk sharing.[4] Therefore, agents would want to use bonds mainly to hedge against relative consumption risk coming from tradable sector shocks. But whether they can do so, depends crucially on how relative bond returns are affected by non-tradable sector shocks.

If relative bond returns respond strongly to non-tradable sector shocks, a portfolio that insulates consumers from fluctuations in tradable sector output can make them more vulnerable to fluctuations in non-tradable output due to 'adverse valuation effects'. This in turn would limit

[3] Markets are incomplete as there are two bonds and four independent sources of risk - shocks to tradable and non-tradable output in each country. We solve the optimal portfolio using the methodology developed by Devereux and Sutherland (2008a).

[4] Cole and Obstfeld (1991) show that terms of trade adjustment can offset supply shocks when all goods are tradable, preferences are symmetric and trade elasticity is close to unity. In our model, we are far from the Cole and Obstfeld economy, therefore terms of trade does not ensure high risk sharing against tradable sector shocks.

the degree of risk sharing that can be provided by bonds. On the other hand, if relative bond returns are weakly related to non-tradable sector shocks, as is the case in most specifications of our model, agents can enjoy a high degree of risk sharing conditional on tradable sector shocks without increasing their exposure to non-tradable shocks, which brings the two bond economy closer to the complete markets economy.

In our model, monetary policy specification has important implications for portfolio allocation and the degree of risk sharing because it determines the nominal exchange rate, and relative bond returns are given by the surprises in the nominal exchange rate. We consider two simple monetary policy rules, domestic tradable price stabilisation and CPI stabilisation, which imply different properties of relative bond returns. Under the former, nominal exchange rate and relative bond returns are determined by the *terms of trade*, whereas under the latter they are given by the *real exchange rate*.

We find that trade in bonds generally leads to higher risk sharing when relative bond returns are determined by the terms of trade as opposed to the real exchange rate. This is because real exchange rate responds more strongly to non-tradable sector shocks, which prevents agents from choosing a portfolio that could insure them fully against the relative consumption risk coming from tradable sector shocks.[5] While the high risk sharing result is robust to different values of trade elasticity when relative bond returns are equal to the terms of trade, this is not the case when relative bond returns are given by the real exchange rate. Our numerical results show that, under CPI stabilisation, the cross-correlation between relative consumption and real exchange rate can be high or low depending on the value of trade elasticity. But under domestic tradable price stabilisation, the correlation is almost perfect regardless of this parameter.

In light of these results, we enrich the shock structure in our two-sector model and consider demand shocks as well as supply shocks. Our focus is on the implications of this additional source of uncertainty on equilibrium portfolio allocation and, through that, on the international transmission of supply shocks. In other words, we explore whether the presence of demand shocks can generate enough market incompleteness such that the transmission of supply shocks can still be negative as in Benigno and Thoenissen (2008) and Corsetti *et al* (2008) even under some endogenous portfolio choice. As demand shocks, we consider shocks to the predictable component of sectoral productivity shocks - 'news shocks' as in Beaudry and Portier (2004), Jaimovich and Rebelo (2008) and Croce and Colacito (2010) among others.[6]

[5] Real exchange rate consists of the terms of trade and the relative price of non-tradables. Because relative price of non-tradables is directly linked to the relative supply of non-tradables, real exchange rate is affected more strongly by non-tradable sector shocks compared to the terms of trade.

[6] We want to stress that the demand shocks we consider work in a different way compared to Stockman and Tesar (1995) type 'taste shocks', which are basically shocks to the marginal utility of consumption. Heathcote and Perri (2007) show that these shocks can be used to generate a realistic negative correlation between relative consumption and real exchange rate but their explanation of the anomaly does not rely on market incompleteness.

Our numerical results show that only under certain parameter and policy settings demand shocks can reduce the degree of risk sharing implied by bonds without comprising the model's ability to match other business cycle facts. The intuition for how demand shocks work is as follows. Demand shocks move relative consumption risk in the same direction as supply shocks, but they affect relative bond returns in the opposite direction. Therefore, relative supply and demand shocks require different signs for optimal bond portfolios, which in turn limits the degree of risk sharing ensured by bonds.

For instance, consider the case where demand shocks require a long position in foreign bonds, while supply shocks require the opposite. If demand shocks are sufficiently large, the optimal portfolio will be a long position in foreign currency, which will make home agents worse-off conditional on supply shocks. Given a long position in foreign currency, a negative supply shock that appreciates the domestic currency brings about capital losses, reducing net wealth of agents at a time they need to increase their consumption. This example illustrates the role of adverse valuation effects in accounting for the anomaly in the presence of endogenous portfolio choice.[7]

Our paper is closely related to the literature on country portfolios. Heathcote and Perri (2007), Kollmann (2006), Collard *et al* (2008), Engel and Matsumoto (2009) and Coeurdacier *et al* (2010) propose different models that can generate realistic portfolio positions under effectively complete markets. There is also a range of papers that analyse equilibrium portfolios under incomplete markets. Coeurdacier *et al* (2007) specify an incomplete market model with supply, demand and redistributive shocks and trade in stocks and bonds to match the basic stylized facts on international portfolios. Hnatkovska (2010) analyses endogenous portfolio choice under incomplete markets in a model with tradable and non-tradable sectors and examines the dynamics of portfolio choice to reconcile the home bias in equity holdings with the high turnover and high volatility of international capital flows. Using different modelling frameworks, Coeurdacier and Gourinchas (2009) and Benigno and Nisticó (2009) also study endogenous bond and equity portfolios under incomplete markets. However, they mainly focus on different hedging motives behind equilibrium portfolio positions, e.g. whether home equity bias is driven by non-diversifiable labour income risk or real exchange rate risk, rather than analysing the implications of portfolio allocation for international risk sharing and consumption-real exchange rate anomaly.

The rest of the paper proceeds as follows. In sections 2 and 3, we lay out a two-country two-sector endowment model and solve the model analytically to show how the comovement of relative consumption and real exchange rates is affected by endogenous portfolio choice in the presence of anticipated and unanticipated shocks. Section 4 gives the quantitative results of a calibrated

[7] Ghironi *et al* (2010) also focus on the role of valuation channel for international risk sharing. They show that valuation effects can dampen or amplify the response of consumption differential to productivity and government spending shocks in a two-country one-sector DSGE model where there is international trade in equity.

production model with capital accumulation. Section 5 concludes.

2 A two-country two-sector endowment economy

We first develop a basic two-country open economy endowment model. There is a home and a foreign country, with each country endowed with a tradable and a non-tradable good. Endowments in each country are stochastic. Households maximize utility over infinite horizon under different asset market configurations: complete markets where agents can trade in a full-set of state-contingent claims, incomplete markets where international asset trade is restricted to a single non-contingent bond and an intermediate case where agents in each country can trade in two nominal bonds denominated in home and foreign currency. The structure of the model is related to the production economies described in BT, CDL and Stockman and Tesar (1995).

2.1 Preferences and Good Markets

Representative agent in home country maximizes the expected present discounted value of the utility:

$$U_t = E_t \sum_{s=t}^{\infty} \delta_s \frac{C_s^{1-\rho}}{1-\rho} \tag{1}$$

where C is consumption and δ_s is the discount factor, which is determined as follows:

$$\delta_{s+1} = \delta_s \beta(C_{As}), \;\; \delta_0 = 1 \tag{2}$$

where C_A is aggregate home consumption and $0 < \beta(C_A) < 1$. To achieve stationarity under incomplete market specification, we assume $\beta_C(C_A) \leq 0$, which implies that agents discount the future more as aggregate consumption increases, i.e. agents bring consumption forward when aggregate consumption is high. Following Devereux and Sutherland (2008a), we assume that the individual takes C_A as given when optimising and specify the discount factor as follows:

$$\beta(C_A) = \omega C_A^{-\eta} \tag{3}$$

with $0 \leq \eta < \rho$ and $0 < \omega \bar{C}_A^{-\eta} < 1$ (for $\eta = 0$ we have the constant discount factor).

C represents a consumption index defined over tradable C_T and non tradable C_N consumption:

$$C_t = \left[\gamma^{\frac{1}{\kappa}} C_{T,t}^{\frac{\kappa-1}{\kappa}} + (1-\gamma)^{\frac{1}{\kappa}} C_{N,t}^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}, \tag{4}$$

where κ is the elasticity of intratemporal substitution between C_N and C_T and γ is the weight that the households assign to tradable consumption. The tradable component of the consumption index is in turn a CES aggregate of home and foreign tradable consumption goods, C_H and C_F:

$$C_{T,t} = \left[\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{5}$$

where θ is the elasticity of intratemporal substitution between C_H and C_F and ν is the weight that the households assigns to home tradable consumption. We allow for a home bias in tradable goods by assuming $\nu > \frac{1}{2}$. We adopt a similar preference specification for the foreign country except that variables are denoted with an asterisk. The consumption price index (CPI), which is defined as the minimum expenditure required to purchase one unit of aggregate consumption for the home agent is given by:

$$P_t = \left[\gamma P_{T,t}^{1-\kappa} + (1-\gamma) P_{N,t}^{1-\kappa}\right]^{\frac{1}{1-\kappa}} \tag{6}$$

Meanwhile, the traded goods price index, which is defined as the minimum expenditure required to purchase one unit of a traded good is given by:

$$P_{T,t} = \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{7}$$

We assume that the law of one price holds, i.e. $P^*_{H,t} = P_{H,t}/S_t$,and $P_{F,t} = P^*_{F,t} S_t$, where S_t denotes the nominal exchange rate defined as the price of foreign currency in terms of domestic currency. The presence of nontradable goods and home bias in tradables consumption leads to deviations from purchasing power parity. We define the real exchange rate as $Q = \frac{SP^*}{P}$.

Good market clearing requires $Y_{H,t} = C_{H,t} + C^*_{H,t}$, $Y^*_{F,t} = C^*_{F,t} + C_{F,t}$, $Y_{N,t} = C_{N.t}$ and $Y^*_{N,t} = C^*_{N,t}$ where C_H and C_F (C^*_F and C^*_H) should satisfy the intratemporal optimisation decisions of home (foreign) households. Endowments of tradable and non-tradable goods follow AR(1) processes of the form:

$$\log Y_{H,t} = \delta_T \log Y_{H,t-1} + u_{H,t}, \qquad \log Y^*_{F,t} = \delta_T \log Y^*_{F,t-1} + u_{F,t} \tag{8}$$

$$\log Y_{N,t} = \delta_N \log Y_{N,t-1} + u_{N,t}, \qquad \log Y^*_{N,t} = \delta_N \log Y^*_{N,t-1} + u^*_{N,t} \tag{9}$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_N < 1$, $u_{H,t}, u^*_{F,t}, u_{N,t}, u^*_{N,t}$ are i.i.d. shocks with $Var(u_H) = Var(u_F) = \sigma^2_T$ and $Var(u_N) = Var(u^*_N) = \sigma^2_N$.

2.2 Asset Markets

Previous literature establishes the link between international risk sharing and the asset market structure. Backus and Smith (1993) and Kollmann (1995) show that complete markets imply a counterfactual perfect correlation between relative consumption and real exchange rates. Benigno and Thoenissen (2008) and Corsetti *et al* (2008) set out the conditions under which it is possible to get a negative correlation between relative consumption and real exchange rates in an incomplete market set-up where only a single non-contingent bond is internationally traded. Here our aim is to see whether their results go through when we allow for endogenous portfolio choice in its simplest form - allowing for trade in two nominal bonds rather than a single non-contingent bond. Hence, we consider three different asset market structures to compare their implications for real exchange rate and relative consumption correlations.

2.2.1 Complete Markets

Complete market set-up can be characterized either by assuming that agents in each country can trade in a complete set of state-contingent assets, as in Chari *et al* (2002) or Heathcote and Perri (2002) to cite a few, or by assuming that there are as many independent assets, bonds and equities, as states of nature, i.e. the spanning condition holds, as in Devereux and Sutherland (2008a), Coeurdacier (2009) among others. Here we follow the former approach and do not characterise equilibrium portfolios associated with the complete market equilibrium. We are mainly interested in the risk sharing implications of complete markets, which we will later compare with the implications of incomplete markets.

The following risk sharing condition holds under complete markets:

$$\frac{U_C(C_t^*)}{U_C(C_t)} = \frac{S_t P_t^*}{P_t} \tag{10}$$

which states that marginal utilities of consumption adjusted by the respective CPI's are equalised across countries for each date and state. Backus and Smith (1993) and Kollmann (1995) show that the perfect correlation between relative consumption and real exchange rates implied by equation (10) under standard preferences, is strictly rejected in the data. Indeed, in the data relative consumption and real exchange rates are negatively correlated for most of the countries (see Table 1).

2.2.2 Incomplete Markets: Non-contingent Bond Economy

In this setting, home and foreign agents hold an international bond, $B_{H,t}$, which pays in units of home currency. The flow budget constraint of the representative home country consumer is given by:

$$B_{H,t} = R_{H,t} B_{H,t-1} + P_{H,t} Y_{H,t} + P_{N,t} Y_{N,t} - P_t C_t \tag{11}$$

where $R_{H,t}$ is the home country nominal interest rate, $P_{H,t}Y_{H,t}$ and $P_{N,t}Y_{N,t}$ are the home currency values of tradable and non-tradable good endowments. In this case, there is no portfolio choice problem. International trade in the non-contingent bond only allows for international borrowing and lending and does not provide any other hedging opportunity. This is the standard incomplete markets set-up used in the open economy macro literature.[8]

Maximisation of expected lifetime utility with respect to (11) implies the usual bond Euler equation for the home agent:

$$U_C(C_t) = \beta(C_t)E_tU_C(C_{t+1})R_{H,t+1}\frac{P_t}{P_{t+1}} \tag{12}$$

Foreign agent's optimal choice of home bonds is given by:

$$\begin{aligned} U_C(C_t^*) &= \beta(C_t^*)E_tU_C(C_{t+1}^*)R_{H,t+1}\frac{S_t}{S_{t+1}}\frac{P_t^*}{P_{t+1}^*} \\ U_C(C_t^*) &= \beta(C_t^*)E_tU_C(C_{t+1}^*)R_{F,t+1}^*\frac{P_t^*}{P_{t+1}^*} \end{aligned} \tag{13}$$

where $R_{F,t}^*$ is the nominal interest rate on foreign bond expressed in terms of foreign currency. In the non-contingent bond economy, the risk sharing condition given by equation (10) no longer holds. Benigno and Thoenissen (2008) and Corsetti *et al* (2008) show that this set-up can account for the consumption-real exchange rate anomaly. We review the main elements of their analysis in section (3.2) and show under what conditions this set-up can account for the anomaly.

2.2.3 Incomplete Markets: International Trade in Home and Foreign Currency Bonds

In this set-up we consider a small deviation from the single bond economy and allow for a second bond to be internationally traded. Agents in each country can now trade in bonds denominated in home and foreign currency. Given that the number of independent assets that can be traded internationally is less than the number of shocks, the spanning condition is not satisfied, i.e. markets are incomplete. The flow budget constraint of the home agent in nominal terms is given by:

$$B_{H,t} + S_tB_{F,t} = R_{H,t}B_{H,t-1} + R_{F,t}^*S_tB_{F,t-1} + P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} - P_tC_t \tag{14}$$

where $B_{H,t-1}$ is the home agent's holdings of internationally traded home bond and $B_{F,t-1}$ is the home agent's holdings of internationally traded foreign bond purchased at the end of period $t-1$

[8] In Benigno and Thoenissen (2008), home agents can trade in both home currency and foreign currency-denominated bonds, while foreign agents can only trade in foreign currency-denominated bonds. Thus international asset trade is restricted to foreign bonds. Stationarity is ensured by assuming international trade of foreign bonds is subject to intermediation costs. This setup has the same implications as our non-contingent bond economy setup with international trade in home bonds.

for holding into period t. $R_{H,t}$ and $R^*_{F,t}$ are the risk-free returns on home and foreign bonds.

Letting $\alpha_{H,t} \equiv B_{H,t}$, $\alpha_{F,t} \equiv S_t B_{F,t}$ and defining $NFA_t \equiv \alpha_{H,t} + \alpha_{F,t}$ as the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets of home agents) we can write (14) as a net foreign asset accumulation equation[9]:

$$NFA_t = NFA_{t-1}R_{H,t} + \alpha_{F,t-1}R_{x,t} + P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} - P_t C_t \tag{15}$$

where $R_{x,t} = R_{F,t} - R_{H,t}$ is the excess return on foreign bond relative to home bond expressed in home currency units, with $R_{F,t} = R^*_{F,t}\frac{S_t}{S_{t-1}}$.[10]

Note that once α_F is determined, α_H, α^*_H and α^*_F will also be determined as $\alpha_H = NFA - \alpha_F$ by definition and $\alpha^*_H = -\alpha_H$, $\alpha^*_F = -\alpha_F$ from market clearing conditions. Thus, we only focus on α_F in what follows.

The main difference between the asset accumulation equations (15) and (11) is the excess return on the portfolio, $\alpha_{F,t-1}R_{x,t}$, which implies state-contingent valuation effects. Therefore, in the set-up with endogenous bond portfolios, agents can smooth consumption not only across time through borrowing and lending in international financial markets, but also across different states of the world to some extent. As we discuss in detail below, the extent of insurance across states provided by trade in bonds depends on the loadings of excess return on different sources of risk.

Consumers' first order conditions imply that as well as the Euler equations given by (12) and (13), there is also a home Euler equation for foreign bond. These imply the following optimal portfolio choice equations should hold in each country:

$$E_t\left[m_{t+1}R_{x,t+1}\right] = 0 \quad E_t\left[m^*_{t+1}R_{x,t+1}\frac{S_t}{S_{t+1}}\right] = 0 \tag{16}$$

where home and foreign stochastic discount factors are given by $m_{t+1} = \beta(C_t)\frac{P_t}{P_{t+1}}\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}$ and $m^*_{t+1} = \beta(C^*_t)\frac{P^*_t}{P^*_{t+1}}\frac{C^{*-\rho}_{t+1}}{C^{*-\rho}_t}$, respectively, and $R_{x,t+1}$ is the excess return on foreign nominal bond, taking home bond as a reference as defined above.

To solve the model in the presence of endogenous portfolio choice under incomplete markets, we use the approximation techniques proposed in Devereux and Sutherland (2008a) and Tille and van Wincoop (2007). We approximate our model around the symmetric steady state in which steady-state inflation rates are assumed to be zero.

[9] Net foreign assets of home agent is defined as net claims of home country on foreign country assets, i.e. $NFA_t = \alpha_{F,t} - \alpha^*_{H,t}$. Since bonds are assumed to be in net zero supply $\alpha_{H,t} = -\alpha^*_{H,t}$. It follows that $NFA_t = \alpha_{H,t} + \alpha_{F,t}$.

[10] A similar budget constraint holds for the foreign agent, where foreign variables are denoted with an asterisk, *. Thus, $\alpha^*_{H,t-1}$ and $\alpha^*_{F,t-1}$ denote the foreign country's holdings of home and foreign bonds, expressed in units of home currency. Bonds are assumed to be in net zero supply in each country. Thus, equilibrium in asset market requires that total bond holdings of home and foreign agents should equal zero, i.e. $\alpha_{H,t} + \alpha^*_{H,t} = 0$ and $\alpha_{F,t} + \alpha^*_{F,t} = 0$.

The second order approximation of the optimal portfolio choice equations in (16) together with the property of the model that expected excess returns are zero up to a first order approximation, i.e. $E_t\left[\hat{R}_{x,t+1}\right] = 0 + O(\varepsilon^2)$, gives an orthogonality condition between excess returns and the relative stochastic discount factors denominated in the same currency, which pins down optimal steady-state portfolios:

$$Cov_t\left[(\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1}), \hat{R}_{x,t+1}\right] = 0 + O(\varepsilon^3) \tag{17}$$

As shown by Devereux and Sutherland (2008a), to evaluate (17) and determine the portfolio shares, it is sufficient to take a first-order approximation of the remaining equilibrium conditions for which the only aspect of portfolio behaviour that matters is the steady-state foreign bond portfolio, $\bar{\alpha}_F$.

2.3 Policy rules

We close the model by considering two simple policy rules. Although prices are fully flexible in our model, the way we specify policy rules matters as long as we have a nominal asset. This is because the return differential between home and foreign bonds is given by the rate of (unexpected) nominal exchange depreciation, which is affected by the policy rule in a flexible price setting. Consequently, equilibrium portfolio shares will be affected, which will then feed back into the model (see Devereux and Sutherland, 2008b and De Paoli *et al*, 2010).

We focus on two cases: in the first one, policy authorities stabilize their own tradable prices ($P_{H,t} = 1$,and $P_{F^*,t} = 1$) and in the second one they stabilize domestic consumer prices ($P_t = 1$,and $P^*_t = 1$).[11] Nominal exchange rate is equal to the terms of trade in the former, while it is given by the real exchange rate in the latter.[12]

3 Relative consumption and real exchange rate under alternative asset markets

In this section we first describe the general equilibrium behaviour of relative consumption and real exchange rate in response to sectoral supply shocks under complete markets and illustrate the Backus-Smith-Kollmann condition. Next, we go over the mechanisms put forth by Benigno and Thoenissen (2008) and Corsetti *et al* (2008) that can account for the consumption-real exchange rate anomaly when international asset trade is limited to a single non-contingent bond. Then, we

[11] Benigno and Thoenissen (2008) close their model by assuming that monetary policy is characterized by CPI targeting whereas Corsetti *et al* (2008) take the domestic CPI as numeraire, which are essentially equivalent.

[12] Having a nominal bond with a CPI targeting rule is equivalent to having a real bond (or CPI indexed bond) with any policy rule in terms of equilibrium portfolio and model solution.

analyse how the link between relative consumption and real exchange rate changes when we move from single bond economy to a two bond economy with endogenous portfolio choice.

3.1 Complete markets: Backus-Smith-Kollmann condition

Assuming CRRA preferences, log-linearisation of the risk sharing condition in (10) gives:

$$\hat{C}_t - \hat{C}_t^* = \frac{\hat{Q}_t}{\rho} \tag{18}$$

which implies that consumption should be higher in the country where it is cheaper to consume.

It is useful to characterize the full general equilibrium solution to relative consumption and real exchange rate under complete markets to compare it with the solution under different configurations of incomplete markets.

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* &= \frac{\gamma\kappa(2\nu-1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{\Gamma_2}{\Gamma_1}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*) \\ \hat{Q}_t &= \rho\left(\frac{\gamma\kappa(2\nu-1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{\Gamma_2}{\Gamma_1}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*)\right) \end{aligned} \tag{19}$$

where $\Gamma_1 = 4\theta\nu(1-\nu)(1+\gamma(\kappa\rho-1)) + \kappa(2\nu-1)^2 > 0$ and $\Gamma_2 = (1-\gamma)(\kappa(2\nu-1)^2 + 4\theta\nu(1-\nu)) > 0$ for all possible parameter values. The only state variables of the complete market model are the exogenous state variables, i.e. the stochastic endowment processes in each sector and country. Net foreign asset accumulation does not matter for equilibrium dynamics under complete markets. Real exchange rate and relative consumption are perfectly correlated as can be seen from (19).

3.2 Incomplete markets: Non-contingent bond economy

Under incomplete markets, the risk-sharing condition no longer holds in levels but in expected future changes in relative consumption and real exchange rate. Combining the home and foreign Euler equations with respect to the international asset gives:

$$E_t(\Delta\hat{C}_{t+1} - \Delta\hat{C}_{t+1}^*) = \frac{1}{\rho}E_t\Delta\hat{Q}_{t+1} \tag{20}$$

Since the risk sharing condition now holds in expected future changes, there will be deviations from the Backus-Smith condition, which can be expressed as $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$. Country-specific shocks will create large fluctuations in relative wealth provided that there are significant deviations from this condition.

To simplify the analytical expressions we assume that shocks are permanent, i.e. $\delta_T = \delta_N = 1$, so that the general equilibrium solution for relative consumption and real exchange rate dynamics reads:[13]

$$\hat{C}_t - \hat{C}_t^* = \psi_c^{nc}\widehat{NFA}_{t-1} + \frac{\gamma(2\theta\nu - 1)}{1 + 2\nu(\theta - 1)}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\gamma)(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*) \tag{21}$$

$$\begin{aligned}\hat{Q}_t &= -\psi_q^{nc}\widehat{NFA}_{t-1} \\ &\quad - \left[\frac{(1-\gamma)(2\theta\nu - 1) - \kappa(2\nu - 1)}{\kappa(1 + 2\nu(\theta - 1))}\right](\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{1-\gamma}{\kappa}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*)\end{aligned} \tag{22}$$

where $\psi_c^{nc} = \frac{4(1-\beta)\theta\nu}{\beta(1+2\nu(\theta-1))}$ and $\psi_q^{nc} = \frac{(1-\beta)}{\beta}\frac{(\kappa(2\nu-1)^2 + 4\nu(1-\nu)(1-\gamma)\theta)}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}$. $\psi_c^{nc} > 0$ and $\psi_q^{nc} > 0$ for $\theta > 1 - \frac{1}{2\nu}$.

In an incomplete market model, net foreign asset position is an endogenous state variable as reflected by the policy functions in (21) and (22)[14]. Relative consumption and real exchange rate are positively related conditional on non-tradable sector shocks. However, they might move in opposite directions conditional on tradable sector shocks depending on the value of trade elasticity, θ, which in turn can account for the consumption-real exchange rate anomaly as shown by BT and CDL.

To illustrate how the transmission of tradable sector supply shocks changes with trade elasticity, we decompose the real exchange rate into two components- the terms of trade, TOT_t, and the relative price of non-tradables across countries, P_t^N :

$$\hat{Q}_t = \gamma(2\nu - 1)\widehat{TOT}_t + (1-\gamma)\hat{P}_t^N \tag{23}$$

where $\widehat{TOT} = \hat{P}_F^* + \hat{S} - \hat{P}_H$ and $\hat{P}^N = \hat{P}_N^* + \hat{S} - \hat{P}_N$.[15] Equation (23) shows clearly that in this model real exchange rates fluctuate due to the presence of home bias in consumption ($\nu > \frac{1}{2}$) and non-traded goods ($\gamma < 1$).

The general equilibrium solution for terms of trade and relative non-tradables price assuming permanent shocks are as follows:

$$\widehat{TOT}_t = -\psi_T^{nc}\widehat{NFA}_{t-1} + \frac{1}{1 + 2\nu(\theta - 1)}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \tag{25}$$

[13] For the analytical derivations, we assume a constant discount factor, i.e. $\eta = 0$.

[14] For a sufficiently high elasticity of substitution between home and foreign goods ($\theta > 1 - \frac{1}{2\nu}$), higher net foreign assets brought from previous period implies higher consumption at home country ($\psi_c^{nc} > 0$) and a more expensive home consumption basket ($\psi_q^{nc} > 0$).

[15] More often, non-tradable prices in each country are expressed relative to the tradable prices, to highlight the Balassa-Samuelson effect:

$$\hat{Q}_t = (2\nu - 1)\widehat{TOT}_t + (1-\gamma)\widehat{RPN}_t \tag{24}$$

where terms of trade is defined as above and relative price of non-tradables is defined as $\widehat{RPN}_t \equiv (\hat{P}_{N,t}^* - \hat{P}_{T,t}^*) - (\hat{P}_{N,t} - \hat{P}_{T,t})$.

where $\psi_T^{nc} = \frac{(1-\beta)(2\nu-1)}{\beta\gamma(1-\nu)(1+2\nu(\theta-1))}$.[16]

$$\hat{P}_t^N = -\psi_N^{nc}\widehat{NFA}_{t-1} - \left[\frac{(2\theta\nu-1)-\kappa(2\nu-1)}{\kappa(1+2\nu(\theta-1))}\right](\hat{Y}_{H,t}-\hat{Y}_{F,t}) + \frac{1}{\kappa}(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*) \tag{26}$$

where $\psi_N^{nc} = \frac{(1-\beta)[\kappa(2\nu-1)^2+4\theta\nu(1-\nu)]}{\beta\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}$.

Using the analytical expressions given in equations (21) to (26), we can characterise five regions of trade elasticity, each of which implies a different transmission mechanism in response to tradable sector shocks on impact. Figure 1 illustrates these regions.

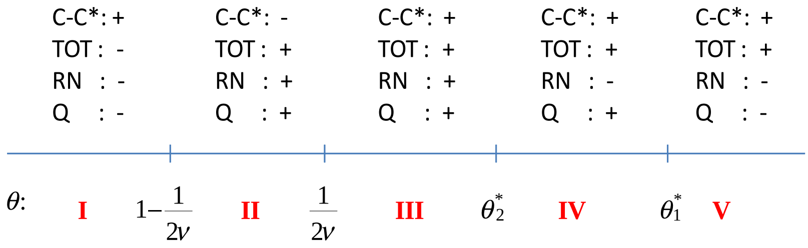


Figure 1: Impact responses to a positive tradable endowment shock with respect to trade elasticity for $\nu > \frac{1}{2}$. $\theta_1^* \equiv \frac{1}{2\nu} + \frac{\kappa}{1-\gamma}\frac{2\nu-1}{2\nu}$ and $\theta_2^* = \frac{1+\kappa(2\nu-1)}{2\nu}$

There are two regions of θ for which a positive tradable sector supply shock leads to an increase in relative consumption and a fall in real exchange rate - hence a negative conditional correlation on impact. These regions are region I, where $\theta < 1 - \frac{1}{2\nu}$, and region V, where $\theta > \theta_1^* \equiv \frac{1}{2\nu} + \frac{\kappa}{1-\gamma}\frac{2\nu-1}{2\nu}$. In both of these regions, an unanticipated increase in the tradable endowment of the home country implies a large increase in the relative wealth of home agents, which in turn leads to higher consumption and higher prices in the home country. As we describe in detail below, the main difference between the two regions is that in the former, the increase in relative wealth appreciates both the terms of trade and the relative price of non-tradables, while in the latter it only appreciates the relative non-tradables.[17]

[16]Note that terms of trade is independent of non-tradable sector shocks because we assume, for ease of exposition, that the persistence of non-tradable endowments, δ_N, is equal to 1. As we show later, terms of trade is independent of non-tradable sector shocks also when $\gamma = 1$ or $\nu = \frac{1}{2}$ or $\kappa\rho = 1$ (utility is separable in tradable and non-tradable consumption).

[17]CDL shows that there is a sixth region, which gives a transmission mechanism similar to the one described by region I for high values of θ. The main idea is that if endowments are expected to reach a permanently higher level over time, demand exceeds supply in the short-run, increasing relative consumption and appreciating the terms of trade. Because in our set-up shocks bring endowment immediately to its permanent level, we do not get this region. But, we do get it in the production economy version of this two-sector model, which we show in the numerical results section.

Figure 1 shows that there is another region, region II, given by $1 - \frac{1}{2\nu} < \theta < \frac{1}{2\nu}$, where relative consumption and real exchange rate are negatively correlated conditional on tradable endowment shocks. In this region, negative conditional correlation is due to the fact that relative consumption falls in response to a positive tradable sector shock while the real exchange rate depreciates. In what follows we focus our attention on regions I and V, which imply a positive relation between relative consumption and relative income.

Region I: The case of low trade elasticity

In this region, characterised by $\theta < 1 - \frac{1}{2\nu}$, the mechanism that accounts for the consumption-real exchange rate anomaly is the one emphasized by Corsetti *et al* (2008): Under incomplete markets, home agents become relatively wealthier following a positive home supply shock. Given that consumption is home biased, this positive wealth effect leads to a stronger increase in consumption of home goods, increasing the relative price of home goods. Since price elasticity of tradables is very low, a rise in the relative price of home goods cannot generate substitution away from home goods to foreign goods, thus the income effect dominates the substitution effect and terms of trade appreciates. The strong rise in relative home wealth also appreciates the relative price of non-tradables. In this region, 'negative transmission' of a positive supply shock does not rely on the presence of a non-tradable sector.

To see this more clearly, consider the case where all goods are tradable, such that real exchange rate dynamics are solely driven by the terms of trade. Equation (23) shows that when $\gamma = 1$, $\hat{Q}_t = (2\nu - 1)\widehat{TOT}_t$. If trade elasticity is sufficiently low such that $\theta < 1 - \frac{1}{2\nu}$, terms of trade appreciates in response to a positive supply shock at home (see equation (25)), which entails an appreciation of the real exchange rate for $\nu > \frac{1}{2}$. On the other hand, the same shock leads to an increase in relative consumption for $\theta < 1 - \frac{1}{2\nu}$, implying a negative correlation between $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$.

Region V: High trade elasticity

In this region, given by $\theta > \theta_1^*$, the mechanism that generates the conditional negative correlation between relative consumption and real exchange rates is the one emphasized by Benigno and Thoenissen (2008): In the absence of complete markets, a positive supply shock in the home tradable sector implies that home agents become relatively wealthier, which in turn increases the demand for non-tradables in the home country. Given the fixed supply of non-tradables, this increase in demand puts an upward pressure on the price of home non-tradables, more so if the elasticity of substitution between tradables and non-tradables, κ, is low so that the (negative) substitution effect on the demand for non-tradables is weaker than the (positive) income effect. The rise in the relative non-tradable price, in turn, appreciates the real exchange rate [See equations (26) and (22)]. For this mechanism to yield an unconditional negative cross correlation between relative consumption and real exchange rate, it is crucial that tradable sector shocks are sufficiently larger than non-tradable sector shocks.

To build some intuition for why this mechanism is valid for high trade elasticity, consider the other regions where trade elasticity is lower than θ_1^*. For region IV, i.e. $\theta_2^* < \theta < \theta_1^*$, wealth effects of an uninsured positive tradable endowment shock are strong enough to appreciate the relative price of non-tradables. On the other hand, due to home bias in consumption, any increase in the supply of home tradable goods should be absorbed mostly by home agents. When trade elasticity is lower, this implies that the price of home goods should fall by much more to clear the market. Hence, in this region, the depreciation in the terms of trade dominates the appreciation in the relative non-tradables price and the real exchange rate depreciates following the shock, resulting in a positive transmission.

In region III, i.e. for $1 - \frac{1}{2\nu} < \theta < \theta_2^*$, the depreciation of the terms of trade in response to a favourable supply shock is large enough to generate a negative income effect, which in turn would curb the demand for non-tradables and give rise to a depreciation in the relative non-tradable price rather than an appreciation, again leading to a positive transmission where relative consumption and real exchange rate both rise following an increase in tradable goods endowment.

3.3 Incomplete markets: International trade in home and foreign currency bonds

In this section we consider a small departure from the single non-contingent bond economy and look at the risk sharing implications of international trade in nominal bonds denominated in home and foreign currency in the presence of sectoral endowment shocks in each country. Endogenous trade in bonds lets agents hedge ex-ante against the relative consumption risk caused by country-specific shocks. Given that there are two independent assets and four different sources of relative consumption risk (tradable and non-tradable sector shocks in each country), this asset market structure represents an incomplete market set-up. Therefore we would expect the degree of risk sharing provided by trade in nominal bonds to fall somewhere in between the degree of risk sharing provided by trade in a single non-contingent bond and that provided by trade in a complete set of contingent claims. Then the main question is whether the two bond set-up is closer to the single bond set-up so that country-specific supply shocks can still generate changes in relative wealth strong enough to account for the consumption-real exchange rate anomaly, or whether it is closer to the complete market set-up which implies a counterfactual high correlation between relative consumption and real exchange rate.

To answer this question we first solve for the optimal bond portfolio and characterise the policy functions for relative consumption and real exchange rate consistent with this portfolio position. Then we compare relative consumption and real exchange rate responses to supply shocks under this set-up with those under the non-contingent bond and complete market set-ups. We show that whether the risk sharing implications of trade in two nominal bonds is closer to one or the other depends crucially on the properties of relative bond returns, which are in turn determined by the monetary policy specification.

3.3.1 Partial equilibrium analysis of optimal bond portfolio

In order to demonstrate the hedging motives of investors, we first derive a partial equilibrium expression for optimal bond positions as in Benigno and Nistico (2009) and Coeurdacier and Gourinchas (2009). Specifically, we use the first order approximation to the model equations to evaluate the portfolio orthogonality condition given by (17). The partial equilibrium solution for optimal steady-state foreign bond holdings can be written as:

$$\tilde{\alpha}_F = -\frac{1}{2(1-\beta)} \left\{ \begin{array}{c} \gamma \frac{Cov_t[\Lambda^T_{Y,t+1}, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} + (1-\gamma) \frac{Cov_t[\Lambda^N_{Y,t+1}, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} \\ + \left(\frac{\rho-1}{\rho}\right) \frac{Cov_t[\Lambda_{Q,t+1}, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} \end{array} \right\} \tag{27}$$

where $\Lambda^T_{Y,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j \Delta \hat{Y}^R_{T,t+1+j}$ and $\Lambda^N_{Y,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \beta^j \Delta \hat{Y}^R_{N,t+1+j}$ denote relative (non-financial) income risk in both sectors and $\Lambda_{Q,t+1} \equiv E_{t+1} \sum_{j=0}^{\infty} \beta^j \Delta \hat{Q}_{t+1+j} - E_t \sum_{j=0}^{\infty} \beta^j \Delta \hat{Q}_{t+1+j}$ denotes real exchange rate risk. We define $\hat{Y}^R_T$ and $\hat{Y}^R_N$, as $\hat{Y}^R_{T,t} \equiv \hat{Y}_H - \hat{Y}^*_F - \widehat{TOT}_t$ (relative tradable income adjusted for relative tradable prices) and $\hat{Y}^R_{N,t} \equiv \hat{Y}_{N,t} - \hat{Y}^*_{N,t} - \hat{P}^N_t$ (relative non-tradable income adjusted for relative non-tradable prices).

Equation (27) shows that the foreign bond portfolio, $\tilde{\alpha}_F$, depends on the loadings of relative bond returns on relative income and real exchange rate risk. It is optimal to go long in foreign bond (and short in home bond) if foreign bonds pay more when relative income is lower at home or when home consumption basket is more expensive. That is, $Cov_t(\Lambda^T_{Y,t+1}, \hat{r}_{x,t+1}) < 0$, $Cov_t(\Lambda^N_{Y,t+1}, \hat{r}_{x,t+1}) < 0$ and $Cov_t(\Lambda_{Q,t+1}, \hat{r}_{x,t+1}) < 0$ for $\rho > 1$ imply a long position in foreign currency bonds, i.e. $\tilde{\alpha}_F > 0$.[18]

Using the property of the model that expected returns are zero up to a first order approximation, i.e. $E_t \hat{r}_{x,t+1} = 0 + O(\varepsilon^2)$, we can write relative bond returns, $\hat{r}_{x,t+1}$, as the surprises in the nominal exchange rate:

$$\hat{r}_{x,t+1} = \hat{S}_{t+1} - E_t \hat{S}_{t+1} + O(\varepsilon^2) \tag{28}$$

Therefore, loading factors and equilibrium portfolios depend crucially on the behaviour of the nominal exchange rate, which in turn is determined by policy specification.

[18] Note that terms of trade and relative non-tradable price affect relative consumption risk through two channels; first by affecting the value of non-financial income in each country and second by affecting the price of the consumption basket.

3.3.2 Portfolio allocation and risk sharing under domestic tradable price stabilisation

Assuming monetary policy in each country stabilises respective domestic tradable prices, excess return on foreign bonds is given by the terms of trade:

$$\hat{P}_{H,t} = \hat{P}^*_{F,t} = 0 \Rightarrow \hat{S}_t = \widehat{TOT}_t \Rightarrow \hat{r}_{x,t} = \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t \tag{29}$$

In this case, due to the monetary policy rule, nominal bonds act like bonds indexed to domestic tradable price index.

To get the analytical solution for the bond portfolio, we characterise closed form expressions for the two components of the portfolio orthogonality condition, real exchange rate adjusted relative consumption and relative bond returns, in terms of the structural shocks and the excess return on portfolio $\tilde{\alpha}_F \hat{r}_{x,t}$. Assuming $\delta_T = 1$, $\delta_N = \delta < 1$ we get the following:[19]

$$\begin{aligned} \hat{C}_t - \hat{C}^*_t - \frac{\hat{Q}_t}{\rho} &= \psi_{rcq}\widehat{NFA}_{t-1} + \frac{\Gamma_3}{\kappa\rho(1+2\nu(\theta-1))}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\ &\quad + \frac{(1-\beta)(1-\gamma)(\kappa\rho-1)}{(1-\beta\delta)\kappa\rho}(\hat{Y}_{N,t} - \hat{Y}^*_{N,t}) \\ &\quad + \frac{(1-\beta)\Gamma_1}{\kappa\rho\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F \hat{r}_{x,t} \end{aligned} \tag{30}$$

where $\psi_{rcq} = \frac{(1-\beta)\Gamma_1}{\beta\kappa\rho\gamma(1-\nu)(1+2\nu(\theta-1))}$ and $\Gamma_3 = (2\theta\nu - 1)(\gamma\kappa\rho + 1 - \gamma) - \kappa(2\nu - 1)$. $\Gamma_3 > 0$ for $\theta > \theta^*_3 \equiv \frac{1}{2\nu} + \frac{1}{2\nu}\frac{\kappa(2\nu-1)}{\gamma\kappa\rho+1-\gamma}$. Note that $1 - \frac{1}{2\nu} < \theta^*_3 < \theta^*_1$ (see Figure 1).

$$\begin{aligned} \hat{r}_{x,t} &= \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t = \frac{1}{1+2\nu(\theta-1)}(u_{H,t} - u_{F,t}) \\ &\quad + \frac{\beta(1-\gamma)(1-\delta)(2\nu-1)(\kappa\rho-1)}{(1-\beta\delta)\Gamma_1}(u_{N,t} - u^*_{N,t}) \\ &\quad - \frac{(1-\beta)(2\nu-1)}{\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F \hat{r}_{x,t} \end{aligned} \tag{31}$$

Consider first real exchange rate adjusted relative consumption and excess returns under the zero-portfolio solution ($\tilde{\alpha}_F = 0$) to build intuition for the optimal bond position. The zero-portfolio solution corresponds to the solution that would arise when agents can only trade in a single non-contingent bond. First note that for $\nu > \frac{1}{2}$, hedging against non-tradable endowment shocks requires a short position in foreign bonds irrespective of the substitutability between tradables and non-tradables or any other parameter. On the other hand, optimal hedge against tradable

[19] We first consider the case with $\delta_N = \delta < 1$, instead of setting $\delta_N = 1$ as we do in the analysis of the non-contingent bond economy. We do this to understand how relative bond returns (terms of trade) responds to non-tradable shocks. Because when $\delta_N = 1$, terms of trade is independent of non-tradable sector shocks.

endowment shocks depends crucially on the value of trade elasticity in line with the arguments following Figure 1. For values of θ in region I, a positive tradable endowment shock leads to an increase in $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ and fall in $\hat{r}_{x,t}$, pulling the equilibrium portfolio towards a long position in foreign bonds. For values of θ that lie in region V, both $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ and $\hat{r}_{x,t}$ increase following a positive tradable endowment shock, which makes it optimal to go short in foreign bonds.[20]

In what follows, to simplify algebra and facilitate the discussion of different cases, we focus on the case where both tradable and non-tradable endowment shocks have unit root, $\delta_T = \delta_N = \delta = 1$ as we do in the analysis in section (3.2). Solving equations (30), (31) and the portfolio orthogonality condition given in (17) under this assumption implies the following optimal bond portfolio:

$$\tilde{\alpha}_F = -\tilde{\alpha}_H = -\frac{\gamma(1-\nu)\Gamma_3}{(1-\beta)(\gamma\kappa\rho + (1-\gamma))} \tag{32}$$

where $\Gamma_3 \gtrless 0$ for $\theta \gtrless \theta_3^*$. Therefore, the sign of the optimal bond portfolio depends on the value of trade elasticity. For θ belonging to region I, optimal portfolio is long in foreign currency whereas for θ in region V, it is the opposite.[21] Although there are four shocks affecting each country and only two assets that can be internationally traded, optimal bond portfolio does not depend on the relative variance of different shocks. This is because under the assumption that $\delta = 1$, terms of trade is independent of non-tradable endowment shocks as shown in equation (31). Hence, agents can choose a portfolio to insure themselves perfectly against tradable sector shocks, without being subject to unwanted valuation effects conditional on non-tradable endowment shocks.[22] For more general parameter values, terms of trade loads on relative non-tradable income shocks, hence equilibrium portfolio becomes a complicated object that depends on the relative variance of tradable versus non-tradable income shocks. However, as we discuss below, even in this case, portfolios will be biased more towards hedging against tradable income shocks as terms of trade loads weakly on non-tradable income shocks even when tradable and non-tradable goods are complements in consumption.

Implications of optimal bond portfolio for relative consumption and real exchange rate correlations under domestic tradable price stabilisation

Optimal portfolio allocation has important implications for the relative consumption and real exchange rate dynamics in response to tradable endowment shocks. The solution for relative con-

[20] Note that for $\theta = \theta_3^*$, $\Gamma_3 = 0$ and there is perfect risk-sharing conditional on tradable endowment shocks even under zero-portfolio. When $\nu = \frac{1}{2}$, $\Gamma_3 = 0$ for $\theta = 1$. This is the knife-edge case described by Cole and Obstfeld: If $\nu = \frac{1}{2}$ and $\theta = 1$, terms of trade ensures complete risk sharing conditional on tradable sector shocks irrespective of the assets that are traded.

[21] This follows from the fact that for $\nu > \frac{1}{2}$, $1 - \frac{1}{2v} < \theta_3^* < \theta_1^*$.

[22] Web appendix shows the decomposition of the equilibrium portfolio given in (32) in terms of the loadings of excess returns on relative non-financial income risk by sector and real exchange rate risk in line with (27).

sumption and real exchange rate in this case becomes:

$$\hat{C}_t - \hat{C}_t^* = \psi_c^{nc} NFA_{t-1} + \frac{\gamma\kappa(2\nu - 1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\gamma)(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*) \tag{33}$$

$$\hat{Q}_t = -\psi_q^{nc} NFA_{t-1} + \frac{\rho}{\Gamma_1}\gamma\kappa(2\nu - 1)(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{(1-\gamma)}{\kappa}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*) \tag{34}$$

where ψ_c^{nc} and ψ_q^{nc} are as defined in section (3.2). Comparison of equations (33) and (34), with equations (21) and (22), which give the solution in the case of a single bond, shows clearly that $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ are no longer negatively correlated conditional on tradable endowment shocks. Indeed, the response of $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ to tradable endowment shocks in this two bonds set-up is exactly the same as that under the complete market set-up given by equations (19). On the other hand, due to the fact that terms of trade is independent of non-tradable endowment shocks, agents cannot use bonds to hedge against these shocks and hence relative consumption and real exchange rate response to non-tradable sector shocks is the same as that under the single bond set-up.

Hence, when excess returns are given by the terms of trade, trade in two nominal bonds ensures perfect risk sharing across countries conditional on tradable endowment shocks for all possible values of θ. Thus, when central bank stabilises the domestic tradable price index, a slight departure from a single bond economy to a two bonds economy kills the wealth effects associated with tradable income shocks.

A more general parameter set-up where terms of trade loads on non-tradable sector shocks

How do the risk-sharing implications of bonds change when terms of trade loads on non-tradable endowment shocks, that is when $\delta < 1$? A closer inspection of equation (31) suggests that even under a general parameter setting, terms of trade loads more strongly on tradable sector shocks compared to non-tradable shocks. This is intuitive as the terms of trade is directly linked to relative supply of tradables whereas it is only indirectly affected by changes in the relative supply of non-tradables through the complementarity/substitutability between tradables and non-tradables. Thus, bonds would be mainly used to hedge against the risks they can span more effectively, implying high insurance in response to tradable income shocks, which implies high insurance overall.[23]

[23] Numerical results for the endowment economy with stationary shocks ($\delta < 1$) show that when excess returns are given by the terms of trade, the cross-correlation between relative consumption and real exchange rate is robustly high (i.e. 0.999) regardless of the calibration of parameters.

3.3.3 Portfolio allocation and risk sharing under consumer price stabilisation

When monetary policy in each country stabilises the respective consumer price index, excess return on foreign bonds is given by the real exchange rate:

$$\hat{P}_t = \hat{P}_t^* = 0 \Rightarrow \hat{S}_t = \hat{Q}_t \Rightarrow \hat{r}_{x,t} = \hat{Q}_t - E_{t-1}\hat{Q}_t \tag{35}$$

In this case, nominal bonds act like CPI-indexed bonds because of the monetary policy specification. For $\delta_T = 1$ and $\delta_N = 1$, excess return on foreign bonds is given by:

$$\begin{aligned} \hat{r}_{x,t} \quad = \quad & \hat{Q}_t - E_{t-1}\hat{Q}_t = -\left[\frac{(1-\gamma)(2\theta\nu - 1) - \kappa(2\nu - 1)}{\kappa(1 + 2\nu(\theta - 1))}\right](u_{H,t} - u_{F,t}) \\ & + \frac{1-\gamma}{\kappa}(u_{N,t} - u_{N,t}^*) - \frac{(1-\beta)[4\theta\nu(1-\nu)(1-\gamma) + \kappa(2\nu-1)^2]}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t} \end{aligned} \tag{36}$$

The other component of the portfolio orthogonality condition, real exchange rate adjusted relative consumption, is still given by equation (30), where $\tilde{\alpha}_F\hat{r}_{x,t}$ is suitably adapted to the new policy specification and $\delta = 1$ is imposed to make it compatible with (36).

To build intuition for the optimal bond position, we consider the zero-portfolio solution once again. As we established during our discussion of the non-contingent bond economy, real exchange rate appreciates in response to a positive supply shock in home tradables sector for θ taking values in regions I and V. For these values of θ, real exchange rate adjusted relative consumption also increases in response to the same shock. Therefore, hedging against the consumption risk coming from tradable sector shocks require a long position in foreign currency for values of θ in region I and and region V (Figure 1).

The optimal hedge against non-tradable income shocks depends on whether tradable and non-tradable goods are substitutes or complements in consumption. Under the former specification, i.e. $\kappa\rho < 1$, relative consumption adjusted by the real exchange rate falls in response to a positive non-tradable income shock (see equation (30)), while the opposite is true for $\kappa\rho > 1$. When the two goods are complements, demand for tradables also increase following a positive non-tradable supply shock. Given that the supply of tradable goods is fixed, this leads to an excess demand for tradables, which appreciates the terms of trade and leads to a fall in real exchange rate adjusted consumption differential. On the other hand, under the zero portfolio solution, real exchange rate depreciates in response to an increase in relative home non-tradable income irrespective of any parameter specification (see equation (36)). Therefore, hedging against the consumption risk coming from non-tradable sector shocks requires a long position in foreign currency when $\kappa\rho < 1$, and a short position when $\kappa\rho > 1$.

Since $\hat{r}_{x,t}$ is a complicated expression even for permanent shocks, we impose the additional restriction that preferences for tradable goods are symmetric ($\nu = \frac{1}{2}$) to be able to display analytical results for optimal portfolio allocation and show its implications for risk sharing. Note that for $\nu = \frac{1}{2}$, real exchange rate movements are driven only by movements in the relative price of non-tradables, i.e. $\hat{Q}_t = (1-\gamma)\hat{P}_t^N$.

Evaluating the portfolio orthogonality condition using (30) and (36) under the parameter restrictions $\delta_T = \delta_N = 1$ and $\nu = \frac{1}{2}$, we get the following optimal foreign bond position:[24]

$$\tilde{\alpha}_F = \frac{\gamma\left[(\theta-1)^2(\gamma\kappa\rho + (1-\gamma))\frac{\sigma_T^2}{\sigma_N^2} - \theta^2(1-\gamma)(\kappa\rho-1)\right]}{2(1-\beta)(1-\gamma)\theta^2\rho} \tag{37}$$

For the reasons discussed above, assuming complementarity between tradables and non-tradables, i.e. $\kappa\rho < 1$, is sufficient to have a long position in foreign bonds. If tradable sector shocks are sufficiently large compared to σ_T^2/σ_N^2, optimal portfolio will still be a long position in foreign currency also for $\kappa\rho > 1$.[25]

Implications of optimal bond portfolio for relative consumption and real exchange rate correlations under consumer price stabilisation

Given the optimal portfolio allocation in (37), relative consumption and real exchange rate dynamics are as follows:

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* &= \psi_{rc}\widehat{NFA}_{t-1} \\ &+\frac{1}{\Gamma_4}\left[\begin{array}{c} \gamma\kappa\rho\theta(\theta-1)(\hat{Y}_{H,t}-\hat{Y}_{F,t}) \\ +\left(\theta^2(1-\gamma)+(\theta-1)^2(\gamma\kappa\rho+1-\gamma)\frac{\sigma_T^2}{\sigma_N^2}\right)(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*) \end{array}\right] \end{aligned} \tag{38}$$

$$\begin{aligned} \hat{Q}_t &= -\psi_q\widehat{NFA}_{t-1} \\ &+\frac{\rho(1-\gamma)}{\Gamma_4}\left[-\theta(\theta-1)(\hat{Y}_{H,t}-\hat{Y}_{F,t})+\theta^2(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*)\right] \end{aligned} \tag{39}$$

where $\Gamma_4 \equiv (\gamma\kappa\rho + 1 - \gamma)\left(\theta^2 + (\theta-1)^2\frac{\sigma_T^2}{\sigma_N^2}\right) > 0$ for all possible parameter values. Equations (38) and (39) show that relative consumption and real exchange rate are negatively correlated conditional on tradable endowment shocks for all possible values of θ, given our parameter restrictions $\delta_T =$

[24] To compare this foreign currency position with the one obtained under domestic tradable price stabilisation, impose $\nu = \frac{1}{2}$ in equation (32):

$$\tilde{\alpha}_F = -\frac{\gamma(\theta-1)}{2(1-\beta)}$$

The optimal foreign bond position under domestic tradable price stabilisation is thus negative for $\theta > 1$.

[25] See the web appendix for a discussion of the loading factors that show the breakdown of the optimal portfolio according to different hedging motives.

$\delta_N = 1$ and $\nu = \frac{1}{2}$. This is because when relative bond returns are given by the real exchange rate, bonds are almost equally good in hedging against the relative consumption risks coming from tradable and non-tradable sector shocks. Therefore, optimal bond portfolio in this case is torn between hedging against tradable and non-tradable shocks, which in turn implies that the consumer cannot insure fully against any of these shocks. This gives rise to international wealth transfers that imply lower risk sharing compared to the case where relative bond returns are equal to the terms of trade.[26]

Even though the parameter restrictions we impose here might seem somewhat limited, analytical results help us compare the equilibrium outcomes in the single bond economy with that in the two bonds economy and facilitates the understanding of the hedging properties of bonds under the two simple policy rules we consider. These results highlight the parameters that are important for optimal portfolios and the transmission of shocks and guide us in the calibration of the model in the numerical analysis.

To summarise, moving away from trade in a single non-contingent bond to trade in two bonds makes a huge difference for international risk sharing and transmission of supply shocks. When monetary policy rules are such that relative bond returns are associated with the terms of trade, sectoral supply shocks do not create a meaningful tension on equilibrium portfolios and hence agents can ensure high risk sharing by taking the correct portfolio position. On the other hand, when relative bond returns are given by the real exchange rate, trade in bonds ensures less risk sharing because the real exchange rate loads equally well on both tradable and non-tradable income risks, which implies that having a portfolio to hedge against one source of shock would imply unwanted valuation effects conditional on the other. Whether this set-up can generate reasonable portfolio positions alongside a negative relative consumption-real exchange rate correlation is a quantitative question which we explore later in section (4).

3.4 Adding demand shocks to the model

In the previous section, we showed that even a small move away from the non-contingent bond set-up leads to very high risk sharing in response to supply shocks, especially when agents can have claims to terms of trade (can trade in nominal bonds when domestic tradable price index

[26] We should acknowledge that the parameter restrictions we impose here, particularly the restriction that $\nu = \frac{1}{2}$, make it easier to get the negative comovement between real exchange rate and relative consumption conditional on tradable income shocks. This is because when $\nu = \frac{1}{2}$, real exchange rates move only due to relative non-tradable prices, which reflect the income effect more strongly. When $\nu > \frac{1}{2}$ and $\theta > 1 - \frac{1}{2\nu}$ such that terms of trade depreciates in response to tradable endowment shocks, it will be more difficult to get the real exchange rate to appreciate following the appreciation in relative non-tradable prices as there will be an offsetting effect coming from terms of trade. Nevertheless, numerical results show that this set-up can still generate a negative correlation between relative consumption and real exchange rate conditional on tradable sector shocks for $\nu > \frac{1}{2}$ and θ in region V.

is stabilised in each country). The insight from the analysis on the nominal bonds under CPI-targeting is that we can limit the risk sharing implied by endogenous asset trade if excess returns load equally well on all sources of risks and different risks imply different portfolio positions. In this case, equilibrium portfolios will depend on the relative size of shocks and valuation effects will have the potential to impede risk sharing depending on the type of shock that hits the economy.

In this section, we introduce shocks to the anticipated component of tradable endowments- "news shocks", which act as demand shocks in our two-sector endowment model and show how these shocks can change the risk sharing properties of nominal bonds conditional on supply shocks.[27] We present analytical results only for the case of tradable price targeting since this is the setting under which trade in two bonds brings the equilibrium close to that under complete markets. The intuition we build for this case can be used to understand the case of CPI stabilisation. We discuss the role of demand shocks in detail in the numerical results section.

3.4.1 News shocks

We assume that tradable endowment process now has a predictable component in each country. $u_{H,t}$ and $u_{F,t}$ are unanticipated home and foreign tradable endowment shocks at time t, $z_{H,t}$ and $z_{F,t}$ are information that arrive at time t about the $t+1$ values of home and foreign tradable endowments. When there is positive news today (an increase in $u_{zh,t}$), agents anticipate home tradable endowment to be higher in the next period. The formulation we use is similar to Croce and Colacito (2010):[28]

$$\begin{aligned} \log Y_{i,t} &= \delta_T \log Y_{i,t-1} + \log z_{i,t-1} + u_{i,t} \\ \log z_{i,t} &= \delta_z \log z_{i,t-1} + u_{Zi,t} \text{ for } i = H, F \end{aligned} \tag{40}$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_z < 1$, $u_{H,t}, u_{F,t}, u_{ZH,t}, u_{ZF,t}$ are i.i.d. shocks with $Var(u_H) = Var(u_F) = \sigma_T^2$ and $Var(u_{ZH}) = Var(u_{ZF}) = \sigma_Z^2$. The stochastic processes for non-tradable endowments are still given by equations (9).[29]

[27] We also derive analytical results for i-pod shocks as in Coeurdacier *et al* (2008), which can be found in the web appendix.

[28] Croce and Colacito (2010) consider endowment processes which grow at a constant rate and follow an integrated process of order 1 in each country. Schmitt-Grohe and Uribe (2008) introduce a more general shock structure in which each structural shock has an unanticipated and anticipated component which can be known up to three quarters in advance. They specify a fully-fledged closed economy RBC model with stationary and non-stationary neutral productivity shocks, non-stationary investment productivity shocks and government spending shocks. Their estimates show that the most important news are the shocks to the stationary component of productivity anticipated 3 quarters in advance. Since in our model a period corresponds to one year, specifying one-period ahead anticipation shocks is roughly consistent with this finding.

[29] We initially introduce "news" only to the tradable sector, because trade in nominal bonds under domestic tradable price stabilisation ensures too much risk sharing conditional on tradable endowment shocks. In the numerical part, we consider news to both sectors.

To understand how the presence of news shocks affects optimal portfolios, consider the general equilibrium expressions for the two components of the portfolio orthogonality condition given by (17), where we again assume that $\delta_N = \delta_T = 1$ for ease of exposition:

$$\begin{aligned}
\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho} &= \psi_{rcq}\widehat{NFA}_{t-1} + \frac{\Gamma_3}{\kappa\rho(1+2\nu(\theta-1))}\left[(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{\beta}{1-\beta\delta_z}(\hat{z}_{H,t} - \hat{z}_{F,t})\right] \\
&+ \frac{(1-\gamma)(\kappa\rho-1)}{\kappa\rho}(\hat{Y}_{N,t} - \hat{Y}^*_{N,t}) \\
&+ \frac{(1-\beta)\Gamma_1}{\gamma\kappa\rho(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}
\end{aligned} \tag{41}$$

$$\begin{aligned}
\hat{r}_{x,t} &= \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t = \frac{1}{1+2\nu(\theta-1)}\begin{bmatrix} (u_{H,t} - u_{F,t}) \\ -\frac{\beta(2\nu-1)\Gamma_3}{(1-\beta\delta_z)\Gamma_1}(u_{ZH,t} - u_{ZF,t}) \end{bmatrix} \\
&- \frac{(1-\beta)(2\nu-1)}{\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}
\end{aligned} \tag{42}$$

where Γ_1 and Γ_3 are as defined before.[30] Note that the coefficients on unanticipated tradable and non-tradable shocks and the excess return on the portfolio ($\tilde{\alpha}\hat{r}_{x,t}$) are identical to the ones given in equations (30) and (31). Shocks to the anticipated component of tradable endowments affect real exchange rate adjusted relative consumption in the same way as unanticipated shocks, only discounted by $\frac{\beta}{1-\beta\delta_z}$. In other words, for $\theta > \theta_3^*$ such that $\Gamma_3 > 0$, or for $\theta < 1 - \frac{1}{2\nu}$, $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ rises in response to an increase in both the anticipated and unanticipated components of tradable endowments.[31]

On the other hand, as shown by equation (42), terms of trade responds differently to anticipated and unanticipated shocks. For $\nu > \frac{1}{2}$ and $\theta > \theta_3^*$, a positive shock to the predictable component of tradables endowment, which increases relative consumption gap in favour of home agents, appreciates the terms of trade. This is because after receiving the positive news about future endowment, home agents increase their demand for tradables in the current period. Given that the supply of tradables is still fixed when agents receive the news, this leads to an excess demand for tradables in the current period, which in turn appreciates the terms of trade as consumption is home biased. Since news about future supply conditions increase current demand and appreciate the terms of trade, news shock act as a demand shock.[32]

Due to the fact that real exchange rate adjusted consumption differential and excess returns are

[30] See Table 2 for a summary of the definitions of convoluted parameters.

[31] The extent to which anticipated shocks affect $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ is determined crucially by δ_z. As δ_z increases, $\frac{\beta}{1-\beta\delta_z}$ increases, amplifying the response of relative consumption to anticipated shocks.

[32] Note that when $\theta < 1 - \frac{1}{2\nu}$, both anticipated and unanticipated endowment shocks work as demand shocks, because terms of trade appreciate following an unanticipated increase in tradable endowment in this region of θ.

positively correlated conditional on unanticipated shocks but negatively correlated conditional on anticipated shocks, relative variance of the two shocks will determine the sign of optimal portfolio as displayed below:

$$\tilde{\alpha}_F = -\frac{\gamma(1-\nu)\Gamma_3}{(1-\beta)(\gamma\kappa\rho + 1 - \gamma)}\left(1 - \frac{\Gamma_3}{\Gamma_1}\frac{\beta^2(2\nu-1)\frac{\sigma_Z^2}{\sigma_T^2}}{(1-\beta\delta_z)^2}\right) \tag{43}$$

As shown in (43), the optimal bond portfolio in the presence of news shocks is the optimal bond portfolio given in (32) plus an expression that depends on the relative variance of anticipated shocks with respect to unanticipated shocks to tradables endowment. Therefore, for $\nu > \frac{1}{2}$, $\theta > \theta_3^*$ and a sufficiently high σ_Z^2/σ_T^2, i.e. $\sigma_Z^2/\sigma_T^2 > \frac{\Gamma_1(1-\beta\delta_z)^2}{(2\nu-1)\Gamma_3\beta^2} \equiv RV_1^*$, it is optimal to have a long position in foreign bonds rather than a short position which would be optimal to hedge againts unanticipated endowment shocks. This would then imply adverse valuation effects in the face of unanticipated shocks to tradable endowments and potentially impede risk sharing. In this case, endogenous trade in nominal bonds will not be enough to hedge perfectly against any of these two shocks. Thus there will be deviations from the perfect risk sharing condition, which might potentially give rise to a negative correlation between relative consumption and real exchange rate conditional on unanticipated supply shocks in the tradables sector.

Real exchange rate-relative consumption correlations in the presence of news shocks

The general equilibrium expressions for $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ are very complicated especially after plugging in the optimal portfolio. Thus to show the risk sharing implications of nominal bonds in the presence of news shocks, we report the solution for the relative consumption and real exchange rate as the zero-portfolio (or non-contingent bond) solution plus the response to the excess return on the portfolio, $\tilde{\alpha}_F \hat{r}_{x,t}$, which is characterized by equations (42) and (43) in equilibrium.

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* \;=\;& \psi_c^{nc}\widehat{NFA}_{t-1} + \frac{\gamma(2\theta\nu-1)}{1+2\nu(\theta-1)}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\ & + \frac{\beta}{1-\beta\delta_z}\frac{\gamma 4\theta\nu(1-\nu)}{1+2\nu(\theta-1)}\frac{\Gamma_3}{\Gamma_1}(\hat{z}_{H,t} - \hat{z}_{F,t}) \\ & + (1-\gamma)(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*) + \frac{4\theta\nu(1-\beta)}{(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t} \end{aligned} \tag{44}$$

$$\hat{Q}_t = -\psi_q^{nc}\widehat{NFA}_{t-1} - \left[\frac{(1-\gamma)(2\theta\nu-1)-\kappa(2\nu-1)}{\kappa(1+2\nu(\theta-1))}\right](\hat{Y}_{H,t}-\hat{Y}_{F,t}) \tag{45}$$
$$-\frac{\beta}{1-\beta\delta_z}\frac{[4\theta\nu(1-\nu)(1-\gamma)+\kappa(2\nu-1)^2]}{\kappa(1+2\nu(\theta-1))}\frac{\Gamma_3}{\Gamma_1}(\hat{z}_{H,t}-\hat{z}_{F,t})$$
$$+\frac{1-\gamma}{\kappa}(\hat{Y}_{N,t}-\hat{Y}^*_{N,t}) - \frac{(1-\beta)[4\theta\nu(1-\nu)(1-\gamma)+\kappa(2\nu-1)^2]}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}$$

Table 3 gives the signs of the responses of relative consumption and real exchange rate to anticipated and unanticipated endowment shocks under certain parameter restrictions to illustrate how the introduction of demand shocks might affect the comovement of these variables through an adverse valuation channel. We construct Table 3 under the assumptions that $\delta_T = \delta_N = 1$, $\nu > \frac{1}{2}$, $\theta > \theta_1^*$ and $\sigma_Z^2/\sigma_T^2 > RV_1^*$ so that $\tilde{\alpha}_F > 0$ as suggested by the news shocks.[33]

Under the zero-portfolio/non-contingent bond economy solution ($\tilde{\alpha}_F = 0$), real exchange rate and relative consumption are negatively correlated in response to both supply and demand shocks in the tradable sector. As shown in Table 3, $\hat{Q}_t$ is negatively related to $\tilde{\alpha}_F\hat{r}_{x,t}$, which means that the real exchange rate appreciates with an increase in the excess return on portfolio.

Therefore, for a given short position in foreign bonds- as in the case of only unanticipated shocks- an increase in home tradable endowment that depreciates the terms of trade ($\hat{r}_{x,t} \uparrow$), leads to a negative valuation effect ($\tilde{\alpha}_F\hat{r}_{x,t} \downarrow$), which in turn offsets any positive wealth effect that would arise under the non-contingent bond economy in response to this shock and hence improve international risk sharing.

However, if news shocks are sufficiently large, optimal bond portfolio switches sign, i.e. $\tilde{\alpha}_F > 0$, and a positive tradable endowment shock that depreciates the terms of trade, implies a positive wealth transfer to the home agent, ($\tilde{\alpha}_F\hat{r}_{x,t} \uparrow$), which in turn appreciates the real exchange rate even more than it would under the non-contingent bond economy and impede risk sharing. Therefore, for sufficiently large news shocks, real exchange rate and relative consumption are negatively correlated conditional on tradable sector supply shocks as well as demand shocks.

As we explore numerically in the next section, even if news shocks are not large enough to overturn the sign of the optimal portfolio, they can still limit risk sharing conditional on unanticipated endowment shocks by changing the size of the optimal portfolio.

[33] We remind the reader that $\theta > \theta_1^*$ implies $\theta > \theta_3^*$ because $\theta_1^* > \theta_3^*$ for $\nu > \frac{1}{2}$.

4 Numerical analysis in a calibrated two-country, two-sector RBC model

In this section, we calibrate a two-country, two-sector production economy model with capital accumulation along the lines of Benigno and Thoenissen (2008) and Corsetti *et al* (2008) and look at the quantitative implications of introducing a second internationally traded asset for optimal portfolios and relative consumption-real exchange rate correlation alongside standard business cycle moments.

We first describe the model briefly, then proceed to the calibration and the discussion of numerical results under various asset market set-ups when there are only unanticipated sectoral productivity shocks. Numerical results confirm the intuition provided by the analytical results regarding the endowment economy that trade in two international bonds brings the equilibrium closer to complete market equilibrium hence implies too much risk sharing compared to the Backus-Smith-Kollmann evidence. Finally we consider implications of introducing news shocks alongside unanticipated shocks.

4.1 Model

The model we use for quantitative analysis follows closely Benigno and Thoenissen (2008). Each country specialises in the production of a tradable and a non-tradable intermediate good. Final goods are obtained by combining domestic and foreign tradable inputs with domestic non-tradable inputs. All trade between the two countries is in intermediate goods and final goods are only used for domestic consumption. Capital and labour are immobile across countries.

4.1.1 Producers

Final good producers combine home and foreign intermediate goods, C_T and C_N, according to the CES function given by equation (4) to yield the final home consumption good $Y \equiv C$. Tradable intermediate inputs, C_T, are obtained by combining home and foreign intermediates according to (5). The intratemporal elasticity of substitution between tradable and non-tradable inputs is given by κ, while θ governs the substitutability between home and foreign tradable inputs. There is home bias in the demand for tradable inputs, i.e. $\nu > \frac{1}{2}$. Price indices corresponding to final output and the output of tradable goods are given by equations (6) and (7).

Intermediate goods firm in each sector choose labour, capital and investment to maximise the expected discounted value of profits:

$$\max_{K_{i,t+1},L_{i,t},X_{i,t}} E_0 \sum_{t=0}^{\infty} \delta_t \frac{U_C(C_t,(1-L_t))}{U_C(C_0,(1-L_0))} \frac{P_0}{P_t} [P_{i,t}Y_{i,t} - P_t w_t L_{i,t} - P_{H,t} X_{i,t}] \tag{46}$$

subject to the production function in each sector,

$$Y_{i,t} = F(A_{i,t}, K_{i,t-1}, L_{i,t}) = A_{i,t} L_{i,t}^{\alpha_i} K_{i,t-1}^{1-\alpha_i}, \tag{47}$$

where the subscript i, for $i = H, N$,marks variables associated with tradable and non-tradable sectors. Y_i denotes the output in sector i, w_t is the real wage, $X_{i,t}$ denotes investment by intermediate firms producing sector i. A_i denotes sector-specific total factor productivity, L_i and K_i are labour and capital input used in sector i. It is assumed that investment is in units of the domestic tradable good, hence investment price in both sectors is given by P_H. Aggregate capital accumulation equation is:

$$K_t = (1 - \delta) K_{t-1} + X_t \tag{48}$$

Aggregate capital and investment are given simply by $K_t = K_{H,t} + K_{N,t}$ and $X_t = X_{H,t} + X_{N,t}$. Intermediate firms' labour demand functions imply the following wage equation

$$\alpha \frac{P_{H,t}}{P_t} A_t \left(\frac{K_{H,t-1}}{L_{H,t}} \right)^{1-\alpha} = w_t = \alpha_N \frac{P_{N,t}}{P_t} A_{N,t} \left(\frac{K_{N,t-1}}{L_{N,t}} \right)^{1-\alpha_N}$$

while optimal investment is determined simply by:

$$P_{H,t} = E_t m_{t+1} \left\{ P_{i,t+1} MPK_{i,t+1} + P_{H,t+1} (1 - \delta) \right\}, \ i = H, N.$$

where m_t is the stochastic discount factor of domestic agents defined as $m_t = \frac{\beta(C_t, 1-l_t) U_C(C_{t+1}, 1-L_{t+1})}{U_C(C_t, 1-L_t)} \frac{P_t}{P_{t+1}}$.

4.1.2 Consumers

Consumers behave similarly to what is described in the endowment economy. Representative agent in home economy maximizes the expected present discounted value of utility,

$$U_t = E_t \sum_{s=t}^{\infty} \delta_s U(C_s, (1 - L_s)) \tag{49}$$

where utility now depends on leisure, $1 - L$, as well as consumption, C. We modify the endogenous discount factor δ_s accordingly:

$$\delta_{s+1} = \delta_s \beta(C_{As}, 1 - L_{As}), \ \delta_0 = 1 \tag{50}$$

where C_A is aggregate home consumption and L_A is aggregate leisure and $0 < \beta(C_A, 1 - L_A) < 1$. To achieve stationarity under incomplete market specification, we assume $\beta_C(C_A, 1 - L_A) \leq 0$ and $\beta_{1-L}(C_A, 1 - L_A)$.

As before, we solve the model under alternative asset market structures. Consumer's first order conditions and net foreign asset accumulation equations under each market structure is as described in subsection (2.2), where marginal utility functions are adjusted accordingly, i.e. $U_C(C)$ is replaced by $U_C(C, 1-L)$ and net foreign asset accumulation equations are modified to account the fact that agents also spend their income on investment, $P_{H,t}X_t$. In addition to optimal consumption and portfolio decisions characterised by the first order conditions given in subsection (2.2), there is an optimal labour supply decision given by:

$$w_t = \frac{u_{1-L}\left(C_t, (1-L_t)\right)}{u_C\left(C_t, (1-L_t)\right)}$$

Similar equations hold for the foreign country.

4.1.3 Market clearing

Market clearing for intermediate goods requires:

$$\begin{array}{ll} Y_{H,t} = F(A_{H,t}, L_{H,t}, K_{H,t-1}) = C_{H,t} + C^*_{H,t} + X_t & Y_{N,t} = F(A_{N,t}, L_{N,t}, K_{N,t-1}) = C_{N,t} \\ Y_{F,t} = F(A_{F,t}, L_{F,t}, K_{F,t-1}) = C^*_{F,t} + C_{F,t} + X^*_t & Y^*_{N,t} = F(A^*_{N,t}, L^*_{N,t}, K^*_{N,t-1}) = C^*_{N,t} \end{array}$$

while for final goods we have $Y_t = C_t$ and $Y^*_t = C^*_t$.

Factor market clearing implies,

$$\begin{array}{ll} L_H + L_N = L & L_F + L^*_N = L^* \\ K_H + K_N = K & K_F + K^*_N = K^* \end{array}$$

while asset market clearing is as described before for the endowment economy. We close the model by two different policy rules as before.

4.2 Calibration

We calibrate the model along the lines of BT and CDL assuming symmetry across countries. Our baseline calibration is given by Table 4. Most of the paramater values are the same as the ones used by BT. We are considering three different trade elasticity values, i.e. $\theta = 0.25, 2.5, 8$, to discuss how the introduction of a second internationally traded asset affects each of the transmission mechanisms that can account for the anomaly when there is only one internationally traded bond.

Following BT and CDL and most of the international RBC literature, we assume that preferences are non-separable in consumption and leisure. We use specification used by Backus *et al* (1992) and CDL:[34]

$$U(C, 1-l) = \frac{\left[C^{\omega}(1-l)^{1-\omega}\right]^{1-\rho} - 1}{1-\rho}, \quad 0 < \omega < 1, \ \rho > 0, \tag{51}$$

We calibrate the consumption share in utility, ω, such that at the steady-state, agents devote one-third of time to work. Risk aversion parameter is equal to 2. As in CDL, we specify the endogenous discount factor in line with the period utility function.

$$\beta(C, 1-l) = \frac{1}{1 + \psi[C^{\omega}(1-l)^{1-\omega}]}$$

where we set the Uzawa convergence parameter, ψ, such that the steady state discount factor, $\overline{\beta}$, is equal to 1/1.04, consistent with a steady-state real interest rate of 4% per year.

We set the parameters pertaining to the consumption basket in the following way. The share of tradable goods in final consumption, γ, is 0.55, while the share of home goods in tradable consumption, ν, is 0.72. The calibration of this parameter is the same across both BT and CDL.

We assume an elasticity of substitution between traded and non-traded goods, κ, of 0.44, as suggested by Stockman and Tesar (1995) and adopted by BT[35]. For $\rho = 2$, this implies that utility is non-separable between traded and non-traded goods. Given that $\kappa\rho < 1$, our benchmark calibration implies traded and non-traded goods are complements.

The share of labour input in the production of tradable and non-tradable intermediates are set equal to each other at $\alpha_H = \alpha_N = 0.67$ and the rate of depreciation of capital is set to 10% per annum.

In calibrating the processes for tradable and non-tradable sector productivity shocks, we mainly rely on BT, who estimate these processes for the US relative to EU15 and Japan using annual data between 1979-2002. We calibrate the persistence of tradable sector productivity shocks slightly higher to 0.88 (BT calibration sets it to 0.84) while keeping the rest of the calibration as in their paper.[36] The persistence of non-tradable productivity shocks are set to 0.30 and tradable sector

[34] BT calibrates the utility function as in Stockman and Tesar (1995) who use the following form:

$$U(C, 1-l) = \frac{C^{1-\rho}(1-l)^{\eta}}{1-\rho}$$

[35] CDL use a higher value of $\kappa = 0.74$ following Mendoza (1991). Ostry and Reinhart (1992) estimate this parameter to be higher in the range of 0.66-1.44. We provide a sensitivity analysis with respect to κ later in the paper.

[36] The utility function used by BT following Stockman and Tesar (1995), implies a slightly higher volatility of relative consumption compared to the utility function we use here. This in turn yields somewhat lower consumption-real exchange rate correlations for a given shock calibration. To make-up for this difference between the two preference specifications, we slightly increase the persistence of tradable sector shocks to make the wealth effects of these shocks more important and to emphasise their mechanism. (See Baxter and Crucini (1995) and Baxter (1995) on how higher

shocks are bigger than non-tradable sector shocks, with a variance-covariance matrix given in Table 1.

After solving the model in terms of the state variables, we use the autoregressive processes for the shocks to generate simulated time series of length T (T=600) for the variables of interest. We repeat this procedure J (J =200) times and then compute the average of the moments from logged and HP-filtered series excluding the first 100 periods of simulation.

4.3 Results with unanticipated productivity shocks

We first look at the performance of the model in a single bond set-up when there are only unanticipated sectoral productivity shocks in line with BT and CDL. As shown by our analytical results for the endowment economy version of this model, the comovement between relative consumption, real exchange rate and its components depends crucially on the value of the trade elasticity, θ. Figure 2 shows the impact responses of real exchange rate, terms of trade, relative price of non-tradables and relative consumption conditional on a 1% increase in tradable sector productivity in the non-contingent bond economy for different values of the trade elasticity parameter, θ.[37] There are six different regions of θ (divided by vertical lines and colored in white and grey to ease identification), which imply different signs of comovement between relative consumption and relative prices on impact. The upper panel shows four of these six different regions that lie to the left of $\theta = 1$ and the lower panel shows the last two regions that cover values of θ greater than 1.

Regions of trade elasticity that we focus on for our calibration are regions I, V and VI, which all imply an increase in relative consumption and an appreciation in the real exchange rate following an increase in tradable sector productivity- implying a negative conditional correlation on impact. CDL emphasises regions I and VI, while BT analysis is valid for region V where θ takes values between 0.93 and 4.6 when parameters other than θ are calibrated according to Table 4.

In section (3.2), we explain the different transmission mechanisms that occur when θ itakes values in regions I and V. The intuition is similar for production economies hence we do not repeat it here. But it is worth to say a few words about the transmission mechanism that occurs in region VI. As CDL explain, for very high degrees of substitutability between home and foreign goods, a sufficiently persistent shock can increase the relative wealth of domestic agents such that in the

shock persistence makes market incompleteness more important in international RBC models).

[37]Impact responses to a non-tradable sector productivity shock do not yield a negative transmission between relative consumption and real exchange rate except for a very limited range of low θ parameters (for θ between 0.31 and 0.36 a positive NT shock appreciates the real exchange rate by appreciating the terms of trade while increases relative consumption at home). Figures are available from authors on request.

short-run the increase in the demand for home goods exceeds the increase in the output, which peaks later due to the dynamics of capital. Hence, terms of trade appreciates on impact, while relative consumption increases. However, terms of trade appreciation in this region is quite limited compared to that in region I.[38]

Next, we briefly discuss how the different transmission mechanisms highlighted in Figure 2 reflect into Backus-Smith correlations and other second moments. Table 5 reports various business cycle statistics for three different values of θ belonging to regions I, V and VI under alternative asset markets. Results for the non-contingent bond economy are given in the first column of each θ panel in Table 5.

Region V: Benchmark calibration

Naturally, the business cycle statistics that we obtain under the calibration with $\theta = 2.5$ are similar to those reported by BT.[39] The model is able to generate a negative cross-correlation between relative consumption and real exchange rate that is around -0.07. Comparing this with a correlation of 0.76 which arises under complete markets (fourth column of first panel in Table 5) shows that market incompleteness really matters in this set-up.[40] The mechanism that generates the negative correlation between relative consumption and real exchange rate for this calibration also implies a negative correlation between the real exchange rate and terms of trade. This is because real exchange rate and terms of trade move in opposite directions in response to a tradable productivity shock for values of θ inside region V as depicted in Figure 2 and tradable sector shocks are dominant in driving the business cycle according to our calibration.

An apparent drawback is the low volatility and persistence of the real exchange rate. Because the law of one price holds for traded goods, only sources of volatility in real exchange rate are the fluctuations in terms of trade and relative price of non-tradables. Due to a relatively high value of trade elasticity, terms of trade volatility is limited. Although large wealth effects that are present under incomplete markets make relative non-tradables prices more volatile compared to complete markets, this effect does not raise real exchange rate volatility much.

The model for this calibration cannot account for the quantity puzzle, which refers to the failure

[38] Note that the two mechanisms that are highlighted in CDL would still be present in a one-sector model with only tradable goods as they rely on the role of the terms of trade in generating a negative correlation between relative consumption and real exchange rate.

[39] Although the model and calibration we use here are in the same spirit as BT, they are not equivalent. For example, we specify a different utility function, we use endogenous discount factor to make the model stationary and we set investment adjustment costs to zero since the volatility of investment relative to GDP is already around 3 without any adjustment costs in the non-contingent bond economy.

[40] The fact that the consumption-real exchange rate correlation is below unity under complete markets is due to the non-separability of consumption and leisure in the utility function.

of a general class of international RBC models in generating higher cross-country correlations between GDPs compared to consumption levels. Comparing the first and fourth columns of the first panel of Table 5 shows that market incompleteness goes in the right way as it reduces the cross-country consumption correlations with respect to complete markets, but it is not sufficient to account for the puzzle.[41] Also, net exports, which are countercyclical in the data, are weakly procyclical for $\theta = 2.5$ in the non-contingent bond set-up. Large wealth effects following a tradable sector productivity shock increase the demand for imported goods at home but the complementarity between tradable and non-tradables limits this demand to some extent as non-tradables supply is fixed. This in turn, makes it harder for the model to generate countercyclical net exports.[42]

Region I: Low trade elasticity

The calibration with $\theta = 0.25$ yields a large negative correlation (-0.90) between relative consumption and real exchange rate in the non-contingent bond economy (see first column of panel 2 in Table 5) in line with the transmission mechanism highlighted in the first region depicted in Figure 2. The correlation between the terms of trade and real exchange rate shoots up to 0.98, which is quite high compared to 0.32 implied by the data.

The non-contingent bond economy with low trade elasticity performs better than that with $\theta = 2.5$ in terms of real exchange rate volatility, though volatility still remains quite below its empirical counterpart. With low trade elasticity, the cross correlation between home and foreign consumption is lower than that of home and foreign GDP, but it is negative, which is not supported by the data. Also with low θ, net exports become strongly countercyclical mainly due to large terms of trade appreciation that makes imports more expensive during good times.

Region VI: High trade elasticity

The terms of trade appreciation for θ belonging to region VI is much more limited compared to the terms of trade appreciation for θ belonging to region I (See Figure 2). This leads to a Backus-Smith-Kollmann correlation of around -0.28, which is more in line with the data than -0.90 implied by $\theta = 0.25$. Also, the fact that the terms of trade depreciates over the long-run for high θ implies a more realistic real exchange rate-terms of trade correlation (0.18) compared to the other two trade elasticity parameters. However, high trade elasticity makes the quantity puzzle much worse, resulting in a much higher correlation between home and foreign consumptions than home and foreign GDPs. It also leads to a counterfactual negative correlation between home and foreign investment.

[41] CDL show that modelling distribution sector can account for the quantity puzzle whether risk sharing is complete or not. It also increases the volatility of terms of trade and real exchange rate.

[42] Indeed, changing the value of κ to 0.83 reduces the correlation of net exports and GDP to 0.02, while the cross-correlation between relative consumption and real exchange rate becomes -0.09.

4.3.1 Implications of a second internationally traded bond for international risk sharing and business cycles

As we discussed before, portfolio choice affects international risk sharing and transmission of shocks through the valuation effect that enters net foreign asset accumulation. Using the goods market clearing conditions and approximating up to first order, change in the net foreign asset position can be written as:

$$\underbrace{\widehat{NFA}_t - \frac{1}{\beta}\widehat{NFA}_{t-1}}_{\Delta NFA_t} = \underbrace{\frac{\bar{C}_H^*}{\bar{Y}}(\hat{P}_{H,t} - \hat{P}_t + \hat{C}_{H,t}^*) - \left(\frac{\bar{C}_F}{\bar{Y}}(\hat{P}_{F,t} - \hat{P}_t + \hat{C}_{F,t})\right)}_{CA_t} + \underbrace{\tilde{\alpha}_F(\hat{r}_{x,t})}_{VAL_t} + O(\varepsilon^2) \quad (52)$$

For the level of approximation we use here, valuation effect is given by the excess return on the steady-state foreign bond portfolio. We know that the steady-state portfolio is determined by the orthogonality condition given by (17). For the utility function specified in (51), this condition can be written as:

$$Cov_t\left[\left(a_1(\hat{C}_{t+1} - \hat{C}_{t+1}^*) + a_2(\hat{l}_{t+1} - \hat{l}_{t+1}^*) - \hat{Q}_{t+1}\right)\hat{r}_{x,t+1}\right] = 0 + O(\varepsilon^3) \quad (53)$$

where $a_1 \equiv 1-\omega(1-\rho)$ and $a_2 \equiv (1-\omega)(1-\rho)\frac{\bar{l}}{1-\bar{l}}$ and $a_1 > 0$, $a_2 < 0$ for $\rho > 1$. Thus, hedging against fluctuations in relative marginal utilities of consumption means hedging against fluctuations in relative consumption and relative labour supplies adjusted by the real exchange rate. It is optimal to have a long position in foreign bonds, if the excess on foreign bond, $\hat{r}_{x,t}$, is higher when consumption is lower in the home country and/or when total hours worked is higher in the home country. Excess returns are determined according to policy rules as described in equations (29) and (35).

Region V: Benchmark calibration

First, consider the baseline calibration with $\theta = 2.5$. To understand the equilibrium portfolio position, it is useful to analyse the components of equation (53), namely the response of relative marginal utilities of consumption adjusted by the real exchange rate and excess return under the zero-portfolio solution (non-contingent bond economy). Figure 3 plots the impulse responses of these variables for $\theta = 2.5$ under four different asset market structures.

For now, let us just focus on the straight line that depicts the non-contingent bond economy solution (NC economy) to understand the equilibrium portfolio. Following a positive tradable sector shock in the home country, relative consumption and hours worked increase. Home agents work more compared to foreign agents because wages are higher in the home country following the increase in productivity. While the increase in relative consumption implies a fall in relative marginal utility, the increase in relative labour effort implies a rise, limiting the overall fall in relative marginal utility on impact.[43] Given the dynamics of the real exchange rate and the terms of trade under $\theta = 2.5$, which we explain in detail above, hedging against tradable sector shocks require a long position in foreign bonds under CPI stabilisation, but a short position when domestic tradable prices are stabilised.

What is the optimal hedge against non-tradable sector shocks? The lower panel of Figure 3 shows that shocks to non-tradable sector productivity do not generate large deviations from the efficient risk sharing condition, i.e. the response of relative marginal utilities of consumption to a non-tradable sector under the non-contingent bond economy is close to that under the complete markets. Therefore, optimal hedge against these shocks is a near-zero portfolio. This creates a tension in the determination of equilibrium portfolio. As our calibration gives a larger weight to tradable sector shocks, equilibrium portfolios would be biased towards hedging against tradable sector shocks. But depending on the strength of the response of excess returns to a non-tradable sector shock, a portfolio that is a good hedge against tradable sector shocks can be a bad hedge against non-tradable sector shocks, which in turn would limit the size of the portfolio and impede risk-sharing conditional on both shocks.

Table 5 reports the optimal foreign currency bond position as a share of GDP along with other business cycle statistics for the two bonds economy under the two policy rules we consider (columns 2 and 3 in the first panel of Table 5). Under CPI stabilisation, the model implies a large long position in foreign bonds (around 6.6 times GDP) and a positive but low consumption-real exchange rate correlation around 0.19. We can see from the second column of the first panel of Table 5 that the partial insurance provided by this trading opportunity limits the volatility of relative non-tradables price and the volatility of the real exchange rate compared to the non-contingent bond set-up. Nevertheless, Balassa-Samuelson effect still operates to some extent as we can see from the negative correlation between real exchange rate and terms of trade and the negative correlation between relative non-tradables price and relative consumption.

Figure 3 shows that the impulse responses to a tradable sector shock in this case (labelled by 2 bonds (rx=Q)) lies in between the impulse responses of the NC economy and complete markets, highlighting the partial insurance against tradable shocks. But, interestingly, impulse responses in the lower panel of Figure 3 show that, having access to two international bonds makes the fluctuations in relative marginal utilities of consumption even larger than they are under the non-contingent bond economy conditional on non-tradable sector shocks. Hence, the lower panel of Figure 3 illustrates very nicely how valuation effects can actually go in the wrong way when market incompleteness matters.

[43] Hence in this case, non-separability of consumption and leisure limits the size of the total risk to be hedged.

Under domestic tradable price stabilisation, the model implies an equally large short position in foreign bonds, but a high consumption-real exchange rate correlation (0.74) which is very close to the correlation implied by complete markets (0.76). Indeed, comparing columns 3 and 4 of the first panel in Table 5 shows that allowing agents to have claims to the terms of trade almost completes the markets despite the fact that relative marginal utilities of consumption are subject to two different sources of risk (relative T and NT productivity shocks). Also, Figure 3 shows how the impulse responses obtained under this set-up (labelled by 2 bonds (rx=TOT)), sit on top of the complete market impulses for both shocks. Hence we confirm the intuition provided by the analytical results within the context of a more general production economy. This result is interesting as it shows that risk sharing can be higher when bonds cannot load on all sources of uncertainty in the economy.

Region I: Low trade elasticity

The result that trade in bonds under tradable price stabilisation almost completes markets also holds here (compare the third and fourth columns of the second panel in Table 5). What is more, trade in bonds implies a high positive correlation between relative consumption and real exchange rate also under the CPI stabilisation.

Figure 4 shows impulse responses to tradable and non-tradable productivity shocks for $\theta = 0.25$. Again, focus on the plots for the non-contingent bond economy to understand the portfolio implications of the model. For this calibration, home terms of trade appreciates on impact following both sectoral shocks, which in turn strengthens the increase in the relative wealth of home agents compared to the calibration with $\theta = 2.5$. This means that the marginal utility gap (the deviation from efficient risk sharing) is bigger under low θ for both shocks, i.e. there is more risk to be shared through the bond portfolio for low θ.

In fact, for low θ, hedging against non-tradable shocks also requires a non-zero portfolio. This is because when tradable and non-tradable goods are complements, an increase in non-tradable goods consumption goes hand in hand with an increase in the demand for tradables. Given that tradable consumption is home biased and trade elasticity is low, this increased demand for tradables given an initally fixed supply leads to a home terms of trade appreciation. This, in turn, leads to higher wealth in the domestic country and widens the gap between the marginal utilities of consumption across the two countries conditional on non-tradable shocks.

When excess return on bonds is given by the real exchange rate, hedging against tradable sector shocks implies a long position in foreign bonds whereas hedging against non-tradable sector shocks implies a short position. This is because real exchange rate appreciates in response to a positive tradable sector shock that lowers relative marginal utility of consumption at home but depreciates in response to a positive non-tradable sector that affects relative marginal utility in a similar way (see Figure 4). The resulting portfolio is a long position in foreign bonds around 6 times the GDP, which is comparable to that obtained under $\theta = 2.5$.

On the other hand, when nominal bonds give claims to the terms of trade, it is optimal to have a long position in foreign bonds to hedge against both sources of shocks because for each shock, home terms of trade appreciates (foreign bonds pay less) precisely when marginal utility is lower in the home country. Thus, optimal portfolio switches sign compared to the case of $\theta = 2.5$ and shrinks in size to 1.7 as a share of GDP (Since terms of trade volatility is higher with low θ, a smaller portfolio can achieve higher risk sharing). Despite the smaller portfolio position, consumption-real exchange rate correlation goes up to 0.97, which is close to the value under complete markets (see the second panel of Table 5).

It is interesting to note that for low elasticity values, impulse responses to tradable sector shocks with trade in two bonds under *both* policy rules are almost identical to those under complete markets. The main difference in the risk sharing implications of bonds across the two policy rules is with regards to non-tradable sector shocks: Tilting the bond portfolio towards tradable sector shocks, implies larger unwanted valuation effects in response to non-tradable sector shocks under CPI stabilisation (Figure 4). But this is not enough to generate a low consumption-real exchange correlation. These results suggest that it is actually harder to account for the consumption-real exchange rate anomaly in the presence of endogenous portfolio choice when θ is low.

Region VI: High trade elasticity

As in the case of trade elasticities belonging to regions I and V, trade in home and foreign bonds under domestic tradable price stabilisation brings the model very close to the complete market outcome also in region VI (see the last panel of Table 5). Trade in bonds under CPI stabilisation leads to a consumption-real exchange rate correlation of around 0.27 which is lower than what is implied by trade in bonds under tradable PPI stabilisation but still higher than the empirical counterpart. Not surprisingly, the implied portfolio positions are extreme and are far from matching the data just as the Backus-Smith-Kollmann correlations are. The fact that the terms of trade volatility falls dramatically with high trade elasticity means that agents should hold a much larger foreign currency position to ensure a given degree of risk sharing.[44]

[44] There is a special case where θ is set such that the terms of trade response to a tradable sector productivity shock is almost zero which means that relative bond returns cannot load on the relative consumption risk created by relative tradable sector shocks. For our calibration this occurs for values of θ between 4 and 5 as can be seen from Figure 2. In particular, for $\theta = 4.6$, relative consumption-real exchange rate correlation is around -0.20 both in the non-contingent bond economy and the two bonds economy with tradable PPI stabilisation, whereas the implied foreign bond position as a share of GDP is -73.

4.3.2 Sensitivity analysis

Our finding that trade in nominal bonds ensures too much risk sharing is robust to different calibrations of key parameters. We already discuss the role of trade elasticity, θ, for optimal portfolios and degree of risk sharing with reference to Figures 2, 3, 4 and Table 5. In Figure 5, we plot consumption-real exchange rate correlation alongside optimal foreign bond positions for different values of intratemporal elasticity of substitution between tradables and non-tradables, κ, under alternative asset market and policy combinations. As mentioned in section 4.2, values of κ generally used in the literature varies between 0.44 and 1.44. In this range, the non-contingent bond set-up yields a negative consumption-real exchange rate correlation. For high values of κ, i.e. for κ larger than 3, relative price of non-tradable goods adjusts less in response to supply shocks hence the correlation turns positive even in the absence of any portfolio choice. The foreign bond portfolio as a share of GDP is quite sensitive to κ when excess return is given by the real exchange rate. For high values of κ, real exchange rate depreciates with respect to a positive tradable sector shock, while relative consumption increases. Hence it becomes optimal to have a short position in foreign bonds rather than a long position. On the other hand, κ has a limited impact on the dynamics of the terms of trade and hence on optimal portfolio under domestic tradable price stabilisation. Under this policy rule, trade in bonds yields a consumption-real exchange rate cross-correlation that is very close to the complete market outcome regardless of the value of κ.

Figure 6 analyses the effects of varying the share of non-traded goods in the consumption of final goods, γ. For very low values of γ, consumption-real exchange rate correlation is very high because most of the final goods are non-tradable and relative price of non-tradable goods moves in a way to offset the changes in the relative supply of non-tradables as we mention above. As γ increases, tradable sector shocks become more important hence we get the mechanism that generates the negative correlation between relative consumption and real exchange rate. As γ becomes very high, the Balassa-Samuelson effect diminishes and correlation picks up again. This U-shaped pattern is valid for all asset market structures. For any value of γ, trade in bonds complete the markets when excess returns are given by the terms of trade while correlations implied by trade in bonds under CPI stabilisation are closer to those that arise with trade in a single non-contingent bond. Equilibrium portfolios increase in absolute value as the share of tradable goods increases. When γ is close to 1, real exchange rate is determined mainly by the terms of trade hence the optimal bond portfolio under CPI stabilisation also becomes negative.

In Figure 7, we present sensitivity analysis with respect to different values of home bias in consumption, ν. The cross-correlation rises after a certain value of consumption home bias. Optimal foreign currency portfolio approaches to zero as ν approaches to 1, i..e complete home bias. Figure 8 repeats this exercise for the relative variance of non-tradable sector shocks with respect to tradable sector shocks. As we increase the relative size of non-tradable shocks, cross-correlation increases under all asset market structures. Optimal foreign currency position falls under CPI stabilisation but it is not affected under domestic tradable PPI stabilisation because terms of trade does not respond significantly to non-tradable sector shocks.

4.4 Results with anticipated productivity shocks

Next, we analyse the consequences of introducing news shocks alongside unanticipated productivity shocks in tradable and non-tradable sectors. As we discussed before in the analytical section, news about future productivity work as a typical demand shock, increasing consumption and prices at the same time. Therefore, relative consumption and real exchange rate would generally be negatively correlated conditional on news shocks, which would potentially help in accounting for the anomaly in the presence of some endogenous portfolio choice.[45] We are mainly interested in the effect of news shocks on optimal portfolios and risk sharing. Provided that anticipated and unanticipated shocks pull the equilibrium portfolio towards different directions, we can generate a meaningful market incompleteness to account for the anomaly.

We specify the exogenous processes for sectoral productivity shocks that incorporate news as follows:

$$\log A_{H,t} = \delta_T \log A_{H,t-1} + \log z_{H,t-1} + u_{H,t}, \quad \log A_{N,t} = \delta_N \log A_{N,t-1} + \log z_{N,t-1} + u_{N,t}$$
$$\log A_{F,t} = \delta_T \log A_{F,t-1} + \log z_{F,t-1} + u_{F,t}, \quad \log A_{N^*,t} = \delta_N \log A_{N^*,t-1} + \log z_{N^*,t-1} + u_{N^*,t}$$
$$\log z_{i,t} = \delta_{z_i} \log z_{i,t-1} + u_{Zi,t} \text{ for } i = H, F, N, N^*$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_N < 1$, $0 \leq \delta_z < 1$. We first consider a calibration where news shocks are persistent and small which is along the lines of Croce and Colacito (2010). Table 6 reports the business cycle statistics obtained from a model which is calibrated according to Table 4 for different values of trade elasticities, where persistence of news to tradable and non-tradable sector productivity are set equal to the persistence of unanticipated productivity shocks in these sectors, i.e. $\delta_{z_H} = \delta_{z_F} = \delta_T = 0.88$, $\delta_{z_N} = \delta_{z_{N^*}} = \delta_N = 0.30$ and the relative variance of news to

[45] Opazo (2006) looks at the role of expectation shocks in accounting for the Backus-Smith puzzle in a single bond economy with only tradable goods.

unanticipated shocks in each sector is 0.01, i.e. $\sigma^2_{z_H}/\sigma^2_T = \sigma^2_{z_F}/\sigma^{*2}_T = \sigma^2_{z_N}/\sigma^2_N = \sigma^2_{z_{N^*}}/\sigma^{*2}_N = 0.01$.

Comparing Table 6 with Table 5 for $\theta = 2.5$, shows that small and persistent news shocks make the consumption-real exchange rate correlation more negative, -0.16, under the non-contingent bond economy without worsening the model's performance to fit other business cycle statistics. In fact, introduction of news shocks makes the model more compatible with the data as it turns the correlation between the real exchange rate and terms of trade from negative to positive and reduces the correlation between terms of trade and relative consumption. Because news shocks are small in our calibration, they do not reduce the comovement of consumption, investment and hours worked with GDP in a significant way.

In line with our intuition and the analytical results presented before, introducing news shocks does not change the risk sharing properties of bonds under CPI stabilisation whereas it makes a big difference under domestic tradable price stabilisation. This is because under the latter, excess return is given by the terms of trade, which covaries negatively with relative consumption risk conditional on anticipated shocks, but positively conditional on unanticipated shocks. This tension makes the short position in foreign currency smaller and implies a negative consumption-real exchange rate correlation of -0.08. Hence, in the presence of small and persistent news shocks, trade in bonds that give claims to terms of trade can no longer replicate the complete market outcome.

As the second and third panels of Table 6 shows, news shocks are more effective for $\theta = 2.5$ (or in general for θ belonging to region V), because under $\theta = 0.25$ and $\theta = 8$, unanticipated shocks to tradable sector productivity affect the terms of trade in a similar way to news shocks, i.e. they also work as demand shocks, hence news shocks cannot reduce consumption-real exchange rate correlation to low levels with endogenous trade in bonds.

For larger news shocks, optimal foreign currency position switches sign under tradable price stabilisation, i.e. it becomes optimal to have a long position in foreign currency rather than a short position, and consumption-real exchange rate correlation becomes more negative but this comes at the cost of creating too much volatility in GDP. Sensitivity analysis with respect to the variance and persistence of news shocks are available from authors upon request.

5 Conclusion

In this paper, we review and compare different mechanisms that rely on good market frictions and market incompleteness to account for the consumption-real exchange rate anomaly. We show that the performance of these models worsen considerably when we move away from a single bond economy and allow for ex-ante risk sharing in the form of home and foreign currency bonds. Irrespective of the value of trade elasticity, relative consumption-real exchange rate correlations increase dramatically to the values implied by complete markets when agents can trade in bonds which give claims to the terms of trade. Although trade in bonds leads to less risk sharing when relative bond returns are given by the real exchange rate, correlations implied by this asset-market and policy combination are much higher than that in the data. A common characteristic of optimal portfolios among different policies and trade elasticity values is that they are implausibly large. Therefore, two-sector models with sectoral productivity shocks fail in both generating realistic portfolio positions and a low degree of risk sharing when we allow for porfolio choice between two assets.

We explore the role of news shocks in generating meaningful market incompleteness in the presence of endogenous portfolio choice and show that only under certain trade elasticity and policy combinations anticipated and unanticipated shocks can create a significant tension on equilibrium bond portfolios and reduce the degree of risk sharing implied by bonds.

Our work suggests that allowing for more sources of uncertainty can potentially improve the performance of this class of models in accounting for the consumption-real exchange rate anomaly while generating realistic portfolio positions provided that they satisfy certain conditions. First of all, these additional shocks should imply a low correlation between relative consumption and real exchange rate in the zero-portfolio solution (non-contingent bond economy) to start with. Because, as long as optimal portfolios are chosen to minimise deviations from risk sharing as in our set-up and most of the recent portfolio literature, the unconditional correlation between relative consumption and real exchange rate in the presence of endogenous portfolio cannot be lower than the non-contingent bond economy outcome. Secondly, different shocks should pull portfolios towards different directions. If hedging against all sources of uncertainty in the model require a similar portfolio position, risk sharing would be high even if there are fewer assets than shocks. Finally, these additional shocks should be empirically relevant and should not have counterfactual implications for other business cycle statistics. Our experiments with other shocks such as i-pod shocks and investment shocks suggest that finding shocks that satisfy these properties is a tedious task that might not have much value-added.

Having said that, one direction for further research might be to introduce portfolio choice in an estimated DSGE model with many shocks and look at the portfolio implications and consumption-real exchange rate correlations in such a set-up. Another direction is to introduce asset market imperfections alongside market incompleteness to limit asset trade and the degree of risk sharing as in Kollmann (2009). Our experience with exogenously specified transaction costs along the lines of Tille and van Wincoop (2007) show that transaction costs should be very large for this class of models to match the observed portfolios alongside a negative correlation between relative consumption and real exchange rate. But, certainly this is an avenue that needs to be investigated more thoroughly.

Another direction we can go to account for the low consumption-real exchange rate anomaly while matching the international portfolios that we observe in the data is to look at alternative explanations of the anomaly that do not rely on market incompleteness, but on non-separable preferences. Raffo (2010), Karabarbounis (2010), Stathopoulos (2010) and Croce and Colacito (2010) are examples to papers that follow this approach without considering portfolio choice. These models suggest that relative consumption and real exchange rate can be negatively correlated under complete markets. This strand of literature can be reconciled with the general equilibrium portfolio literature that is successful in accounting for the observed portfolio positions in models which do not display large deviations from risk sharing.

6 Tables and Figures

Table 1: International portfolios and relative consumption-RER correlations (vis-a-vis US) for selected industrial countries

	Financial Globalisation (A+L)/GDP		Net FC exposure as % of GDP		Equity Home Bias		Cor(C-C^{US},Q) Hp-filtered		Cor(C-C^{US}, Q) First-differenced	
	1990	**2004**	**1990**	**2004**	**1990**	**2004**	**1970-1990**	**1991-2006**	**1970-1990**	**1991-2006**
Australia	95.5	218.4	-2.7	17.6	0.88	0.84	-0.26	-0.80	-0.13	-0.62
Austria	130.3	387.9	-7.8	-28.7	0.74	0.44	0.01	-0.65	-0.07	-0.19
Belgium	394.8	802.5	36.9	24.9	0.64	0.57	-0.17	-0.28	-0.12	0.02
Canada	122.1	211.1	-2.2	52.9	0.79	0.72	-0.53	-0.70	-0.16	-0.52
Denmark	195.8	398.7	-27.7	56.4	0.89	0.62	-0.08	-0.59	-0.24	-0.28
Finland	92.1	396.1	-23.5	50.3	0.99	0.61	-0.27	-0.53	-0.06	-0.63
France	128.5	415.1	17.3	37.0	0.86	0.73	-0.13	-0.37	-0.11	-0.27
Germany	118.6	325.6	18.7	19.2	0.85	0.52	-0.32	-0.28	-0.34	0.00
Greece	74.2	194.0	-9.6	10.4		0.95	-0.32	-0.76	-0.13	-0.57
Italy	73.9	222.5	-2.4	9.9	0.84	0.59	-0.12	-0.48	-0.04	-0.32
Japan	111.7	141.9	10.3	58.1	0.97	0.87	0.14	-0.23	0.19	-0.08
Netherlands	260.0	767.4	59.2	87.8	0.65	0.32	-0.45	0.59	-0.41	0.40
New Zealand	133.6	224.8	-27.0	-19.2		0.66	-0.15	-0.92	-0.18	-0.91
Norway	110.1	337.8	3.2	103.8	0.90	0.52	0.19	-0.39	0.01	-0.29
Portugal	85.3	404.0	13.2	2.1	0.89	0.66	-0.60	-0.19	-0.56	0.01
Spain	62.7	285.0	12.3	7.1	0.97	0.85	-0.64	-0.55	-0.45	-0.42
Sweden	147.8	422.8	-11.6	95.1	0.89	0.58	-0.55	-0.43	-0.28	-0.45
Switzerland	378.1	956.6	119.3	317.2	0.66	0.51	0.09	-0.29	0.06	-0.02
UK	349.0	713.3	52.1	99.5	0.77	0.65	-0.56	-0.05	-0.51	0.10
US	80.1	192.2	14.9	46.8	0.90	0.74				
Median	**120.4**	**362.8**	**6.8**	**41.9**	**0.87**	**0.64**	**-0.26**	**-0.43**	**-0.13**	**-0.28**

Source: Portfolio data are from Lane and Milesi-Ferretti (2006), Lane and Shambaugh (2010), CPIS, GFD and authors' calculations. Consumption, exchange rates and prices are from OECD Outlook Database, consumption is real private consumption index (2000=100) and real exchange rates are constructed using consumer price indices.

Table 2: Definitions of some of the convoluted parameters used in text

$$\Gamma_1 \equiv 4\theta\nu(1-\nu)(1+\gamma(\kappa\rho-1)) + \kappa(2\nu-1)^2 > 0$$
$$\Gamma_2 \equiv (1-\gamma)(\kappa(2\nu-1)^2 + 4\theta\nu(1-\nu)) > 0$$
$$\Gamma_3 \equiv (2\theta\nu-1)(\gamma\kappa\rho+1-\gamma) - \kappa(2\nu-1) > 0 \text{ iff } \theta > \theta_3^*$$
$$\Gamma_4 \equiv (\gamma\kappa\rho+1-\gamma)\left(\theta^2 + (\theta-1)^2\frac{\sigma_T^2}{\sigma_N^2}\right) > 0$$
$$\theta_1^* \equiv \frac{1}{2\nu} + \frac{\kappa}{1-\gamma}\frac{2\nu-1}{2\nu}$$
$$\theta_2^* \equiv \frac{1}{2\nu} + \kappa\frac{2\nu-1}{2\nu},\ \theta_2^* < \theta_1^* \text{ given that } 1-\gamma < 1.$$
$$\theta_3^* \equiv \frac{1}{2\nu} + \frac{\kappa}{\gamma\kappa\rho+1-\gamma}\frac{2\nu-1}{2\nu},\ \theta_3^* < \theta_1^* \text{ for } \nu > \frac{1}{2}$$
$$RV_1^* \equiv \frac{\Gamma_1(1-\beta\delta_z)^2}{(2\nu-1)\Gamma_3\beta^2}$$

Table 3: Relative consumption and real exchange rate responses to supply and demand (news) shocks with endogenous portfolio choice

Assumptions: $\nu > \frac{1}{2},\ \theta > \theta_1^*, \tilde{\alpha}_F > 0$	*Solution in terms of shocks and the portfolio valuation effect*		*Portfolio valuation effect*
	$\hat{C}_t$-$\hat{C}_t^*$	$\hat{Q}_t$	$\tilde{\alpha}_F \hat{r}_{x,t}$
$\hat{Y}_{H,t} - \hat{Y}_{F,t}$	+	-	+
$\hat{z}_{H,t} - \hat{z}_{F,t}$	+	-	-
$\hat{Y}_{N,t} - \hat{Y}^*_{N,t}$	+	+	*0*
$\tilde{\alpha}_F \hat{r}_{x,t}$	+	-	*NA*

Note: Table 3 is constructed under the assumptions that $\nu > \frac{1}{2}$, $\theta > \theta_1^*$ *and* $\sigma_Z^2/\sigma_T^2 > RV_1^*$ *which ensures that* $\tilde{\alpha}_F > 0$.

Table 4: Baseline calibration

Parameter	Description	Baseline values
$\bar{\beta} = \omega \bar{C}^{-\eta}$	Steady-state discount factor	0.96
η	Uzawa convergence parameter	
ρ	CRRA	2
ω	Consumption share in utility	0.34
θ	Elas. of subs. across dom. and foreign goods	0.5, 2.5, 8
κ	Elas. of subs. across tradable and non-tradable intermediates	0.44
ν	Preference for domestic intermediates in tradable goods production	0.72
γ	Preference for tradable goods in final consumption	0.55
$\alpha = \alpha_N$	Labour share in production	0.67
δ	Depreciation rate	0.10
λ	Productivity shocks-persistence and spillovers	$\begin{bmatrix} 0.88 & 0 & 0.22 & 0 \\ 0 & 0.88 & 0 & 0.22 \\ 0 & 0 & 0.30 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix}$
$V(u)$	Variance-covariance matrix of productivity shocks (in percent)	$\begin{bmatrix} 0.0376 & 0.0159 & 0.0072 & 0.0044 \\ 0.0159 & 0.0376 & 0.0044 & 0.0072 \\ 0.0072 & 0.0044 & 0.0051 & 0.0021 \\ 0.0044 & 0.0072 & 0.0021 & 0.0051 \end{bmatrix}$

Table 5: Business cycle statistics in the model with only sectoral TFP shocks-different trade elasticities

		θ=2.5				Low θ, θ=0.25				High θ, θ=8			
	data	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market
Std dev of GDP	1.57	1.80	1.82	1.85	1.85	1.73	1.69	1.69	1.69	1.87	1.94	1.98	1.98
Std dev rel. to GDP													
Real exchange rate (RER)	6.16	0.37	0.30	0.29	0.30	1.39	0.38	0.43	0.41	0.44	0.36	0.37	0.36
Terms of trade (TOT)	2.12	0.31	0.37	0.42	0.42	1.31	1.10	1.08	1.08	0.22	0.24	0.24	0.24
Rel. price of non-traded	1.46	0.92	0.82	0.80	0.81	1.84	0.64	0.72	0.70	0.95	0.83	0.87	0.85
Consumption	0.76	0.38	0.37	0.36	0.35	0.57	0.41	0.40	0.40	0.37	0.35	0.33	0.33
Investment	4.33	3.22	3.21	3.19	3.19	3.14	3.17	3.17	3.17	3.60	3.63	3.63	3.64
Hours worked	0.31	0.41	0.42	0.44	0.44	0.37	0.36	0.36	0.36	0.43	0.48	0.50	0.50
AR(1) coefficients													
Real exchange rate	0.67	0.44	0.54	0.39	0.36	0.43	0.68	0.43	0.45	0.47	0.56	0.45	0.47
GDP	0.50	0.50	0.50	0.51	0.51	0.49	0.49	0.49	0.49	0.53	0.53	0.54	0.54
Consumption	0.66	0.47	0.46	0.47	0.47	0.50	0.47	0.48	0.48	0.47	0.46	0.47	0.47
Cross corr btw H and F													
GDP	0.35	0.38	0.35	0.31	0.31	0.50	0.56	0.57	0.57	0.28	0.19	0.14	0.14
Consumption	0.06	0.69	0.72	0.83	0.84	-0.18	0.61	0.70	0.70	0.69	0.74	0.86	0.85
Investment	0.07	0.29	0.26	0.24	0.24	0.47	0.50	0.50	0.50	-0.05	-0.13	-0.16	-0.17
Cross corr btw													
RER and Relative consumption	-0.45	**-0.07**	**0.19**	**0.74**	**0.76**	**-0.90**	**0.60**	**0.97**	**0.98**	**-0.28**	**0.27**	**0.70**	**0.71**
RER and Terms of trade	0.32	-0.16	-0.20	-0.10	-0.09	0.98	0.80	0.75	0.76	0.18	0.03	-0.03	-0.04
TOT and Relative consumption	-0.74*	0.65	0.41	0.41	0.44	-0.94	0.53	0.86	0.84	0.19	0.15	0.25	0.23
Cross corr btw GDP and													
Real net exports	-0.26	0.09	0.27	0.43	0.44	-0.40	-0.44	-0.44	-0.44	-0.05	0.11	0.17	0.17
Real exchange rate	-0.09	-0.37	-0.35	-0.22	-0.21	-0.49	0.27	0.29	0.30	-0.48	-0.50	-0.44	-0.45
Consumption	0.78+	0.85	0.79	0.74	0.74	0.86	0.85	0.86	0.86	0.84	0.70	0.65	0.65
Investment	0.93+	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.92	0.91	0.91	0.91
Hours worked	0.86+	0.95	0.95	0.95	0.95	0.93	0.91	0.91	0.91	0.96	0.94	0.94	0.94
Foreign bond position/GDP	0.47		6.64	-6.47			6.22	1.69			10.37	132.52	

Notes: Data column contains statistics calculated by Benigno and Thoenissen (2008). Home country refers to the United States, and foreign is a weighted average of Japan and EU-15 and the period for calculation spans 1970-2002. Both the data and simulated moments are of annual frequency, logged and HP-filtered. Data statistics marked by * are taken from Corsetti et al (2008) while those marked by + are taken from Raffo (2010). Data for foreign bond position as a share of GDP is net foreign currency exposure as share of GDP for the US taken from Lane and Shambaugh (2010).

Table 6: Business cycle statistics with sectoral anticipated and unanticipated TFP shocks-different trade elasticities

		θ=2.5				Low θ, θ=0.25				High θ, θ=8			
	data	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market
Std dev of GDP	1.57	1.83	1.86	1.84	1.88	1.74	1.71	1.72	1.71	1.91	2.00	2.00	2.03
Std dev rel. to GDP													
Real exchange rate (RER)	6.16	0.40	0.30	0.38	0.31	1.73	0.40	0.48	0.44	0.45	0.36	0.39	0.37
Terms of trade (TOT)	2.12	0.35	0.41	0.38	0.43	1.93	1.11	0.96	1.11	0.25	0.27	0.26	0.26
Rel. price of non-traded	1.46	0.94	0.82	0.91	0.83	2.06	0.68	0.88	0.72	0.96	0.82	0.90	0.86
Consumption	0.76	0.38	0.37	0.38	0.35	0.62	0.41	0.42	0.40	0.37	0.34	0.33	0.33
Investment	4.33	3.23	3.22	3.23	3.21	3.19	3.23	3.22	3.23	3.58	3.62	3.63	3.63
Hours worked	0.31	0.42	0.44	0.43	0.45	0.38	0.38	0.38	0.38	0.44	0.50	0.49	0.51
AR(1) coefficients													
Real exchange rate	0.67	0.47	0.60	0.46	0.39	0.44	0.63	0.39	0.44	0.48	0.58	0.48	0.49
GDP	0.50	0.51	0.51	0.51	0.52	0.49	0.49	0.49	0.49	0.54	0.55	0.55	0.55
Consumption	0.66	0.47	0.46	0.47	0.47	0.49	0.47	0.48	0.48	0.47	0.46	0.47	0.47
Cross corr btw H and F													
GDP	0.35	0.36	0.31	0.34	0.28	0.49	0.54	0.53	0.54	0.24	0.13	0.13	0.10
Consumption	0.06	0.65	0.69	0.67	0.83	-0.31	0.59	0.57	0.68	0.62	0.70	0.82	0.84
Investment	0.07	0.27	0.24	0.26	0.22	0.44	0.46	0.45	0.46	-0.05	-0.15	-0.16	-0.18
Cross corr btw													
RER and Relative consumption	-0.45	**-0.16**	**0.20**	**-0.08**	**0.76**	**-0.92**	**0.58**	**0.34**	**0.97**	**-0.36**	**0.28**	**0.44**	**0.71**
RER and Terms of trade	0.32	0.01	-0.09	0.05	-0.07	0.97	0.78	0.63	0.76	0.26	0.09	0.01	-0.01
TOT and Relative consumption	-0.74*	0.42	0.28	0.31	0.45	-0.91	0.54	0.65	0.84	-0.04	0.04	0.38	0.30
Cross corr btw GDP and													
Real net exports	-0.26	0.13	0.32	0.23	0.45	-0.38	-0.42	-0.41	-0.42	0.04	0.18	0.18	0.22
Real exchange rate	-0.09	-0.33	-0.31	-0.28	-0.21	-0.36	0.20	-0.02	0.26	-0.43	-0.47	-0.46	-0.45
Consumption	0.78[+]	0.81	0.74	0.78	0.72	0.76	0.83	0.86	0.83	0.79	0.63	0.68	0.62
Investment	0.93[+]	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.90	0.90	0.90	0.90
Hours worked	0.86[+]	0.94	0.94	0.94	0.95	0.92	0.89	0.89	0.89	0.95	0.94	0.94	0.94
Foreign bond position/GDP	0.47		10.22	-2.95			6.74	2.20			13.74	69.32	

Notes: Data column contains statistics calculated by Benigno and Thoenissen (2008). Home country refers to the United States, and foreign is a weighted average of Japan and EU-15 and the period for calculation spans 1970-2002. Both the data and simulated moments are of annual frequency, logged and HP-filtered. Data statistics marked by * are taken from Corsetti et al (2008) while those marked by + are taken from Raffo (2010). Data for foreign bond position as a share of GDP is net foreign currency exposure as share of GDP for the US taken from Lane and Shambaugh (2010).

Figure 1 is inserted in text.

Figure 2: Impact responses of relative consumption and relative prices to a 1% tradable productivity shock with respect to trade elasticity

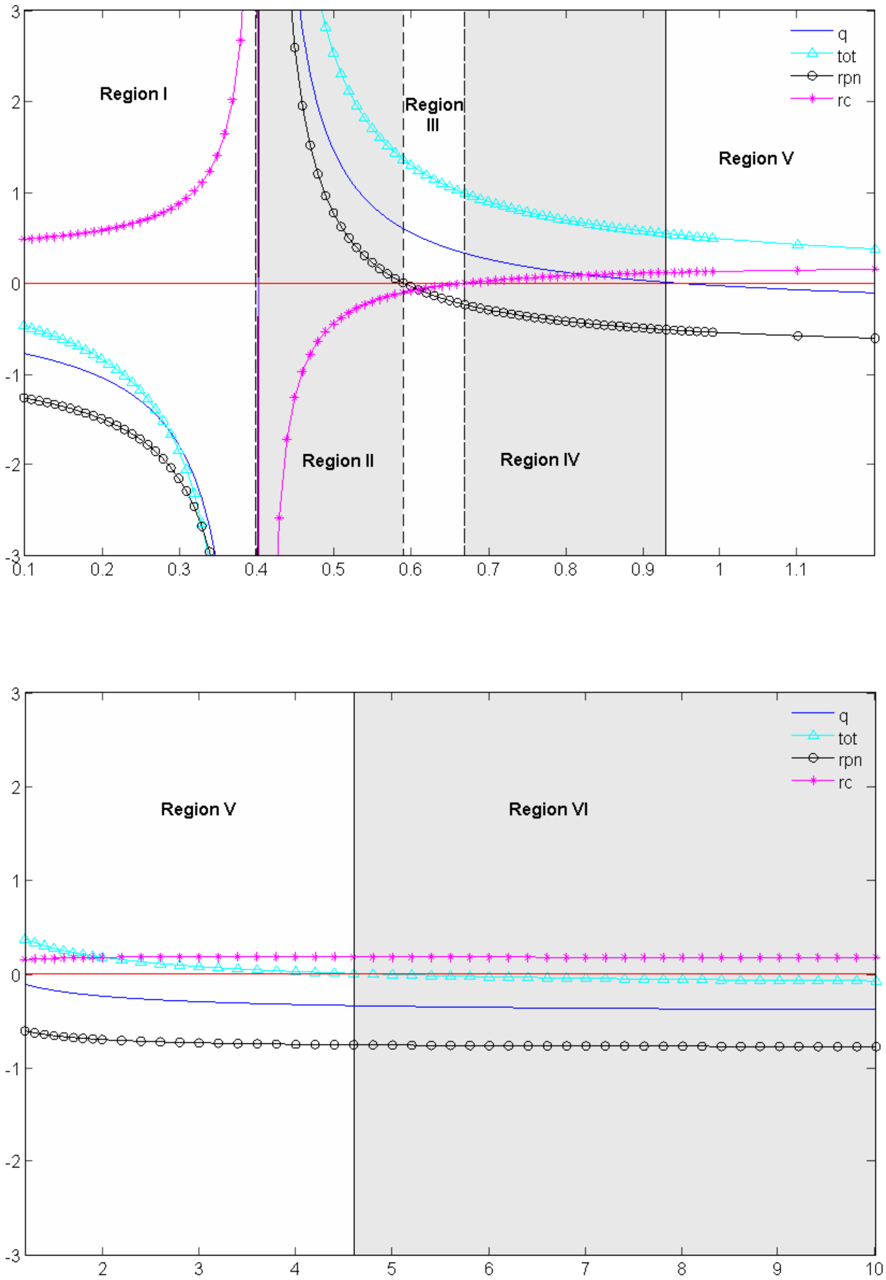
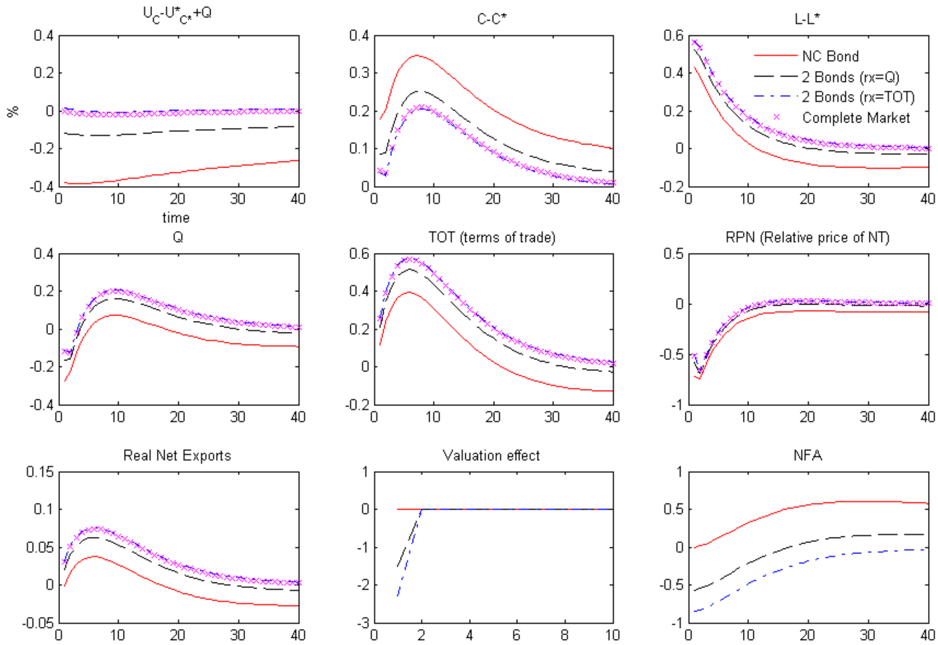


Figure 3: Impulse responses to sector-specific productivity shocks with $\theta = 2.5$

Tradable sector productivity shock



Non-tradable sector productivity shock

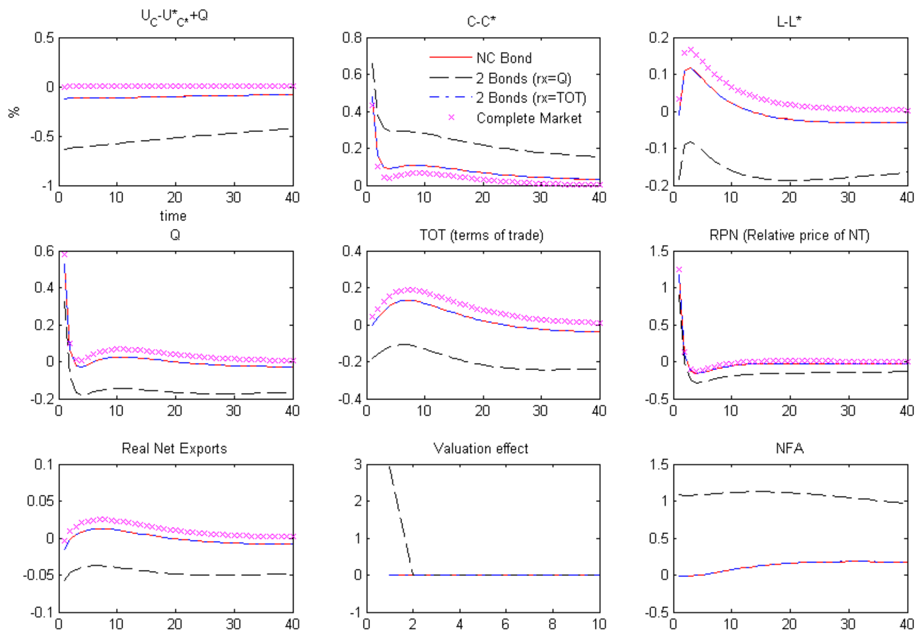
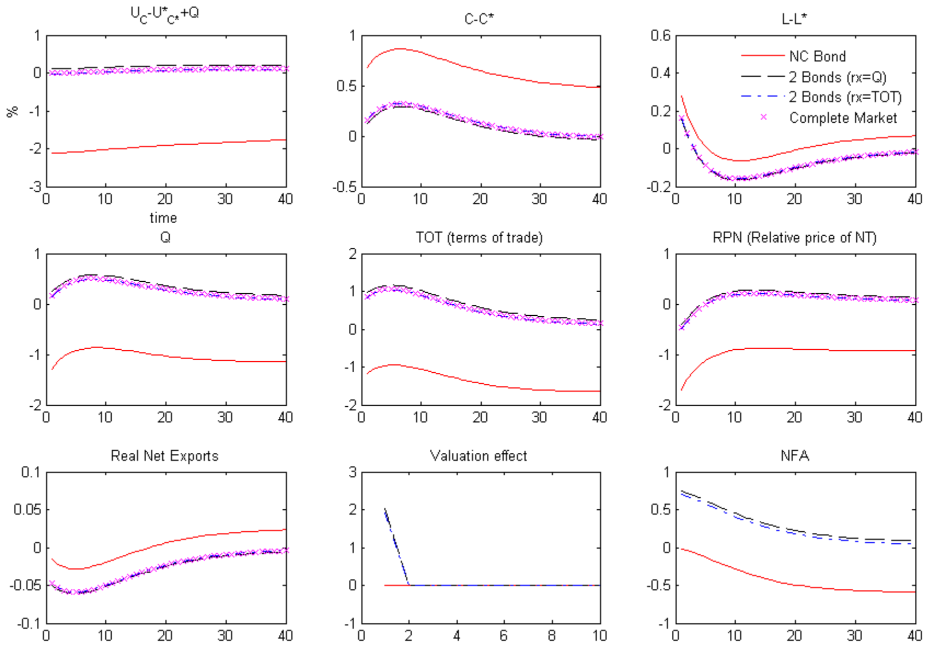


Figure 4: Impulse responses to sector-specific productivity shocks with $\theta = 0.25$

Tradable sector productivity shock



Non-tradable sector productivity shock

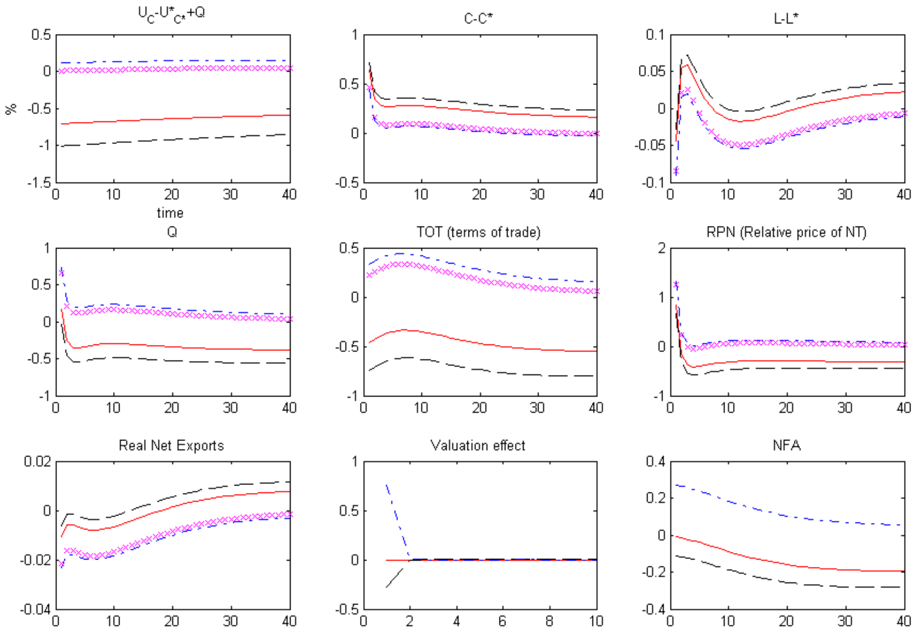


Figure 5: Sensitivity with respect to the elasticity of substitution between tradables and non-tradables, κ, in the benchmark model with unanticipated productivity shocks in each sector

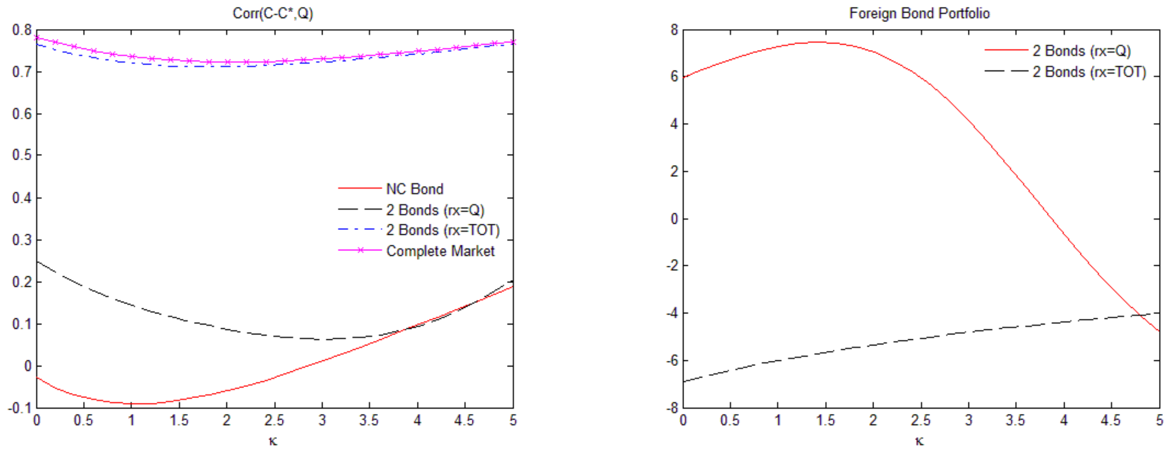


Figure 6: Sensitivity with respect to the share of non-tradables in the consumption of final goods, γ, in the benchmark model with unanticipated productivity shocks in each sector

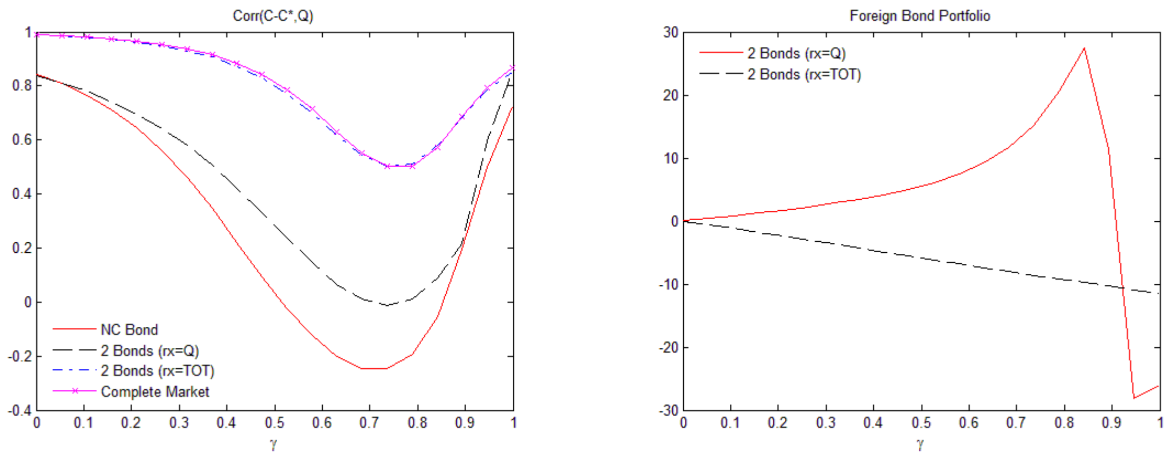


Figure 7: Sensitivity with respect to the degree of home bias in consumption, ν, in the benchmark model with unanticipated productivity shocks in each sector

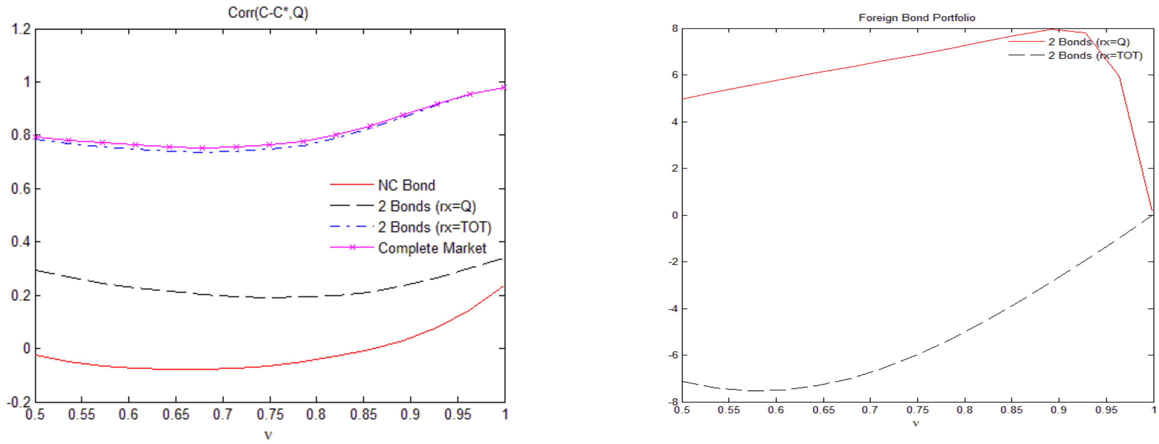
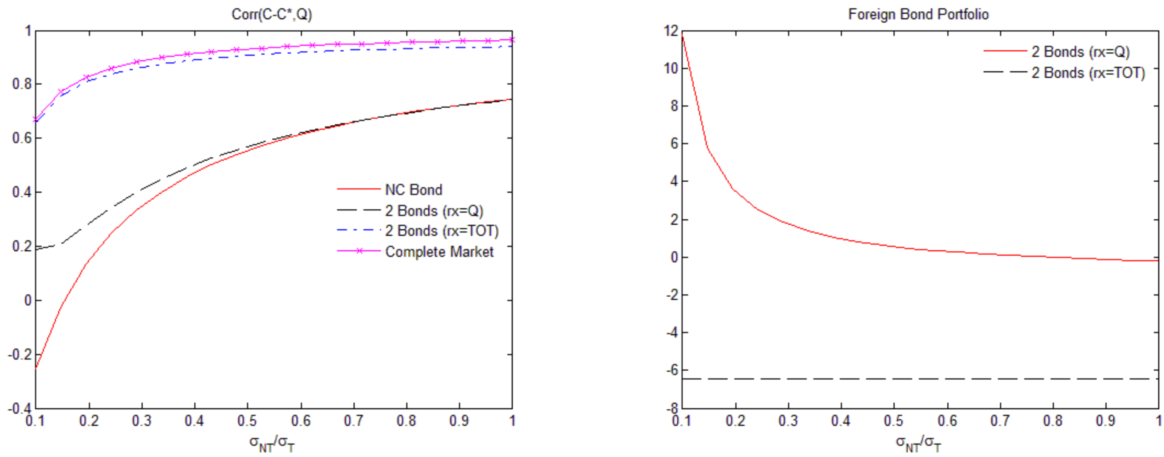


Figure 8: Sensitivity with respect to the relative variance of non-tradable shocks, σ^2_{NT}/σ^2_T, in the benchmark model with unanticipated productivity shocks in each sector



7 Appendix

7.1 Loading factors that determine optimal portfolio under domestic tradable price stabilisation

To understand the hedging motives behind the optimal bond portfolio, we use the partial equilibrium expression in (27) to decompose (32). We show how excess returns (terms of trade in this case) load on different components of relative consumption risk, namely relative income risk in tradable and non-tradable sectors and real exchange rate risk.

Loadings of terms of trade on relative income risk in the tradables sector:

$$\beta_Y^T \equiv \frac{Cov_t[\Lambda_{Y,t+1}^T, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = [4\theta\nu(1-\nu) - 1] + \frac{\kappa(2\nu-1)^2}{\gamma\kappa\rho + (1-\gamma)} > 0 \text{ if } \theta > \frac{1}{4\nu(1-\nu)} \tag{54}$$

$\theta > \frac{1}{4\nu(1-\nu)}$ is a sufficient condition for $\beta_Y^T > 0$.[46] In other words, for sufficiently large θ, an increase in relative tradable income is associated with a terms of trade depreciation. Intuitively, when the price elasticity of tradables is high, relative price of home goods falls to increase home and foreign demand for home tradables goods and clear the excess supply of tradables in the market. $\beta_Y^T > 0$ implies that foreign bonds pay relatively more when relative tradable income is high, making it optimal to have a short position in foreign bonds as a hedge against tradable income risk.

Loadings of terms of trade on relative income risk in the non-tradables sector:

$$\beta_Y^N \equiv \frac{Cov_t[\Lambda_{Y,t+1}^N, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = -\frac{\gamma(2\nu-1)(\kappa\rho-1)}{\gamma\kappa\rho + (1-\gamma)} \gtreqless 0 \text{ iff } \kappa\rho \lesseqgtr 1 \tag{55}$$

Note that $\beta_Y^N = 0$ if $\nu = \frac{1}{2}$ or $\kappa\rho = 1$. For $\nu > \frac{1}{2}$, the sign of β_Y^N depends on the sign of $(\kappa\rho - 1)$. In other words, assuming tradables consumption is biased towards home goods, when tradables and non-tradables are gross complements, i.e. $\kappa\rho < 1$, terms of trade depreciates in the states of the nature where relative non-tradable income is high, implying a short position in foreign bonds. On the other hand, when tradable and non-tradable goods are gross substitutes, i.e. $\kappa\rho > 1$, terms of trade appreciates when relative non-tradable income is high, making it optimal to have a long position in foreign bond.

To build intuition for the result note that relative non-tradable income, $\hat{Y}_{N,t}^R$, consists of two components: the relative supply and the relative price of non-tradable goods, i.e. $\hat{Y}_{N,t}^R \equiv \hat{Y}_{N,t} - \hat{Y}_{N,t}^* - \hat{P}_t^N$ where $\hat{P}_t^N = \hat{P}_{N,t}^* + \hat{S}_t - \hat{P}_{N,t}$. Since the terms of trade is independent of non-tradable endowment shocks under the assumption that $\delta_N = 1$,(see equation (31)), excess return only loads on the non-tradable income risk coming from tradable endowment shocks, which affect $\hat{Y}_{N,t}^R$ through $\hat{P}_t^N-$ the relative price of non-tradables. Now, consider a positive shock to home tradables endowment that depreciates the terms of trade and increases the consumption of tradables. For $\kappa\rho < 1$, consumption of non-tradables also increase because of the complementarity between the two goods. This in turn implies that the relative price of home non-tradables goes up, increasing the value of the fixed endowment of non-tradable goods. Therefore, for $\kappa\rho < 1$, a rise in relative non-tradable income is associated with a terms of trade depreciation, i.e. a rise in excess return, making it optimal to short foreign bonds.

[46] For the case of no consumption home bias, i.e. $\nu = \frac{1}{2}$, $\theta > 1$ is necessary and sufficient for $\beta_Y^T > 0$. For $\nu > \frac{1}{2}$, $\frac{1}{4\nu(1-\nu)} > 1$, so θ should be sufficiently larger than 1 to have $\beta_Y^T > 0$.

Loadings of terms of trade on real exchange rate risk

$$\beta_Q \equiv \frac{Cov_t[\Lambda_{Q,t+1}, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = \frac{\gamma\kappa\rho(2\nu - 1)}{\gamma\kappa\rho + (1-\gamma)} \gtrless 0 \text{ iff } \nu \gtrless \frac{1}{2}$$

When there is home bias in tradables consumption, $\nu > \frac{1}{2}$, terms of trade and real exchange rate are positively correlated. Thus, foreign bonds pay more in the states of the nature where home consumption basket is cheaper, making it optimal to have a short position in foreign bonds.

To summarize, under the conditions $\nu > \frac{1}{2}$, $\theta > \frac{1}{4\nu(1-\nu)}$ and $\kappa\rho < 1$, different hedging motives all require a short position in foreign bonds and there is no tension between different hedging motives.

7.2 Loading factors that determine optimal portfolio under consumer price stabilisation

To have a better understanding of the hedging motives behind the optimal bond position , we decompose the relative consumption risk generated by tradable and non-tradable shocks into (sectoral) relative income risk and real exchange rate risk components according to the partial equilibrium formulation given in equation (27).

Loadings of terms of trade on relative income risk in the tradables sector:

$$\beta_Y^T \equiv \frac{Cov_t[\Lambda_{Y,t+1}^T, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = -\frac{(\gamma\kappa\rho + 1 - \gamma)}{(1-\gamma)} \frac{(\theta-1)^2}{\theta^2} \frac{\sigma_T^2}{\sigma_N^2} < 0$$

Excess return, i.e. real exchange rate, and relative tradable income risk are negatively correlated for all possible parameter values. An increase in the relative supply of tradables makes home agents relatively wealthier and appreciates the relative price of non-tradables and therefore the real exchange rate. Therefore, the optimal hedge against the tradables income risk arising from tradable sector shocks is to have a long position in foreign bonds. The presence of non-tradable shocks limit this position, because under the parameter restrictions we impose, relative tradables income is independent of non-tradable supply shocks (because terms of trade is independent). Having a zero bond position is therefore the optimal hedge against non-tradable sector shocks. In other words, taking a long position in foreign bonds to hedge against the tradables income risk caused by shocks to tradable endowment makes the agents vulnerable to non-tradable endowment shocks, which would have no effect on relative tradables income for a zero bond portfolio. This explains why β_Y^T, is decreasing in σ_N^2/σ_T^2 in absolute value terms.

Loadings of terms of trade on relative income risk in the non-tradables sector:

$$\beta_Y^N \equiv \frac{Cov_t[\Lambda_{Y,t+1}^N, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = -\frac{(\rho-1) - \gamma(\kappa\rho - 1)}{\rho(1-\gamma)} \lesseqgtr 0 \text{ iff } \kappa\rho - 1 \lesseqgtr \frac{\rho - 1}{\gamma}$$

For $\rho > 1$, a sufficient condition for $\beta_Y^N < 0$ is $\kappa\rho < 1$. If these conditions are satisfied, real exchange rate depreciates (foreign bonds pay higher) when relative non-tradables income is low, making it optimal to have a long position in foreign bonds. To see this, again consider the effects of tradable and non-tradable supply shocks on relative non-tradables income, i.e. $\hat{Y}_{N,t}^R \equiv \hat{Y}_{N,t} - \hat{Y}_{N,t}^* - \hat{P}_t^N$.

An increase in the supply of home tradable goods appreciates the relative price of non-tradables due to wealth effects and raise the value of home non-tradable income compared to foreign. Therefore, conditional on tradable endowment shocks, relative non-tradables income and real exchange rate are negatively correlated, making it optimal to have a long position in foreign bonds.

Now, consider an increase in the supply of home non-tradable goods. If tradables and non-tradables are complements, home agents want to increase the consumption of tradables alongside the consumption of non-tradables. Given fixed supply of tradables, non-tradables price will have to fall even more to clear the excess supply of non-tradables. In this case, relative non-tradable income will fall and relative non-tradables price will depreciate in the home country ($\Lambda_{Y,t+1}^N \downarrow$ and $\hat{P}_t^N \uparrow$) , making it optimal to have a long position in foreign bonds. But when the substitutability between tradables and non-tradables is sufficiently high, an increase in the relative supply of non-tradables only require a small depreciation in relative non-tradables price ($\Lambda_{Y,t+1}^N \uparrow$ and $\hat{P}_t^N \uparrow$), implying a short position in foreign bonds.

Loadings of terms of trade on the real exchange rate risk

$$\beta_Q \equiv \frac{Cov_t[\Lambda_{Q,t+1}, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = 1 > 0$$

By definition, excess returns load perfectly on real exchange rate risk, therefore for $\rho > 1$, it is optimal to short foreign bonds to hedge against real exchange risk. When home consumption is more expensive ($\Lambda_{Q,t+1} \downarrow$), home bonds are a better hedge as home currency is more valuable in real terms ($\hat{r}_{x,t+1} \downarrow$).

To summarise; under the conditions $\nu = \frac{1}{2}, \rho > 1$ and $\kappa\rho < 1$, relative income risk in each sector require a long position in foreign bonds, whereas the real exchange rate risk requires a short position. But $\kappa\rho < 1$ ensures that optimal portfolio is a long position (see equation (37)), which in turn implies that relative income risk dominates the real exchange rate risk under these conditions.[47]

[47] Note that for the same restrictions, i.e. $\nu = \frac{1}{2}$, $\rho > 1$ and $\kappa\rho < 1$, there are no conflicting hedging motives when relative bond returns are given by the terms of trade. Relative consumption risk is driven only by the relative income risk in the tradable sector, $\beta_{Y,T}$, which implies a short position in foreign currency for $\theta > 1$.

7.3 Preference (I-pod) shocks

As an alternative demand shock, we introduce preference shocks as in Coeurdacier *et al* (2007) by modifying the consumption of tradables in the following way:

$$C_{T,t} = \left[\nu^{\frac{1}{\theta}} (\Psi_{H,t} C_{H,t})^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} (\Psi_{F,t} C_{F,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{56}$$

where $\Psi_{H,t}$ and $\Psi_{F,t}$ are shocks that reflect changes in world preferences for home and foreign produced tradable goods, respectively. As also mentioned by the authors, these shocks can also be thought as capturing changes in the quality of home and foreign goods, which is more of a supply-side interpretation. The tradables price index that is consistent with the modified tradables consumption is the following:

$$P_{T,t} = \left[\nu (P_{H,t}/\Psi_{H,t})^{1-\theta} + (1-\nu)(P_{F,t}/\Psi_{F,t})^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{57}$$

Foreign consumption of tradables and the associated price index are affected by $\Psi_{H,t}$ and $\Psi_{F,t}$ in a similar way as the home variables. Goods market clearing conditions in the tradables sector change accordingly:

$$Y_{H,t} = \Psi_{H,t}^{\theta-1} \left\{ \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta} \left(\frac{P_{T,t}}{P_t}\right)^{-\kappa} \gamma\nu C_t + \left(\frac{P^*_{H,t}}{P^*_{T,t}}\right)^{-\theta} \left(\frac{P^*_{T,t}}{P^*_t}\right)^{-\kappa} \gamma(1-\nu) C^*_t \right\} \tag{58}$$

$$Y^*_{F,t} = \Psi_{F,t}^{\theta-1} \left\{ \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\theta} \left(\frac{P_{T,t}}{P_t}\right)^{-\kappa} \gamma(1-\nu) C_t + \left(\frac{P^*_{F,t}}{P^*_{T,t}}\right)^{-\theta} \left(\frac{P^*_{T,t}}{P^*_t}\right)^{-\kappa} \gamma\nu C^*_t \right\}$$

I-pod shocks are assumed to follow AR(1) processes similar to endowment shocks:

$$\log \Psi_{H,t} = \delta_\Psi \log \Psi_{H,t-1} + u_{\Psi,t}, \qquad \log \Psi_{F,t} = \delta_\Psi \log \Psi_{F,t-1} + u^*_{\Psi,t}$$

Since these preference shocks affect tradable goods prices in each country, they affect the consumer prices and hence the real exchange rate. Log-linearisation of the price indices and the decomposition of the real exchange rate shows this clearly:

$$\hat{Q}_t = \gamma(2\nu - 1)\left[\widehat{TOT}_t + (\hat{\Psi}_{H,t} - \hat{\Psi}_{F,t})\right] + (1-\gamma)\hat{P}^N_t \tag{59}$$

As before, real exchange rate depreciates following a depreciation in the terms of trade for $\nu > \frac{1}{2}$, or a depreciation in the relative non-tradables price for $0 < \gamma < 1$. But now it also depends on relative ipod shocks: for a given $\widehat{TOT}_t$ and $\hat{P}^N_t$, real exchange rate depreciates when there is a positive quality shock in the home country.

Coeurdacier *et al* (2007), and Coeurdacier and Gourinchas (2009) note that due to difficulties in measuring quality changes, the observed real exchange rate might be different from the welfare-based real exchange rate given by equation (59). Here we present some analytical results assuming that these shocks are perfectly measured as in Coeurdacier *et al* (2007).

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho} &= \frac{\Gamma_3}{\kappa\rho(1+2\nu(\theta-1))}\left[(\hat{Y}_{H,t}-\hat{Y}_{F,t})+(\hat{\Psi}_{H,t}-\hat{\Psi}_{F,t})\right] \\ &+\frac{(1-\gamma)(\kappa\rho-1)}{\kappa\rho}(\hat{Y}_{N,t}-\hat{Y}^*_{N,t}) \\ &+\frac{(1-\beta)\Gamma_1}{\gamma\kappa\rho(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t} \end{aligned} \tag{60}$$

$$\hat{r}_{x,t} = \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t = \frac{1}{1+2\nu(\theta-1)}(u_{H,t}-u_{F,t}) \tag{61}$$

$$-\frac{2\nu(\theta-1)}{1+2\nu(\theta-1)}(u_{\Psi h,t}-u_{\Psi f,t}) - \frac{(1-\beta)(2\nu-1)}{\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t} \tag{62}$$

As shown in equation (60), an increase in the world demand for home goods affects the real exchange rate adjusted relative consumption in the same way as a positive supply shock. As discussed above, $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ moves in the same way in response to a positive supply or demand shock for $\theta > \theta_3^*$ ($\Gamma_3 > 0$) or $\theta < 1 - \frac{1}{2\nu}$. On the other hand, the response of the terms of trade to supply and demand shocks goes in opposite ways provided that $\theta > 1$. That is, an increase in the world preference for home goods ($\hat{\Psi}_{H,t} \uparrow$) implies an appreciation of domestic terms of trade and thus a fall in the excess return on foreign bonds if the elasticity of substitution between home and foreign tradables, θ, is greater than 1. Therefore, for $\theta > Max(1, \theta_2^*)$, it is optimal to have a long position in foreign bonds to hedge against preference shocks, but a short position to hedge against tradable endowment shocks. As before, relative variance of the two shocks will determine the optimal foreign currency position:

$$\tilde{\alpha}_F = -\frac{\gamma(1-\nu)\Gamma_3}{(1-\beta)(\gamma\kappa\rho+(1-\gamma))\left(1-\frac{2\nu(\theta-1)\Gamma_1\frac{\sigma_\Psi^2}{\sigma_T^2}}{(\gamma\kappa\rho+(1-\gamma))\left(2\nu(\theta-1)\frac{\sigma_\Psi^2}{\sigma_T^2}-1\right)}\right)}$$

For $\theta > 1$, $\frac{\sigma_\Psi^2}{\sigma_T^2} < \frac{1}{2\nu(\theta-1)}$ is a sufficient condition to ensure that optimal portfolio is a short position in foreign bonds as in the case of only supply shocks (see equation (32)). But for sufficiently large $\frac{\sigma_\Psi^2}{\sigma_T^2}$, optimal bond portfolio switches sign as in the case with news shocks.

Real exchange rate-relative consumption correlations in the presence of ipod shocks

$$\begin{aligned}\hat{C}_t - \hat{C}_t^* &= \psi_c^{nc}\widehat{NFA}_{t-1} + \frac{\gamma(2\theta\nu-1)}{1+2\nu(\theta-1)}\left[(\hat{Y}_{H,t}-\hat{Y}_{F,t}) + (\hat{\Psi}_{H,t}-\hat{\Psi}_{F,t})\right] \\ &\quad +(1-\gamma)(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*) + \frac{4\theta\nu(1-\beta)}{(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}\end{aligned} \tag{63}$$

$$\begin{aligned}\hat{Q}_t &= -\psi_q^{nc}\widehat{NFA}_{t-1} - \left[\frac{(1-\gamma)(2\theta\nu-1)-\kappa(2\nu-1)}{\kappa(1+2\nu(\theta-1))}\right]\left[(\hat{Y}_{H,t}-\hat{Y}_{F,t}) + (\hat{\Psi}_{H,t}-\hat{\Psi}_{F,t})\right] \\ &\quad +\frac{1-\gamma}{\kappa}(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*) - \frac{(1-\beta)[4\theta\nu(1-\nu)(1-\gamma)+\kappa(2\nu-1)^2]}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}\end{aligned} \tag{64}$$

Just like news shocks, preference shocks can reduce the effectiveness of nominal bonds in hedging against supply shocks if they are large enough. Table 7 shows how the comovement between real exchange rate and relative consumption is affected by the valuation channel when the optimal portfolio is a long position in foreign bonds due to the presence of ipod shocks. Apart from the parameter restrictions under which they are constructed, and the difference in the nature of the demand shocks, Table 7 is identical to Table 3. The same explanations follow.

Table 7: Relative consumption and real exchange rate responses to supply and demand (i-pod) shocks with endogenous portfolio choice

Assumptions: $\nu>\frac{1}{2}, \theta>\max(1,\theta_1^*), \tilde{\alpha}_F>0$	*Solution in terms of shocks and the portfolio valuation effect*		*Portfolio valuation effect*
	$\hat{C}_t$-$\hat{C}_t^*$	$\hat{Q}_t$	$\tilde{\alpha}_F\hat{r}_{x,t}$
$\hat{Y}_{H,t}-\hat{Y}_{F,t}$	+	-	+
$\hat{z}_{h,t}-\hat{z}_{f,t}$	+	-	-
$\hat{Y}_{N,t}-\hat{Y}^*_{N,t}$	+	+	*0*
$\tilde{\alpha}_F\hat{r}_{x,t}$	+	-	*na*
Note: Table 7 is constructed under the assumptions that $\nu>\frac{1}{2}$, $\theta>\max(1,\theta_1^*)$ *and* σ_Ψ^2/σ_T^2 *is sufficiently big such that* $\tilde{\alpha}_F>0$.			

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